Incentivizing Resilience in Financial Networks

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Introduction

- **Systemic Risk (SR):**
  - Property of systems of interconnected components:
    
    Failure of a single entity (or small set of entities) can result in a cascade of failures jeopardizing the whole system.
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  ⇒ Failure to manage systemic risk (SR) can be extremely costly for society (e.g. financial crisis of 2007-2008)
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- This happens in financial (i.e. interbank) systems:
  
  $\Rightarrow$ Failure to manage systemic risk (SR) can be extremely costly for society (e.g. financial crisis of 2007-2008)

- Regulations proposed fail to address the fact that SR is a network property (BASEL III. e.g. Tobin taxes, capital requirements)
Insolvency Cascades in Networks

- A financial network is really a network of exposures.
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A financial network at time $t$ is a pair $(\bar{A}_t, \vec{E}_t)$
- $\bar{A}_t$: adjacency matrix of a weighted, directed network
- $\vec{E}_t$: vector of equities of institutions in the network
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The systemic impact of bank $i$ at time $t$:

$$SI^i(\bar{A}_t, \vec{E}_t) = \sum_{j \neq i} \mathbb{1}_{\{j \text{ fails | } i \text{ fails}\}} E^j_t.$$  

→ Total value of financial system lost as a result of bankruptcy of bank $i$
Quantifying Systemic Risk

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- An algorithm can compute $1\{j \text{ fails} \mid i \text{ fails}\}$
  → à la DebtRank (Battiston et al. (2012), Thurner and Poledna (2013))
Quantifying Systemic Risk

- Expected Systemic Loss:

\[
ESL(\vec{A}_t, \vec{E}_t) = \sum_{j=1}^{n} \mathbb{P}\{j \text{ defaults}\} \cdot SI^j(\vec{A}_t, \vec{E}_t)
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- Different topologies have different effects on size of insolvency cascades (e.g. Boss et al. (2004), Gai & Kapadia (2010), Amini et al. (2013), Poledna et al. (2015))


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- Some work focuses on minimizing \( ESL(\bar{A}_t, \bar{E}_t) \), given a certain topology (e.g. by injecting capital in a certain set of banks)

- Less work focuses on controlling the network topology (e.g. Poledna & Thurner (2016))
**Question**: How can we control formation of a financial network?

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→ Then we can match lenders to borrowers.
Controlling Formation of a Financial Network

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High ‘systemic risk’ loan: Bank 12 inherits the high systemic impact $SI^4(A_t, \vec{E}_t)$ of bank 4
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Matching markets: Designed to resolve a range of complex economic problems

- Example 1: Students to Schools (Roth, 1984, 1999)
- Example 2: Kidney donors to receivers (Roth et al., 2003)
- Example 3: Online matching platforms (e.g. Airbnb, Uber)

→ We need an equilibrium concept based on stable matchings (Gale & Shapley (1962))
Simple model of a credit system

At each discrete time $t \in \{0, 1, 2, \ldots [T]\}$, each bank $i \in \mathcal{N}$ receives a liquidity shock $\epsilon^i_t$

$$
\epsilon^i_t = \begin{cases} 
+1 & \text{with prob. } y/2 \quad \text{(bank } i \text{ in supply of liquidity)} \\
-1 & \text{with prob. } y/2 \quad \text{(bank } i \text{ in demand of liquidity)} \\
0 & \text{with prob. } 1 - y
\end{cases}
$$

where $y \in [0, 1]$. 
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where \( y \in [0, 1] \).

Induces a set of lenders and a set of borrowers:
Simple bilateral contracts

- Each borrower $j$ has an exogenous failure probability $\rho_j$ and a reservation rate $\bar{r}_j$.
- Each lender has an exogenous baseline lending rate $r_i$ and adds a (fair) risk premium $h_i(\rho_j)$:

$$r_{ij} = r_i + h_i(\rho_j)$$

Any borrower $j$ prefers borrowing from lenders with lower rates (up to a maximal rate) if $r_{1j} < r_{2j} < r_{3j} < \bar{r}_j < r_{4j}$ → Preference list $P_j = 1, 2, 3$. Risk premium makes lenders indifferent as to who they lend to.
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- Any borrower $j$ prefers borrowing from lenders with lower rates (up to a maximal rate)
  If $r_{1j} < r_{2j} < r_{3j} < \bar{r}_j < r_{4j}$
  \[ \rightarrow \text{Preference list } P^j_\beta = 1, 2, 3. \]
- Risk premium makes lenders indifferent as to who they lend to
Let \( P = \{ P_5^\beta, P_6^\beta, P_7^\beta, \ldots \} \) be the set of all preference lists

We call the triplet \((\mathcal{L}_t, \mathcal{B}_t, P)\) a market for liquidity at time \( t \).
Two-sided matching

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We call the triplet \((\mathcal{L}_t, \mathcal{B}_t, P)\) a market for liquidity at time \( t \).

Outcome at each \( t \), is a matching \( \mu_t \):

\[
\mu_t(1) = 7, \quad \mu_t(2) = 8, \quad \text{etc}...
\]
Equilibrium Concept: Stable Matching

Definition (Stable Matching)

A matching $\mu^*_t$ is stable if:

(I) No set of borrowers $\vec{b} \in B_t$ could agree to swap their counter-parties.

