Dependency of Crop Production between Global Breadbaskets: A Copula Approach for the Assessment of Global and regional Risk Pools

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Abstract:
As recent events have shown, simultaneous crop losses in different parts of the world can cause serious risks to global food security. However, to date, little is known about the spatial dependency of lower than expected crop yields from global breadbaskets. This is even more so the case for extreme events, i.e. were one or more breadbaskets are experiencing far below average yields. Without such information risk management approaches cannot be applied and vulnerability to extremes may remain high or even increase in the future around the world. We tackle both issues from an empirical perspective focusing on wheat yield. We model the interdependencies between historically observed wheat yield deviations in 5 breadbaskets – (USA, Argentina, India, China and Australia) with copula approaches that can incorporate increasing tail dependencies. In doing so, we are able to attach probabilities to inter-regional as well as global yield losses. To address the robustness of our results we apply three different methods for constructing multivariate copulas: vine copulas, ordered coupling using a mini-max approach and hierarchical structuring. Notwithstanding evidence of global climatic teleconnections that may influence crop production, we demonstrate empirically that wheat production losses are independent between global bread baskets, which strengthens the case for inter-regional risk pooling strategies such as crop insurance. Improved estimation of dependency in crop production at an intra-regional scale provides the basis for more accurate pricing of regional risk sharing instruments.

Keywords: Wheat yield losses, copula approach, risk pooling.
1 INTRODUCTION

The increasing global interconnectivity and mutual interdependence of economic and ecological systems can amplify vulnerability and risk of disasters \(^1\). The global food system demonstrates the effects of increasing interdependence and the possibility of systemic risk. Owing to the globalization of the grain-market, a food shock due to drought (or other downside risks) in one or more major food producing areas can lead to a significant increase in the world food prices and might threaten food security, especially in poorer countries \(^2\). For example, between 2005 and 2008, international wheat and maize prices tripled \(^3\) leading to the 2007/2008 food price crisis. Low-income groups in developing countries were especially affected as they spend 60 – 80% of their income on food \(^4\). Analyzing the food price crisis in 2008, several factors were found to be responsible for the fast increase in grain prices including increased energy prices, shrinking world grain reserves, a rise in demand through increasing population and wealth, financial speculation in commodity markets, decline in growth of crop production due to decreased investment in agricultural R&D and the use of grains for biofuel production \(^3,5,6\). The major cause, however, is considered to be adverse weather conditions \(^7\). Especially, the droughts in major crop producing areas including Australia, the EU, Ukraine and Russia in the years before 2008 had decreased the supply and made the prices very sensitive to small changes in supply \(^6\). Similar to the 2007/08 crisis, 2010 was another year of severe droughts in different parts of the world. Russia suffered from the worst drought in a century \(^8\), which destroyed more than 13.3 million hectares of crops, more than 30% of the area cultivated in the affected regions \(^9\). In August 2010, Russia announced a grain export ban that was meant to curtail price hikes and speculation on grain products, but which proved to be ineffective. At the same time,
2010 was the driest year on record in Southwest Australia where crop production is the major driver of the economy. In 2010, wheat production reached only half of the production in 2011-2012 (10). In south-western China, a drought with the longest period without rain during winter season and the lowest percentage rainfall anomaly in the past 50 years occurred in 2009-2010 (11). The drought in spring 2010 led to major decline in summer-harvested crops (12). The confluence of all these weather extreme events in the same year resulted in price shocks: between June 2010 and January 2011 the global wheat price more than doubled (13) which led to a threat to food security in many regions. Some scientists suggest that the increase in food prices contributed to the unrest that triggered the Arab spring (8,14).

The examples given above, together with a projected increase in extreme weather events (1) show the need to understand better the consequences of extreme weather events occurring in more than one location around the world. Moreover, attention needs to be given to how to manage such risk from a global or regional perspective (15). Regarding the latter, risk pooling is now seen as one promising way to manage risk on larger scales (1) and several applications can be found throughout the world, including the Caribbean Catastrophe Insurance Facility (CCRIF), the African Risk Capacity pool (ARC), as well as the European Union Solidarity Fund (EUSF). However, for a risk pool to work, the risks faced by members of a risk pool should be (statistically) independent, or in other words, the pool should aggregate highly uncorrelated risks. If independence or low dependence is ensured, the risk-reduction occurs due to the law of large numbers, which as a consequence, makes the variance of the aggregated risk less than the variance of the risks taken individually (16,17). Variance is important as, for a given mean, it determines the potential loss below some critical threshold. Regarding the global food system and the feasibility of global and/or regional risk pooling one therefore has to
analyze the dependency structure of losses between and within the regions because different risk instruments, such as structural adaptation or insurance schemes, need to be applied conditional on the results.

So far, the global food system has kept up with an increasing population and rising food demand but due to the recent experiences discussed above and under the threat of climate change, understanding the risks to global food supply together with a functioning system of global food trade becomes essential for bringing resilience to the system and guaranteeing future food security.

Unfortunately, as recent research has shown, seemingly uncorrelated risk (e.g., for very frequent events) may become very much more correlated in case of extreme events and therefore this dependence structure needs to be taken explicitly into account in the analysis (18).

One emerging method dealing with this issue and now increasingly included in applied research is the copula method (16,19). Copulas are useful for modelling dependencies between continuous random variables. Using a copula model allows the selection of marginal distributions separately from the modeling of their interdependence. In other words, while the marginal distributions contain the information on the separate risks, the copula contains the information about the structure of the dependency.

