PRIMARY ENERGY SUBSTITUTION MODELS*

On The Interaction Between Energy And Society

C. Marchetti

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P r e f a c e :

This paper describes an attempt to develop a "synthetic" model of primary energy substitution, using certain rules which proved fruitful in describing the substitution of other commodities.

This model will be used for forecasting, and for checking the validity of certain objectives set for R&D in the field of energy.

*From a lecture, delivered in Moscow, November 1974.

Trends in energy demand

The first point in forecasting energy demand is obviously to look at historical trends, over a century at least, and try to extract the signal out of the white noise and various medium-scale perturbations that occur along the way. Although the long-term extrapolation of these trends may require a more subtle analysis of social and economic trends, they are good to be kept in mind.

The ones reported in Fig. 1 and Fig. 2 have something special. They include wood and farm waste. This is necessary to get a proper basis for extrapolation because part of the growth of commercial energy sources is due to substitution of wood and farm waste.

As you see, apart from the big dip, coinciding with the great recession, "healed" then by World War II and some "overheating" coinciding with world war I and preceding the 1930's recession, the 2% secular trend is followed quite tightly for the world, even taking into account the compression due to the log display.

In the case of the U.S. we also have a well defined trend with the bumps in somehow different positions. The higher value of 3% does not appear particularly significant as the U.S. population has grown roughly 1% faster than the rest of the world in the period considered (1860-1960).

The second point is to look inside the envelope of total energy demand for trends in primary fuels demand. I did this exercise at IIASA, using a methodology completely different from the "modelling" which is so popular in many places of the world, and whose contradictory results, when used to forecast over long ranges, cast many doubts on its reliability.

I started from the somehow iconoclastic hypothesis that the different primary energy sources are commodities competing for a market, like different brands of soap or different processes to make steel, so that the rules of the game may after all be the same. These rules are best described by Fischer and Pry (1)(2), and can be resumed in saying that the fractional rate at which a new commodity penetrates a market is proportional to the fraction of the market not yet covered:

$$1) \quad \frac{1}{F} \frac{dF}{dt} = \alpha (1-F)$$

or:

$$2) \quad \ln (F/1-F) = \alpha t + C$$

where: F = fraction of market penetrated
α and C constants, characteristic of the particular commodity and market.

In Figs. 3, 4 and 5 some cases of market penetration are reported, showing the extraordinary precision by which those curves fit the statistical data (which often are not very precise). All of them refer to a competition between two products. In the case of energy we have three or four energy sources competing most of the time so I had to extend a little the treatment with the extra rule that one of the fractions is defined as the difference to 1 of the sum of the others. This fraction follows approximately an equation of type (2) most of the time, but not always. It finally shows saturation and change in coefficients.

The fraction dealt with in this way corresponds to the oldest of the growing ones. The rule can be expressed in the form: First in - first out.

Fig. 6 shows the plotting of statistical data for the U.S. in the form ln (F/1-F) vs. time.

More than a century of data can be fitted in an almost perfect way using only two constants, which come out to be two dates, for each of the primary energy sources (wood, coal, oil, gas). The whole destiny of an energy source seems to be completely predetermined in the first childhood.

As we can see by analyzing the curves and the statistical data in greater detail, these trends - if we can call them that way - go unscathed through wars, wild oscillations in energy prices and depressions. Final total availability of the primary reserves also seems to have no effect on the rate of substitution. The only real departures from the curves are due to strikes in the coal industry, but the previous trend is rapidly resumed and the effects of the strike somehow "healed". On the point of availability it seems that the market regularly moved away from a certain primary energy source, long before it was exhausted, at least at world level. The extrapolation of these trends indicates that the same thing is likely to happen in the future, e.g. that oil reserves will never be exhausted because of the timely introduction of other energy sources.

When I started showing around those curves, people said they were fascinated, then that the fit was too good to be true, then that one should find the explanation before accepting and using them. Nothing to say about the first two points but the third one is in principle unacceptable: laws work or don't work, and the only reason to accept a rule as a law is because all sorts of tests applied to it show that it works.

What most model makers do, starting from elementary relations and by functional and progressive aggregations going to macroscopic variables (e.g. demand) is very similar to what is done in statistical mechanics in order to "induce" e.g. thermodynamic laws from mechanistic principles. But thermodynamics is completely autonomous from the interpretation, in the sense that its "truth" is internal to the set of macroscopic measurements from which it has been derived.

Now, putting philosophy aside, I played the game of forecasting (i.e. of backcasting) within the historical period. E.g., I took the data for the U.S. from 1930 to 1935 and tried to forecast oil coverage of the U.S. market up to 1970. As Fig. 7 shows, the predicted values even for the saturation period fit the statistical data better than 1%, which after all is the minimum error that can be expected from this kind of statistics. This means that the contribution of oil to the U.S. energy budget, e.g. in 1965, was completely predetermined 30 years before, with the only assumption that a new primary source of energy (e.g. nuclear) was not going to play a major role in the meantime. As the history of substitutions shows, however, the time a new source takes to make some inroad in the market is very long indeed, about a hundred years to become dominant starting from scratches, so that also this assumption appears really unimportant for predictions up to 50 years ahead.

As our game worked so well in the last hundred years, why not make a try for the next hundred years, just to see what happens?

The results are shown in Figs. 8, 9 and 10, and some quite important consequences can be drawn from them.

The first one is that substitution has a certain internal dynamics largely independent from external factors like final reserves of a certain primary energy source. So the coal share of the market has started decreasing in the U.S. around World War I in spite of the fact that coal reserves were in a sense infinite.

The second is that substitution proceeds at a very slow pace, let us say of the order of a hundred years to go from 1% to 50%. The "acceleration of the times" which we all perceive does not show up in the statistics. Perhaps the increasing number of changes is giving us that sense of acceleration even if the rate of each individual change stays constant and low.

This fact rules out the possibility of having fusion or solar energy covering a sizable fraction of the energy market before the year 2050 and leaves us with the narrow choice: go nuclear or bust. A resurgence of coal appears improbable too, and I found very nasty reactions on that point from everybody except from coal people who appeared in a sense relieved from a mission well above their forces.

The problem, however, of how to consider a SNG plant, a coal consumer or a primary energy producer, as in fact it is seen from the market, is still an open question. This leaves some ambiguity in the interpretation of the curves in the case of important intertransformation of fuels.

Figs. 11 and 12 show the same curves for the world, with different hypotheses concerning the timing and rate of introduction of new sources of primary energy.

These curves relate to fractions. To get absolute values, one has to multiply them by the total level of energy consumption. Fig. 13 gives the result for the world, using a 2% secular rate of growth, Fig. 14 for the U.S., using two different rates of growth. In both cases, the amount consumed in 1970 is taken as a unity.

In case of scarcity it appears that energy saving is much more efficient than substitution.
Phasing out of a source does not necessarily mean reduced production in absolute terms.

The following step is to integrate this consumption all over the cycle of a certain primary fuel and compare it with the resources. I did this exercise and found that after all the world will not be short of oil, if nuclear energy will keep the present rate of penetration and perhaps even if not, but that there may be problems with natural gas. As everybody has his own figures for the reserves, I prefer not to raise a row on this point and leave it to you to make comparisons and draw conclusions. After all, the scope of this presentation is essentially methodological.

Productivity vs. energy

People in the world rightfully try to improve their lot, and the numerical indicator for this is GNP. So the linkage between GNP and energy consumption, and the possibility of making this linkage looser as it appears now, are of the utmost importance both in order to better understand and plan the working of our society and perhaps have a guess on the evolutionary trends.

Although I will not be able to draw final conclusions, I hope the next figures will show you that there is much purpose in the research and the linkage is not as rigid and indissoluble as much of the pertinent literature tends to indicate.

History as usual is a good mine for digging and I will start giving a little hint. Fig. 15 shows Europe in 1890, a very homogeneous system for technique, cultural and societal organization. But strangely, GNP vs. energy consumption organizes over two different lines. In the first one you have the nations who don't have coal mines, in the second the ones who have them. For the same GNP, the ratio of energy consumption between the two groups of nations is 4!! Large differences appear also if you compare widely different systems as Pakistan and Sweden or nations at different times.

Apart from energy, the other inputs to a productivity function are raw materials, know-how, capital and societal organization, and one may expect a certain degree of substitutability between them. The most convincing analysis in that sense has been made by H. Millendorfer and C. Gaspari (3) and I report here some of the results.

One of the most obvious indicators of the level of know-how is literacy and in fact the correlations between GNP and literacy work well, as shown in Fig. 16.

The very interesting point is, however, that the nations of the world, bunched into a certain number of parallel lines essentially five in all, indicating another factor at work which we may call "efficiency parameter" or "societal efficiency". The different groups are geographically identified in the following Fig. 17 . Societal efficiency seems to correlate strongly with religion.

Inside each of the groups, the productivity function becomes:

3) $y = C_z m^{1/4} e^b F_s + 0.8q$

where

- y the GNP per capita in U.S. dollars
- C_z the zonal constant, or societal efficiency
- m the indicator for the material input (per capita electricity consumption)
- b the indicator for the immaterial input (literacy, or engineers/10,000 popul., when this indicator is saturated)
- q mineral resources, expressed in per capita value of production
- F_s is a "stress function" indicating the non-complete substitutability of the material and immaterial inputs. $F_s = 1$ for $m^{1/4} = e^b$ and bends somehow the isoquanten as it appears in Fig. 18 . It is fitted through one parameter only.

$$F_s = \left| \frac{1}{2} \left(\frac{m^{1/4}}{e^b}\right)^{-\rho} + \frac{1}{2} \left(\frac{e^b}{m^{1/4}}\right)^{-\rho} \right|^{-1/\rho}$$

ρ fitted by regression

The results of the calculations are given in the following table:

Table 1

	calc.	obs.		calc.	obs.
Canada	2540	2380	Great Britain	1830	1700
Australia	1970	1970	Switzerland	2150	2310
Belgium	1770	1740	U.S.A.	3870	3670
Denmark	1850	1950	Sweden	2230	2500
France	1780	1950	Holland	(2250)	1520
W.Germany	1760	1750			

for 1969 - in U.S. dollars per capita

The only real departure is for Holland. One interpretation being that it really belongs to the "Catholic" group, i.e. to the second one, with a lower societal efficiency.

The results are graphically displayed in Fig. 18 where it appears very clearly how different nations have organized themselves, and how high GNP with low material input, e.g. energy can be obtained via a high level of engineers, i.e. of know-how.

It is unfortunate for Japan to have such a low level of societal efficiency, revealing perhaps the difficulty of adapting its society to an economic system developed by a protestant society.

One might, in abstract, speculate on the consequences of trying to adapt western technology to the Japanese society, the reverse of the option taken a century ago.

C o n c l u s i o n :

A new approach in the analysis of the internal dynamics of primary energy substitution and of energy use is attempted.

The results are very encouraging and promise a deeper insight into the subtle links between energy use and society operation.

