Construction of a guiding positional strategy using program packages method for a closed-loop guidance problem by a fixed time

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Problems with incomplete information - an approach by Yu. S. Osipov, A. V. Kryazhimskiy

«The problem of constructing optimal closed-loop control strategies under uncertainty is one of the key problems of the mathematical control theory. Its solution would give a new impetus to the theory's development and create the foundation for its new applications.» Arkady Kryazhimskiy (2013)

- Yu. S. Osipov. *Control Packages: An Approach to Solution of Positional Control Problems with Incomplete Information.* Usp. Mat. Nauk 61:4 (2006), 25–76.
- A. V. Kryazhimskiy, Yu. S. Osipov. *Idealized Program Packages and Problems of Positional Control with Incomplete Information*. Trudy IMM UrO RAN 15:3 (2009), 139–157.
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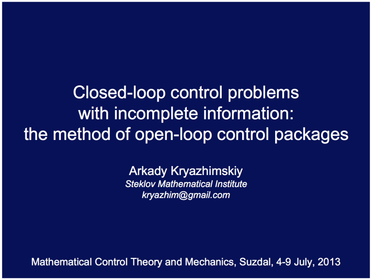


Guaranteed positional guidance problem **at** the (pre-defined) time

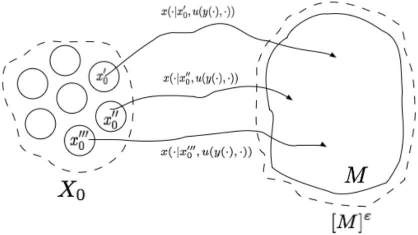
$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + c(t), t_0 \le t \le \vartheta \quad (1)$$

Open-loop control (program) $u(\cdot)$ is measurable.

$u(t) \in P \subset \mathbb{R}^r$, P is a convex compact set

$x(t_0) = x_0 \in X_0 \subset \mathbb{R}^n$, X_0 is a **finite** set

$x(\vartheta) \in M \subset \mathbb{R}^n$, M is a **closed and convex** set



Observed signal $y(t) = Q(t)x(t)$, $Q(\cdot) \in \mathbb{R}^{q \times n}$ is left piecewise continuous

Problem statement

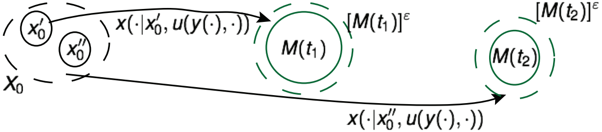
Based on the given arbitrary $\varepsilon > 0$ choose a closed-loop control strategy with memory, **whatever the system's initial state** x_0 **from the set** X_0, *the system's motion $x(\cdot)$ corresponding to the chosen closed-loop strategy and starting at the time t_0 from the state x_0 reaches the state $x(\vartheta)$ belonging to the ε-neighbourhood of the target set M at the time ϑ.*

Guaranteed positional guidance problem **by** the (pre-defined) time

$W \subset (t_0, \vartheta]$ is a given finite set of **admissible guidance times (AGT)**, and for each $t \in W$ a convex closed non-empty **target** set $M(t) \subset \mathbb{R}^n$ is given.

Problem statement

Based on an arbitrary given $\varepsilon > 0$ it is required to construct such a closed-loop strategy, that for any admissible initial state $x_0 \in X_0$ the motion $x(\cdot)$ of the system (1), starting from this state at the time t_0 and being driven by the constructed strategy, is guided on the ε-neighbourhood of the target set $M(t_{x_0})$ at some time $t_{x_0} \in W$



Homogeneous signals

Homogeneous system, corresponding to (1)

$$\dot{x}(t) = A(t)x(t)$$

For each $x_0 \in X_0$ its solution is given by the Cauchy formula:

$$x(t) = F(t, t_0)x_0;\ F(t, s)\ (t, s \in [t_0, \vartheta])\text{ is the fundamental matrix.}$$

