# MODELS OF URBAN ECONOMIC GROWTH: A REVIEW

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The processes by which cities come to exist and grow have been of great interest to both academics and policy-makers. As with any such topic, the complexity, richness, and profusion of urbanization phenomena have led researchers with different disciplinary and cultural perspectives to offer a multitude of explanatory theories and models. Policy-makers who try to distill something from the academic can be left very confused by this broad array of ideas.

The purpose of the present paper is, in one sense, limited. It is beyond the scope here to assimilate and synthesize the complete range of ideas available about urbanization. Instead, we shall concentrate on the contributions of economists towards a model of urban growth. Further, we shall emphasize models relevant to current metropolitan growth in North America. Although the range of this study is limited in this way, we are better able to concentrate on specific economic mechanisms when other cultural and economic factors are held approximately constant. The currency and locational specificity of these concepts hopefully make this paper useful to contemporary urban policy-makers in these two countries.

## I. THE CONTEXT

There is much statistical evidence, as exemplified by Berry (1973; chapter 1), that metropolitan regions are experiencing sustained above-average population growth. Recent

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empirical evidence however suggests that the very largest U.S. metropolitan areas may not now be able to achieve this above-average growth[1]. With only small differentials in regional rates of natural increase, we first note that the ability or inability of a metropolitan region to achieve above-average growth is reflected in its ability to attract positive net in-migration.

Secondly, we note that this migration has come from several sources. As shown in Table 1, currently out of every twenty in-migrants to large Canadian cities, approximately nine come from abroad, eight come from other Canadian urban areas, and two come from rural areas[2]. In the case of the United States, exactly comparable statistics are not available but what is (see Table 2) suggests a much smaller role for immigration while maintaining similar ratios of metropolitan to other U.S. originating in-migrants. Both tables indicate that off-farm migration is no longer an important source of urban growth.

It is within the context of the steady growth of at least a broad size class of metropolitan areas and the correspondence of differential migration rates that most urban economic growth models have been formulated. Thus, explaining why cities grow and why in-migration occurs would seem to be the driving force behind the creation of an urban model. It is somewhat surprising then to find, as we soon shall, that most of the models currently available treat the causes of growth and in-migration very simplistically.

The picture can be made even bleaker than this. It has been suggested that even if a growth model offering a rich explanation could be formed, it would be difficult to test empirically. The few recent instances in which a large city has actually declined in size would make it difficult

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[1]Refer to U.S. Department of Commerce (1974), especially Table 3, which indicates that SMSA's of over 2,000,000 population have grown slowly on average since 1970.

[2]Note that the census definitions of in-migrants, urban areas, and rural areas are used.

Table 1. In-migrants to Canadian CMA's over 1,000,000 population, 1966-1971, by place of origin.

| | |
|---|---|
| Total In-migrants | 1,014,780 |
| From Outside Canada | 449,210 |
| Origin Not Stated | 88,095 |
| Stated Origin in Canada | 477,475 |
| From Another CMA | 209,475 |
| From Other Urban Area | 171,975 |
| From Rural Area | 96,025 |

Source: Statistics Canada. 1971 Census of Canada. Bulletin 92-746. November 1974. Pages 19-15, 19-39, and 19-41.

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Table 2. In-migrants to United States SMSA's over 1,000,000 population, 1965-1970, by place of origin.

| | |
|---|---|
| Total In-migrants | 14,085,700 |
| From Outside U.S.A. | 1,513,600 |
| Origin Not Stated | 4,205,500 |
| Stated Origin in U.S.A. | 8,366,600 |
| Other SMSA's | 5,997,600 |
| Other Areas | 2,369,000 |

Source: U.S.D.C. Bureau of the Census. 1970 Census of Population. Bulletin PC(2)-2C. March, 1973. Pages 1 and 23.

to derive statistical inferences about the significance of particular causal factors. While not denying the validity of such a claim, the development of more thorough models must proceed. The naivety of current models must be replaced even if conflicting theories cannot be tested for some time to come.

## II. THE MAIN ISSUE

The over-riding emphasis in current models of urban economic growth is on the urban labour market. In particular, emphasis is placed on those exogenous forces which come to affect the supply of and demand for labour in a particular city. Those models which emphasize supply factors tend to play up the role of migration in affecting growth. Those which emphasize demand aspects tend to ignore migration. Therefore a classification of models as supply or demand-oriented serves also to classify models according to the role assigned to migration in affecting urban growth.

### (a) Demand and Supply Models of Urban Growth.

Engle (1974) identifies two polar cases of demand and supply-oriented models. Pure demand models, seen as intellectual descendents of Keynesian macro-economics, presume perfectly elastic factor supplies. Labour (and capital) in-migration to a city occurs exactly and only so as to maintain a given real factor price. Thus, a city's growth is limited only by its factor demand. By constructing a Keynesian model for instance in which certain kinds of local expenditures are endogenous, urban growth is made dependent only on exogenous expenditures such as local investment, government spending, and export (outside the city) demand.

At the other polar extreme, pure supply models presume that factors are not completely price elastic. The amount of labour is seen to be only somewhat responsive to local wage variations with migration and natural increase, based

partly on a non-wage incentive affecting the total supply. In their simplest form, these models presume that the city's producers face a perfectly elastic demand for exports at a given output price. The output level of the city is then limited only by the availability of factors.

These two kinds of models are evidently polar extremes. Any one city may have partially inelastic factor supplies and partially elastic demands placed on its outputs. Different cities may be approximated better by models at different points in the spectrum depending on the market conditions facing them. The labour market characteristics of the two polar solutions and their differences are illustrated in Figure 1.

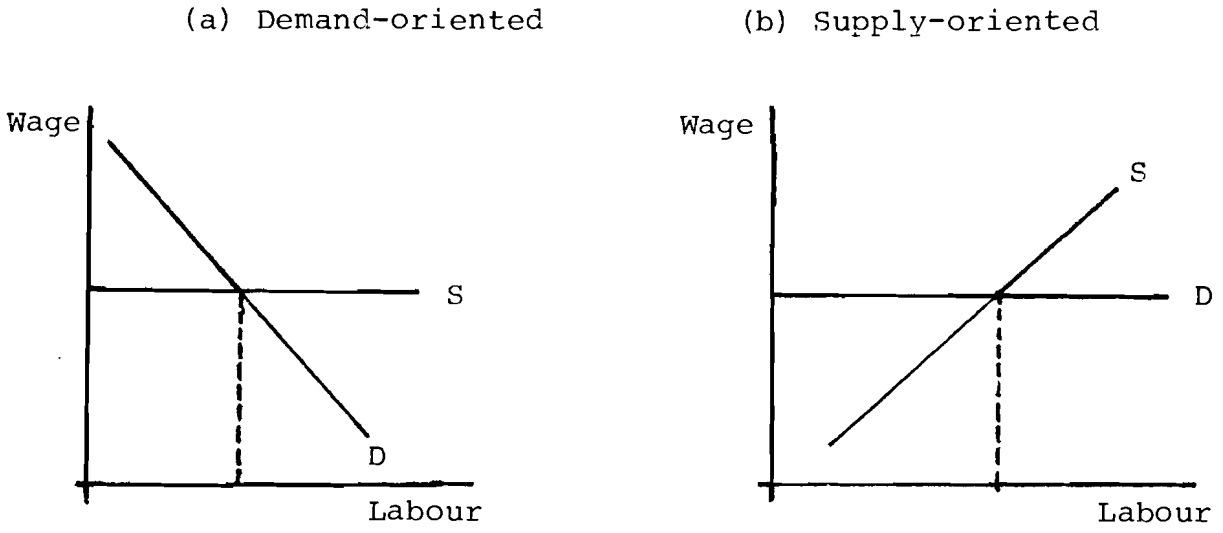


Figure 1. Labour market characteristics in the two polar cases.

(b) Conceptual Problems with a Demand-Supply Orientation.

While the polar demand and supply-oriented models seem to provide an interesting classification scheme, they are not without considerable shortcomings. This is noted immediately when one tries to define what the supply and demand curves for labour in Figure 1 really represent. Consider first the

supply curve for which there are at least two alternative concepts. The first is that this curve denotes the amount of labour forthcoming from the current residents of a town at any particular wage. The shape of the curve would reflect the work-leisure trade-offs of these residents. The second concept is that the supply curve of labour reflects the total amount of labour which would be made available to employers in a town at any given wage rate. This would be defined as the sum of the labour made available by current residents and prospective in-migrants attracted by that wage less the labour withdrawn by out-migrants repelled by that same wage. The second concept is more useful here because it enables a simple definition of growth-inducing migration. Since the labour supply curve under this second concept includes all wage-related migration, shifts in the supply curve represent only those portions of natural increase or migration which are in response to non-wage changes in relative local conditions. Thus, migration which implies a movement along the supply curve is seen as being induced by demand variations while migration which shifts the whole supply schedule are independent of demand variations and, therefore, potentially growth-inducing. For this reason, we use the second concept of the labour supply function in the remainder of this paper.