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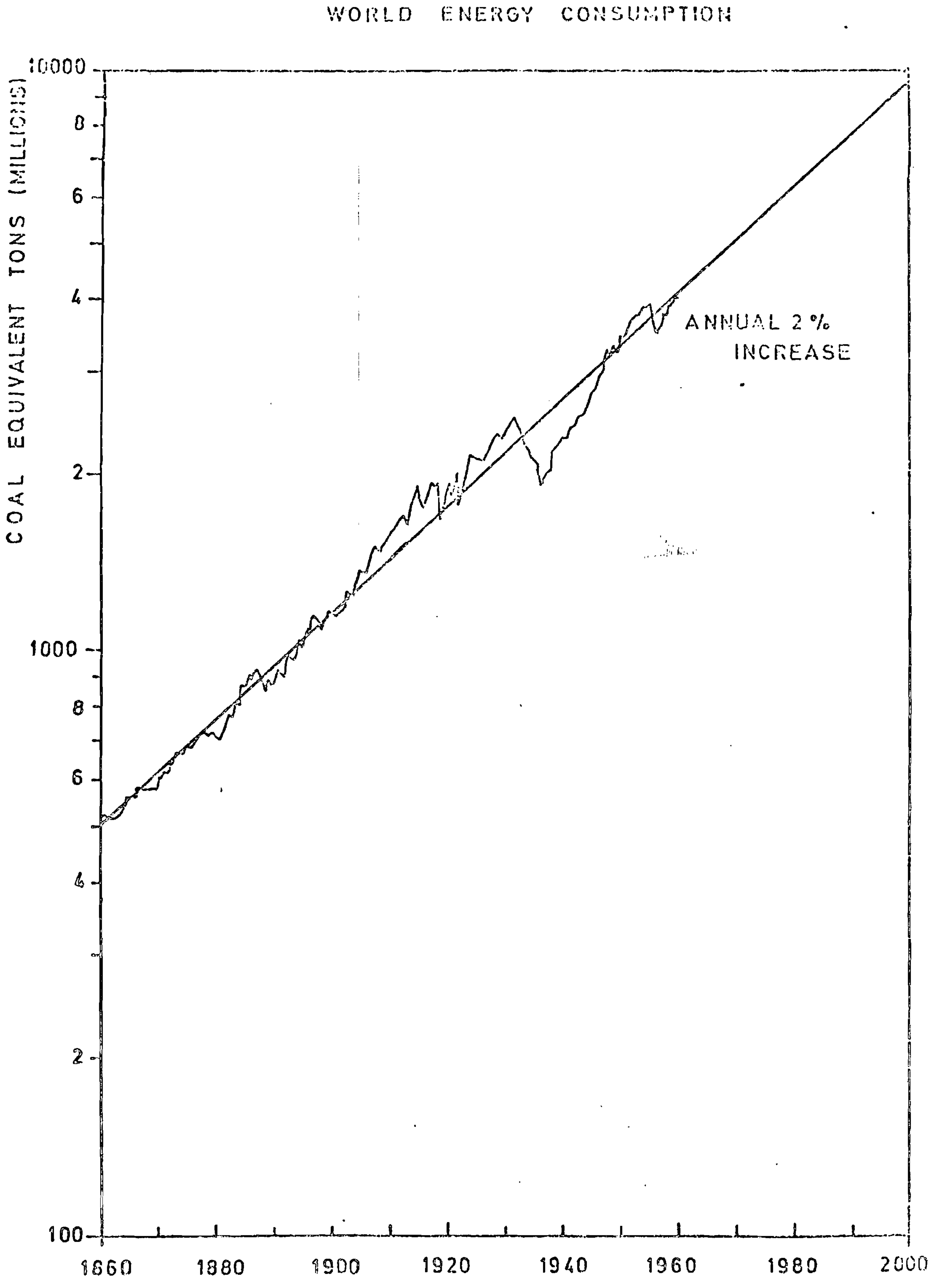
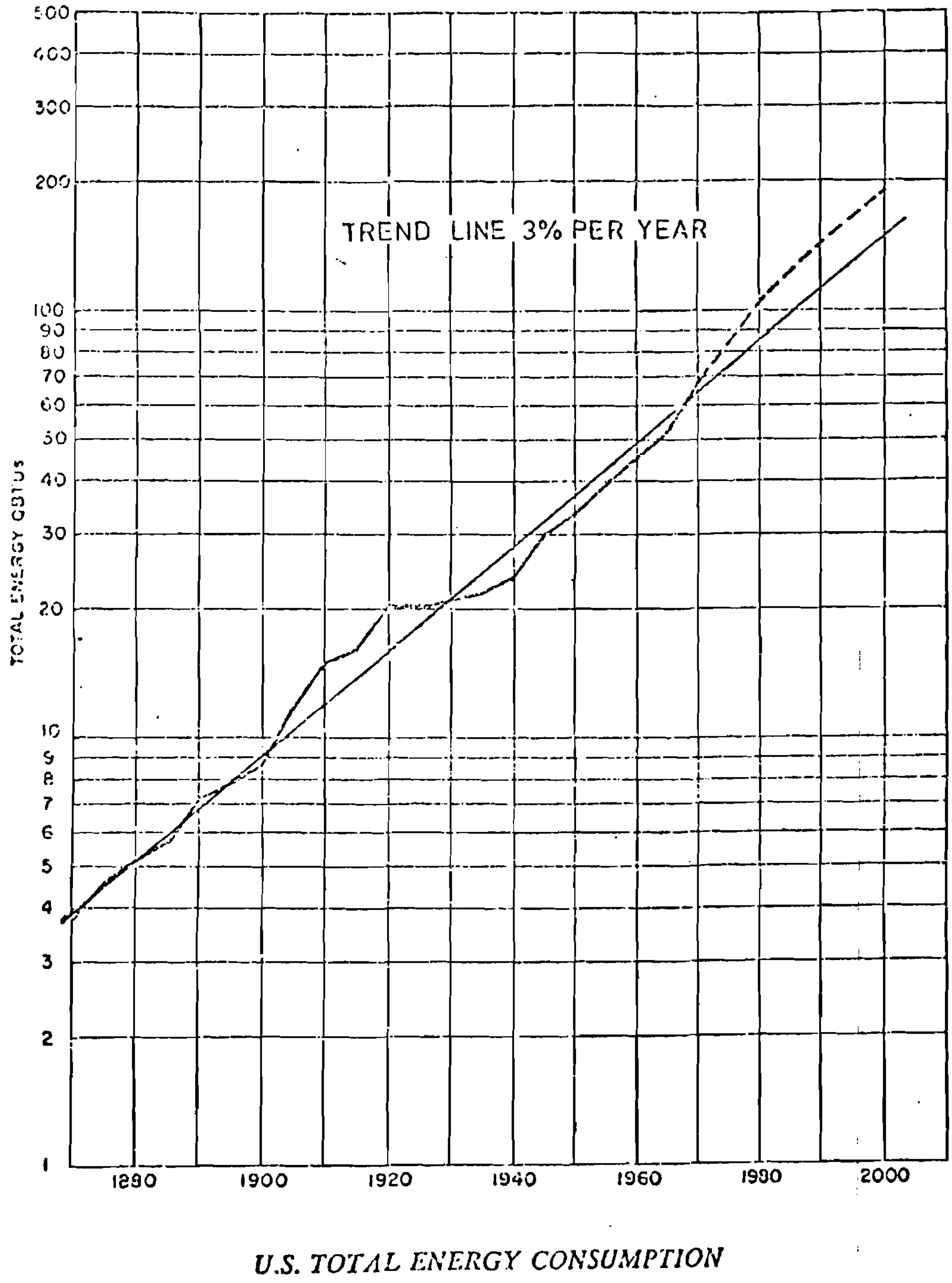


Fig. 1 - World energy consumption, including wood and farm waste. The trend line has a 2% slope.



<u>Fig. 2</u> - Adapted from R.E. Lapp, "The Logarithmic Century".

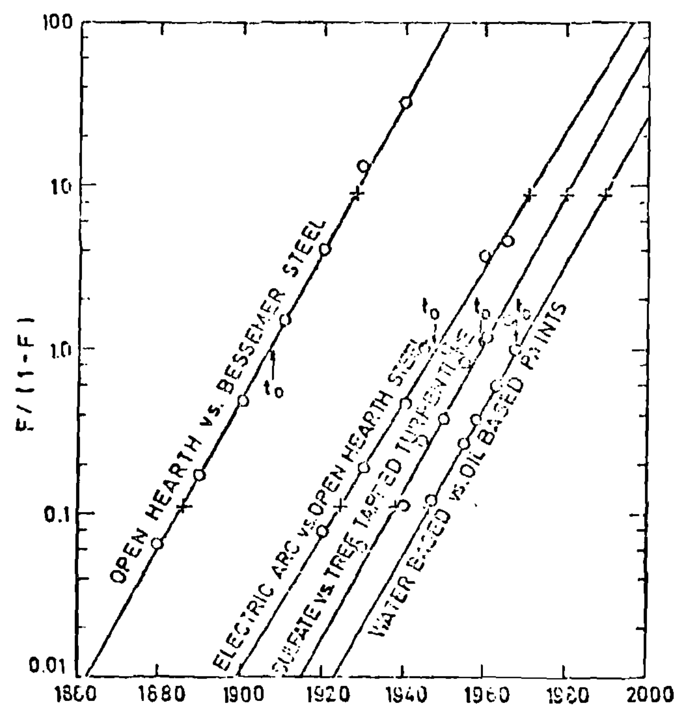


Fig. 3 - Market penetration curves in the US for:

a) open-hearth vs. Bessemer steel
b) electric arc vs. open hearth steel
c) sulphate turpentine vs. natural turpentine
d) water based vs. oil based paints

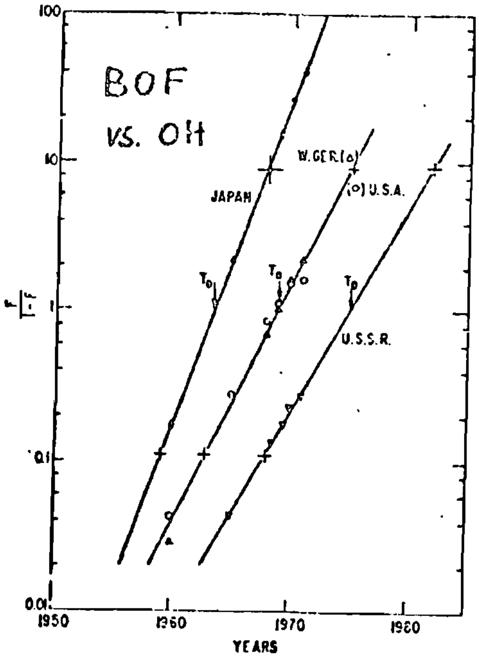
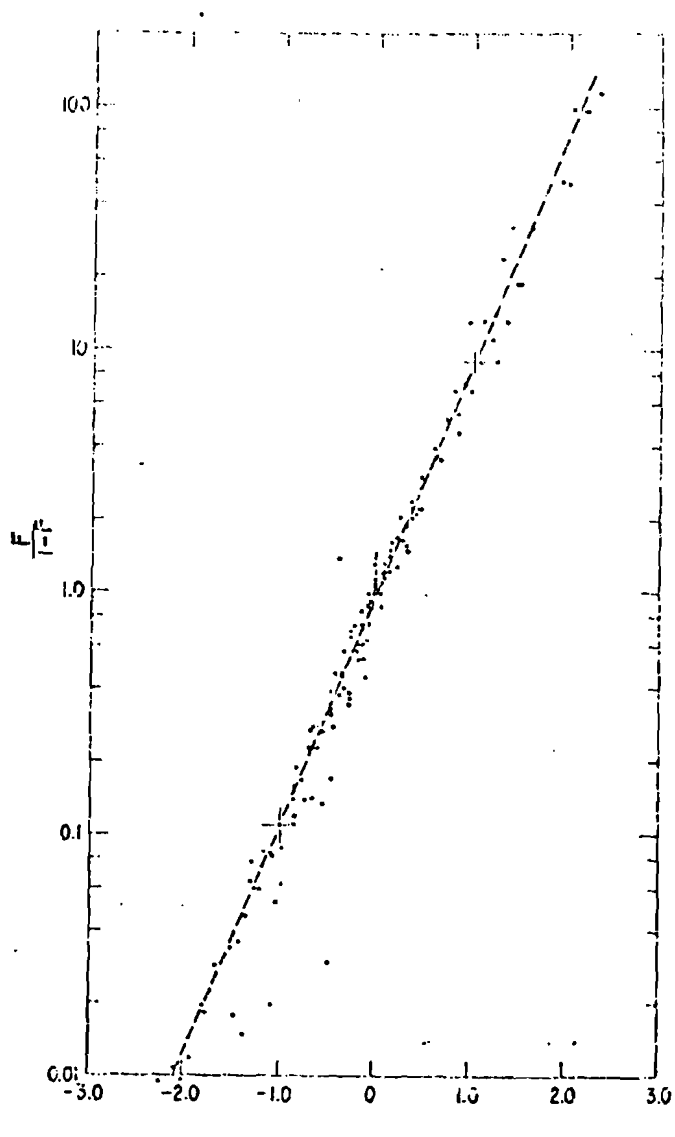


Fig. 4 - Market penetration curves for oxygen steel (BOF) vs. open hearth and Bessemer steel in four countries (Japan, US, West Germany, Russia).
The same law appears to hold also for a socialist economy. Japan appears to be the first to use intensively this technique, originally developed in Austria during the Second World War.



<u>Fig. 5</u> - Normalized plotting for 17 cases of market penetration. This shows that in spite of a certain amount of noise the trends are respected even over very long periods of time.

