NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

ON SYSTEMS OPTIMIZATION

V.M. Glushkov

March 1981
CP-81-10

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

On systems optimization

V.M. Glushkov

Institute of cybernetics

Kiew USSR

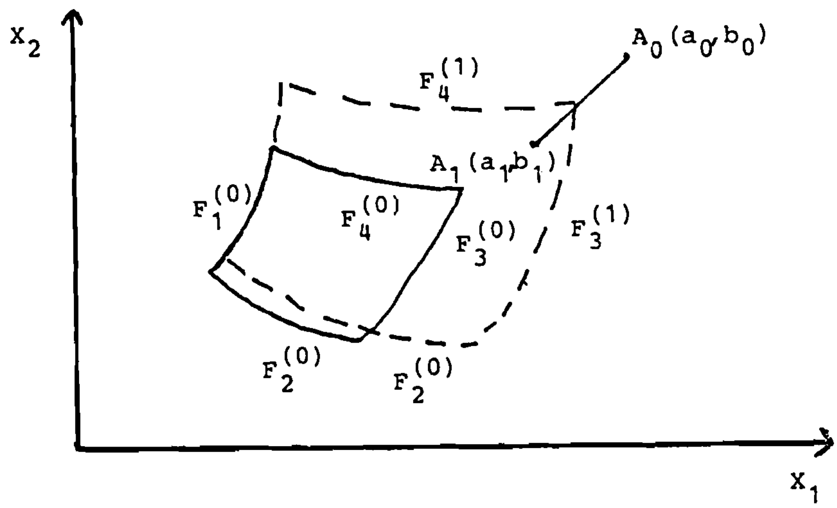
The modern theory and practice of optimization relies on the classical statement of optimization problems. It it known that the essence of such a statement consists of finding the point (or set of points) p in the preassigned ***invariable*** feasible set P, where the given ***scalar*** objective function $f(p)$ assumes its extreme value.

This statement is unsatisfactory for a vast number of economic planning problems and design-engineering problems in at least in two respects. For one thing, the objective function $f(x)$ in such problems is the *vector* function rather than the scalar one. Furthermore, it proves to be practically irreducible to the scalar function by a particular *a priori* procedure (for instance weighting of various components of an initial vector function). For another, the feasible set P can vary during the optimization process. What is more, its ***purposeful*** change is the *very meaning* of the optimization process for a given class of problems.

As the laws of admissible changes in the feasible set P are usually specified by a *system of models*, the described approach to the optimization problems is naturally referred to as the *systems approach*. It should be noted that under the systems approach the *constraints*, determining the feasible set in the space of specific parameters, generally change as a result of a sequence of *solutions* chosen from a discrete set of possible solutions. Consider, also, that the set itself is usually not comprehensively specified when optimization begins and is completed during interaction between persons (planners and designers) not fully versed in formalized methods of making new decisions and optimization.

Examine one of the formalized statements which is characteristic of the systems optimization problem. To provide greater impact, let us employ its graphical representation and discuss the two-criteria case. Further assume, that the choice of values for these criteria determines uniquely the corresponding solution.

In other words, the desired solution is sought for in the space K of optimization criteria designated by x_1 and x_2 (see Figure).



The solution starts with choosing some point A_0 with coordinates a_0 and b_0 in the given space K , that is , the desired solution. Then the *initial constraints* $F_1^{(0)}(x_1,x_2)\geq 0, \ldots$ $F_n^{(0)}(x_1,x_2)\geq 0$ specifying the initial *feasible set* P_0 are constructed. An immediate test shows whether the point A_0 belongs to the set P_0 or not. In principle, the normal (classical) procedure of optimization either by one of the criteria x_1 or x_2 or by their particular combination is applicable in the first case. However, the radically different method is usually employed in the systems approach. Namely, in accordance with the highest level model M, which controls the choice of criteria, the point A_0 *is removed beyond the boundaries of the feasible set* P_0 as is shown in the Figure.

Next, we isolate constraints not satisfied at the point A_0. In the case under consideration, such are the constraints $F_3^{(0)}$ and $F_4^{(0)}$. Then models M_3 and M_4 that define these constraints are employed and particular *solutions* changing the correspondent constraints in the *wanted direction* (if such a change happens to be feasible) are tested in the dialog mode. Here the wanted direction is that which decreases the absolute value of negative *discrepancies* $F_i^0(a_0,b_0)$ in the given case $F_3^{(0)}(a_0,b_0)$ and $F_4^{(0)}(a_0,b_0)$.