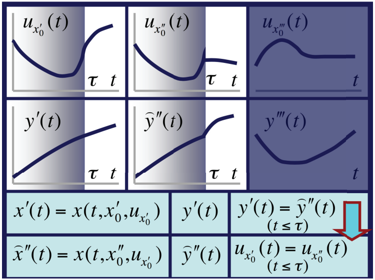
Homogeneous signal, corresponding to an admissible initial state $x_0 \in X_0$:

$$g_{x_0}(t) = Q(t)F(t, t_0)x_0\ (t \in [t_0, \vartheta],\ x_0 \in X_0).$$

Let $G = \{g_{x_0}(\cdot) | x_0 \in X_0\}$ be the set of all homogeneous signals and let $X_0(\tau | g(\cdot))$ be the set of all admissible initial states $x_0 \in X_0$, corresponding to the homogeneous signal $g(\cdot) \in G$ *till time point* $\tau \in [t_0, \vartheta]$:

$$X_0(\tau | g(\cdot)) = \{x_0 \in X_0 : g(\cdot)|_{[t_0, \tau]} = g_{x_0}(\cdot)|_{[t_0, \tau]}\}.$$

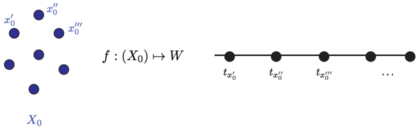
Program package is an open-loop controls family $(u_{x_0}(\cdot))_{x_0 \in X_0}$, satisfying **non-anticipatory condition**: for any homogeneous signal $g(\cdot)$, any time $\tau \in (t_0, \vartheta]$ and any admissible initial states $x_0', x_0'' \in X_0(\tau | g(\cdot))$ the equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in [t_0, \tau]$.



A program package $(u_{x_0}(\cdot))_{x_0 \in X_0}$ **is guiding by the time**, if for any $x_0 \in X_0$ there is $t_{x_0} \in W$ such, that $x(t_{x_0} | x_0, u_{x_0}(\cdot)) \in M(t_{x_0})$. **Package guidance problem by the time** is solvable, if there exists guiding by the time program package.

Package guidance problem with a family of AGT

Admissible guidance times (AGT) family is an arbitrary family $\omega = (t_{x_0})_{x_0 \in X_0}$ of the elements of the set W.
Program package $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **guiding with the AGT family** $\omega = (t_{x_0})_{x_0 \in X_0}$, if for any $x_0 \in X_0$ holds $x(t_{x_0} | x_0, u_{x_0}(\cdot)) \in M(t_{x_0})$.
Package guidance problem with the AGT family ω is solvable, is there exists a program package, guiding with the AGT family ω.



Lemma 1

1) Program package is guiding by the time if and only if it is guiding with some AGT family.
2) Package guidance problem by the time is solvable if and only if a package guidance problem is solvable with some AGT family.

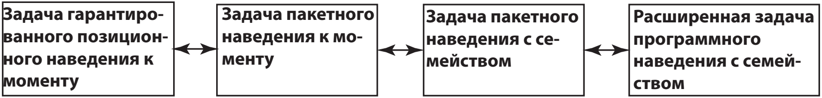
Problems equivalence

Lemma 2

Let the package guidance problem by the time be not solvable. Then the guaranteed positional guidance problem by the time is also not solvable.

Theorem 1

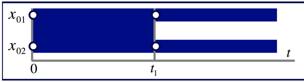
The guaranteed positional guidance problem by the time is solvable if and only if the package guidance problem by the time is solvable.



Homogeneous signals splitting

For an arbitrary homogeneous signal $g(\cdot)$ let

$$G_0(g(\cdot)) = \left\{ \tilde{g}(\cdot) \in G : \lim_{\zeta \to +0} (\tilde{g}(t_0 + \zeta) - g(t_0 + \zeta)) = 0 \right\}$$



be the set of **initially compatible** homogeneous signals and let

$$\tau_1(g(\cdot)) = \max \left\{ \tau \in [t_0, \vartheta] : \max_{\tilde{g}(\cdot) \in G_0(g(\cdot))} \max_{t \in [t_0, \tau]} |\tilde{g}(t) - g(t)| = 0 \right\}$$

be its **first splitting moment**.

