MULTIDIMENSIONAL MATHEMATICAL DEMOGRAPHY: AN OVERVIEW

Kenneth C. Land
Population Research Center, the University of Texas, USA

Andrei Rogers
International Institute for Applied Systems Analysis, Laxenburg, Austria

RR-82-35
October 1982

Reprinted from Kenneth C. Land and Andrei Rogers, editors, *Multidimensional Mathematical Demography.* New York: Academic Press, 1982

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
Laxenburg, Austria

Reprinted with permission from Kenneth C. Land and Andrei Rogers, editors, *Multidimensional Mathematical Demography.* New York: Academic Press, 1982 (ISBN 0-12-435640-0), pages 1–41.

PREFACE

Generalizations of the classical models of mathematical demography to include multiple states of existence in the course of the life cycle have appeared with increasing frequency during the past decade. The new methods for manipulating data, constructing life tables, and generating population projections have fostered innovative empirical studies of, for example, interregional migration, marriage and divorce, and labor force participation. And they have established a need for a systematic assessment of this growing body of research. Responding to this need, the authors of this essay convened a conference on multidimensional mathematical demography at the University of Maryland at College Park, Maryland, USA in March 1981. Supported financially by the US National Science Foundation, the meeting brought together demographers, mathematicians, sociologists, and statisticians to report on and evaluate the current state of the art of the newly established field. The essay reproduced in this reprint, written by the organizers of the conference, presents an overview of multidimensional demography and outlines the results of the conference. It appears as the introductory chapter of the conference proceedings volume just published by Academic Press.

A list of related publications issued by IIASA is included at the back of this report.

ANDREI ROGERS
Chairman
Human Settlements and Services Area

1

Multidimensional Mathematical Demography: An Overview

Kenneth C. Land and Andrei Rogers

1. INTRODUCTION

A large and significant body of theory, methods, and applications in demography is concerned with the transitions that individuals experience during their lifetime, as they pass from one state of existence to another: for example, transitions from being single to being married, from being alive to being dead, from being in school to having graduated, from being out of the labor force to being in the labor force, from living in one region to being a resident of another. A unifying analytic thread that runs throughout these substantively diverse problems is their description by a set of two or more "living" states (marital statuses, schooling statuses, labor force statuses, geographic regions), among which the members of a population make transitions, plus the absorbing state of death into which all individuals eventually enter. The analysis considers the evolution that arises as a consequence of the transitions that occur over successive periods of time and age.

Not only are these and similar problems of intrinsic substantive interest to demographers, but they often relate to

(a) other fundamental demographic phenomena (e.g., fertility);

(b) patterns of social and economic change (e.g., in family and household structures or in regional employment and economic growth);

(c) legal questions (e.g., estimating the expected number of remaining years of working life for a worker who is disabled on the job); and

(d) assorted social policy issues (e.g., ascertaining the impacts of employment and retirement trends on social security and pension systems).

The large numbers of uses and users of disaggregated multidimensional population projections have led national statistical agencies all over the world to expand their provision of such detailed totals. The U.S. Federal Government, for example, regularly issues a number of projections that are based on the Census Bureau's national population projections. These deal with fertility, mortality, immigration, school enrollment, educational attainment, family and household totals and composition, and the income distribution of households (Fig. 1). The only link between these different projection series is that in practice the exogenously projected rates are all applied to the same age-sex-race-specific population. No attempt has as yet been made to ensure that the rates used in projecting each series are consistent with those used in other series or with the level of the projected population. Thus, fertility rates may not be consistent with the proportion of ever-married women, and the size and composition of households may not be consistent with the level of educational attainment of household members. Yet the need for such consistencies is becoming ever more apparent as these projections are increasingly used to support planning and policy making.

Until relatively recently, demographers and agencies, such as the U.S. Bureau of the Census, have sought to introduce multidimensionality into their numerical projections by applying, more or less directly, the basic *single-decrement life table* and the associated *single-dimensional population projection model* of conventional demographic theory, as described, for example, in Keyfitz (1977) and in Shryock and Siegel (1973). But these *single-state* (living at age x) *models* exhibit one or both of the following deficiencies (Rogers, 1980, p. 497). First, single-state models

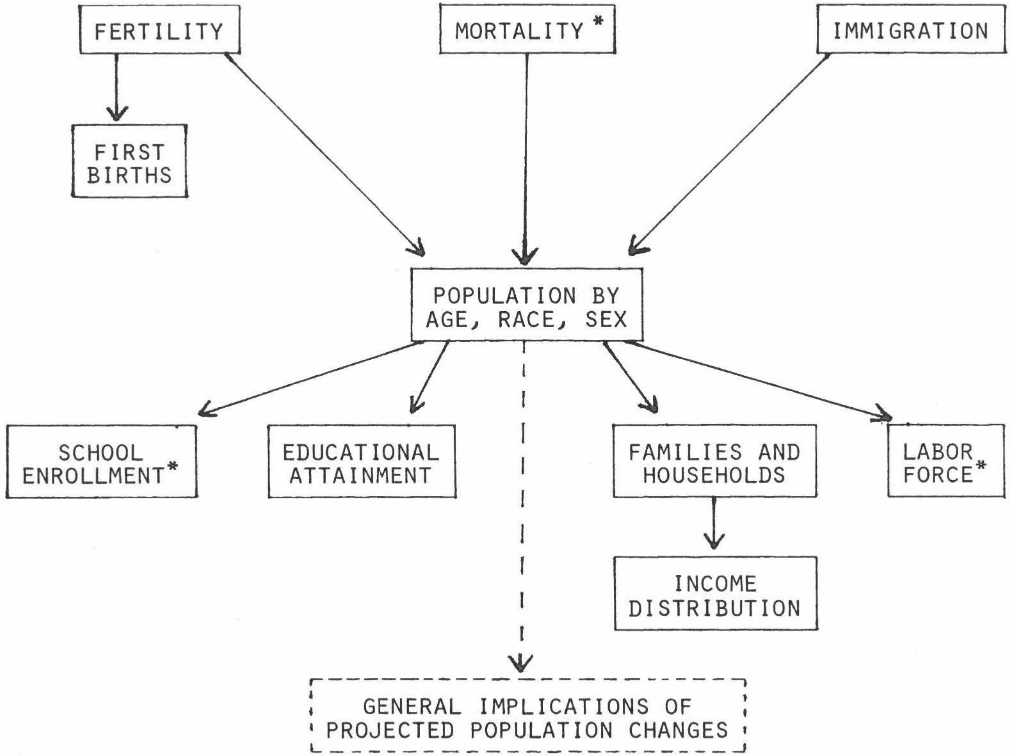

*Fig. 1. Demographic characteristic projections based on the Census Bureau's national population projections (Long, 1980). *Projections done by or in cooperation with other federal agencies.*

cannot incorporate interstate transfers differentiated by origins and destinations, and must therefore analyze changes in population stocks (i.e., the number of persons occupying various states at distinct points in time) by reference to *net* flows among the states, for example, net migration. Second, single-state models cannot follow individuals across several changes of state and therefore cannot disaggregate current or future stocks and flows by initially or previously occupied states.