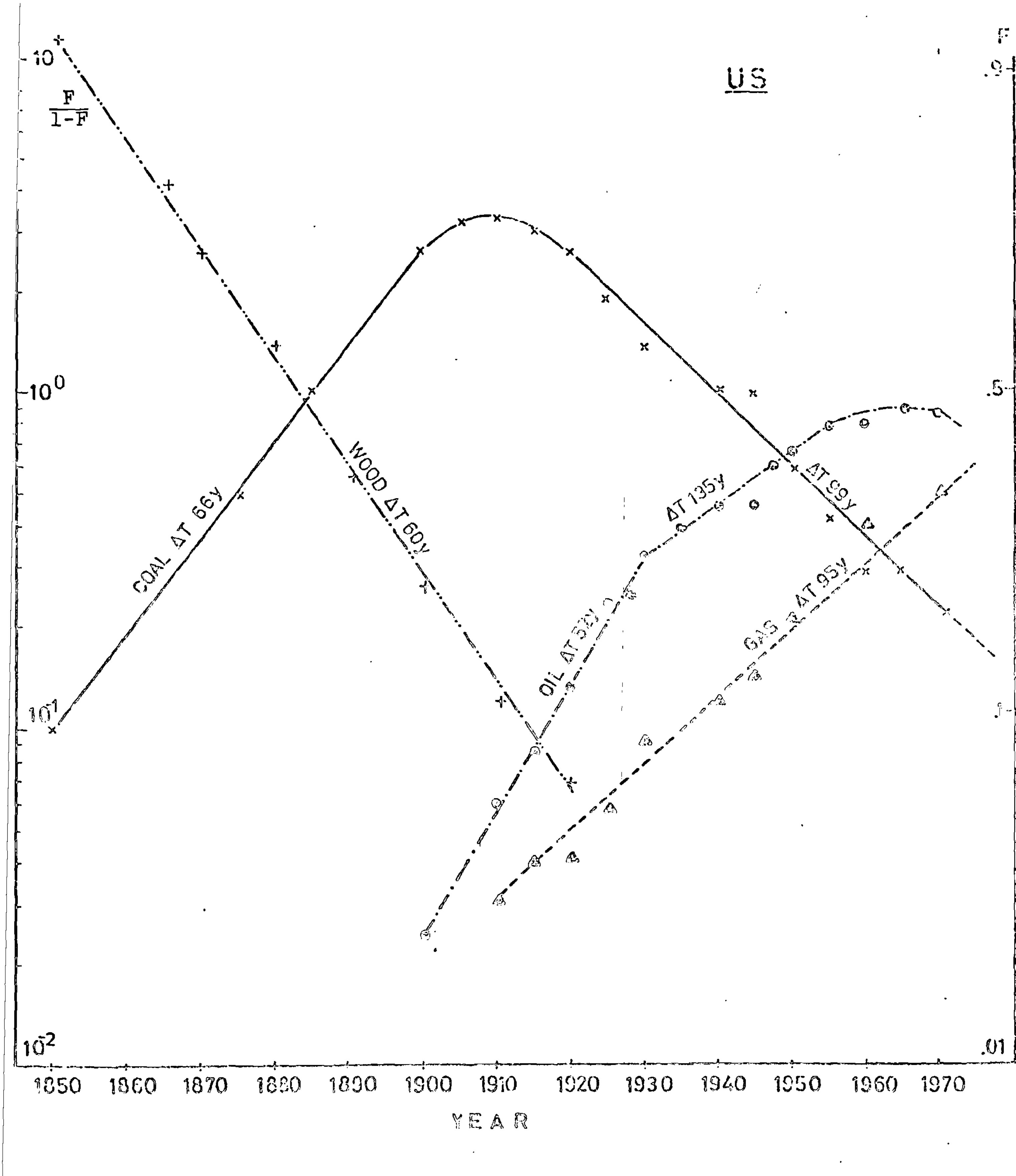


Fig. 6 - Fitting of the statistical data on primary energy consumption in the U.S. Straight lines are represented by equations of type (2). Rates of penetration are indicated by the time to go from 1% to 50% of the market (ΔT years). The knee in the oil curve and the saturation regions can be calculated by the rule "first in- first out".

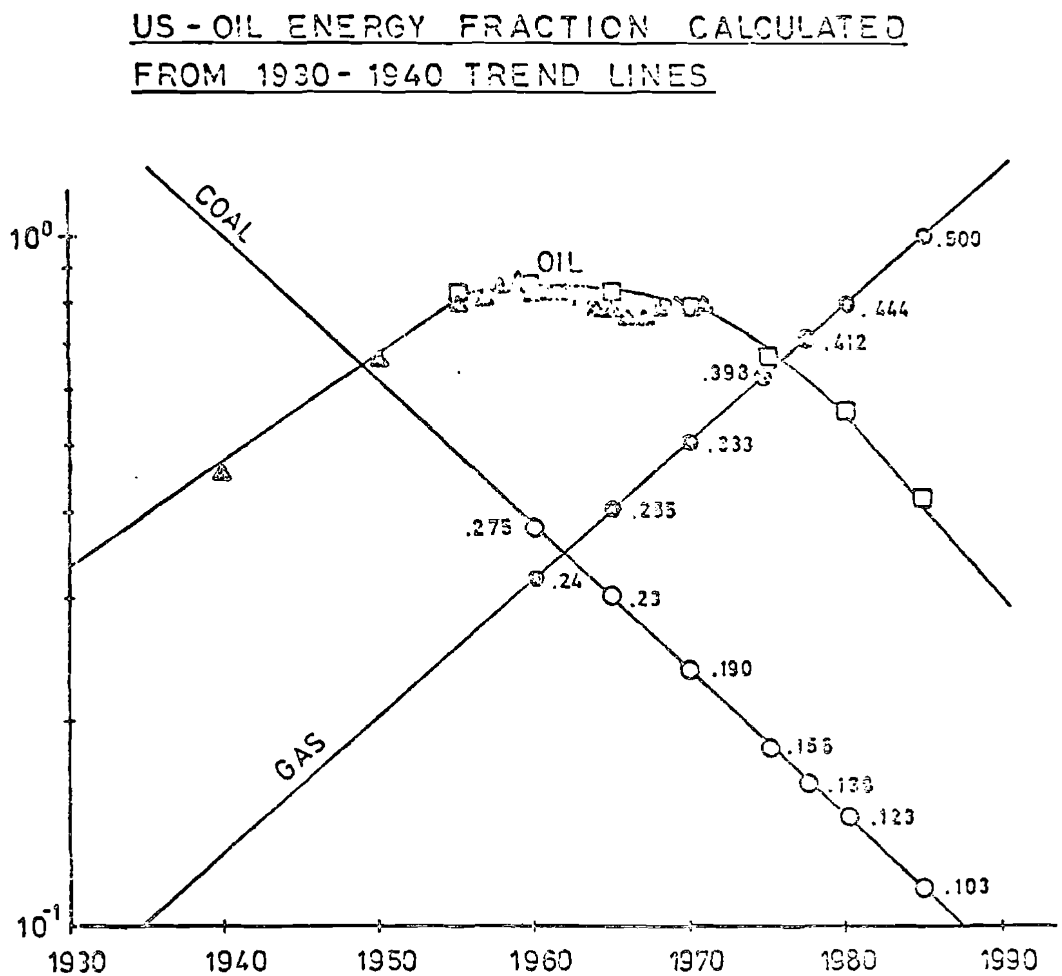


Fig. 7 - Forecasting US oil consumption as fraction of total energy consumption from 1930-1940 trends.

□ calculated values

Δ statistical data

Other symbols and figures represent intermediate steps in the calculation, the graph having been drawn from my notebook.

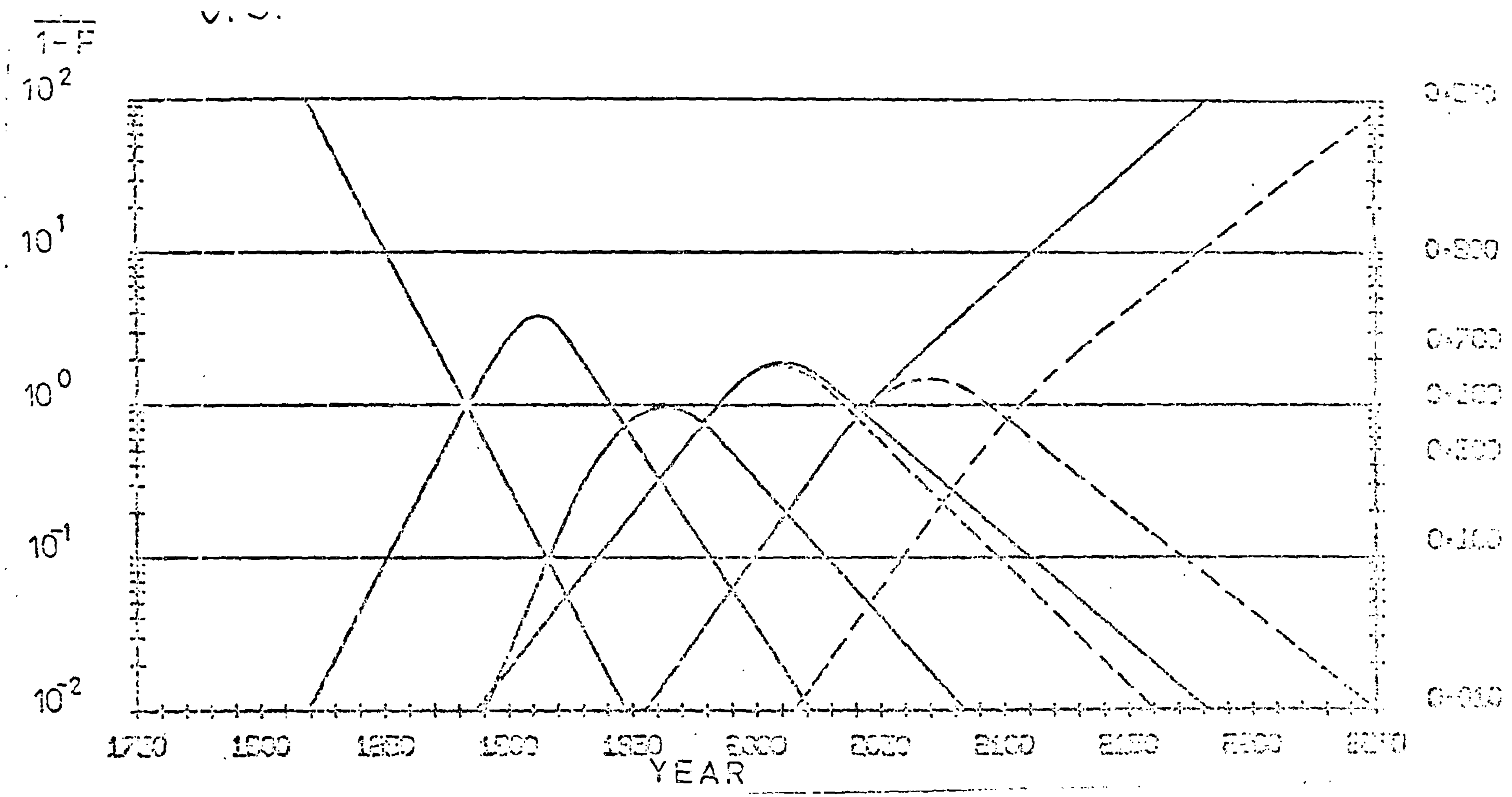


Fig. 8 - US energy consumption from various sources. Actual fractions are given on the right side of the figure. The effect of the introduction of a new source of energy (solar-fusion) is indicated by the dashed lines. Its effect appears to be minimal on conventional sources, and dramatic only on the nuclear.

This figure and the following ones are reported for illustration of the method only and are not intended to have predictive value.

