WORKING PAPER

TIME PREFERENCE AND THE LIFE CYCLE:
THE LOGIC OF LONG-TERM HIGH RISK VS.
SHORT-TERM LOW RISK INVESTMENT

Robert U. Ayres
Shunsuke Mori

April 1987
WP-87-33

IIASA
International Institute
for Applied Systems Analysis

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FOREWORD

The integrating 'meta-model' underlying IIASA's Technology-Economy-Society Program is the life cycle, which appears to be applicable both to technologies and to industries. One of our most important research tasks is to increase our understanding of the dymamics of the life cycle.

In general, the life cycle begins with a major innovation or 'breakthrough'. But most economists have always had difficulty explaining such breakthroughs, insofar as they require longer-term, high-risk investments. The authors of this paper suggest that investment behavior is a function of the life cycle itself. If so, one can perhaps begin to understand how the end of one cycle leads into the next.

Thomas H. Lee
Program Leader,
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Abstract

This paper argues that time-preference functions (or 'discount rates') for R&D should properly be considered to be functions of the economic environment. In particular, during periods of accelerating growth and general increasing prosperity it is appropriate and rational to prefer a marginal dollar in the present to a marginal dollar in the future. Conversely, during periods of saturating growth and deteriorating prospects, the converse holds: it is rational to prefer a marginal dollar in the future to one in the present. Periods of increasing general prosperity--rising tide--are likely to be associated with the early phases of an industry 'life cycle'. Periods of declining prosperity, by contrast, may occur towards the end of the life cycle.

The implications for R&D policy are derived in terms of a simple model. The results suggest that at the beginning of the life cycle the optimal R&D policy is to invest in short-term, low risk ventures (i.e. product or process improvements). Late in the cycle, however, the optimal policy reverses to long-term high-risk projects. In simple terms: a firm in a declining industry needs to find a new product or business to replace the old one. Unfortunately, the appropriate behavior is discouraged by most existing B/C methodologies.

TIME PREFERENCE AND THE LIFE CYCLE: THE LOGIC OF LONG-TERM HIGH RISK VS. SHORT-TERM LOW RISK

Robert U. Ayres
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Introduction

It can perhaps be taken for granted, in what follows, that project (or venture) evaluation and selection are core activities in a modern business organization. Some of the reasons will be discussed later. This paper addresses a critical issue in venture evaluation/selection methodology. In principle, many interrelated factors must be considered, as indicated by the schematic diagram of and R&D allocation model (Figure 1). A number of large corporations have developed and use such models. At the heart of any such model or procedure, however, is a quantitative comparison between alternative ventures in terms of their attractiveness as investment opportunties. Typically, the problem is conceptualized as the allocation of a fixed budget among the annual requirements of a portfolio of ventures, in rank-order of priority, until the available funds are exhausted. The schematic diagram (Figure 1), complex as it is, glosses over many difficulties, such as exactly how to calculate the effect of R&D expenditures on earnings or on the impact of changes in product mix on sales.

These rather obvious difficulties may tend to disguise a much more fundamental problem, namely how to meaningfully compare projects that absorb funds at different rates for different periods, and generate profits in different amounts at very different times in the future. In short, how does one compare a modest, inexpensive, short-term project with a very ambitious, expensive long-term one?

Two more or less equivalent approaches are in general use. One approach is to reduce each project, regardless of time variation, to an equivalent `present value'. This is done by (1) calculating a time-varying pattern of future income or profits, (2) discounting each future increment to a present value, and (3) summing them up to a total. This sum can be compared to the total projected investment (also discounted), and the ratio or return-on-investment (ROI) can then be computed.

The second approach, which is less common, is to project an annual percentage yield on the capital investment over a target lifetime (allowing for capital replacement) and compare that to the yield on funds invested in long-term bonds or other financial instruments over the same period. Again, a simple ratio can be derived. Although discounting *per se* is not explicitly required in the second approach, it is obvious that to reduce a time-varying future income stream to a supposedly equivalent annual rate of return does not really avoid the problem, but rather shoves it under the rug. It is more intellectually honest (and usually more realistic) to confront the issue of discounting--or time-preference--directly and explicity.

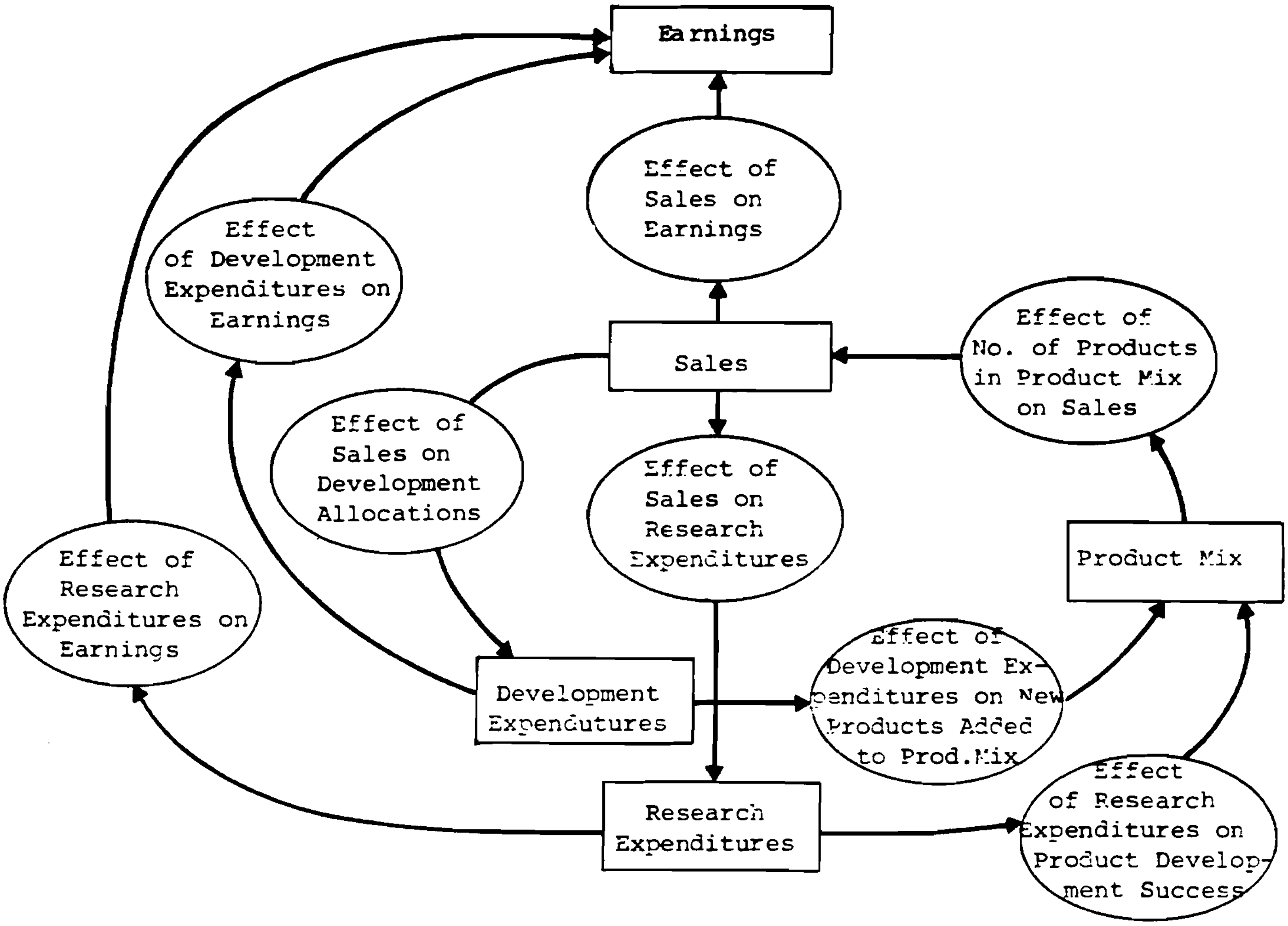

Figure 1. Simplified diagram of R&D resource allocation model.
Source: Blackman, 1973

The Theory of Discounting

The standard formal theory of benefit/cost analysis is predicated on the notion of discounting to compensate for opportunity cost. Specifically, it is traditionally argued that a marginal dollar of income in the future must be discounted with respect to a marginal dollar in the present, since the latter could be invested in a savings bank (or equivalent) where it will earn interest. The arguments in the economics literature tend to revolve around the "right" choice of a discount-rate for public vs. private sector projects, and for individuals (mortal) vís à vís firms (immortal). There seems to be no doubt among these authors that some choice (roughly in the range 3%-8%) is "correct".

Arrow [Arrow 76] has identified two fundamental justifications for discounting:

(i) Pure utility time preference = time preference for goods = interest rate

(ii) Growth-of-consumption-decrease-of-marginal-utility.

The first of these is the economic equivalent of increasing entropy, or "time's" arrow. It states that the marginal dollar delivered in the future has less utility than a present dollar, irrespective of other factors. Pigou [Pigou 20] attributed it to a "weakness in our telescopic faculty" causing us to see "future pleasures, as it were, on a diminished scale" (pp. 24-25). It is widely assumed to be an empirical fact. However, this can be challenged, in the light of recent evidence discussed later. The second justification is simply that, if our descendents (or we ourselves) are going to be richer in the future than we are today, it does not make sense to trade a present dollar for a future one on a one-for-one basis.

We argue that the "pure utility" justification is, at a deeper level, also based on the assumed growth of consumption. Certainly, financial investments cannot earn interest, on the average, unless real economic growth is occurring at the same time. The exact theoretical relationship between GNP growth and real interest rates is open to some debate, but the two seem to track together fairly closely. In any event, most authors writing on the subject seem to agree that the appropriate discount rate is closely related to a long-term interest rate. This has two direct implications:

(a) The discount rate is a positive constant δ in the decreasing multiplicative exponential function

$$\exp(-\delta t).$$

(b) The constant δ can be chosen "once for all", in principle.

A number of authors including [Ramsey 28], [Page 77] and [Georgescu-Roegen 79] have argued that δ should be zero, on ethical grounds, to ensure that interests of future generations (e.g. in the environment) be given the same weight as interests of those now alive and "voting". This is another way of stating the view, set forth by Jefferson, that the environment is a common property of all generations, held "in-usufruct" by the living. However, in general, most economists would choose a positive non-zero discount rate somewhere between .03 (3%) and .08 (8%), depending on assumptions about inflation and taxes. Arrow, for example, stated in his 1976 paper that 6 or 7 percent "sounds like the right kind of rate" on a constant value basis (zero inflation). Elsewhere (ibid) he divides this (roughly) equally between the two components noted above, i.e. in the "neighborhood of 3%" for "pure" time utility and 4-5% for growth-of-consumption (aggregated).

As regards "pure" time utility, Arrow (op. cit.) argues that it is an empirical (and presumably universal) fact. He does not attempt to explain it. On the other hand, if it is not a universal phenomenon but, rather, an occasional one, an explanation would be helpful. Indeed, discounting behavior on the part of the individuals can easily be explained by finite job and life expectancy. For middle-aged and older people, especially, a preference for immediate consumption would be natural in view of the non-zero possibility of not being alive to enjoy it later. The same principle applies (even more strongly, perhaps) in the case of corporate managers with limited job tenure. A project with costs in the present but yielding benefits in the more remote future may actually be a liability to the mobile mid-level manager who is evaluated in the basis of current performance at the "bottom line" and who will be not be in his position long enough to claim the credit for the payoff. Not suprisingly, managers expecting to be transferred away in 2 or 3 years are unlikely to invest in anything with a payoff 5 or 10 years away.

Thus the phenomenon of "pure" time utility in the sense of short-term orientation can easily be explained in principle as a consequence of short job tenure and/or finite mortality. Indeed, the apparently inferior management performance of major U.S. based multi-national corporation in recent decades **vis à vis** comparable Japanese (or European) firms with longer and more secure management tenure can be regarded as evidence that the phenomenon is quite real (e.g. [Hayes & Abernathy 81]).

Of course, it goes without saying that a hypothetical impersonal and immortal management intelligence identified only with the well-being of the firm, or the social entity, would not be affected by such considerations. To be sure, real organizational decisions are not made by impersonal immortal decision-makers. Nevertheless some economists might argue that the "free, competitive market" is itself an example of an impersonal and immortal decision-making intelligence. Financial and commodity markets are probably the freest and most competitive of all markets. We also have a clearly articulated theory, due to Hotelling [Hotelling 30] to the effect that the shadow price or scarcity rent (above extraction costs) of exhaustible minerals should rise at the prevailing market rate of interest on other (monetary) investments. In other words, the opportunity cost of leaving minerals in the ground (i.e. extracting them later rather than immediately) should be equal to the implicit market discount rate.

Hotelling's model is simple enough to carry conviction. If there is any "pure" time utility (apart from the increasing prosperity or growth-of-consumption factor) it should be reflected by mineral commodity markets. Yet, the empirical record seems to show otherwise. In particular, Dresch [Dresch 84] finds that resource prices in almost all cases have risen more slowly than the market rate of interest, over a period of many decades. It is therefore argued here that for an impersonal, immortal decision-maker, at least, there is no such thing as pure time utility. In any discussion of decision-making in terms of the utility of an immortal entity (i.e. a firm) it can properly be ignored.

The market rate of interest, in real terms, can be regarded as a rough measure of the "expected increasing prosperity" factor, as applied to the economy as a whole. However, as noted already in the discussion of "pure" time preference, factor may be quite different, both for individuals and for firms in specific circumstances. Frequently, an expectation of the "continuously increasing prosperity" is unrealistic. Indeed, for many--if not most--individuals approaching retirement age it is contrary to fact. Most working people must look forward to a period, in old age, of sharply reduced income and an increased probability of high medical or health-related expenses that may not be fully covered by insurance or social security. Thus people are motivated to save "for a rainy day", even if the savings depreciate in real terms due to hyper-inflation as in much of the 1970's. This behavior can only be explained by an effectively negative discount rate for some people at certain times in their lives. Nor does the "increasing prosperity" assumption hold true in general for firms.[1]

In fact, we propose that the concept of a well-defined or unique discount rate should be discarded, because of its misleading connotations. In its place, the notion of a time preference function applicable to specific circumstances suggests itself. The following model is proposed: Consider a decision maker (DM) and a lottery with unit payoffs at various future times. Let $W(t)$ be a function that defines the utility of receiving a unit payoff (e.g. winning a lottery) at future time t relative to the utility of a unit payoff at time zero ($W(o) = 1$). It is clear that $W(t)$ depends on the perceived value of receiving a unit payoff in the future as compared to the present. This depends not on the payoff itself, but on the DM's expected level of prosperity at future time t. If the DM is a person, this would depend on his/her expected income from job and/or secure investments. If the DM is the CEO of a firm representing the interests of its stockholders, $W(t)$ depends on the expected

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[1] Indeed, Arrow himself acknowledges (op. cit.) that it is really only true for the aggregate of all investments over a very long period--or for society as a whole. By implication, it is not always true for either individuals or for firms.

future profitability of the firm's existing (core) business.