Several studies have applied the copula method to analyze yield losses and their implications for agricultural insurance. Vedenov (20) used the Gaussian copula to estimate the joint distribution of corn yields at two aggregation levels, the farm- and the county-level. He found that the dependence between farm and county yield changed in the lower tail of the distribution. Zhu et al. (21) applied the Gaussian and the t-copula to model the dependence structure between crop yield and prices and found a higher
dependency in the distribution tails. Xu et al. \(^{(22)}\) examined the magnitude of spatial
dependence between weather indices used for different regions in Germany. The
purpose of their study was to quantify the likelihood of a crop insurance payoff due to
the joint occurrence of bad weather at different locations. Bokusheva \(^{(23)}\) used the copula
method to measure the dependence between yields and weather indices in Kazakhstan.
She first used a regression analysis to test the sensitivity of crop yields to several
weather indices and then applied the copula method to estimate joint tail dependence
between crop yields and weather indices. Okhrin et al. \(^{(24)}\) found systemic risk in index-
based insurance for several regions in China and a decline of the risk premium with
increasing trading area for the insurance. They recommend risk pooling between
regions in order to decrease the required buffer fund and therefore risk premiums.
Larsen et al. \(^{(25)}\) found an inverse relationship between geographic conditions and yield
dependencies in 380 counties of the US wheat belt using copulas and Goodwin and
Hungerford \(^{(26)}\) used different copula models to estimate premium rates for revenue
insurances, insuring both yield and price losses, for soybean and corn in four counties in
Illinois. In Spain, Ahmed and Serra \(^{(27)}\) used the copula methodology to show for the
apple and orange sector, that agricultural revenue insurance reduces premium rates
compared to yield insurance schemes. Compared to the linear correlation approach,
copula estimations are considered to be more reliable as they allow to measure extreme
dependence between random variables \(^{(23)}\). Vedenov \(^{(20)}\) recommends using copulas for
optimal crop coverage selection and for crop insurance rating.

Our study contributes to efforts to increase current and future food security by focusing
on production risk and possible ways to decrease such risk explicitly taking spatial
dependencies into account. In doing so we examine the dependence structure of wheat
yield losses within and between five major wheat producing areas, so called
breadbaskets, in the US, Argentina, India, China and Australia. We chose wheat as an exemplary crop as, measured in acreage, it is the most extensively grown food crop globally. Overall wheat production for human consumption is the second largest after rice\(^{(28)}\).

The paper is organized as follows. Section 2 introduces the breadbaskets, Section 3 describes the copula methodology applied in this study and Section 4 gives a detailed description of the results. Afterwards, we base our results within a broader discussion on risk pooling and end with a summary and outlook to the future in Section 6.

## 2 WHEAT YIELD DATA

For the selection of global breadbaskets, the Spatial Allocation Model (SPAM)\(^{(29)}\), a crop production data set, was used to identify the major wheat producing regions of the world. For the analysis, states and provinces were chosen as they provide sufficient accuracy to examine yield loss correlation structures within a region and, at the same time, there is data available and accessible. SPAM production was aggregated on a state/province scale and the highest crop producing units in the US, Argentina, China, India and Australia were selected. These results were compared with suggestions from scientific papers, governmental information and grey literature. With the premise that the states/provinces of a breadbasket have to be adjacent, global breadbaskets were defined as shown in Figure 1. Production in each breadbasket accounts for at least 60% of its national wheat production. A sixth very important wheat producer is Russia, but due to limited data availability, Russia was excluded from the analysis. The five breadbaskets selected for this study account for 35% of global wheat production in 2011.
In what follows we give a short summary of each of the selected global bread baskets in Argentina, U.S., China, India and Australia, data sources employed as well as their relation to the global scale.

The Argentinian breadbasket includes Entre Rios, Santa Fe, Buenos Aires and Cordoba. Wheat in the Argentinian breadbasket accounts for 68% of the national Argentinian wheat production (30). Data were obtained from the agricultural ministry of Argentina (31). In 2013, Argentina was the 13th largest wheat producer in the world and the largest in South America. It is a net exporter of wheat. Wheat in Argentina is usually planted from May to end of July and harvested between mid-November and mid-January (32).

The US breadbasket includes the states Washington, Montana, Idaho, Nebraska, North Dakota, South Dakota, Minnesota, Kansas, Oklahoma and Texas. Wheat production in these states accounts for 63% of national production (30) which is the third largest in the world. Behind corn and soybeans, wheat is the third most important crop produced in the US. The country is the world’s biggest wheat exporter with half of its production being exported. 70-80% of wheat grown in the US is winter wheat which is planted between September and end of October and is harvested between end of May and late
August, depending on the region (33). Data used in this study are taken from the United States Department of Agriculture (34).

The Chinese breadbasket consists of the provinces Shandong, Hebei, Henan, Jiangsu and Anhui. Wheat production in this region covers 84% of the entire Chinese wheat production (30). China is the largest wheat producer in the world but due to its large population, domestic consumption is very high and therefore, China is also the seventh largest importer of wheat (33). 94% of China’s wheat production consists of winter wheat. Growing season for winter wheat in China is usually October to May (35). Agriculture is an important sector in China, contributing 10% to GDP in 2013. Looking at the provinces relevant to our study, the share of agriculture of Gross Regional Product (GRP) are 12.4% in Hebei, 6.2% in Jiangsu, 12.3% in Anhui, 8.7% in Shandong and 12.6% in Henan (36). Data were obtained from the National Bureau of Statistics of China (36).

