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A MODEL TO ASSIST PLANNING THE PROVISION OF HOSPITAL SERVICES

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

One of these submodels, DRAM 1 (Disaggregated Resource Allocation Model) is designed to simulate the allocation of one resource between several patient types. In this paper, written jointly by the author of DRAM 1, R. Gibbs, and by a colleague in Canada, J. Rousseau, data from Quebec Province in Canada has been used to simulate how a given number of hospital bed-days will be allocated between the competing demands of patients of different types. It is hoped that the predictions from this model will be of value to the decision maker involved in forming health care policies.

Related publications in the Health Care Systems Task are listed at the end of this report.

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ABSTRACT*

One of the most important health service issues concerns the level of provision of acute hospital beds. To assist resolution of this issue, a model is proposed which simulates how hospital beds are used in terms of admission rates and lengths of stay for different categories of disease. The model can be used to predict the likely effects of changes in the provision of beds. Thus if it is proposed to increase the supply of beds the model will estimate, by disease type, how much of an increase this will cause in admission rates and lengths of stay.

The ability of the model to accurately simulate this type of behavior is illustrated by an application in Quebec Province, Canada.

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*The opinions expressed by the authors are their personal views and should not be ascribed to either the University of Montreal or the Department of Health and Social Security. Part of this paper was presented at the June 1979 Modeling Health Care Systems Workshop at IIASA. The entire paper has been submitted for publication in the Journal of the Institute for Management Science.

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INTRODUCTION

One of the most important questions faced by health service planners in many countries is "how many acute hospital beds should be provided?" The acute bed question is important not only because this sector of health services consumes a large proportion of health service finance but also because it is regarded by the population as the key life-saving arm of the service. This paper describes an illustrative application of a mathematical model to examine this question in the context of Quebec Province, Canada.

The function of the model in this type of application is to estimate the likely consequences of alternative levels of provision of hospital beds in terms of the numbers of patients of different types who could be admitted and their lengths of stay in hospital. By making judgments about what levels of admission rates and lengths of stay are acceptable, the planner can use the results of the model to assess the appropriate level of provision of hospital beds. Thus the model does not remove the need for judgment by a planner by producing some kind of unique optimum solution, but it does provide a planner with information about the consequences of his decision and so, hopefully, it enables a better decision to be made.

The model works by simulating how a given number of hospital bed-days will be allocated between the competing demands of patients of different types; for this reason the model is termed DRAM (Disaggregated Resource Allocation Model). It was built at IIASA as one of a suite of connected models for tackling a range of issues in the strategic planning of health services. The study described in this paper employs the first version of the model DRAM 1, which simulates the allocation of one resource between several patient types. Later versions simulate more complex resource allocation processes, involving several resource types and several forms of treatment.

The text which follows is in four main sections. First, the assumptions in the model are stated. Second, the formulation, solution, and parameter estimation of the model are described. Third, an application of the model to hospital data from Quebec Province, Canada, is presented. Fourth, there is a discussion of how the model can be developed to cover wider planning issues and how it is intended to apply it further both in Canada and in England.

ASSUMPTIONS

There are two main assumptions about the Health Care System (HCS) that are made in the model:

1. The demand for HCS services always rises to meet the supply of services.
2. Faced with the problem of allocating limited services between competing demands the actors in the HCS (doctors, nurses, patients, and others) behave collectively in a manner that can be represented as an attempt to maximize a utility function of admission rates and lengths of stay whose parameters can be inferred from data on how they have allocated services in the past.

There is a large body of empirical evidence for the first assumption. For example, a number of studies in different countries (e.g., Harris 1975, Feldstein 1967, Roemer 1959) have shown that, for a wide range of clinical conditions and specialities both admission rates and lengths of stay are elastic to the overall supply of acute hospital beds, i.e., the more beds the greater the admission rates and lengths of stay. Nowhere, apparently, have the demands for beds been saturated and its seems, as Rousseau (1977) has observed, that within the limits of what society can afford to supply they will remain unsaturated. (Of course a very important question when considering greater provision of hospital services is "what benefit, if any, will higher admission rates and longer lengths of stay have for the long term health of the population served?". This question is however beyond the scope of this paper since it raises many profound medical, epidemiological, and societal issues which are, as yet, far from being solved).

It is difficult to test the second assumption directly and so our confidence in it has to depend on how well output from the model fits empirical data - a point to which we shall return later in this paper. Here we shall merely elaborate the assumption. We envisage the actors in the HCS at the point of delivery of health care as having:

1. A concept of an ideal pattern of admission rates and lengths of stay that they would attain if beds were unlimited
2. A set of priorities and preferences for deciding which patients to admit and when to discharge them given the limited number of beds available

We assume that the HCS achieves an equilibrium by balancing the desirability of treating more patients of one type against the desirability of treating more of other types and against the undesirability of discharging patients too soon. The function of the model is to simulate the equilibrium-seeking behavior of the actors in the HCS and so predict the likely point of equilibrium for any given aggregate provision of beds.