It is traditional in business plans and reports to stockholders to project ever-increasing growth and profitability. But this kind of projection often disguises real, and predictable, problems. For example, a corner grocer with a long established `niche' in some neighborhood may learn that a chain store is planning to locate a branch store across the street. Or, a drug company with a profitable proprietary drug may have to face up to the expiration of its patent protection and the entry of low cost generic competitors into the market. In fact, few business enterprises are safe from competition by a new technology or from a better-financed or lower cost "brand x" competitor. When such competition becomes visible on the horizon the profitability of current operations can be reasonably expected to decline. <u>Under these conditions, the utility of a lottery with a payoff in the future can be greater than the utility of a lottery with a payoff in the present.</u> (Hereafter the term "lottery" can be replaced by "R&D project", and the "payoff" is a net contribution to corporate income[2]).

<u>Management Options in Relation to the Life Cycle</u>

At first glance the conclusion of the previous section appears to contradict one of the most standard assumptions of economics: that a dollar received in the present will never decline in utility because it can <u>always</u> be reinvested at positive real interest rates, e.g. by purchasing government bonds or T-bills. In reality, however, there is no assurance that real interest rates will always be positive. Moreover, for a non-financial business firm with no existing debt to repay the implied choice is not necessarily available: current profits are subject to tax and must be declared and distributed (in part, at least) to stockholders. Moreover, for such a firm purely financial investments are not generally acceptable, except for reasonable reserves. The choice is usually (1) to invest in expanding the existing business, (2) to invest in short-term product or process improvements, (3) to invest in long-term major innovations, either in production technology or new products, (4) to diversify by acquiring or merging with other profitable existing businesses.

In a perfect financial market, the last alternative is equivalent either to a financial investment (in the absence of "special" synergies such as countercyclic behavior or vertical

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[2]We assume for clarity that any payoff must be <u>consumed</u> (not reinvested) if it is received. In some cases a transfer of surplus funds forward in time is possible. Thus a near-term payoff that is simply held "in escrow" (earning interest) might be regarded as equivalent to a long-term payoff. However, this comparison is misleading, since the corresponding reverse transfer (backward in time) is not possible. In any case, for a firm, income must be credited (and taxed) in the year received. In general, it cannot be carried forward and regarded as income in a later year.

integration yielding economies of scale or scope) or to a successful R&D project. In fact, there is a good reason to believe most mergers are profitable only to the investment bankers, lawyers and brokers. There is a growing body of evidence based on <u>ex post</u> studies suggesting that the synergies are illusory and the risks greatly outweigh the benefits [Louis 82]. For purposes of this discussion, however, the "diversification option" is ruled out of further consideration. The viable options are, in brief (1) expand without innovation, (2) short-term incremental product improvement or cost cutting, (3) long-term major innovation with greater risk and greater returns.

Consider now two possibilities: suppose, first, that the DM for the firm anticipates a "rising tide" of continuously growing revenues and profits based on vigorous expansion of the market as a whole, rising incomes, etc. In this case, he probably has no problem deciding what to do with excess cash. If the industry is operating at or near capacity the obvious strategy is to invest in expanding capacity and holding market share. Or, if there is some excess current capacity, cash can be invested in short-term R+D to improve the product and make it more attractive, or to cut production costs. But now suppose, on the other hand the following scenario: the DM's advisors forecast a future period of declining profitability, or even of losses, due to rising competition, higher costs, shrinking markets, or some combination, even taking into account all known opportunities for product or process improvement. The problem is now of a different sort, namely to <u>replace</u> the existing business. Since the merger/acquisition option has been excluded, this leaves only the long-term R+D option. At least, so it would appear on the basis of a qualitative argument.

Can the argument be quantified? A fairly simple approach is suggested in the following paragraphs. The first step is to characterize the utility of a marginal unit of profit as a function of future time, as a function of perceived future market conditions. Basically, if continued prosperity is expected (scenario I), the utility of a marginal dollar declines with time, as shown in Figure 2. On the other hand, if an "end to prosperity" is expected, the situation is portrayed in Figure 3 (scenario II).

The concept of a natural `life cycle' for products, technologies and industries is now quite widely accepted. The cycle begins with conception (i.e. innovation) and runs through successive stages of `childhood', `adolescence', `maturity' and `senescence'. Each stage has characteristic behavior patterns. The apparent aging process in terms of industries was noted and discussed extensively by economists as far back as the 1930's [Alderfer & Michl 42]. The international trade and locational dimension was emphasized by Vernon [Vernon 66], and the technological aspect has been elaborated particularly by Albernathy & Utterback [Albernathy & Utterback 75]. A detailed characterization of the life cycle concept need not be included here. Suffice it to say that scenario I is a reasonable description of the view at an early stage of the life cycle (e.g. childhood or adolescence). On the other hand, scenario II is a description corresponding to the late mature or senescent stages. It is clearly vital for senior management to determine where in the cycle each of its businesses is at any given time.

Scenario I (Figure 2) may be `normal' for society as a whole in the very long run. At least, it seems applicable since the industrial revolution. However scenario II (Figure 3) is really the normal situation for most large firms, in the sense that a continuation of existing activities ("static operations") cannot be expected to result in continuously increasing revenues and profits for the indefinite future (Figure 4). Of course, competent and successful organization plans (and expects) to fill the "sales gap" either by improvement of existing products and operations or by innovation/acquisition of new products. But it is important to recognize that the necessary actions to fill the sales gap--which require risky investments--will not be taken if increasing prosperity is taken for granted.

In fact, the "sales gap" is a normal feature of the scene from the executive suite of the multi-product, multi-business corporation. The job of the DM, of course, is to see to it that the gap forecast is, in some sense, a self-denying one. In other words, his primary responsibility is to initiate the actions needed to fill the projected gap. He is unlikely to be successful in doing so, however, if he is using a decision-making (benefit/cost) formula that implicitly assumes a continuation of the present trend (of increasing prosperity) by heavily discounting the future. The situation is akin to riding a bicycle: stability requires forward motion. If the rider takes continued stability for granted and stops pedalling, the bicycle will slow down and topple over.

As a working hypothesis, it seems reasonable to assert that, looking ahead to a future time t, the stockholder's real time preference function is something like

$$W(t) = (\pi(t)/\pi_0)^{-1} \quad , \quad W(0) = 1 \qquad (1)$$

where $\pi(t)$ represents the expected stream of profits from pre-existing products or activities, over time.

The Choice of R+D Project

Apart from issues of time preference, there are two major factors that must be considered in selecting among possible R+D projects. They are:

(1) the probability (P_1) of technical success (i.e. it works) in relation to R&D expenditures and time

(2) the probability (P_2) of market success (i.e. it sells), assuming it is a technical success.

With regard to (1) above, it is clear that, other factors remaining equal, the probability of technical success is an

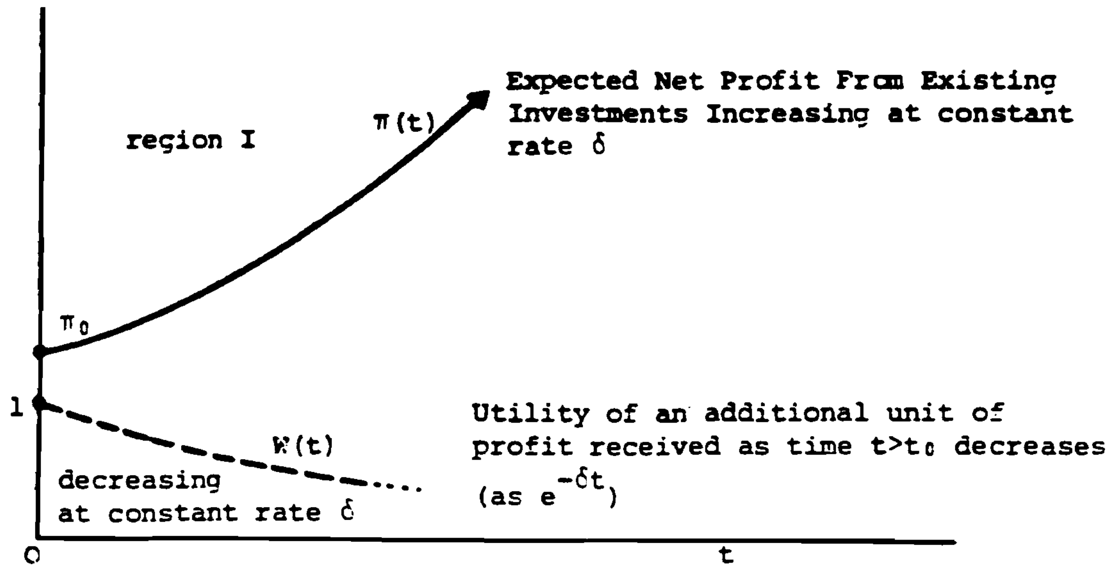

Figure 2. Scenario I: Increasing prosperity and growth (Early in the life cycle).

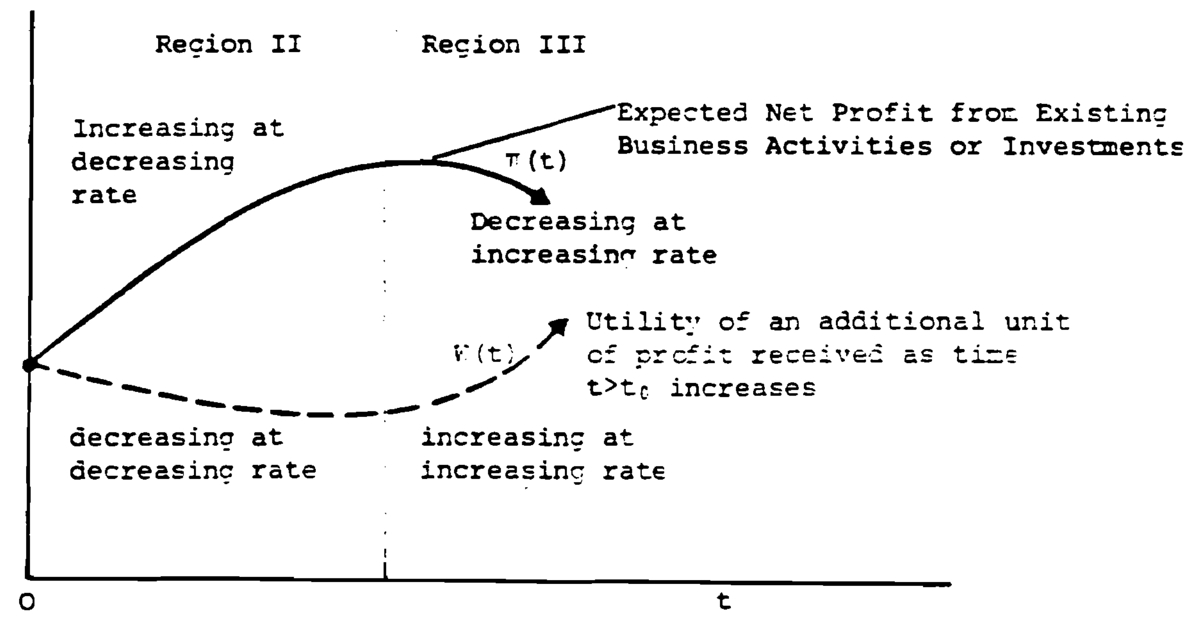

Figure 3. Scenario II: End of prosperity (Late in the life cycle).

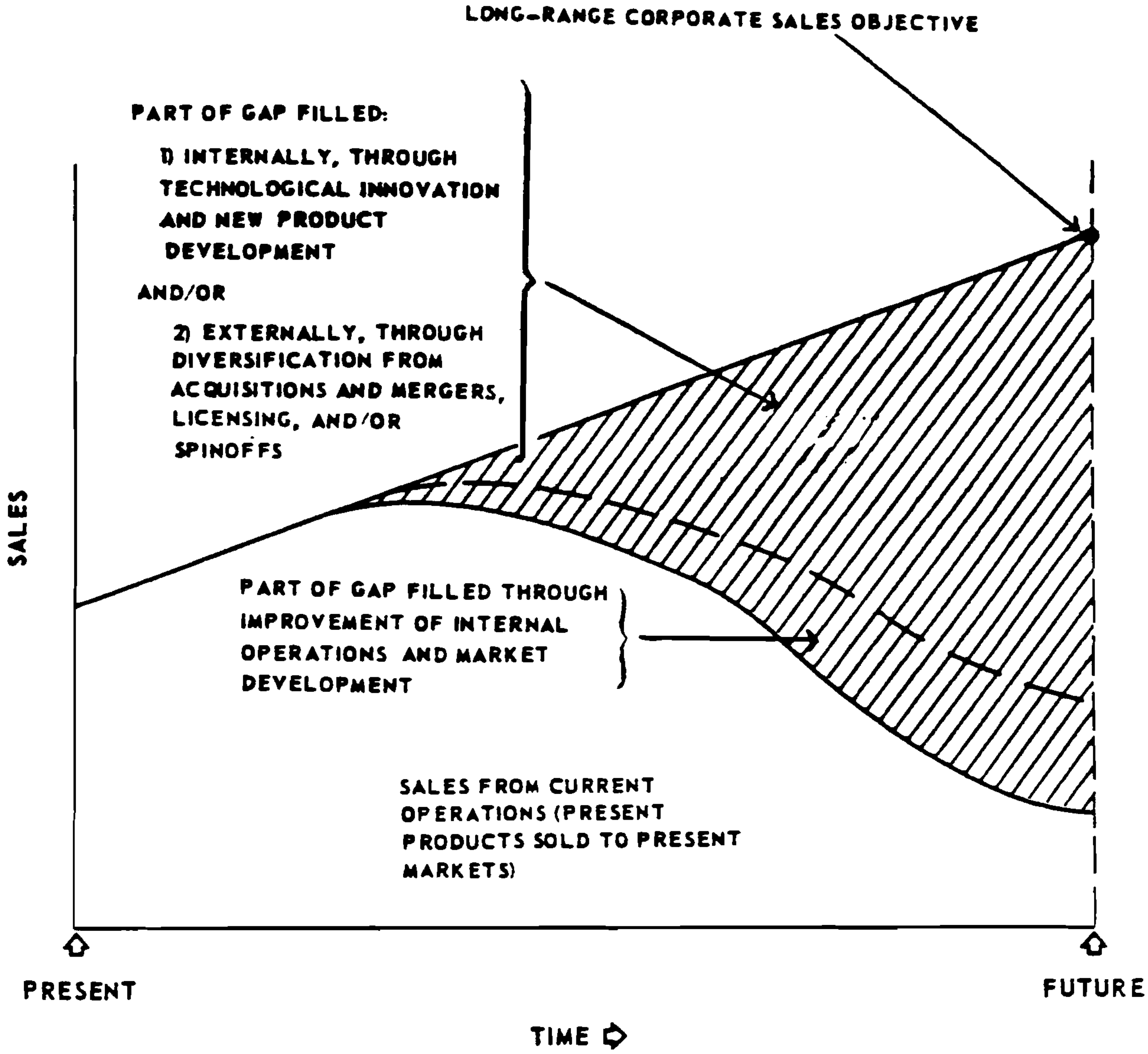

Figure 4. Sales GAP diagram for a multi-product enterprise

Source: Blackman, 1973

increasing function of research intensity.