Early efforts by actuarial scientists and demographers to generalize the single-state life table model led to the development of *multiple-decrement life tables*. They incorporate two or more forms of decrement from an initial status, such as mortality by cause of death, or attrition from the status of being single by mortality and nuptiality (Jordan, 1967, pp. 271-290; Preston *et al.*, 1972, pp. 13-20). More complex tables with secondary decrements (Jordan, 1967, pp. 291-304) or *hierarchical increment-decrement models* (Hoem, 1970a,b; Oechsli, 1975) were defined by chaining together a series of multiple-decrement tables in such a way as to allow successive transitions among living states, and hence increments into subsequently occupied states, but *no reentries* (with a given age interval) into a state previously occupied (e.g., from single to married to divorced back to married). Until about a decade ago, these hierchical increment-decrement models were the primary tools, other than single-decrement life tables, used by demographers for the construction of nuptiality tables, tables of working life, and tables of educational life (for a survey of this literature, see Shryock and Siegel, 1973, pp. 455-459). All such tables, however, suffer from the limited ability of these models to accommodate *reentrants* into states as well as decrements.[1]

The earliest extensions of the single-state population projection model focused on multiple states of residence and therefore were called *multiregional projection models* (Rogers, 1966, 1968; Feeney, 1970; Le Bras, 1971). Similar generalizations concerned with other classifications such as parity and occupational mobility appeared shortly thereafter (Goodman, 1969; Coleman, 1972). It then became clear that projections of populations classified by multiple states of existence could be carried out using a common methodology of multistate projection in which the core model of population dynamics was a multidimensional generalization either of the continuous age-time model of Lotka (Le Bras, 1971) or the discrete-age-time Leslie Model (Rogers, 1966; Goodman, 1969).

During the early 1970s, work on multistate life tables and on multistate projection models progressed rapidly, and the two streams of research were fused together to produce a consistent generalization of classical demographic techniques that unified many of the methods for dealing with transitions between multiple states of existence. This generalization of conventional analytical demography has produced

[1] *For thorough discussions of limitations in the "old" tables of working life constructed on the basis of hierarchical models and estimated from prevalence rates, see Hoem and Fong (1976), Schoen and Woodrow (1980), and Willekens (1980).*

(1) the specification of a general *nonhierarchical increment-decrement life table* (IDLT) *model* and an associated *multistate population projection model* capable of differentiating interstate transfers by both origin and destination states and that can accommodate reentrants into states;

(2) the development of *estimation and computational algorithms* to allow such models to be applied to the rather sketchy information demographers often confront; and

(3) the *empirical application* of these models to a growing range of substantive topics.

Because multistate models can be viewed as superimposing a set of two or more life (e.g., social, economic, health) statuses on the natality (birth)--alive at age x--mortality (death) continuum of the classic single-decrement life table model, thus combining the age dimension with one or more status dimensions, they also are called *multidimensional models*.[2] We (and the authors of chapters in the volume) use these terms interchangeably. In order to aid the reader in understanding the contributions of the chapters in the present volume to the state of the art in this field, we now turn to a brief account of the more recent historical development of multistate demographic models and their connections to other fields, such as mathematical statistics and biometrics.

[2] *Since they also combine the age dimension with one or more status dimensions, simple and hierarchical multiple-decrement life table models are multistate models, according to this definition. But the critical members of this class of models are the nonhierarchical models. The structure of the latter subclass is more complex, and it is in their specification, estimation, and use in projection that the critical advances of the past decade have been made.*

2. RECENT DEVELOPMENTS IN MULTISTATE DEMOGRAPHIC MODELS

2.1. *INCREMENT-DECREMENT LIFE TABLE MODELS*

The simplest possible mathematical framework suitable for specifying the stochastic process underlying a nonhierarchical multistate IDLT model is the *classic* discrete-state, continuous-time *Markov chain* (Doob, 1953, pp. 235-255). A formal model of this type was studied in the context of disability insurance as long ago as Du Pasquier (1912, 1913). A similar Markov chain model was specified and applied to the study of recovery, relapse, death, and loss of patients by Fix and Neyman (1951). Sverdrup (1965) specified a three-state version of this model (two intercommunicating living states plus the absorbing state of death) and made a more systematic study of its statistical estimation and test procedures. While these are not the only analysts who developed statistical estimators applicable to this model, most other works (e.g., Meier, 1955; Zahl, 1955; Billingsley, 1961; Albert, 1962; Chiang, 1964) are based on an observation plan that assumes that all individuals can be observed over a fixed period [0,T]. Hoem (1971) noted that this assumption often is violated in demographic data (e.g., by censored or incomplete observations), extended Sverdrup's theory to a countably-infinite state-space, and suggested the application of the three-state model to the study of labor-force participation. But such an application was not published until Hoem and Fong (1976) constructed tables of working life for Denmark.

Given the existence of a model with long-standing foundations in actuarial science, mathematical statistics, and biomedical research, it might be assumed that mathematical demographers would regard the problem of specifying a model for multidimensional life tables as essentially resolved. For several reasons, however, this is not the case. First, because a nontrivial empirical application of the classic model did not appear until a few years ago, there was no common agreement that this model could be applied to the sketchy transition information typically available to demographers. Second, applications of multistate models especially in interregional migration studies often must deal with state spaces wherein the number of living states k is greater than two. However, the simple closed-form expressions for the transition probabilities and forces used by Hoem and Fong (1976) are based on a model with only two living states (e.g., in the labor force, not in the labor force) and nondifferential mortality into the absorbing state of death, and no explicit expressions are feasible when a model contains more than four intercommunicating living states.[3] Third, empirical applications of this classic model usually are based on the simplest possible time(age)-inhomogeneous Markov chain, namely, a chain that postulates *constant* instantaneous transition forces (intensities) within the age intervals over which the model is estimated. While this may be a tenable assumption for most demographic processes when the

[3]*By "explicit," we mean expressions that involve only a finite number of algebraic operations. Thus, according to this convention, the matrix exponential (infinite series) solution of the classic homogeneous model for $k > 4$ intercommunicating states (applied, for example, to competing risks of illness and death by Chiang, 1964 and the marital status by Krishnamoorthy, 1979) does not produce explicit expressions for the transition probabilities and forces.*

estimation-age-intervals are relatively short (e.g., single years), many mathematical demographers would not regard it as a sound basis for producing a sufficiently accurate IDLT when the age intervals are longer (e.g., 5 or 10 yr). In other words, while the piecewise-constant transition-forces multistate ife table model may be a reasonable specification for estimating an unabridged IDLT, it is less satisfactory for estimating an *abridged* IDLT.

These considerations help explain why demographers working on empirical multidimensional problems proceeded, in the early 1970s, to forge alternative multistate IDLT models and methods. For example, in the process of conducting research on inter-regional population growth and distribution, Andrei Rogers (1973a,b, 1975) developed and applied multiregional generalizations of the classic single-decrement life table. At about the same time, Robert Schoen performed a life table analysis of marriage, divorce, and mortality data (Schoen and Nelson, 1974) and investigated generalizations of the corresponding methods for constructing IDLTs (Schoen, 1975).

