Model formulation

Our length-structured model describes the population at time t through a vector $\mathbf{n}_t$ with $m = 55$ components, which contain the current abundances in the 55 length classes (with cuts at 1 cm, 2 cm, …). A transition matrix $\mathbf{L}_t$ describes the effect of the various processes that change the population structurein time steps of one month. These steps enable the incorporation of catch and stocking actions, which occur in different months in Lake Irrsee. Given an initial population $\mathbf{n}_0$, the population at any desired time can thus be projected by iteratively applying

$$\mathbf{n}_{t+1} = \mathbf{L}_t \mathbf{n}_t,$$

where the transition matrix needs to be recalculated at every time step to account for changing population densities, habitat temperatures etc. Dropping the index t where confusion is unlikely, we now discuss in detail the structure of $\mathbf{L}$ and the processes considered.

Matrix structure

The transition matrix has the structure

$$\mathbf{L} = \begin{pmatrix} s_{1,1} & f_2 & \cdots & f_m \\ s_{2,1} & s_{2,2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ s_{(m-1),1} & s_{(m-1),2} & \cdots & 0 \\ s_{m,1} & s_{m,2} & \cdots & s_{m,m} \end{pmatrix}.$$

The matrix elements $s_{m,n}$ are the probabilities for a fish in length class n to appear in length class m in the next month, and so describe probabilistic growth and survival. In particular, fish may also remain in the same length class, the probability for which is encoded in the diagonal elements. Fish cannot shrink, which is expressed by most superdiagonal elements being 0. The exceptions are the elements f_j in the first row of the matrix, which describe the effective fecundities of fish in the 55 length classes.

Biphasic growth

Growth probabilities from one length class to another are derived from a biphasic growth function with consideration of a growth variability with $\pm 20\%$ (as observed for whitefish in gillnet samples) growth temperature T_{g} (i.e., the average temperature during the growth period between May and October with consideration of the oxythermal habitat for coldwater fish with $O_2 > 3\ \mathrm{mgl}^{-1}$ and $T < 21.2$ °C; Stefan et al. 1995) and population density as expressed through the total biomass B of whitefish. The basic biphasic growth function f_{A} is composed of a non-linear von Bertalanffy growth function (VBGF) for older and bigger whitefish, and linear growth for juveniles in the first year of life (age A smaller than age of young-of-the-year A_{yoy}), i.e.,

$$f_{\mathrm{A}} = \begin{cases} \alpha A & \text{if } A < A_{\mathrm{yoy}}, \\ L_{\infty}(1 - \exp(-k(A - A_0))) & \text{if } A \geq A_{\mathrm{yoy}}, \end{cases}$$

where the initial asymptotic length L_{∞}, growth coefficient k and the age offset A_0 are estimated from length-at-age data of the Irrsee population.

Temperature and density dependence

Temperature dependence is incorporated directly into k and indirectly into L_{∞} of the growth model. The temperature-dependent growth coefficient k_{T} is calculated by using the growth temperature (i.e., $T_{\mathrm{g}} = 9.48$°C) in Lake Irrsee and minimum, maximum and optimum growth temperatures (i.e., $T_{\min} = 2$°C, $T_{\max} = 22$°C and $T_{\mathrm{opt}} = 14.1$°C; Table S1) with

$$k_{\mathrm{T}} = k_{\mathrm{opt}} \frac{(T_{\mathrm{g}} - T_{\min})(T_{\mathrm{g}} - T_{\max})}{(T_{\mathrm{g}} - T_{\min})(T_{\mathrm{g}} - T_{\max}) - (T_{\mathrm{g}} - T_{\mathrm{opt}})^2}.$$

The temperature-dependent asymptotic length $L_{\infty,\mathrm{T}}$ is thereafter derived through the relationship

$$L_{\infty,\mathrm{T}} = L_{\infty}\sqrt{\frac{k}{k_{\mathrm{T}}}},$$

which is subsequently used to calculate a temperature-dependent and density-dependent asymptotic length $L_{\infty,\mathrm{T,B}}$ with

$$L_{\infty,\mathrm{T,B}} = \frac{L_{\infty,\mathrm{T}}}{1+aB^{b}},$$

where the effective biomass B of the populations is used. The starting value for the effective biomass of the studied whitefish population is assumed to be 60% of the observed total fish biomassin hydro-acoustic surveys in the year 2000 resulting in 30.98 kg ha^{-1} (Wanzenböck et al. 2003)

The strength of density dependence is determined by parameter a (i.e., $a = 10^{-8}$and type of density dependence by parameter b (i.e., $b = 1$), which are estimated from the data. Incorporating these parameters into the VBGF we get the length-at-age at the current temperature and biomass $L_{\mathrm{A,T,B}}$ as

$$L_{\mathrm{A,T,B}} = L_{\infty,\mathrm{T,B}}(1 - \exp(-k_{\mathrm{T}}(A - A_0))),$$

which is also used to calculate the size of the young-of-the-year and the slope of the associated linear growth model.

The estimated temperature-dependent and density-dependent biphasic growth trajectory is used to calculate growth increments for each length-class of the matrix model

$$\Delta L_{\mathrm{A}} = L_{\infty,\mathrm{T,B}} \exp\big(-k_{\mathrm{T}}(A - A_0)\big) \exp\big(k_{\mathrm{T}}(\mathrm{A} + 1)\big) - 1,$$

and growth probabilities between length-classes of the matrix model through integration of the log-normal monthly growth increments of the 55 length classes.