However, this second concept involves a commitment to a particular time frame. The first concept is apparently a short-run one. When producers change their wage offer, one might expect the local labour force to make a reasonably quick adjustment in terms of their labour offer. However, migrants need considerably more time to gain information about job and wage offers, make a decision on relocation, and actually complete the move. Thus, the second concept is more concerned with a longer-run time frame than the first.

A similar problem emerges when we attempt to define what the demand schedule represents. The same problem with respect to a time frame emerges. A short-run viewpoint is that the demand schedule for labour is derived from (i) the demand function for the city's output and (ii) the existing stocks of other factors of production (notably fixed capital). A longer-run viewpoint is that the demand schedule is derived assuming that these other factors of production can be varied to suit the profit-maximizing firm's requirements. Since we have adapted a long-run time frame for the labour supply curve, it seems appropriate to work in the same frame with the demand curve.

Are the conceptual problems resolved neatly if we choose to adapt a long-run time frame in viewing the urban labour market? There is reason to be negative in responding to this question. Research on locational interdependency as typified by the Koopmans-Beckmann problem suggests that the spatial concentration of industry may be similar to a non-stationary stochastic process. If so, the growth of cities may be cumulatively responsive to sudden and short-run changes in local conditions. Thus, short-run changes may accumulate to produce behaviour which in the long run does not approximate either the demand or supply-oriented model. This means that a study of short run behaviour within models of long run growth may well be necessary.

An additional conceptual problem is posed by an orientation towards the long-run. This problem also has some ramifications for the importance of short-run analysis. The problem is that, as we move from the short to the long run, the elasticity of both the supply and demand curves for labour should increase. In the short-run, both curves may be quite inelastic. With time the ability to physically move capital and labour increases and the curves should become more elastic. If we then admit the possibility that both

curves can within a sufficient time frame become perfectly elastic, the concept of an equilibrium city size becomes indeterminate. At any point in time, the size of a city is determinate only by virtue of the dynamics of the urban labour market.

(c) The Concept of Growth-Inducing Migration.

The central issue of this paper is concerned with how and why, in empirical and theoretical terms, is migration important in causing urban growth. Most of the models to be discussed attempt to examine how migration relates to the urban growth process. Little is said about why growth-inducing migration occurs at all. Therefore, we begin by reviewing briefly three different theories as to why such migration occurs.

One of these theories holds that non-wage incentives, in and of themselves, account for this kind of migration. Hirsch (1973; chapter 9) among others speaks of "household-initiated" urbanization in which it is the amenities of living in a particular area which attracts migrants. These newcomers are willing to remain unemployed or underpaid for a time in the belief either that the amenities outweigh any wage loss or that new jobs will be created and wages rise in the near future.

A second answer is to view growth-inducing migration not as a deliberate act but as the consequence of a sluggish response to wage changes. Richardson's (1973; chapter 4) review suggests the following important reasons for inertia in migration flows. (i) The information channels through which wage and job availability data are passed back to areas of out-migration are at best imperfect. Migration generated on the basis of perceived labour market conditions thus appear to be sluggishly responsive to actual conditions.

(ii) Initial movements by out-migrants from an area tend to affect the destination patterns of later out-migrants through a "friends-and-neighbours" or "stem-family" process. The availability of temporary lodging, job information and contacts, and spiritual encouragement at the home of friends may lend more migrants to come to an area than would have been justified on the basis of an initial wage or job opening difference.

A third reason for non-wage migration has been advanced by David (1974). He suggests that people may move from one area to another even when the local average wages are equal if the local dispersions of wages about these means are different. Some kinds of risk-bearing migrants will be attracted by the possibility of a substantially higher wage in another town even if the average wage there is equal to or less than their old one.

All of these answers rest on implicit notions about the dynamics of the urban labour market. Migrants cannot usually wait forever to get a job. There are psychic, monetary, and time costs involved in moving and searching which the prospective migrant has to weigh against an uncertain gain in income. At the same time, those producers who have a reasonably elastic demand for labour need time to adjust their production schedules to use the increased labour available.

In the remaining sections of this paper, we will be examining a number of urban growth models to see what is assumed to cause economic growth. Particular emphasis is placed, where possible, on the roles of growth-inducing migration and accompanying labour market mechanisms. In Section III, we examine several demand-oriented models; both theoretical and empirical. In Section IV, we consider a similar array of "mixed" models; models which are neither pure demand nor supply models but somewhere in between.

In the final Section V, several conclusions and research suggestions are outlined.

We do not attempt in the next two sections a general review of all the problems associated with urban (or regional) growth models. The interested reader is referred to Richardson (1973; chapter 2) and Engle (1974) for such treatments.

## III. DEMAND-ORIENTED MODELS

In this section, we examine some pure demand models of urban growth. These include two export-base models, an income model, and three econometric growth models. The objective here is to show up the similarities among the models in their treatment of urban growth. One model is extended to show a lead-in to mixed models.

### (a) A Simple Export-Base Model.

In one of its simpler forms, the export-base hypothesis recognizes two sectors of employment in a city at full employment, N, basic or export-oriented employment, B, and nonbasic or local oriented employment, S.[3] The kernel of this theory lies in the hypothesis that nonbasic employment, S, is linearly related to the aggregate population, P, of the city. Another hypothesis is that aggregate population, P, of the city is linearly related to total employment, N, of the city. In review, these conditions state

$$N = B + S \tag{1.a}$$

$$S = \alpha_0 + \alpha_1 P \qquad 0 \leq \alpha_1 \leq 1 \tag{1.b}$$

$$P = \beta_0 + \beta_1 N \qquad \beta_1 \geq 1 \tag{1.c}$$

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[3]Exports are those products shipped from the city to any other place.

These can be shown to assert

$$N = \gamma_0 + \gamma_1 B \qquad (2.a)$$

where

$$\gamma_0 = (\alpha_0 + \alpha_1\beta_0)/(1 - \alpha_1\beta_1) \qquad \gamma_1 = \beta_1/(1 - \alpha_1\beta_1) \quad . (2.b)$$

Given that $\alpha_1\beta_1 < 1$, it is seen from (2.a) that total employment, N, is some multiple (greater than unity) of basic employment[4].

It is not difficult to imagine a plausible set of assumptions which could form the basis of this model. Assume first that a city is one of a large number in a region and that workers migrate to any particular city in search of higher wages. In an equilibrium, the capitalized value of the wage difference between any pair of cities would be at most the net cost of migration. Any increase in the wage in one city would bring about a very large (effectively infinite) in-migration of workers. In this sense, the supply of labour in any one city is infinitely elastic at a certain wage rate. Note that both the local and export sectors must pay this same wage. Assume secondly that the local sector has constant returns to scale in production, is made up of a large number of firms, and can purchase non-labour inputs in competitive markets. Thus, the local sector's labour requirements increase proportionately with output. Assume finally that each worker has the same demand function for the output of the local sector. These assumptions ensure that (i) the local sector has fixed marginal

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[4] The condition that $\alpha_1\beta_1 < 1$ states that with a unit increase in population, the marginal change in service employment, $\alpha_1$, must be less than or equal to the marginal change in total employment, $(1/\beta_1)$.

costs of production, (ii) the price of local sector output remains fixed as city size increases, and (iii) local sector employment increases proportionately with employment in the basic sector.

Under the above assumptions, the export-base model is readily seen to be a demand-oriented model. Each sector has a downward sloping demand curve for labour. Each faces the same horizontal supply curve as shown in Figure 2. This is structurally equivalent to Figure 1(a).

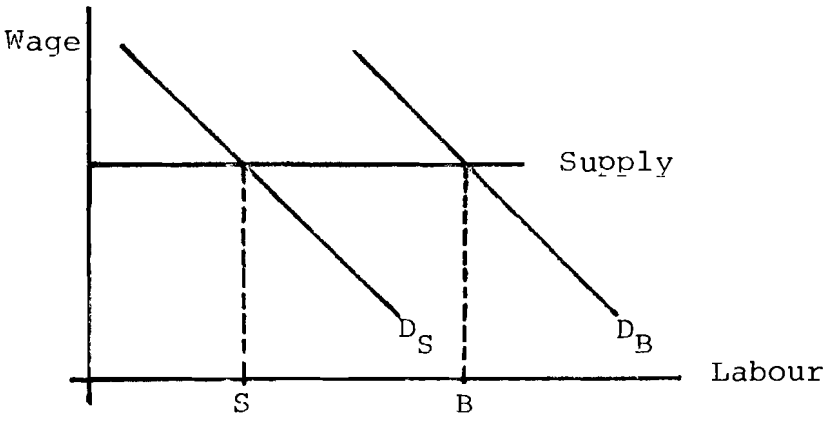


Figure 2. Labour market in the export-base model.