For each $i = 1, 2, \ldots$ let

$$G_i(g(\cdot)) = \left\{ \tilde{g}(\cdot) \in G_{i-1}(g(\cdot)) : \lim_{\zeta \to +0} (\tilde{g}(\tau_i(g(\cdot)) + \zeta) - g(\tau_i(g(\cdot)) + \zeta)) = 0 \right\}$$

be the set of all homogeneous signals from $G_{i-1}(g(\cdot))$ equal to $g(\cdot)$ in the right-sided neighbourhood of the time-point $\tau_i(g(\cdot))$ and let

$$\tau_{i+1}(g(\cdot)) = \max \left\{ \tau \in (\tau_i(g(\cdot)), \vartheta] : \max_{\tilde{g}(\cdot) \in G_i(g(\cdot))} \max_{t \in [\tau_i(g(\cdot)), \tau]} |\tilde{g}(t) - g(t)| = 0 \right\}$$

be the **$(i + 1)$-th splitting moment** of the homogeneous signal $g(\cdot)$.

Initial states set clustering

Let

$$T(g(\cdot)) = \{\tau_j(g(\cdot)) : j = 1, \ldots, k_{g(\cdot)}\}$$

be the set of all splitting moments of the homogeneous signal $g(\cdot)$ and let

$$T = \bigcup_{g(\cdot)\in G} T(g(\cdot))$$

be the set of all splitting moments of all homogeneous signals. T is finite and $|T| \leq |X_0|$. Let us represent this set as $T = \{\tau_1, \ldots, \tau_K\}$, $t_0 < \tau_1 < \ldots < \tau_K = \vartheta$. For every $k = 1, \ldots, K$ let the set

$$\mathcal{X}_0(\tau_k) = \{X_0(\tau_k | g(\cdot)) : g(\cdot) \in G\}$$

be the **cluster position** at the time-point τ_k, and let each its element $X_{0j}(\tau_k)$, $j = 1, \ldots, J(\tau_k)$ be a **cluster of initial states** at this time-point; $J(\tau_k)$ is the number of clusters in the cluster position $\mathcal{X}_0(\tau_k)$, $k = 1, \ldots, K$.

Lemma 1

Open-loop control family $(u_{x_0}(\cdot))_{x_0\in X_0}$ is a program package if and only if for any $k = 1, \ldots, K$, any $X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k)$, $j = 1, \ldots, J(\tau_k)$ and arbitrary initial states $x_0', x_0'' \in X_{0j}(\tau_k)$ the equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in (\tau_{k-1}, \tau_k]$ in case $k > 1$ and for almost all $t \in [t_0, \tau_1]$ in case $k = 1$.

Extended space

Let $\mathcal{R}^h$ $(h = 1, 2, \ldots)$ be a finite-dimensional Euclidean space of all families $(r_{x_0})_{x_0 \in X_0}$ from $\mathbb{R}^h$ with a scalar product $\langle \cdot, \cdot \rangle_{\mathcal{R}^h}$ defined as

$$\langle r', r'' \rangle_{\mathcal{R}^h} = \langle (r'_{x_0})_{x_0 \in X_0}, (r''_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} = \sum_{x_0 \in X_0} \langle r'_{x_0}, r''_{x_0} \rangle_{\mathbb{R}^h} \quad ((r'_{x_0})_{x_0 \in X_0}, (r''_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h).$$

For each non-empty set $\mathcal{E} \subset \mathcal{R}^h$ $(h = 1, 2, \ldots)$ let us define its *lower* $\rho^-(\cdot|\mathcal{E}) : \mathcal{R}^h \mapsto \mathbb{R}$ and *upper* support functions $\rho^+(\cdot|\mathcal{E}) : \mathcal{R}^h \mapsto \mathbb{R}$:

$$\rho^-((l_{x_0})_{x_0 \in X_0} | \mathcal{E}) = \inf_{(e_{x_0})_{x_0 \in X_0} \in \mathcal{E}} \langle (l_{x_0})_{x_0 \in X_0}, (e_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} \quad ((l_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h),$$

$$\rho^+((l_{x_0})_{x_0 \in X_0} | \mathcal{E}) = \sup_{(e_{x_0})_{x_0 \in X_0} \in \mathcal{E}} \langle (l_{x_0})_{x_0 \in X_0}, (e_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} \quad ((l_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h)$$