The states forming the Indian breadbasket are Uttar Pradesh, Madhya Pradesh, Rajasthan, Maharashtra, Gujarat, Bihar, Haryana and Punjab. Together, they produce 88% of India’s national wheat. In 2013, India accounted for 13% of global wheat production and was the second largest producer after China (30). Same as China, large domestic consumption makes India a net wheat importer. India grows mostly winter wheat which is planted between October and end of December and harvested between March and May (32). Data for India was taken from the Ministry of Agriculture and Farmers Welfare, Govt. of India (37).

The Australian breadbasket includes News South Wales and Southern Australia. Together, they produce 80% of Australia’s wheat (30). Australia is the 8th largest producer of wheat and an important wheat exporter. Most of its wheat is winter wheat
which is planted between May and end of July and harvested between October and end of December \(^{(32)}\). Australian wheat data used in this study was taken from the \(^{(38)}\).

Annual historical wheat yield data on a state or provincial scale from 1967 to 2012 was used. The wheat yield time series were detrended and the deviations from the trend were taken to estimate the correlation structure as described in the following section.

### 3 STATISTICAL ANALYSIS METHODOLOGY

The copula methodology is based on Sklar’s theorem \(^{(39)}\) which states that the joint distribution function of any continuous random variables \((X,Y)\) can be written as:

\[
H(x,y) = C[F_X(x), F_Y(y)] \quad x,y \in \mathbb{R} \quad (1)
\]

with \(F_X(x)\) and \(F_Y(y)\) as marginal probability distributions and \(C = [0,1]^2 \rightarrow [0,1]\) as copula. Sklar \(^{(39)}\) showed that \(C\) is uniquely defined if \(F_X\) and \(F_Y\) are continuous. Assuming that the marginal distributions are continuous and have probability density functions \(f_X(x)\) and \(f_Y(y)\), the bivariate joint probability density function will have the form

\[
f_{X,Y}(x,y) = c(F_X(x), F_Y(y)) f_X(x) f_Y(y) \quad (2)
\]

with \(c\) as the density function of \(C\), defined as

\[
c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \quad (3)
\]

for \(0 \leq u, v \leq 1\). The advantage of this approach is that the copula, and thereby the dependence characteristics between \(F_X\) and \(F_Y\), can be chosen independently from the marginal distributions. There are many different copula families described in literature which can be divided into four classes: Archimedean, elliptical, extreme value and others \(^{(19)}\). Archimedean copulas such as the Clayton, Frank, and Gumbel-Hougaard copulas are widely adopted because of their simple form \(^{(40)}\) and for elliptical copulas, the Gaussian
copula and the student t-copula are widely adopted examples. However, any multivariate distribution function can generally be used as the basis for a copula \(^{41}\).

Usually, four steps have to be taken to build a multivariate copula model. In the first step, an adequate copula structure has to be chosen. The selection can be based on expert knowledge or is implied by the structure of the data. A multivariate copula structure can be a hierarchical Archimedean copula (HAC) \(^{24,42,43}\), a vine copula \(^{44–46}\) or a pairwise-coupling process using the minimax approach \(^{47}\). For complex datasets no single methodology is guaranteed to yield robust results. We therefore present and compare three different types of copula structure selection methods which represent varying degrees of complexity and also implicit assumptions within the modeling structure (and should shed some light on uncertainty ranges of the results): For the analysis within the wheat breadbaskets, pairwise-coupling with a minimax approach using different copulas is shown as well as a regular vine tree structure which can include all types of copulas. In the end, hierarchical structuring is used to combine all five breadbaskets. In the second step, appropriate copula families have to be selected which can be done graphically through scatterplots in the bivariate case or analytically through different goodness-of-fit tests such as the Kendall \(^{48}\) or the Vuong and Clarke test \(^{49,50}\). Then, the copula parameters are estimated and in the end, the model has to be evaluated with tests such as the AIC or BIC criterion. We start with the copula structuring methods applied in our analysis. Our analysis was conducted using CRAN R including the packages ‘copula’, ‘CDVine’ and ‘VineCopula’.

### 3.1. Pairwise coupling using a minimax structuring approach

One possibility to order the random variables, in this case yield deviations in the different states or provinces within one breadbasket, is pairwise ordered coupling. Here,
yield deviations are defined as deviations from a logistic trend. Observations are called \( y_{i,t} \) and we use the following model

\[
y_{i,t} = f_i(t) + \Delta y_{i,t}
\]  

(4)

where \( f_i(t) \) is a 4-parameter logistic regression function

\[
f_i(t) = a_i + \frac{b_i}{1 + c_i \exp(-d_i t)}
\]

(5)

For pairwise coupling, assume that a breadbasket has 5 states with yield deviations \( \Delta y_i \) in each state \( i \). Let the yield deviation \( \Delta y_1 \) in state 1 influence yield deviation \( \Delta y_2 \) in state 2. Yield deviation \( \Delta y_2 \) in state 2 then influences yield deviation \( \Delta y_3 \) in state 3 which influences yield deviation \( \Delta y_4 \) in state 4 and so on. Then, 2-dimensional copula densities \( c_{1,2}, c_{2,3}, c_{3,4} \) and \( c_{4,5} \) are estimated and combined through conditional copulas in the following way:

\[
c(u_1, u_2, u_3, u_4, u_5) = c_{1,2}(u_2|u_1) \cdot c_{2,3}(u_3|u_2) \cdot c_{3,4}(u_4|u_3) \cdot c_{4,5}(u_5|u_4)
\]

(6)

The tree structure of this copula is shown in Figure 2.