There is no meaningful sense in which the supply of labour can be increased under these assumptions and, therefore, the supply of labour can play no role in affecting city size. By making a set of assumptions about the movement of capital, similar to those above for labour, it can also be shown that city growth is independent of the supply of capital as well.

What causes a city to grow in this model? It is apparent that the growth of exports causes the growth of employment but there is no theory or mechanism by which the level of exports is explained. The city exists at the whim of exogenously-defined variables and has no means, for example, of enabling its own growth.

Finally, we note that, with the assumptions made, the model represents a long-run equilibrium solution. There is no role here for urban labour market dynamics because migration implicitly brings the market into continuous equilibrium.

(b) Czamanski's Baltimore Model.

Czamanski (1965) proposed an improvement to the export-base model in which he introduced lead-lag relationships between employment and population growth. The particular lengths of leads and lags are derived on his work with forecasting the growth of the Baltimore SMSA and are based only on qualitative theoretical arguments.

In this improvement, the population-employment relationship (1.c) becomes

$$P_t = \emptyset_0 + \emptyset_1 N_{t-2} \qquad (3.a)$$

where the subscript refers to a year. Thus, (3.a) asserts that population follows employment growth with a lag of two years. Equation (1.a) is now represented by the current identity

$$N_t = G_t + C_t + S_t \qquad (3.b)$$

where basic employment in year 't', $B_t$, is sub-divided into geographically-based employment, $G_t$, and employment in industries complementary to the geographically-based sector, $C_t$. From (1.b) is made the new hypothesis that local-oriented employment growth follows population growth with a lag of one year.

$$S_t = \alpha_0 + \alpha_1 P_{t-1} \quad . \qquad (3.c)$$

Finally, complementary employment is linearly related to employment in the geographically based sector

$$C_t = \varepsilon_0 + \varepsilon_1 G_t \quad . \tag{3.d}$$

The system of equations (3.a) through (3.d) reduces to the third-order difference equation

$$P_t = \Psi_0 + \Psi_1 G_{t-2} + \Psi_2 P_{t-3} \tag{4.a}$$

where

$$\Psi_0 = \emptyset_0 + \emptyset_1(\alpha_0 + \varepsilon_0) \qquad \Psi_1 = \emptyset_1(1 + \varepsilon_1)$$
$$\Psi_2 = \emptyset_1 \alpha_1 \quad . \tag{4.b}$$

Menchik (1971) terms (4.a) a dynamic equilibrium-seeking model if $-1 < \emptyset_1 \alpha_1 < 1$ which has an equilibrium solution associated with a particular level of $G_t$, say $G_t = G$, of

$$\lim_{e \to \infty} P_e = \left(\Psi_0/(1 - \Psi_2)\right) + \left(\Psi_1/(1 - \Psi_2)\right) G \quad . \tag{4.c}$$

This equilibrium solution is equivalent in form to the solutions of the earlier version of the economic base model, namely (2.a).

In effect, (4.a) is merely a partial-adjustment model in which population growth tends toward an export-base solution but in which the structural parameter $\Psi_2$ serves to determine how quickly the population size tends toward that solution.

Casting the export-base model in partial adjustment form is an important extension in two main respects. First, this model introduces short-run labour market dynamics into a previously static equilibrium model. Although there is no explicit notion of labour force here, Czamanski's construct permits a flexible relationship between current population and employment and the most reasonable assumption is that this reflects varying degrees of unemployment. Unemployment exists in the model presumably because it takes time for people to gather enough information about the local job market to decide whether to out-migrate. Coupled with a sluggish response by workers is the delayed response of the local service sector as hypothesized in (3.c). Thus, although labour force shortages and surpluses can occur, these do not affect the growth rate of employment.

The second important respect is that, by extending the model slightly, it now is possible to relate basic employment to city size. Suppose we hypothesize that, in any time period, the geographically-based sector makes available a number of jobs which is linearly related to the previous period's population.

$$G_t = \delta_0 + \delta_1 P_{t-1} \quad . \tag{4.d}$$

Then (4.a) becomes

$$P_t = (\Psi_0 + \Psi_1\delta_0) + (\Psi_1\delta_1 + \Psi_2)\, P_{t-3} \quad . \tag{4.e}$$

This extended model is a three-period, first-order difference equation which if $\Psi_1\delta_1 + \Psi_2 > 1$ indicates that city size will increase at an asymptotically-constant, three-year growth rate. The larger are any of $\delta_1$, $\emptyset_1$, $\varepsilon_1$, or $\alpha_1$, the faster will the city grow. There will also, of course, be no

equilibrium size corresponding to (4.c)[5].

This extension of Czamanski's model is no longer an export-base model in the usual sense because there is no meaningful long-run sense in which a city's size is limited by its level of export demand. Since the key to this extension is (4.d), what set of plausible assumptions would be consistent with such a model? As in the local sector, assume that the export sector has constant returns to scale and can purchase non-labour inputs in competitive markets.

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[5]This extended model is very similar to the macro-urban model of Niedercorn (1963). Niedercorn's model also attempts to relate employment in the export sector, $G_t$, and total population, $P_t$. Its structure is as follows:

$$g_t = \alpha_{11} g^*_{t-1} + \alpha_{12} g_{t-1}$$

$$p_t = \alpha_{20} + \alpha_{21} g_t$$

where $$g_t = (G_t - G_{t-1})/G_{t-1}$$

$$p_t = (P_t - P_{t-1})/P_{t-1}$$

$$g^*_{t-1} = (\alpha_{31} P_{t-1} - G_{t-1})/G_{t-1} \quad .$$

Thus, the first hypothesis above is that the export employment growth rate, $g_t$, is a function of the previous period's growth rate and the discrepancy, $g^*_{t-1}$, between actual export employment in the previous period and some fixed proportion of the population. The population growth rate, $p_t$, is tied in the second equation to the export employment growth rate.

As Mills (1972; pp. 65-66) points out, the dynamic properties of this model are not easily established. The model, like the extended version of Czamanski's, does not possess a static equilibrium solution. Unlike Czamanski's model, however, this model also rules out the possibility that export employment and total population could increase at the same average rate. The responsiveness of $p_t$ to $g_t$, the accelerator mechanism, is the key to understanding these dynamic properties. The larger is $\alpha_{21}$ relative to $\alpha_{20}$, the more rapid is the divergence between $g_t$ and $p_t$ with time.

Assume further that the city's exporters operate in a competitive national market where they are again price takers. Then, the producers have an infinitely elastic demand for labour at a given wage rate[6]. It is this combination of infinitely elastic demand for and supply of labour which makes a notion of equilibrium city size indeterminate.

What causes urban growth in Czamanski's model? In the original version without (4.d), urban growth is equated in the long-run with export growth. As in the initial export-base model, no hypotheses are offered to explain why these exports occur. The augmented version with (4.d) is different. A city also grows in this version because its level of exports increases but export growth in turn is tied to city size. In this case, labour force growth occurring in anticipation of employment becomes a self-fulfilling prophecy. Significantly, this occurs without an economies-of-scale or other centralization forces as often popularly argued.

This augmented version leaves us in the dark on at least one main issue. The equations in the model imply the significance of people's anticipations. Job-seekers in (3.a) attempt to anticipate the number of new jobs to be created in a time period. Employers, particularly in (3.c) and (4.d), try to anticipate the number of workers required and available. If both groups were able to perfectly anticipate the other's response, the growth of a city would be indeterminate here. However, the model tells us nothing about how these anticipations (i) come to be formed or (ii) evolve with experience. If one is to pursue models, such as this one, which emphasize pure labour market dynamics then explaining the formation of anticipations would seem to be one main issue.

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[6]This infinitely elastic labour demand is presumed only for the longer run (more than one year). In the short run (less than one year), the existence of unemployment means that the firms must require time to adjust their production levels.

(c) Income Models.

Pure Keynesian models of urban growth are another common kind of demand-oriented model. Empirical models are relatively scarce, however, because data on the central income and expenditure variables are difficult to come by. In this section, we consider the model suggested by Anderson (1970) as an illustrative example while recognizing that there are many variants on basic Keynesian models. An empirically tested model for Ohio of similar (though more disaggregated) structure is discussed in L'Esperance et al. (1969). Also, Moody et al. (1970) have used a similar model to study interregional linkages in the United States.