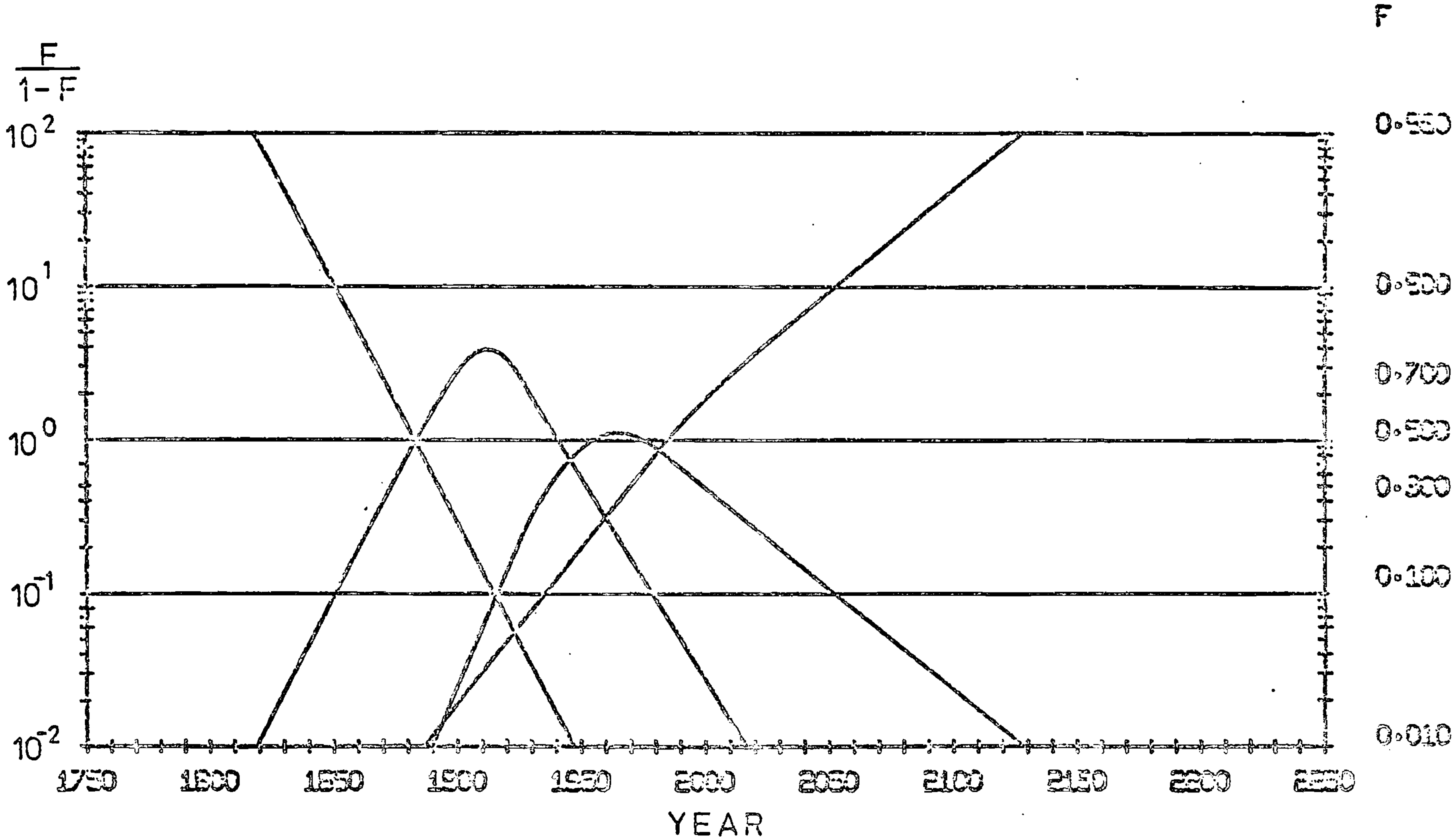


Fig. 9 - The assumption that no nuclear energy, or new sources will be introduced leads to the absurd situation where all energy input in the US will rely on natural gas.

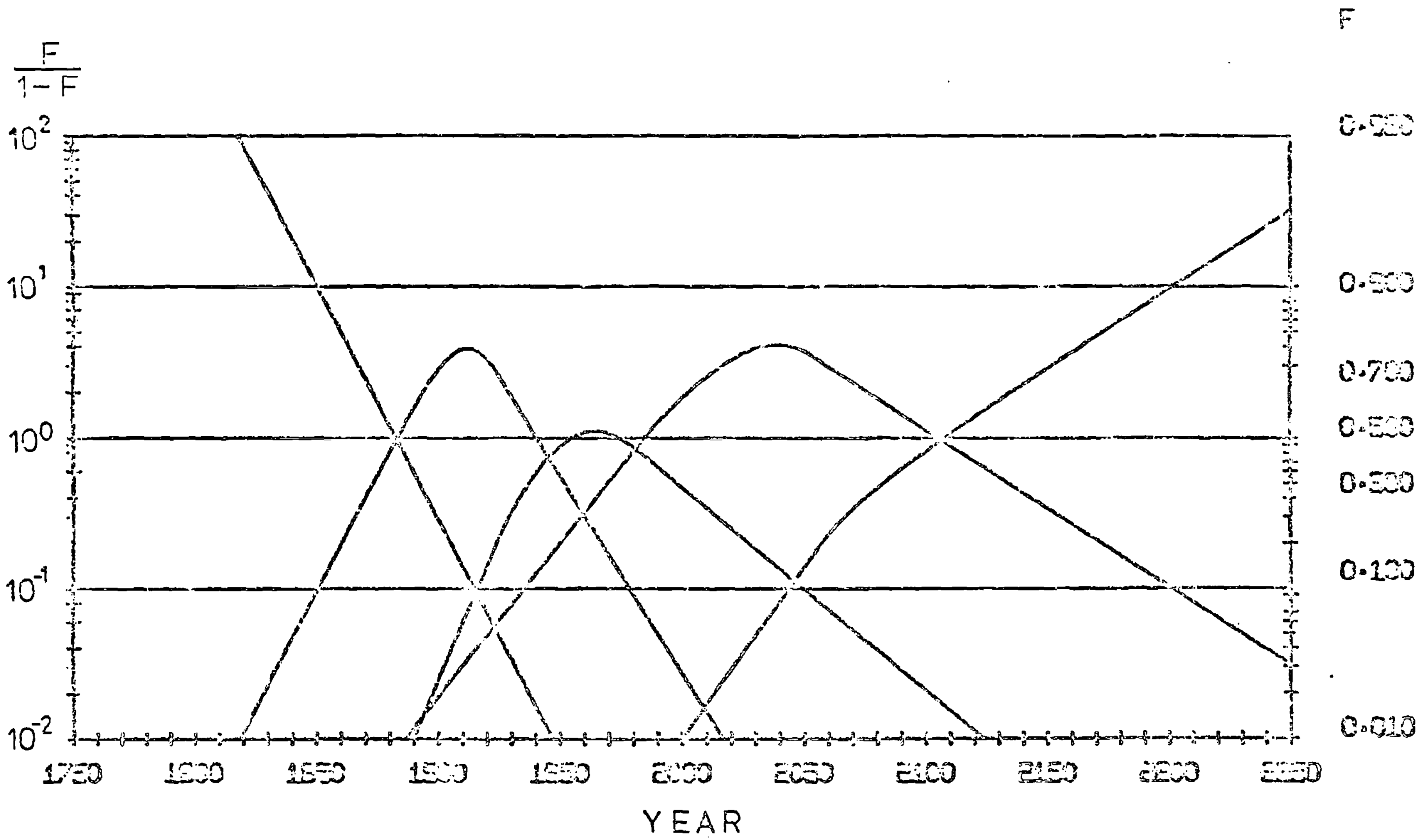
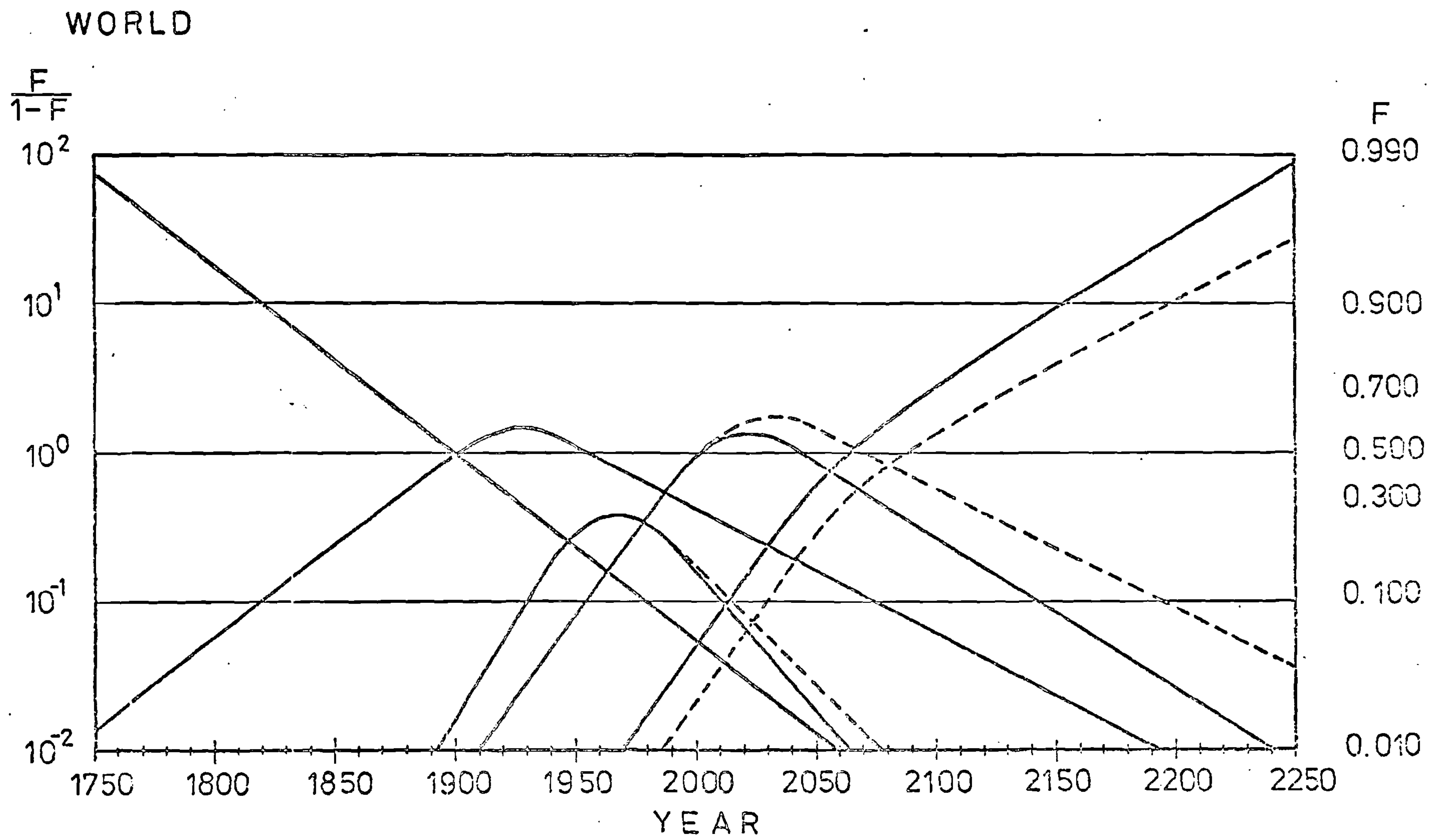


Fig. 10 - Even the assumption of a moratorium for nuclear energy up to the year 2000 leads to a situation of incompatibility with resources. The introduction of nuclear energy appears a perfectly timed device to make ends meet.



<u>Fig. 11</u> - World energy consumption from various sources (fractional). The curves correspond to wood, coal, oil, gas, nuclear, in the order. The dashed lines indicate the effect of a delay in the introduction of nuclear energy. Only gas consumption appears to be heavily affected.

