Individual radial growth model
for uneven-aged mixed oak forests in central Korea

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Abstract

This study suggested an individual radial growth model for three oak species (*Q. ariabilis*, *Q. acutissima* and *Q. mongolica*) in mixed and uneven-aged oak stands in central South Korea. The site quality dependent model showed relatively better statistical performance than the site quality independent model.

Different components, such as tree age, tree size, competition and growing site, were employed to build and evaluate the individual radial growth model. The components of age, tree size and competition were proven to have significant effects on the tree radial growth. The age and competition had negative effects on the radial growth, while the tree size had a positive effect. The age effect on the radial growth was greater for larger trees and with lower competition. The tree size was also expected to have a greater effect on younger trees and with lower competition. The competition had greater negative effects on younger and bigger trees. The classical site index, derived from the mean age and dominant height, was proven to not be applicable to the individual radial growth model. Among the topographical factors, only the aspect index was proven to be statistically significant for explaining the radial growth. These analyses of the effects of the different components on the radial growth of oak species were found to be reasonable and consistent with well-known silvicultural experiences.

Keywords: Individual tree, radial growth model, mixed stand, uneven-aged stand, oak stands

1. Introduction

The management of uneven-aged and mixed stands is of increasing importance for sound and sustainable forest management. Stand level growth models have proven to be useful for managing even-aged and pure stands, and such models are widely used, especially in Korea (Kim 1963; Kim and Lee 1970; Lee 1971; Yoo and Noh 1987; Yoo et al. 1986). However, they are of limited use in uneven-aged and mixed stands, where many different species, ages and sizes of trees exist (Lee et al. 2004). A crucial factor for managing uneven-aged and mixed forests is knowledge relating to growth at the individual tree level for different tree species (Lee et al. 2004). Analyzing the growth of mixed stands, therefore, requires a modeling approach related to the growth of individual trees. Individual tree
growth models need to simulate each individual tree as a basic unit (Vanclay 1994; Gadow and Hui 1999; Bartelink 2000).

Most oak stands, which account for approximately 26% of the total forested area in South Korea (Korea Forest Service 2008), are naturally regenerated (Lee et al. 2004). *Quercus variabilis, Quercus accutissima* and *Quercus mongoloca* are dominant tree species in oak stands in South Korea, and are recognized as potential natural species requiring intensive management. Their growth rates are comparable with other tree species, particularly red pine (*Pinus densiflora*), but only few individual tree growth models have been developed for oak species, especially for mixed and uneven-aged stands in South Korea. Lee et al. (2004) developed a diameter growth model for mixed stands of red pine and oak in Korea. However, this model dealt with the growth of two different tree species, no models are currently available for different oak species in Korean forests.

This study aimed at developing an individual tree growth model for three oak species (*Q. variabilis, Q. accutissima* and *Q. mongoloca*) in mixed and uneven-aged oak stands in the central region of Korea, which takes the tree age, size, competition and site condition into consideration.

2. Materials

A total of 40 temporary plots, covering between 0.01 and 0.08 ha, were installed in a mixed and uneven-aged oak forest zone (*Q. variabilis, Q. acutissima* and *Q. mongolica*), which represented the core of the natural distribution of oak in central Korea. The stands were naturally regenerated, without a history of silvicultural treatment. The plots were selected to cover a large range of tree ages and sizes, as well as competition and topographic situations. The diameter at the breast height (dbh) and the height of all the trees were measured in each sample plot. The tree coordinates were established by measuring the distance and azimuth from the primary subject tree of a sample plot to the neighboring trees. The increment cores were taken from 5 to 8 sample trees in each plot, with the tree rings measured to within 1/100 mm using a tree ring measurement system. A total of 217 sample trees, consisting of 94 *Q. variabilis*, 66 *Q. acutissima* and 57 *Q. mongolica*, were also made available for this study.
In order to avoid measurement errors in the annual tree ring widths, as well as the
influence of short-term climatic fluctuations, the periodic annual increment for the
preceding 10 years was used as the annual radial increment ($\Delta r$) in the analysis. The basic
statistics for the data set are given in Table 1.