It should be remembered that on numerous occasions the constraints F_i turn out to be *interdependent* ,so a change in one of them induces changes in a certain part of the other constraints. *The solution choice control* exercised for changing the constraints is aimed at the minimization of a *penalty function* $g_0(a_0,b_0)$. The *maximum absolute value of negative discrepancies* $\lambda_i F_i^{(0)}(a_0,b_0)$ (where λ_i are positive weighting coefficients) is usually chosen as such a function. If the discrepancies do not exist, then by definition $g_0(a_0,b_0) = 0$.

In general, the control results in a number of solutions c$R_1,\ldots,R_m$ reducing the penalty function value which after the m-th solution is denoted by $g_m(a_0,b_0)$. Changing the constraints of each of the adopted decisions is responsible for the adequate change in the feasible set. The figure illustrates two such changes. The first one alters the constraints $F_3^{(0)}$ and $F_2^{(0)}$ and substitutes them by the constraints $F_3^{(1)}$ and $F_2^{(1)}$ respectively. The second affects only the constraint $F_4^{(0)}$ substituting it by the constraint $F_4^{(1)}$ The feasible set P_2, resulting from the above two changes, is bounded by the lines $F_1^{(0)},F_2^{(1)},F_3^{(1)}$ and $F_4^{(1)}$ while the corresponding value of the penalty function equals $g_2(a_0,b_0) \neq 0$. Note, that it is impossible to choose the finite feasible set in advance, as the sequence of sets $P_0,P_1,\ldots$ may not be completely ordered with respect to the imbedding. Moreover, the laboriousness of the construction of new constraints, when one would have to waste a great deal of effort in changing insignificant constraints, interferes with completion of this work in good time.

If (as in our Figure) $g_2(a_0,b_0) = 0.$, and there are no solutions which decrease the penalty function value further, then we return to the highest model M which controls the choice of the desired problem solution $A(a,b)$. A succession of solutions $D_1,D_2,\ldots$, will change the initial problem solution $A_0(a_0,b_0)$ by step-by-step substitution of the latter by

$$A_1(a_1,b_1),\quad A_2(a_2,b_2),\quad A_3(a_3,b_3),\ldots$$

until the successive point $A_k(a_k,b_k)$ falls within the feasible set (in Figure k=1). The solutions to change are chosen from the feasible set of solutions for the purpose of minimizing the penalty function. The described process is similar to the classic optimization process except that the steps are not chosen arbitrarily, but rather to match the admissible (with respect to the model M) solutions.

At last, once the point A_k is in the final feasible set P_m an additional procedure of optimization by some combinations of criteria x_1 and x_2 may be applied within this feasible set. Such a procedure differs from the classic one by the fact that the choice of optimization steps is not arbitrary, but is controlled by the highest level model M. If some constraints, changeable in the desired direction, impede further improvement of the chosen criterion, then the optimization process may be continued with the inclusion of successive solutions to attain such changes.

It should be emphasized that the unambiguous definition of the problem solution by choosing all values of the optimization criteria is not so infrequent as it may seem at first sight. It is applied, for instance, to econometric planning problems, where the net output of diversified products is the (vector) criterion, while the gross output is the problem solution(Glushkov75a).

In the absence of such unambiguousness the set where the solution is sought can have coordinates other that those corrseponding to the optimization criteria. Then the above optimization process becomes more complicated owing to the fact that points $A_i(a_i, b_i)$ are substituted by hyperplanes. The definition of the penalty function also acquires more sophistication: any distance from the chosen hyperplane to the next feasible set in space with specified compressions (extensions) along axes corresponding to the optimization criteria may serve as that.

In the most general case, arbitrary punctiform sets can appear instead of the hyperplanes. Statements are possible by which criteria values are not determined uniquely on those sets, and the corresponding weighting functions are specified (by the highest level model M) on the sets to discriminate between more or less preferable solutions. However, the important systems optimization feature inherent in all the approaches is an interaction between models of different levels, aside from the multicriteria nature and changeability of the feasible set. In the case of economic planning problems, decisions are made by managers at different levels, and in the case of design-engineering problems, by designers handling various parts of the project.

The described principles have found application in "Displan", one of the specific optimization systems, developed by the author.

Glushkov75a. V.M. Glushkov, *Macroeconomic models and principles underlying design of all- Union automated system.*, Statistika, Moscow (1975).