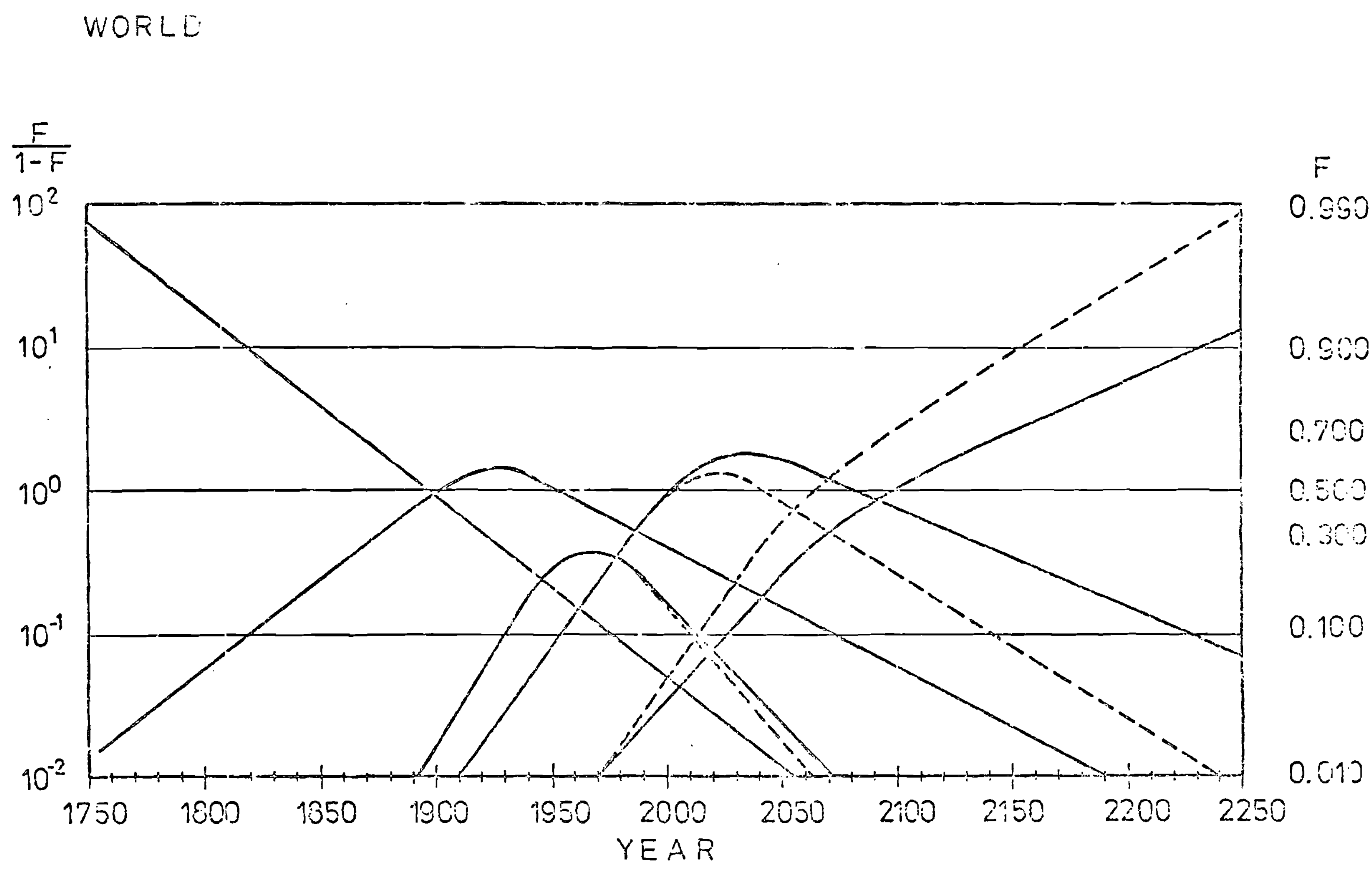
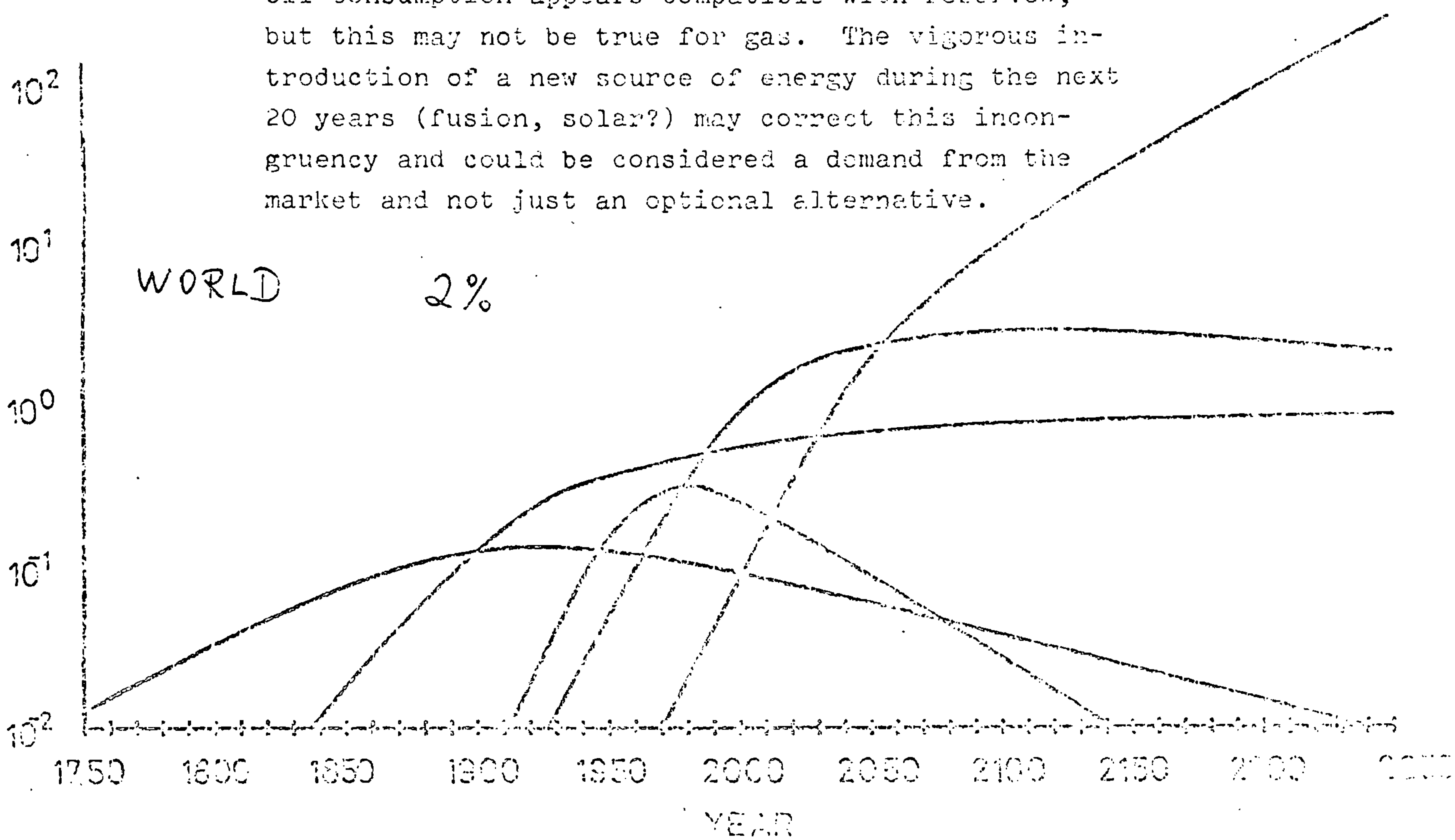


Fig. 12 - Definitions as in Fig. 11.
Effect of an accelerated nuclear program (dashed lines). Again only gas consumption appears to be heavily affected.

Fig. 13 - World energy consumption in absolute terms(1970=1). Secular growth rate 2%. The curves correspond, in the order, to wood, coal, oil, gas, and nuclear energy. It may be observed that with the hypotheses adapted, the absolute level of coal consumption will reach an asymptotic value and stay constant. Total oil consumption appears compatible with reserves, but this may not be true for gas. The vigorous introduction of a new source of energy during the next 20 years (fusion, solar?) may correct this incongruency and could be considered a demand from the market and not just an optional alternative.



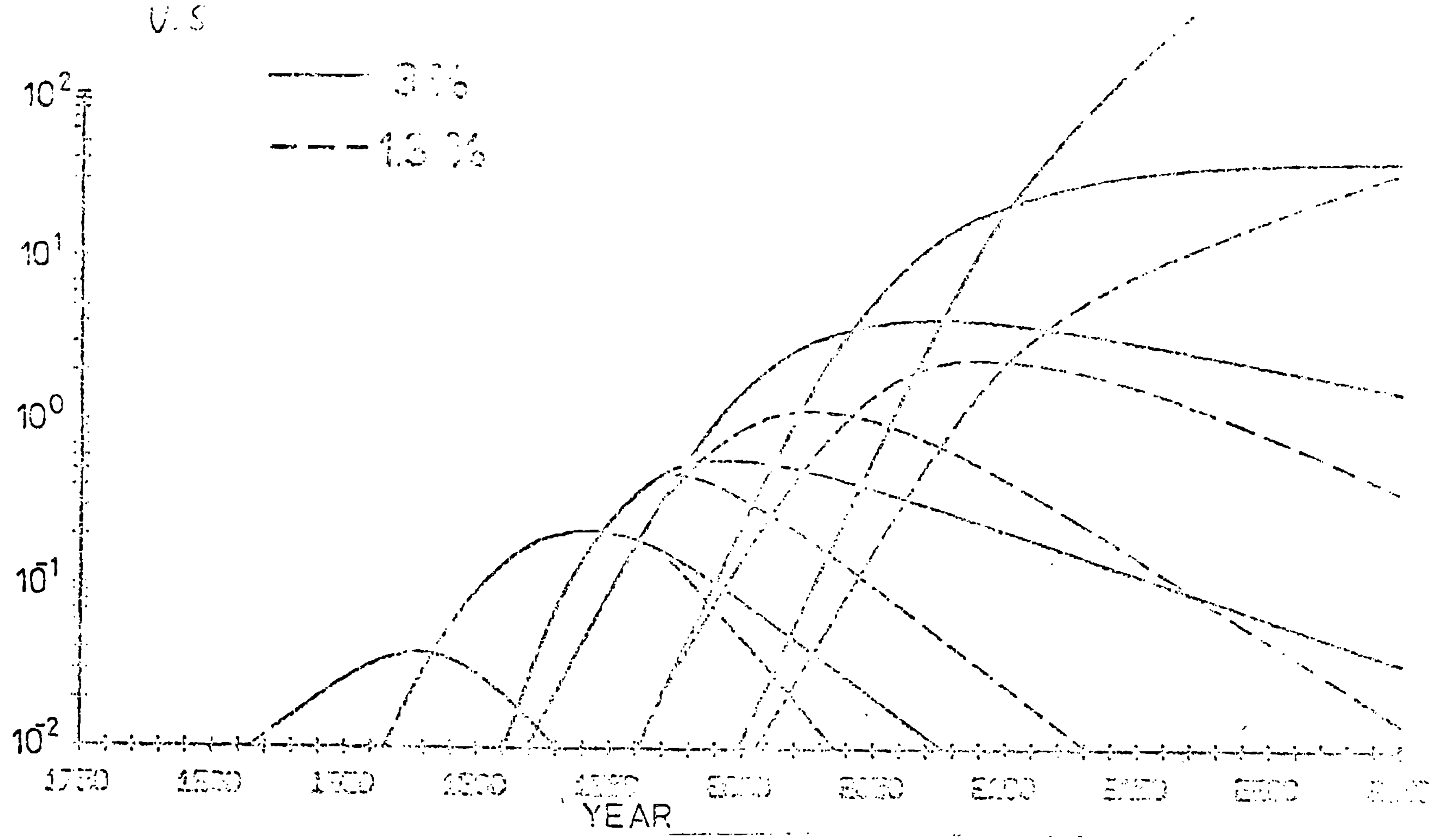
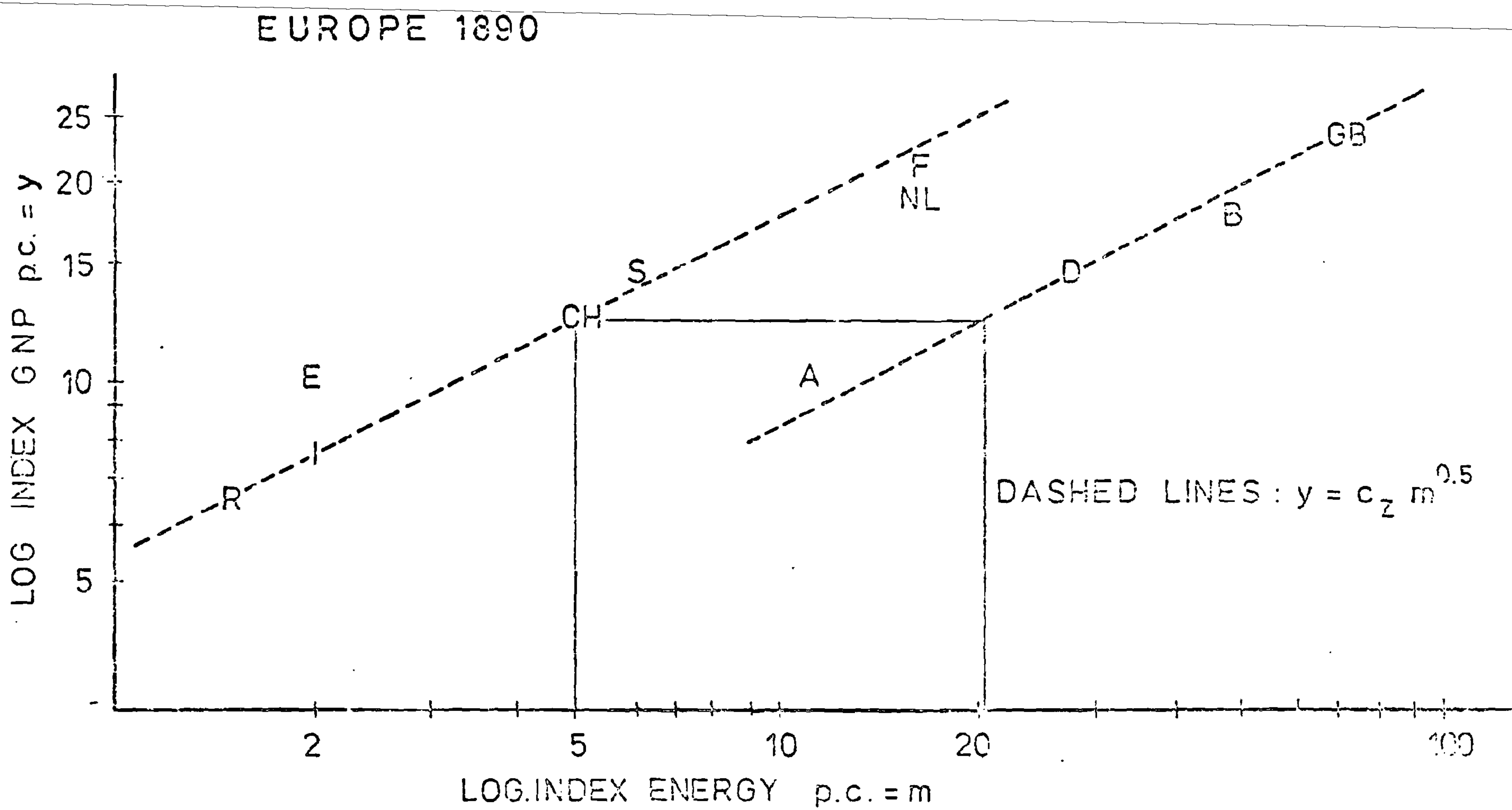


Fig. 14 - US energy consumption in absolute terms(1970=1). A new source of energy is assumed to be introduced around the year 2000. The effect of energy saving, through a reduction of the rate of growth, is indicated by the dashed lines. The value 1.5 for the secular growth has been chosen with the hypothesis that the ratio of per capita consumption in the US and the world remains constant as in the last 100 years, and that the world secular growth in energy consumption stays at 2%.