![Figure 2. Pairwise ordered coupling.](image)

In a next step, an ordering technique has to be chosen. In this paper we employ the minimax approach based on Timonina et al. (47). First, the metric used for the structuring process has to be chosen. This can be, e.g., geographical distance between states or the
non-parametric correlation coefficient Kendall’s tau (τ) which was used for this study. Kendall’s τ is calculated for each pair of states which results in a matrix shown in Equation 7.

\[
T = \begin{pmatrix}
1 & \tau_{12} & \ldots & \tau_{1N} \\
\tau_{21} & 1 & \ldots & \tau_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\tau_{N1} & \tau_{N2} & \ldots & 1
\end{pmatrix}
\] (7)

Columns and rows represent states and Kendall’s \(\tau_{ij}\) is the non-parametric correlation between state \(i\) and \(j\). Second, the ordering technique minimax is applied by choosing \(N\) pairs of states from matrix \(T\). Therefore, the highest correlation \(\tau_{ij}\) is chosen and the ordering starts with states \(i\) and \(j\). All correlations with these two states, \(\tau_{i,1}, \tau_{i,2}, \ldots, \tau_{i,N}\) and \(\tau_{1,j}, \tau_{2,j}, \ldots, \tau_{N,j}\) are screened and for each pair \(\tau_{i,n}\) and \(\tau_{n,j}\) (with \(n=1,2,\ldots,N\) states) the minimum is chosen. Between all minima \(\min[\tau_{i,n}, \tau_{n,j}]\) the maximum is chosen, \(\max[\min[\tau_{i,n}, \tau_{n,j}]]\), which will be basin \(k\) (with \(k\neq i, k\neq j\)). \(\tau_{i,k}\) or \(\tau_{k,j}\) will be the new starting point and the selection process for the next basin starts over again. Detailed information about the minimax technique can be found in Timonina et al. (47).

After the ordering structure is determined, the copula family for the entire model has to be selected. Similar to Timonina et al. (47), we test four types of copulas for the minimax approach, the Gumbel, Clayton, Gaussian and Frank copula, which possess different characteristics concerning their tail dependence. We then use the AIC (51) and BIC (52) goodness-of-fit test for each pair-copula in the structuring process and choose the copula family with the best overall fit (see supplementary material). As demonstrated in Equation 6, conditional copulas are created to model the dependency between yield deviations in the different states within one breadbasket. A conditional copula is a partial derivative of the original copula \(C_\theta(u,v)\) over \(v\):

\[
C_\theta(u|v = v_0) = \frac{\partial C_\theta(u,v)}{\partial v} \bigg|_{v=v_0}
\] (8)
where \( u \) and \( v \) are placeholders for the cumulative distribution functions \( F_{\Delta y_i} \) and \( F_{\Delta y_j} \) of yield deviations in state \( i \) and \( j \). A random number \( u \) following the conditional Gaussian copula given \( v \) is given by Equation 9.

\[
u = \Phi(N(\theta \Phi^{-1}(v), 1 - \theta^2))\tag{9}\]

with \( \Phi \) as Gaussian distribution and \( \theta \) as copula parameter which can be calculated using the relationship to Kendall’s \( \tau \): \( \sin(\pi \tau / 2) = \theta \). Unfortunately, the Gumbel copula is not directly invertible as shown in Timonina et al. \(^{47}\) and therefore, a numerical iteration has to be used which is explained in detail in the appendix. Gaussian, Clayton and Frank copulas are directly invertible. By sequentially linking the conditional copulas as in Equation 6, a conditional copula with \( d \) dimensions, depending on the number of states in a breadbasket, is calculated. For getting random joint yields this random generation process is repeated 10 000 times resulting in a 10 000 x \( d \) matrix with dependent yield deviations.

3.2. Vine copula approach

Another ordering approach for multivariate copula models are vine copulas \(^{44,53,54}\). A vine copula is a flexible graphical model which uses a cascade of conditional and unconditional bivariate pair-copulas to decompose the multivariate probability density. The pair-copulas are ordered in so-called tree structures with three most common ways of ordering being lines trees, called D-vines, star trees, called C-vines, or R-vines which are most flexible. Figure 3a and b show a C-vine and an R-vine respectively for a five-dimensional copula.
As each copula pair can be chosen independently an enormous amount of combinations and thereby dependence structures are possible. For a $d$-dimensional copula, there are $\frac{d(d-1)}{2}$ bivariate copulas that can be estimated in $\frac{d!}{2} \cdot 2^{(d-2)}$ possible R-vine trees or in $d!/2$ possible C-vine trees\(^{(46)}\). A vine structure has $(d-1)$ trees with nodes $N_i$ and edges $E_i$ joining the nodes. A tree structure is built considering the proximity condition\(^{(53)}\) which states that if an edge connects two nodes in tree $j+1$, the corresponding edges in tree $j$ share a node. In order to select the structure of one tree, there are different selection approaches. Aas et al.\(^{(44)}\) choose the variables with the strongest bivariate dependencies, measured with Kendall’s $\tau$, for the first tree. Other possible edge weights are the p-value of a goodness-of-fit test or directly the copula parameter $\theta$. This paper...
selects trees using maximum spanning trees with Kendall’s τ as edge weights \(^{(55)}\). For each tree, the spanning tree which maximizes the sum of absolute Kendall’s τ’s is selected by solving the following optimization problem:

\[
\max_{E = [j,k] \in n \text{spanning tree}} \sum_{j,k} |\hat{\tau}_{j,k}|
\]  

with \(\hat{\tau}_{j,k}\) as pairwise empirical Kendall’s τ. A spanning tree, i.e. a circle free graph, connects all nodes.