Keynesian models emphasize the determinants of different components of total regional income or expenditure. In Anderson's model, total regional income ($Y_t$) in year 't' is broken into five major components; regional consumption ($C_t$), gross investment ($I_t$), government expenditure ($G_t$), exports ($X_t$), and imports ($M_t$).

$$Y_t = C_t + I_t + G_t + X_t - M_t \quad . \tag{5.a}$$

Personal disposable income of residents ($YPD_t$) is the difference between regional income and a collection of leakages including depreciation, taxes, and the net outflow of factor payments. These leakages are summed and termed $D_t$.

$$YPD_t = Y_t - D_t \quad . \tag{5.b}$$

Regional consumption is tied to current and lagged personal disposable income in Anderson's model.

$$C_t = \alpha_{10} + \alpha_{11} YPD_t + \alpha_{12} YPD_{t-1} \quad . \tag{5.c}$$

Further, he hypothesizes that the disposable wage income of residents is linearly related to total regional income and to a time trend.

$$YL_t = \alpha_{20} + \alpha_{21}Y_t + \alpha_{22}t \quad . \tag{5.d}$$

The disposable non-wage income of residents is then given by an identity

$$YNL_t = YPD_t - YL_t \quad . \tag{5.e}$$

Gross investment is a function of this current and lagged non-wage income as well as current and lagged autonomous investment (A).

$$I_t = \alpha_{30} + \alpha_{31}YNL_t + \alpha_{32}YNL_{t-1} + A_t + \alpha_{33}A_{t-1} \quad . \tag{5.f}$$

Finally, Anderson hypothesizes that imports are a function of regional income.

$$M_t = \alpha_{40} + \alpha_{41}Y_t \quad . \tag{5.g}$$

It is not difficult to establish a solution to this model. Suppose we wish to solve this model for $Y_t$. By judicious substitution, the following form can be derived.

$$Y_t = \beta_0 + \beta_1 D_t + \beta_2 t + \beta_3(A_t + G_t + X_t) + \beta_4 D_{t-1} + \beta_5 A_{t-1} + \beta_6 Y_{t-1} \quad . \tag{6.a}$$

The $\beta$ terms are functions of the original $\alpha$ parameters. As is the case with each other endogenous variable, it is now

seen that $Y_t$ is the solution to a first-order difference equation involving seven current and lagged exogenous variables. If $-1 < \beta_6 < 1$, this is a dynamic equilibrium-seeking model.

What causes the growth of income in this model? If (6.a) is a dynamic equilibrium-seeking model, the levels of $D_t$, $D_{t-1}$, $A_t$, $A_{t-1}$, $G_t$, $X_t$, and t are the critical determinants. With the exception of 't', these represent either exogenous demands for the output of or leakages from the local economy. Whereas in the export base model the sole source of growth was exports, the income model allows for autonomous investment, government spending, taxes, and capital outflows as well as exports to affect growth. Thus, the model also has little to say about why growth occurs because it does not suggest how these exogenous variables come to change.

What assumptions permit this model to operate? The model indicates nothing about wages and prices although one might expect several of the exogenous variables to respond to them. One plausible route is to assume, as done earlier for the export base model, an infinitely elastic supply of all inputs within the urban region as well as constant returns to scale[7]. By making these assumptions, fixed wages and prices can be assumed. The city's factor markets would then be equivalent to those in the export base model as illustrated in Figure 2. In this case, the income model is clearly a demand-oriented one. This model has no role for

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[7]It is somewhat difficult to reconcile this statement with the treatment of capital in an income model. On the one hand, the assumption is that additions to the capital stock are made to exactly meet output requirements. On the other hand, gross investment in this model, as determined by (5.f), depends on exogenous investment as well as a pool of investable funds (YNL). There is no reason to believe that (5.f) generates the appropriate increment to capital stocks. This problem becomes more pressing when capital stocks are introduced explicitly into the model as is soon seen in the case of Bell's model.

growth-inducing migration or labour market dynamics since it has nothing to do with the labour market at all.

(d) Bell's Model of Massachusetts.

We now turn to a discussion of three models based on the principles of the export base and income models. All of these income, population, and employment models have been empirically estimated. The first is the model of a region somewhat larger than a city (Massachusetts). However, the predominance of the Boston SMSA within this region and a number of unique features of this model make it an appropriate starting point. We consider it in more detail than the subsequent models because its structure is illustrative.

Bell's model rests on the export base hypothesis. Using location quotients, Bell estimated income produced ($X_t$) by exporting industries. This is hypothesized to be a linear function of GNP for the United States.

$$X_t = \alpha_{10} + \alpha_{11} GNP_t \qquad \alpha_{11} > 0 \quad . \tag{7.a}$$

The income produced in supplying local consumption ($S_t$) is related to the income received by residents of the region ($Y_t$)

$$S_t = \alpha_{20} + \alpha_{21} Y_t \qquad \alpha_{21} > 0 \quad . \tag{7.b}$$

The total income produced in the region ($GRP_t$) is identically the sum of export and local consumption income

$$GRP_t = X_t + S_t \quad . \tag{7.c}$$

Total income received differs from total income produced mainly by the income accruing to 'foreign' (outside Massachusetts) capital. Following Borts and Stein's (1964) findings,

it is hypothesized that these are in fixed proportion

$$Y_t = \alpha_{31} GRP_t \qquad \alpha_{31} > 0 \quad . \tag{7.d}$$

This completes the income block of Bell's model and it is seen that GNP, through its role in affecting exports, determines all variables in this section.

The determination of factor demands and supplies constitutes two more blocks in this model; one for capital and one for labour. The economy is broken into two sections here; manufacturing and non-manufacturing. In the manufacturing sector, growth of the capital stock ($K_{m,t}$) is assumed to be determined by a partial adjustment model related to an optimal capital stock ($K^*_{m,t}$)[8].

$$\frac{K_{m,t}}{K_{m,t-1}} = \left( \frac{K^*_{m,t}}{K_{m,t-1}} \right)^{\alpha_{40}} \qquad 0 < \alpha_{40} < 1 \quad . \tag{7.e}$$

This optimal capital stock is a function of the export sector's output (produced income) level as well as time.

$$K^*_{m,t} = \alpha_{50} (\alpha_{51})^t X_t^{\alpha_{52}} \qquad \alpha_{50}, \alpha_{52} > 0 \quad , \quad 0 < \alpha_{51} < 1 \quad . \tag{7.f}$$

In effect $K^*_{m,t}$ is the demand for capital while $K_{m,t}$ is the supply in this sector. In the non-manufacturing sector,

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[8] We may note in Bell's (1967; page 120) equation (5') that the coefficient $\alpha_{40}$ has an estimated value of 1.023 which is too large for a partial adjustment process. The likelihood of an overcompensating equilibrium-seeking process is raised although the theoretical basis of this is unclear.

capital is assumed to be less durable. It is tied directly to the income produced in that sector.

$$K_{nm,t} = \alpha_{60} S_t^{\alpha_{61}} \qquad \alpha_{60}, \alpha_{61} > 0 \quad . \tag{7.g}$$

The total capital stock, $K_t$, is made up of the sum of these two components

$$K_t = K_{m,t} + K_{nm,t} \quad . \tag{7.h}$$

The demand for labour, $L_t$, is estimated as a derived demand. By manipulating a Cobb-Douglas production function with Hicks-neutral technical change, the following labour demand is obtained.

$$L_t = \alpha_{70} \alpha_{71}^{-t/1-\beta} K_t^{-\beta/1-\beta} GRP_t^{1/1-\beta} \qquad 1 > \beta > 0 \quad , \quad \alpha_{71} > 1 \quad . \tag{7.i}$$

Here, $\beta$ represents the produced income elasticity of capital.

The supply of labour is determined by net natural increase and migration. Bell begins by defining the expected population, $P_{e,t}$, to be the previous period's population, $P_t$, augmented by natural increase, a constant factor of $\gamma$ here. Thus, where $M_t$ is the net in-migration during period 't', we have

$$P_{e,t} = \gamma P_{t-1} \qquad \gamma > 1 \text{ generally, and} \tag{7.j}$$

$$P_t = P_{e,t} + M_t \quad . \tag{7.k}$$

Bell then defines the expected labour force, $N_{e,t}$, at time 't' to be

$$N_{e,t} = \delta P_{e,t} \qquad 0 < \delta < 1 \quad , \tag{7.l}$$

where $\delta$ is the participation rate. Further, he hypothesizes the level of net in-migration to be a function of expected labour force and the level of employment

$$M_t = \alpha'_{80} + \alpha'_{81}(L_{t-1} - N_{e,t-1}) \qquad \alpha'_{81} > 0 \quad . \tag{7.m}$$

Finally, the number of workers available, $N_t$, is found by applying the labour force participation rate, $\delta$, to (7.k). Using (7.j) through (7.m), this yields a reduced form as follows

$$N_t = \alpha_{80} + \alpha_{81}P_{t-1} - \alpha_{81}\alpha_{82}P_{t-2} + \alpha_{82}L_{t-1} \qquad \alpha_{81}, \alpha_{82} > 0 \quad , \tag{7.n}$$

where $\alpha_{80}$, $\alpha_{81}$, and $\alpha_{82}$ represent functions of the old $\gamma$, $\delta$, $\alpha'_{80}$, and $\alpha'_{81}$. Thus, the supply of labour is seen, by successive substitutions from (7.j) through (7.m) into (7.n), to be a function of the time stream of employment levels.