<u>Fig. 15</u> - GNP vs. energy consumption per capita in Europe in 1890. The separation between coal exporting countries (Austria, Germany, Belgium and Great Britain), and coal importing countries is very sharp. This reveals that the link between GNP and energy consumption is not so rigid as many studies on the subject tend to assume.

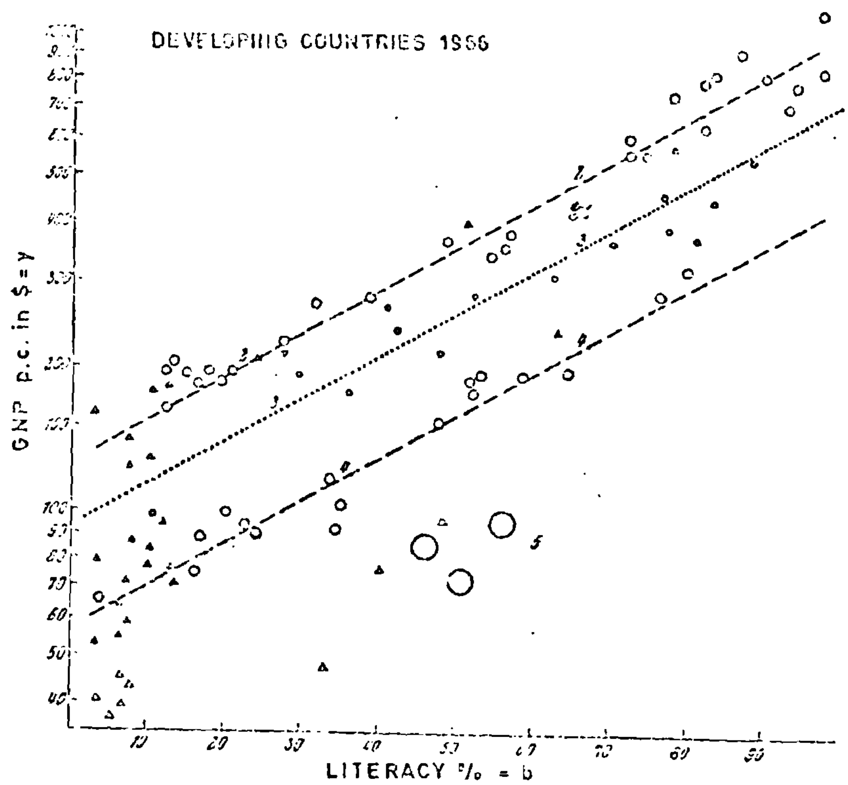


Fig. 16 - Analysis of GNP vs. literacy, sediments the countries of the world into four layers. A fifth one is not included because the indicator is saturated. The proper indicator in this case is percentage of engineers in the population.

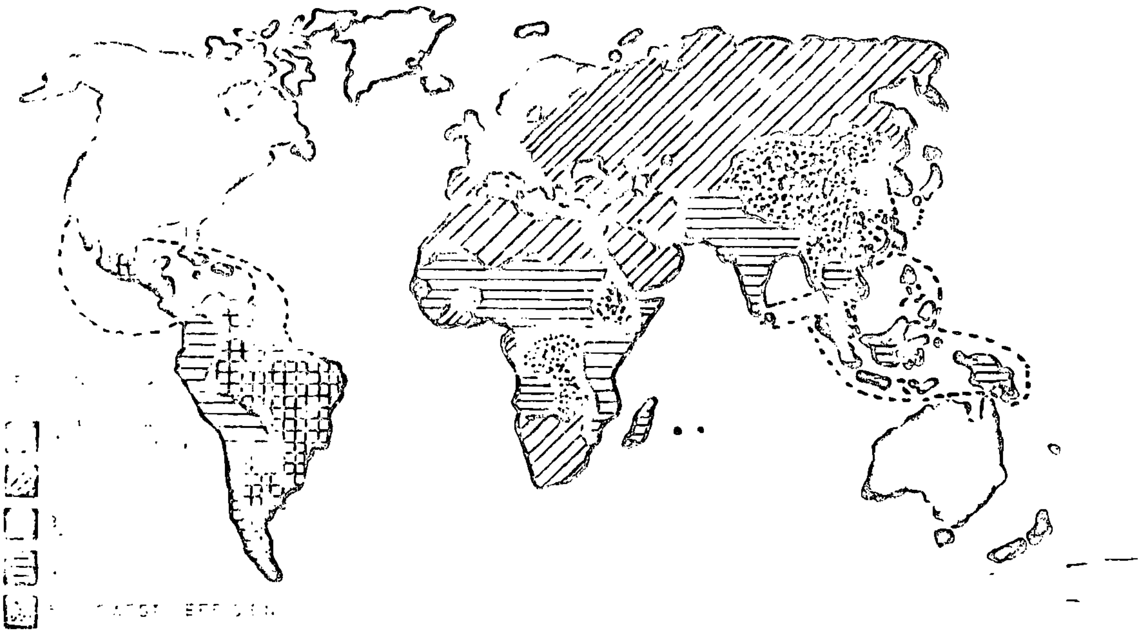


Fig. 17 - World map of the regions with equal "societal organization" coefficient. The ratio of the coefficients of levels 2 and 3, or levels 3 and 4, is about 1.4. This means level 3 needs 40% more input than level 2 for the same GNP.

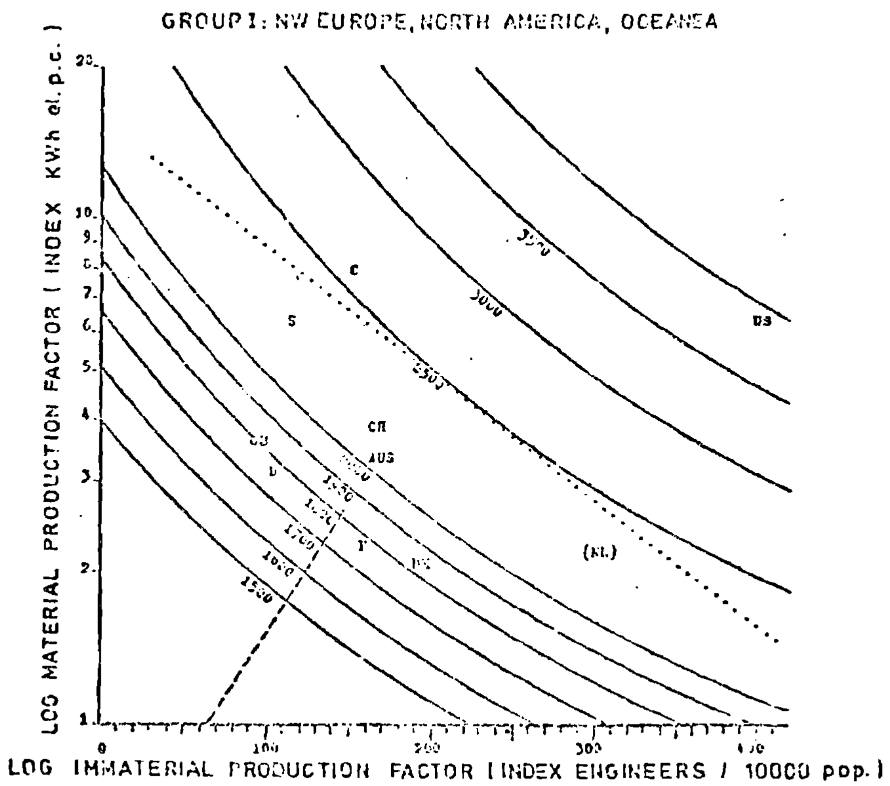


Fig.18 - Iso-GNP as a function of the two indicators for material and immaterial inputs. Dashed line indicates their balance, i.e. $m^{1/4} = e^b$. Dotted line has been drawn for $F_s = 1$ and shows the effect of incomplete substitutability of the production factors.

It is very interesting to note that the US and Sweden have roughly the same material index, and the much higher GNP per capita of the US appears to be due essentially to a higher immaterial production factor.

Appendix

Methods of Calculation

N. Nakicenovic

We will describe here the first attempt to implement the ideas presented in the paper for determining the market penetration behavior of energy sources on the basis of the historical data.

First, by definition, the sum of all fractional market shares must be always equal to 1:

$$\sum_{i=1}^{n} F_i(t) = 1 \tag{1}$$

where $F_i(t)$ is the fraction of market penetration of the source i and n is the number of energy sources. Expression (1) must hold for all t.

As shown in the paper, the logistic functions appear to give a good description of the market share of a given product. These logistic functions can be written as:

$$\ln \left[\frac{F_i(t)}{1-F_i(t)} \right] = \alpha_i^o t + c_i^o \tag{2}$$

The superscript ^{o} for parameters α_i and c_i will indicate that those parameters are defined on the basis of historical data.

(1) can be rewritten:

$$F_i(t) = \frac{\exp(\alpha_i^o t + c_i^o)}{1+\exp(\alpha_i^o t + c_i^o)} \tag{3}$$

Our intention is to project the market penetration trends of the primary energy sources over the time interval longer than the time period for which data are available.

This projection in the future leads to situations where one or more energy sources are penetrating the market at a higher rate than other energy sources leaving the market. Thus, in such situations expressions (1) and (3) would be contradictory. The expression (1) is a statement by definition and cannot be violated. Therefore, the logistic function in expression (3) cannot hold for the whole time interval in question and for all energy sources. In fact, it will hold universally only for those energy sources which are leaving the market from the beginning.

The method used is that the oldest still growing energy source must decrease, i.e. the expression (3) would not hold for that, oldest, energy source from that time point on, and will be equal to the difference between 1 and the sum of all other energy sources. By oldest we mean the energy source which is anterior to all other energy sources. This chosen energy source then enters a transition period after which the market share starts decreasing. When this decrease starts, the originally second oldest still growing energy source must be labeled "oldest".

We assume that there are n energy sources competing on the market so that any energy is denoted by $i\varepsilon[1,n] \leq 1$. We want to consider a time period longer than the historical period for which data are avilable:

$$t\varepsilon[t_{INI}, t_{FIN}] \subseteq \mathbb{R}^{+}.$$

At the beginning we choose, as described above, the oldest still growing energy source j by:

$$j=\{ i|i = [1,n] \subseteq \mathbb{N},\ t=t_{INI},\ \alpha_i^o \geq 0,\ \alpha_{i-1}^o < 0 \} \qquad (4)$$

As already described above, for all other energy sources that are leaving the market, i.e. $\alpha_i^o < 0$, expression (3) always holds. For the oldest still growing energy source j, we define the market penetration by:

$$F_j(t)=\begin{cases} \dfrac{\exp(\alpha_j^o t + c_j^o)}{1+\exp(\alpha_j^o t+c_j^o} & \text{for } t_{INI} \leq t < t_{bi} \text{ (ascending phase)} \\ 1 - \sum\limits_{i\neq j} F_i(t) & \text{for } t_{bj} \leq t < t_{ej} \text{ (transition phase)} \\ \dfrac{\exp(\alpha_j^* t + c_j^*)}{1+\exp(\alpha_j^* t+c_j^*)} & \text{for } t_{ej} \leq t \leq t_{FIN} \text{ (descending phase)} \end{cases} \qquad (5)$$

In the following we will explain the quantities introduced by (5). First we define t_{bj} and t_{ej} as these are instrumental in defining the transition phase:

$$t_{bj} = \min \{ t \mid t \in [t_{INI}, t_{FIN}] \subseteq \mathbb{R}^+, \frac{\exp(\alpha_j^O t + c_j^O)}{1+\exp(\alpha_j^O t + c_j^O)} + \sum_{i \neq j} F_j(t) > 1 \} \quad (6)$$

and:

$$t_{ej} = \{ t \mid t \in [t_{bj}, t_{FIN}] \subseteq \mathbb{R}^+, \frac{d[\ln(\frac{F_j(t)}{1-F_j(t)})]}{dt} = c \text{ or } c \in \mathbb{R}, F_j(t) = F_j(t_{bj}) \} \quad (7)$$

where c is a constant.

t_{bj} is, therefore, the beginning of the transition period for energy source j. It denotes the time point where the rate of market penetration of energy source j starts to fall. t_{ej} is the end of this transition period, when the market penetration of energy source j follows the logistic function again (see Fig.1).

