A Metric for the Prognostic Outreach of Scenarios

Learning from the Past to Establish a Standard in Applied Systems Analysis

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This talk covers

1. Motivation
2. Framing conditions and definitions
3. Why diagnostic and prognostic uncertainty are different and independent
4. Learning in a prognostic context
5. Toward application: an accurate and precise system
6. Insights and outlook
1. Motivation

Our motivation is two-fold:


2. and to contribute to the unresolved question of *How limited are prognostic scenarios?*

We are still moving at a theoretical level but we already encounter important insights and windfall profits!
1. Motivation (2)

An easy-to-apply metric or indicator is needed that informs non-experts about the time in the future at which a prognostic scenario ceases to be (for whatever reasons) in accordance with the system’s past.

This indicator should be applicable in treating a system / model coherently (from beginning to end)!
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1. Motivation (1)

Jonas et al. (2014):
The mode of bridging diagnostic and prognostic uncertainty across temporal scales relies on two discrete points in time: ‘today’ and 2050.

Now we want to become continuous ...
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Now we want to become continuous...
2. Framing conditions and definitions

Net Storage in the Atmosphere

Sphere of Activity under the KP

FF Industry
Kyoto Biosphere
“Non-Kyoto” Biosphere

Impacting?

Globe or Group of Countries or individual Country

Jonas and Nilsson (2007: Fig. 4); modified
2. Framing conditions and definitions

Net Storage in the Atmosphere

Sphere of Activity under the KP

FF Industry  Kyoto Biosphere  “Non-Kyoto” Biosphere

Only $F_{FF\_C}$, $F_{\text{terr}\_C}$ and $F_{\text{oc}\_C}$ can be discriminated top-down globally!

Jonas and Nilsson (2007: Fig. 4); modified
2. Framing conditions and definitions

Jonas and Nilsson (2007: Fig. 6); modified
2. Framing conditions and definitions

Bottom-up / top-down (full C) accounting is not in place. We cannot yet verify $\Delta C$ fluxes at the country scale!
2. Framing conditions and definitions

Moss & Schneider (2000: Fig. 5; see also Giles, 2002); IPCC (2006: Vol. 1, Fig. 3.2)
3. Diagnostic vs prognostic uncertainty

Diagnostic uncertainty
→ can increase or decrease depending on whether or not our knowledge of accounting emissions becomes more accurate and precise!

Prognostic uncertainty
→ under a prognostic scenario always increases with time!
3. Diagnostic vs prognostic uncertainty

Meinshausen et al. (2009: Fig. 2)
3. Diagnostic vs prognostic uncertainty

Meinshausen et al. (2009: Fig. 3)
3. Diagnostic vs prognostic uncertainty

Probability of exceeding 2 °C:

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Emissions</th>
<th>Probability of exceeding 2 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative total CO₂ emission 2000-49</td>
<td>886 Gt CO₂</td>
<td>8-37% 20%</td>
</tr>
<tr>
<td></td>
<td>1,000 Gt CO₂</td>
<td>10-42% 25%</td>
</tr>
<tr>
<td></td>
<td>1,158 Gt CO₂</td>
<td>16-51% 33%</td>
</tr>
<tr>
<td></td>
<td>1,437 Gt CO₂</td>
<td>29-70% 50%</td>
</tr>
<tr>
<td>Cumulative Kyoto-gas emissions 2000-49</td>
<td>1,356 Gt CO₂ equiv.</td>
<td>8-37% 20%</td>
</tr>
<tr>
<td></td>
<td>1,500 Gt CO₂ equiv.</td>
<td>10-43% 26%</td>
</tr>
<tr>
<td></td>
<td>1,678 Gt CO₂ equiv.</td>
<td>15-51% 33%</td>
</tr>
<tr>
<td></td>
<td>2,000 Gt CO₂ equiv.</td>
<td>29-70% 50%</td>
</tr>
<tr>
<td>2050 Kyoto-gas emissions</td>
<td>10 Gt CO₂ equiv. yr⁻¹</td>
<td>6-32% 16%</td>
</tr>
<tr>
<td></td>
<td>(Halved 1990) 18 Gt CO₂ equiv. yr⁻¹</td>
<td>12-45% 29%</td>
</tr>
<tr>
<td></td>
<td>(Halved 2000) 20 Gt CO₂ equiv. yr⁻¹</td>
<td>15-49% 32%</td>
</tr>
<tr>
<td></td>
<td>36 Gt CO₂ equiv. yr⁻¹</td>
<td>39-82% 64%</td>
</tr>
<tr>
<td>2020 Kyoto-gas emissions</td>
<td>30 Gt CO₂ equiv. yr⁻¹</td>
<td>(8-38%) (21%)</td>
</tr>
<tr>
<td></td>
<td>35 Gt CO₂ equiv. yr⁻¹</td>
<td>(13-46%) (29%)</td>
</tr>
<tr>
<td></td>
<td>40 Gt CO₂ equiv. yr⁻¹</td>
<td>(19-56%) (37%)</td>
</tr>
<tr>
<td></td>
<td>50 Gt CO₂ equiv. yr⁻¹</td>
<td>(53-87%) (74%)</td>
</tr>
</tbody>
</table>

Meinshausen et al. (2009: Tab. 1)
3. Diagnostic and prognostic uncertainty

Massari Coelho et al. (2012: Fig. 10)
4. Learning in a prognostic context

(a) Emissions

Time

today
4. Learning in a prognostic context

(a) Emissions

(b) Emissions

Time
today
4. Learning in a prognostic context
4. Learning in a prognostic context

Task: Find optimum between 'order of the signal’s dynamics' and both the extension and the opening of uncertainty wedge!
4. Learning in a prognostic context

Task: Find optimum between 'order of the signal’s dynamics’ and both the extension and the opening of uncertainty wedge!

<table>
<thead>
<tr>
<th>Learning (L) / Forecasting (F)</th>
<th>Uncertainty (Precision)</th>
<th>With</th>
<th>Without</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>✓ (F)</td>
<td>✓ (F)</td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>✓ (F)</td>
<td>✓ (F)</td>
<td></td>
</tr>
<tr>
<td>? (L)</td>
<td>? (L)</td>
<td>? (L)</td>
<td></td>
</tr>
</tbody>
</table>
4. Learning in a prognostic context

Andriana (2015:Slide 15); modified
5. Toward application: accurate + precise system

Assume that we have learned from a RL exercise

- that each historical data record has a memory and exhibits (but not necessarily) a linear dynamics;
- that each data record’s uncertainty (learning) wedge unfolds linearly into the future (until when?)
- and that our data records exhibit linear interdependencies [eg: $T = T(C)$ ; $C = C(E)$ ; $E = E(t)$ ]
5. Toward application: accurate + precise system

Assume that we have learned from a RL exercise

- that each historical data record has a memory and exhibits (but not necessarily) a linear dynamics;
- that each data record's uncertainty (learning) wedge unfolds linearly into the future (until when?);
- and that our data records exhibit linear inter-dependences:
  \[ Y = Y(t) \]
  \[ \Delta Y \propto a_y \cdot t \]
  \[ E = E(t) \]
5. Toward application: accurate + precise system

\[ E(t) \rightarrow C(t) \rightarrow T(t) \]

\[ E = m_{Et} t ; \quad C = m_{Ct} t ; \quad T = m_{Tt} t \]

\[ E_u = f_u m_{Et} t \]

\[ E_t = f_t m_{Et} t \]

\[ \Delta E = \Delta f_{Et} m_{Et} t = \Delta f_{Et} E \]

\[ \sigma_E^2 = \left( \frac{\partial E}{\partial m_{Et}} \right)^2 \sigma_{m_{Et}}^2 + \left( \frac{\partial E}{\partial t} \right)^2 \sigma_t^2 \quad \Rightarrow \quad \sigma_E^2 = \sigma_{m_{Et}}^2 t^2 \]

\[ \Delta E = 2\sigma_E = 2\sigma_{m_{Et}} t = \Delta f_{Et} m_{Et} t = \Delta f_{Et} E \]

\[ \Delta f_{Et} = \frac{\Delta E}{E} = \frac{2}{2} \frac{\sigma_E}{E} = \frac{\sigma_{m_{Et}}}{m_{Et}} \]