<Table 1>

3. Methods

3.1. Radial growth model

3.1.1 Model components

Individual tree growth has been modeled using either the basal area (Wykoff 1990;
Quicke et al. 1994; Monserud and Sterba 1996; Jõgiste 2000; Rose Jr and Liynch 2001) or
diameter increment (Pukkala 1989; Lee 1996; Lee et al. 1999; Gourlet-Fleury and Houllier
2000; Rautiainen et al. 2000; Lee et al. 2004). West (1979) concluded there was no reason
for expressing the growth as either the diameter or basal area increment (Lee et al. 2004).
In this study, the annual radial increment at the breast height ($\Delta r$) was selected as the
dependent variable for the radial growth model. It was also assumed that the annual radial
growth of a tree depended on the tree age and size, as well as the competition and site quality
at the stand level, as suggested by equation 1 in Lee et al. (2004).

$$growth = f(Age, Size, CI, Site)$$

(1)

The tree age has generally been considered an important variable for radial growth, and
was included as an independent variable in our model. For the tree size variable, the dbh
was employed as an independent variable, because assessing the height and crown variables
in the field is expensive and can be associated with a high level of measurement bias (Lee
et al. 2004).

In even-aged forests, the effect of the site quality on tree growth is generally accounted
for by the site index, which is derived from the stand age and dominant height (Schröder
2000; Lee et al. 2004). The stand age and dominant height are generally incorporated to
indirectly consider the effect of the site quality on tree growth (Lee 1996; Lee et al. 1999;
Lee et al. 2004). Generally, a simple and unique index for assessing the site quality in uneven-aged and mixed stands is unavailable. Therefore, an alternative approach is to directly include specific attributes of the growing site (Wykoff 1990; Hasenauer and Monserud 1996; Monserud and Sterba 1996; Sterba et al. 2002; Lee et al. 2004). Wykoff (1990) used the aspect, slope, elevation, habitat type and geographic location as indicators of the site quality for stands composed of mixed species. Hasenauer and Monserud (1996), Monserud and Sterba (1996), Sterba et al. (2002), and Lee et al. (2004) also incorporated the elevation, slope and aspect into their individual tree growth models for mixed stands. In this study, two models were built and evaluated, as follows; 1) a site quality-independent model (SQiD), and 2) a site quality-dependent model (SQD). For the site quality dependent model (SQD-1), the dominant height and age were initially included as independent variables for representing the site index. Topographic variables; the aspect, elevation, and slope, were also introduced to explain the site quality in the site quality dependent model (SQD-2).

A competition component, assuming values in the interval [0, 1], was used as an influencing factor on the radial growth. In this study, Hegyi’s competition index (HgCI, eq (2)) was applied, which was calculated from the DBH ratio and distance between the subject tree and competitor trees (Hegyi 1974).

\[ HgCI_i = \frac{\sum_{j=1}^{n} d_j / d_j}{\text{Dist}_{ij}} \]  

where;
- \( d_i \) = diameter of the subject tree (cm) at breast height
- \( d_j \) = diameter of the competitor tree (cm) at breast height
- \( \text{Dist}_{ij} \) = distance between subject tree and competitor tree (m)

In this study, an expanded iterative search method was applied to select the competitor trees (Figure 1), which consisted of two steps: in the first step, the neighbors to the subject tree (Si) were selected as the primary competitor trees (PCj), using the iterative search method developed by Lee and Gadow (1997). This search method differentiates between active competitors facing the reference tree and passive competitors positioned behind an active competitor, when viewed from the subject tree. In the second step, the neighbors to each primary competitor tree (PCj) were selected as secondary competitor trees (SCjk).
using the same search method.

<Figure 1>

3.1.2 Model Structure

The following radial growth model (eq. 3), suggested by Lee et al. (2004), was selected for modeling the individual radial growth of three oak species;

\[
\Delta r_{t,j} = a_0 A_{t,j}^{a_1} D_{t,j}^{a_2} e^{b \times C_{t,j}^{a_3}} \times S_{t,j}^{a_4},
\]

where \( \Delta r_{t,j} \) is the annual radial increment at the breast height of tree \( j \), at time \( t \) (cm), \( A_{t,j} \) the tree age (years), \( D_{t,j} \) the tree diameter at the breast height (cm), \( C_{t,j} \) Hegyi’s competition index for tree \( j \) at time \( t \), \( S_{t,j} \) the stand level site quality, with \( S_{t,j} = 1 \) for the SQ-independent model (SQiD), \( S_{t,j} = A_o t^{c_1} H_o t^{c_2} \) for the SQ-dependent model with dominant height and age (SQD-I), \( H_o \) the stand dominant height (mean height of the 3 to 5 highest trees; m), \( A_o \) the stand dominant age (mean age of the 3 to 5 highest trees; in years), \( S_{t,j} = e^{c_3 A_l t^{c_4} + c_5 E_l t^{c_6} + c_7 S_l t^{c_8}} \) the SQ-dependent model with topographic factors (SQD-2), \( A_l \) the aspect index; where \( A_l = 2 \times |1 – \text{aspect}/180| \), with values ranging from 0 (south) to 2 (north), \( E_l \) the elevation (m) and \( S_l \) the slope (°).