Both of these analysts replaced the piecewise-constant transition forces assumption of the classic model with a specification on the survivorship functions, namely, that they change (increase or decrease) linearly with distance into an age interval.[4] The resulting model has since become known as "the linear model" (for a brief review of this model, see Ledent, 1978 and Section 3.2 of the chapter by Land and Schoen in this

[4] *Schoen's (1975) algorithm actually is more general and potentially allows for other forms of the survivorship functions. But it is the linear model that he develops most fully.*

volume). But, because the form of data typically available to demographers for the study of migration in the United States (survivorship proportions from decennial censuses) differs from that typically available for the study of nuptiality (occurrence/exposure rates from vital event registers), the estimation methods of Rogers are somewhat different from those of Schoen.[5] Further, both Rogers, in his Option 1 method, and Schoen developed their estimation techniques initially in scalar form (Roger's Option 2 method, which focuses on the use of survivorship proportions, was expressed initially in matrix form).[6] However, after seeing Schoen's (1975) scalar expressions for the estimators of the "linear" version of this algorithm, Rogers and Ledent (1976) were able to derive a matrix estimator of interstate transition probabilities analogous to the scalar formula for survival probabilities in single-decrement theory when the survival function is assumed to be linear.[7] Nonetheless, be-

[5] *Even though Roger's Option 1 method deals with data in the form of occurrence/exposure rates, his assumption that individuals made only one state transition per estimation-age-interval (Rogers 1975, p. 59) makes his estimators differ from those of Schoen (1975).*

[6] *All life table functions originate from a set of transition probabilities, defined for all ages. In constructing such tables from the normal data on vital events and survivorship proportions, demographers frequently adopt one of two approaches: one that focuses on observed rates or one that considers observed proportions surviving. In Rogers (1975, p. 81) these two approaches are called the Option 1 and the Option 2 methods, respectively.*

[7] *Recall that this single-decrement formula for, say, 5-yr age intervals is*

$$_5p_x = \ell_{x+t}/\ell_x = (1 - (5/2)\,_5M_x)/(1 + (5/2)\,_5M_x)$$

(cf., Keyfitz, 1977, p. 20). The analogous formula for the linear IDLT is given, for example, as Eq. (3.13c) in the chapter by Land and Schoen in this volume.

cause neither Rogers nor Schoen had fully specified an underlying instantaneous process for their estimation algorithms, it was not clear exactly what was the underlying parametric counterpart of this matrix estimator. This question was addressed by Schoen and Land (1979), who specified a general continuous-time (age)-inhomogeneous Markov chain model for IDLTs and correspondingly modified the estimation algorithm of Schoen (1975).

Although a focus on transition probabilities and their underlying intensity functions has characterized the contributions of probabilists and statisticians to the construction of IDLTs, mathematical demographers have also directed their attention to other life table functions, such as expectations of life at various exact ages and age-group-specific survivorship proportions for use in population projection exercises. In Rogers (1973a,b, 1975), multistate generalizations for these functions were compactly expressed in matrix form, showing the resemblance to their corresponding conventional single-state counterparts. Further work on the use of survivorship proportions to calculate multistate life tables (the Option 2 method) was carried forward by Rees and Wilson (1977) and Ledent (1978, 1980). The latter author also contrasts the survivorship proportion approach with the more standard occurrence-exposure rate methods.

2.2. MULTISTATE POPULATION PROJECTION MODELS

An important and fundamental application of the survivorship probabilities and proportions provided by a multistate life table is to population projection. With the development of IDLTs, it became possible to generalize the demographer's conventional methods for estimating the elements of a population projection matrix to the multistate case in a consistent manner (Rogers, 1973a,b, 1975). The marriage of multistate life tables with multistate projection models and their expression in matrix form to show transparently their natural correspondence with widely accepted conventional single-state methods established multidimensional mathematical demography as a serious branch of analytical demography (Keyfitz, 1979; Rogers, 1980).

The distribution of a multistate population across its constituent states and the age compositions of its state-specific subpopulations are determined by the interactions of fertility, mortality, and propensities of interstate transfer. Individuals are born, age with the passage of time, reproduce, move between different states of existence, and ultimately die. Such a general perspective of the population projection process suggests a wide range of substantive applications. Regional population projections, generated simultaneously for a system of several interacting regional populations, instead of region-by-region, illustrated the first serious application of the new methodology (Rogers, 1975; Willekens and Rogers, 1978). More recently, multistate projection models of labor force totals have been proposed (Willekens, 1980), and multistate projections of the U.S. population by age and marital status are currently being developed (Espenshade, 1980). Thus, it appears that many of the categories of projections listed in Fig. 1, which generally have been calculated by applying extrapolated proportions and ratios to a projected population base, now can be produced using the models of multidimensional demography.

Such models seem to offer a fruitful direction of research for the internal consistency in projection exercises sought by government statistical agencies such as the U.S. Bureau of the Census:

> Perhaps the most striking results of this overview of projections methodologies are the lack of a mechanism for assuring consistency between projected variables and the apparent arbitrariness of many of the assumptions used to project (or more appropriately, to extrapolate) the proportions and ratios applied to the projected population base. Our interest at this point is to identify the most fruitful areas of research that may lead to specifying linkages between variables in the system, to estimating the parameters specified, and to devising a system or model for projecting these parameters (Long, 1980, pp. 14-15).

Studies of the asymptotic dynamics of the multistate projection model have shown that its ergodic properties can be analyzed by means of straightforward generalizations of the stable growth theory of conventional single-dimensional demography. It has been established, for example, that a multistate population system that is closed to external migration and subjected to an unchanging multistate schedule of mortality, fertility, and migration ultimately will converge to a stable constant age-by-state distribution that increases at a constant stable rate of growth (Rogers, 1966, 1975; Le Bras, 1971). Le Bras (1977) extended this proof to the case of weak ergodicity; and Liaw (1978) has demonstrated that, as in the case of the conventional single-state population projection model, the dominant root accounts for the part of an observed population distribution that is stable. The other positive roots transmit the redistributional effects of interstate transfers, and the negative and complex roots generate fluctuations in population totals and age profiles known as "waves."

3. THE CONFERENCE ON MULTIDIMENSIONAL MATHEMATICAL DEMOGRAPHY

There can be little doubt that the foregoing developments have greatly enriched the field of mathematical demography and expanded the range of application of its traditional analytic models. But these developments have left a number of questions unanswered. For instance, what is the precise relationship of the "linear" IDLT specification to the classic "constant-forces" model? More generally, how does each of these specifications relate to the underlying continuous-time Markov chain model that has come to be accepted as the mathematical basis for IDLTs? Are there other possible specifications that are better than these? What are the statistical properties of these and other models? For instance, does the linear model exhibit problems of embeddability, identification, and estimation with structural zeros similar to those that have been discovered for the constant-forces model when applied to panel data? Can multistate models be extended to incorporate two sexes? Is there a multidimensional stochastic generalization of the ergodic theorems of classic stable population theory? How can recent developments on stochastic process models and methods on mathematical statistics and mathematical sociology be used to refine the models of multistate demography?

It was to address these and related theoretical and methodological questions and to stimulate further work on empirical applications that the editors of this volume organized and directed, on March 23 - 25, 1981, a conference on multidimensional mathematical demography.[8] The Conference brought together mathematical demographers, who had made prior contributions to multidimensional models, with other demographers, mathematical sociologists, and mathematical statisticians. This book is the product of that Conference.