G. Wyatt [Wyatt 86] mentioned the tradeoff between technical success probability and research duration in the context of the following simple model, where it is assumed that a research program consists of n research projects, each of which continues for only one period and has a certain independent success probability. Determining the total research program period, say z ($\leq$ n), and distributing research projects among them, many research programs can be generated. The research program is terminated when at least one of the projects succeeds or time z is reached.

Expected research program duration (z) and expected cost (EC) can be then formulated and some numerical examples exhibit a convex time-cost tradeoff curve between them.

Although the above formulation implies an optimal reseach schedule, it neglects interdependence of research projects and the learning effects, as Wyatt noted. But when the present value of R&D research is discussed, these dynamic effects may play an important role as well as time preference behavior of the entrepreneur.

Here, in order to focus on the interactions between time preference function and existing knowledge and learning effect, we employ a sequential research program where cost or research intensity, say j, is time constant. Therefore in our model, EC is always equal to (z)j.

Though success may never be assured, its probability also increases with elapsed time because new knowledge may become available at no cost by "diffusion" from external sources. In fact, the same total effort is perhaps more likely to lead to success if it is spread over a longer period, because of the diffusion factor.

Sometimes a "crash" program can shorten the time required for research, e.g. by testing many alternative approaches simultaneously (in parallel) rather than in sequence. The sequential approach tends to be much less expensive, however, than the parallel approach since the lessons learned by the researchers from each unsuccessful trial can make the next attempt more efficient. It is assumed that enough is known _a priori_ about the research problem to define a program and determine an 'optimal' R&D effort with a fixed annual budjet j. The research then continues at this level until the problem is solved or the project is terminated. The length of time needed to solve the problem is also a function of its intrinsic difficulty or its "technological distance" from the state-of-the-art (SOA). It is also reasonable to postulate[a] that the cumulative probability of success $P_1(t)$ by time t is given by the differential equation

$$\frac{dP_1}{dt} = jP_1(1-P_1) \qquad (2)$$

whence

$$P_1(t) = [1 + \exp j\ (T-t)]^{-1} \qquad (3)$$

Here j is a measure of the research intensity and T is the time when the probability of success reaches 0.5. For some short-term "improvements" the initial probability of success P_1 can be greater than 0.5, which implies a negative value of T.

The probability density P(t) of a breakthrough occurring between time t and time t+ t is given by the derivative

$$\rho_1(t) = \frac{dP_1}{dt} = \frac{je^{j(T-t)}}{[1+\exp\cdot j(T-t)]^2} \qquad (4)$$

The expected cost of research at constant annual rate of expenditure j is given by summing up the probability of (continued) failure in each period times the cost of continued research for the next period. The probability of continued

[a]However, it should be pointed out that other formulations are also plausible. In particular, Mansfield has developed a probabilistic model of the R&D choice process [Mansfield 68]. However Mansfield's study did not address the tradeoff between long-term, high risk projects and short-term, low risk projects.

failure up to time t is $[1 - P_1(t)]$, so expected cost EC is

$$EC = j \int_0^{t_F} (1 - P_1(t))dt \qquad (5)$$

Substituting (3) into (5), the integration can be carried out exactly, yielding

$$EC = jt_F + \ln\left(\frac{1+e^{-jT}}{1+e^{-j(T-t_F)}}\right) = \ln\left(\frac{1+e^{jT}}{1+e^{j(T-t_F)}}\right) \qquad (6)$$

Turning now to the calculation of benefits, a collection of assorted investment projects can then be evaluated and compared in terms of expected payoff (or profitability) <u>if successful</u> vs. probability of market success. Market success is a combination of technical and other factors. If one assumes the existence of an equilibrium risk-return tradeoff[4] it is evident that any projects offering unusually high return in relation to perceived risk will be quickly selected out and thus removed from the list of candidates. One with too high a risk, on the other hand, will never be selected. Figure 6 illustrates typical relationships. The "best" projects will be those with the highest value V for a given combined probability of success or (equivalently) those with the highest probability of success for a specified payoff value. Either way of looking at it, the best projects are to be found on the "envelope" illustrated in Figure 6. One of the key points to note is that potential products or processes with largest maximum payoff's are also likely to be technologically farthest away from the State-of-the-Art (SOA), hence inherently riskiest. Along the envelope, therefore, one would expect V to be an increasing function of technological difficulty or 'distance'; such as

$$V = V_0 \quad , \qquad J \leq 0 \qquad (7a)$$

$$V = V_0 + AJ^{\alpha} \quad , \qquad J > 0 \qquad (7b)$$

where A, α are parameters.

A convenient surrogate measure of technological 'distance' (J) for our purposes, is

$$J = jt_F \qquad (8)$$

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[4]See, for example [Conrad & Plotkin 68].

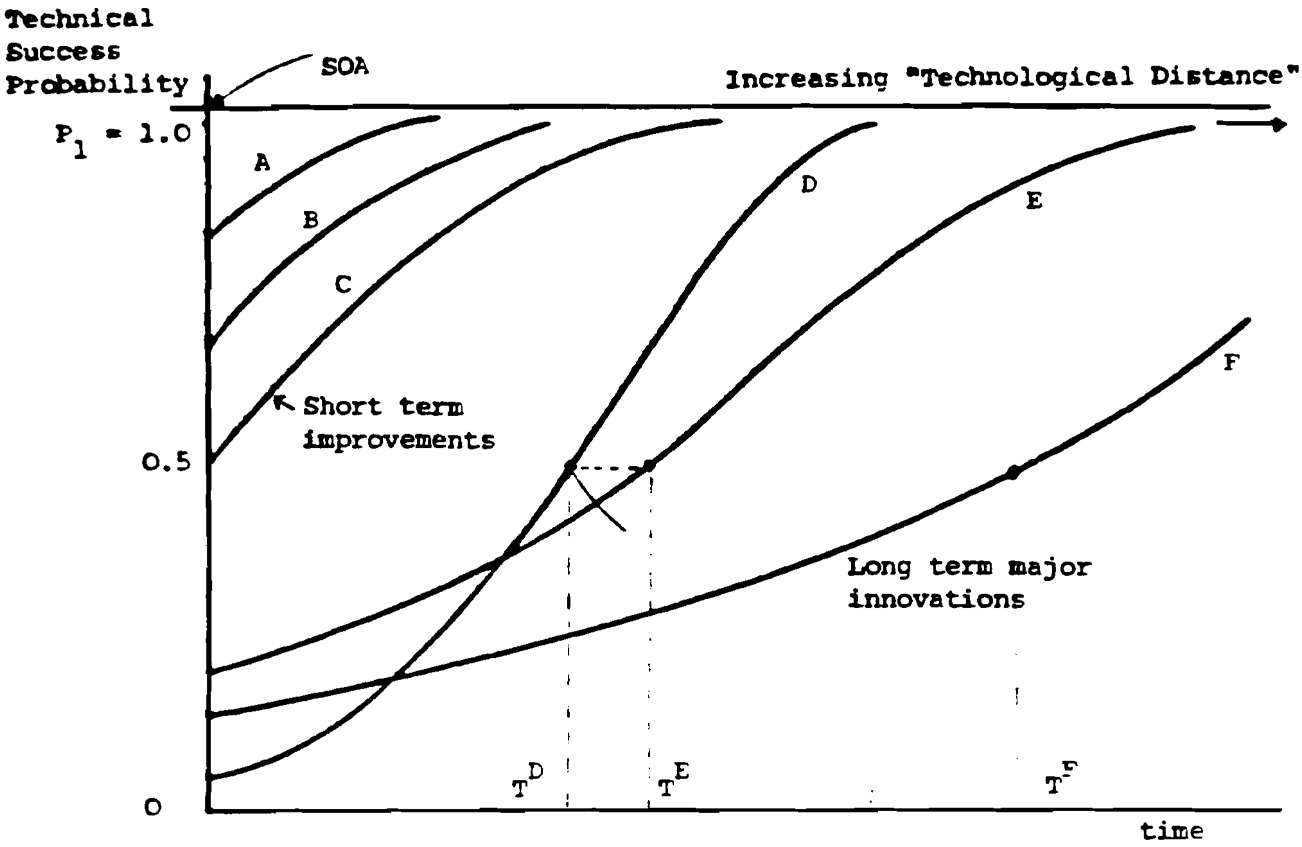

Figure 5. Cumulative Probability of technical success vs. time/cost, various trajectories

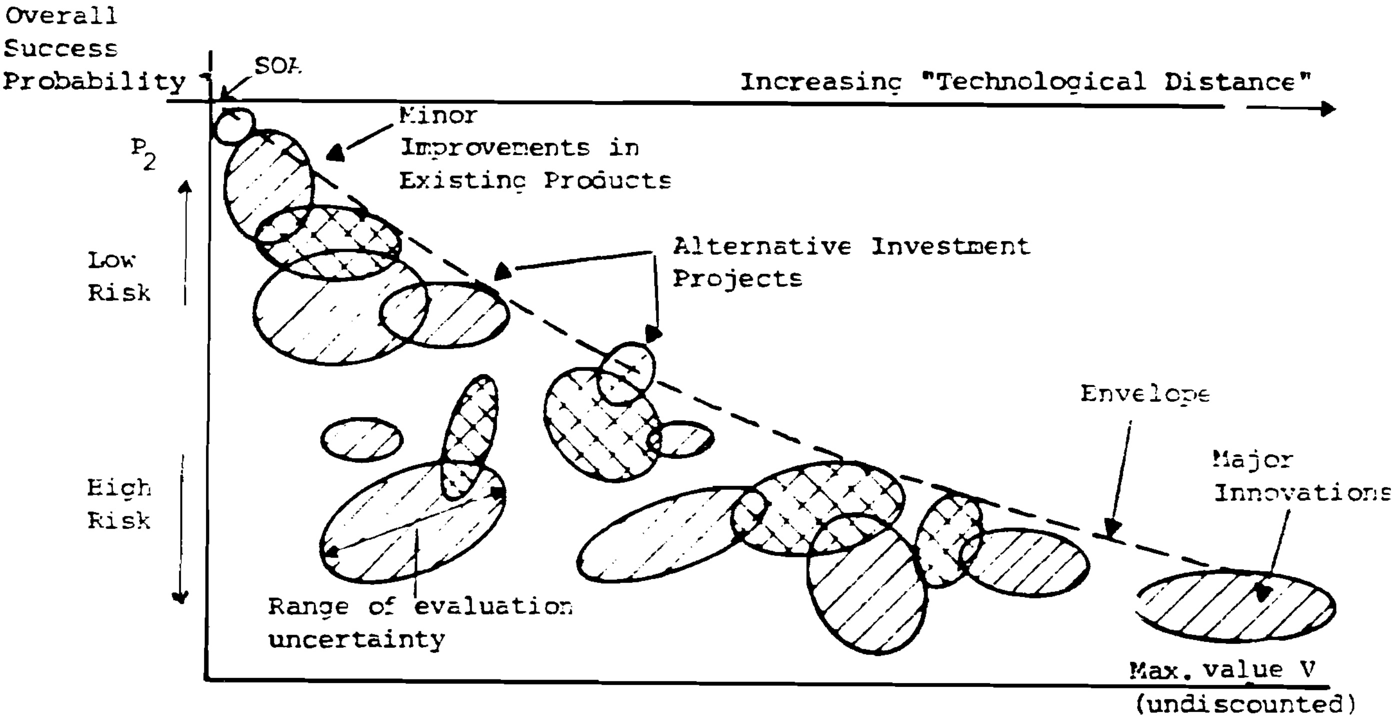

Figure 6. Perceived tradeoff between risk and value

Thus J is, in effect, the cumulative cost of research if continued to time t_F. Evidently $V(J)$, as defined above and approximated by (7), already incorporates the probability of market success (P_2). However, for what follows the specific form of (7) need not be specified. The analysis which follows is independent of the relationship between 'timeless' value, market risk and technological difficulty.

One can now calculate the expected benefits EB of an R+D project; starting at time t = 0 and terminating at time t_F, viz.

$$EB = VP_2\{\int_0^{t_F} \rho_1(t)\ W(t)dt + P_1(0)\ W(0)\}$$

$$= VP_2\{\int_0^{t_F} (\frac{dP_1(t)}{dt})\ W(t)dt + P_1(0)\} \tag{9}$$

where $W(t)$ depends inversely on $\pi(t)$ (equation 1) and $\pi(t)$ is the <u>expected</u> continuing stream of profits from pre-existing sources or static operations. The second term represents immediate success possibility.

