APPLICATION OF THE GENERALIZED REACHABLE SETS METHOD TO WATER RESOURCES PROBLEMS IN SOUTHWESTERN SKÅNE, SWEDEN

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resource management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This paper is part of a collaborative study on water resources problems in South Western Skane, Sweden, pursued by IIASA in collaboration with the Swedish National Environmental Protection Board and the University of Lund. The paper reports on the results of an application of the Generalized Reachable Sets (GRS) method to the multiobjective water resources allocation problem in the region of South Western Skane. It is a companion paper to the earlier IIASA Working Paper WP-81-145 which describes the GRS method itself.

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CONTENTS

INTRODUCTION

This collaborative research was carried out as a part of investigations undertaken by the International Institute for Applied Systems Analysis (IIASA) on the methods for the rational allocation of natural resources. One of the problems which was used for testing these methods is allocation of water resources in the Southwestern Skåne Region of Sweden [1]. In this region, during low-flow periods, especially in dry years, the problem of water allocation between agricultural and municipal users arises. For the solution of this problem, economic, social and environmental consequences of the allocation decisions should be taken into account. The work discussed herein is based on the model of the Kävlinge River System [2], but a different multiobjective optimization method is applied.

THE METHOD

The Generalized Reachable Sets (GRS) approach was developed for analysis of mathematical models with exogenous variables (see detailed description in [3]). This approach makes it possible to present explicitly the information contained in a model in an aggregated form. Within the framework of a man-computer system, the GRS approach can be applied for analysis of the multiobjective decision-making problems at the screening level of such analysis. This approach allows us to describe the set of all objective function values which are reachable under feasible alternatives. The description has the form of an intersection of a finite number of hemispaces. If the relationships used for model formulation, as well as the objective functions, are linear algebraic, then the set of reachable values of objective functions can be described precisely. If convex functions are used in the model formulation, then the approach can provide an approximation of the reachable set. If the model is nonconvex, then it is possible to obtain a convex approximation of the reachable sets.

In the following, a formal definition of the GRS is presented for a linear static model. The model description has the form of a finite number of linear inequalities

$$A\, x \leqq b, \; x \in E^n, \tag{1}$$

with A and b being a matrix and a vector specified; $x \in E^n$ is a vector of variables. The objective functions are specified in the form

$$f = F\, x \tag{2}$$

with F being a matrix specified; $f \in E^r$ is a vector of objective functions. The GRS for this model is the set

$$G_f = \{f\colon f = F\, x, \; A\, x \leqq b\} \tag{3}$$

This definition gives a description of the GRS in an implicit form. The approach suggested herein allows one to obtain an explicit description of the GRS in the form

$$G_f = \{f\colon D_f \leqq d\} \; . \tag{4}$$

To compute the matrix D and vector d, convolution methods for linear inequalities introduced by Fourier and developed further by a number of other authors are used. The GRS is constructed in advance to allow the decision maker participation in a real-time man-computer dialogue leading to investigation of all feasible alternatives of the system under study. It is necessary to underline that the GRS approach does not substitute for other alternative multiobjective methods, but it can be used as a screening technique to formulate weights, reference objectives, etc. The main area of the GRS application is the analysis of multiobjective problems, but the aggregation and interfacing of mathematical models as well as stability investigation in the simulation system may also be carried out with the aid of the GRS approach.

THE MODEL OF THE SYSTEM

In this paper the GRS method is applied for analysis of water resources allocation in the Kävlinge River System, during the summer months of low precipitation. The difficulties in allocation of water quantity are combined with the water quality problems arising from the use of fertilizers in the agricultural sector; the chemicals are partly brought to the Kävlinge River with irrigation return water.

The scheme of the Kävlinge River System is presented in Figure 1. The Kävlinge River flows out of the Vomb Lake. Water release to the Kävlinge River from the Vomb Lake is regulated. The Vomb Lake serves as a source of municipal water supply for the city of Malmö. There are three aggregated agricultural regions using water from the Kävlinge River System for irrigation. At the control point near the river estuary, flow and the concentration of the pollutants in the Kävlinge River are monitored.

To make application of the GRS method possible, the original model [2] was slightly modified. The agricultural production is described by means of N irrigation technologies. Let x_{ij} be the area brought under irrigation in the j-th region, $j = 1,2,3$, with i-th type of irrigation technology, $i = 1,\ldots,N$.

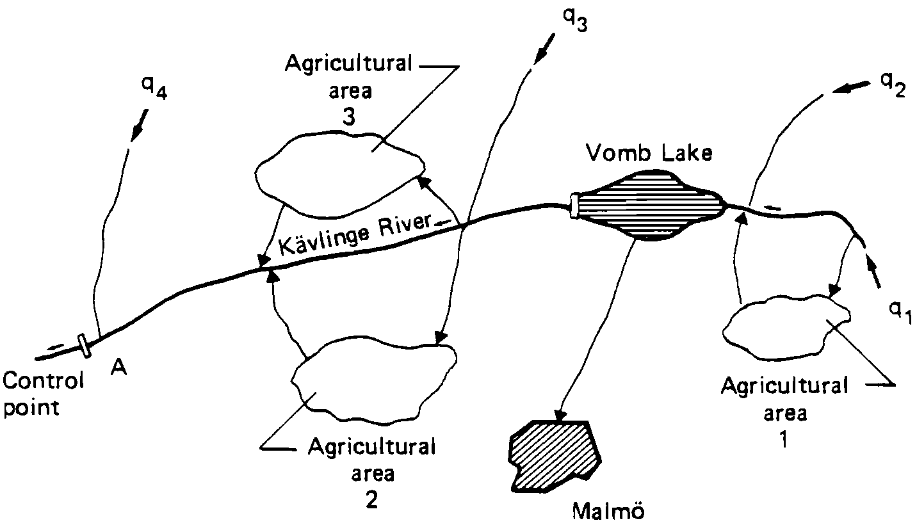

Figure 1. General scheme of the Kävlinge River System.

