

Collaborative Paper

A SIMPLE METHOD OF MEASURING THE
INCREASE OF LIFE EXPECTANCY WHEN
A FIXED PERCENT OF DEATHS FROM
CERTAIN CAUSES ARE ELIMINATED

Z. Nanjo

December 1980
CP-80-35

International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

In this paper, Professor Nanjo, from the Fukushima Medical College, Japan, generalizes Keyfitz's method for measuring the increase of life expectancy due to a marginal reduction in any one cause of death. He relaxes Keyfitz's assumption that the number of deaths in each age group is decreased at a fixed rate and goes on to derive a mathematical formulation that leads to an improved approximation.

Recent publications in the Health Care Systems Task are listed at the end of this report.

Andrei Rogers
Chairman
Human Settlements
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ABSTRACT

The effect that one type of medical improvement will have on life expectancies is often computed using a life table. In classical methods, such as Greville's, the increase in life expectancy has been dealt with by assuming that deaths from a particular cause have been eradicated. Keyfitz derived a parameter that measures the increase in life expectancy by a marginal reduction in any cause of death. The parameter is additive in several causes and useful for various studies of causes of death.

This paper is a generalization of Keyfitz's idea and deals with a case where some percent of the deaths from a particular cause are eliminated, not necessarily uniformly in all age intervals.

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A SIMPLE METHOD OF MEASURING THE INCREASE OF
LIFE EXPECTANCY WHEN A FIXED PERCENT OF DEATHS
FROM CERTAIN CAUSES ARE ELIMINATED

1. INTRODUCTION

Life tables are often used in the analysis of the increase in life expectancy when certain death causes are eradicated. With this tool we are able to obtain the difference between the life expectancy for all death causes and the one calculated on the assumption that deaths from a certain death cause have been eliminated. There are several methods for calculating these life tables, among them Greville's methods (Greville, 1948 and 1954), and those of Preston et al. (1972) are well-known. However, when we use these traditional methods, the following points are questioned.

(a) Usually

$$d_A + d_B < d_{A+B}$$

when d_A , d_B , and d_{A+B} denote, respectively, the increase of life expectancy assuming that death causes A, B, and A+B have been eradicated. In this case equality does not hold. That is to say, the increase of life expectancy is not additive with the two causes of death.

- (b) Greville's and other traditional methods discuss the case in which deaths from a certain cause have been eradicated. We are more concerned here with the case in which some percent of the deaths from several causes are decreased. In the former case, for example, one hundredth of an increase in life expectancy, assuming that deaths from cause A have been eradicated, cannot be used as the increase of life expectancy when one percent of deaths from death cause A has been eliminated.
- (c) The traditional methods assume that several death causes are independent.

In our case the assumption in (c) cannot be avoided. Points (a) and (b), however, have been discussed by Keyfitz (1977a). He has derived a parameter that measures how much life expectancy is increased due to a marginal reduction in any cause of death. This parameter is additive when several causes of death are considered.

Our method is a generalization of Keyfitz's idea. When some percent of the deaths from certain causes are eliminated, not necessarily uniformly in all the age intervals, we can easily get the life expectancy, based on a given life table, by using the sets of parameters that have been obtained beforehand. Our method is also additive for several causes and has interesting applicability to the study of causes of death.

2. GENERALIZATION OF KEYFITZ'S RESULTS

To begin with, I will explain briefly Keyfitz's idea (Keyfitz, 1977a,b). If the chance of dying in the time interval dx of a year for a person who has reached age x has been $\mu(x)dx$, suppose that this is changed to $\mu'(x)dx = \mu(x)(1+\delta)dx$. In this case, δ will be a small negative quantity, typically -0.01 , representing 1 percent improvement in all causes at all ages. The probability of living to age x then changes from

$$l(x) = \exp[-\int_0^x \mu(x) dx]$$

to

$$l'(x) = \exp[-\int_0^x \mu(x)(1+\delta) dx] = l(x)^{1+\delta}$$

this being approximately

$$l(x)[1 + \delta \ln l(x)]$$

in the neighborhood of $\delta = 0$ by Taylor's expansion. The life expectancy at birth changes from

$$\overset{\circ}{e}_0 = \int_0^\omega l(x) dx$$

to

$$\overset{\circ}{e}'_0 = \int_0^\omega l'(x) dx$$

Therefore we have

$$\begin{aligned} (\overset{\circ}{e}'_0 - \overset{\circ}{e}_0) / \overset{\circ}{e}_0 &= \int_0^\omega [l'(x) - l(x)] dx / \int_0^\omega l(x) dx \\ &= -\delta \left[\int_0^\omega -l(x) \ln l(x) dx / \int_0^\omega l(x) dx \right] = -\delta H, \end{aligned}$$

where

$$H = -\int_0^\omega l(x) \ln l(x) dx / \int_0^\omega l(x) dx$$

This expression shows that the effect of the elimination of 1 percent of deaths on the life expectancy will increase $\overset{\circ}{e}_0$ by H percent.