Equation (9) can first be integrated by parts, yielding

$$EB = VP_2\{W(t_f)P(t_f) - \int_0^{t_F} P_1(t)\dot{W}(t)dt\} \tag{10}$$

It is convenient to approximate $W(t)$ by a 5 parameter function:

$$W(t) = C_0 + C_1e^{-jt} + C_2e^{-2jt} + D_1e^{jt} + D_2e^{2jt} \tag{11}$$

where

$$W(0) = C_0 + C_1 + C_2 + D_1 + D_2 = 1 \tag{12}$$

and

$$\dot{W}(t) = (-j)[C_1e^{-jt} + 2C_2e^{-2jt} - D_1e^{jt} -2D_2e^{2jt}] \tag{13}$$

It is convenient, hereafter, to define

$$X = e^{j(T-t)} \tag{14}$$

$$A = e^{jT} \tag{15}$$

$$B = {}^{-jt_F} = e^{-J} \tag{16}$$

It is shown in the Appendix that

$$EB = V \quad \{(\frac{1}{1+AB})(C_0 + C_1B + C_2B^2 + \frac{D_1}{B} + \frac{D_2}{B^2})$$

$$- \frac{2C_2}{A}(B-1) - (\frac{C_1}{A} - 2\frac{C_2}{A^2} - AD + 2A^2D_2)\ln\ (\frac{1+AB}{1+A})$$

$$- (D_1 - 2AD_2)(A \ln B + \frac{1}{B} - 1) - D_2(\frac{1}{B^2} - 1)\} \tag{17}$$

Substituting (15 and (16) into (6),

$$EC = \ln(\frac{1+A}{1+AB}) \tag{18}$$

Obviously the net benefits NB are given by

$$NB = EB - EC \tag{19}$$

from (17) and (18).

An investor or corporate decision-maker (DM) is presumably confronted by given values of the term VP_2, which is determined by some combination of the possibilities of technology and the attributes of the market place. Similarly, the DM is confronted by a particular form of W(t), which can be defined in terms of given values of the parameters $\{C_0\ C_1\ C_2\ D_1\ D_2\}$. The characterization of V(t) is a function of perceptions about the stage of the product or technology 'life cycle' and its implications for the future prospects of the exsiting business (Figures 3, 4). Early in the life cycle it may be reasonable to expect "continued growth and prosperity" for the foreseeable future. On the other hand, in the mature stage of the cycle, as markets approach saturation and/or competing technologies become established, it is realistic to anticipate the approach of much harder times, i.e. "the end of prosperity". At the beginning of the life cycle V(t) is a <u>decreasing</u> function, whence $D_1 = D_2 = 0$. Towards the end of the cycle, on the other hand, V(t) is an <u>increasing</u> function, which implies $D_2 > 0$ or $D_1 > 0$. Either way, V(t) is not a decision variable for the DM.

In principle, three decision variables or 'controls' accessible to the decision-maker are j, T and t_F. In practice, only values of $t_F \geq T$ ($1/B \geq A$) need be considered. A rational profit maximizing corporate leader (DM) wants to maximize net benefits NB with repsect to these three variables. However, as the problem is formulated here, only two are independent. Thus, it is sufficient to maximize with respect to T and t_F or, equivalently, A and B.

The conditions for an extreme value of net benefits as a function of A, B (or T, t_F) are:

$$\frac{\partial(NB)}{\partial A} = 0 \tag{20}$$

$$\frac{\partial(NB)}{\partial B} = 0 \tag{21}$$

or

$$\frac{\partial(EB)}{\partial A} = \frac{\partial(EC)}{\partial A} \tag{22}$$

$$\frac{\partial(EB)}{\partial B} = \frac{\partial(EC)}{\partial B} \tag{23}$$

Unfortunately, the algebraic expressions are quite complex and non-linear (transcendental) and they cannot be solved in closed form. Thus, in general, a simulation approach has been undertaken. The results are summarized hereafter. Plots of V(t) and net benefits (NB) vs. t_F for various choices of V(t) and values of t are presented in the Appendix.

Results and Conclusions

The quantitative analysis yields a very clear qualitative conclusion in terms of optimal R&D investment policy. The result is summarized below:

Life Cycle Stage	Time Preference W(t)	Optimal R&D Policy
Early (adolescence)	Decreasing (positive discount)	Short-term (Improvement) (T small, t_F small) See Figures 7,8
Intermediate (maturity)	Constant (no discount)	Short-term (Improvement) (T small) See Figure 9
Late (senescence)	Increasing (negative discount)	Long-term (T large, t_F large) See Figure 10

From other evidence, it appears that in the earlier stage of the cycle product improvement will predominate, whereas in the later stage, process improvement will be emphasized [Abernathy & Utterback 75]. It is remarkable that the optimal R&D policy is consistently the short term one, until a late stage of the life cycle, when the "end of prosperity" is clearly foreseeable.[5] But, for increasing functions W(t) the situation is reversed. (The policy reversal conditions can be explored more closely with the help of the simulation model).

Let us consider the situation successively in 3 stages of the life cycle, beginning with an early (childhood/adolescent) view. For example, suppose we assume a family of declining functions W(t), as shown in Figure 7a. Cases 1 through 5 correspond to discount rates with increasingly positive values. Let us now assume an R&D project with an initial probability of technical success of 0.5 (T=0). The resulting curves for the net benefits NB, as a function of t_F, are shown in Figure 7b. Note that each NB curve reaches a maximum value for some value of t_F, and then declines and finally becomes negative.

The optimum strategy for a venture capitalist making a one-time investment is to provide enough money for the venture to continue until the maximum value of expected NB before disocunting. The situation is more complicated if projects can be net evaluated each year and "turned off" at any time, based on new information. In this case, however, all past expenditures are regarded as "such" costs and only expected future costs and benefits need be considered.[6] In the case of annual re-

[5]In principle, of course. In practice managers of mature and senescent industries often fail to recognize this point, even after it is evident to outsiders.

[6]Annual re-evaluation gives the large firm investing in a portfolio of in-house R&D projects a significant theoretical advantage over the venture capitalist who must make longer-term commitments.

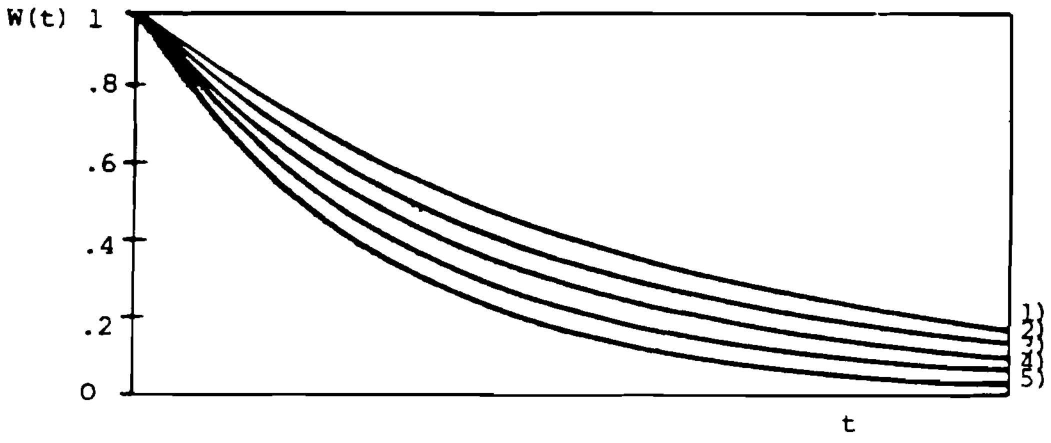

Figure 7a. A family of time-preference functions for scenario I (Early in the life cycle, positive discounting.

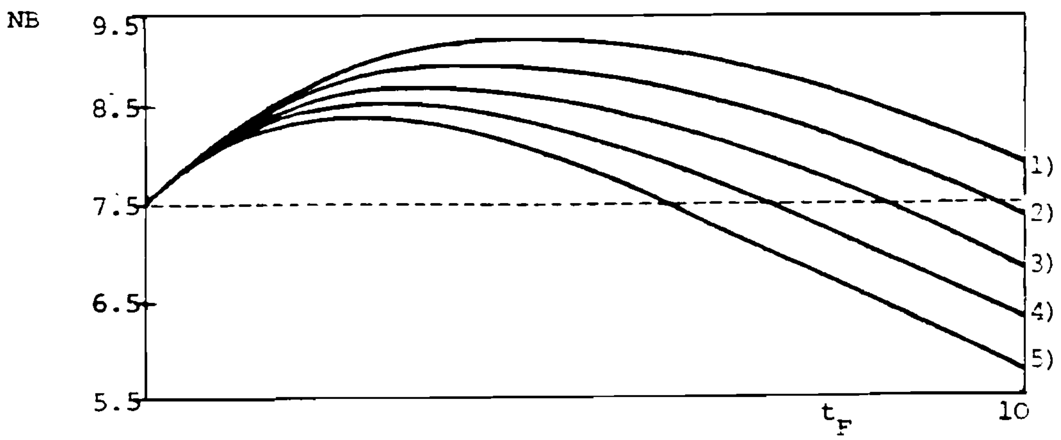

Figure 7b. Expected net benefit curves for various time-preference functions, assuming T = 0.

$C_0 = D_1 = D_2 = 0$ $V \cdot P_2 = 15$ $J = 0.2$ $T = 0$

1) $C_1 = 1$, $C_2 = 0$
2) $C = 0.75$, $C_2 = 0.25$
3) $C_1 = 0.5$, $C = 0.5$
4) $C_1 = 0.25$, $C_2 = 0.75$
5) $C_1 = 0$, $C_2 = 1.0$.

evaluation, the criterion for continuation is also somewhat more complex and need not be discussed here.

The effect of varying the initial probability of technical success is next shown in Figures 8a and 8b. (The W(t) function shown in Figure 8a is not identical to any of those in Figure 7a, although it most nearly resembles case 5). Qualitatively, it can be seen that the greater the initial probability of technical success the higher the maximum value of expected net benefits, and the shorter the optimum period of R&D. This result leaves no doubt that the optimal R&D policy during the early part of the life cycle is <u>short-term</u> and <u>low risk</u>.

We now consider the effect of moving through the life cycle. As time passes, the rate of market and profits growth slows and a time might come when a few large, stable, oligopolistic (or regulated) firms have a commodity-like makret that is no longer growing significantly, but which is also "safe". In this case the situation is intermediate: neither scenario I nor scenario II is applicable, and the best time-preference function is simply $W(t) = 1$.

The expected net-benefit (NB) curves for this case are shown in Figure 9. Comparing Figure 9 and Figure 8b, it can be seen that there is not much difference for the case $P_T(0) = 0.88$ ($jT = 2$), which has it maximum value for rather small values of t_F, but projects with smaller initial success probabilities becomes much more attractive. Nevertheless, if $W(t) = 1$, the short-term low-risk project is still preferable, other factors remaining equal.

The stable intermediate case considered above may persist for some time, but in general the life cycle moves inexorably on to an unstable situation more like scenario II. One possible time-preference function for this case is depicted in Figure 10a. The corresponding expected net benefit curves are shown in Figure 10b.

A very remarkable preference "reversal" phenomenon is observed by comparing Figure 10b with Figure 9 or Figure 8b. Providing the firm is financially able to continue investing for long enough, the optimum R&D project is one that has its maximum expected value later, rather than earlier. <u>This implies that late in the life cycle a risky, long-term project of given 'value' can be preferred to a safe short-term project.</u>

This result must be interpeted with some care, of course. If the planning horizon (Z) were allowed to become indefinitely large, so that t_F is unlimited, the above result seems to imply that $P_T(0)$ should become infinitesimal. This is obvious nonsense, because in such a case NB would also remain negative for an indefinitely long time! In reality, therefore, the maximum planning horizon is a constraint on the problem and $t_F < Z$. Subject to this caveat, what we have shown is that in a simple mathematical 'model' world, external circumstances (i.e. the life-cycle) can strongly influence attitude to risk. In fact, the conventional idea that 'risk aversion' or 'risk-seeking' are unchanging characteristics of decision makers must now be challenged.

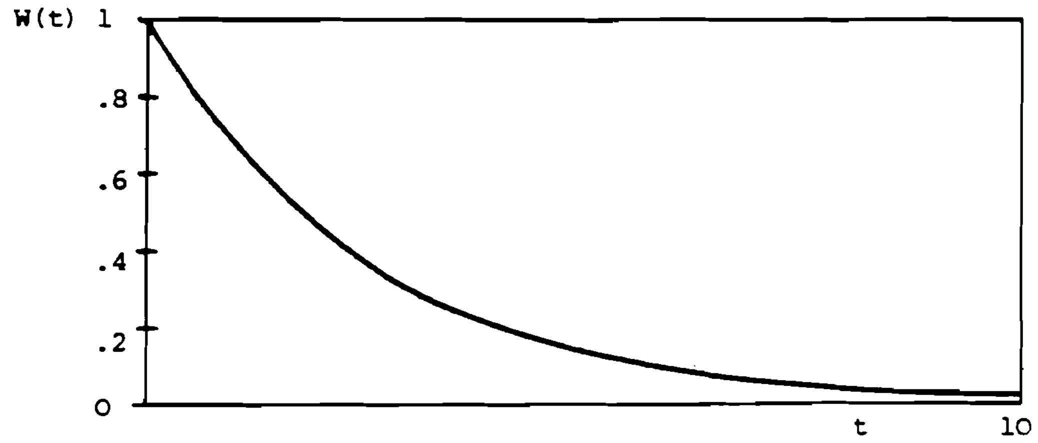

Figure 8a. A time preference function for scenario I (Early phase), positive discounting.