The assumptions of the model have been described above in relation to the allocation of acute hospital beds but appear to be valid for a range of health services, e.g., ambulatory clinics and doctor's time. We should expect the model to be applicable to these also.

THE MODEL

This model was originally built by Gibbs (1978a) at the International Institute for Applied Systems Analysis, Laxenburg, Austria. It draws heavily from two similar resource allocation models--first a model built and applied in England--see MacDonald, et al (1974) and Gibbs (1978b)--and second a model built and applied in Quebec by Rousseau (1977). Although its scope is much more limited than the English model, which covers the allocation of many services and many alternative forms of care, it has the advantage, as we shall see, of computational simplicity. The following is a brief description of the model.

Formulation of the Model

Definition

Subscript

i = Patient category (e.g., by disease type)

Variables

x_i = Number of patients of type i admitted to hospital per thousand population

u_i = Average length of stay for patients of type i who are admitted (days)

Data

X_i = Ideal, maximum, number of patients per thousand of population of type i who need hospital treatment

U_i = Ideal average length of stay (days)

B = Total number of bed-days per thousand population available for occupation

α_i, β_i are strictly positive constants

Hypothesis

The HCS chooses the x_i, u_i so as to maximize a utility function, Z, where:

$$Z = \sum_i g_i(x_i) + \sum_i x_i h_i(u_i)$$

$$g_i(x_i) = - \frac{U_i X_i}{\alpha_i} \quad \frac{x_i}{X_i} \quad ^{- \alpha_i}$$

and

$$\lambda_i(u_i) = \frac{U_i}{\beta_i} \quad I - \frac{u_i}{U_i} \quad ^{- \beta_i}$$

subject to the constraint

$$\sum_i x_i u_i = B$$

The form of the functions $g_i(x_i)$ and $h_i(u_i)$ which represent the utility of admission rate and length of stay are show in Figure 1.

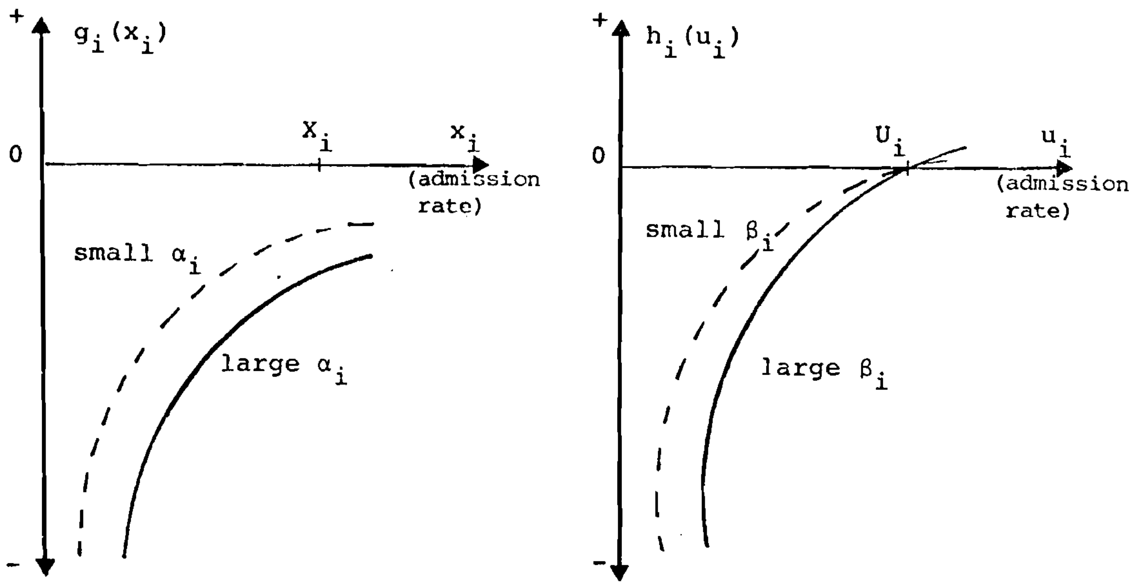

Figure 1. The utility of admission rate and length of stay assumed in the model.

These particular functional forms were chosen because they possess certain suitable properties including the following.

- They are monotonically increasing with decreasing gradients.
- At the ideal admission rate X_i, and ideal length of stay U_i, the marginal utility of increasing either admissions or stay equates to corresponding marginal requirement for bed-days, (i.e., $h_i'(U_i) = 1$ and $g_i'(X_i) = U_i$; for lower values of the arguments marginal utility is greater than the marginal bed requirement and for higher values marginal utility is less.
- The larger the value of α_i (or β_i), the greater is the marginal decrease in utility associated with a given reduction in admission rate (or stay) below the ideal level X_i (or U_i).

With these properties the model simulates the HCS allocating beds in the following manner, which is consistent with the two assumptions about HCS behavior made earlier.