Extended open-loop control control

Let $\mathcal{P} \subset \mathcal{R}^m$ be the set of all families $(u_{x_0})_{x_0 \in X_0}$ of vectors from P.
Extended open-loop control control is a measurable function $t \mapsto (u_{x_0}(t))_{x_0 \in X_0} : [t_0, \vartheta] \mapsto \mathcal{P}$.
Let us identify arbitrary programs family $(u_{x_0}(\cdot))_{x_0 \in X_0}$ and an extended open-loop control $t \mapsto (u_{x_0}(t))_{x_0 \in X_0}$.

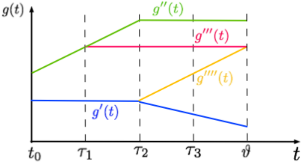
For each $k = 1, \ldots, K$ let $\mathcal{P}_k$ be an **extended admissible control set** on $(\tau_{k-1}, \tau_k]$ in case $k > 1$ and on $[t_0, \tau_1]$ in case $k = 1$ as a set of all vector families $(u_{x_0})_{x_0 \in X_0} \in \mathcal{P}$ such that, for each cluster $X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k), j = 1, \ldots, J(\tau_k)$ and any $x_0', x_0'' \in X_{0j}(\tau_k)$ holds $u_{x_0'} = u_{x_0''}$.

Extended open-loop control control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **admissible**, if for each $k = 1, \ldots, K$ holds $(u_{x_0}(t))_{x_0 \in X_0} \in \mathcal{P}_k$ for almost all $t \in (\tau_{k-1}, \tau_k]$ in case $k > 1$ and for almost all $t \in [t_0, \tau_1]$ in case $k = 1$;

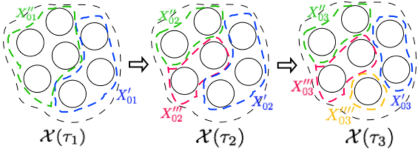
Lemma 2

Extended open-loop control control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is a control package if and only if it is admissible.

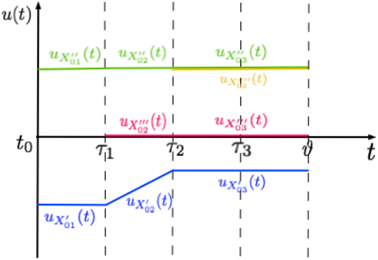
Homogeneous signals, cluster positions and extended open-loop control controls



Homogeneous signals splitting



Initial states set clustering



Extended open-loop control control

Extended program guidance problem with AGT family

Extended system (in the space $\mathcal{R}^n$):

$$\begin{cases} \dot{x}_{x_0}(t) = A(t)x_{x_0}(t) + B(t)u_{x_0}(t) + c(t) \\ x_{x_0}(t_0) = x_0 \end{cases}$$

$$(x_0 \in X_0)$$

Extended target set for the AGT family $\omega = (t_{x_0})_{x_0 \in X_0}$ is a set $\mathcal{M}(\omega)$ of all families $(x_{x_0})_{x_0 \in X_0} \in \mathcal{R}^n$ such that $x_{x_0} \in M(t_{x_0})$ for all $x_0 \in X_0$.
Extended admissible control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **guiding the extended system with the AGT family** $\omega = (t_{x_0})_{x_0 \in X_0}$, if $(x(t_{x_0}|x_0, u_{x_0}(\cdot)))_{x_0 \in X_0} \in \mathcal{M}(\omega)$.
Extended program guidance problem with the AGT family ω is solvable, if there exists an extended program guidance problem with the family ω.