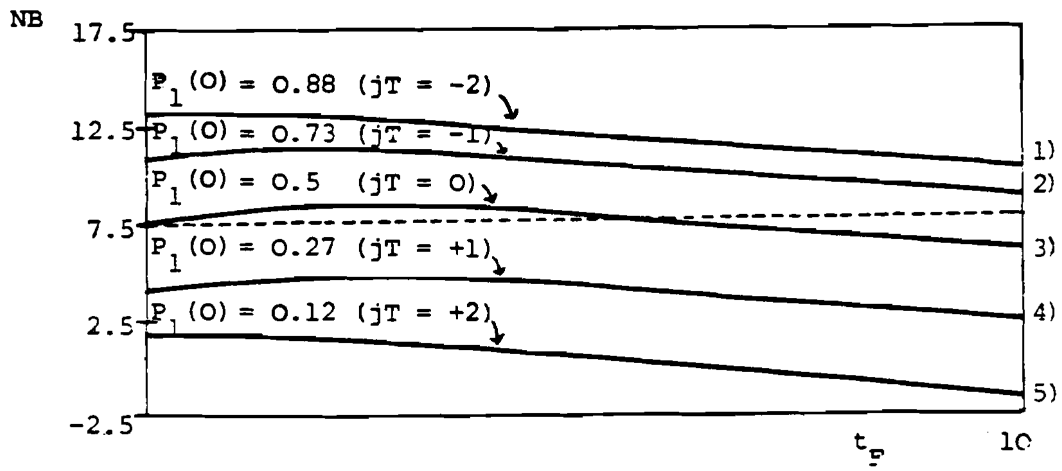

Figure 8b. Expected net benefits for various values of $P_1(0)$ from 0.12 to 0.88.

$C_1 = 1.0 \qquad C_0 = D_1 = D_2 = C_2 = 0.0 \qquad j = 0.4 \qquad V \cdot P_2 = 15$

1) $jT = -2$ $\quad P_1(0) = 0.88$
2) $jT = -1$ $\quad P_1(0) = 0.73$
3) $jT = 0$ $\quad P_1(0) = 0.5$
4) $jT = 1$ $\quad P_1(0) = 0.27$
5) $jT = 2$ $\quad P_1(0) = 0.12$

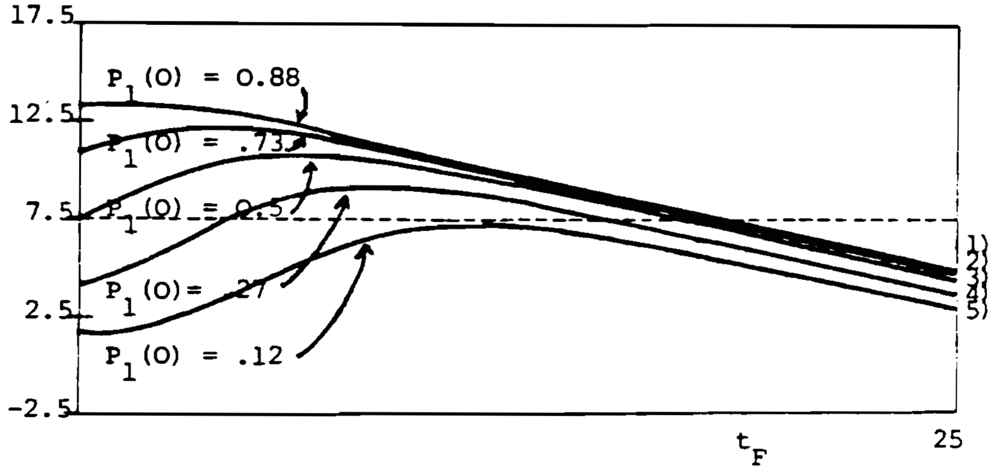

Figure 9. Net benefit curves for constant $W(t) = 1$ (Intermediate phase). Various values of $P_1(0)$ from 0.12 to 0.88

$C_0 = 1, \quad C_1 = C_2 = D_1 = D_2 = 0 \quad (W(t) = 1.0)$

$j = 0.4, \quad V \cdot P_2 = 15$

1) $jT = -2$, 2) $jT = -1$, 3) $jT = 0$
4) $jT = 1$, 5) $jT = 2$.

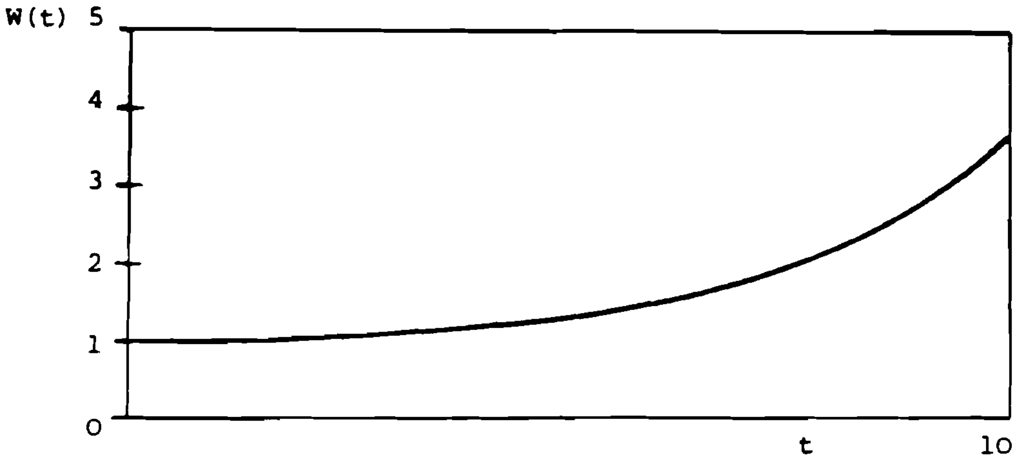

Figure 10a. A time-preference function for scenario II (Late phase), negative discounting.

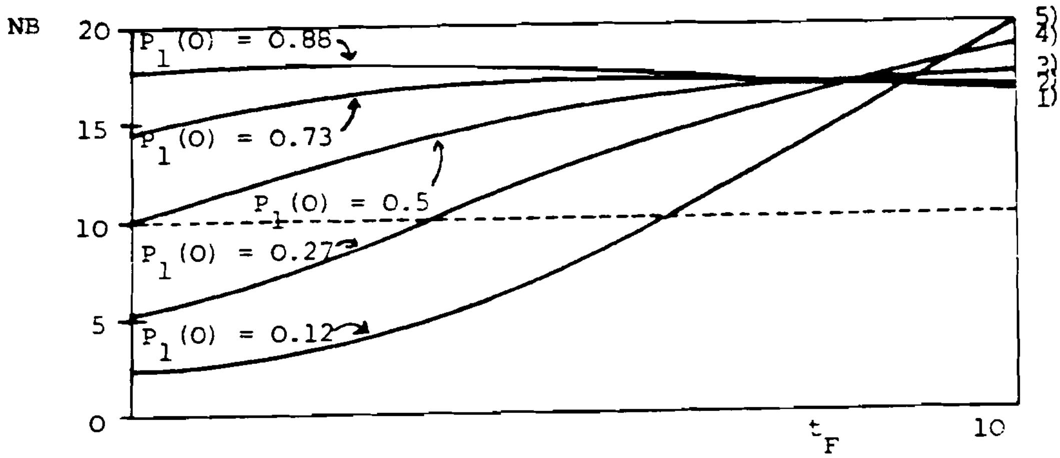

Figure 10b. Expected net benefit for various values of $P_1(0)$ from 0.12 to 0.88.

$j = 0.4 \qquad P_2 \cdot V = 20$

$C_0 = 0.95, \qquad C_1 = C_2 = D_2 = 0, \qquad D_1 = 0.005$

1) $jT = -2 \qquad P_1(0) = 0.88$
2) $jT = -1 \qquad P_1(0) = 0.73$
3) $jT = 0 \qquad P_1(0) = 0.5$
4) $jT = 1 \qquad P_1(0) = 0.27$
5) $jT = 2 \qquad P_1(0) = 0.12$

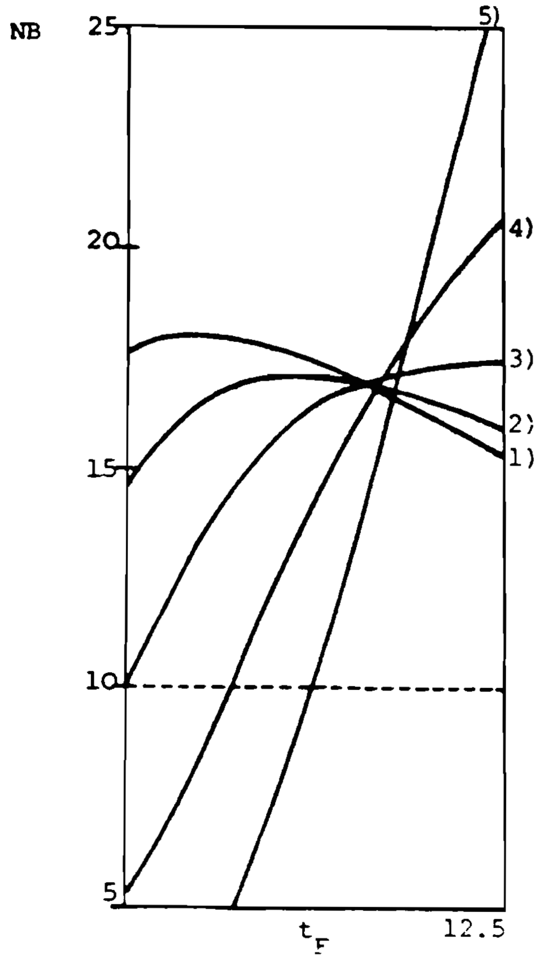

Figure 10c. Expected net benefits for various values of $P_1(0)$ continued.

Evidently, W(t) can be chosen in many ways other than those shown above. Some additional simulations are included in the Appendix, to illustrate the sensitivty of the results to the choice of parameters.

The logic underlying the model discussed in this paper is much more general than the specific application to R&D. Applications in other fields will be considered in subsequent papers. In conclusion, we emphasize that long-term, high risk ventures can never be justified by a benefit/cost methodology using time-preference functions equivalent to positive discount rates. The increasingly mindless use of packaged `models' (such as DCF) by business economists is an ominous development, in this context.

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APPENDIX

Derivation of Equation (17)

$$W(t) = C_0 + C_1 e^{-jt} + C_2 e^{-2jt} + D_1 e^{jt} + D_2 e^{2jt}$$

$$W(o) = Co + C_1 + C_2 + D_1 + D_2$$

$$\dot{W}(t) = (-j)\left[C_1 e^{-jt} + 2C_2 e^{-2jt} - D_1 e^{jt} - 2D_2 e^{2jt}\right]$$

$$\int_0^{t_F} \frac{dP_1(t)}{dt} w\,(t)\,dt$$

$$= \left[P_1(t)W(t) - \int P_1(t)\dot{W}(t)\,dt\right]_0^{t_F}$$

$$= P_1(t_F)W((t_F) - P_1(o)$$

$$- \int_0^{t_F} P_1(t)W(t)dt$$

$$P_1(t) = \frac{1}{1+e^{j(T-t)}}$$

Here, substituting $A = e^{jT}$, $X = e^{j(T-t)}$ and $B = e^{-jt_F}$

(-> $dx = (-j)\ X\ dt$)

and rearranging terms, we can obtain

$$\int_0^{t_F} P_1\,(t)\,\dot{W}\,(t)\,dt$$

$$= \int_A^{AB} \left(\frac{1}{1+x}\right) (-j) \left[\frac{C^1X}{A} + \frac{2C_2X^2}{A^2} - D_1A\left(\frac{1}{X}\right) - 2D_2A^2\left(\frac{1}{X^2}\right)\right]$$

$$\left(-\frac{1}{j}\right)\left(\frac{1}{x}\right) dx$$

$$= 2\left(\frac{C_2}{A}\right)(B-1) + \left(\frac{C_1}{A} - 2\frac{C_2}{A^2} - AD_1 + 2A^2D_2\right)Ln\left(\frac{1+AB}{1+A}\right)$$

$$+ (AD_1 - 2A^2D_2)LnB + (D_1 - 2AD_2)\left(\frac{1}{B} - 1\right)$$

$$+ D_2\left(\frac{1}{B^2} - 1\right) .$$

The above result and the following equation:

$$P_2(t_F)W(t_F) - P_1(o)$$

$$P_2(t_F)W(t_F) - P_1(o)$$

$$= \frac{1}{1+AB}\left(C_o + C_1B + C_2B^2 + \frac{D_1}{B} + \frac{D_2}{B^2}\right) - \frac{1}{1+A}$$

derive expected benefit as

$$EB = \left[\frac{1}{1+AB}\left[C_o + C_1B + C_2B^2 + \frac{D_1}{B} + \frac{D_2}{B_2}\right]\right.$$

$$- 2\left(\frac{C_2}{A}\right)(B-1) - \left(\frac{C_1}{A} - 2\frac{C_2}{A_2} - AD_1 + 2A^2D_2\right)Ln\left(\frac{1+AB}{1+A}\right)$$

$$- (D_1 - 2AD_2)\left(ALnB + \frac{1}{B} - 1\right)$$

$$\left.- D_2\left(\frac{1}{B_2} - 1\right)\right] V \quad P_2$$

Some mathematical properties of net benefit

$$NB = EB - EC$$

$$= V\,P_2 \left[\frac{1}{1+AB} [C_0 + C_1 B + C_2 B^2 + \frac{D_1}{B} + \frac{D_2}{B^2}] \right.$$

$$-2(\frac{C_2}{A})\ (B-1) - (\frac{C_1}{A} - 2\frac{C_2}{A^2} - AD_1 + 2A^2 D_2) Ln\ (\frac{1+AB}{1+A})$$

$$-(D_1\ - 2AD_2)\ (ALnB + \frac{1}{B} - 1)$$

$$\left. -D_2(\frac{1}{B^2} - 1) \right]$$

$$-\left[jt_F + Ln\ (\frac{1+A}{1+AB}) \right]$$

if t_F --> $+\infty$ or B --> + o, then NB can be approximated by

$$NB \cong VP_2 \left[\frac{D_1}{B} + \frac{D_2}{B^2} - (D_1 - 2AD_2)\ \frac{1}{B} - (D_1\ -2AD_2) A\ LnB \right.$$

$$\left. - \frac{D_2}{B^2} \right] - jt_F$$

$$= VP_2\ (2AD_2\ \frac{1}{B} - (D_1\ - 2AD_2) A\ LnB) + LnB\ ,$$

Where A>0.

In case of D_2>0, then as is shown below, NB diverges to positive infinite value as t_F increases (or B converges to zero).

[proof] Let define y as

$$y = \frac{1}{B} + jLnB = Ln\ e^{\frac{1}{B}} + LnB^{\alpha}$$

$$= Ln(B^{\alpha} e^{\frac{1}{B}}) \quad .$$

There exists positive integer $M > \alpha$ and

$$e^{\frac{1}{B}} > 1 + (\frac{1}{B}) + \frac{1}{2}(\frac{1}{B})^{2} + --- \frac{1}{M!}(\frac{1}{B})^{M}$$

holds.