- The HCS tries to attain admission rates x_i, and lengths of stay u_i, as close as possible to the ideal levels X_i and U_i, but cannot achieve this because the number of bed-days available in practice is less than the number needed (i.e., $B < \sum_i X_i U_i$).
- Accordingly all admission rates and lengths of stay are in practice less than the corresponding ideal levels, though none of them is zero.
- Some disease categories have a higher priority for admission than others (e.g., acute appendicitis would usually have priority over bronchitis) and so their admission rates more closely approach the ideal levels (the higher priority of these diseases is represented in the model by higher values of the α_i).
- Similarly for some diseases there is less scope for discharging a patient before his length of stay has reached the ideal (these diseases have higher values of the β_i.

Solution

The constrained maximization problem above can be readily solved using the Lagrange multiplier technique. It is shown in Gibbs (1978) that the solution is given by:

$$u_i = u_i(\lambda) = U_i^{-\frac{1}{\beta_i+1}} \tag{1}$$

$$x_i = x_i(\lambda) = X_i \left\{ \beta_i^{-1} \left[(\beta_i+1)\lambda^{\frac{\beta_i}{\beta_i+1}} - 1 \right] \right\}^{-\frac{1}{\alpha_i+1}} \tag{2}$$

where λ, the Lagrange multiplier, can be found by solving the equation $f(\lambda) = 0$ where

$$f(\lambda) = -B + \sum_i x_i(\lambda)u_i(\lambda) \tag{3}$$

by the Newton-Raphson method.

This solution is attractive since $u_i(\lambda)$ and $x_i(\lambda)$ are analytic functions of λ only and equation (3) is so amenable to solution by the Newton-Raphson method that in practice a satisfactory solution is obtained with a small number of iterations. The solution algorithm has been written in a fairly simple Fortran program that can (and has) been readily implemented on different types of computer installations and requires only a small amount of computer time to run.

For example, in one application with 16 disease categories, it takes only 1.3 seconds of CPU time on a CDC Cyber 173, to obtain the optimum solution. A test with 99 disease categories was performed in 5.2 seconds of CPU time.

Parameter Estimation

The problem of estimating the parameters of the model for applications to national Health Care Systems varies between different countries depending upon the nature of the HCS. In some countries where the HCS is centrally planned values for the parameters X_i and U_i, the ideal admission rates and lengths of stay, are obtained from epidemiological studies and expert opinions as a formal part of the planning system. In other situations, such as Quebec, these parameter values are not available exogenously. Here the task is to estimate both these parameters, the X_i and U_i, and also the power parameters α_i and β_i, in the terms of the utility function dealing with admission rates and lengths of stay. We shall now describe how parameters may be estimated in this latter situation.

We shall assume that we have data available for actual admission rates and lengths of stay in a single past time-period for a geographical entity, such as Quebec Province, and for sub-divisions within it, such as the 11 health service regions of Quebec Province. Let

x_{ir} = actual admission rate
u_{ir} = actual length of stay
B_r = actual aggregate availability of bed-days per thousand population
} in region r

$\bar{x}_{i.}$ = average admission rate
$\bar{u}_{i.}$ = average length of stay
$\bar{B}_{.}$ = average aggregate availability of bed-days per thousand population
} in the Province as a whole

If we now assume that the Provincial average data $\bar{x}_{i.}$ and $\bar{u}_{i.}$, correspond to what the model solution would be for simulating the allocation of the Provincial average bed availability $\bar{B}_{.}$, we may invert equations (1) and (2) and obtain the following expressions for the parameters X_i and U_i.

$$U_i = \bar{u}_{i.} \lambda^{\frac{1}{\beta_i+1}} \tag{4}$$

$$X_i = \bar{x}_{i.} \left\{ \beta_i^{-1} \left[(\beta_i+1) \lambda^{\frac{\beta_i}{\beta_i+1}} - 1 \right] \right\}^{\frac{1}{\alpha_i+1}} \tag{5}$$

To complete the parameter estimation we need expressions for the α_i, β_i and λ. We do this by introducing the concept of elasticity.

Let γ_i = elasticity of admission rate
η_i = elasticity of length of stay
} with respect to aggregate bed supply

(thus γ_i is the percentage increase in admission rate η_i for length of stay for category i due to a 1% increase in aggregate bed supply.

Least squares estimates $\hat{\gamma}_i$ and $\hat{\eta}_i$, of these elasticities may be obtained using the regional data x_{ir}, u_{ir}, and B_r to estimate the following regression equations, (where each region supplies one observation):

$$\log x_{ir} = \hat{\gamma}_i \log B_r + \text{const} \tag{6}$$

$$\log u_{ir} = \hat{\eta}_i \log B_r + \text{const} \tag{7}$$

By requiring that the model solution--the x_i and u_i--should respond to changes in bed availability, B, in a manner consistent with these empirically observed elasticities Gibbs (1978a) derives the following expressions for the α_i and β_i:

$$\beta_i = \frac{c}{\hat{\eta}_i} - 1 \tag{8}$$

$$\alpha_i = \frac{c\beta_i}{\hat{\gamma}_i} \quad (\beta_i+1) - \lambda^{-\frac{\beta_i}{\beta_i+1}} \tag{9}$$