Theorem 2

1) An admissible extended open-loop control is a guiding program package with the AGT family ω if and only if it is guiding the extended system with this family.
2) Package guidance problem with the AGT family ω is solvable if and only if the extended program guidance problem is solvable with this family .

Additional denotations

Let Ω be the set of all AGT families $(t_{x_0})_{x_0 \in X_0}$. For each $\omega = (t_{x_0})_{x_0 \in X_0} \in \Omega$ let us introduce the corresponding **attainability set** $\mathcal{A}(\omega) = \{(x(t_{x_0}|x_0, u_{x_0}(\cdot)))_{x_0 \in X_0} : (u_{x_0}(\cdot))_{x_0 \in X_0} \in \mathcal{U}_{ext}\}$ of the extended system.
For an arbitrary $x_0 \in X_0$ and an arbitrary $l \in \mathbb{R}^n$ let us introduce the function $p(\cdot, \cdot)$:

$$p(l, x_0, t_{x_0}) = \langle l, F(t_{x_0}, t_0)x_0 \rangle + \left\langle l, \int\limits_{t_0}^{t_{x_0}} F(t_{x_0}, t)c(t)dt \right\rangle \quad (l \in \mathbb{R}^n,\ x_0 \in X_0).$$

Let us denote

$$D(t_{x_0}, t) = B^{\mathrm{T}}(t)F^{\mathrm{T}}(t_{x_0}, t) \quad (x_0 \in X_0,\ t \in [t_0, \vartheta]).$$

For each family $\omega = (t_{x_0})_{x_0 \in X_0}$ let us introduce the set

$$\bar{X}_k(\omega) = \{x_0 \in X_0 : t_{x_0} \in (\tau_{k-1}, \tau_k]\} \quad (k = 1, \ldots, K)$$

and for any family of vectors $l = (l_{x_0})_{x_0 \in X_0} \in \mathcal{L}(\omega)$ let it be

$$l_{x_0,\omega}(t) = \begin{cases} l_{x_0}, & t \le t_{x_0} \\ 0, & t > t_{x_0} \end{cases} \qquad (t \in [t_0, \vartheta],\ x_0 \in X_0).$$

Additional denotations

For an arbitrary family $(l_{x_0})_{x_0 \in X_0}$ of elements of some linear space and an arbitrary function $f(\cdot)$, defined on this space let us use the following short notations:

$$\begin{aligned}
\Sigma^1 f(\Sigma^{1,k}_{x_0} l_{x_0}) &= \sum_{X_{0j}(\tau_1) \in \mathcal{X}(\tau_1)} f\left(\sum_{x_0 \in X_{0j}(\tau_1) \cap \bar{X}_k(\omega)} l_{x_0} \right) \\
(k &= 1, \ldots, K, j = 1, \ldots, J(\tau_1)),
\end{aligned}$$

$$\begin{aligned}
\Sigma^r f(\Sigma^{r,k}_{x_0} l_{x_0}) &= \sum_{X_{0j}(\tau_r) \in \mathcal{X}(\tau_r)} f\left(\sum_{x_0 \in X_{0j}(\tau_r) \cap \bar{X}_k(\omega)} l_{x_0} \right) \\
(r, k &= 1, \ldots, K, \;\; k \geq r, j = 1, \ldots, J(\tau_r)),
\end{aligned}$$

$$\begin{aligned}
\Sigma^1 f\left(\sum_{k=1}^{i} \Sigma^{1,k}_{x_0} l_{x_0} \right) &= \sum_{X_{0j}(\tau_1) \in \mathcal{X}(\tau_1)} f\left(\sum_{k=1}^{i} \sum_{x_0 \in X_{0j}(\tau_1) \cap \bar{X}_k(\omega)} l_{x_0} \right) \\
(i &= 1, \ldots, K, j = 1, \ldots, J(\tau_1)),
\end{aligned}$$

$$\begin{aligned}
\Sigma^r f\left(\sum_{k=r}^{i} \Sigma^{r,k}_{x_0} l_{x_0} \right) &= \sum_{X_{0j}(\tau_r) \in \mathcal{X}(\tau_r)} f\left(\sum_{k=r}^{i} \sum_{x_0 \in X_{0j}(\tau_r) \cap \bar{X}_k(\omega)} l_{x_0} \right) \\
(r, i &= 1, \ldots, K, \;\; i \geq r).
\end{aligned}$$