The irrigated area in each region is restricted by total area available

$$\sum_{i=1}^{N} x_{ij} = a_j \quad , \quad j = 1,2,3. \tag{5}$$

The variables x_{ij} are nonnegative

$$x_{ij} \geqq 0, \quad i = 1,\ldots, N, \quad j = 1,2,3. \tag{6}$$

The agricultural production in the j-th region is described by means of following indices:

y_{j1} - yield effect due to irrigation and fertilization in the j-th region (kg);

y_{j2} - monthly irrigation water withdrawals to the j-th region (m^3);

y_{j3} - amount of fertilizer applied (kg);

y_{j4} - monthly return flow (m^3);

y_{j5} - chemicals in return flow (kg).

These indices are calculated with the use of coefficients a_{kij} where k is the index number, i is the type of irrigation technology and j denotes the region

$$y_{jk} = \sum_{i=1}^{N} a_{kij}\, x_{ij}, \quad j = 1,2,3, \quad k = 1,\ldots,5. \tag{7}$$

The relationships (5) to (7) describe the agricultural production. The coefficients a_{kij} are specified in Table 1 on the basis of information presented in [2] for N = 7.

It should be noted that in the first technology column, irrigation and fertilization are absent. This results in the absence of yield effect. The pollution coefficients a_{5ij} are based on the assumption that about 15% of the fertilizers is brought back to the river with the irrigation return flow.

Table 1. The values of a_{kij} coefficients.

$j = 1$

k \ i	1	2	3	4	5	6	7	Unit
1	0	4000	5500	8000	4500	6800	9500	kg/ha
2	0	300	300	300	550	550	550	m^3/ha
3	0	0	80	150	0	80	150	kg/ha
4	0	60	60	60	110	110	110	m^3/ha
5	0	0	12	22.5	0	12	22.5	kg/ha

$j = 2,3$

k \ i	1	2	3	4	5	6	7	Unit
1	0	4000	5500	9200	4500	6800	10800	kg/ha
2	0	300	300	300	550	550	550	m^3/ha
3	0	0	80	180	0	80	180	kg/ha
4 j=2 j=3	0	30 60	30 60	30 60	55 110	55 110	55 110	m^3/ha
5	0	0	12	27	0	12	27	kg/ha

The total agricultural areas in each of three regions are:
$a_1 = 3\,000$ ha; $a_2 = 2\,500$ ha; $a_3 = 2\,300$ ha.

The coefficients for the second and third regions are equal (except for the return flow). The coefficients of irrigation are specified under the assumption that the value of precipitation in this month equals 10 mm.

Let us describe now the water and pollution balances. Let q_1, q_2, q_3 and q_4 be mean monthly inflows to the system (1.8, 1.5, 0.8 and 0.7 m^3/s respectively). The actual storage volume in the Vomb Lake is equal to:

$$S = S_0 + (q_1 + q_2)T - (Z_k + Z_M)T + y_{14} - y_{12} \quad (8)$$

with T being the length of the time period (1 month = 2.59 x 10^6 sec), S_0 being the initial storage volume of the lake (3 x 10^7 m^3), Z_k being the release from the lake to the Kävlinge River (m^3/s), Z_M being the water intake for Malmö (m^3/s). Here the assumption is made that the values of inflows, releases, withdrawals and intakes are constant during the month under study.

The flow rate in the Kävlinge River at the control point A denoted by ν_A (m^3/s) is

$$\nu_A = Z_k + q_3 + q_4 + \frac{1}{T}(y_{24} + y_{34}) - \frac{1}{T}(y_{22} + y_{32}) \quad (9)$$

The pollution flow at the point A denoted by ω_A (kg/s) is

$$\omega_A = \frac{1}{T}(y_{25} + y_{35}) + \psi_3 \cdot q_3 + \psi_4 \cdot q_4 + \omega_V \quad (10)$$

with ψ_3 and ψ_4 being the initial pollution concentrations in the third and fourth inflows respectively, ω_V being the pollution outflow from the Vomb Lake. The value of the ω_V (kg/s) is calculated as

$$\omega_V = (1 - \phi)\ (\psi_1 \cdot q_1 + \psi_2\, q_2 + \frac{1}{T} \cdot y_{15}) \quad (11)$$

with ψ_1 and ψ_2 being the initial concentrations in the first and the second inflows respectively, the coefficient ϕ describing reduction of the pollution in the Vomb Lake. ($\psi_1 = \psi_2 = 10^{-3}$ kg/m^3;

$\psi_3 = 2 \times 10^{-3}$ kg/m^3; $\psi_4 = 1.5 \times 10^{-3}$ kg/m^3; $\phi = 0.9$).

The following constraints on the water and pollution balances must be taken into account: the nonnegativity restrictions,

$$Z_k \geqq 0 \; , \qquad (12)$$

$$Z_M \geqq 0 \; , \qquad (13)$$

$$S \geqq 0 \; , \qquad (14)$$

the physical constraints on water withdrawals,

$$y_{12} \leqq T \cdot q_1 \; , \qquad (15)$$

$$y_{22} + y_{32} \leqq T(Z_k + q_3) \; , \qquad (16)$$

and constraints related to environmental requirements

$$\nu_A \geqq \nu_A^* \; , \qquad (17)$$

$$\frac{\omega_A}{\nu_A} \leqq \omega_A^* \qquad (18)$$

The constraint (17) requires that the flow in the Kävlinge River at point A denoted by ν_A be not less than $\nu_A^* = 6$ m^3/s. The constraint (18) show that pollution concentration at point A should not be greater than $\omega_A^* = 10$ g/m^3. Furthermore, there is a constraint indicating that water intake for Malmö cannot exceed the amount required ($Z_M^* = 2$ m^3/s):

$$Z_M \leqq Z_M^* \; . \qquad (19)$$

Finally, the volume of water storage in the Vomb Lake should not be greater than S^* which is optimal from environmental and recreational points of view ($S^* = 29 \times 10^6$m^3):

$$S \leqq S^* . \qquad (20)$$

The objectives of the system performance are the same as in (2).

1. Maximize agricultural yield effects due to irrigation and fertilization

$$J_1 = y_{11} ; \quad J_2 = y_{21} ; \quad J_3 = y31$$

2. Minimize water deficit in Malmö

$$J_4 = z_M^* - z_M$$

3. Minimize the excess over the flow required at the control point A

$$J_5 = \nu_A - \nu_A^*$$

4. Minimize the deviation from the optimal level of the Vomb Lake

$$J_6 = b(S^* - S)$$

with b being the coefficient relating water level to the storage volume of the lake ($b = 6 \times 10^{-7}\ 1/m^2$).