Therefore

$$Ln(B^{\alpha} e^{\frac{1}{B}}) > Ln(B^{\alpha} + B^{\alpha-1} + \frac{1}{2} B^{\alpha-2} + --- \frac{1}{M!} B^{\alpha-M})$$

is obtained

The last term of the right hand side diverges to positive infinite as $B \dashrightarrow +0$ and other terms converge to zero.

Therefore in case of $D_2 > 0$, there is no optimal t_F for any value of A.

[Q.E.D]

When $D_2 = 0$ and $D_1 > 0$ then NB can be approximated by

$$NB = VP_2 \ (-AD_1) LnB + LnB$$

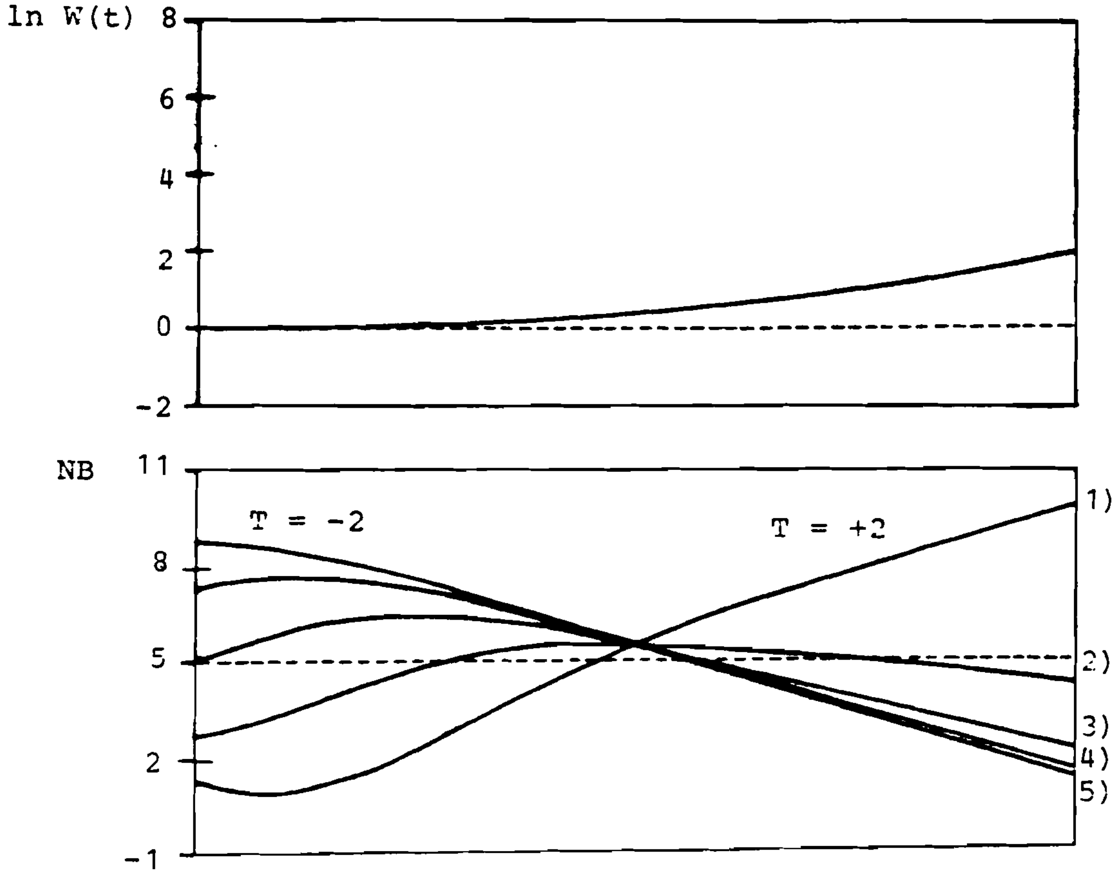
$$= (1 - VP_2 AD_1) LnB \ .$$

Therefore we can obtain the following three cases.

$$\left.\begin{array}{llll} \text{if } VP_2AD_1 > 1 & \text{then} & NB \dashrightarrow + \\ \phantom{\text{if }} VP_2AD_1 = 1 & \text{then} & NB \dashrightarrow \alpha \\ \phantom{\text{if }} VP_2AD_1 < 1 & \text{then} & NB \dashrightarrow - \end{array}\right\} \begin{array}{l} \text{as } B \dashrightarrow 0 \\ (t_F \dashrightarrow + \end{array}$$

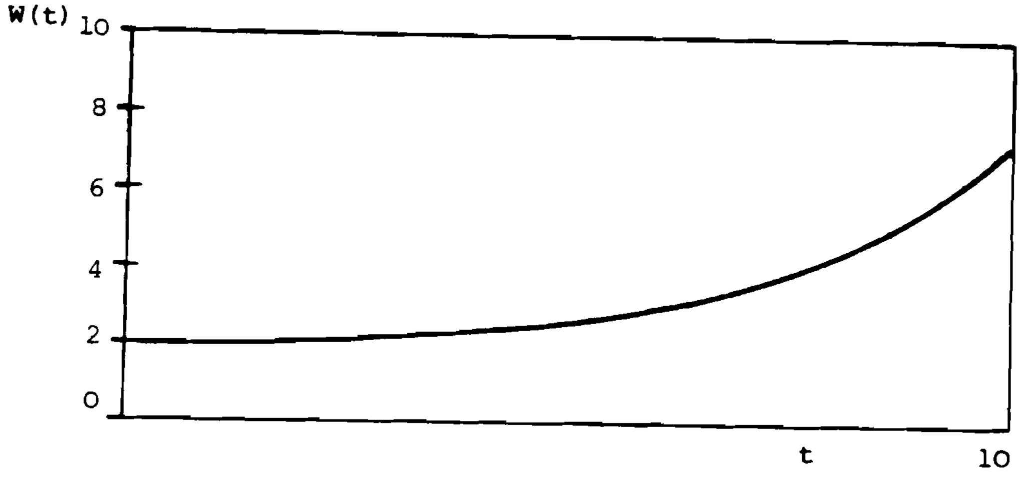
As is suggested in the above, there may be many cases on behavior of NB.

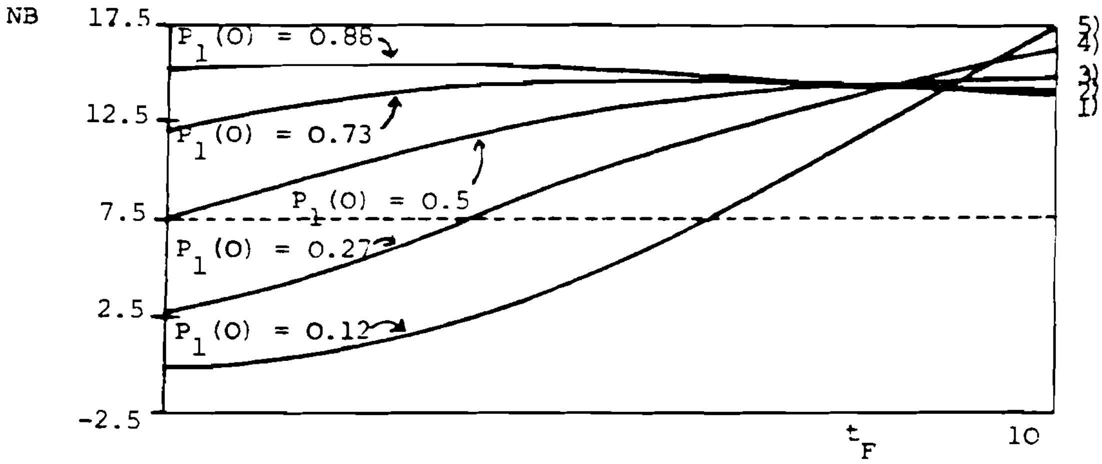
Some examples of behavior of NB



Case $D_1 = 0.025$, $D_2 = C_1 = C_2 = 0$, $C_0 = 0.975$, $P_2 \cdot V = 10$

1) jT = 2, 2) jT = 1, 3) jT = 0, 4) jT = -1, 5) jT = -2.





$C_0 = 0.95$, $C_1 = C_2 = 0$, $D_1 = 0.05$, $D_2 = 0$

$j = 0.4$, $V \cdot P_2 = 15$.

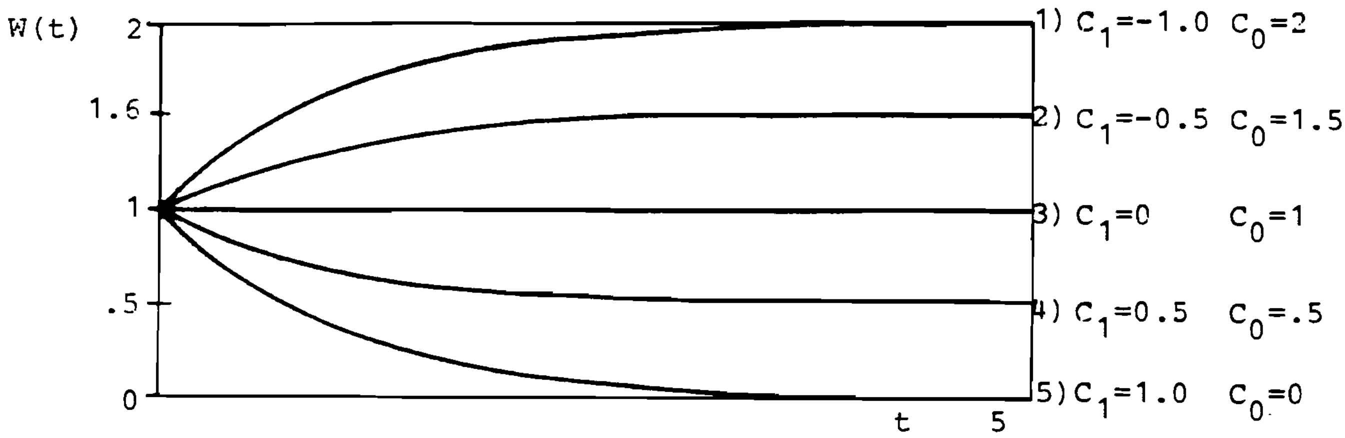
1) $jT = -2$, 2) $jT = -1$, 3) $jT = 0$

4) $jT = 1$, 5) $jT = 2$.

Main Case A

Saturating W (t)

(W(t) → constant as t → ∞).



C_2=0

D_1=D_2=0

Case $T = 0$, $V \cdot P_2 = 10$, $j = 1$

$C_2 = 0$, $D_1 = D_2 = 0$

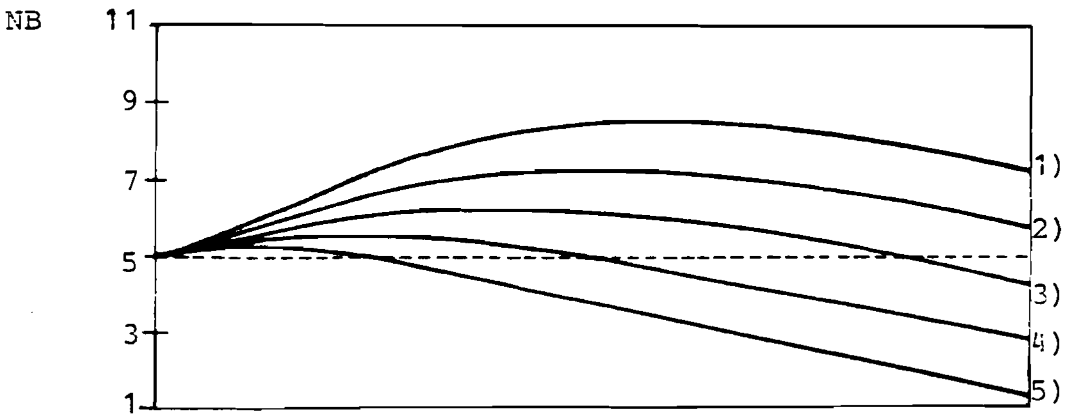
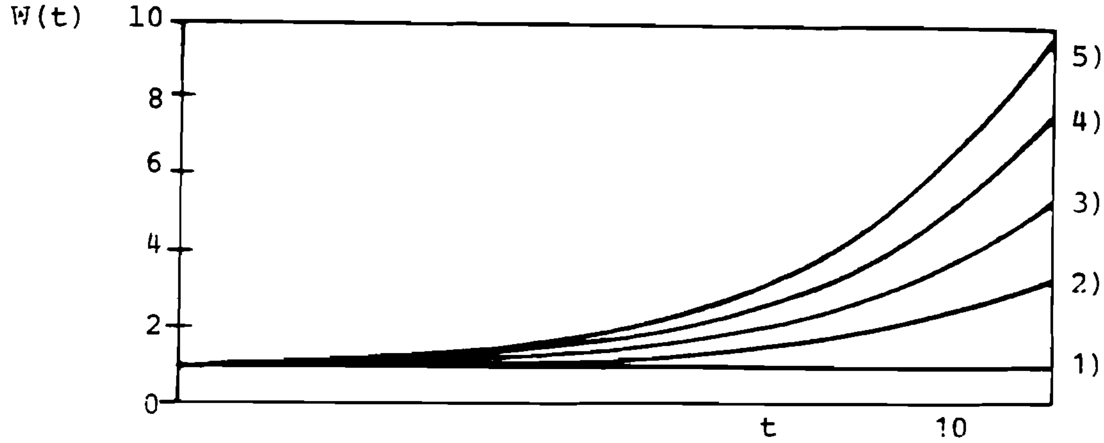
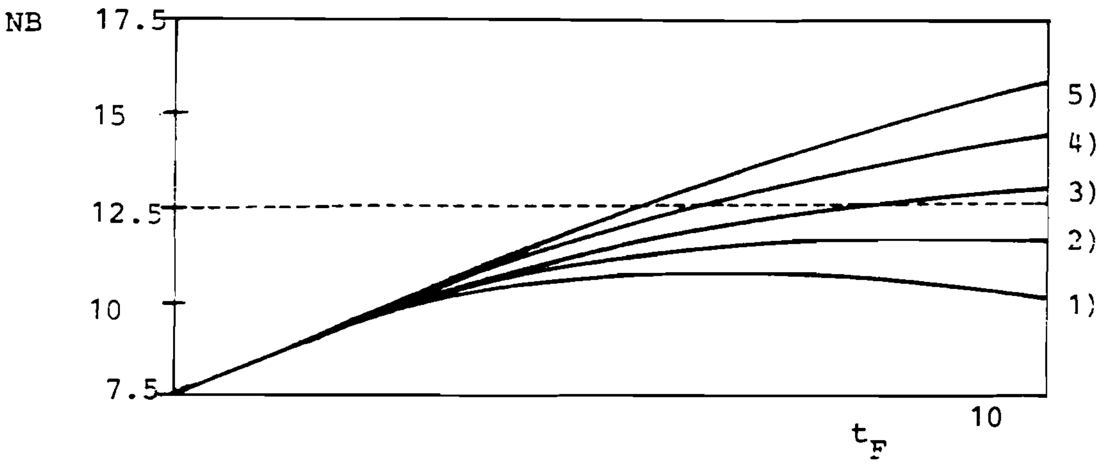