Compared to the pairwise ordering approach using minimax, the vine copula approach can include different types of copulas for each bivariate copula. The copulas are selected as described above and the copula parameters are estimated.

### 3.3. Hierarchical structuring

The third way of copula structuring discussed in this paper is hierarchical structuring. After combining the different states in each breadbasket through pairwise coupling using the minimax approach as shown in Figure 2, the copulas themselves are structured in a hierarchical way as shown in Figure 4 for two copulas.
The function for Figure 3 can be written as follows:

\[
C(u_{11}, u_{21}, u_{31}, u_{41}, u_{51}, u_{12}, u_{22}, u_{32}) = \tilde{C}(C_{11,21,31,41,51}(u_{11}, u_{21}, u_{31}, u_{41}, u_{51}), C_{12,22,32}(u_{12}, u_{22}, u_{32}))
\]  

(11)

In the case of five breadbaskets, five different copulas are structured in a hierarchical way. The ordering approach chosen for the five copulas is again pairwise coupling using the minimax approach. Kendall’s τ is used as metric for the structuring process and is estimated using the linearly detrended production in each breadbasket with breadbasket production being the sum of the production in each state.

4 RESULTS

We start with a simple bivariate copula example, i.e. the states Uttar Pradesh and Haryana in the Indian breadbasket. First, the marginal univariate distribution parameters are estimated. For the two states both detrended (using a logistic regression function as in (5)) yield deviations follow a normal distribution (tested via the Shapiro-Wilk test of normality) and statistical parameters for Uttar Pradesh (UP) and Haryana (H) are mean=0, sd=0.12 and mean=0 and sd=0.20, respectively. The correlation between the two states was measured with Kendall’s τ=0.52. As copula family the Gumbel copula was chosen for which the parameter \( \theta \) has the following relationship with Kendall’s τ: \( \theta = \frac{1}{1-\tau} \). The Gumbel copula is

\[
C_\theta(u, v) = \exp \left\{ -\left[ (-\ln u)^\theta + (-\ln v)^\theta \right]^{1/\theta} \right\}
\]  

(12)
Figure 5 shows the contour plot of a Gumbel copula joining wheat yield deviations of Uttar Pradesh and Haryana as well as the underlying univariate distributions.

**Figure 5.** Gumbel copula for joint modeling of wheat yield deviation in Uttar Pradesh and Haryana.

Using this kind of information one is able to calculate risks of joint yield losses. For example, the probability that in the same year wheat yields in both states are below the mean by more than one standard deviation is \( P(UP \leq -0.118, H \leq -0.2) = C_\theta[F_{UP}(-0.118), F_H(-0.2)] = 0.075 \). Note, if we assume independence between the two states, the risk would be underestimated by a factor of three, e.g., \( P(UP \leq -0.118, H \leq -0.2) = F_{UP}(-0.118) \cdot F_H(-0.2) = 0.024 \). The implications of underestimating this correlated risk are tremendously important for risk pooling and policy makers who have to take decisions on agricultural policies such as subsidies or post disaster risk finance mechanisms. The
importance of including such dependency structures for risk modeling becomes even
more apparent in the subsequent multivariate models which are discussed next.

To start with, the pairwise coupling was applied to all states within one breadbasket
leading to a Kendall’s τ correlation matrix for all states (which can be interpreted in the
same way as discussed for the bivariate example given above). The copula structure was
determined using the minimax approach. Afterwards, the correlation structure for
wheat yield deviation was determined and estimated using Gumbel copulas, and yields
were transformed into total production via the formula

\[ p_k(t) = \sum(f_i(t) + \Delta\tilde{y}_{it}) \cdot a_i \]  \hspace{1cm} (13)

with \( i = 1, \ldots, 8 \) Indian states and \( t = \) time. \( p_k \) is total production in breadbasket \( k \) and \( a_i \) is
harvested area in ha in state \( i \) in 2012 which was held constant for each state in order to
avoid production changes due to increased acreage instead of increased yield. \( \Delta\tilde{y}_{it} \)
denote random draws from the estimated joint distribution using the estimated
marginals and the estimated copula.

Average yield is time-dependent as yields are increasing over time due to technological
improvements which was captured in the logistic trend function \( f(t) \) (and a linear trend
in the Argentinian breadbasket). Based on the logistic (or linear) trend and the sampled
dependent residuals three production curves were calculated including future
production estimates for today, in 10 years and in 20 years from 2012 onwards. In
contrast to ordered pairwise coupling using the minimax approach, we already indicated
that R-vines are more flexible concerning the choice of copula families, e.g., for each
copula pair, a different copula type may be chosen. To find the best fitting copula family
for each pair, the Akaike Information Criterion (AIC) \(^{56}\) was used here. The R-vines
were applied to yield deviations in all breadbaskets and production curves were produced as described in Equation 13. Figure 6a shows the results of the analysis of the lower tail of the distribution of production, using an R-vine compared with results from the previous approach.
Figure 6. Production curves up to the lower 10 percentile of the production distribution for the Indian breadbasket a) comparing the with minimax structured copula using Gaussian copulas with an R-vine for three time steps. b) comparing the with minimax structured copula using Gaussian copulas and an R-vine with the independent and fully dependent case with today's average production levels.