3.2. Statistical performance evaluation

The model was refitted several times with the initial values estimated from the previous fits to ensure stability of the parameter estimates, using the SAS NLIN procedure (SAS Institute 1998). The statistical performance of the model was evaluated using the root of the mean square of error (\( \sqrt{MSE} \)), the coefficient of determination (\( R^2 \)), Akaike’s Information Criteria (\( AIC \); Burnham and Anderson 2002) and the significance level of the estimated coefficients.
4. Results and Discussion

4.1. Statistical performance of the models

Table 2 shows the parameter estimates and related statistics for all the models (SQiD, SQD-1 and SQD-2). The models showed relatively good performance for explaining the variations in radial growth, with $R^2$ values ranging from 0.45 to 0.54 for *Q. variabilis*, 0.53 to 0.66 for *Q. acutissima* and 0.47 to 0.59 for *Q. mongolica*. The statistical evaluation of the models using $\sqrt{\text{MSE}}$, $R^2$ and $AIC$ indicated that the site quality dependent models were slightly superior to the site quality independent model. Lee et al. (2004) also proved that the dbh models for red pine and oaks could be significantly improved by the addition of site quality variables to the models. Our model employed relatively simple variables for tree size and competition, which can be easily obtained from simple measurements. Therefore, the radial growth model in this study might be less accurate than those requiring information on the tree height or crown to calculate the competition index (Biging and Dobbertin 1995; Pretzsch 1995; Hasenauer and Monserud 1996; Monserud and Sterba 1996; Bachmann 1997), but with relatively simple components this model can be more widely used in situations where information on the tree height and crown are not available.

4.2. Statistical feasibility of the parameters

4.2.1. Tree age

The tree age parameters were found to be significant in all models at a significance level of 0.001 or 0.01. The negative coefficient for tree age indicated that the annual radial growth decreased with increasing tree age when the other variables remain constant, which was consistent with the general experience of other individual tree growth models (Quicke et al. 1994; Lee 1996; Lee et al. 1999; Jõgiste 2000; Lee et al. 2004). Figure 2 shows the effects of tree age and size (dbh) on radial growth in the SQiD model. The competition index was fixed at a value of 1.5. The three dimensional graphs shown in Fig. 2 are useful for evaluating the combined effects of two independent variables (McFadden and Oliver 1988;
Quicke et al. 1994; Jõgiste 2000; Lee et al. 2004). The negative effect of tree age on radial
growth becomes increasingly distinct with younger age, bigger size (Figure 2) and lower
competition (Figure 3). For *Q. acutissima*, the negative effect of tree age was noticeably
lower compared to the other two oak species, suggesting *Q. acutissima* should be less
sensitive to the changing age of a tree.

4.2.2. Tree size

The coefficients of the tree size component, DBH, were significant at the relative lower
0.05 or 0.01 level, except for *Q. variabilis* in SQD-1. Lee et al. (2004) also reported a relative
lower significant level for the DBH parameter of *Quercus* species. The DBH coefficients
were positive, with the exception of those for *Q. variabilis* in the SQD-1 model, as
confirmed in many other studies (Wykoff and Monserud 1988; Wykoff 1990; Quicke et al.
Sterba et al. 2002; Lee et al. 2004). The positive effect of the tree size on the radial growth
can be explained by the size, which reflects the effects of previous competition and/or vigor
on tree growth (Lee et al. 2004). The positive effect of size on radial growth distinctly
appears at a younger tree age (Figure 2) and with a lower competition index (Figure 4). This
implies that larger trees grow faster when there is less competition. The positive effect of
tree size was similar for all oak species with respect to the competition effect.

4.2.3. Competition

All competition coefficients were significant at the 0.001 level. The negative coefficients
indicate that the radial growth would be expected to decrease with increasing competition,
as the radial increment was found to decrease with increasing competition in other studies
(Wykoff 1990; Holmes and Reed 1991; Quicke et al. 1994; Biging and Dobbertin 1995;
Hasenauer and Monserud 1996; Lee 1996; Monserud and Sterba 1996; Jõgiste 2000; Sterba
et al. 2002; Lee et al. 2004). The negative effect of competition on tree growth was larger
with lower competition and age (Figure 3), and bigger trees (Figure 4). This indicates that
younger and bigger trees are more sensitive to competition. The negative effect of
competition was similar for all oak species with respect to the size effect.