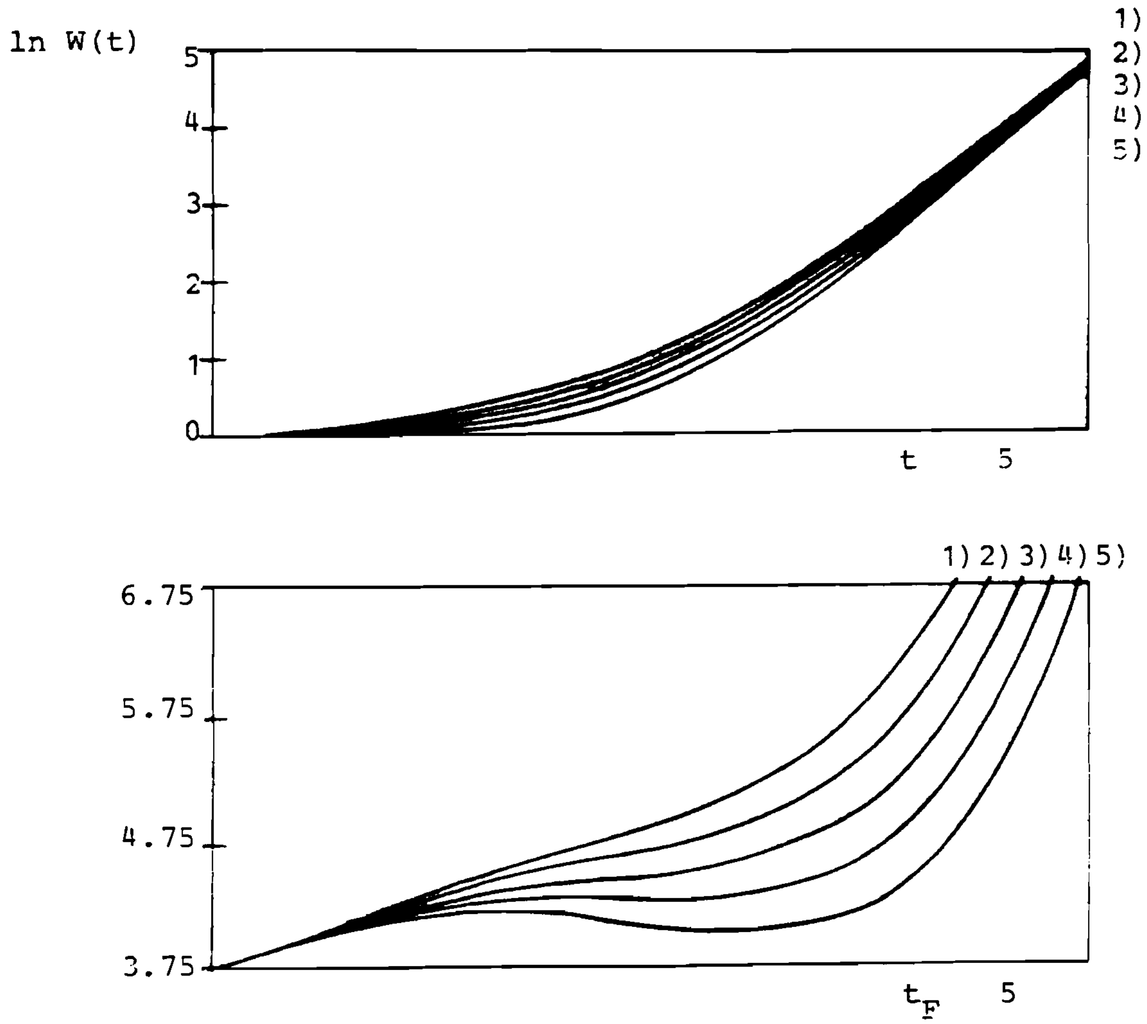
Figure A-1





$D_2 = C_1 = C_2 = 0$, $jT = 0$, $j = 0.4$, $V \cdot P_2 = 15$.

1) $D_1 = 0.0$, $C_0 = 1$
2) $D_1 = 0.04$, $C_0 = 0.96$
3) $D_1 = 0.08$, $C_0 = 0.92$
4) $D_1 = 0.12$, $C_0 = 0.88$
5) $D_1 = 0.16$, $C_0 = 0.84$.



Case $T = 0$, $V \cdot P_2 = 7.5$, $j = 1$

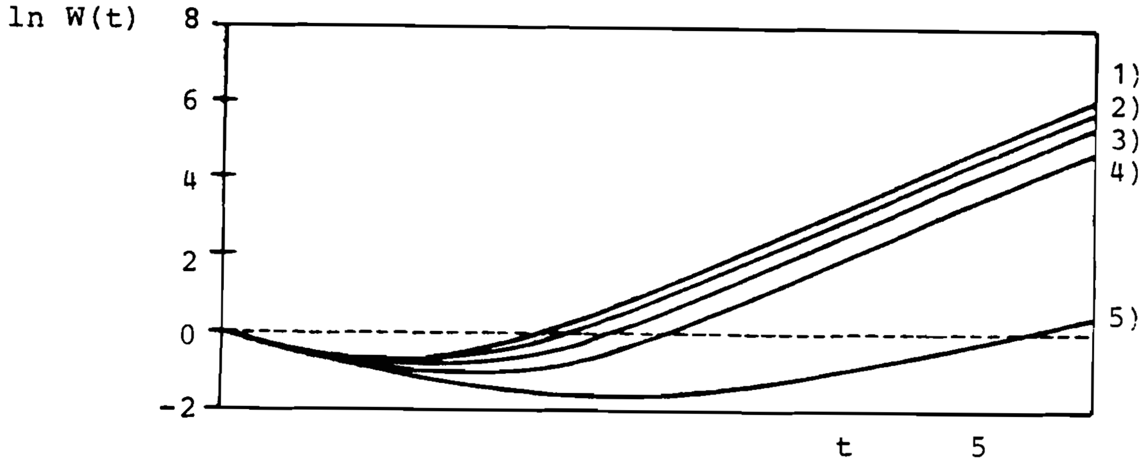
$C_1 = C_2 = 0$, $D_2 = 0.005$, $D_1 = 0.1$ .... 1)

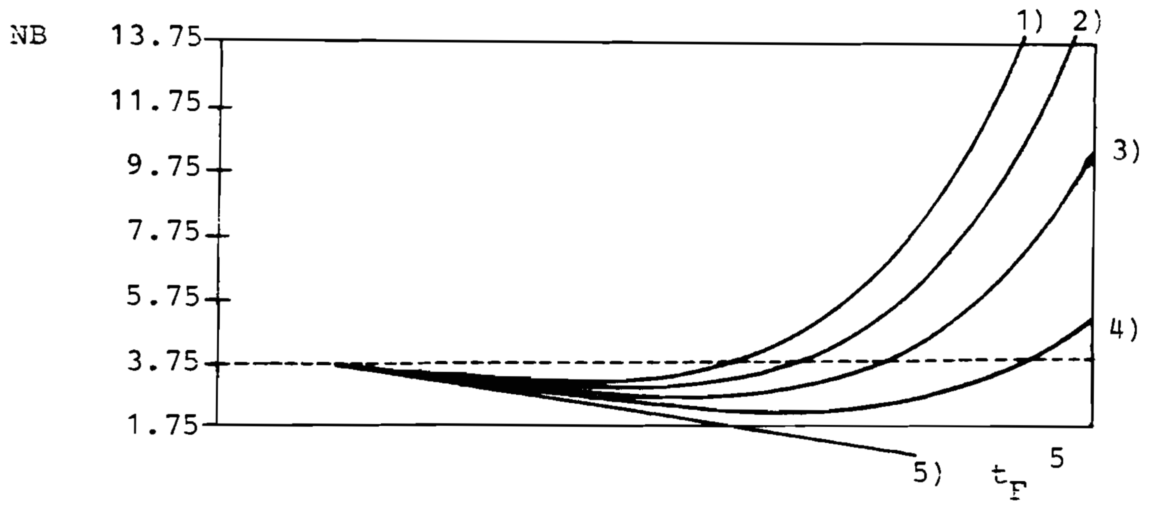
$= 0.075$ .. 2)

$= 0.05$ ... 3)

$= 0.025$ .. 4)

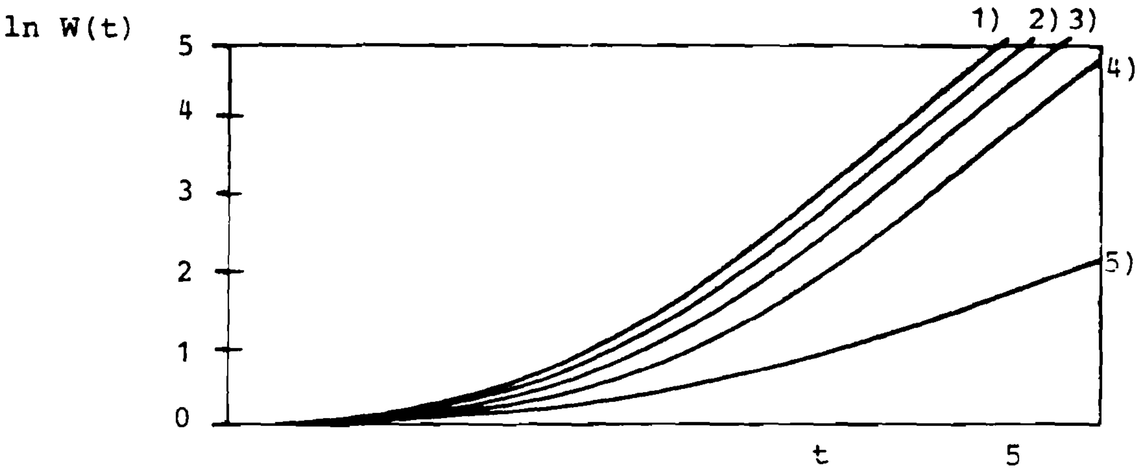
$= 0.0$ .... 5)

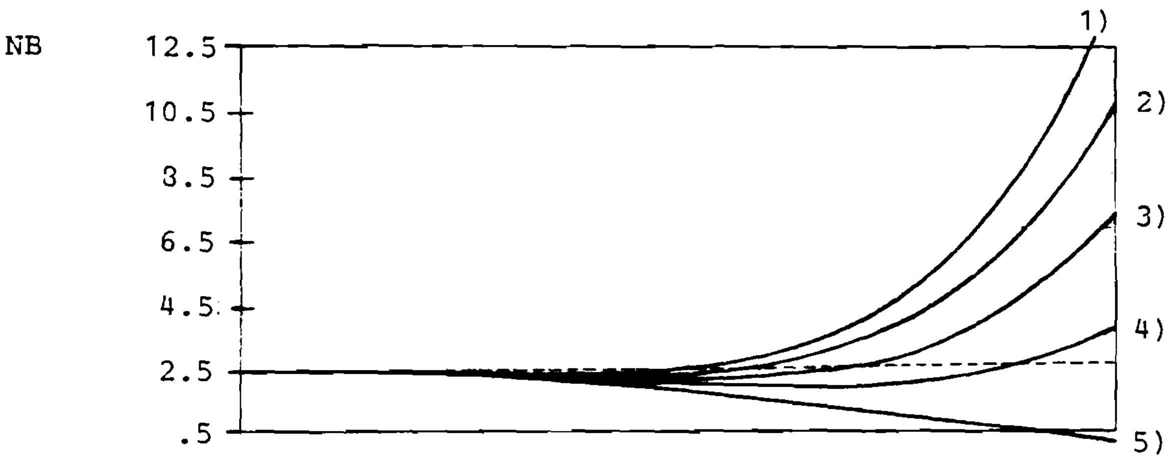




Case $T = 0$, $V \cdot P_2 = 7.5$, $j = 1$

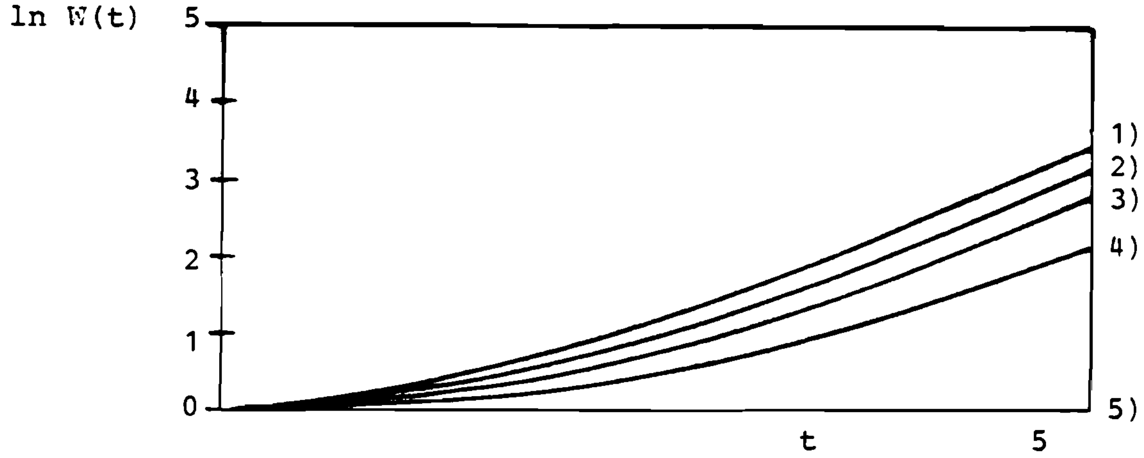
$C_1 = 1$, $C_2 = 0$, $D_1 = 0.01$,	D_2	$= 0.2$	C_0	$= -0.03$	...1)
		$= 0.015$		$= -0.025$	..2)
		$= 0.01$		$= -0.02$	...3)
		$= 0.005$		$= -0.0015$	..4)
		$= 0.0$		$= -0.01$	...5)