We merge an accurate-precise system with classical statistics!
\( \Delta f_{Et} \) combines Unc (learn) + Dyn (mem) knowledge!
Similarly for \( C = C(t) \):

\[ C = m_{ct} t \]

\[ T = m_{nt} t \]

The linearly interdependent cases \( C = C(E) \) and \( T = T(C) = T(C(E)) \):

\[ C = m_{ce} E = m_{ce} m_{et} t = m_{ct} t; \]

\[ T = m_{tc} C = m_{tc} m_{ce} E = m_{tc} m_{ce} m_{et} t = m_{nt} t \]

Find:

\[ \Delta E = \Delta f_{et} m_{et} t = \Delta f_{et} E \]

\[ \Delta C = \Delta f_{ct} m_{ct} t = \Delta f_{ct} C = \sqrt{\Delta f_{ce}^2 + \Delta f_{et}^2} C \]

\[ \Delta T = \Delta f_{nt} m_{nt} t = \Delta f_{nt} T = \sqrt{\Delta f_{tc}^2 + \Delta f_{ce}^2 + \Delta f_{et}^2} T \]

...  

That is:

\[ \Delta f_{ct} = \sqrt{\Delta f_{ce}^2 + \Delta f_{et}^2} , \]

\[ \Delta f_{nt} = \sqrt{\Delta f_{tc}^2 + \Delta f_{ce}^2 + \Delta f_{et}^2} \]

...
5. Toward application: accurate + precise system

Similarly for $C = C(t)$:

<table>
<thead>
<tr>
<th>Function</th>
<th>Variance</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f = aA$</td>
<td>$\sigma_f^2 = a^2 \sigma_A^2$</td>
<td>$\sigma_f = a \sigma_A$</td>
</tr>
<tr>
<td>$f = aA + bB$</td>
<td>$\sigma_f^2 = a^2 \sigma_A^2 + b^2 \sigma_B^2 + 2ab \sigma_{AB}$</td>
<td>$\sigma_f = \sqrt{a^2 \sigma_A^2 + b^2 \sigma_B^2 + 2ab \sigma_{AB}}$</td>
</tr>
<tr>
<td>$f = aA - bB$</td>
<td>$\sigma_f^2 = a^2 \sigma_A^2 + b^2 \sigma_B^2 - 2ab \sigma_{AB}$</td>
<td>$\sigma_f = \sqrt{a^2 \sigma_A^2 + b^2 \sigma_B^2 - 2ab \sigma_{AB}}$</td>
</tr>
<tr>
<td>$f = AB$</td>
<td>$\sigma_f^2 \approx f^2 \left[ \left( \frac{\sigma_A}{A} \right)^2 + \left( \frac{\sigma_B}{B} \right)^2 + 2 \frac{\sigma_{AB}}{AB} \right]$</td>
<td>$\sigma_f \approx</td>
</tr>
<tr>
<td>$f = \frac{A}{B}$</td>
<td>$\sigma_f^2 \approx f^2 \left[ \left( \frac{\sigma_A}{A} \right)^2 + \left( \frac{\sigma_B}{B} \right)^2 - 2 \frac{\sigma_{AB}}{AB} \right]$</td>
<td>$\sigma_f \approx</td>
</tr>
<tr>
<td>$f = aA^b$</td>
<td>$\sigma_f^2 \approx \left( abA^{b-1} \sigma_A \right)^2 = \left( \frac{f \sigma_A}{A} \right)^2$</td>
<td>$\sigma_f \approx</td>
</tr>
<tr>
<td>$f = a \ln(bA)$</td>
<td>$\sigma_f^2 \approx \left( a \frac{\sigma_A}{A} \right)^2$</td>
<td>$\sigma_f \approx \left</td>
</tr>
<tr>
<td>$f = a \log_{10}(A)$</td>
<td>$\sigma_f^2 \approx \left( a \frac{\sigma_A}{A \ln(10)} \right)^2$</td>
<td>$\sigma_f \approx \left</td>
</tr>
<tr>
<td>$f = ae^{bA}$</td>
<td>$\sigma_f^2 \approx f^2 (b \sigma_A)^2$</td>
<td>$\sigma_f \approx</td>
</tr>
<tr>
<td>$f = a^{bA}$</td>
<td>$\sigma_f^2 \approx f^2 (b \ln(a) \sigma_A)^2$</td>
<td>$\sigma_f \approx</td>
</tr>
<tr>
<td>$f = A^B$</td>
<td>$\sigma_f^2 \approx f^2 \left[ \left( \frac{B}{A} \frac{\sigma_A}{A} \right)^2 + (\ln(A) \sigma_B)^2 + 2 \frac{B \ln(A)}{A} \sigma_{AB} \right]$</td>
<td>$\sigma_f \approx</td>
</tr>
</tbody>
</table>