To provide a focal point for Conference discussions, several participants were asked to prepare papers dealing with questions such as those raised above. Discussions on the first day were devoted primarily to essays on theoretical developments in, and empirical applications of, multidimensional demographic models, while those on the second day centered on multidimensional life table models and methods. Essays on the relationships of stochastic process models and methods (from mathematical statistics and mathematical sociology) to multidimensional demography were examamined on the morning of the third day. The Conference concluded with summaries of the principal sessions and a general discussion of needed research and next steps in multistate demography.

The diversity of disciplinary backgrounds and research interests of the Conference participants fostered a series of lively, intense, and fruitful discussions. It would be neither possible nor illuminating to give a detailed summary of these

[8] *The Conference was funded by Grant Number SES 80-16789 from the Sociology Program, Division of Social and Economic Science, National Science Foundation. It was held at the Center for Adult Education of the University of Maryland in College Park.*

discussions in this short chapter. Rather, in the remainder of this chapter, we shall focus on a description of the general issues and themes around which the Conference papers and discussions were organized and a statement of some possible next steps in research in multidimensional mathematical demography.

4. ORGANIZATION OF THE VOLUME

The papers collected in this volume are a selection of those presented at the "Conference on Multidimensional Mathematical Demography." They may be conveniently grouped into four major themes: data problems, life tables, population dynamics, and heterogeneity.[9]

4.1. DATA PROBLEMS

Empirical studies in multistate demography often begin with data, set out in tabular form, which describe changes in *stocks* that have occurred over two or more points in time. These changes arise as a consequence of increments and decrements associated with *events,* such as births and deaths, and with *flows* of individuals between different states of existence.

[9] *A list of participants and their current organization affiliations is included at the end of the volume. Four additional papers presented at the Conference were not revised for publication in this volume: "Estimating Individual-Level Transition Probabilities for Multistate Life Tables" by James S. Coleman; "Simplified Multiple Contingency Calculations" by Nathan Keyfitz and Andrei Rogers; "Constructing Multiregional Life Tables Using Place-of-Birth Specific Migration Data" by Jacques Ledent; and "Relations Between Individual Life Cycles and Population Characteristics" by Samuel H. Preston.*

The latter can be viewed either as events or as changes of state between two points in time (Ledent, 1978).

When all of the appropriate elements in such tables have been filled in with numbers, they generally are referred to as *accounts* (Stone, 1971, 1981; Rees and Wilson, 1977; Rees, 1980; Land and McMillen, 1981). And when, as is often the case, some data are unavailable, ingenuity and sophisticated fudging are used to supply the missing entries. Prominent among such techniques are various row and column balancing methods that have been successfully implemented in economics (input-output matrices), transportation planning (origin-destination traffic flows), and statistics (contingency tables). In the second chapter of this volume, Frans Willekens unifies much of this work showing that the underlying strategy in all of it is a search for missing elements in a flow matrix that preserves, in some sense, the structure of the whole data set. To accomplish this, Willekens adapts techniques used in demographic accounting and log-linear models of contingency table analysis.

To implement his procedure, Willekens focuses on two sets of accounts: an observed flow matrix, with its marginal totals, structural zeros, and subset of known elements, and an estimated flow matrix, with *all* of its flows specified. The elements in the latter are selected to reflect patterns of association that resemble those found in a similar but different problem setting, or they may reflect historical data or patterns suggested by cross tabulations of intermediate or explanatory variables.

Multidimensional demographic models require data on population flows for purposes of applying the methods of estimation, that recently have been developed. But, for many phenomena, censuses and sample surveys provide only aggregate data. Consequently, techniques such as those presented by Willekens may become critical for converting existing data into a form that can be used in multistate demographic analyses.

Age-specific patterns of demographic events such as fertility or mortality exhibit remarkably persistent regularities (e.g., Coale and Demeny, 1966; Coale and Trussell, 1974). The age profiles of these schedules seem to be repeated, with only minor differences, almost everywhere. As a result, demographers have found it possible to summarize and capture such regularities by means of hypothetical schedules called *model* schedules.

Model schedules have two important applications in demography: (1) they may be used to infer the empirical schedules of populations for which the requisite data are unavailable or inaccurate; and (2) they can be applied in analytical studies of population dynamics arising out of growth regimes that may be defined in terms of a relatively limited set of model schedule parameters. Because the data requirements of multidimensional population analyses increase exponentially with the number of dimensions, the role of model schedules in such analyses is likely to be fundamental.

In Chapter 3, Andrei Rogers and Luis Castro focus on the development of hypothetical (synthetic) model schedules that reflect regularities in age profile found in empirical schedules of migration rates. They define two alternative perspectives for creating such synthetic schedules for use in situations where only inadequate or defective data on internal gross migration flows are available. The first associates variations in the parameters and variables of the model schedule to each other and then to age-specific migration rates; the second embodies different relationships between the model schedule parameters in several standard schedules and then associates the logits of the migration rates in the standard to those of the population in question. Preliminary tests of the proposed model schedules indicate that, although the quality of fits are satisfactory in describing internal migration flows in developed countries, further work will be needed if such approaches are to be of practical use in Third World population settings.

4.2. LIFE TABLES

The life table has been a central concept in classical demography. Its use to describe the facts of mortality in terms of probabilities and their combined impact on the lives of a hypothetical cohort of individuals born at the same moment has been so successful that, in the words of Keyfitz (1977, p. 3) "... we are incapable of thinking of population change and mortality from any other starting point." The natural starting point for thinking about multidimensional population change, therefore, is the multistate life table, its theoretical derivation, and its empirical calculation.

Chapters 4 - 6 deal with the methodology of constructing multidimensional life tables. Jan M. Hoem and Ulla Funck Jensen lead off this section with a critical overview of current multistate life table theory and estimation methods. Arguing from a probability theory/mathematical statistics perspective, they take the position that the proper place at which to begin the construction of an IDLT is with the specification of its state space and transition intensities. Furthermore, Hoem and Jensen maintain that the fundamental assumptions of a model specification should be made in those terms, and not in terms of transition probabilities or survival functions (as in the linear model), or other "derived" quantities. Given estimates of the transition intensities, the method they recommend for IDLT construction is to compute the transition probabilities and other quantities as a solution to the Kolmogorov equations. Hoem and Jensen also construct examples that show how the transition intensities in the linear model may violate fundamental theoretical requirements, such as nonnegativity constraints. Finally, they make a number of observations about general demographic methodology and present some results concerning observational plans and statistical inference in multistate life tables.

In Chapter 5, Kenneth C. Land and Robert Schoen identify their own set of shortcomings in existing methods of estimating IDLTs. One of the most serious of these is that existing methods *either* are capable of incorporating transition forces that increase, remain constant, or decrease within estimation-age-intervals *or* are capable of being put in explicit algebraic form, but they do not have both desirable features. To fill this gap in existing methods, Land and Schoen develop a new estimation method that is based on the specification of quadratic transition probabilities or gross flow functions. They also review the derivation of their general algorithm for estimating IDLTs (Schoen and Land, 1979) and show how the constant-forces, linear, and quadratic models can be estimated as special cases of this algorithm. In addition, Land and Schoen derive algebraic expressions for the classes of rational polynomial transition force functions corresponding to the linear and quadratic models. Referring to the pathologies concerning these induced transition forces discovered by Hoem and Jensen, Land and Schoen point out that these are nothing more than embeddability and estimation-with-structural-zero-constraints problems, and that the latter appear also in the approach of Hoem and Jensen when applied to similar data situations (e.g., in the application of the constant-forces model to panel data; see Singer and Spilerman, 1976b).