5. Minimize the flow rate of pollutants at the control point A.

$$J_7 = \omega_A .$$

THE RESULTS

For the given set of data, the analysis of the model shows that constraints (12), (15), (16), (18) and (20) are consequences of other relationships of the model. In the case of constraints (15) and (16) this implies that the maximum irrigation water withdrawals can be satisfied by the available water resources.

For the constraint (18) it means that any feasible alternative of water allocation will result in pollution concentration that is less than the allowable level ω_A^*. For the constraints (12) and (20) this means that in order to obtain sufficient flow at the control point A it is always necessary to release water to the Kävlinge River and to reduce the actual storage volume of the Vomb Lake below the optimal one.

The dependance of the yield effect in the first region y_{11} on the irrigation water withdrawal y_{12} and the amount of fertilizer y_{13} are shown in Figures 2 and 3. In Figure 2 the sets of pairs $\{y_{11}, y_{12}\}$ are presented, for fixed values of y_{13} while no additional constraints are imposed on other variables. Each set in Figure 2 corresponds to the values of y_{13} shown in Table 2.

Table 2. Fertilizer application rates for analysis of the dependence of yield effects in the first agricultural region on irrigation water withdrawals (see Figure 2).

Set number	1	2	3	4	5	6
y_{13} (x 10^5 kg)	4.3	3.8	3.0	2.0	1.0	0.0

For example, set No. 4 (y_{13} = 2 x 10^5 kg) shows that using y_{12} = 1.5 x $10^6 m^3$ of water it is possible to obtain the yield effect between 1.8 x 10^7 kg and 1.9 x 10^7 kg. Point M in Figure 2 corresponds to the maximum yield effect (y_{11} = 2.8 x 10^7 kg). Since we are looking for the maximum yield effect of the given resources, curve ABCD is the most interesting subset of the set No. 4 considered. We describe this curve (the set of efficient points) by the function y_{11} (y_{12}). The dependence of this function on y_{13} (as a parameter) can be seen in Figure 2. The breaking points of curves $y_{11}(y_{12})$ correspond to the transitions from one efficient irrigation technology to another. As may be easily recognized in Figure 2, the marginal utility of water resources declines sharply. For example, the yield effects in points C and D are roughly the same, so it is reasonable to save water that would be necessary to reach point D.

In Figure 3, the sets of pairs $\{y_{11}, y_{13}\}$ are presented, while the values of y_{12} are fixed now. Therefore, Figure 3

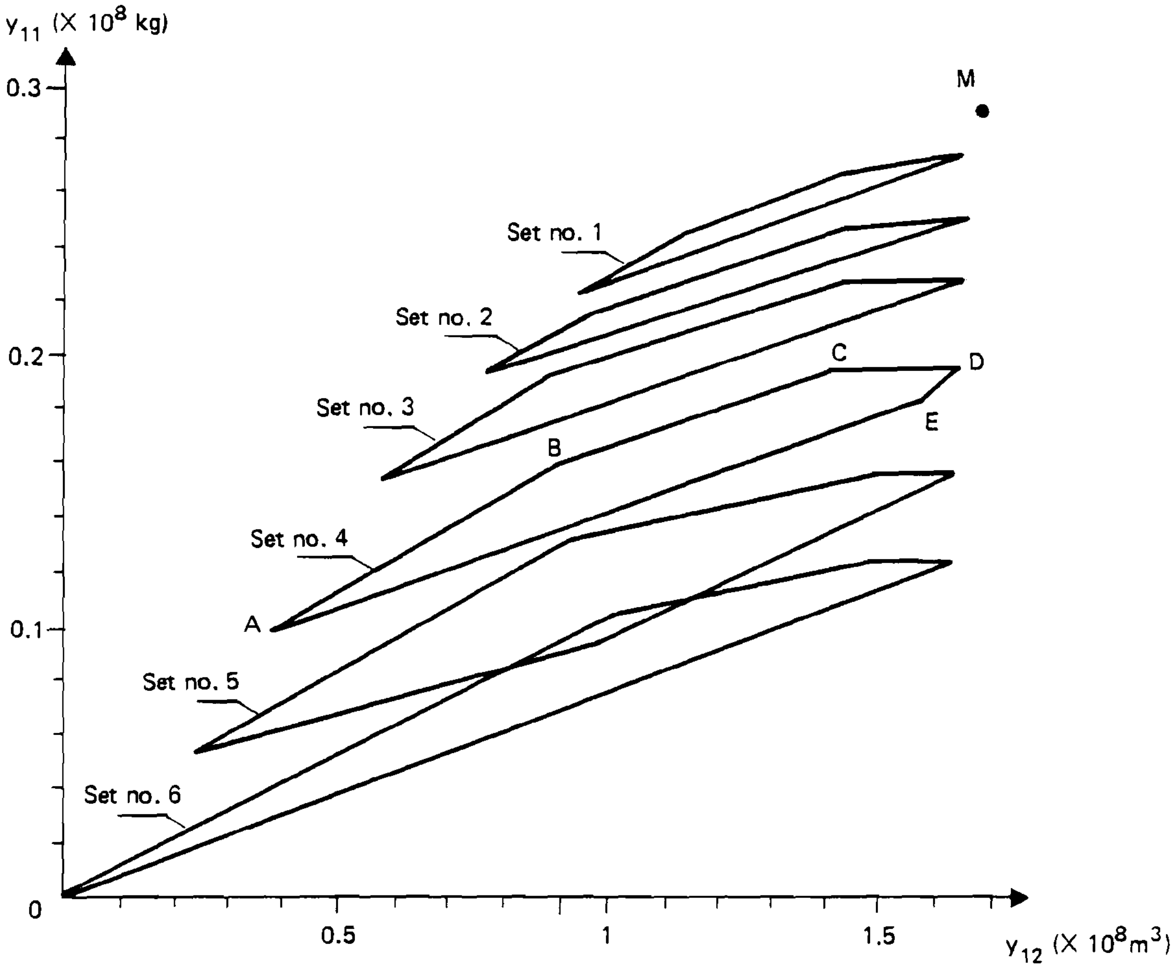

Figure 2. The dependence of yield effects (y_{11}) in the first agricultural region on irrigation water withdrawals (y_{12}) for six fertilizer application rates (y_{13}) (see Table 2).