Also in the final block of equations we define two additional variables. One equation relates the wage rate, $W_t$, to a time trend. The other defines the unemployment level, $U_t$.

$$W_t = \alpha_{90}\alpha_{91}^t \qquad \alpha_{91} > 1 \quad , \tag{7.o}$$

$$U_t = N_t - L_t \quad . \tag{7.p}$$

This part of the block has no significance for the working of the model because these two variables do not feed back to influence any other variables.

The causal structure of this model is quite interesting. It is described graphically in Figure 3. The exogenous variables determine the income block. These then determine variables in the capital stock block. These in turn determine the labour market block. From this Figure 3, we can now see exactly which other variables are affected by a change in the value of any variable in the model.

Let us consider for example the effect of a random increase in the total capital stock, $K_t$. There is no repercussion back to other variables in the capital block nor to any in the income block. Thus, an exogenous increase in the capital stock does not increase the level of produced income for instance. However, this exogenous increase does affect labour market variables. The initial effect is to decrease the level of employment and increase unemployment. In the subsequent period, this would lead to a decrease in net in-migration which implies a smaller population for the city than would otherwise have resulted.

Suppose, on the other hand, that the supply of labour, $N_t$, is exogenously increased. This could occur either because the population or the labour force participation rate increased. In either case, the immediate effect is an increase in the level of unemployment. Only in the second subsequent period do we see an effect, through $N_{e,t-1}$, on migration which reduces city size. There are, further, no effects back on the capital stock or on the level of income.

What causes the growth of income, employment, and population in this model? As Bell argues, the level of GNP drives the export level which drives the remainder of the model. Thus, the model asserts that income growth can only occur as a result of growth in GNP.

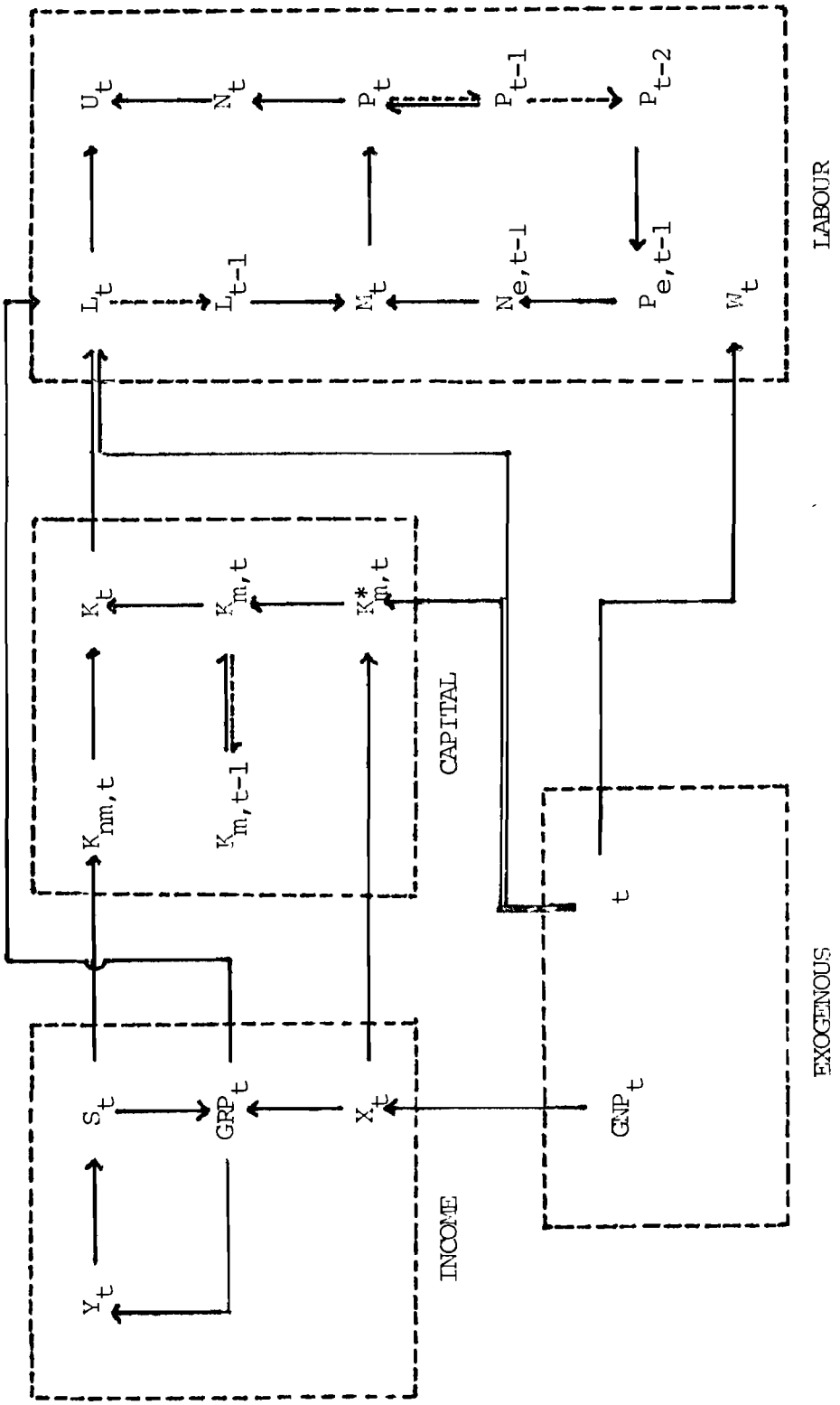


Figure 3. Causal Structure in Bell's Model of Massachusetts.

Notes: --- indicates causality operates with a time lag.

Source: See text.

Surpluses or shortages of factors do not affect the growth rate of income. We have seen that an over-supply of capital tends over time to affect the growth rates of employment and population. Shortages or surpluses of population and labour force, however, have no effect on the growth of employment, capital stocks, or income. Thus, although this model is completely different from Czamanski's Baltimore model, it too does not allow labour in-migration to affect employment growth. As in the Baltimore model, the labour force which is in excess of the amount required by the level of export demand tends to out-migrate after about two years. There is no role for growth-inducing migration in either model.

(e) The Glickman and Hall-Licari Models.

Two other annual econometric models to date have been constructed specifically for metropolitan regions. These are the models by Glickman (1971) of Philadelphia and by Hall-Licari (1974) of Los Angeles. These models are very similar in structure and it is helpful to discuss their structures simultaneously while making comparisons back to Bell's model.

Each model can be divided into a number of recursive blocks of variables as was done for the Bell version. There are three blocks in the Glickman model and two in the Hall-Licari prototype. In both cases, the first block relates to the manufacturing sector. Unlike Bell, the two later models do not use location quotients to identify export industries. Instead, they simply assume that the manufacturing sector closely approximates this industry.

The manufacturing sector, as a proxy for all export industry, is strongly tied to national conditions in both models. In each case, manufacturing value-added, $Q_1$, is

related to GNP in a linear relationship

$$Q_{1,t} = \alpha_{10} + \alpha_{11} GNP_t \quad . \tag{8.a}$$

Hall-Licari hypothesize that manufacturing investment, $I_1$, is related to manufacturing value-added and the manufacturing capital stock in the previous period, $K_{1,t-1}$.

$$I_{1,t} = \alpha_{20} + \alpha_{21} Q_{1,t} + \alpha_{22} K_{1,t-1} \quad . \tag{8.b}$$

Glickman has a similar hypothesis, but also allows lagged investment as an independent variable

$$I_{1,t} = \alpha'_{20} + \alpha'_{21} Q_{1,t} + \alpha'_{22} K_{1,t-1} + \alpha'_{23} I_{1,t-1} \quad . \tag{8.b'}$$

In both cases, there is a simple identity relating current manufacturing investment stocks to the rate of depreciation, d, and to manufacturing investment.

$$K_{1,t} = (1 - d)\ K_{1,t-1} + I_{1,t} \quad . \tag{8.c}$$

Finally, employment in the manufacturing sector, $E_{1,t}$, is related by both to the value added in that sector, although Glickman adds in a time trend proxy for efficiency.

$$E_{1,t} = \alpha_{30} + \alpha_{31} Q_{1,t} + \alpha_{32} t \quad . \tag{8.d}$$

This completes the first block of each model[9]. An illustrative

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[9] Strictly speaking, Hall-Licari's equation for the manufacturing wage could be put in this first block although Glickman's, because of a feedback, could not. Little is lost by putting this equation in the second block of both models.

causal outline using Glickman's version is presented in Figure 4.

There are striking similarities between these two models and that of Bell if we equate the manufacturing and export sectors. In all three models, produced income or value added in exports is tied directly to GNP. In all cases, manufacturing investment is seen as a kind of adjustment process where the capital stock slowly adapts to the requirements of the level of manufacturing output. In all cases, there is no feedback from any other part of the model to the export sector. Value added, capital stock, and employment variables in the export sector are independent, for instance, of labour availability. The only significant difference among the three models is that the Bell version allows a capital surplus or deficiency to have an effect on labour demand while the other two do not.