$\Delta f_{CT} = \sqrt{\Delta f_{CE}^2 + \Delta f_{EI}^2}$,  

$\Delta f_{TI} = \sqrt{\Delta f_{TC}^2 + \Delta f_{CE}^2 + \Delta f_{EI}^2}$


...
To understand this result, look at $C = C(E)$, for which we found:

$$\Delta f_{ct} = \sqrt{\Delta f_{ce}^2 + \Delta f_{ct}^2}$$

Rewrite as

$$\Delta f_{ce}^2 + \Delta f_{ct}^2 - \Delta f_{ct}^2 = 0 \iff \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

which describes a second-order cone:
To understand this result, look at $C = C(E)$, for which we found:
5. Toward application: accurate + precise system

Serial-parallel interdependence:

\[ \begin{align*}
E_1 & \rightarrow C_1 \rightarrow T \\
E_2 & \rightarrow C_2 \\
\end{align*} \]

Deriving \( \Delta f_n \) is easy and straightforward (particularly in the case of uncorrelated variables)!

The analytical expression for \( \Delta f_{tt} \) also holds for a system, where the second emissions source \( E_2 \) is replaced by a sink (R: removal):

\[ \begin{align*}
E & \rightarrow C_1 \rightarrow T \\
R & \rightarrow C_2 \\
\end{align*} \]

meaning that the learning does not change while the two systems differ:

\[ C = C_1 + C_2 \quad \text{versus} \quad C = C_1 - C_2. \]

That is, a sink reduces a source but their uncertainties still add up!
5. Toward application: accurate + precise system

a) Emissions

b) Emissions - Removals

Jonas & Nilsson (2007: Fig. 9); modified
6. Insights and outlook

1. The risk of exceeding a 2050 global temperature target (eg, 2 °C) appears to be greater than assessed by the IPCC!

The correct approach would have been to deal with cumulated emissions and removals individually to determine their combined risk of exceeding the agreed temperature target.

RL allows exactly this to be done: RL overcomes this shortfall and allows the effect of learning about emissions and removals individually to be grasped.
6. Insights and outlook

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6. Insights and outlook

2. We anticipate that, in the case of success, the way of constructing prognostic models and conducting systems analysis will have to meet certain quality standards:

• Better diagnostic data handling (retrospective learning)!
• Specifying the models’ outreach limits!
• Safe-guarding complex models by means of meta-models which fulfill the above!