where

$$c = -\frac{B}{\lambda f'(\lambda)} \tag{10}$$

and

$$f'(\lambda) = \frac{df(\lambda)}{d\lambda} \tag{11}$$

Equations (4), (5), (8), (9) and (10) are insufficient to uniquely define the parameter values since λ and $f'(\lambda)$ are unknown. However suitable values may be obtained by an iterative procedure with starting values for λ and $f'(\lambda)$ selected from a priori determined ranges--see Gibbs (1978a). It has been found that the model output obtained from using such parameter values is not sensitive to these starting values. Thus the procedure is satisfactory for simulating HCS behavior. However because of the degrees of arbitrariness in the procedure the absolute values obtained for X_i and U_i are not in themselves significant and <u>cannot</u> be interpreted as the ideal admission rates and lengths of stay perceived by the actors in the HCS. Nevertheless for predicting how actual admission rates and lengths of stay will respond to different levels of aggregate bed availability these parameter values are found to be adequate.

This parameter estimation procedure has been incorporated into the Fortran program mentioned earlier for solving the model. Thus to operate the program for this type of model application the user merely has to supply values for the elasticities $\hat{\gamma}_i$ and $\hat{\eta}_i$, and the Provincial average quantities $\bar{x}_{i.}$, $\bar{u}_{i.}$ and $\bar{B}_{.}$. From this the program can be used to simulate the response to the HCS to any input value for available bed days, B, the parameter values being calculated as intermediate quantities within the program.

In effect such a simulation is a prediction of admission rates and lengths of stay that would result from a given aggregate bed availability B; this prediction is conditional to the preferences and priorities of the actors in the HCS remaining the same as those that prevailed in the time period and location from which the data was drawn. It does not necessarily represent the allocation of bed days that is optimal from the point of view of a planner or a member of the population served who might well disagree with the preferences and priorities in the HCS. Nevertheless we believe that such a conditional prediction is valuable and relevant, at least in countries where the HCS is not centrally planned to a high degree, the central planner has little power and often little desire to alter the prevailing preference system. However he usually has much more power to influence the aggregate availability of services such as acute hospital beds. Hence we consider that a model such as this, which simulates how the HCS will respond to central decisions on aggregate supply is appropriate and useful to planners. If a central planner considers that he <u>does</u> have power to implement certain changes in the preference system, however, this could be represented in the model by inserting suitable values exogenously for those parameters that the planner expects to be able to change.

AN APPLICATION

We shall now describe a test application of the DRAM 1 model to data from Quebec Province, Canada. The purpose of this exercise is to investigate the realism and reliability of the model. The model is used to simulate the allocation of beds in a past year and the outputs obtained from the model are then compared with data on the allocations that actually occurred.

THE DATA

The principal source of data used was the Quebec hospital form AH-101 from the Ministry of Social Affairs. The year 1975 was selected because it was the most recent year (without strikes) for which the data was complete at the time of the study. The computerized form AH-101 includes for each Quebec resident hospitalized, in or outside the province, both personal data (age, sex, municipality of residence) and medical data (discharge diagnosis, surgical procedure, and duration of stay).

The data on patients was classified by disease according to the 18 category International Classification of Diseases (ICD) "A" Code. Category V, mental problems, was however excluded because only a minority of the patients in this category were hospitalized in acute hospital beds; moreover, this portion varied heavily from one region to the other. Categories XI and XV were regrouped. Both refer to childbirth, and its complications (including miscarriage) and in practice it seemed that the differentiation between these categories was not consistent from one region to the other.

To test the model, the 12 sociosanitary regions, as defined by the Quebec government were used. This choice was justified because the planning of resources was done on a regional basis. Table 1 summarizes the principal demographic and medical supply characteristics of these regions. Region 10, Nouveau Quebec is excluded from the subsequent analysis. It is the northern part of the province (as large as France) with extremely low population density and practically non-existent medical services (6 physicians, 110 beds). We realize from Table 1 that the regions are very different from one and other. The island of Montreal (region 6a) is a large urban area with a very high density of physicians. Cantons de l'Est (region 5) and Quebec (region 3) have both a high density of physicians and a high hospital bed supply. Finally Cote Nord (region 9), a low density populated area, has very few physicians but a high hospital bed supply. Some regions (regions 6b, 6c, 7, 8, 9) also use a relatively large proportion of bed-days in other regions.

Table 1. Principal demographic and medical supply characteristics of the 12 Quebec region (for 1975).