Similarly, suppose the age specific death rate from the cause i changes from

$$\mu^{(i)}(x)$$

to

$$\mu^{(i)}(x)(1+\delta)$$

The probability of living to age x then changes from $l(x)$ to

$$l(x)[l^{(i)}(x)]^\delta \doteq l(x)[1 + \delta \ln l^{(i)}(x)]$$

where $l^{(i)}(x)$ is the probability of living to age x in the face of risks from the cause i alone. Therefore, in the same way as we mentioned above, we have

$$(\overset{\circ}{e}_0' - \overset{\circ}{e}_0)/\overset{\circ}{e}_0 = -\delta H^{(i)}$$

where

$$H^{(i)} = -\int_0^{\omega} l(x) \ln l^{(i)}(x) dx / \int_0^{\omega} l(x) dx$$

This shows that the effect of the elimination of 1 percent of deaths from the i -th cause on the life expectancy will increase $\overset{\circ}{e}_0$ by $H^{(i)}$ percent.

Keyfitz's idea is based on an assumption that the number of deaths at each age group is decreased at a fixed rate. This assumption is, however, often unsuitable for real situations. Here, therefore, let us make a general assumption that the rate of decrease in deaths at each age group is not necessarily fixed.

According to Greville, it is known that the probability of living n year after age x in a life table from which the cause i is completely eliminated is closely approximated from

$$n P_x^{(-i)} = n P_x^{1-n \gamma_x^{(i)}}$$

where n^P_x relate to the life table with all causes present, and

$$n^{\gamma}_x^{(i)} = n^{D_x^{(i)}} / n^{D_x}$$

in which n^{D_x} and $n^{D_x^{(i)}}$ represent respectively the number of deaths from all the causes and from the cause i alone in the age group $(x, x+n-1)$. In this case, if the number of deaths from the cause i is not eradicated but decreased by $-100\delta_x\%$, it can be shown that the probability of living in the age interval is

$$n^P_x = \left[1 + n^{\gamma}_x^{(i)} \delta_x \right] \quad (\delta_x < 0)$$

as in Greville's expression (Greville, 1948).

Therefore, if the deaths from the cause i are decreased at the rate of $-\delta_0, -\delta_5, \dots, -\delta_{80}, -\delta_{85}$ for the age interval $0 \sim 4, 5 \sim 9, \dots, 80 \sim 84, 85+$ respectively, the probability of living to age x is

$$\begin{aligned} l'(x) &= \left[5^P_0 \left(1 + 5^{\gamma}_0^{(i)} \delta_0 \right) \right] \left[5^P_5 \left(1 + 5^{\gamma}_5^{(i)} \delta_5 \right) \right] \dots \left[5^P_{x-5} \left(1 + 5^{\gamma}_{x-5}^{(i)} \delta_{x-5} \right) \right] \\ &= \left[5^P_0 \cdot 5^P_5 \cdots 5^P_{x-5} \right] \cdot \left[5^P_0 \left(1 + 5^{\gamma}_0^{(i)} \right)^{\delta_0} \right] \cdots \left[5^P_{x-5} \left(1 + 5^{\gamma}_{x-5}^{(i)} \right)^{\delta_{x-5}} \right] \end{aligned} \quad (1)$$

This is equivalent to Keyfitz's expression, $l(x) [l^{(i)}(x)]^\delta$, if $\delta_0 = \delta_5 = \dots = \delta$. Here, using Taylor's theorem as above to hold linearity, we have

$$\begin{aligned} l'(x) &\doteq 5^P_0 5^P_5 \cdots 5^P_{x-5} \left(1 + \delta_0 \ln 5^P_0 \left(1 + 5^{\gamma}_0^{(i)} \right) \right) \left(1 + \delta_5 \ln 5^P_5 \left(1 + 5^{\gamma}_5^{(i)} \right) \right) \cdots \\ &\quad \left(1 + \delta_{x-5} \ln 5^P_{x-5} \left(1 + 5^{\gamma}_{x-5}^{(i)} \right) \right) \\ &\doteq (5^P_0 \cdots 5^P_{x-5}) \left(1 + \delta_0 \ln 5^P_0 \left(1 + 5^{\gamma}_0^{(i)} \right) + \dots + \delta_{x-5} \ln 5^P_{x-5} \left(1 + 5^{\gamma}_{x-5}^{(i)} \right) \right) \end{aligned} \quad (2)$$

This is a closely approximated expression when each δ_k is small.

Thus

$$\overset{\circ}{e}'_0 - \overset{\circ}{e}_0 = \int_0^{\omega} l'(x) dx - \int_0^{\omega} l(x) dx \quad (3)$$

can be expressed in the linear form

$$(-\delta_0) c_0^{(i)} + (-\delta_5) c_5^{(i)} + \dots + (-\delta_{85}) c_{85}^{(i)} \quad (3a)$$

where $c_x^{(i)}$ can be calculated from equation (2). Therefore, from this expression we can obtain the quantity of increase in the life expectancy at birth $\overset{\circ}{e}'_0 - \overset{\circ}{e}_0$ at once, if we have coefficients $c_0^{(i)}, \dots, c_{85}^{(i)}$ for the cause of death i beforehand.