The remaining blocks of the Glickman and Hall-Licari models are quite similar. One of the few differences is that the Glickman model has two blocks of variables (an omnibus second block feeding a third government block) in addition to the initial manufacturing block. In the Hall-Licari model, the government-sector variables feed back into the second block so that no separable, recursive sub-blocking is possible. Hall-Licari emphasize this difference as it allows government policy variables in their model to affect the growth of non-manufacturing output, income, and employment variables in the second block. The only exogenous policy variable in the Glickman model is the property tax rate which helps to determine local public expenditures and revenues, but has no effect on any other variables in the model. Even more extreme is the Bell model which has no direct policy variables at all.

One other important difference among these models concerns the treatment of population and labour force. In Glickman's model, the population variable is driven by a

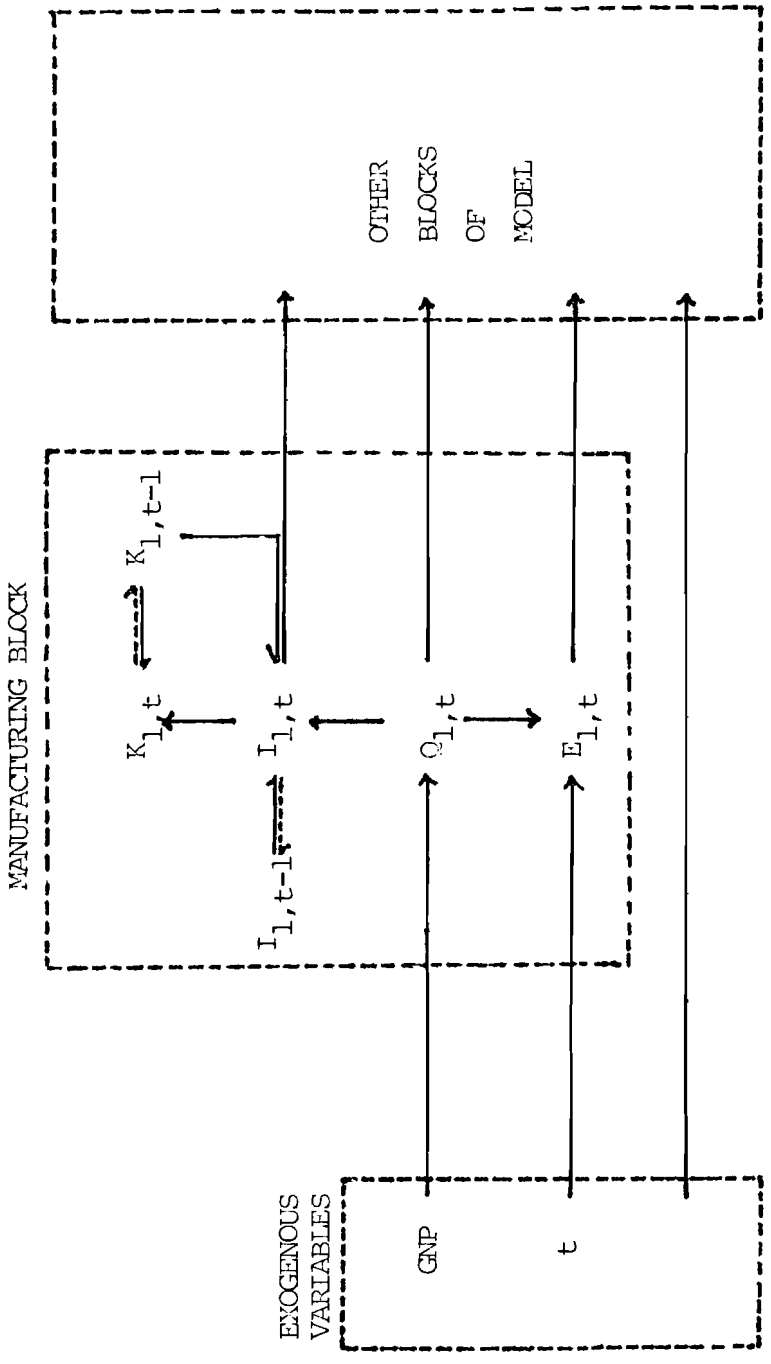


Figure 4. Causal Structure in the Glickman Model's Manufacturing Block.

time proxy and by labour force. Since there are no feedbacks to the rest of the model, an exogenous increase in population (without a concurrent exogenous increase in labour force) has no effect on employment, labour force, or output. In the Hall-Licari model, population is an exogenous variable which, together with total employment, determines labour force. Thus, population growth leads to labour force growth and to changes in other parts of the model. Note, however, that this has been done at the expense of making population and migration strictly exogenous to the model.

In both models, it is the change in labour force which comes to affect most variables in the models. Both models use the same kind of mechanism. Initially, the effect of an increase in labour force is to increase the level of unemployment. This increased unemployment is seen to lower money wages in different industry sectors. Since there is no direct feedback from wage to employment levels, employment initially remains fixed so that the total real wage bill falls[10]. This reduces personal income which lowers the output of the non-manufacturing sectors and reduces their employment levels. As a first round effect then, an increase in labour force leads initially to a decline in non-manufacturing activity and thereby to a decline in total output, income, and employment. Because of the internal complexity of the second block in each model, it is difficult to evaluate the full, as opposed to initial, impact of an increase in labour force. Numerical solutions seem to be the only method available. Hall-Licari estimate full impact elasticities,

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[10] In the Hall-Licari and Glickman models, an exogenous increase in labour force also leads to a decrease in the regional price deflator used in converting from money wages to the real wage bill. This tends to lessen the decline in the real wage bill that would occur if the deflator remained constant. This secondary effect seems to be of a small enough magnitude in empirical work to omit in this discussion.

as illustrated in Table 3, using 1970 values for their model. These bear out the predominance of these initial effects.

Table 3. Single-year impact elasticities for the labour force variable in Hall-Licari model using 1970 Los Angeles SMSA base.

| Endogenous Variable | Elasticity |
|---|---|
| Number of Unemployed | 23.618 |
| Total Real Wage Bill | -.106 |
| Total Money Wage Bill | -.200 |
| Total Employment | -.032 |
| Average Money Wage | -.174 |
| Regional Price Deflator | -.106 |
| Personal Income | -.097 |
| Consumer Expenditures | -.083 |
| Gross Regional Product | -.069 |

Source: Adapted from Hall-Licari (1974), pages 342, 343, and 349.

The inability of a labour force increase to generate an increase in employment or output may seem to be surprising. As Engle (1974) argues, however, it is a consequence of the structural assumptions made. In both the Hall-Licari and Glickman models, the effect of a labour force increase is, through an increase in unemployment, to decrease the wage rates in certain industry sectors. There is no means by which wage rate changes can attract employers to the idea of making more jobs available. No explanation is offered by either of these researchers as to why such a relationship was not included. Although Bell makes the wage rate independent of the level of unemployment, the three models are similar in their disregard of growth-inducing migration.

In view of the differences among all three econometric models, it seems useful to now ask ourselves just what kind of structural mechanisms should be present for growth-inducing migration. As one alternative, should the effect of a labour surplus on employment levels be through variations in the wage rate. The answer put forward by some is that institutional factors such as national labour unions increasingly make wage changes uniform among all cities. This seems to be consistent with the empirical finding by Bell (1967; pp. 114-116) that local labour unemployment has no effect on wage levels. The argument is also supported by Freund (1973; pp. 284-288) who could find no significant relationship with annual data covering manufacturing in 35 American SMSA's from 1961 to 1967. Even the Hall-Licari wage equations do not have statistically significant coefficients for the unemployment rate. In view of this, Glickman's solitary significant results should be viewed with some apprehension.

The other alternative is to think of a labour surplus as acting directly on the equilibrium demand for labour rather than on the wage rate. Such an hypothesis is consistent with the polar supply-oriented growth model of Figure 1(b)[11]. It seems surprising to find that these hypotheses have not been included for testing in any of the three econometric models.

We may conclude by summarizing how the last two econometric models structuralize the process of urban growth and how they handle the possibility of growth-inducing migration. We have seen that both models have a manufacturing sector in which output and employment growth depends strictly on national growth and a time trend. No other variable,

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[11] Viewed in this light, the wage equations of the Glickman and Hall-Licari models may be quite inconsistent with the demand-oriented model of Figure 1(a).

endogenous or exogenous, affects the growth of this sector. In both models, growth in the non-manufacturing sectors is at least partly determined by this manufacturing sector. Both models permit other exogenous variables, notably national wages, to affect the non-manufacturing sectors although the Hall-Licari model additionally asserts a specific role for government variables such as federal revenue sharing. Both models, however, permit no direct role for growth-inducing migration and even suggest, through the effect of unemployment on wages, that migration will lead to a decline in total employment.