Regions	Population in 1000	Density of pop. per sq. mile	Density of physicians[a] (per 100 000 pop.)	% of specialists	Number of acute beds per 1000 pop.	% of total bed-days used outside the region
1 - Bas-St-Laurent-Gaspesie	223	16.9	87.9	37%	5.7	23%
2 - Saguenay-Lac-St-Jean	277	6.7	92.4	47%	6.1	7%
3 - Quebec	962	56.6	158.4	54%	6.2	3%
4 - Trois-Rivieres	412	23.3	96.8	49%	4.7	16%
5 - Cantons de l'Est	225	68.0	178.7	59%	6.8	5%
6a- Montreal-Ile	2207	7745.5	258.4	59%	6.6	2%
6b- Montreal-Laurentides	430	60.6	73.5	37%	2.7	50%
6c- Montreal-Rive-Sud	910	196.6	96.8	41%	2.0	49%
7 - Outaouais	258	12.5	91.5	38%	3.7	44%
8 - Nord-Ouest	136	6.1	71.3	32%	3.0	26%
9 - Cote Nord	101	1.0	57.4	21%	6.2	30%
10- Nouveau Quebec	15	.04	40.0	33%	7.6	64%

[a] These numbers include all physicians that receive some amount of money from RAMQ, the national insurance board.

CALIBRATION

In order to run the model for each of the remaining 11 regions an input value was required for B_r, the number of bed-days available for occupation per thousand population per year. Because of the large numbers of patients hospitalized outside their region of residence it was decided to set this value equal to the total number of bed-days used by residents of a region, including usage both within and outside their home region. Similarly the data on regional admission rates and lengths of stay are calculated from all hospitalizations of residents of a region.

The estimates, $\hat{\gamma}_i$ and $\hat{\eta}_i$, of the elasticities of admission rates and lengths of stay with respect to total bed availability were calculated from the data for all 11 regions using regression equations (6) and (7) as described above. Results are shown in Table 2. Following this, the parameters X_i and U_i, the ideal admission rates and lengths of stay are derived from equations (4), (5), (8), (9) and (10) using data on average values of bed availability, admission rates, and lengths of stay for Quebec Province as a whole.

RESULTS

The DRAM 1 was run for each of the 11 regions. In each of these runs the input data was identical except for one item: the value of B_r of regional bed availability. In order to assess the performance of the model we shall compare the model's predictions of regional admission rates and average lengths of stay, by disease category, with data on the actual values that occurred in practice. We shall also make a further test on the model's performance by comparing the accuracy of its predictions with a very simple model based on the following 3 assumptions.

-- All bed-days available to a region are used.

-- The regional average lengths of stay, by category, are equal to the corresponding values of Quebec

Table 2. Estimates of elasticities of admission rates and lengths of stay with respect to total bed availability; standard errors in [].

ICDA chapter	Title	Admission rate elasticity	Length of stay elasticity
I	Infectious and parasitic	2.02 [0.25]	-0.21 [0.27]
II	Neoplasm	0.01 [0.14]	0.02 [0.14]
III	Endocrinal Metabolic and nutritional	1.28 [0.17]	-0.01 [0.11]
IV	Blood	0.96 [0.25]	-0.57 [0.15]
VI	Eye, ear and nervous system	0.72 [0.13]	0.34 [0.30]
VII	Circulatory	0.58 [0.14]	0.40 [0.28]
VIII	Respiratory	1.52 [0.15]	0.36 [0.10]
IX	Digestive	0.84 [0.23]	-0.11 [0.21]
X	Urinary and genital	0.96 [0.15]	0.15 [0.13]
XII	Skin	1.07 [0.23]	0.31 [0.16]
XIII	Organ of movement	0.69 [0.25]	0.40 [0.24]
XIV	Congenital	0.45 [0.18]	0.53 [0.17]
XVI	Ill-defined	1.36 [0.36]	0.34 [0.12]
XVII	Accidents and trauma	1.22 [0.25]	-0.32 [0.16]
XVIII	Supplementary	0.57 [0.32]	0.29 [0.12]
XI and XV	Childbirth and complications	0.47 [0.18]	0.26 [0.07]

Province as a whole (i.e., length of stay is unaffected by total bed availability).

-- The regional admission rates by category, are directly proportional to the total bed availability (i.e., if bed availability increases by a given percentage all admission rates increase by this same percentage).

This very simple model is equivalent to a special case of the DRAM 1 in which all the admission rate elasticities are unity and all the length of stay elasticities are zero; for this reason we term it the 'one-zero' model.

Let us start by examining the DRAM 1's predictions for region 2, which has a bed availability 34% above that of the Quebec Province average. The results are displayed in Table 3 and reveal a good fit with the data on actual admission rates and average lengths of stay. For disease categories such as I and VIII for which the estimated admission rate elasticities are high, DRAM 1 correctly predicts values of admission rates considerably above the Quebec average rates. Moreover these predictions are considerably closer to the actual values than the prediction of the 'one-zero' model. Similarly for a category such as II, for which the estimated admission rate elasticity is very low, DRAM 1 correctly predicts admission rates close to the Quebec average rates whereas the 'one-zero' model wrongly predicts higher values.

The predictions, though good, do not correspond exactly with the actual values. This is scarcely surprising since we would not expect aggregate bed-availability to be the sole determinant of admission rates. Other factors such as variations in morbidity may have effects; for example the fact that the admission rate for category II (neoplasms) is below the Quebec average despite the fact that the bed availability of the region is well above the Quebec average may be a consequence of this region having a lower than average morbidity for this desease group.