Finally we define α_j^* and c_j^* by:

$$\alpha_j^* = \frac{d[\ln(\frac{F_j(t)}{1-F_j(t)})]}{dt} \quad (8)$$

and:

$$c_j^* = \ln \left(\frac{F_j(t)}{1-F_j(t)}\right) - \alpha_j^* t$$

at time point $t = t_{ej}$ (9)

Beyond t_{ej} the energy source market penetration follows then the logistic function in the descending mode, i.e. $\alpha_j^* < 0$.

All of the quantities on the right hand side of expression (5) are now defined, and therefore also the market penetration behavior of the energy source j.

The energy source j thus defined, however, is only the first energy source that must leave the market due to definition in expression (1). As the time goes on we might still encounter the problem that we must reduce the current, oldest still growing energy source. Accordingly, we go through the process source by source. Generally, we therefore have:

$$j=\{i \mid i\varepsilon[1,n]\subseteq\mathbb{N},\ t=[t_{INI},t_{FIN}]\subseteq\mathbb{R}^{+},\ \alpha_j(t)\geq 0,\alpha_{i-1}(t)<0\} \qquad (4^*)$$

where:

$$\alpha_j(t)=\begin{cases}\alpha_j^o \text{ if } \alpha_j^o < 0 \text{ for } t_{INI}\leq t\leq t_{FIN}\\ \alpha_j^o \text{ if } \alpha_j^o > 0 \text{ for } t_{INI}\leq t<t_{bj}\\ \alpha_j^* \text{ if } \alpha_j^o \geq 0 \text{ for } t_{ej}\leq t\leq t_{FIN}\end{cases} \qquad (10)$$

and similarly:

$$c_j(t)=\begin{cases}c_j^o \text{ if } \alpha_j^o < 0 \text{ for } t_{INI}\leq t\leq t_{FIN}\\ c_j^o \text{ if } \alpha_j^o \geq 0 \text{ for } t_{INI}\leq t<t_{bj}\\ c_j^* \text{ if } \alpha_j^o \geq 0 \text{ for } t_{ej}\leq t\leq t_{FIN}\end{cases} \qquad (11)$$

Finally expression (5) becomes:

$$F_j(t)=\begin{cases}\dfrac{\exp[\alpha_j(t)t+c_j(t)]}{1+\exp[\alpha_j(t)t+c_j(t)]} & \text{for } t<t_{bj} \text{ or } t\geq t_{ej}\\ 1-\sum_{i\neq j} F_j(t) & \text{for } t_{bj}\leq t<t_{ej}\end{cases} \qquad (5^*)$$

To summarize, it follows from the relations presented above that $\alpha_i(t)$ and $c_i(t)$ are piecewise defined for those energy source which penetrate the market and later must leave it. They are not defined over the time interval $[t_{bi}, t_{ei}]$. During this interval equation (1) is employed to govern the behavior of the energy source in transition. Those energy sources which have the descending market penetration from the beginning of the time period analyzed, have $\alpha_i(t)$ and $c_i(t)$ defined over the whole time period. The definition for this is: $\alpha_i(t_{INI}) = \alpha_i^o < 0$.

We developed a computer code for expressions (4) through (11). The initial values of parameters α_i^o and c_i^o were determined on the basis of historical data. The program, then generated the values of $F_i(t)$ for any chosen interval of time between $t > 0$ and $t < \infty$.

In order to express consumption of different energy energy sources in absolute terms, a base year $t_o = 1970$ is given as input and a constant growth rate (r) for total energy consumption is given. The consumption in the base year for each source has been taken as unity for this source.

The source code for this program has been written in Fortran language. It has been implemented on the PDP11/45 with DOS version V07 operating system and later with UNIX operating system. The graphical software used in the program is a version of Calcomp plotting library specifically modified for PDP11/45.

Attached is the flow chart of this program.

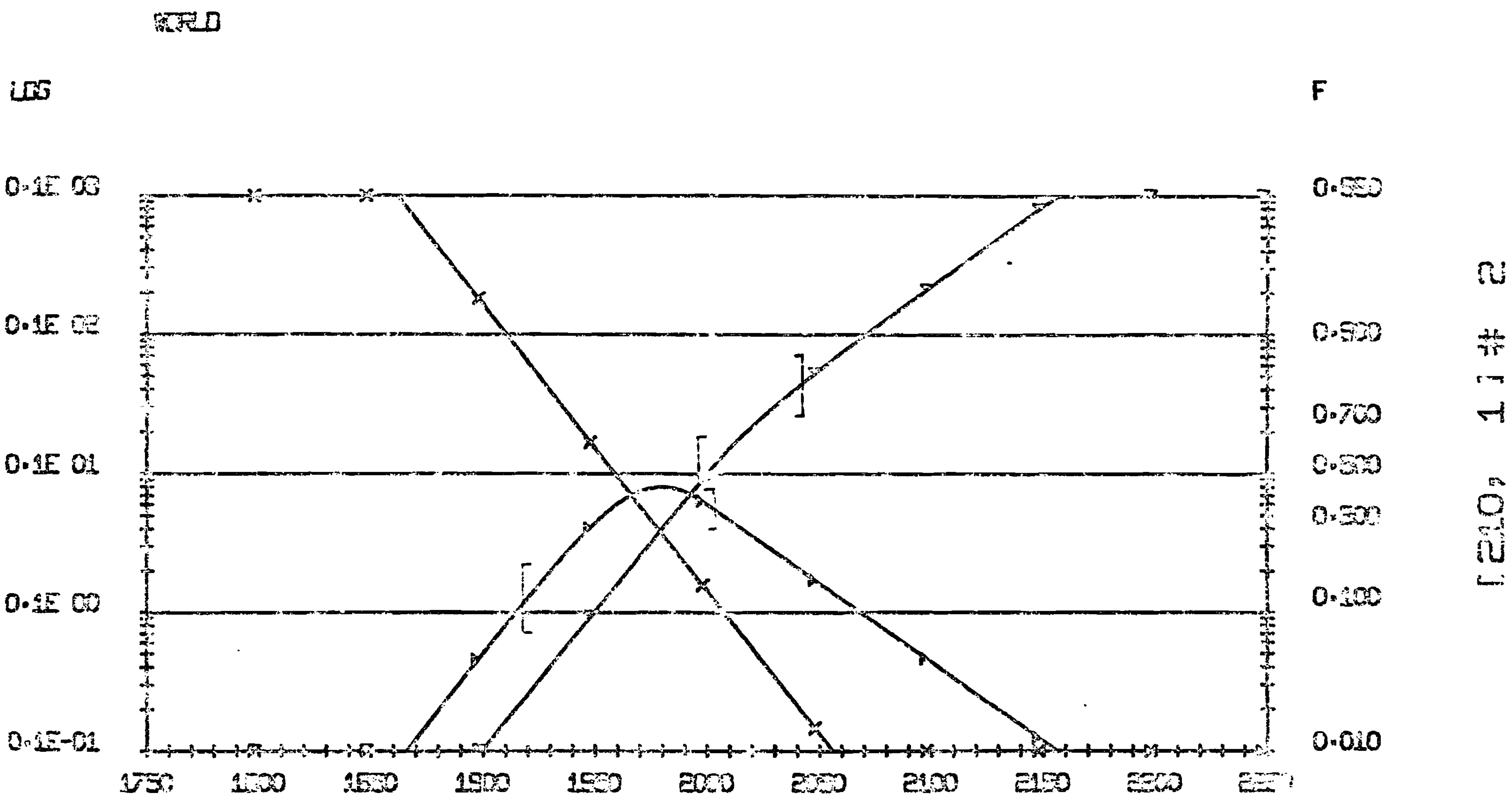


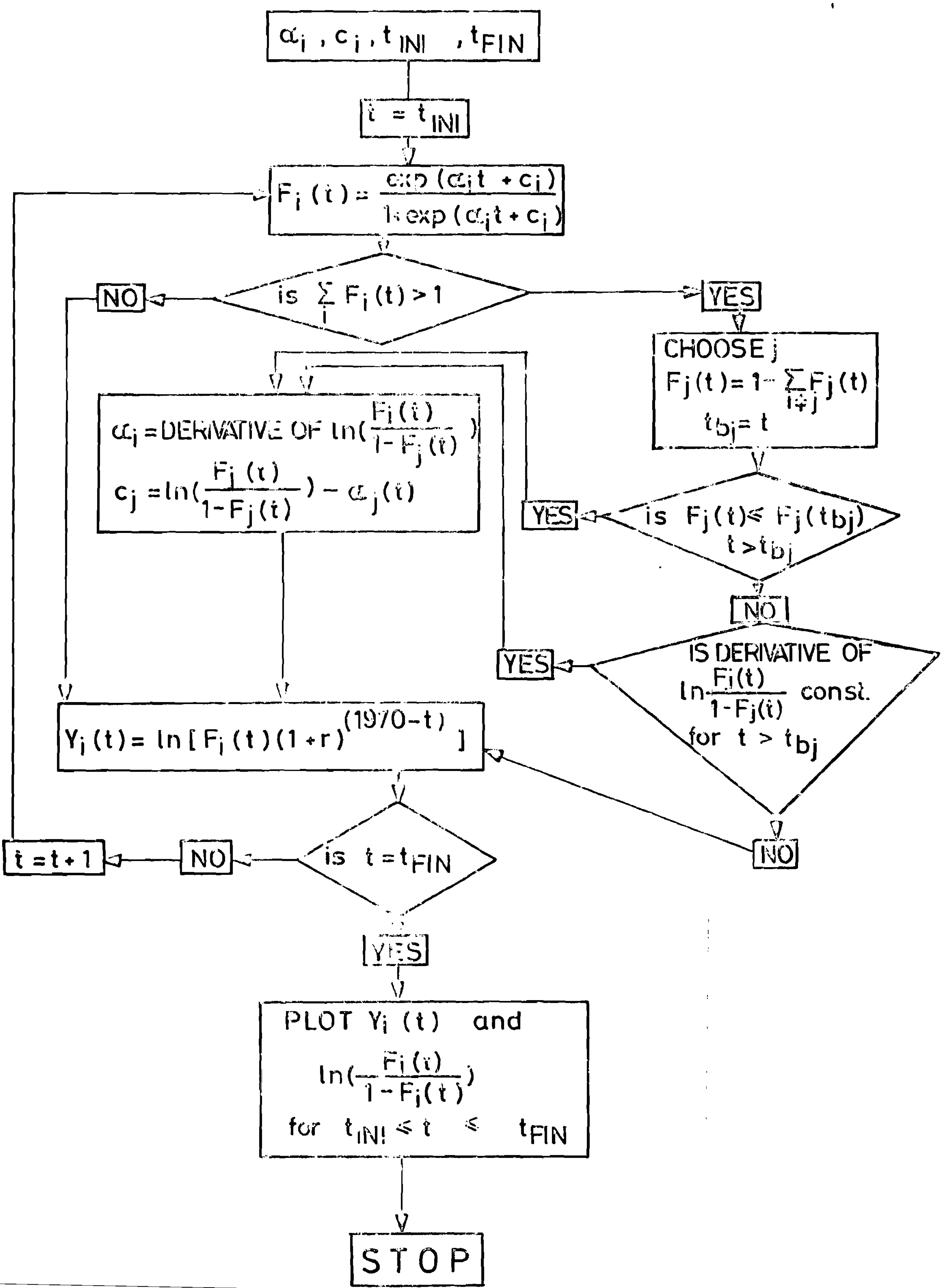
Figure 1: There are 3 energy sources: For the first energy source(+) $\alpha_1(t) < 0$ for all t

For the second energy source (◺) $\alpha_2(t) > 0$ for $t < tb_2$

For the third energy source (Δ) $\alpha_3(t) > 0$ for $t < tb_3$

For time period $t \epsilon [tb_2, te_2)$: $F_2(t) = 1 - F_1(t) - F_3(t)$, elsewhere $F_2(t) = \frac{\exp(\alpha_2(t)t + c_2(t))}{1 + \exp[\alpha_2(t)t + c_2(t)]}$

Transition period in brackets.