As we are mainly interested in yield losses, the figure above shows only the lower tail of the distributions. In more detail, the plot shows production curves for today, for in 10 and for in 20 years. The results between the two structuring methods differ: most importantly the production distributions for R-vines have fatter tails than the distributions where the minimax approach with Gaussian copulas was used. In other words production losses are more likely under an R-vine structure than if the Gaussian copulas are applied. Whilst there is no definitive way to establish which way of copula structuring fits better, the R-vines structure is more flexible and general so is more attractive. Figure 6b compares the lower tail of wheat production in the Indian breadbasket for today’s average production levels using minimax with Gaussian copulas, an R-vine, an independent copula and a copula assuming full dependence between the Indian states. The Gaussian copula using minimax ordering and the R-vine structure lie between the independent and the fully dependent case. For example, with a probability of 5% or a 20 year return period, the production in the Indian breadbaskets will be 76.1 million tons if the breadbaskets were fully dependent, 78.5 million tons using the R-vine approach, 79.1 million tons applying a structured copula and 80.5 million tons if we don’t consider correlations between the states. The difference between the R-vine curve and the independent copula is 2 million tons of wheat which equals 2.2% of the actual wheat production in the Indian breadbasket in 2012, 91.2 million tons. The difference between the minimax structured copula and the independent case are 1.4 million tons which equals 1.5% of the actual production. This shows that, if correlations between wheat yields within the breadbaskets are not considered in risk analysis, the risk of
production losses is underestimated, which is important for crop insurance schemes and agricultural policy decisions. Using the example above, the risk of a production of 80.5 million tons is more than three times higher assuming all states are independent compared to the R-vine curve.

As explained above for India, copula models for production risk for all regions have been developed (see supplementary material). In the next step, these copula models were combined through another multivariate copula, now up to the global level. In that way, dependencies between but also within the regions can be captured. Pairwise coupling using minimax was chosen to build an ordering structure for the breadbasket models consisting of multivariate copula distributions as estimated before. The correlations between linearly detrended production (with production as product of yield and area with area held constant at 2012 levels) in the breadbaskets was used to estimate copulas. For the copulas within breadbaskets, the results of pairwise coupling with minimax using the according copula families were again used for all breadbaskets. For the hierarchical structuring, pairwise ordering with minimax was applied and Gaussian copulas were used to estimate the conditional multivariate copula model as they proved to be the best fit. The copula density function for five breadbaskets combined in hierarchical structuring is shown in Equation 14:

\[
c(u_{US}, u_{IN}, u_{ARG}, u_{CH}, u_{AUS}) = c_{US, ARG} (c_{US} | c_{ARG}) \cdot c_{IN, ARG} (c_{IN} | c_{ARG}) \cdot c_{CH, IN} (c_{CH} | c_{IN}) \\
\cdot c_{AUS, CH} (c_{AUS} | c_{CH})
\]

with \( c_{US} = c_{US}(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}) \)

\[
= c_{1,2} (u_2 | u_1) \cdot c_{2,3} (u_3 | u_2) \cdot c_{3,4} (u_4 | u_3) \cdot c_{4,5} (u_5 | u_4) \cdot c_{5,6} (u_6 | u_5) \cdot c_{6,7} (u_7 | u_6) \\
\cdot c_{7,8} (u_8 | u_7) \cdot c_{8,9} (u_9 | u_8) \cdot c_{9,10} (u_{10} | u_9)
\]
\[ c_{ARG} = c_{ARG}(u_{11}, u_{12}, u_{13}, u_{14}) \]
\[ = c_{11,12} (u_{12}|u_{11}) \cdot c_{12,13} (u_{13}|u_{12}) \cdot c_{13,14} (u_{14}|u_{13}) \]
\[ c_{IN} = c_{IN}(u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}, u_{21}, u_{22}) \]
\[ = c_{15,16} (u_{16}|u_{15}) \cdot c_{16,17} (u_{17}|u_{16}) \cdot c_{17,18} (u_{18}|u_{17}) \cdot c_{18,19} (u_{19}|u_{18}) \]
\[ \cdot c_{19,20} (u_{20}|u_{19}) \cdot c_{20,21} (u_{21}|u_{20}) \cdot c_{21,22} (u_{22}|u_{21}) \]
\[ c_{AUS} = c_{AUS}(u_{23}, u_{24}) \]
\[ = c_{23,24} (u_{24}|u_{23}) \]
\[ c_{CH} = c_{CH}(u_{25}, u_{26}, u_{27}, u_{28}, u_{29}) \]
\[ = c_{25,26} (u_{26}|u_{25}) \cdot c_{26,27} (u_{27}|u_{26}) \cdot c_{27,28} (u_{28}|u_{27}) \cdot c_{28,29} (u_{29}|u_{28}) \]

where \( i = 1,2,\ldots,29 \) are the 29 states in five breadbaskets used for this analysis.

The correlations between detrended wheat productions of the five breadbaskets are shown in Table 1. Compared to correlations within the regions, correlations between breadbaskets are found to be quite low.