(II) The lending rates are below the borrowers’ reservation rates (i.e. $r_{ij} \leq \bar{r}_j$)

→ In words: No bank could benefit from behaving differently
Lemma (Equilibrium Multiplicity under Bilateral Contracting)

Any matching $\mu_t$ such that the lending rates are below the borrowers' reservation rates (i.e. $r_{ij} < \bar{r}_j$) is stable.

- Many networks can emerge in equilibrium!
  - Results from borrowers having **homogenous** preferences
    (the all prefer the lender $i$ with lowest baseline rate $r_i$)

- How to compare the different equilibrium matchings?
  - Need a notion of efficiency.
An equilibrium matching $\mu_{t}^{*,eff}$ is systemic risk-efficient if it minimizes systemic risk given a certain transaction volume $\nu$:

$$\mu_{t}^{*,eff} \in \arg\min_{\mu_{t}^{*}: Vol(\mu_{t}^{*})=\nu} ESL(\vec{A}^{*}, \vec{E}_{t}).$$
Revisiting the toy example

Systemic Risk Efficient Equilibrium
Revisiting the toy example

Systemic Risk *Inefficient* Equilibrium

High ‘systemic risk’ loan
Question: Can we select a systemic risk-efficient matching that is sustained as a *unique* equilibrium?

Answer: Yes, by creating *heterogeneous* preferences by means of a transaction-specific tax.

\[ T = \{ \tau_{ij} \} \]  

\[ \tau_{ij} \geq 0 \]  

is a mark-up applied to the interest rate paid by bank \( j \) when it borrows from bank \( i \):

\[ r_{Tij} = r_i + h_i(\rho_j) + \tau_{ij} \]  

→ Idea introduced in Poledna & Thurner (2016)

Each borrower can now prefer a different lender.
**Question:** Can we select a systemic risk-efficient matching that is sustained as a *unique* equilibrium?

**Answer:** Yes, by creating *heterogeneous* preferences by means of a transaction-specific tax.

- \( \mathcal{T} = \{\tau_{ij}\} \): a matrix of transaction-specific taxes, \( i \in \mathcal{L}_t \) and \( j \in \mathcal{B}_t \)
- \( \tau_{ij} \geq 0 \) is a mark-up applied to the interest rate paid by bank \( j \) when it borrows from bank \( i \):

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**Equilibrium Selection and Uniqueness**

**Idea:** leave desired matches untaxed and tax the undesired matches

![Diagram showing the selection process](image)
Idea: leave desired matches untaxed and tax the undesired matches

Low ‘systemic risk’ loan

\( T_{1,12} = 0 \)
$\mathcal{T}$ re-orders the preferences of each borrower over the set of lenders

→ allows a regulator to create heterogeneous preferences, i.e. each borrower $j$ can now have a different preference list $P^j_\beta$ with optimal match on top.

**Proposition (Systemic Risk under Systemic Risk Tax)**

*For some desired volume $\nu$, there exists $\mathcal{T}$ such that $\mu^*_t$ is unique and systemic risk efficient. We call this $\mathcal{T}$ a Systemic Risk Tax (SRT).*
A Tobin-like tax is a particular case of the SRT $\mathcal{T}$

- Borrowing rate under SRT $\mathcal{T}$:

\[ r_{ij}^{\mathcal{T}} = r_i + h_i(\rho_j) + \tau_{ij} \]

where $\tau_{ij} = 0$ for desired matches and $\tau_{ij} > 0$ for undesired ones
A Tobin-like tax is a particular case of the SRT $\mathcal{T}$

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- Borrowing rate under Tobin-like tax:
  
  $$ r_{ij}^\kappa = r_i + h_i(\rho_j) + \kappa $$

  where $\kappa > 0$ for all matches.
SRT versus Tobin-like tax

A Tobin-like tax is a particular case of the SRT $\mathcal{T}$

- Borrowing rate under SRT $\mathcal{T}$:

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- Borrowing rate under Tobin-like tax:

$$r_{ij}^{\kappa} = r_i + h_i(\rho_j) + \kappa$$

where $\kappa > 0$ for all matches.

→ Makes all transactions more costly, without re-ordering the borrowers’ preference lists.
→ Cannot select a particular systemic risk efficient equilibrium.
SRT versus Tobin-like Tax

Proposition (Tobin-like tax versus Systemic Risk Tax)

There always exists $T$ such that $ESL(\bar{A}_t^*, T, \bar{E}_t) \leq ESL(\bar{A}_t^*, \kappa, \bar{E}_t)$ and $Vol(\mu_t^*, T) \geq Vol(\mu_t^*, \kappa)$

In words: SRT can achieve higher trading volume and lower systemic risk
Regulator’s Optimization Problem

Solve this problem on a dynamically evolving complex network:

- Banks receive liquidity shocks and trade that liquidity in the form of interbank loans
- At each $t$, regulator solve following one-period-ahead optimization problem

$$\hat{T} \in \arg\min_{T} ESL(\bar{A}_t^*, T, \vec{E}_t)$$

$\forall T: Vol(\mu_t^*, T) = \nu$

→ Optimize matching of lenders and borrowers, given a certain transaction volume
Regulator’s Optimization Problem

Expected systemic loss ($ billions)
- no tax
- Tobin-like tax
- SRT

Cumulative volume (# transactions)
- no tax
- Tobin-like tax
- SRT

**Companion papers:**