αi , ci , tINI , tFIN
t = tINI
Fi (t) = exp (αi t + ci) / 1+ exp (αi t + ci)
is Σi Fi (t) > 1
NO
YES
CHOOSE j
Fj (t) = 1 − Σi≠j Fj (t)
tbj = t
αj = DERIVATIVE OF ln(Fj (t) / 1 − Fj (t))
cj = ln(Fj (t) / 1 − Fj (t)) − αj (t)
is Fj (t) ≤ Fj (tbj)
t > tbj
YES
NO
IS DERIVATIVE OF
ln Fj(t) / 1 − Fj(t) const.
for t > tbj
YES
NO
Yi (t) = ln [Fi (t) (1 + r)^(1970 − t)]
is t = tFIN
NO
t = t + 1
YES
PLOT Yi (t) and
ln(Fi (t) / 1 − Fi (t))
for tINI ≤ t ≤ tFIN
STOP

```
      DIMENSION Y(6),F(6),A(6),B(6),ARE(6),AA(6),AB(6)
      DIMENSION G(6,501),TL(6,501),REF(6)
      DIMENSION TITLE(10),TYPE(6,4),YLOGM(6)
      READ (5,60) N
      READ (5,60) MAX
      READ (5,60) MIN
      READ (5,60) IREFY
      READ (5,50) RG
      READ (5,50) RG1
      READ (5,50) ERROR
      READ (5,60) NP
      READ (5,90) (TITLE(I),I=1,10)
      DO 20 I=1,N
      READ (5,130) (TYPE(I,J),J=1,4)
20    CONTINUE
      WRITE (6,40)
      WRITE (6,170) NP
      WRITE (6,40)
      DO 1 I=1,N
      READ (5,100) X1, F1
      WRITE (6,140) X1, F1
      READ (5,100) X2, F2
      WRITE (6,140) X2, F2
      Y1=ALOG(F1)
      Y2=ALOG(F2)
      CALL FUNC(Y1,Y2,X1,X2,A(I),B(I))
1     CONTINUE
      WRITE (6,40)
40    FORMAT (1H )
50    FORMAT (F5.3)
60    FORMAT (I4)
70    FORMAT (E8.1)
80    FORMAT (' E=LOG(F/(1-F))=A*T+C')
90    FORMAT (10A2)
100   FORMAT (F5.0,F5.3)
110   FORMAT ('F')
120   FORMAT (4A2,'  TOTAL AMOUNT CONSUMED =',E14.7)
130   FORMAT (4A2)
140   FORMAT (1H ,3(10X,E14.7))
150   FORMAT (10X,'CONSUMED BEFORE ',I4,'  =',E14.7)
160   FORMAT (10X,'CONSUMED AFTER  ',I4,'  =',E14.7)
170   FORMAT (10X,'PLOT # ',I2)
180   FORMAT (10X,'TOTAL CONSUMPTION IN ',I4,' IS 1')
190   FORMAT (10X,'GROWTH RATE (RG) BEFORE ',I4,'  =',F5.3)
195   FORMAT (10X,'GROWTH RATE (RG) AFTER  ',I4,'  =',F5.3)
200   FORMAT (1H ,10X,F6.0,6(10X,F6.4))
210   FORMAT (' R=F*(1+RG)**(',I4,'-T)')
220   FORMAT (1H ,I4,10X,2(10X,E14.7))
      XMAX=FLOAT(MAX)
      XMIN=FLOAT(MIN)
      X1MIN=XMIN-50.
      X3MIN=XMIN-70.
      M1=(MAX-MIN)/10
      YMAX=.1E+3
      YMIN=.1E-1
      YMAXL=ALOG(YMAX)
      YMINL=ALOG(YMIN)
      YL=ALOG(.5*YMIN)
      XM=-1
      RG2=RG
```

```
12    CONTINUE
      CALL P1130
      CALL FPLOT(1,3.,3.)
      CALL SCALF(1.,1.,0.,0.)
      CALL FPLOT(1,3.,0.)
      CALL YLOGA(YMIN,YMAX,.01,4,5.,0.,XMIN,.02,SCY)
      CALL FGRID(0,XMIN,YMINL,10.,M1)
      DO 8 I=MIN,MAX,50
      XI=FLOAT(I)-15.
      CALL FCHAR(XI,YL,.12,.15,0.)
      WRITE (7,60) I
8     CONTINUE
      CALL FCHAR(X3MIN,YMINL,.12,.15,0.)
      WRITE (7,70) YMIN
      BE=YMIN
      DO 7 I=1,4
      BE=BE*10.
      BEL=ALOG(BE)
      CALL FCHAR(X3MIN,BEL,.12,.15,0.)
      WRITE (7,70) BE
7     CONTINUE
      TITI=ALOG(YMAX*5.)
      CALL FCHAR(X3MIN,TITI,.12,.15,0.)
      IF (XM.GT.0.) GO TO 30
      WRITE (7,80)
      GO TO 31
30    CONTINUE
      WRITE (7,210) IREFY
31    CONTINUE
      IF (XM.GT.0.) GO TO 37
      YHIG=YMIN
      DO 25 I=1,4
      YHIG=YHIG*10.
      YHIGL=ALOG(YHIG)
      CALL FPLOT(1,XMIN,YHIGL)
      CALL FPLOT(2,XMAX,YHIGL)
25    CONTINUE
37    CONTINUE
      CALL FPLOT(1,XMAX,YHIG)
      TITH=ALOG(YMAX*15.)
      CALL FCHAR(XMIN,TITH,.12,.15,0.)
      WRITE (7,90) (TITLE(K1),K1=1,4)
      IF (XM.GT.0.) GO TO 13
      YI=0.
      FI=0.
      J=1
      SLOPE=100.
      DO 2 I=MIN,MAX
      SUM=0.
      X=FLOAT(I)
      II=I-MIN+1
      IF (A(J).LT.0.) J=J+1
      DO 3 M=1,N
      Y(M)=A(M)*X+B(M)
      F(M)=[illegible](1./(1.+EXP(Y(M))))
      SUM=SUM+F(M)
3     CONTINUE
      IF (SUM.EQ.1.) GO TO 6
      F(J)=1.-SUM+F(J)
      IF (F(J).LT.1.E-6) GO TO 4
```

```
      AB(J)=AREAB/REF(J)
      AA(J)=AREAA/REF(J)
      IF (TL(J,MAX-MIN+1).LE.0.) TL(J,MAX-MIN+1)=YMIN
      TL(J,MAX-MIN+1)=ALOG(TL(J,MAX-MIN+1))
27    CONTINUE
      WRITE (6,40)
      DO 10 K=1,N
      CALL FPLOT(-2,XMIN,G(K,1))
      K5=50
      DO 11 I=MIN,MAX
      II=I-MIN+1
      IF (II.EQ.K5) GO TO 21
      GO TO 22
21    CONTINUE
      K5=K5+50
22    CONTINUE
      X=FLOAT(I)
      CALL FPLOT(0,X,G(K,II))
11    CONTINUE
      CALL FPLOT(1,XMAX,G(K,II))
10    CONTINUE
      XMIN1=XMIN+20.
      CALL FPLOT(1,XMAX,YMINL)
      CALL YLOGA(YMIN,YMAX,.01,4,5,,0.,XMIN,.02,SCY)
      CALL FCHAR(XMIN1,YMINL,.12,.15,0.)
      FMIN=YMIN/(1.+YMIN)
      WRITE (7,50) FMIN
      DO 17 I=1,9,2
      YVA=FLOAT(I)/10.
      YLA=ALOG(YVA/(1.-YVA))
      CALL FCHAR(XMIN1,YLA,.12,.15,0.)
      WRITE (7,50) YVA
17    CONTINUE
      CALL FCHAR(XMIN1,YMAXL,.12,.15,0.)
      FMAX=YMAX/(1.+YMAX)
      WRITE (7,50) FMAX
      CALL FCHAR(XMIN1,TIT1,.12,.15,0.)
      WRITE (7,110)
      XMO=XMIN+150.
      CALL FPLOT(1,XMAX,YMINL)
      XM=1.
      CALL SCALF(1.,1.,0.,0.)
      CALL FPLOT(1,1.,-3.)
      GO TO 12
13    CONTINUE
      DO 14 K=1,N
      CALL FPLOT(-2,XMIN,TL(K,1))
      K5=50
      DO 15 I=MIN,MAX
      II=I-MIN+1
      IF (II.EQ.K5) GO TO 23
      GO TO 24
23    CONTINUE
      K5=K5+50
24    CONTINUE
      X=FLOAT(I)
      CALL FPLOT(0,X,TL(K,II))
15    CONTINUE
      CALL FPLOT(1,XMAX,TL(K,II))
14    CONTINUE
```

```
      Y(J)=ALOG(F(J)/(1.-F(J)))
      SLOPEN=Y(J)-YI
      IF (SLOPEN.EQ.0.) GO TO 36
      SDIFF=SLOPE/SLOPEN
      ERT1=1.-ERROR
      ERT2=1.+ERROR
      IF (SDIFF.GE.ERT1.AND.SDIFF.LE.ERT2.AND.SLOPEN.GT.0.) YLOGM(J)=
 36   CONTINUE
      IF (SLOPE.LT.0..AND.F(J).LE.YLOGM(J)) GO TO 5
      IF (SDIFF.GE.ERT1.AND.SDIFF.LE.ERT2.AND.SLOPE.LT.0.) GO TO 5
      YI=Y(J)
      FI=F(J)
      SLOPE=SLOPEN
      GO TO 6
  4   CONTINUE
      F(J)=1.E-6
      Y(J)=ALOG(F(J)/(1.-F(J)))
      YI=Y(J)
      FI=F(J)
  5   CONTINUE
      CALL FUNC(Y(J),YI,X,X-1.,A(J),B(J))
      J=J+1
      YI=0.
      FI=0.
      SLOPE=100.
  6   CONTINUE
      DO 9 K=1,N
      G(K,II)=Y(K)
      IF (Y(K).GT.YMAXL) G(K,II)=YMAXL
      IF (Y(K).LT.YMINL) G(K,II)=YMINL
      IF (I.GT.IREFY) RG=RG1
      XM=(1.+RG)**(I-IREFY)
      IF (I.EQ.IREFY) REF(K)=F(K)
      XM=F(K)*XM
      IF (XM.LT.YMIN) XM=YMIN
      TL(K,II)=XM
  9   CONTINUE
  2   CONTINUE
      I1=MAX-1
      DO 27 J=1,N
      AREA=0.
      AREAB=0.
      AREAA=0.
      AREA25=0.
      I25=0
      DO 19 I=MIN,I1
      II=I-MIN+1
      XM=(TL(J,II)+TL(J,II+1))/2.
      IF (TL(J,II+1).LT..01) XM=0.
      IF (TL(J,II).LE.0.) TL(J,II)=YMIN
      TL(J,II)=ALOG(TL(J,II))
      AREA=AREA+XM
      IF (I.GE.IREFY) GO TO 28
      AREAB=AREAB+XM
      GO TO 29
 28   CONTINUE
      AREAA=AREAA+XM
 29   CONTINUE
 19   CONTINUE
      ARE(J)=AREA/REF(J)
```

```
      CALL SCALF(1.,1.,0.,0.)
      CALL FPLOT(1,1.,-3.)
      CALL P1130
      CALL FPLOT(1,3.,3.)
      CALL SCALF(1.,1.,0.,0.)
      XMO1=1.
      XMO2=0.
      CALL FCHAR(XMO1,0.,.12,.15,0.)
      WRITE (7,180) IREFY
      CALL FCHAR(XMO1,-.3,.12,.15,0.)
      WRITE (7,190) IREFY,RG2
      CALL FCHAR(XMO1,-.6,.12,.15,0.)
      WRITE (7,195) IREFY,RG1
      DO 16 J=1,N
      HIG=FLOAT(J)
      HIG1=HIG-.3
      HIG2=HIG1-.3
      CALL FPLOT(-2,XMO2,HIG)
      CALL FPLOT(1,XMO2,HIG)
      CALL FCHAR(XMO1,HIG,.12,.15,0.)
      WRITE (7,120) (TYPE(J,K3),K3=1,4),ARE(J)
      CALL FCHAR(XMO1,HIG1,.12,.15,0.)
      WRITE (7,150) IREFY,AB(J)
      CALL FCHAR(XMO1,HIG2,.12,.15,0.)
      WRITE (7,160) IREFY,AA(J)
16    CONTINUE
      HIG=FLOAT(N+1)
      CALL FCHAR(XMO2,HIG,.24,.3,0.)
      WRITE (7,170) NP
      CALL SCALF(1.,1.,0.,0.)
      CALL FPLOT(1,3.,-3.)
      CALL FPLOT(2,3.1,-3.)
      CALL FPLOT(1,3.1,-3.)
      STOP
      END
```

```
SUBROUTINE FUNC(Y1,Y2,X1,X2,A,B)
A=(Y1-Y2)/(X1-X2)
B=Y2-A*X2
RETURN
END
```