Both Hoem and Jensen and Schoen and Land deal with estimation problems created by mobility data in the form of events (*moves*) such as are typically obtained from population registers. In practice, however, there exists alternative sources, e.g., population censuses and surveys, which yield mobility data in the form of *movers*, i.e., interstate transfers defined by a comparison of the states in which individuals were present at two different points in time. Both Hoem and Jensen and Schoen and Land recognize the existence of these alternative forms of mobility data from which to estimate IDLTs and adapt their methods thereto. In addition, Jacques Ledent, in the final chapter of this section, focuses primarily on the problem of estimating transition probabilities from the latter type of data. Ledent first reviews two existing approaches and then attempts to develop them further.

As revised by Ledent, both estimation procedures require the following input data: (a) mortality rates as conventionally measured; and (b) mobility measures obtained by an appropriate transformation of the raw data on *movers*. The first of these procedures, originating from Roger's (1975) Option 1 method, calls for adequately estimated mobility propensities, whereas the second, following from Rogers's (1975) Option 2 method, requires transition probabilities conditional on survival. Of the two alternative approaches, Ledent appears to prefer the second one, because it relies on some additional information about stayers. Also, the latter method is more readily applicable to the calculation of increment-decrement life tables for open systems, e.g., to multiregional population systems that experience *international* (external) migration.

In general, these three chapters represent extensions of the existing theoretical and methodological streams in multistate demography summarized earlier herein. Based on the premise that estimation methods must be tailored to each type of available data, the paper by Ledent deals with the case of data coming in the form of counts of individuals who have moved. Methodological rather than theoretical in nature, it revises existing procedures of estimation from such data, with a special concern for ensuring agreement between some life table statistics and their observed counterparts.

The chapters by Hoem and Jensen and Land and Schoen are especially helpful in identifying the relationships of the constant-forces and linear survival function specifications to each other and to the corresponding Markov chain model. In brief, it now is clear that both specifications assume the same basic continuous-time(age)-inhomogeneous Markov chain. But, whereas the constant-forces approach deals with the age-inhomogeneity by dissecting an age range into age intervals that are sufficiently small that the transition forces can be approximated by constants, the linear specification approximates the solution of the Kolmogorov equations (over possibly longer age intervals) by linear functions. Thus, for example, the linear function defined by Eq. (3.45) of the chapter by Land and Schoen can be regarded as a Taylor polynomial approximation (to the linear term) of the (generally unknown and nonlinear) solution of the "true" Kolmogorov equations that generated the data. Similarly, the quadratic function defined by Land and Schoen's Eq. (3.38) can be regarded as a second-order Taylor polynomial approximation.

Clearly, these linear and quadratic approximations yield simple, algebraically explicit computation formulas. Furthermore, in the absence of embeddability and structural zeros problems, they appear to produce somewhat more accurate numerical estimates in *abridged* IDLTs than does the constant-forces model. Nonetheless, as the width of an estimation-age-interval decreases, the exponential, linear, and quadratic estimators will approach each other. For, in this case, the higher-order terms of the rational polynomial transition force functions of the former estimators will decrease toward zero so that the force functions will deviate less and less from a constant level over the age interval.

In our view, the ultimate conclusions to be drawn from these three chapters about "proper" methods of multistate life table estimation depend critically on the type of table to be constructed and the forms of data available for estimation. Clearly, the strong points of the classic constant-forces model are its well-developed foundation in mathematical statistics and its corresponding ability to deal with problems of statistical inference in sample data. In addition, the assumption of constant forces is least critical when the estimation-age-intervals of an IDLT can be made "small" relative to the local variability of the transition forces being modeled. Thus, we have no hesitation in recommending the use of this specification when the objective is the construction of an *unabridged* IDLT from data in which age intervals can be chosen optimally relative to the constant-forces assumption and for which statistical inferences are relevant.

On the other hand, when the objective is the construction of an *abridged* IDLT from population-level statistics or census data, particularly data in which the estimation-age-intervals are fixed in rather wide lengths, the polynomial gross flow specifications have two salient features. First, their more flexible specifications on the transition forces may yield more accurate estimates of transition probabilities than does the constant-forces specification. Second, they have the advantage of computational simplicity. Of course, the statistical theory for such specifications, embedded as it is in the theory of analytic graduation (Hoem, 1972b), may be less familiar to demographers than is that for the classical model. But statistical inferences traditionally have been more peripheral in the context described in this paragraph than that described in the preceding paragraph.

In the middle ground between these two extremes, the choice of estimation method is less clearcut. But, again, an optimal decision depends on whether the objective is an abridged or unabridged table, whether the age data are grouped or not, and on the relative importance of statistical inference versus computational simplicity. Note that these views on the estimation of *abridged* IDLTs are not greatly different from those of Hoem and Jensen (Section 4.4). The main difference is that the methods recommended here emphasize simple parametric forms of the solutions of the Kolmogorov equations, whereas those of Hoem and Jensen emphasize simple parametric forms for the transition forces. Since the emphasis in an abridged table is on producing accurate estimates of the transition probabilities, the former may seem more natural in this context.

4.3. POPULATION DYNAMICS

The chapters in the third group are concerned both with theoretical developments and with substantive applications of multidimensional demographic models. They deal with the evolution of multistate populations exposed to a given regime of growth and interstate mobility. Leading off is Robert Schoen's essay on the incorporation of the interaction between the sexes, in the form of nuptiality and fertility, in life table and stable population models. Using the harmonic mean as a mechanism for distributing the consequences of interactions between the sexes among the several states of the model, Schoen shows how the classic "problem of the sexes" is mathematical demography (Keyfitz, 1977, pp. 293-336) can be accommodated in multistate models. When the one-sex/two-sex dimension is combined with the stationary population/stable population and decrement/increment-decrement dimensions, ten distinct life table models arise. Schoen demonstrates how all ten models can be specified and constructed in terms of the same four sets of equations, and discusses some of the properties of each model.

As an illustration, a two-sex (fertility) increment-decrement stable population model is presented using birth, death, and migration rates for the United States and California, 1970.

Multistate population projection models disaggregate conventional population projections into a number of state-specific categories, such as state of current residence and state of birth, status at an earlier age, and duration of occupancy in the current state. If interstata transition probabilities vary significantly according to the chosen categories, then the disaggregated multistate projection models should produce more accurate results than aggregated models. In Chapter 8, Dimiter Philipov and Andrei Rogers explore the consequences of introducing several such state-specific categorizations of multiregional populations.

A number of studies have reported higher than average probabilities of migration to a given destination among those returning to their place of birth or region of previous residence (e.g., Ledent, 1981). Philipov and Rogers incorporate this characteristic into a multistate projection model that distinguishes between *native* and *alien* populations in each region of a multiregional system. Introducing higher transition probabilities for return migrants, they show that such native-dependent projections produce spatial distributions that differ significantly from those generated by a native-dependent multistate projection model. The latter consistently underestimate the fraction of natives in each regional population.