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>USA</th>
<th>Argentina</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>India</td>
<td>------</td>
<td>0.20*</td>
<td>-0.17</td>
<td>0.20*</td>
</tr>
<tr>
<td>China</td>
<td>------</td>
<td>0.04</td>
<td>-0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>USA</td>
<td>------</td>
<td>0.23*</td>
<td>-0.13</td>
<td></td>
</tr>
<tr>
<td>Argentina</td>
<td>------</td>
<td></td>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>------</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The cumulative production distribution curve for wheat in all five breadbaskets, based on average production in 2012, can be estimated via the numbers above and is shown in Figure 7. Average production for 2012 in the five breadbaskets is estimated 236 million
tons, nearly one third of the total global wheat production in 2012 (671 million tons according to FAOSTAT\textsuperscript{57}), which shows that the breadbaskets used in this analysis are a representative selection for global wheat production. The actual observed production in this area was 244.87 million tons which means that 2012 was a good, above-average year for global wheat production.

![Graph showing wheat production curve](attachment:image.png)

**Figure 7.** Production curve for wheat production in five global breadbaskets.

As we are interested in occurrences of low production rather than high production, we focus again on the lower tail of the distribution. Table 2 shows quantiles and return periods for low production based on Figure 7.

**Table II.** Wheat production in the global breadbaskets.

<table>
<thead>
<tr>
<th>Return period</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantiles</td>
<td>0.2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>Production in million tons</td>
<td>231.21</td>
<td>228.75</td>
<td>226.58</td>
<td>224.25</td>
<td>222.66</td>
<td>221.06</td>
</tr>
</tbody>
</table>
For example, a one in 50 year event would be a production of 224.25 million tons of wheat which is a negative deviation from the mean of 11.75 million tons which is more than the entire wheat production in the Argentinian breadbasket (8.5 million tons). We are now able to answer the question stated at the very beginning of the article, namely if risk pooling is a possible way to reduce production risk on the global or regional level. The advantage of risk pooling on a global level can be shown by the following numerical example. We use the 1% Value at Risk (16), which can be interpreted here as the wheat production which will be exceeded in 99%, as a measure for an extremely poor harvest. Alternative ways of estimating extreme yield losses are described in Ben-Ari et al. (58). If we calculate the 1% VaR for each breadbasket and define the difference from the average production as production loss, the overall, aggregated production loss in all five breadbaskets is 26.59 million tons. However, if we pool risks, the loss from a 1% VaR global wheat production is only 13.4 million tons. This finding shows that risk aggregation is indeed feasible on the global level providing that all breadbaskets are aggregated. However, due to the interdependencies found between states within a breadbasket, risk pooling would be less favorable on the regional level.

5 DISCUSSION

Our results in this study have important implications for risk reduction strategies of wheat production losses, e.g. through insurance or other pooling arrangements. Results for production within and between breadbaskets showed that, while production losses between regions are mostly independent, states within a breadbasket experience systemic production loss risks. A risk is systemic when one of the main conditions for insurability is not satisfied: stochastically independence of risks across insured individuals (59). In agriculture, systemic risk of crop failure comes from geographically
extensive weather extremes such as droughts, which impact a large number of farms across a wide region. Because of systemic risk, crop insurers cannot pool risks across individuals and therefore, crop insurance markets are not efficient and rely on government subsidies (59). One solution, suggested by Quiggin (60), is reinsurance in the international market. However, due to the systemic nature of crop failure risks, the international private insurance and reinsurance industry is unwilling or unable to offer affordable crop insurance (59). Even governments, especially from smaller countries, are sometimes unable to provide post-disaster support (61). A possible solution could be mutual, intergovernmental risk pooling. A recent example is the Caribbean Catastrophe Risk Insurance Facility (CCRIF) which is a cooperation between Caribbean governments, international institutions and donors which can provide immediate liquidity in a case of a hurricane or earthquake (62). Such a mutual risk financing mechanism could be applied to crop insurances as well. As this study has shown, systemic risk of crop failure is a problem within the breadbaskets. Between breadbaskets, however, the independence of yield losses suggests that insurance schemes could be developed for mutual risk financing between the breadbaskets. Through inter-regional risk pooling, post-disaster liabilities of governments and international donors could be decreased. If the global insurance model is well designed, crop insurance might even become cost-effective and premium rates affordable for farmers.

In this study, future wheat yield projections were obtained by extrapolation of a trend which was determined from the time series of yield data. However, there is significant uncertainty around the extrapolation of a statistically determined trend. We consider 20 years to be the limit for credible extrapolation. The logistic trend accounts for technological change which increases yields and improves its resilience to heat stress or plant diseases but does not consider climatic change which has a detrimental impact on
wheat yields in many regions of the world \((63)\). Other possibilities of projecting a yield trend are using results of crop models such as EPIC \((64)\) or LPJml \((65,66)\) which base their yield estimations on plant phenology and future climate scenarios. Especially the consideration of climate impacts on wheat yield is important as many studies are projecting wheat yield losses due to rising temperatures as a consequence of climate change in many regions of the world \((63,67,68)\).