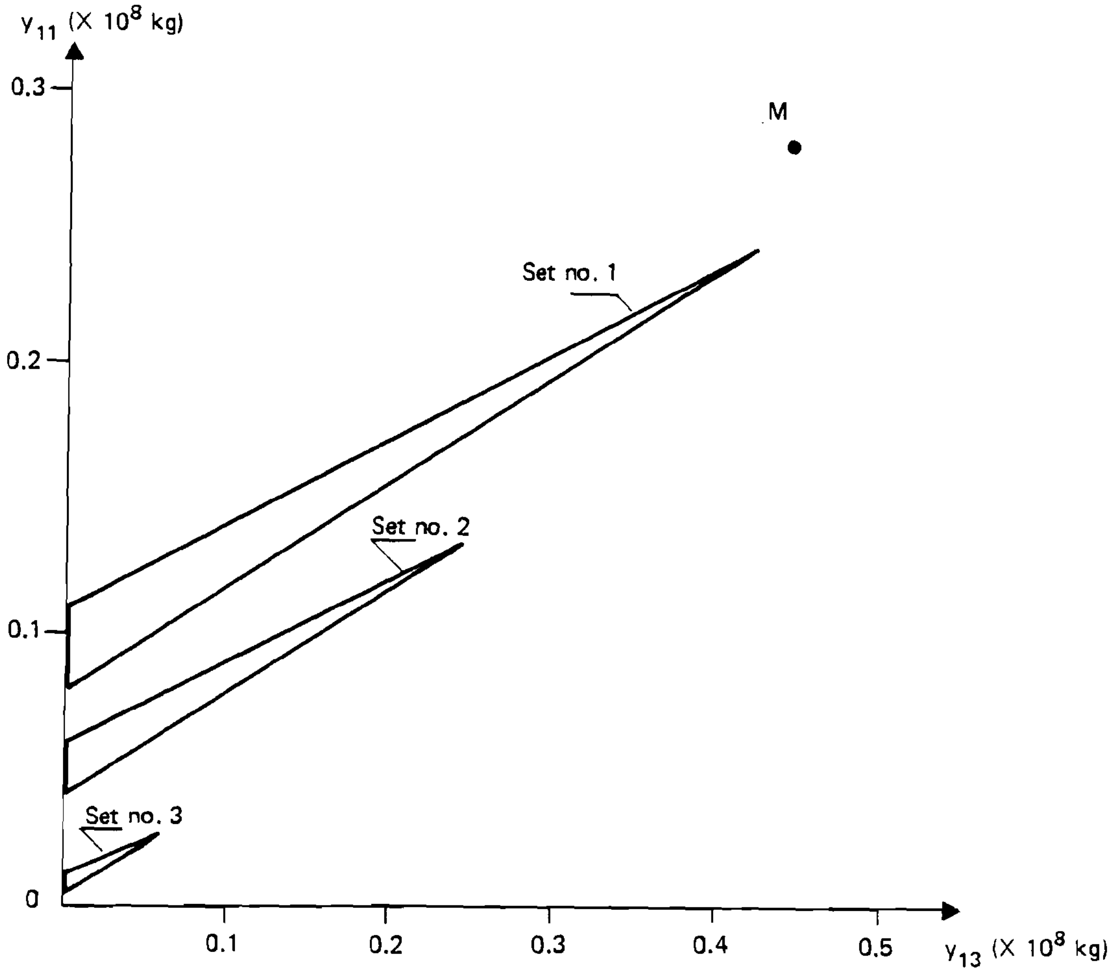

Figure 3. The dependence of yield effects (y_{11}) in the first agricultural region on fertilizer application rates (y_{13}) for three irrigation water withdrawal levels (y_{12}).

contains the same information as Figure 2 but this information is expressed in the alternative form. Set No. 1 corresponds to $y_{12} = 1.0 \times 10^6\ m^3$, set No. 2 to $y_{12} = 0.5 \times 10^6 m^3$, and set No. 3 to $y_{12} = 0.1 \times 10^6\ m^3$. Point M has the same meaning as in Figure 2. It is easy to recognize that the marginal utility of fertilizers is approximately constant in this case.

The dependence of the yield effect on the resources of the second and the third regions are of the same type, and corresponding figures are not presented in this paper.

Now we shall discuss the interdependence between yield effects in agricultural regions, water intake for Malmö and the level of the Vomb Lake (Figures 4-6). In Figure 4 the dependence of the deviations from the optimal level of the Vomb Lake upon yield effect in the first agricultural region are presented, with the values of yield effects in other regions and water intake for Malmö being fixed. The curves are parts of the GRS's boundaries. The correspondence between different curves and the values of fixed variables is presented in Table 3.

Table 3. Yield effects in the second and third agricultural regions and water deficit in Malmö, for analysis of the dependence of deviations from the optimal level of Vomb Lake on yield effects in the first agricultural region (see Figure 4)

Curve number	1	2	3	4	5	6
$J_2 (\times 10^8$ kg)	0.2	0.2	0.2	0.25	0.27	0.0
$J_3 (\times 10^8$ kg)	0.2	0.2	0.2	0.23	0.245	0.0
Water deficit in Malmö	50%	25%	0	0	0	0

It is easy to recognize that the level of the lake is sharply declining when the value of J_1 becomes greater than 0.24×10^8 kg. The water deficit in Malmö makes it possible to increase the level of the lake (curves 1 and 2) but certainly