Concluding this group of essays, Joel Cohen's chapter considers the ergodic properties of multiregional population projection models with changing rates and stochastic patterns of behavior. In mathematical demography, ergodic theorems define long-run behavior that is independent of initial conditions. Weak ergodic theorems describe populations experiencing changing rates, and stochastic ergodic theorems assume that such rates are selected from a set of possible rates by some stochastic process. Building on extensions of his previous work in single-state ergodic theory, Cohen (1976, 1977a,b) develops four weak ergodic theorems and a stochastic ergodic theorem that assumes that a Markov chain selects the rates of transition from a set of alternatives.

4.4. HETEROGENEITY

Most of the models used in multidimensional demography assume that moves from one state to another are independent of each other, suppose that all of the individuals occupying a particular state at a given moment are homogeneous, and consider the evolution that would occur if the various probabilities of interstate transition were to remain constant over a time period. Yet it is widely accepted that the reverse is almost always a more accurate description of reality.

Blumen *et al.* (1955), in an early stochastic analysis of occupational mobility, posited a model of "movers" and "stayers" as a means for accommodating heterogeneity in simple Markov chain models. Their pioneering investigation stimulated a generation of studies, to which the chapter by Pavel Kitsul and Dimiter Philipov is the most recent addition. Kitsul and Philipov are motivated by the problem of analyzing mobility data collected over unit time intervals of different length. For example, in the case of interregional migration, registration systems in several countries (such as Sweden) can produce flow matrices every year. Censuses, on the other hand, usually provide such data over a fixed period (five years, say). How can the two alternative descriptions of the same mobility phenomenon be reconciled?

To address this problem analytically, the authors distinguish two homogeneous populations of movers: one with a high intensity of moving and the other with a low intensity. Representing the mobility process as a mixture of two Markovian processes, they introduce a few simplifying assumptions, which allow them to fit their model to British migration data for the five-year period from 1966 - 1971 and also for the single year 1970. They then demonstrate how their model can be used to transform data collected over one unit of time into comparable information covering a time period of different length.

Another stream of research that has grown out of the original Blumen *et al.*, investigations into the effects of population heterogeneity on mobility processes pertains to the use of semi-Markov, rather than Markov, specifications (see, for example, Ginsberg, 1971, 1972a,b; Hoem, 1972a). In Chapter 11, Charles J. Mode reviews a number of junctures at which semi-Markov process can be related to IDLTs from a sample path perspective. Underlying both IDLT methodology and semi-Markovian processes is the notion of a set of states among which an individual moves over a period of time. The set of states visited by an individual and the sojourn times in these states constitute the person's sample path. Mode discusses a class of stochastic processes based on probability distributions defined directly on the sample paths and relates these to problems of estimating IDLTs from microdata on sample paths.

Heterogeneity is also the focus of the chapter by James J. Heckman and Burton Singer, the final contribution to this volume. The two authors consider strategies for analyzing population heterogeneity in demographic studies using models that contain mixtures of Markov and semi-Markov processes. To illustrate the critical importance of this topic, Heckman and Singer show how different assumed choices of distributions of unobservable variables lead to substantively contradictory inferences in a structural model of waiting-time durations. They then derive a nonparametric estimator for mixing measures as a strategy to bypass the more traditional, but dangerous, *ad hoc* assumptions about mixing distributions used in most conventional modeling of duration data. Clearly, population heterogeneity is a data modeling problem of continuing relevance in multidimensional mathematical demography. Investigations such as Heckman's and Singer's, and those reported in their references, will therefore be of growing importance to the development and refinement of multistate demographic models.

5. CONCLUSIONS AND NEXT STEPS

The chapters of this volume, and the literature to which they refer, demonstrate that the field of multidimensional mathematical demography has come of age. A body of theoretical models, grounded in the mathematics of time-inhomogeneous Markov chains, now exists. Associated with this are several empirical methods, based on actuarial and statistical principles, for fitting these models to real data and for using their outputs to project the future evolution of multidimensional populations. Finally, a number of impressive empirical applications of nonhierarchical increment-decrement life tables and population projection models have been made. In some instances, these applications have appeared in substantive areas where no multistate analyses had existed before (e.g., interregional migration). In others, the new applications represent substantial improvements over the techniques that were previously available (e.g., nuptiality, labor-force participation).

It is remarkable that these accomplishments span little more than a decade. Clearly, this has been a very active period in the development and application of multidimensional generalizations of the models of classical mathematical demography. Furthermore, since many of the individuals who made contributions to this field during the past decade still are active researchers, and since others in related areas of demography, mathematical statistics, sociology, and geography, have been made aware of this area of applications and its problems, it is reasonable to expect that the near future will also exhibit a rapid rate of innovations. What are some promising lines of inquiry along which such developments may be expected to occur? Based in part on discussions of this topic by participants in the Conference, we see several important directions of theoretical-methodological research and of substantive applications.

A first, and most obvious, theme for future theoretical-methodological inquiry pertains to extensions and generalizations of ideas and methods summarized and developed in the chapters of this volume. For instance, given the computational simplifications and other desirable features of the polynomial gross flows methods for abridged IDLTs developed in the chapter by Land and Schoen, it might be useful to develop extensions of these specifications to IDLTs generated by semi-Markov processes (the approach that commences with a specification of the force functions has been extended to a semi-Markov framework by Hoem, 1972a). Such generalizations would help demographers to deal with the origin- and/or duration-dependence known to affect some mobility processes. Similarly, it is clear that studies of the effects of population heterogeneity in unobservables, such as those summarized by Heckman and Singer and Kitsul and Philipov, have a strategic importance for multidimensional demography. The extension of the life table model to capture the interactions between the sexes, as described by Schoen, opens up numerous theoretical and methodological issues. One of the most important questions is whether two-sex models exhibit weak or stochastic ergodicity. That is, can multistate ergodic theorems, such as those presented by Cohen chapter in this volume, be modified to apply to two-sex models? Since the rates defined in these models exhibit a complicated interactive interdependence, this question seems to require a nontrivial transformation of existing theory. Finally, several of the issues of statistical estimation and projection developed by Hoem and Jensen, Ledent, and Philipov and Rogers will provide a continuing source of problems for the attention of mathematical demographers and statisticians. As in any area of scientific inquiry, these issues are essentially open-ended and in need of continual development and refinement.

The second methodological innovation that we expect to unfold in the near future is an application to multistate models of methods of controlling for population heterogeneity in observable variables that have been developed in fields related to multidimensional demography. For example, in the context of single-decrement life tables, proportional hazards models have been created by statisticians and used by mathematical demographers to deal with heterogeneity in the presence of concomitant information on covariates (see, for example, Cox, 1972; Holford, 1976, 1980; Laird and Oliver, 1981; Manton and Stallard, 1981; Menken *et al.*, 1981). Other methods for coping with population heterogeneity have been developed by mathematical sociologists and statisticians in the context of applications of Markov chains to microdata from panel studies and event histories (see, for example, Coleman, 1964, 1981; Singer, 1981; Singer and Spilerman, 1976a,b; Cohen and Singer, 1979; Singer and Cohen, 1980; Tuma *et al.*, 1979). The latter methods seem especially applicable to IDLTs with little modification, at least in the case of piecewise-constant transition forces. For other specifications, new methodological developments may be required.