Solvability criterion

For each pair of families $(l_{x_0})_{x_0 \in X_0} \in \mathcal{R}^n$ и $\omega \in \Omega$ let it be

$$\begin{aligned}
\gamma((l_{x_0})_{x_0 \in X_0}, \omega) \quad = \quad & \sum_{x_0 \in X_0} p(l_{x_0}, x_0) + \\
& + \sum_{k=1}^{K-1} \int_{\tau_{k-1}}^{\tau_k} \Sigma^k \rho^- \left(\Sigma_{x_0}^{k,k} D(t_{x_0}, t) l_{x_0,\omega}(t) + \sum_{r=k+1}^{K} \Sigma_{x_0}^{k,r} D(t_{x_0}, t) l_{x_0} \middle| \; P \right) \\
& + \int_{\tau_{K-1}}^{\tau_K} \Sigma^K \rho^- \left(\Sigma_{x_0}^{K,K} D(t_{x_0}, t) l_{x_0,\omega}(t) \middle| P \right) dt - \sum_{x_0 \in X_0} \rho^+ \left(l_{x_0} | M(t_{x_0}) \right).
\end{aligned}$$

Theorem 3 (Extended problem of program guidance solvability criterion [1])

The extended program guidance problem with the AGT family $\omega = (t_{x_0})_{x_0 \in X_0}$ is solvable if and only if

$$\max_{(l_{x_0})_{x_0 \in X_0} \in \mathcal{L}(\omega)} \gamma((l_{x_0})_{x_0 \in X_0}, \omega) \leq 0. \tag{2}$$

Construction of the guiding program package with an AGT family

Let the solvability criterion (2) hold. Let us introduce the function $\hat{\gamma}(\cdot,\cdot,\cdot) : \mathcal{R}^n \times \Omega \times [0,1] \mapsto \mathbb{R}$:

$$\hat{\gamma}((l_{x_0})_{x_0\in X_0},\omega,a) = \sum_{x_0\in X_0} p(l_{x_0},x_0) +$$

$$+ \quad a\sum_{k=1}^{K-1}\int_{\tau_{k-1}}^{\tau_k} \Sigma^k\rho^-\left(\Sigma_{x_0}^{k,k}D(t_{x_0},t)l_{x_0,\omega}(t)+\sum_{r=k+1}^{K}\Sigma_{x_0}^{k,r}D(t_{x_0},t)l_{x_0}\middle|\, P\right)dt +$$

$$+ \quad a\int_{\tau_{K-1}}^{\tau_K} \Sigma^K\rho^-\left(\Sigma_{x_0}^{K,K}D(t_{x_0},t)l_{x_0,\omega}(t)\middle| P\right)dt - \sum_{x_0\in X_0}\rho^+\left(l_{x_0}|M(t_{x_0})\right).$$

The program package $(u^0_{x_0}(\cdot))_{x_0\in X_0}$ **is zero-valued with family** $\omega = (t_{x_0})_{x_0\in X_0}$, if $u^0_{x_0}(t) \equiv 0$ $(t\in[t_0,t_{x_0}], x_0\in X_0)$.