There are limits of interpretation and usefulness of the approach discussed here which need to be discussed. Firstly, data used for this analysis are historical observed wheat data without a differentiation between wheat types such as winter or spring wheat or between irrigated and rain-fed wheat. Owing to limited data availability, this global analysis therefore could not go more into detail about specific crop types as well as specific technological applications in the different breadbaskets. However, for countries in which there are data available on irrigated wheat yields versus rain-fed wheat yields, such as the US, separate analysis of crop loss correlations for rain-fed and irrigated wheat within a breadbasket could lead to new and interesting conclusions. In China, for instance, Wang et al. \((69)\) found a positive economic effect of temperature on irrigated, but negative effect on rain-fed crop producers. In general, underlying causes of wheat yield losses such as droughts or pests and their effects on crop yields could be investigated further to be able to give more concrete policy recommendations. This could be done via explicit global crop modeling approaches which currently focus on average changes rather than on variability of crop production. This has serious shortcomings, as without any risk information measures to reduce risk and especially instruments who will not fail in case of very extreme events cannot be assessed appropriately. Our research can be seen as one possible step towards the idea of global risk pooling of production risk and the pre-conditions that need to be satisfied, focusing
on dependency issues. The use of copulas within such kind of extreme risk assessment is emerging and provides great potential for a more nuanced approach for decision making, e.g., via risk-layering frameworks (70).

It should be also mentioned that we could not explicitly include large climate patterns such as the El Nino Southern Oscillation (ENSO) which affects hydrological processes around the globe and subsequently has influence for natural hazards including hurricanes and droughts. A recent study by Ward et al. (71) showed that climate variability from ENSO can and should be incorporated into disaster risk assessments and policies. As one possible way forward the use of copula approaches for determining yield dependencies within ENSO years may be a useful step forward. This may be especially useful in the light of the predictability of ENSO several seasons in advance, which opens up the opportunity to build and strengthen risk management strategies for these periods. Additionally, for the future projections of crop production and risks, we implicitly assumed that the dependency structure stays the same. While this may stay more or less true in the short run, for the long run this will be not necessarily the case. As was shown in the case of possible future water shortages in the London water system due to climate change, changes in the dependency structure of hydrological variables may have as negative effects as changes in distinct water dimensions alone (72). In our example, a possible change in ENSO occurrences (73,74) and a change of its impact on crop yields (75) might alter the correlation structures. Last but not least, our focus was on crop production and dependencies but we did not incorporated in our analysis global trade networks and subsequent risks. A rich literature on global trade and risks, especially focusing on network effects, now exists (76) and could shed more light on additional threats due to network disruptions.
CONCLUSION

This paper investigated the dependency structure within and between five global wheat breadbaskets based on historic yield data and an advanced modeling approach via copulas. Results showed high correlations within but not very significant correlations between the regions. Through the use of the copula approach it was possible to determine that global risk pooling is feasible even for catastrophe risks. However, within regions systemic risk to crop losses were found, leading to the conclusion that risk pooling would not be advisable on that level, at least if disaster risk has to be tackled, too. Our results open up the possibility to explore risk-pooling mechanisms and mutual risk financing to mitigate systemic risks within breadbaskets in a more detailed fashion, including a risk based approach not yet applied on this level.

The paper explored the use of different structuring approaches as well as different copula families. R-vines were found to be the most accurate structuring and copula estimation approach for this study as the underlying data did not indicate a clear correlation structure. R-vines offer a large variety of copula families which are chosen separately for each copula pair. In that way, complex correlation structures can be modeled in an accurate way. For the estimation of a correlation structure between the five breadbaskets, minimax ordering and the Gaussian copula were chosen. As for pairwise coupling with minimax, there are no goodness-of-fit tests for the entire model available yet, so the accuracy of a model can only be assessed for the bivariate copulas within the multivariate model. Our analysis showed that results are sensitive to the copula structuring approach. Both of the structures fitted the data well according to existing goodness-of-fit tests. Development of improved goodness-of-fit tests could help
to distinguish between alternative structuring approaches so more precisely quantify
the aggregate risk.

To estimate future production curves, the logistic trend was used for extrapolation of
the average yield in each state. In further research, alternative approaches to estimating
future wheat production in the breadbaskets and their correlation structure can be
examined.

The analysis in this paper showed that it is important to include correlation structures in
crop yield risk analysis as otherwise, the risks of production losses are underestimated
which can have severe effects on risk preparedness of governments or crop insurance
schemes. The results of this study are highly relevant for policy makers in the major
wheat producing countries. Especially global risk pooling between the five breadbaskets
could decrease the need for government's post-disaster liabilities and make crop
insurance schemes affordable for farmers.

7 APPENDIX

Generation of a conditional Gumbel copula
The algorithm which generates $u$ conditional on $v$ for the Gumbel copula following
Timonina et al. (47) includes the following steps:

1. $v$ is fixed and equals $v = v_0$
2. $r$ is randomly generated from the interval (0,1)
3. $w = v_0$ is assigned
4. an iteration is done in the following way:

$$ w_{\text{new}} = w - \frac{w \left(- \ln w\right) - rv_0 \left(-\ln v_0\right) \left(\frac{\ln w}{\ln v_0}\right)^\theta}{\theta - 1 - \ln w} $$

while $|w_{\text{new}} - w| > 10^{-6}$
(5) \( u = \exp \left[ -\left( -\ln w_{\text{new}} \right)^{\theta} \left( -\ln v_{0} \right)^{\theta} \right] \) is assigned.

8 REFERENCES

30. FAO. Statistical database. 2015.
32. USDA/NOAA. Joint agricultural weather facility. 2015.
57. FAOSTAT. 2015.