This development of methods for dealing with population heterogeneity in multistate models is related to one of the main substantive innovations that we see forthcoming, namely, the utilization of alternative data and the refinement of existing data sources. Up to now, multistate models have been constructed primarily from aggregate data with little or no cross classification other than by sex, age-interval, and one or two status dimensions (for example, region of residence by region of birth as in Ledent, 1981). But in order to apply the methods of "covariance analysis," additional information will be required on relevant "covariates." This may require the use of microdata sets in place of the aggregate tabulations that have been utilized heretofore.

At the same time, efforts should be made to upgrade the information gathered in vital statistics and other sources in order to take advantage of the power and flexibility of the new methods described in this volume. For instance, while it is now easy to incorporate differential mortality by labor-force participation status into tables of working life, available data typically do not allow this to be done because death certificates do not record the labor-force status of the deceased at the time of death. Similar comments on inadequacies of data on population flows from censuses and current population surveys could be compiled (see, for example, Land and McMillen, 1981). But the general point here is that the capacity of the models seems to have outstripped the data used in multidimensional demography. It is appropriate, therefore, to suggest that census and vital statisticians should consider what modifications of their data collection procedures would allow these models to be used to their full potential.

Because changes in established governmental data collection procedures take time to implement, methods of inferring data from inadequate or inaccurate sources, problems of missing data, and related topics in the design and use of model multistate schedules should become a central branch of multistate modeling in the future. The data requirements for such modeling activities are extensive and, even when available, multistate data are difficult to comprehend and manipulate. In the large majority of cases, however, multidimensional data are simply not available at the level of detail required and must be inferred from available sources by such means as multiproportional adjustment techniques and model schedules.

Another line of substantive research that we expect to grow pertains to an expansion of the range of applications of multistate models. One way in which this will occur is through the construction of multistate models for additional types of transitions (e.g., schooling), situations (e.g., the marriage squeeze), and populations (e.g., a criminal offender population). Other studies will apply multistate models to the study of economic-demographic interactions (e.g., in the tradition of Coale and Hoover, 1958), or, more generally, to the analysis of social change (e.g., as in Land, 1979).

In brief, research in multidimensional mathematical demography during the next decade can be expected to proceed apace along these and related lines. While some developments will be primarily methodological, they almost surely will be motivated by strong connections to the empirical transitions in multistate space that have characterized contributions to this field in the recent past.

REFERENCES

Albert, A. (1962). Estimating the infinitesimal generator of a continuous time, finite state Markov process. *Annals of Mathematical Statistics 33,* 727-753.

Billingsley, P. (1961). "Statistical inference for Markov processes." Chicago: University of Chicago Press.

Blumen, I., Kogan, M., and McCarthy, P. J. (1955). "The Industrial Mobility of Labor as a Probability Process," Vol. 6. Cornell Studies of Industrial and Labor Relations. Ithaca: Cornell University Press.

Chiang, C. L. (1964). A stochastic model of competing risks of illness and competing risks of death. *In* "Stochastic Models in Medicine and Biology" (J. Gurland, ed.), pp. 323-354. Madison: University of Wisconsin Press.

Coale, A. J., and Hoover, E. M. (1958). "Population Growth and Economic Development in Low-Income Countries." Princeton, New Jersey: Princeton University Press.

Coale, A. J., and Demeny, P. (1966). "Regional Model Life Tables and Stable Populations." Princeton, New Jersey: Princeton University Press.

Coale, A. J., and Trussell, T. J. (1974). Model fertility schedules: Variations in the age structure of childbearing in human populations. *Population Index 40,* 185-258.

Cohen, J. E. (1976). Ergodicity of age structure in populations with Markovian vital rates, I: Countable states. *Journal of the American Statistical Association 71,* 335-559.

Cohen, J. E. (1977a). Ergodicity of age structure in populations with Markovian vital rates, II: General states. *Advances in Applied Probability 9,* 18-37.

Cohen, J. E. (1977b). Ergodicity of age structures in populations with Markovian vital rates, III: Finite-state moments and growth rates; Illustration. *Advances in Applied Probability 3,* 462-475.

Cohen, J. E., and Singer, B. (1979). Malaria in Nigeria: Constrained continuous-time Markov models for discrete-time longitudinal data on human mixed-species infections. *Lectures on Mathematics in the Life Sciences 12,* 69-133.

Coleman, J. S. (1964). "Introduction to Mathematical Sociology." New York: Free Press.

Coleman, J. S. (1972). Flow models for occupational structure. *In* "Input-Output Techniques" (A. Brody and A. P. Carter, eds.), pp. 80-93. Amsterdam: North Holland.

Coleman, J. S. (1981). "Longitudinal Data Analysis." New York: Basic Books.

Cox, D. R. (1972). Regression models and life tables (with discussion). *Journal of the Royal Statistical Society (Series B) 34,* 187-220.

Doob, J. L. (1953). "Stochastic Processes." New York: Wiley.

Du Pasquier, L. G. (1912). Mathematische theorie der invalidätatsversicherung. *Mitt. Ver. Schweitzer, Versicherungsmath 7,* 1-7.

Du Pasquier, L. G. (1913). Mathematische theorie der invalidätatsversicherung. *Mitt. Ver. Schweitzer, Versicherungsmath 8,* 1-153.

Espenshade, T. (1980). Personal communication.

Feeney, G. M. (1970). Stable age by region distributions. *Demography 6,* 341-348.

Fix, E., and Neyman, J. (1951). A simple stochastic model of recovery, relapse, death and loss of patients. *Human Biology 23,* 205-41.

Ginsberg, R. B. (1971). Semi-Markov processes and mobility. *The Journal of Mathematical Sociology 1,* 233-262.

Ginsberg, R. B. (1972a). Critique of probabilistic model: Application of the semi-Markov model to migration. *The Journal of Mathematical Sociology 2,* 63-82.

Ginsberg, R. B. (1972b). Incorporating causal structure and exogenous information with probabilistic models: With special reference to choice, gravity, migration, and Markov chains. *The Journal of Mathematical Sociology 2,* 83-104.

Goodman, L. (1969). The analysis of population growth when the birth and death rates depend upon several factors. *Biometrics 25,* 659-681.

Hoem, J. M. (1970a). A probabilistic approach to nuptiality. *Biometrie-Praximetrie 11,* 3-9.

Hoem, J. M. (1970b). Probabilistic fertility models of the life table type. *Theoretical Population Biology 1,* 12-38.

Hoem, J. M. (1971). Point estimation of forces of transition in demographic models. *Journal of the Royal Statistical Society (Series B) 33,* 275-289.

Hoem, J. M. (1972a). Inhomogeneous semi-Markov processes, select actuarial tables, and duration-dependence in demography. *In* "Population Dynamics" (T. N. E. Greville, ed.), pp. 251-296. New York: Academic Press.

Hoem, J. M. (1972b). On the statistical theory of analytic graduation. *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability 1,* 569-600.

Hoem, J. M., and Fong, M. S. (1976). A Markov chain model of working life tables. Working Paper No. 2. Denmark: University of Copenhagen, Laboratory of Actuarial Mathematics.