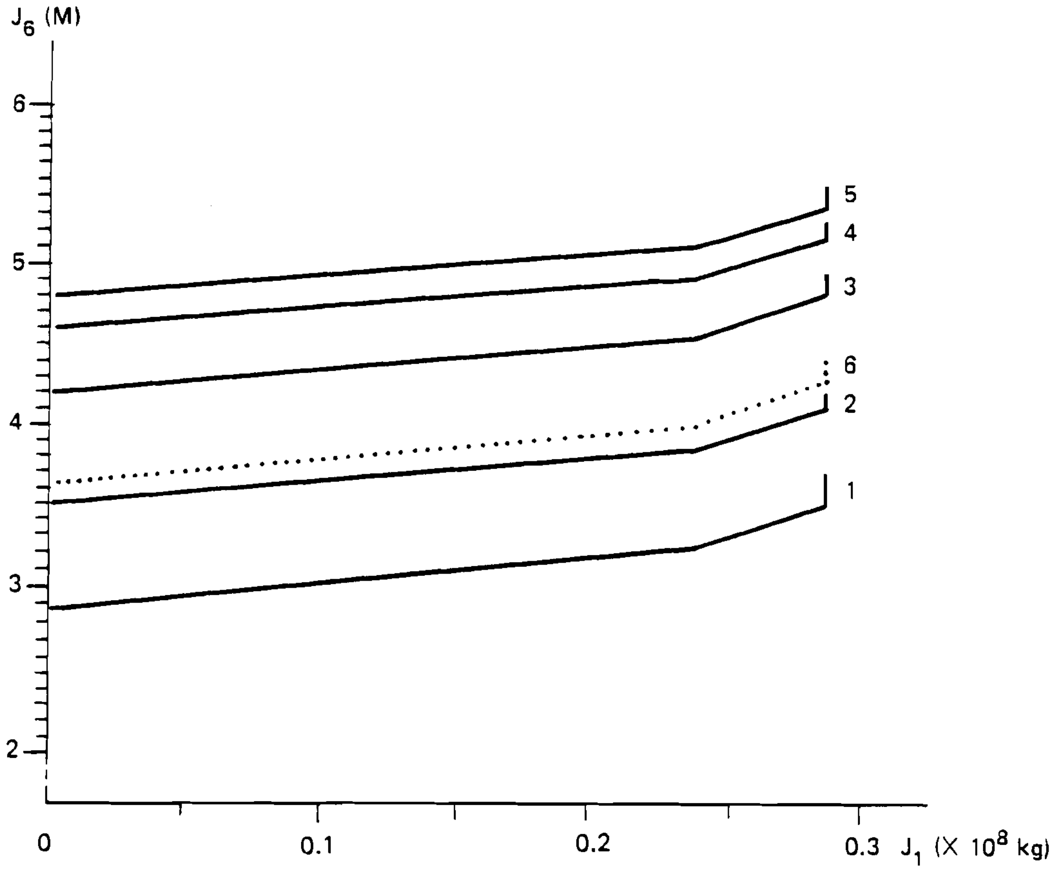

Figure 4. The dependence of deviations from the optimal level of the Vomb Lake (J_6) on yield effects in the first agricultural region (J_1) for fixed values of other variables (see Table 3).

the values of 25% and 50% water deficit are too high. The interdependence between water deficit in Malmö and the level of the Vomb Lake can be studied in more detail using Figures 5 and 6.

In Figure 5, the sets of possible values of the yield effects in the first and second agricultural regions are presented, while the values of the yield effect in the third region as well as the water deficit in Malmö and the deviations of the level of the lake are fixed. The values of the fixed variables are presented in Table 4.

The main feature of Figure 5 is the sharp dependence of the yield effects on the lake level, while its deviation is between four and five meters (sets number 1-4). If the deviation from the optimal level is less than four meters, the irrigation-based agriculture cannot practically exist. The deviations greater than five meters are of virtually no importance to the agricultural production. The yield effects in the first and second region depend sharply on the yield effect in the third region as long as the latter is greater than 0.2×10^8 kg (sets number 5 and 6). The rise of yield effect in the third region up to 0.22×10^8 kg (without any decrease of yield in other regions) can be achieved by the virtually unimportant water deficit in Malmö of 3.5%.

Table 4. Yield effects in the third agricultural region, water deficit in Malmö, and deviations from the optimal level of Vomb Lake--for analysis of mutual dependence of yield effects in the first and second agricultural regions (see Figure 5).

Set number	1	2	3	4	5	6	7	8	9	10
$J_3 (\times 10^8$ kg)	0.2	0.2	0.2	0.2	0.24	0.22	0.18	0.22	0.22	0.2
Water deficit in Malmö	0	0	0	0	0	0	0	3.5%	7.5%	2.5%
Deviation of the lake level (m)	4	4.2	4.5	5	4.5	4.5	4.5	4.5	4.5	4.5