3. OUR METHOD OF COMPUTATION

For $x=0, 5, 10, \dots$ we denote $l(x)$, $l'(x)$ by l_x , l'_x respectively, and then write

$$5d_x = l_x - l_{x+5} \quad (4)$$

$$\beta_x = \gamma_x \ln 5^P_x = 5 \gamma_x^{(i)} \delta_x \ln 5^P_x \quad (5)$$

$$l'_x = l_x (1 + \beta_0 + \beta_5 + \dots + \beta_{x-5}) \quad [\text{cf. (2)}] \quad (6)$$

$$5d'_x = l'_x - l'_{x+5} \quad (7)$$

Let nL_x be the total person-years lived by the stationary population in the age-interval x to $x+n$. Then

$$nA_x = \frac{nL_x - nl_{x+n}}{n d_x} \quad (8)$$

is the average number of years lived in the age-interval x to $x+n$ by those who die in it. Using this n^A_x , we estimate $n^{L'}_x$, as is often done, by

$$n^{L'}_x = n l'_{x+n} + n^A_x \cdot n d'_x \quad (9)$$

Here for the sake of brevity, we use A_x , d_x , d'_x , $\gamma_x^{(i)}$, L_x and L'_x in place of ${}_5A_x$, ${}_5d'_x$, ${}_5\gamma_x^{(i)}$, ${}_5L_x$, and ${}_5L'_x$. In the last age-interval, we use the age-interval of ages over x . For example, $L_{100} = {}_\infty L_{100}$. Then we have

$$\begin{aligned} L'_0 &= 5l'_5 + A_0 d'_0 = 5l_5 (1+\beta_0) + A_0 \{ l_0 - l_5 (1+\beta_0) \} \\ &= 5l_5 (1+\beta_0) + A_0 (d_0 - l_5 \beta_0) \quad (\because l_0 = l'_0) \end{aligned}$$

Similarly

$$\begin{aligned} L'_5 &= 5l'_{10} + A_5 d'_5 = 5l_{10} (1+\beta_0 + \beta_5) + A_5 \{ d_5 (1+\beta_0) - l_{10} \beta_5 \} \\ L'_{10} &= 5l'_{15} + A_{10} d'_{10} = 5l_{15} (1+\beta_0 + \beta_5 + \beta_{10}) + A_{10} \{ d_{10} (1+\beta_0 + \beta_5) \\ &\quad - l_{15} \beta_{10} \} \\ \dots\dots \\ L'_{95} &= 5l'_{100} + A_{95} d'_{95} = 5l_{100} (1+\beta_0 + \dots + \beta_{95}) \\ &\quad + A_{95} \{ d_{95} (1+\beta_0 + \dots + \beta_{90}) - l_{100} \beta_{95} \} \end{aligned} \quad (10)$$

In the last age-interval

$$L'_{100} \doteq L_{100} (1+\beta_0 + \dots + \beta_{95}) / (1+\gamma_{100})$$

This is equivalent to Greville's expression (Greville, 1954) used to get the life expectancy \bar{e}_x at age x in the last age-interval (x, ∞) when a death cause has been eradicated. This

is approximately equal to

$$\begin{aligned}
 & L_{100} (1 + \beta_0 + \dots + \beta_{95}) (1 - \gamma_{100}) \\
 = & L_{100} \left\{ (1 + \beta_0 + \dots + \beta_{95}) - \gamma_{100} - \gamma_{100} (\beta_0 + \dots + \beta_{95}) \right\} \\
 \doteq & L_{100} (1 + \beta_0 + \dots + \beta_{95}) - \gamma_{100} L_{100}
 \end{aligned} \tag{11}$$

if γ_k ($k = 0, 5, 10, \dots$) is small.

Therefore we have

$$\begin{aligned}
 L'_0 - L_0 &= 5l_5(\beta_0) + A_0(-l_5\beta_0) \\
 &= (5-A_0)l_5\beta_0
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 L'_5 - L_5 &= 5l_{10}(\beta_0 + \beta_5) + A_5 \{ d_5\beta_0 - l_{10}\beta_5 \} \\
 &= L_5\beta_0 + 5l_{10}\beta_5 - A_5l_{10}\beta_5 \\
 &= L_5\beta_0 + (5-A_5)l_{10}\beta_5
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 L'_ {10} - L_{10} &= 5l_{15}(\beta_0 + \beta_5 + \beta_{10}) + A_{10} \{ d_{10}(\beta_0 + \beta_5) - l_{15}\beta_{10} \} \\
 &= L_{10}(\beta_0 + \beta_5) + 5l_{15}\beta_{10} - A_{10}l_{15}\beta_{10} \\
 &= L_{10}(\beta_0 + \beta_5) + (5-A_{10})l_{15}\beta_{10} \\
 &\dots
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 L'_{95} - L_{95} &= 5l_{100}(\beta_0 + \dots + \beta_{95}) + A_{95} \{ d_{95}(\beta_0 + \dots + \beta_{90}) - l_{100}\beta_{95} \} \\
 &= L_{95}(\beta_0 + \dots + \beta_{90}) = (5-A_{95})l_{100}\beta_{95}
 \end{aligned} \tag{15}$$

and

$$L'_{100} - L_{100} = L_{100}(\beta_0 + \dots + \beta_{95}) - \gamma_{100}L_{100} \tag{16}$$