Lemma 3

Let the solvability criterion (2) for some family of admissible guidance times $\omega = (t_{x_0})_{x_0\in X_0}$, and zero-valued program package $(u^0_{x_0}(\cdot))_{x_0\in X_0}$ with the family ω is not guiding the extended system. Then exists $\mathbf{a}_* \in (0,1]$ such that

$$\max_{(l_{x_0})_{x_0\in X_0}\in\mathcal{L}(\omega)} \gamma((l_{x_0})_{x_0\in X_0},\omega,\mathbf{a}_*) = 0. \tag{3}$$

Construction of the guiding program package with an AGT family

Theorem 4 (Minimum condition for the extnded problem with an AGT family [1])

Let P be the strictly convex compact set, containing the zero vector inside; the condition (3) holds, and the program package $(u^*_{x_0}(\cdot))_{x_0\in X_0}$ satisfies the conditions $u^*_{x_0}(t)\in \mathbf{a}_* P,\ x_0\in X_0,\ t\in[t_0,\vartheta]$, Let the clusters $X_{0j}(\tau_k)\in\mathcal{X}_0(\tau_k)$, $k=1,\dots,K$, $j=1,\dots,J(\tau_k)$ be regular, and for each of them holds

- *on the segments $[\tau_{k-1},\tau_k]$, $k=1,\dots,K-1$:*

$$\left\langle \sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_k(s)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0},\omega}(\mathbf{t}) + \sum_{r=k+1}^{K}\ \sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_r(\omega)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0}},\ u^*_{X_{0j}(\tau_k)}(t)\right\rangle =$$

$$= \quad \rho^-\left(\sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_k(s)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0},\omega}(\mathbf{t}) \sum_{r=k+1}^{K}\ \sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_r(\omega)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0}}\middle|\,\mathbf{a}_* P\right)$$

- *on the segment $[\tau_{K-1},\tau_K]$:*

$$\left\langle \sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_K(\omega)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0},\omega}(\mathbf{t}),\ u^*_{X_{0j}(\tau_k)}(t)\right\rangle = \rho^-\left(\sum_{x_0\in X_{0j}(\tau_k)\cap\bar X_K(\omega)} D(t_{x_0},t)\mathbf{l}^*_{\mathbf{x_0},\omega}(\mathbf{t})\middle|\,\mathbf{a}_* P\right)$$

The the program package $(u^*_{x_0}(\cdot))_{x_0\in X_0}$ is guiding with the AGT family ω.

Construction of the ε-guiding positional strategy

Let us constructively define the ε-guiding positional strategy. Let us set **the correction times**

$$\sigma_k = \begin{cases} t_0 + \delta, & k = 0, \\ \tau_k + \delta, & k = 1, \ldots, K-1. \end{cases}$$

Construction of the positional guidance problem takes place **during the actual control process of the system, starting from the concrete, but yet unknown point $\hat{x}_0$.**

On each segment $[\tau_k, \sigma_k]$, $k = 0, \ldots, K-1$ the following procedure is applied:

An arbitrary test control is applied

$\Downarrow$

The signal $y(t)$ (non-homogeneous!) is observed

$\Downarrow$

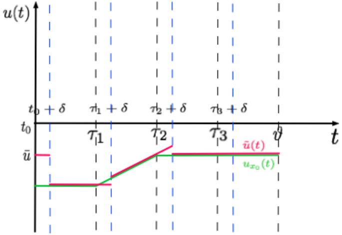
With the Cauchy formula the homogeneous signal is derived

$\Downarrow$

The cluster containing $\hat{x}_0$ is identified from the homogeneous signal

$\Downarrow$

The control (element of the program package) corresponding to the cluster containing $\hat{x}_0$ is applied



Construction of the guiding positional strategy

Lemma 3

Let the package guidance problem with the AGT family $\omega = (t_{x_0})_{x_0 \in X_0}$ be solvable for the system (1). Then such natural $\bar{K} \leq K$ exists, that $\max\limits_{t_{x_0} \in \omega} t_{x_0} \leq \tau_{\bar{K}}$, where K is the number all the splitting moments $\tau_k, k = 1, \ldots, K$ of all the homogeneous signals corresponding to admissible initial states $x_0 \in X_0$.

From this lemma and results obtained in [1] the following theorem follows.

Theorem 4

Let the package guidance problem with the AGT family $\omega = (t_{x_0})_{x_0 \in X_0}$ be solvable for system (1), and let the condition $\bar{K}\delta C \leq \varepsilon$, where C is some positive constant, hold for rather small positive $\delta > 0$. Then the closed-loop strategy $S^* = (\sigma_k, U_k)_{k=0}^{\bar{K}+1}$ corresponding to the guiding program package with the family ω is ε-guiding.