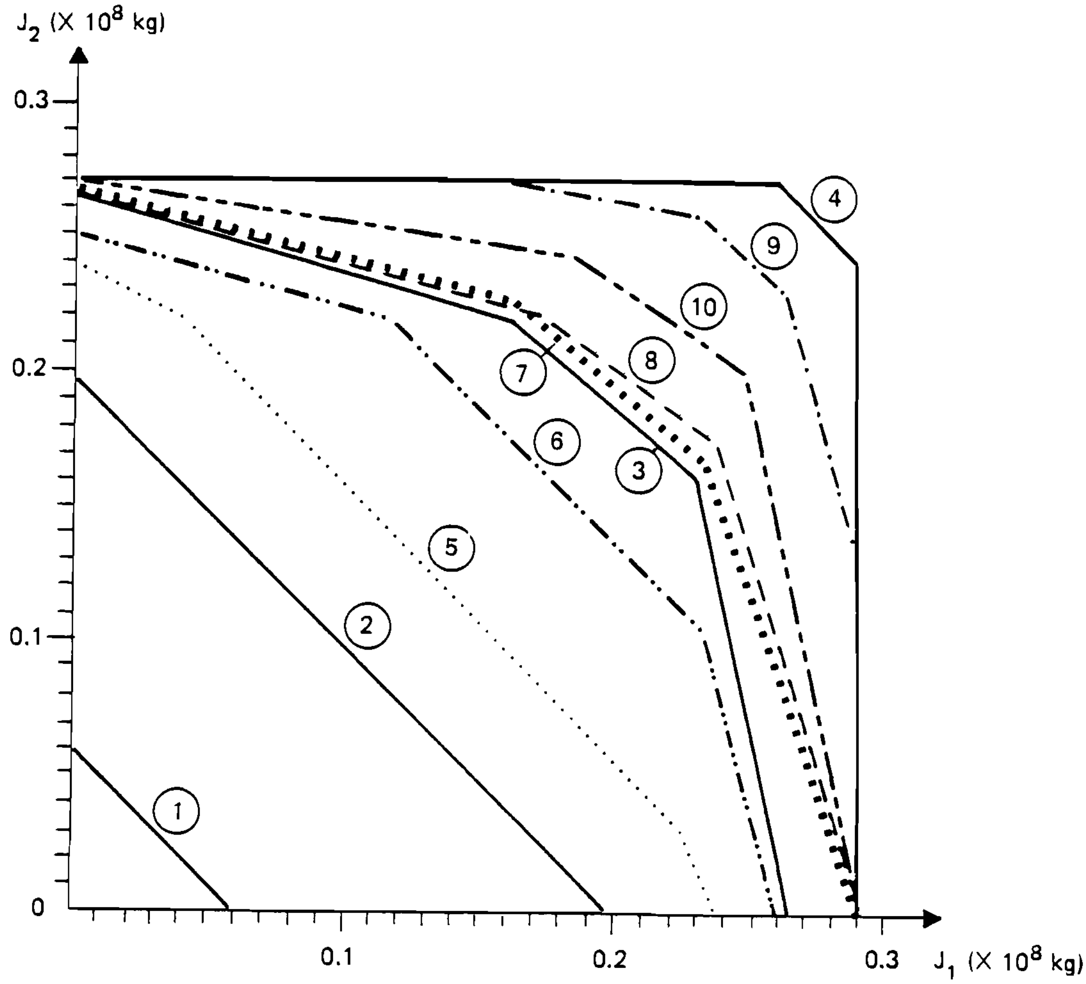

Figure 5. The mutual dependence of yield effects in the first and the second agricultural region (J_1 and J_2, respectively) for fixed values of other variables (see Table 4).

The dependence of water deficit in Malmö on the Vomb Lake's level is presented in Figure 6. Set number 1 corresponds to the small values of $y_{11} = 0.21 \times 10^8$ kg, $y_{21} = 0.2 \times 10^8$ kg, and $y_{31} = 10^8$ kg. Set number 2 corresponds to the large values of $y_{11} = 0.25 \times 10^8$ kg, $y_{21} = 0.25 \times 10^8$ kg, and $y_{31} = 0.24 \times 10^8$ kg.

The values of $J_5 = \nu_A$ and $J_7 = \omega_A$ remain to be examined. In Figure 7 the sets of reachable pairs $\{\nu_A, \omega_A\}$ are presented while the values of J_1, J_2, J_3, J_4, and J_6 are fixed. The correspondance between different sets and the values of other variables is presented in Table 5.

Table 5. Yield effects in three agricultural regions, water deficit in Malmö, and deviations from the optimal leval of Vomb Lake--for analysis of the dependence of an excess flow and the flow rate of pollutants in the control profile A (see Figure 7).

Set number	1	2	3	4	5	6
J_1 (x 10^8 kg)	0.23	0.23	0.20	0.18	0.20	0.23
J_2 (x 10^8 kg)	0.22	0.22	0.20	0.18	0.23	0.20
J_3 (x 10^8 kg)	0.21	0.21	0.20	0.18	0.21	0.21
Water deficit in Malmö	5%	3.5%	5%	5%	5%	5%
Deviation of the lake level (m)	4.5	4.5	4.5	4.5	4.5	4.5

Sets 1, 3 and 4 show that the decrease in agricultural yield will increase the Kävlinge River flow over the required rate by about 2%-3%. The comparison of sets 1 and 2 shows that the increase of water deficit in Malmö up to 5% will increase

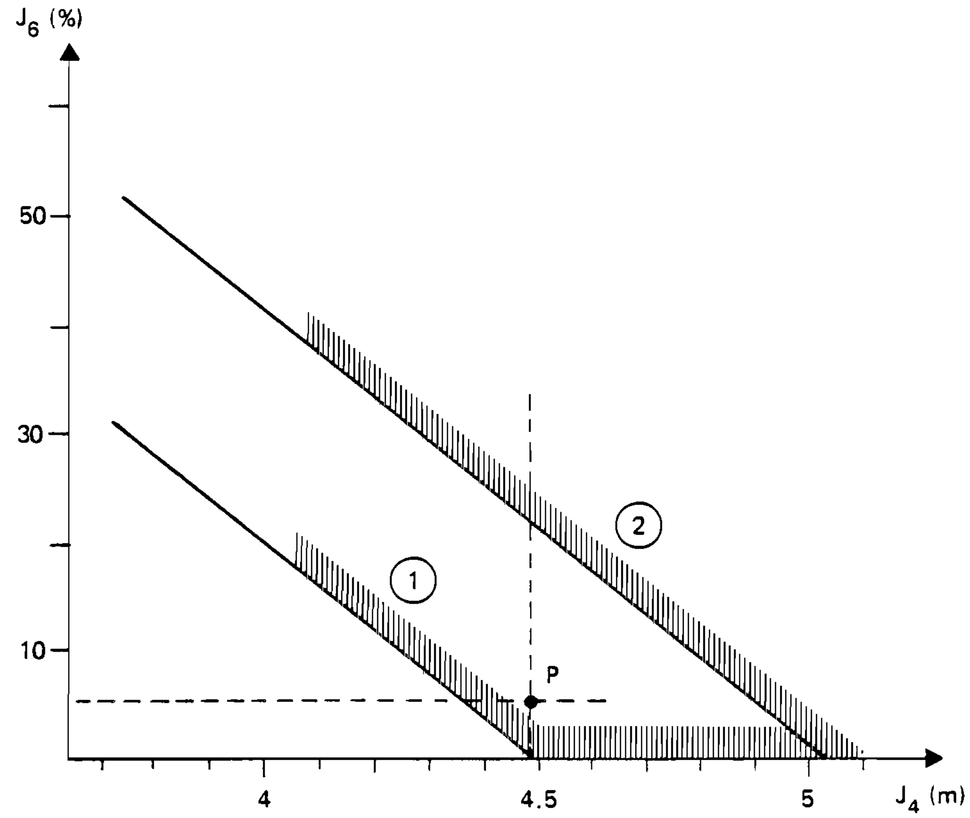

Figure 6. The dependence of water deficit in Malmö (J_6) on the possible deviations from the optimal level of Vomb Lake (J_6) for the small (set 1) and large (set 2) yield effects in all three agricultural areas.