Sum up both sides of the expressions (12) to (16) and divide the total by l_0 . Then if we use

$$T_x = L_x + L_{x+5} + \dots, \quad T'_x = L'_x + L'_{x+5} + \dots,$$

we obtain

$$\begin{aligned} {}^o e'_0 - {}^o e_0 &= \frac{1}{l_0} (T'_0 - T_0) \\ &= \frac{1}{l_0} \left[\beta_0 \{T_5 + (5-A_0) l_5\} \right. \\ &\quad + \beta_5 \{T_{10} + (5-A_5) l_{10}\} \\ &\quad + \beta_{10} \{T_{15} + (5-A_{10}) l_{15}\} + \dots \\ &\quad \left. + \beta_{95} \{T_{100} + (5-A_{95}) l_{100}\} - \gamma_{100} T_{100} \right] \end{aligned} \quad (17)$$

If we write

$$C_x^{(i)} = -\frac{\gamma_x^{(i)} l_{n5} P_x}{l_0} \{T_{x+5} + (5-A_x) l_{x+5}\} \quad (x=0, 5, 10, \dots, 95) \quad (18)$$

$$C_{100}^{(i)} = \gamma_{100}^{(i)} T_{100} / l_0$$

and if we refer to the expression (5), we obtain

$${}^o e'_0 - {}^o e_0 = (-\delta_0) C_0^{(i)} + (-\delta_5) C_5^{(i)} + \dots + (-\delta_{100}) C_{100}^{(i)} \quad (19)$$

In this expression, if $C_{85}^{(i)} + C_{90}^{(i)} + \dots + C_{100}^{(i)}$ is denoted by $C_{85}^{(i)}$ we obtain the expression (3a) in section 2. In the case of ${}^o e'_{60} - {}^o e_{60}$, as in the case of age 0, we start from l_{60} for age 60 and obtain

$$\begin{aligned}
 {}^{\circ}e'_{60} - {}^{\circ}e_{60} = & \frac{1}{l_{60}} [\beta_{60} \{T_{65} + (5-A_{60}) l_{65}\} \\
 & + \beta_{65} \{T_{70} + (5-A_{65}) l_{70}\} + \dots \\
 & + \beta_{95} \{T_{100} + (5-A_{95}) l_{100}\} - \gamma_{100} T_{100}] \tag{20}
 \end{aligned}$$

Therefore if we write

$$\begin{aligned}
 c_x^{(i)} = & -\frac{\gamma_x^{(i)} \ln 5 P_x}{l_{60}} \{T_{x+5} + (5-A_x) l_{x+5}\} \quad (x=60, 65, \dots, 95) \\
 c_{100}^{(i)} = & \gamma_{100}^{(i)} T_{100} / l_{60} \tag{21}
 \end{aligned}$$

we obtain

$${}^{\circ}e'_{60} - {}^{\circ}e_{60} = (-\delta_{60}) c_{60}^{(i)} + \dots + (-\delta_{100}) c_{100}^{(i)} \tag{22}$$

Now if the coefficients $c_x^{(i)}$ are computed for $x = 0, 5, \dots, 85$, or $x = 60, 65, \dots, 85$, it seems to serve our purpose sufficiently. However, there is some doubt about the expression in the treatment of the last coefficient:

$$c_{85}^{(i)} = \gamma_{85}^{(i)} T_{85} / l_0, \text{ or } c_{85}^{(i)} = \gamma_{85}^{(i)} T_{85} / l_{60}.$$

Therefore we computed $c_x^{(i)}$ to as advanced an age as possible, that is, to 100 years of age as shown above. In this case, it should be noted that $c_{100}^{(i)}$ is extremely small. In Japan, the data necessary for this computation are available.

In its practical use, we can use the table that summed up the figures of ages 85 and over from the table calculated in the above-mentioned way (cf. Table in Section 5 and Appendix tables).