Table 3. Results of model predictions by disease category for region 2.

		Admission Rates					Lengths of Stay			
ICDA Chapter	Title	Elasticity	Actual Value	DRAM I prediction	One-zero model prediction	Quebec average	Elasticity	Actual value	DRAM I prediction	One-zero model prediction (Quebec average)
I	Infectious and parasitic	2.02	5.7	5.2	3.9	2.9	-0.21	12.28	10.35	10.35
II	Neoplasm	0.01	7.7	8.0	10.7	8.0	0.02	16.14	16.17	16.06
III	Endocrinal metabolic and nutritional	1.28	3.8	3.7	3.4	2.6	-0.01	13.17	15.05	15.04
IV	Blood	0.96	0.9	1.1	1.2	0.9	-0.57	10.55	12.83	12.83
VI	Eye, ear and nervous	0.72	6.8	7.4	8.1	6.0	0.34	15.31	12.33	11.21
VII	Circulatory	0.58	11.4	12.7	14.5	10.8	0.40	26.58	19.82	17.72
VIII	Respiratory	1.52	21.2	21.2	18.5	13.8	0.36	8.08	8.00	7.24
IX	Digestive	0.84	19.1	20.9	22.1	16.4	-0.11	9.25	9.42	9.42
X	Urinary and genital	0.96	14.4	14.6	14.9	11.1	0.15	8.41	8.47	8.13
XII	Skin	1.07	3.3	3.2	3.1	2.3	0.31	8.86	8.97	8.22
XIII	Organ of movement	0.69	6.1	6.3	7.0	5.2	0.40	11.67	12.85	11.47
XIV	Congenital	0.45	3.1	2.7	3.1	2.3	0.53	10.60	10.36	8.92
XVI	Ill-defined	1.36	4.6	6.5	5.9	4.4	0.34	10.55	10.39	9.44
XVII	Accidents and trauma	1.22	10.0	12.5	11.9	8.8	-0.32	10.29	10.89	10.89
XVIII	Supplementary	0.57	20.4	20.2	23.1	17.2	0.29	5.21	5.59	5.15
XI & XV	Childbirth and complications	0.47	27.9	24.9	29.2	21.8	0.26	5.55	5.62	5.22
All categories		-	166.7	171.1	180.7	134.6	-	10.08	9.82	9.30

The model's predictions for lengths of stay are also good (see Table 3). However most estimated elasticities for length of stay are small and thus the results for both DRAM 1 and the 'one-zero' models are very close to the Quebec average. The results of the model for all disease categories combined are very close to the actual observation. The total admission rate and the average length of stay predicted by DRAM 1 differ by about 2% from the actual values whereas those predicted by the 'one-zero' model differ by about 8% (see bottom row of Table 3).

Unfortunately the results are not as good for all regions. Table 4 summarizes the global (all categories) results for the 11 regions. For regions 1, 6b, 6c, 7, 8, 9, the actual global average length of stay is smaller than predicted by the model and is lower than the Quebec average and correspondingly the actual admission rate is higher than the one predicted by the model. From Table 1 however, we note that these regions experience a great amount of hospitalization outside their regions. Because of the absence of specialized hospitals and physicians in these regions there are several transfers of patients to hospitals in Montreal or Quebec City. When a transfer occurs, two separate records of hospitalization are produced for each case, one at each hospital. In the first hospital in the region of residence, the length of stay may be very short (just to assess or stabilize the patient's condition), while the second length of stay may probably be somewhat over the expected average. This factor distorts the data, increasing the recorded admission rate and reducing the recorded average length of stay, which partly accounts for the apparent error in the prediction of DRAM 1.

Let us examine in more detail the results from one of the regions, 9, where the 'one-zero' model produces more accurate global predictions than DRAM 1. An analysis of results by disease category (see Table 5) shows DRAM 1 performing better than the 'one-zero' model in detail despite being less accurate globally. For admission rates, the DRAM 1 makes a more accurate prediction than the 'one-zero' model for 10 out

Table 4. Model predictions of global (all disease categories) admission rates and lengths of stay, by region.

Bed-days used/1000 population			Total admission rate (upper) and average length of stay (lower)		
Region	Actual value	Difference from Quebec average	Actual value	DRAM I	One-zero model
1	1548	+24%	171.8 9.01	160.0 9.67	166.5 9.30
2	1679	+34%	166.7 10.08	171.1 9.82	180.7 9.30
3	1406	+12%	143.6 9.79	148.0 9.50	151.3 9.30
4	1611	+29%	147.6 10.92	165.3 9.75	173.3 9.30
5	1455	+16%	177.6 8.19	152.1 9.56	156.5 9.30
6a	1084	-13%	114.4 9.48	119.8 9.05	116.6 9.30
6b	942	-25%	112.9 8.35	107.0 8.81	101.4 9.30
6c	1103	-12%	128.2 8.60	121.5 9.08	118.6 9.30
7	1282	+ 2%	156.0 8.22	137.3 9.34	137.9 9.30
8	1573	+26%	180.6 8.71	162.2 9.70	169.2 9.30
9	1846	+48%	207.7 8.89	184.8 9.99	198.6 9.30
Province of Quebec	1252	0%	134.6 9.30	134.3 9.32	134.6 9.30

Table 5. Results of model predictions by disease category for region 9.