<Figure 4>

4.2.4. Site Index

The dominant height parameters in the site quality dependent model (SQD-1) were only
significant for Q. variabilis, while the dominant age parameters were significant for all
species at the 0.001 or 0.05 level. The site index, which can be represented by the dominant
age and height, has been reported as being questionable, as the age and dbh can already
capture the influence of the site on individual tree growth (Lemon and Schumacher 1962;
Cole and Stage 1972, Quicke et al. 1994), and due to the initial suppression of smaller trees
(Peng 2000). Lee et al (2004) also reported a similar problem when using the site index, or
dominant age and height, for the mixed stands of P. densiflora and Q. variabilis. Other
works have also pointed out that the classical site index approach is not applicable to
uneven-aged multi-species forests (Wykoff and Monserud 1988; Sterba et al. 2002).

4.2.5. Topography

The site quality-dependent model, with a topographic index, had the advantage of
accessing the growing site by employing the same criteria for different tree species (Lee et
al. 2004). The topographic parameters in the site quality dependent model (SQD-2) showed
different performances according to the tree species. For Q. variabilis and Q. mongolica
only the aspect parameter was significant at the 0.001 and 0.05 levels. For Q. acutissima,
the aspect and slope were significant at the 0.01 level. It was notable that the aspect
parameters were found to be significant for all three species, which was similar to the result
of Monserud and Sterba (1996), where only the aspect affected oak growth, but different
from that of Lee et al. (2004), which showed a non-significant effect of the aspect on the
radial growth for mixed stands of P. densiflora and Q. variabilis. It was also notable that
the coefficients of elevation were found be insignificant for all species, which was similar to the result of Hasenauer and Monserud (1996), who reported an insignificant influence of this site factor on the growths of stone pine and oak in Austria, but different to that of Lee et al. (2004), where the elevation parameter was significant for *Q. variabilis* at the 0.0001 level. The coefficients of slope for *Q. variabilis* and *Q. mongolica* were insignificant, as reported by Hasenauer and Monserud (1996) and Lee et al. (2004).

The coefficients of the aspect index (*AI*), which increased from south to north, were positive for *Q. acutissima* and *Q. variabilis* and negative for *Q. mongolica*. This might imply that the northern slope favored diameter growth of *Q. acutissima* and *Q. variabilis*, but not for *Q. mongolica*. However, *Q. acutissima* and *Q. variabilis* appear on the southern slope and *Q. mongolica* on the northern slope, when the slope is identified by the main ridge stretching from the top of the mountain (Lee et al. 2001; Lee et al. 2003; Ham et al. 2004). The mean value and range of the AI in Table 1 also satisfactorily represent the spatial distribution of the oak species. The spatial distribution and growth of tree species can be influenced by the micro-topography which is identified by small branch ridge within the same aspect by the big main ridge (Lee et al. 2008; Kim et al. 2009). Considering the actual spatial distribution and AI range (Table 1) of oak species in the study area, the positive effect of AI on *Q. acutissima* and *Q. variabilis* can only imply that these species appear mostly on the southern slope from the main ridge, but their growth could improve with increasing AI, or from south to north formed by the small branch ridge on the southern slope. Similarly, the negative effect of AI on *Q. mongolica* only suggests that the growth of *Q. mongolica*, mostly appearing on the northern slope from the main ridge, can be facilitated with decreasing AI, or from north to south formed by the small branch ridge on the northern slope.

**5. Conclusion**

This study prepared individual radial growth models for different oak species, and showed that the statistical performance of the model can be significantly improved by incorporating variables for site quality in terms of site index and topographical factors. However, the mean age and dominant height, which together represent the site index,
showed low levels of significance or non-significance in explaining the radial growth of oak tree species. Similarly to the site index, the topographical factors appeared to be problematic for representing the site quality. The coefficients of elevation were not significant for all oak species, with only the aspect index proven to have a significant effect on radial growth for all oak species.

The individual radial growth model developed in this study employed tree age and size, competition and growing site as factors influencing radial growth. Relatively easily available variables in the field, such as tree age, dbh, and Hegey’s competition index, which only uses the dbh and distance, dominant height, aspect, slope and elevation, were used for the model components. This can be attributed as the reason our radial growth model might be less accurate than those requiring crown information to represent the competition and soil information when assessing the site quality. However, this model, with relatively simple variables, can be more widely used in situations where information on the crown and soil are not available.

The components of age, tree size and competition have been proven to have a significant effect on the tree radial growth. Age and competition have negative effect on the radial growth, while the tree size has a positive effect. The age effect on the radial growth was greater for larger trees and with lower competition. The tree size was also expected to have a greater effect on younger trees and with lower competition. Competition was found to have a greater negative effect on younger and bigger trees. These analyses of the effects of the different components on the radial growth of oak species were found to be reasonable and consistent with well-known silvicultural experiences.