4. RELATION BETWEEN OUR RESULTS AND THE RESULTS OF KEYFITZ

According to Keyfitz (1968, p.342), it is known that the effect on $\overset{\circ}{e}_0$ of $\Delta_{5q_x} = 5q'_x - 5q_x$ when the probability of dying in the age-interval $(x, x+4)$ changed from $5q_x$ to $5q'_x$ is approximately

$$\overset{\circ}{e}'_0 - \overset{\circ}{e}_0 \doteq - \frac{1_x}{l_0} (\overset{\circ}{e}_{x+5} + 5 - A_x) \Delta_{5q_x} \quad (23)$$

And if we take out only an age-interval $(x, x+4)$ from the expression (17), we obtain

$$\overset{\circ}{e}'_0 - \overset{\circ}{e}_0 = - \frac{1}{l_0} \beta_x \{ T_{x+5} + (5 - A_x) l_{x+5} \} \quad (24)$$

$$= \frac{1}{l_0} \beta_x l_{x+5} (\overset{\circ}{e}_{x+5} + 5 - A_x)$$

The expression (24) is one which was obtained by quite a different idea from (23), but both of them give much the same result. This is shown in the following way. If $\delta_x \gamma_x^{(i)}$ is small

$$5^{P_x} \overset{1+\delta_x \gamma_x^{(i)}}{=} 5^{P_x} (1 + \delta_x \gamma_x^{(i)} \ln 5^{P_x}) \quad (25)$$

Therefore, if we denote $5^{P_x} - 5^{P_x}$ by Δ_{5P_x} , we obtain from the expression (25)

$$\Delta_{5P_x} = \delta_x \gamma_x^{(i)} 5^{P_x} \ln 5^{P_x}$$

i.e.

$$-\Delta_{5q_x} = \delta_x \gamma_x^{(i)} \frac{l_{x+5}}{l_x} \ln 5^{P_x} \quad (\because 5^{P_x} = 1 - 5^{q_x})$$

therefore

$$-1_x \Delta_5 q_x = \delta_x \gamma_x^{(i)} \log 5 P_x l_{x+5} = \beta_x l_{x+5}$$

so we get our result.

5. AN APPLICATION AND COMMENTS

As an example, let me show part of the table (Table 1) of $c_k^{(i)}$ for males, which is taken out of the Table A2 (Appendix) computed by our method, using the life table and the statistics of mortality of Japan for 1970.

Table 1. The coefficients $c_k^{(i)}$ for malignant neoplasms and cerebrovascular disease on Japanese males, 1970

Start of age interval	coefficients	B19 ^a	Causes of death B30 ^b
Total	C	2.02632	2.46930
0	c_0	0.02698	0.00530
5	c_5	0.01705	0.00148
10	c_{10}	0.01338	0.00158
15	c_{15}	0.01855	0.00352
20	c_{20}	0.02115	0.00448
25	c_{25}	0.02680	0.00958
30	c_{30}	0.03874	0.01798
35	c_{35}	0.05910	0.04426
40	c_{40}	0.08999	0.07757
45	c_{45}	0.13889	0.10699
50	c_{50}	0.19812	0.16027
55	c_{55}	0.26542	0.23360
60	c_{60}	0.31843	0.33045
65	c_{65}	0.32470	0.41371
70	c_{70}	0.25360	0.44192
75	c_{75}	0.14763	0.35570
80	c_{80}	0.05458	0.18977
85	c_{85}	0.01322	0.07116

^aB19: Malignant Neoplasms

^bB30: Cerebrovascular Disease

According to the table above, if the number of deaths or death rate from B19 at all ages is decreased by 3 percent, we have only to multiply C by 0.03 to get the increment of $\overset{\circ}{e}_0$:
 $2.02632 \times 0.03 = 0.061$. As another example, if the death rate from B19 in 1970 is decreased by 4 percent at ages over 50 and by 2 percent at the other ages, $0.02 \times (C_0 + \dots + C_{45}) + 0.04 \times (C_{50} + \dots + C_{85}) = 0.072$, that is, $\overset{\circ}{e}_0$ will increase by 0.072 of a year.

And if the death rate from B30 decreases by 3 percent at ages over 60,

$$0.03 \times (C_{60} + C_{65} + \dots + C_{85}) = 0.054$$

that is, $\overset{\circ}{e}_0$ will increase by 0.054 of a year. If we want to take these two cases together, we have only to sum up the two results above, so we have

$$0.072 + 0.054 = 0.126 \text{ year}$$

In our computation we also assume the independence of death causes as do the traditional methods. [Assumption (c) in section 1]. And the effect of reduction of deaths from a death cause upon the life expectancy is extremely small. If the assumption of independence is not built up, the effect will become still smaller. In fact, however, the independence does not exist.

Keyfitz's and our methods should be used for the case of marginal reduction but not for that of the eradication of deaths from a particular cause. However, if these methods are to be used for the latter case, the effect of the eradication of deaths from a cause upon the life expectancy obtained by Keyfitz's method will be a little smaller than that of Greville's method, and the effect obtained by our method will be very much smaller than that of Keyfitz's method. If we take the assumption of independence into account, we can consider our result to be the upper limit of the effect of the elimination of deaths from a cause upon the expectation of life.

6. CONCLUSION

It has been said that the traditional methods are inadequate for measuring the effect of the marginal reduction of deaths from a particular cause upon the expectation of life. We generalized Keyfitz's method, which was devised to improve these methods. By our method we can easily calculate the effect of some percent elimination of deaths from a death cause in any age-interval upon life expectancy. And the appended tables computed for this will be of much use for the study of main death causes. By means of our method, also, we can easily get the parameters which are equivalent to Keyfitz's parameter $H^{(i)}$ for ages over 0 and $H_{60}^{(i)}$ for ages over 60.