ICDA Chapter	Title	Admission Rates: Elasticity	Admission Rates: Actual value	Admission Rates: DRAM I prediction	Admission Rates: One-zero model prediction	Admission Rates: Quebec average	Lengths of Stay: Elasticity	Lengths of Stay: Actual value	Lengths of Stay: DRAM I prediction	Lengths of Stay: One-zero model prediction (Quebec average)
I	Infectious and parasitic	2.02	7.4	6.2	4.3	2.9	-0.21	9.15	10.35	10.35
II	Neoplasm	0.01	7.3	8.0	11.8	8.0	0.02	15.51	16.21	16.06
III	Endocrinal, metabolic and nutritional	1.28	5.0	4.1	3.8	2.6	-0.01	14.03	15.04	15.04
IV	Blood	0.96	1.6	1.23	1.3	0.9	-0.57	8.78	12.83	12.83
VI	Eye, ear and nervous system	0.72	8.4	7.9	8.9	6.0	0.34	9.69	12.70	11.21
VII	Circulatory	0.58	12.3	13.4	16.0	10.8	0.40	14.74	20.52	17.72
VIII	Respiratory	1.52	22.8	24.3	20.3	13.8	0.36	7.91	8.26	7.24
IX	Digestive	0.84	21.5	22.4	24.3	16.4	-0.11	9.04	9.42	9.42
X	Urinary and genital	0.96	16.9	15.8	16.4	11.1	0.15	8.96	8.58	8.13
XII	Skin	1.07	4.3	3.5	3.4	2.3	0.31	9.44	9.21	8.22
XIII	Organ of movement	0.69	7.8	6.7	7.7	5.2	0.40	11.29	13.31	11.47
XIV	Congenital	0.45	3.0	2.8	3.5	2.3	0.53	13.04	10.85	8.92
XVI	Ill-defined	1.36	10.9	7.3	6.5	4.4	0.34	11.74	10.71	9.44
XVII	Accidents and trauma	1.22	16.9	13.9	13.1	8.8	-0.32	8.11	10.89	10.89
XVIII	Supplementary	0.57	29.9	21.3	25.4	17.2	0.29	5.41	5.73	5.15
XI & XV	Childbirth and complications	0.47	31.6	25.9	32.1	21.8	0.26	6.23	5.75	5.22
All categories		-	207.7	184.8	198.6	134.6	-	8.89	9.99	9.30
Sum of [Predicted-Actual]				30.3	33.8					

of 16 categories (1, 2, 3, 7, 8, 9, 12, 13, 16, 17) and the sum of the absolute errors is smaller for DRAM 1. The performance of DRAM 1 in predicting lengths of stay is not as good. For 6 categories it is more accurate than the 'one-zero' model and for 5 categories less accurate. In the paragraph above we suggested that this may not be simply due to a failure of DRAM 1 but, at least partly, to distortions in the data.

Tables 6 and 7 summarize the accuracy of the predictions of the DRAM 1 model and compare it with those of the 'one-zero' model. Table 6 shows results by region. In each case the quantity calculated is the percentage error of the predicted value relative to the actual value. For example, for region 2 DRAM 1 predicts admission rates considerably more correctly than the 'one-zero' model, the average errors being 8% compared to 14% for the 'one-zero' model (see Table 6). In assessing the model's performance over all 11 regions we need to recognize that for region 7 the total bed availability is very close to the Quebec average and so both DRAM 1 and the 'one-zero' models predictions are very close to the Quebec average figures. (This explains why the errors for the 2 models are approximately equal in this region.) Thus we need to direct our attention to the remaining regions. Of these the DRAM 1 predictions of admission rates are more accurate in 8 regions (1, 2, 3, 4, 5, 6b, 6c, and 9) and equal in 2 regions (6a and 8). For lengths of stay, DRAM 1's predictions are more correct in 7 regions (2, 3, 4, 6a, 6b, 6c and 8), equal in one region (1), and slightly less so in 2 (5 and 9).

Turning now to the results by disease category (Table 7) the predictions by DRAM 1 for admission rates are more accurate than those of the 'one-zero' model for 12 categories [especially much better for 2 of these (1 and 2)] and equal for the remaining 4 categories. For lengths of stay, the DRAM 1 predictions are more correct for 10 categories and equal for the remaining 6. Naturally the predictions of DRAM 1 are similar to those of the 'one-zero' model for admission rates for those categories where the estimated elasticity for admission rates is close to unity (4, 10, 12) or for length of stay when the estimated

Table 6. Percentage errors*of model predictions by regions.