## IV. MIXED DEMAND-SUPPLY ORIENTED MODELS

To this point, a number of pure demand models of urban growth have been investigated. We now turn to a series of models which do permit some role for growth-inducing migration. We term these "mixed" models in that while they permit migration to affect urban growth rates, they also have roles for other sources of growth. We begin by considering an untested, theoretical model by Klein. Then, attention is focussed on simple empirical models formulated by Muth and Greenwood. Finally, we re-examine the models by Bell and Hall-Licari to see what would happen if their structures were re-estimated using Muth's formulation.

### (a) The Klein Model.

Klein (1969) presented his model of a regional economy in the form of a theoretical construct. Although never implemented, it is sufficiently different in its structure from the others to make it valuable for study. In particular, it emphasizes a role for prices not found in the previous models [12].

[12] Note that, in this paper, we shall discuss only 8 of the 20 equations making up the Klein model. This reduction simplifies our exposition without doing great damage.

The model is built around a form of the Keynesian identity linking product income and expenditures. Money gross regional product is the product of real gross product, GRP, and the implicit price deflator, p. It is decomposed as follows:

$$pGRP = P_c C + P_i I + G_{SL} + G_F + p_x X - p_m M \quad , \qquad (9.a)$$

where C is real regional consumer expenditures and $P_c$ is the national (assumed equal to regional) consumption price deflator,

I is real regional investment with $P_i$ its national and regional price deflator,

$G_{SL}$ and $G_F$ are money state-local and federal government expenditures respectively,

X is real regional exports with a regional price deflator of $p_x$ and M is real regional imports with a regional price deflator of $p_m$.[13]

The conventional multiplier mechanism is used to drive this model. C is related to the regional personal disposable income (PDI) in money terms divided by the consumption deflator, $P_c$,

$$C = f_1(PDI/P_c) \quad . \qquad (9.b)$$

PDI in turn is the money gross regional product less federal-state-local taxes, T, and depreciation, D.

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[13] A different notation from that of Klein is used to make the model more easily comparable. Note that upper case P's refer to national prices while lower case p's are regional prices. Note also that $p_c = P_c$ and $p_i = P_i$.

$$PDI = pGRP - T - D \qquad (9.c)$$

Real regional investment is related to real GRP, to the national (also regional) rate of return on capital, $P_k$, and to the capital stock in the previous period, $K_{t-1}$.

$$I = f_2(GRP, P_k, K_{t-1}) \quad . \qquad (9.d)$$

The level of real imports, M, is also tied to GRP. In addition, it depends on the regional implicit price deflator, p, and the import price deflator, $p_m$.

$$M = f_3(GRP, p/p_m) \quad . \qquad (9.e)$$

Finally, the level of real exports is tied both to GNP and to the ratio of p and $p_m$.

$$X_i = f_4(GNP, p/p_m) \quad . \qquad (9.f)$$

Although the above six equations do not constitute a recursive block in Klein's model, they usefully illustrate as a set two interesting differences from earlier models. First, we note that, by substitution into (9.a), gross regional product can be made a function of several national and regional variables. The relevant national variables are GNP, $G_F$, $P_c$, $p_m$, and $P_i$ while the regional ones include p, $p_x$, $G_{SL}$, and D. The T variable here includes both national and regional elements. This extends the models discussed earlier by explicitly introducing price changes as a source of economic growth. Secondly, we note that, unlike the earlier models, the export sector does not have a fixed relationship to GNP. Export output can change as relative prices do.

A central issue is then raised as to how these regional price indices respond to changes in the regional economy. The regional implicit price deflator, p, is of particular interest because it is the only endogenous variable explaining export levels in (9.f). Klein hypothesizes that 'p' is a function of the deflator for GNP, P, and the local wage rate, w.[14]

$$p = f_5(P,w) \quad . \tag{9.g}$$

The wage rate in turn is seen as a function of the national unemployment rate (U), the regional unemployment rate (u), and the national-regional consumption price deflator.

$$w = f_6(U,u,P_c) \quad . \tag{9.h}$$

Thus, variations in the local unemployment rate are seen to affect local wage rates which affect the local price level and thereby the level of real exports and regional product. This ability to make the export sector responsive to internal regional conditions is unique among the models considered.

The treatment of population in this model is similar to that of Hall-Licari. Population is treated as strictly exogenous and the labour force is found by applying a fixed participation rate. There is no direct notion of an "optimal" population size supported by a level of exports as found, for instance, in Bell's model. If there were an exogenous increase in population, there would be an increase in labour force and initially an increase in unemployment. This would lead, through the mechanisms discussed above, to an increase

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[14] Klein also has another variable explaining 'p' which he calls "$q_i^m$." This variable is undefined in his paper and it is here assumed to be exogenous.

in the level of exports, total output, and employment. Thus, exogenous migration can in this model induce economic growth.

We have already brought into question the exact mechanism used here to permit growth-inducing migration. The notion that unemployment rates affect wage rates is one which seems to lack empirical verification for large cities. Therefore, there is some question as to whether Klein's model would hold up under empirical testing.

(b) The Models of Muth and Greenwood.

Muth (1968, 1971) was among the first to suggest the structure of a formal theory of supply-oriented urban growth. Together with his theoretical model, he presents a simple two-equation model which he empirically estimates using data for some 78 urban areas in the United States. It is to this model that we now turn our attention.

Muth suggests that the growth in employment and the level of in-migration are, in fact, interdependent. He argues that each is partly determined by the other as well as by other variables. One of the more successful forms used by him is as follows[15].

$$\left[\frac{E_{60}}{E_{50}}\right]^* = \alpha_{10} + \alpha_{11}\left[1 + \frac{M_{60}}{L_{50}}\right]^* + \alpha_{12}\left[1 + \frac{N_{60}}{L_{50}}\right]^* + \alpha_{13}\left[\frac{Y_{60}}{Y_{50}}\right]^* \qquad (10.a)$$

$$\left[1 + \frac{M_{60}}{L_{50}}\right]^* = \alpha_{20} + \alpha_{21}\left[\frac{E_{60}}{E_{50}}\right]^* + \alpha_{22}\left[1 + \frac{N_{60}}{L_{50}}\right]^* + \alpha_{23}\left[1 + \frac{\Delta ML}{L_{50}}\right]^* + \alpha_{24}\left[\frac{U_{50}}{L_{50}}\right]^* \qquad (10.b)$$

[15] Equations (10.a) and (10.b) are adapted from Column 4 of Tables IV and V in Muth (1971; page 304), omitting variables whose coefficients are statistically insignificant.

where E is total civilian employment
M is net in-migration over the previous decade
L is total civilian labour force
N is natural population increase over the previous decade
Y is median family inc me
$\Delta ML$ is the decade change in military personnel
U is the number of unemployed persons
50,60 subscripts refer to 1950 and 1960, and
* denotes a natural logarithm.

$$\alpha_{22}, \alpha_{24} \leq 0$$

$$\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{23} \geq 0 \quad .$$

As in Klein's model, the effect of an exogenous increase in migration is to change the employment levels. However, unlike Klein, Muth does not have this change occur via an apparent wage rate variation.

Greenwood (1973) develops a similar kind of model in which there are five endogenous variables; (i) the level of out-migration (OM), (ii) the level of in-migration (IM), (iii) the growth rate of median personal income ($\Delta$INC), (iv) the growth rate of employment ($\Delta$EMP), and (v) the growth rate of unemployment ($\Delta$UNEMP). He used data for the 100 largest SMSA's in the United States with the migration data based on the 1955-60 period and growth rates referring to the decade change from 1950 to 1960. Each endogenous variable is hypothesized to be a function of some subset of other endogenous variables as well as a set of exogenous variables as illustrated in Figure 5[16]. Note that, like Muth, Greenwood

[16]Note that the depicted structure is a subset of the structure original version hypothesized by Greenwood. For simplicity, non-significant statistical relationships have been omitted as in the case of Muth.