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APPENDIX

Table A1. A list of 12 causes of death according to the international classification of diseases.

B List Number	Detailed list number	
B 4	008,009	Enteritis and other diarrhoeal diseases
B 46 e	535,561,563	Gastritis, duodenitis and chronic gastro-enteritis
B 5	010-012	Tuberculosis of respiratory system
B 6	013-019	Other tuberculosis, including late effects
B 19	140-209	Malignant neoplasms, including neoplasms of lymphatic and haematopoietic tissue
B 26	393-398	Chronic rheumatic heart disease
B 28	410-414	Ischaemic heart disease
B 29	420-429	Other forms of heart disease
B 27	400-404	Hypertensive disease
B 30	430-438	Cerebrovascular disease
B 32	480,486	Pneumonia
B 33 a	490,491	Bronchitis
B 46 d	466	Acute bronchitis and bronchiolitis
B 37	571	Cirrhosis of liver
B 38	580,584	Nephritis and nephrosis
B 45 a	794	Senility without mention of psychosis
B E 47	E810,E823	Motor vehicle accidents
B E 48	E800-E807, E825,E949	All other accidents
B E 49	E950,E959	Suicide

Notes on Tables A2, A3, A4, and A5

1. Data are Complete Life Tables and Death Statistics published by Department of Statistics and Information, Ministry of Health and Welfare, Japan.
2. Causes of death are based on International B List Number.
(cf. Table A1)
3. Each figure in the row of Total is the sum of figures for age 0 to 100+ in the corresponding column.
4. Each figure in the row of 85+ is the sum of figures for age 85 to 100+ in the corresponding column.
5. Each figure in the row of G-Method shows the increment of δe_0 by Greville's method in case of a cause being eradicated, and each figure should be multiplied by 10^2 .
6. Each figure in the last column is the sum of figures in the corresponding row and should be used for a small quantity of change in the death rate from all causes of death.

Table A2. The effect of 1% change in number of deaths by age-interval
and by cause of death on \bar{e}_0 , Japanese Males, 1970.
(This should be used for a small quantity of change)

(x 10⁻²)

Causes of death	B26			B32, B33 a			BE47			A11		
	B4,	B5, B6	B19	B27	B30	B46 d	B37	B38	B45 a	BE48	BE49	Others causes
Total (All)	0.11617	0.33527	2.02632	1.21226	0.19930	2.46930	0.56414	0.29076	0.16577	0.22772	1.45295	0.33354
0	0.04944	0.00177	0.02698	0.01589	0.0	0.00530	0.15624	0.00085	0.00212	0.0	0.18527	0.0
5	0.00133	0.00044	0.01705	0.00245	0.0	0.00148	0.00927	0.00037	0.00422	0.0	0.09369	0.0
10	0.00063	0.00079	0.01538	0.00446	0.00007	0.00158	0.00511	0.0058	0.00460	0.0	0.04367	0.00288
15	0.00056	0.00169	0.01655	0.00972	0.00045	0.00352	0.00637	0.00061	0.00622	0.0	0.15719	0.02201
20	0.00105	0.00315	0.02115	0.01434	0.00046	0.00446	0.00548	0.00091	0.01174	0.0	0.15020	0.04765
25	0.00193	0.00455	0.02680	0.02108	0.00146	0.00958	0.00526	0.00225	0.01474	0.0	0.11548	0.04623
30	0.00103	0.00192	0.01674	0.02651	0.00144	0.01798	0.00485	0.00840	0.01164	0.0	0.10066	0.03429
35	0.00124	0.01629	0.05910	0.03647	0.00364	0.04426	0.00756	0.02329	0.01106	0.0	0.09823	0.03161
40	0.00172	0.02460	0.08999	0.04756	0.00456	0.07737	0.00948	0.03165	0.01106	0.0	0.09751	0.04110
45	0.00179	0.03173	0.15469	0.05451	0.00586	0.10699	0.01296	0.03195	0.01106	0.0	0.09347	0.05028
50	0.00230	0.03555	0.19612	0.08111	0.01642	0.16027	0.01674	0.03794	0.01231	0.0	0.09593	0.06706
55	0.00344	0.03910	0.25542	0.11039	0.01538	0.23300	0.02256	0.04664	0.01263	0.0	0.09210	0.11169
60	0.00526	0.04327	0.31843	0.14592	0.02077	0.33045	0.03975	0.03631	0.01234	0.0	0.13637	0.13637
65	0.00723	0.04720	0.32470	0.16730	0.03044	0.41371	0.05687	0.02935	0.01234	0.0	0.05894	0.19055
70	0.00953	0.03755	0.25550	0.17319	0.03573	0.44192	0.06645	0.02221	0.01066	0.0	0.16755	0.16755
75	0.01220	0.02343	0.14703	0.15048	0.03453	0.35570	0.06531	0.01453	0.02507	0.0	0.03514	0.13492
80	0.01002	0.00977	0.09639	0.09639	0.02276	0.16977	0.04396	0.06642	0.06503	0.0	0.07052	0.13957
85	0.00514	0.01220	0.01165	0.04046	0.00946	0.05970	0.01951	0.00153	0.00186	0.0	0.07149	0.01090
90	0.00131	0.00220	0.00126	0.00910	0.00166	0.01059	0.00444	0.00035	0.00060	0.0	0.00129	0.00393
95	0.00011	0.0002	0.00009	0.00010	0.00085	0.00038	0.00001	0.00003	0.00014	0.0	0.00016	0.00568
100	0.0	0.0	0.00003	0.00013	0.00013	0.00010	0.0	0.00015	0.00001	0.0	0.00010	0.00053
85+	0.00656	0.00248	0.01322	0.01124	0.00113	0.02242	0.00179	0.00224	0.00149	0.00465	0.0149	0.03614
C-Method	0.11687	0.31924	2.19353	1.31925	0.20326	2.98977	0.58113	0.29322	0.16640	0.27232	1.47884	0.33331