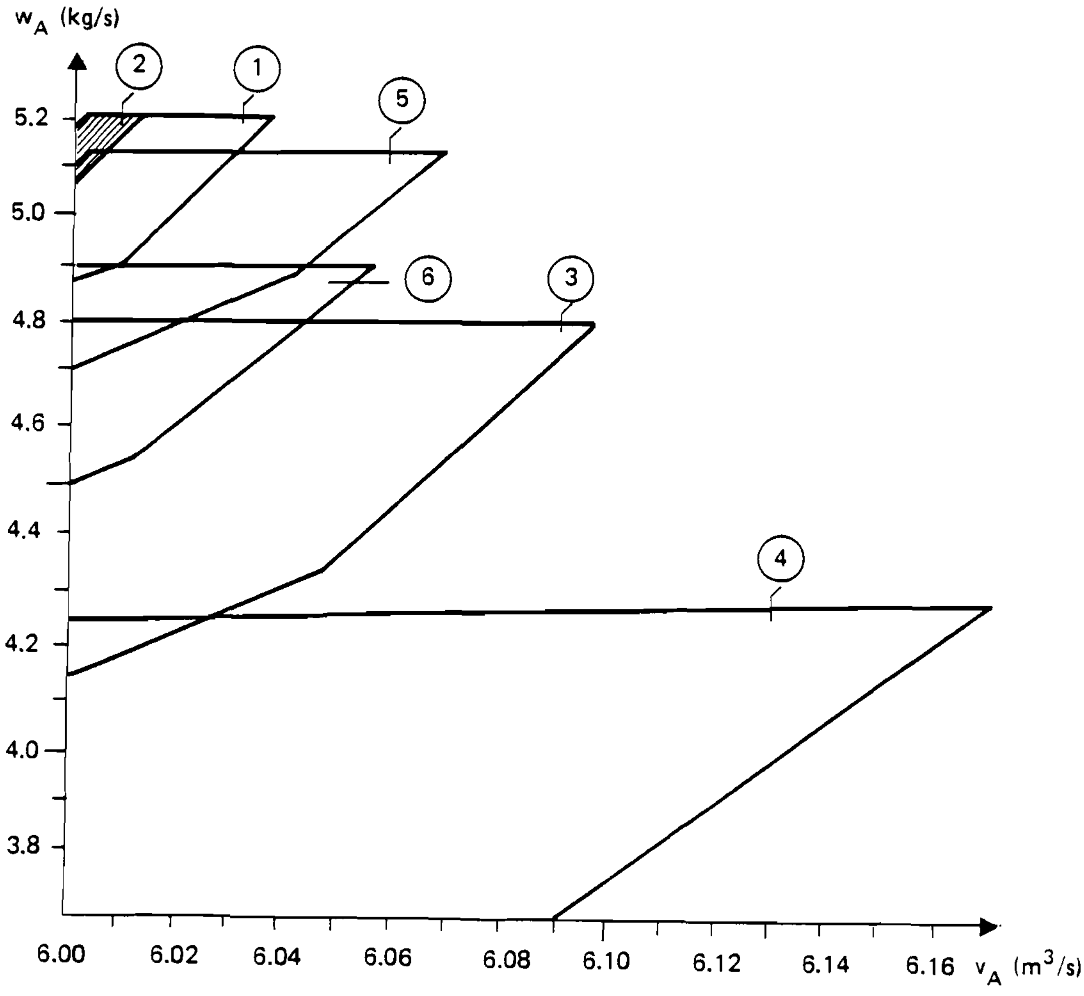

Figure 7. The dependence of an excess flow at the control point A (V_A) and the flow rate of pollutants at the same control point (W_A) for fixed values of other variables (see Table 5).

the flow over the minimum level by about 1%. Sets 5 and 6 show that it is more appropriate to decrease the yields in the second agricultural region than in the first one. The curves in Figure 7 indicate the potential for substitution between river pollution and the flow rates.

Figure 8 represents the interdependence between ν_A and y_{11}. The fixed values of other variables are presented in Table 6.

In Figure 9 the dependence between ω_A and y_{11} is presented. Set 1 corresponds to the following values of other variables: $J_2 = J_3 = 0.2 \times 10^8$ kg; $J_4 = 5\%$; $J_5 = 4.5$ m; and $J_6 = 0$. Set 2 corresponds to the same variables, except that $J_6 = 0.2\ m^3/s$.

Table 6. Yield effects in two agricultural regions, water deficit in Malmö, deviations from the optimal level of Vomb Lake, and flow rate of pollutants in the control profile A--for analysis of the dependence of yield effects in the first agricultural region and the excess flow at the control point A.

Set number	1	2	3
J_2 (x 10^8 kg)	0.2	0.2	0.2
J_3 (x 10^8 kg)	0.2	0.2	0.2
Water deficit in Malmö	5%	5%	5%
Deviation of the lake level (m)	4.5	4.5	4.5
J_7 (kg/s)	4	4.5	4.85