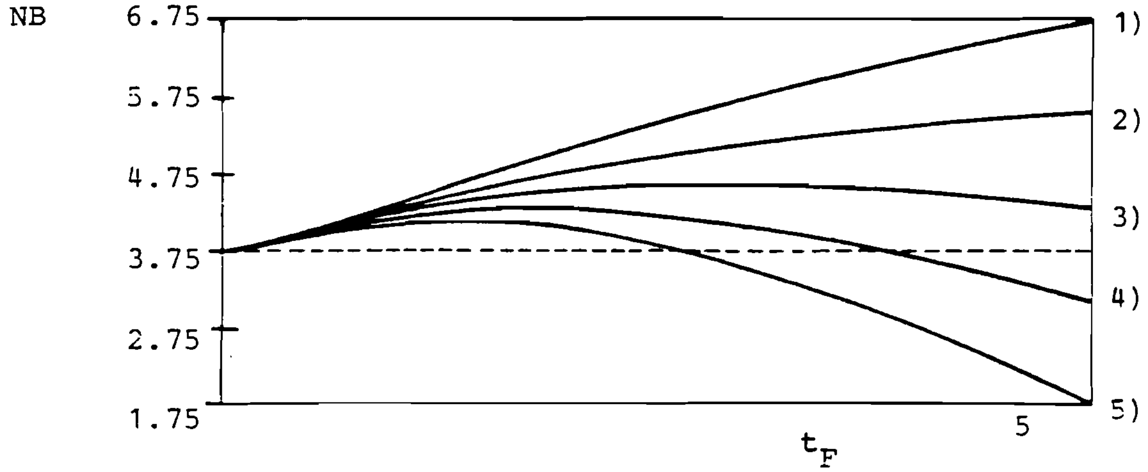




Case $T = 0$, $V \cdot P_2 = 5.0$, $j = 1$

$C_1 = C_2 = 0$, $D_1 = 0.05$, $D_2 = 0.02$ ... 1)
= 0.0015 .. 2)
= 0.01 .... 3)
= 0.005 ... 4)
= 0 .... 5)





Case $T = 0$, $V \cdot P_2 = 7.5$, $j = 1$

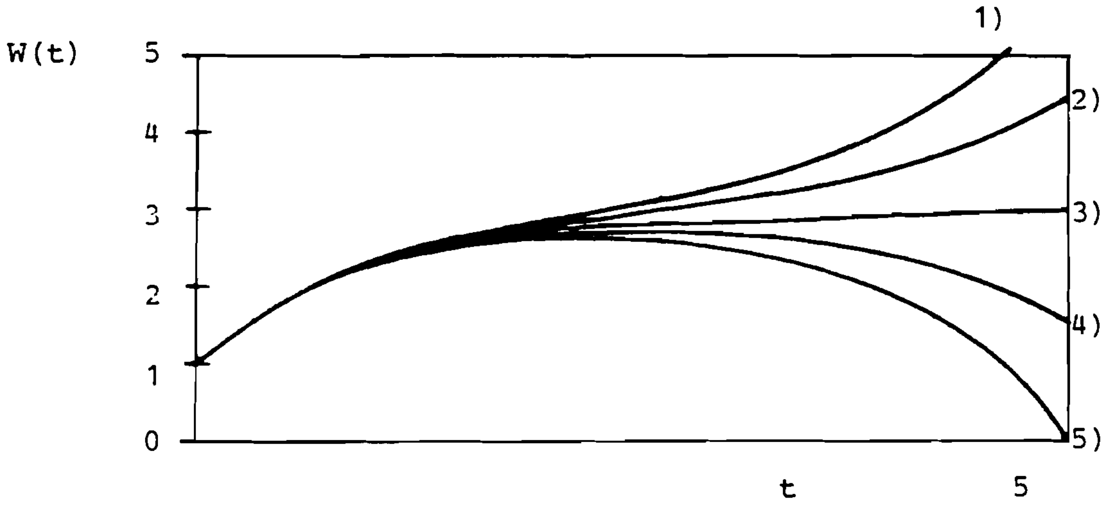
$C_1 = C_2 = 0$, $D_2 = 0$, $D_1 = 0.2$ ... 1)

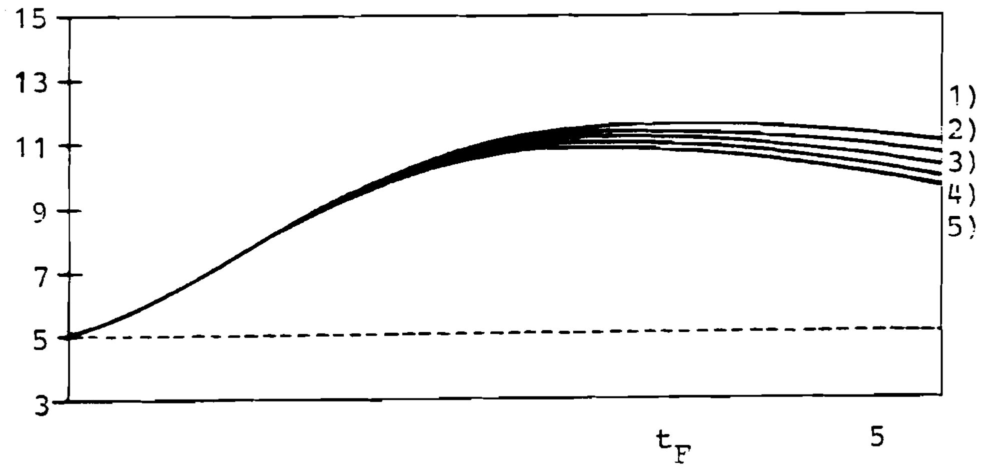
$= 0.15$ ... 2)

$= 0.1$ ... 3)

$= 0.05$ ... 4)

$= 0$ ... 5)

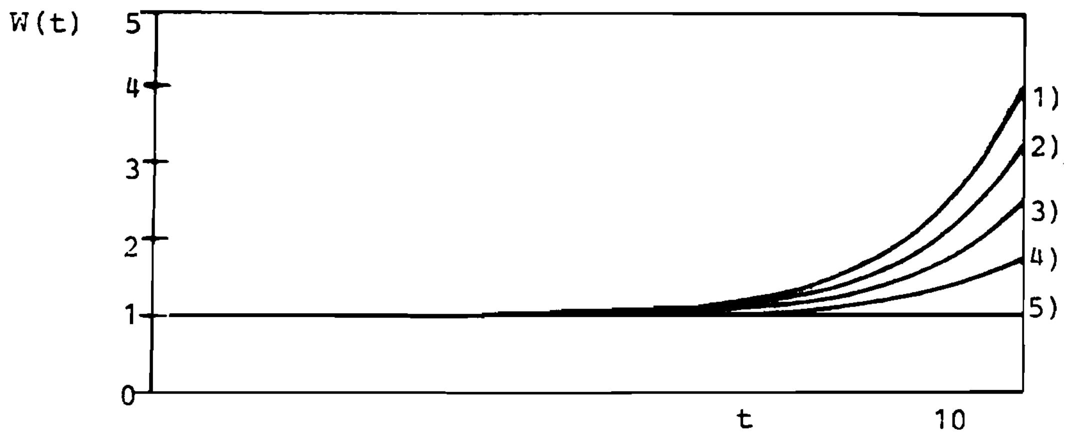


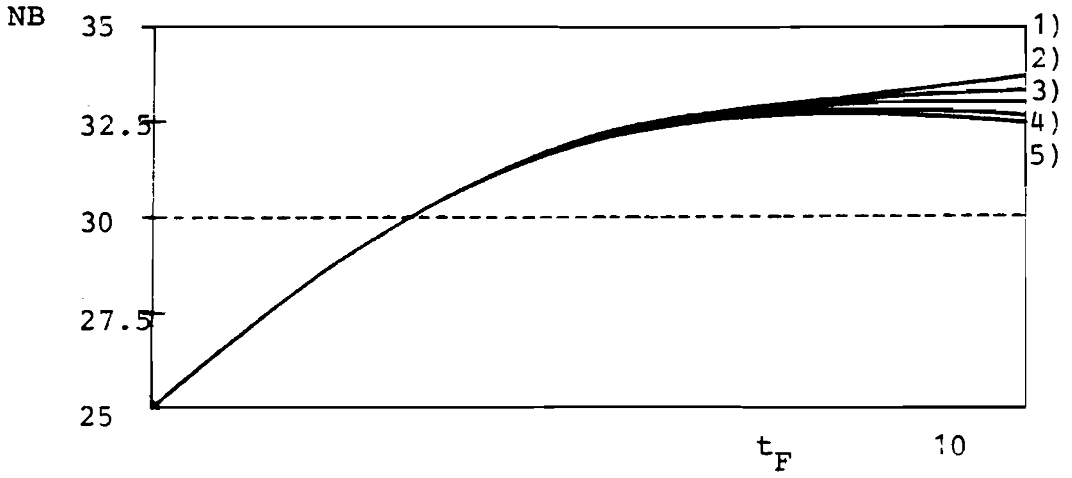


Case $T = 0$, $V \cdot P_2 = 10$, $j = 1$

$C_1 = -2$, $C_2 = 0$, $D_2 = 0$,

$D_1 = 0.02$	$C_0 = 2.98$	.. 1)
$= 0.01$	$= 2.99$	.. 2)
$= 0$	$= 3.0$	... 3)
$= -0.01$	$= 3.01$	.. 4)
$= -0.02$	$= 3.02$	.. 5)

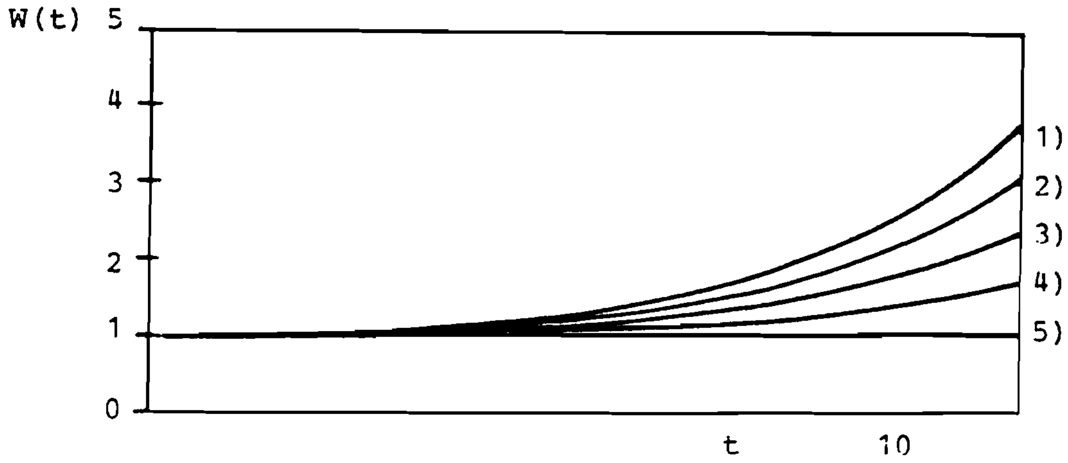


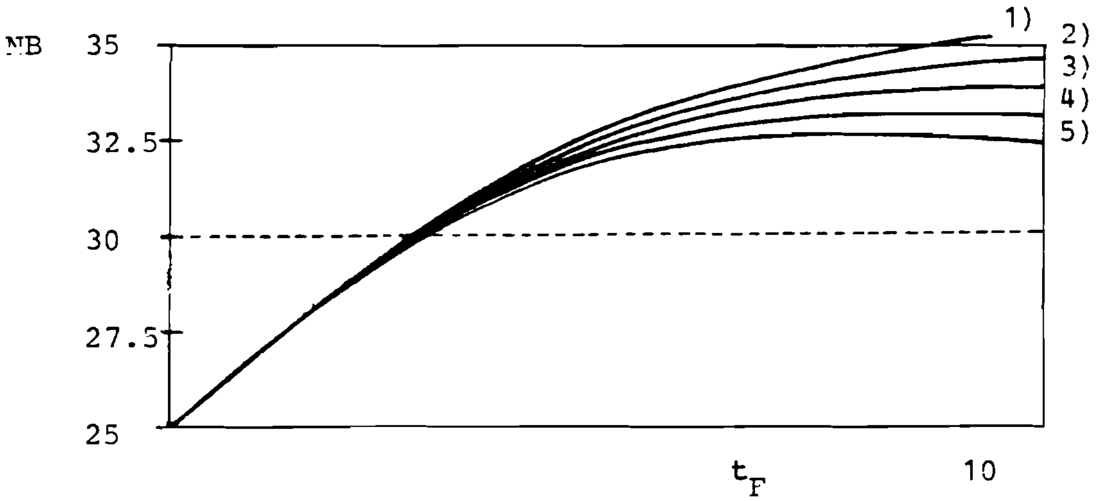


$j = 0.4$, $T = 0$, $P_2 \cdot V = 50$

$C_1 = C_2 = D_1 = 0.$

1) $D_2 = 0.001$, $C_0 = 0.999$
2) $D_2 = 0.00075$, $C_0 = 0.99925$
3) $D_2 = 0.0005$, $C_0 = 0.9995$
4) $D_2 = 0.00025$, $C_0 = 0.99975$
5) $D_2 = 0$, $C_0 = 1.0.$





$j = 0.4, \quad T = 0, \quad P_2 \cdot V = 50$

$C_1 = C_2 = D_2 = 0$

1) $D_1 = 0.05, \quad C_0 = 0.95$
2) $D_1 = 0.0375, \quad C_0 = 0.9625$
3) $D_1 = 0.025, \quad C_0 = 0.975$
4) $D_1 = 0.0125, \quad C_0 = 0.9875$
5) $D_1 = 0, \quad C_0 = 1.0.$