Table A3 . The effect of 1% change in number of deaths by age-interval
and by cause of death on \bar{Q}_{60} , Japanese Males, 1970.
(This should be used for a small quantity of change)

($\times 10^{-2}$)

Total	0.06299	0.20303	1.37040	0.97222	0.7283	2.23590	0.37031	0.13719	0.05237	0.23188	0.22100	0.07606	0.99197	7.18716
Age:														
60	0.00652	0.05367	0.39495	0.18098	0.02576	0.40980	0.04930	0.01530	0.00265	0.07310	0.02443	0.20472	1.48628	
65	0.00697	0.05655	0.40272	0.20750	0.03775	0.51312	0.07053	0.01497	0.00958	0.02641	0.05759	0.23202	1.67072	
70	0.01182	0.04657	0.31494	0.21481	0.04431	0.54612	0.08492	0.02755	0.01322	0.03109	0.04359	0.1539	0.62513	
75	0.01513	0.02906	0.18314	0.18664	0.04283	0.41117	0.08100	0.01602	0.00983	0.06717	0.02743	0.0975	0.17535	1.26949
80	0.01242	0.01211	0.06769	0.11955	0.02823	0.23537	0.05452	0.00756	0.00526	0.08867	0.01352	0.00424	0.16223	0.75277
85	0.00637	0.00273	0.01470	0.05019	0.01173	0.07429	0.02395	0.00150	0.00231	0.06070	0.00493	0.00150	0.03694	0.29254
90	0.00163	0.00032	0.00156	0.01129	0.00209	0.01269	0.00550	0.00031	0.00044	0.1967	0.00074	0.00022	0.00705	0.06571
95	0.00013	0.00002	0.00011	0.00110	0.0013	0.00106	0.00047	0.00011	0.0004	0.0216	0.0005	0.0002	0.00653	0.00653
100	0.0	0.0	0.00003	0.00016	0.0	0.00002	0.00012	0.0	0.0	0.00019	0.00002	0.0	0.00012	0.0006

85+ 0.00814 0.00307 0.01640 0.06273 0.01394 0.08626 0.00004 0.00222 0.00278 0.08271 0.00577 0.00185 0.04482 0.36273

Table A4. The effect of 1% change in number of deaths by age-interval
and by cause of death on \mathbb{Q}_0 , Japanese females, 1970.
(This should be used for a small quantity of change)

(x 10⁻²)