	Bed-days/1000 population	Rate of admission (error)		Length of stay (error)	
Regions	Quebec mean: 1252	DRAM I	One-zero model	DRAM I	One-zero model
1	1548	9	14	13	13
2	1680	8	15	10	14
3	1407	7	10	5	6
4	1611	13	19	9	13
5	1455	16	17	16	14
6a	1084	7	7	5	6
6b	943	8	14	5	9
6c	1103	8	9	5	6
7	1282	12	12	11	10
8	1573	13	13	11	12
9	1847	15	16	16	15

$$*\%error = \frac{\sum_{categories} |(predicted-actual)|}{\sum_{categories} (actual)} \times 100$$

rounded to nearest integer

Table 7. Percentage errors* of model predictions by diagnostic category.

ICDA Chapter	Elasticity	DRAM I	One-zero model	Elasticity	DRAM I	One-zero model
I	2.02	13	25	-0.21	13	13
II	0.01	8	24	0.02	7	7
III	1.28	9	12	-0.01	7	7
IV	0.96	14	14	-0.57	12	12
VI	0.72	7	9	0.34	17	18
VII	0.58	7	10	0.40	14	16
VIII	1.52	8	13	0.36	5	8
IX	0.84	10	11	-0.11	10	10
X	0.96	7	7	0.15	6	7
XII	1.07	12	12	0.31	7	9
XIII	0.69	12	13	0.40	12	14
XIV	0.45	9	14	0.53	8	12
XVI	1.36	19	21	0.34	5	9
XVII	1.22	15	15	-0.32	10	10
XVIII	0.57	17	18	0.29	5	8
XI & XV	0.47	10	11	0.26	3	6

$$\text{*\%error} = \frac{\sum_{\text{regions}} |(\text{predicted}-\text{actual})|}{\sum_{\text{regions}} (\text{actual})} \times 100$$

rounded to nearest integer

elasticity for length of stay is close to zero (or negative) (1, 2, 3, 4, 9, 17). These account for most of the cases where the prediction errors of the 2 models are equal.

When the DRAM 1 model produces predictions of a region's admission rates and lengths of stay, the only region-specific input to the model is the region's aggregate bed-availability B. No account is taken of other factors that may be relevant such as regional differences in morbidity and physician density. Considering this we conclude that the model has performed reasonably well in predicting admission rates and lengths of stay in the 11 regions.

DEVELOPMENT AND APPLICATION

There are two ways in which we think we can improve the ability of DRAM 1 to simulate HCS behavior so that it can be more reliably applied to policy issues. First we hope to identify a different categorization of patients such that each category is more homogeneous with respect to elasticities. The ICD Chapter headings employed in the exercise described above are somewhat heterogeneous in this respect. For example, Chapter IX (diseases of the digestive system) covers a wide range of disease varying from those such as peritonitis for which one would expect a low elasticity of admission rate (i.e., high priority for admission) to those such as inguinal hernia where one would expect a high elasticity. More homogeneous categories would lead to elasticity estimates with lower standard errors than those found in this exercise and correspondingly better fits of model output to historical data.

A second improvement will be to include in the model a representation of the effects of the density and levels of specialization of physicians. This factor is believed to have caused some of the larger errors in the predictions of the model. For example in region 9 both the density and level of specialization of physicians in the region is very low which causes a large number of patients to be transferred outside the region; in region 5 however both density and level of specialization are high which causes a higher overall admission rate and a shorter average length of stay than would be expected on the basis of bed-supply alone. This important factor is included in a more sophisticated version of the model (Hughes, 1978) - DRAM Mark 2 - which simulates how the HCS allocates several resources (e.g., bed-days and physician time).

At the time of writing we are attempting to apply the model to policy issues in England and Quebec. In England the model is being applied by the Operational Research Service of the Department of Health and Social Security. The Department has a policy which places a high priority on the development of services for the mentally ill, the mentally handicapped, and the elderly. At a time when the overall growth in the National Health Service was tightly constrained by the economic situation, this required a deliberate decision to give these people priority over the development of general and acute hospital services. The trend in acute treatment has, nevertheless, been to treat more patients, though in general patients stay a shorter time in hospital and use fewer beds more intensively. This trend is continuing even though the scope for futher improvements is limited. The model is being employed to examine the likely consequences in terms of admission rates and lengths of stay against a background of changes in clinical practice and in the age structure of the population. The same problem is met in Quebec. The Ministry of Social Affairs is concerned about the disparities between the regions in the availability of hospital beds and physicians. With regard to the former it can exercise some control directly through the issuing of priorities to hospitals which limit, and if necessary, reduce the number of beds the hospital can make available to acute patients. The Ministry is considering the use of the model as a first step towards evaluating the consequences of decisions in this area.

In conclusion we feel that we have indicated the ability of DRAM 1 to simulate the way in which certain health service resources are allocated and the relevance of the model for examining significant policy issues in our two countries.

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