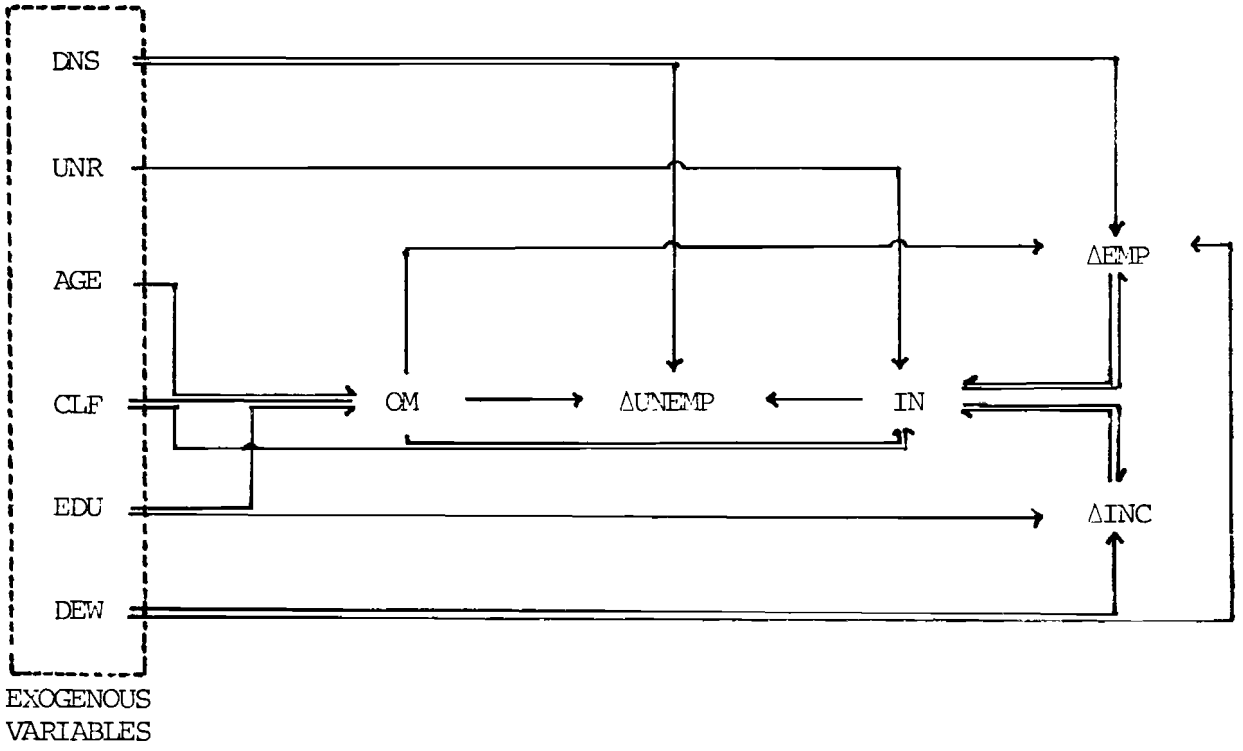


Figure 5. Statistically-significant linkages in Greenwood's Model.

Notes: DNS - North-South Dummy Variable
UNR - SMSA Unemployment Rate
AGE - Median Age of SMSA Population
CLF - Civilian Labour Force in SMSA
EDU - Median Years of Schooling of SMSA Population over 25
DEW - East-West Dummy Variable

Source: Greenwood (1973; page 102). See text concerning adaptation.

sees a direct link between migration and employment without any reference to a wage change linkage.

Both of these models see the sources of urban growth as being manifold. In Muth's model, the growth in jobs and in-migrants feed on one another completely simultaneously. In Greenwood's model, however, there are three recursive blocks; the first contains just OM, the second ΔEMP, IN, and ΔINC, and the final one just ΔUNEMP. Thus, an exogenous change in ΔUNEMP has no effect on the remainder of his model, while a change in OM affects all other variables. An exogenous increase in in-migration, then, brings about increases in employment, income, and unemployment. Note, however, that, among the exogenous forces affecting the endogenous variables in either model, there is no longer a measure of export demand present as found in all previous models.

This illustrates a central difficulty with Muth and Greenwood in that they fail to present a formal theory which might underly the specific models they have chosen to estimate. While both models contain a growth-inducing role for migration, it is not clear why the growth of jobs is limited by anything more than the growth of labour force. What process is being modelled here? One good argument is that the Muth and Greenwood models are really models of the short-run dynamics of the urban labour market where entrepreneurs try to anticipate the change in labour supply and migrants in turn try to anticipate the number of new jobs to be created. However, the five to ten year time frames of these two studies seems to be too long to realistically estimate such a model[17]. Is there any evidence to suggest that, within a shorter time frame, these models would still be empirically valid?

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[17] The extended version of Czamanski's Baltimore model seems to be on a more appropriate, one-year time frame.

(c) Bell and Hall-Licari Revisited.

Some reworking of the Bell model is possible because the original data are presented in his article. The data for the Hall-Licari model was graciously provided by one of the authors. This enables a re-examination of the growth mechanisms underlying both models.

The hypothesis is forwarded that the level of exports (or manufacturing) value-added is responsive to the growth in labour force. The decision to tie in export growth in this way is related in part to the recursive structure of these models. If the level of migration affects the export level, in other words, the whole set of endogenous variables is affected. Certain complexities in the estimation of non-linear simultaneous models are posed by such a relationship and we do not consider them at present[18]. We use Ordinary Least Squares in both cases and derive the following equations to replace (7.a) and (8.a):

$$e_t = -0.01348 + 1.00310\ g_t + 1.50272\ \ell_t \qquad (11.a)$$
$$(4.70530) \quad (1.78486)$$

$$\bar{R}^2 = 0.678 \quad DW = 2.690 \quad N = 15$$

$$q_t = -0.11396 + 0.99539\ g_t + 3.75270\ \ell_t \qquad (11.b)$$
$$(1.61462) \quad (2.01750)$$

$$\bar{R}^2 = 0.568 \quad DW = 1.952 \quad N = 11$$

The two equations share some similarities. The first, using Bell's data, relates the growth rate of export value-

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[18] It is noted that Greenwood (1973; pages 102 and 109) found little qualitative difference between his OLS and Three-Stage Least Squares estimates.

added, e, to the growth rate of GNP, g, and the growth rate of the regional labour force, ℓ. The second equation poses the same model, using the data of Hall-Licari, where the endogenous variable, q, is the growth rate of manufacturing output. Both equations show a near-unity relationship between the manufacturing-export sector growth and national growth. The difference between the coefficients of "ℓ" is somewhat larger but this may be in part due to the difference between "manufacturing" and "export" activity. Additionally, in both models the slope coefficients are either significant or near-significant.

These two equations are in the spirit of the work of Muth and Greenwood. They suggest in addition that the simultaneity between economic and population growth is valid in the short-run time frame of a single year. They still do not, however, provide any new insights as to what kind of theory might underly them.

## V. CONCLUSIONS

Let us now stop and ask ourselves what we have learned from all of the above discussions. Several interesting models of urban growth have been examined. We have seen that all of these models attribute urban growth to one of two sources; either external demand for the city's output (usually related to GNP) or a black box called labour market dynamics. Is that all there is to city growth? The answer to that, by any serious student of urban form, must be no. So, in concluding, it is appropriate to speculate on where research on urban growth should be headed. Three areas seem to offer special promise.

The first area of interest involves an examination of the concept of short-run labour market dynamics, especially on the supply side. This would involve the construction of models which hypothesize the behaviour of the urban job

seekers. What processes are involved in his search for a job? How does he decide where he will look for a job and how long he will look? How does he weigh housing and other factors in making his decision to in- (or out-) migrate to (or from) an area? What is the relationship between migration and population growth both in terms of fertility-mortality changes and in terms of subsequent movements of dependents and others? On the general topic of search theory, starting points here might come from the work of David (1974), Phelps (1970), and Zarembka (1972; pp. 54-62). One operational model of this type is being developed by Cordey-Hayes and Gleave (1974).

The second area of interest concerns some spatial aspects of urban growth. There are a number of questions which could be usefully raised here. One set of questions would be concerned with the two-way linkage between urban growth and a city's hinterland or market area. To what extent, for instance, is migration to a city in any period limited by the nature of the urban system? How do improvements in technology alter the relationship between a city and its hinterland so that hinterland labour is freed to migrate to the city? It may well be that answers to such questions prove to be an important determinant of the growth of particular cities.

Other spatial aspects which might affect urban growth have to do with the interior spatial structure of the city. Simple Alonso-type models suggest, for instance, that there are several kinds of costs which increase with city size. Hoch (1972) finds similar empirical evidence. None of the models examined, however, introduce such costs at all. Variables such as average commuting costs, rent levels, environmental quality, the dispersion of job opportunities, and recreational access costs could be introduced explicitly and endogenously into aggregate growth models.

This issue of rising costs with city size brings into perspective the third major area of interest. If we view the city simply as a concentration of production activities, one principal raison d'etre lies in indivisibilities of production which generate economies of scale. In the theory of urban economic growth, an equilibrium size is reached when further increases to city size increase the costs of such size faster than these economies of scale. This trade-off between economies and costs has not entered any of the models discussed here.

This issue of why cities exist has more implications than just the introduction of a more refined or plausible growth model. It brings out a central conflict with respect to the theory behind a pure supply model of urban growth. As was mentioned earlier, the pure supply model assumes constant returns to scale in production. This would seem to deny a major reason for the existence of cities at all. A careful reconciliation of these two assumptions is required as part of the theoretical structure underlying pure supply models. One might try to argue, for example, that there are economies of scale, but that these are exhausted at the present scale of large cities. If so, then why and to whom does such a city export its goods? If each city services only its hinterland then in what sense can it be viewed as a price taker; another assumption of the pure supply model. What prevents the city from behaving monopolistically within such a market area? Some careful re-thinking has to be done on this whole conflict.

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