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2 Table 1. General description of the variables used in the diameter growth model according to species.

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4 Table 2. Parameter estimates and related statistics for the radial growth models for oak species.

5

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7 Figure. 1. Diagram of a sample plot using the expanded iterative search method ($S_i$: subject tree $i$, $PC_j$: $j^{th}$ primary competitor of $S_i$, $SC_{jk}$: $k^{th}$ secondary competitor of $PC_j$).

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9 Figure. 2. Annual radial growth for different ages and diameters, as simulated using the $SQiD$ model, with a constant CI of 1.5.

10

11 Figure. 3. Annual radial growth for different combinations of Hegyi’s competition index ($HgCI$) and age, as simulated using the $SQiD$ model, with a constant DBH of 15 cm.

12

13 Figure. 4. Annual radial growth for different combinations of Hegyi’s competition index ($HgCI$) and the current DBH, as simulated using the $SQiD$ model, with a constant tree age of 30 years.
Table 1. General description of the variables used in the diameter growth model according to species.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Total (217)</th>
<th>Q. variabilis (94)</th>
<th>Q. accutisima (66)</th>
<th>Q. mongolica (57)</th>
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<tbody>
<tr>
<td></td>
<td>mean</td>
<td>min</td>
<td>Max</td>
<td>S.D.</td>
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<tr>
<td>Age (year)</td>
<td>36.1</td>
<td>11</td>
<td>103</td>
<td>10.3</td>
</tr>
<tr>
<td>DBH (cm)</td>
<td>18.5</td>
<td>8.0</td>
<td>35.3</td>
<td>5.9</td>
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<tr>
<td>Height (m)</td>
<td>14.8</td>
<td>6.0</td>
<td>23.0</td>
<td>3.4</td>
</tr>
<tr>
<td>HgCI</td>
<td>2.7</td>
<td>0.2</td>
<td>8.8</td>
<td>1.3</td>
</tr>
<tr>
<td>PAI (cm)</td>
<td>0.231</td>
<td>0.046</td>
<td>0.521</td>
<td>0.111</td>
</tr>
<tr>
<td>Ao (year)</td>
<td>41</td>
<td>20.3</td>
<td>72.3</td>
<td>9.5</td>
</tr>
<tr>
<td>Ho (m)</td>
<td>14.3</td>
<td>13.2</td>
<td>20.3</td>
<td>2.5</td>
</tr>
<tr>
<td>Aspect (°)</td>
<td>181.4</td>
<td>10</td>
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<td>1.94</td>
<td>0.4</td>
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<tr>
<td>Elevation (m)</td>
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<td>195</td>
<td>460</td>
<td>53.7</td>
</tr>
<tr>
<td>Slope (°)</td>
<td>25.2</td>
<td>7</td>
<td>39</td>
<td>7.9</td>
</tr>
</tbody>
</table>

HgCI: Hegyi's competition index, PAI: Periodic Annual Increment of tree radius, Ao: dominant stand age, Ho: dominant stand height, AI: Aspect index = 2 |1 - Aspect/180|.
Table 2. Parameter estimates and related statistics for the radial growth models for oak species.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Tree species and growth models</th>
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<tbody>
<tr>
<td></td>
<td>Q. variabilis</td>
</tr>
<tr>
<td></td>
<td>SQiD</td>
</tr>
<tr>
<td>$a_0$</td>
<td>5.4314*</td>
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<td>$a_1$</td>
<td>-0.7901***</td>
</tr>
<tr>
<td>$a_2$</td>
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</tr>
<tr>
<td>$b$</td>
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</tr>
<tr>
<td>$c_1$</td>
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<tr>
<td>$c_2$</td>
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<td>$c_3$</td>
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<tr>
<td>$AIC$</td>
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</tr>
</tbody>
</table>

***: significant at a level of 0.001, **: significant at a level of 0.01, *: significant at a level of 0.05, ns: not-significant, $\sqrt{MSE}$: the root of mean square of error, $R^2$: coefficient of determination, AIC: Akaike’s Information Criteria.
Figure 1. Diagram of a sample plot using the expanded iterative search method ($S_i$: subject tree $i$, $PC_j$: $j^{th}$ primary competitor of $S_i$, $SC_{jk}$: $k^{th}$ secondary competitor of $PC_j$).
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Figure 4. Annual radial growth for different combinations of Hegyi’s competition index (HgCI) and the current DBH, as simulated using the SQiD model, with a constant tree age of 30 years.