Holford, T. R. (1976). Life tables with concomitant information. *Biometrics 32,* 587-597.

Holford, T. R. (1980). The analysis of rates and survivorship using log-linear models. *Biometrics 36,* 299-306.

Jordan, C. W. (1967). "Life Contingencies." Second Edition. Chicago: Society of Actuaries.

Keyfitz, N. (1977). "Introduction to the Mathematics of Population, with Revisions." Reading, Massachusetts: Addison-Wesley.

Keyfitz, N. (1979). Multidimensionality in population analysis. *Sociological Methodology 1980,* 191-218.

Krishnamoorthy, S. (1979). Classical approach to increment-decrement life tables: An application to the study of the marital status of United States females, 1970. *Mathematical Biosciences 44,* 139-154.

Laird, N., and Oliver, D. (1981). Covariance analysis of censored survival data using log-linear analysis techniques. *Journal of the American Statistical Association 76,* 231-240.

Land, K. C. (1979). Modeling macro social change. *Sociological Methodology 1980,* 219-279.

Land, K. C., and McMillen, M. M. (1981). Demographic accounts and the study of social change, with applications to the post-World War II United States. *In* "Social Accounting Systems" (F. T. Juster and K. C. Land, eds.). New York: Academic Press.

Le Bras, H. (1971). Equilibre et croissance de populations soumises à des migrations. *Theoretical Population Biology 2,* 100-121.

Le Bras, H. (1977). Une formulation générale de la dynamique des population. *Population* (special issue), 261-293.

Ledent, J. (1978). "Some Methodological and Empirical Considerations in the Construction of Increment-Decrement Life Tables." RM-78-25. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Ledent, J. (1980). Multistate life tables: Movement versus transition perspectives. *Environment and Planning A 12,* 533-562.

Ledent, J. (1981). Constructing multiregional life tables using place-of-birth-specific migration data. *In* "Advances in Multiregional Demography" (A. Rogers, ed.), pp. 35-49. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Liaw, K. L. (1978). Dynamic properties of the 1966-1971 Canadian population systems. *Environment and Planning A 10,* 389-398.

Long, J. (1980). Survey of federally-produced national level projections. Unpublished manuscript. Washington, D.C.: U.S. Bureau of the Census, Population Division.

Manton, K. G., and Stallard, E. (1981). Methods for the analysis of mortality risks across heterogeneous small populations: Examination of space-time gradients in cancer mortality in North Carolina counties. *Demography 18,* 217-230.

Meier, P. (1955). Note on estimation in a Markov process with constant transition rates. *Human Biology 27,* 121-124.

Menken, J., Trussell, J., Stempel, D., and Babakol, O. (1981). Proportional hazards life table models: An illustrative analysis of socio-demographic influences on marriage dissolution in the United States. *Demography 18,* 181-200.

Oechsli, F. W. (1975). A population model based on a life table that includes marriage and parity. *Theoretical Population Biology 7,* 229-245.

Preston, S., Keyfitz, N., and Schoen, R. (1972). "Causes of Death: Life Tables for National Populations." New York: Seminar Press.

Rees, P. (1980). Multistate demographic accounts. *Environment and Planning A12,* 499-531.

Rees, P., and Wilson, A. G. (1977). "Spatial Population Analysis." London: Edward Arnold.

Rogers, A. (1966). The multiregional matrix growth operator and the stable interregional age structure. *Demography 3,* 537-544.

Rogers, A. (1968). "Matrix Analysis of Interregional Population Growth and Distribution." Berkeley, California: University of California Press.

Rogers, A. (1973a). The mathematics of multiregional demographic growth. *Environment and Planning 5,* 3-29.

Rogers, A. (1973b). The multiregional life table. *The Journal of Mathematical Sociology 3,* 127-137.

Rogers, A. (1975). "Introduction to Multiregional Mathematical Demography." New York: Wiley.

Rogers, A. (1980). Introduction to multistate mathematical demography. *Environment and Planning A12,* 489-498.

Rogers, A., and Ledent, J. (1976). Increment-decrement life tables: A comment. *Demography 13,* 287-290.

Schoen, R. (1975). Constructing increment-decrement life tables. *Demography 12,* 313-324.

Schoen, R., and Land, K. C. (1979). A general algorithm for estimating a Markov-generated increment-decrement life table for applications to marital status patterns. *Journal of American Statistical Association 74,* 761-776.

Schoen, R., and Nelson, V. E. (1974). Marriage, divorce and mortality: a life table analysis. *Demography 11,* 267-290.

Schoen, R., and Woodrow, K. (1980). Labor force status life tables for the United States, 1972. *Demography 17,* 297-322.

Shryock, H. S., and Siegel, J. S. (1973). "The Methods and Materials of Demography." Two volumes. Washington, D.C.: U.S. Government Printing Office.

Singer, B. (1981). Estimation of nonstationary Markov chains from panel data. *Sociological Methodology 1981,* 319-337.

Singer, B., and Cohen, J. E. (1980). Estimating malaria incidence and recovery rates from panel surveys. *Mathematical Biosciences 49,* 273-305.

Singer, B., and Spilerman, S. (1976a). Some methodological issues in the analysis of longitudinal surveys. *Annals of Economic and Social Measurement 5,* 447-474.

Singer, B., and Spilerman, S. (1976b). The representation of social processes by Markov models. *American Journal of Sociology (July) 82,* 1-54.

Stone, R. (1971). "Demographic Accounting and Model-Building." Paris: Organization for Economic Cooperation and Development.

Stone, R. (1971). The relationship of demographic accounts to national income and product accounts. *In* "Social Accounting Systems" (F. T. Juster and K. C. Land, eds.). New York: Academic Press.

Sverdrup, E. (1965). Estimates and test procedures in connection with stochastic models for deaths, recoveries, and transfers between different states of health. *Skandinavisk Aktuarietidskrift 40,* 184-211.

Tuma, N. D., Hannan, M. T., and Groeneveld, L. P. (1979). Dynamic analysis of event histories. *American Journal of Sociology 84,* 820-854.

Willekens, F. (1980). Multistate analysis: Tables of working life. *Environment and Planning A12,* 563-589.

Willekens, F., and Rogers, A. (1978). "Spatial Population Analysis: Methods and Computer Programs." RR-78-18. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Zahl, S. (1955). A Markov process model for follow-up studies. *Human Biology 27,* 90-120.

RELATED IIASA PUBLICATIONS

Migration and Settlement: Selected Essays (Reprinted from a Special Issue of Environment and Planning A) Andrei Rogers (Editor)	RR-78-6
Spatial Population Analysis: Methods and Computer Programs Frans Willekens and Andrei Rogers	RR-78-18
Migration Patterns and Population Redistribution (Reprinted from Regional Science and Urban Economics) Andrei Rogers	RR-80-7
Essays in Multistate Demography (Reprinted from a Special Issue of Environment and Planning A) Andrei Rogers (Editor)	RR-80-10
Multidimensionality in Population Analysis (Reprinted from Sociological Methodology 1980) Nathan Keyfitz	RR-80-33
Advances in Multiregional Demography Andrei Rogers (Editor)	RR-81-6
Simplified Multiple Contingency Calculations (Reprinted from The Journal of Risk and Insurance) Nathan Keyfitz and Andrei Rogers	RR-82-30