Model example

Let us consider a dynamical controlled system on the segment $[0,2]$:

$$\begin{cases} \dot{x}_1 = x_2, x_1(0) = x_{01} \\ \dot{x}_2 = u, x_2(0) = x_{02}. \end{cases}$$

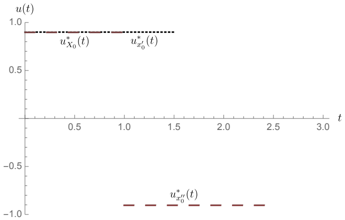
$$X_0 = \left\{ \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} \right\}; \qquad M = \left\{ \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \in \mathbb{R}^2 : |x_1| \le 1, x_2 \in \mathbb{R} \right\}$$

$$u(t) \in P = \{u : |u| \le 1\},\ t \in [0,2]; \qquad Q(t) = \begin{cases} (0,0), t \in [0,1] \\ (1,0), t \in (1,2]. \end{cases}$$

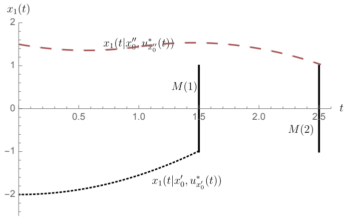
$W = \{\frac{3}{2}, \frac{5}{2}\} \Rightarrow$

1. $\omega_1 = \{\frac{3}{2}, \frac{3}{2}\},\ \gamma^0(\omega_1) > 0;$
2. $\omega_2 = \{\frac{3}{2}, \frac{5}{2}\},\ \gamma^0(\omega_2) < 0;$
3. $\omega_3 = \{\frac{5}{2}, \frac{3}{2}\},\ \gamma^0(\omega_3) < 0;$
4. $\omega_4 = \{\frac{5}{2}, \frac{5}{2}\},\ \gamma^0(\omega_4) < 0.$

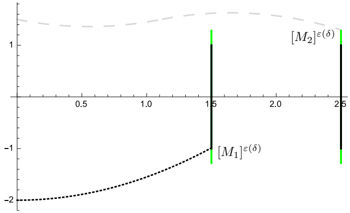
Model example



Guiding program package $(u^*_{x_0}(\cdot))_{x_0 \in X_0}$ with the AGT family ω_2.



System motion corresponding to the Guiding program package $(u^*_{x_0}(\cdot))_{x_0 \in X_0}$ with the AGT family ω_2.

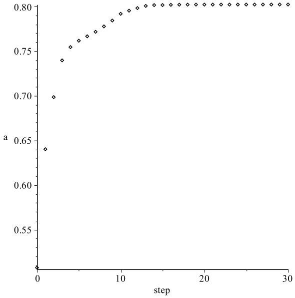


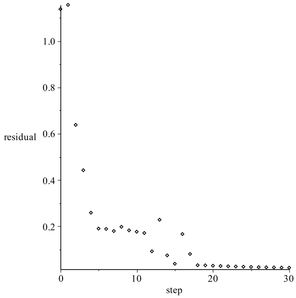
Guding positional strategy controls; $\delta = 0.1, \varepsilon = 0.27$.

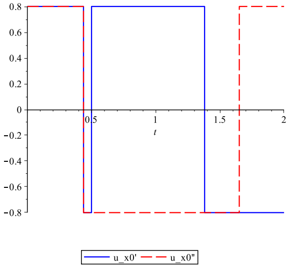
Further steps

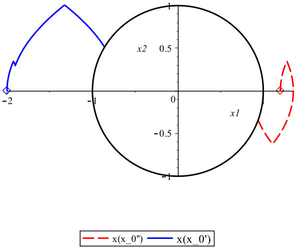
- From the finite X_0 to infinite set – approximation theorem has been proved by P. G. Surkov [3].
- From finite W to the continuous problem convergence – to be done
- Minimal time problem
- Numerical algorithms

Further steps









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Thanks for your attention!