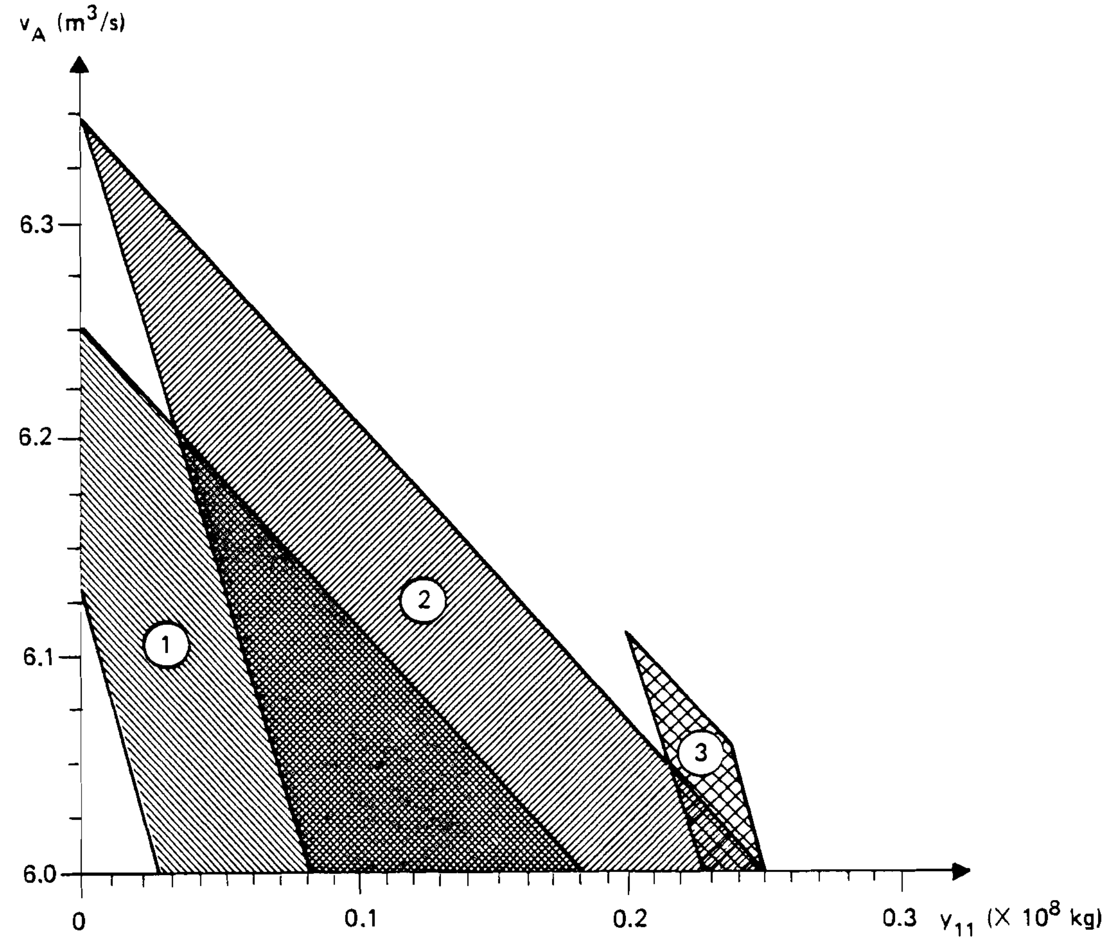

Figure 8. The dependence of yield effects in the first agricultural region (y_{11}) and the excess flow at the control point A (v_A) for fixed values of other variables (see Table 6).

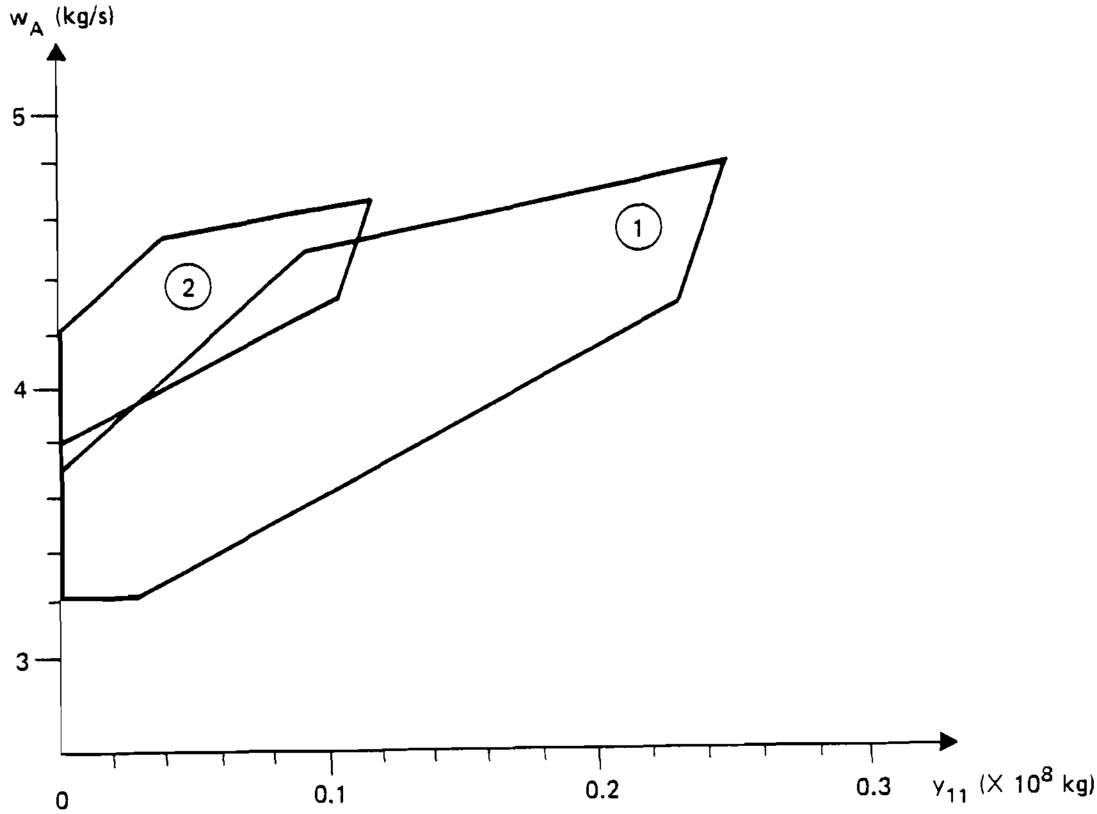

Figure 9. The dependence of yield effects in the first agricultural region (y_{11}) and the flow rate of pollutants in the control profile A (W_A) for fixed values of other variables (see Table 6)

CONCLUDING REMARKS

Based on the analysis of Figures 2 to 9, the following values of the objective functions seem to be most appropriate:

$$J_1 = y_{11} = 0.23 \times 10^8 \text{ kg} ;$$
$$J_2 = y_{21} = 0.20 \times 10^8 \text{ kg} ;$$
$$J_3 = y_{31} = 0.21 \times 10^9 \text{ kg} ;$$
$$J_4 = z_M^* - z_M = 0.05 \times 2 \text{ m}^3/\text{s} = 0.1 \text{ m}^3/\text{s} ;$$
$$J_5 = \nu_A - \nu_A^* = 0 ;$$
$$J_6 = b(S^* - S) = 4.5 \text{ m} ;$$
$$J_7 = \omega_A = 4.5 \text{ kg/s} .$$

Certainly this combination of the values of objective functions is quite arbitrary. The decision maker could find perhaps a different and more appropriate combination of such in a dialogue with the computer, using the cross-sections and projections of GRS presented in Figures 2 through 9 as well as alternative ones presented at his request on the screen of the display.

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