Causes of death	B26			B32			BE47			All				
	B4, B46 e	B5, B6	B19	B28 B29	B27	B30	B33 a B46 d	B37	B38	B45 a BE48	BE49	Others causes		
Total (A52)	0.15153	0.16936	1.93511	1.26074	0.26262	2.42250	0.55871	0.14076	0.16928	0.50540	0.50014	0.29228		
0	0.03985	0.00306	0.92313	0.01518	0.0	0.00325	0.15152	0.06130	0.00187	0.0	0.13505	0.0		
5	0.00176	0.00667	0.01399	0.00216	0.0	0.00692	0.00989	0.00625	0.00235	0.0	0.03972	0.0		
10	0.00057	0.00236	0.00428	0.0	0.00137	0.00670	0.00549	0.00339	0.0	0.01243	0.0	0.04768	0.11941	
15	0.00088	0.00101	0.01555	0.00676	0.0	0.00305	0.01613	0.00551	0.00872	0.0	0.02263	0.0	0.03452	0.07858
20	0.00076	0.00267	0.01938	0.01118	0.00086	0.00463	0.00529	0.00055	0.00871	0.0	0.00690	0.0	0.01947	0.12407
25	0.00134	0.00581	0.03253	0.01470	0.00077	0.00586	0.00663	0.00139	0.00930	0.0	0.01783	0.0	0.04324	0.17957
30	0.00175	0.01659	0.05240	0.01908	0.00108	0.01059	0.00553	0.00218	0.01018	0.0	0.01556	0.0	0.06600	0.19877
35	0.00150	0.01517	0.01944	0.02552	0.00152	0.01784	0.00726	0.00492	0.01114	0.0	0.01943	0.0	0.02078	0.2026
40	0.00155	0.01860	0.01813	0.0325	0.00562	0.00562	0.00942	0.00691	0.01583	0.0	0.02065	0.0	0.02173	0.24534
45	0.00207	0.01913	0.01044	0.04835	0.00459	0.00475	0.00945	0.01121	0.01206	0.0	0.01790	0.0	0.01719	0.34659
50	0.00211	0.01932	0.01973	0.06352	0.00815	0.01250	0.01342	0.01168	0.02212	0.0	0.01906	0.0	0.01742	0.4507
55	0.00448	0.01664	0.23353	0.08515	0.01042	0.16400	0.01935	0.01321	0.00097	0.0	0.02282	0.0	0.01556	0.57543
60	0.00476	0.01750	0.20198	0.1455	0.02160	0.25009	0.02912	0.02066	0.01370	0.0	0.02436	0.0	0.01683	0.71727
65	0.00424	0.01554	0.24627	0.15206	0.03111	0.22331	0.04255	0.01774	0.01634	0.0	0.01610	0.0	0.01719	0.13930
70	0.01415	0.01693	0.22331	0.16533	0.06740	0.45453	0.06884	0.01761	0.01507	0.0	0.03701	0.0	0.01657	0.10579
75	0.02212	0.01223	0.15410	0.19779	0.05648	0.46604	0.09947	0.01327	0.01169	0.0	0.02594	0.0	0.01669	0.18925
80	0.02292	0.01595	0.05651	0.16087	0.04777	0.31467	0.05817	0.03714	0.00807	0.0	0.02385	0.0	0.01292	0.17911
85	0.01513	0.0213	0.01795	0.08620	0.02205	0.12846	0.03370	0.0213	0.01532	0.0	0.01561	0.0	0.01714	0.1677
90	0.00462	0.00633	0.0258	0.02619	0.00496	0.02922	0.01041	0.00633	0.01193	0.0	0.00557	0.0	0.01634	0.51454
95	0.00033	0.0	0.0054	0.00594	0.00345	0.00165	0.00165	0.00011	0.00117	0.0	0.00019	0.0	0.00015	0.00015
100	0.00015	0.0	0.00010	0.00010	0.00049	0.00019	0.00024	0.0	0.0003	0.0	0.0003	0.0	0.0003	0.0003
85+	0.02073	0.00246	0.02097	0.11682	0.02761	0.16161	0.04659	0.02448	0.00468	0.0	0.0273	0.0	0.00282	0.6944
3-Method	0.15323	0.19016	2.03905	1.37906	0.26904	2.88845	0.57280	0.14138	0.16986	0.67131	0.50344	0.29556	2.46367	11.73502

Table A5. The effect of 1% change in number of deaths by age-interval
and by cause of death on Oe_{60} , Japanese females, 1970.
(This should be used for a small quantity of change)

(x 10^{-2})

	Total	0.10569	0.08579	1.11139	1.05828	0.26384	2.27268	0.34898	0.08906	0.07558	0.57373	0.14178	0.08301	0.99863	7.203842
Age	60	0.60541	0.51968	0.29798	0.13929	0.02457	0.28445	0.05312	0.02282	0.01556	0.02336	0.02771	0.01955	0.15843	1.04193
65	0.00937	0.02109	0.28238	0.17297	0.03538	0.39945	0.04616	0.02016	0.01551	0.01149	0.02913	0.02112	0.16920	1.25543	
70	0.01509	0.02153	0.25399	0.21420	0.05391	0.51048	0.06919	0.02603	0.01715	0.01150	0.02910	0.01598	0.21525	1.46891	
75	0.02516	0.01391	0.17527	0.22496	0.06424	0.53007	0.07900	0.01502	0.01352	0.01150	0.02695	0.01367	0.20122	1.49503	
80	0.02507	0.00677	0.07792	0.13298	0.05433	0.55791	0.08912	0.00812	0.00918	0.01754	0.01788	0.01418	0.13383	1.13383	
85	0.01721	0.01243	0.02342	0.09805	0.02508	0.14611	0.05642	0.00433	0.00577	0.01277	0.00812	0.00264	0.6781	0.54523	
90	0.00549	0.00338	0.00293	0.02979	0.00564	0.03323	0.01164	0.00338	0.00635	0.00312	0.00219	0.00051	0.01415	0.17530	
95	0.00971	0.0	0.00038	0.00048	0.00057	0.00392	0.00188	0.00061	0.00013	0.00010	0.00017	0.00009	0.00239	0.22740	
100	0.00016	0.0	0.00012	0.00056	0.00012	0.00056	0.00061	0.0	0.00059	0.00199	0.00009	0.0	0.00050	0.13450	
85+	0.02558	0.02280	0.02335	0.13287	0.03140	0.18382	0.05275	0.02282	0.01643	0.23058	0.01052	0.00520	0.09335	0.79329	

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