Mortality and survival

Thereafter, we introduce temperature dependent instantaneous mortality from Pauly´s equation (Pauly 1980; Quinn & Deriso 1999), which derives the annual instantaneous natural mortality rate from temperature and growth parameters by

$$\ln(M) = -0.0152 - 0.279 \ln(L_{\infty,\mathrm{T,B}}) + 0.6543 \ln(k_{\mathrm{T}}) + 0.4634 \ln(T_{\mathrm{h}}),$$

where $L_{\infty,\mathrm{T,B}}$ is given in cm, M and k_{T} in y^{-1}, and T_{h} in °C, and therefore

$$M = \exp(-0.0152)\, {L_{\infty,\mathrm{T,B}}}^{-0.279} {k_T}^{0.6543} {T_h}^{0.4634},$$

which we convert to the monthly natural survival fraction S_{M} as

$$S_{\mathrm{M}} = \exp\left(\frac{-M}{12}\right)$$

and multiply by growth probabilities of length classes resulting in combined growth and survival probabilities per length class (i.e., $S_{m,n}$). In a second model run, we used another method to derive natural mortality (Jensen 1996) simply through

$$M = 1.5\, k_{\mathrm{T}}.$$

The survival probability of the first length class ($S_{1,1}$; 0 to 1 cm) is treated differently and considered to be rather similar to the egg survival probability due to the yolk-sac stage of larvae, which represents non complete embryonic development. Therefore, the growth probability of the first length class is multiplied by the assumed daily egg mortality of 6% (see Table S1) and a developmental period of 30 days, amounting to a survival probability of 15.62% in the first length class.

Fecundity and reproduction

The reproductive rate in length class m, the effective fecundity f_i, is defined as the number of offspring produced by every individual fish that survives to the first class (i.e., $S_{1,1}$; $0 - 1$ cm length). This effective fecundity depends on the fecundity f, which is the number of eggs per unit weight, the average weight w_i, the probability of egg survival q, the fraction of reproducing individuals m_i, and the sex ratio $r_i$in the length class as

$$f_i = f\, w_i\, m_i\, r_i\, q.$$

Fecundity estimates of the year 2010 are used to generate a stochastic fecundity value through a random selection from a normal distribution around the mean value of $f = 19.6\ \pm\ 1.6$ SD eggs per gram female fish. Fecundity did not differ considerably from earlier estimates in 1995 with $f = 20.9$ eggs per gram female fish and in 2000 with $f = 21.7$ eggs per gram female fish and we assume therefore that fecundity is rather constant in Lake Irrsee.

The average weight is calculated from the measured lengths via the length-weight relationship

$$w_i = \alpha {l_i}^{\beta},$$

where parameters α and β are determined by fitting this function to individual length (l_i) and weight data (w_i) of sampled Irrsee whitefish through a non-linear least squares method. The fraction of reproducing individuals per length class (m_i) is described by a sigmoid function based on observations of gonad ripeness, where

$$m_i = \frac{\alpha}{1 + \exp(-\frac{l_i - \beta}{\gamma})}$$

and parameters α, β and γ are estimated again through a non-linear least squares method.

We assumed an egg survival probability of 0.0205% according to average daily mortality estimates of Wahl & Löffler (2009) with 6% $\mathrm{d}^{-1}$and a developmental period of 100 days in Lake Irrsee.

Catch and stocking

Catch and stocking activities by the angler association of Lake Irrsee are incorporated into the matrix model by subtracting respectively adding numerical vectors of fish with specified length in particular months. Annual harvesting by recreational fisheries amounted to 3,000 individuals on average between 40 cm and 55 cm length, which we distribute over the angling season between March and September. A functional response is additionally used to estimate the possible catch ($C_{\mathrm{B},t}$) in relation to the biomass of catchable fish ($B_{\mathrm{C},t}$), where

$$C_{\mathrm{B},t} = \frac{a\, B_{\mathrm{C},t}}{1 + b B_{\mathrm{C},t}}.$$

This function is fitted to observed catch and biomass data of catchable fish derived from angling statistics and hydro-acoustic surveys to obtain the parameters a and b.

Initialization

The starting distribution in the length classes of the matrix model is derived from average catch-per-unit effort data of the length-frequency distribution in gillnet samples corrected for gillnet selectivity according to the method described in Millar & Holst (1997).

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Table S1: Parameters used in the length-structured matrix model.

Parameter	**Symbol**	**Unit**	**Value**	**Reference**
Mean annual growth temperature	T_{g}	°C	9.48	Irrsee data
Minimum growth temperature	T_{min}	°C	3	Siikavuopio et al. 2010
Maximum growth temperature	T_{max}	°C	22	EIFAC 1994; Stefan et al. 1995
Optimal growth temperature	T_{opt}	°C	14.1	Casselman et al. 2002
Fecundity (eggs per mass)	f	g^{-1}	19.4 ± 1.63 SD	Irrsee data
Egg mortality	q	d^{-1}	0.06	Wahl & Löffler 2009
Sex ratio (female/male)	r	1	1	Irrsee data
Asymptotic length (initial value)	L_{∞}	cm	45.09	Irrsee data
Growth coefficient (initial value)	k	y^{-1}	0.37	Irrsee data
Age offset	A_0	y^{-1}	-0.65	Irrsee data
Whitefish biomass in year 2000	B	kg ha^{-1}	30.98	Irrsee data