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COMPLEX DYNAMICS IN ROMANTIC RELATIONSHIPS

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Abstract

Minimal models composed of two ordinary differential equations are considered in this paper to mimic the dynamics of the feelings between two persons. In accordance with attachment theory, individuals are divided into secure and non-secure individuals, and in synergic and non-synergic individuals, for a total of four different classes. Then, it is shown that couples composed of secure individuals, as well as couples composed of non-synergic individuals can only have stationary modes of behavior. By contrast, couples composed of a secure and synergic individual and a non-secure and non-synergic individual can experience cyclic dynamics. In other words, the coexistence of insecurity and synergism in the couple is the minimum ingredient for complex love dynamics. The result is obtained through a detailed local and global bifurcation analysis of the model. Supercitical Hopf, fold and homoclinic bifurcation curves are numerically detected around a Bogdanov-Takens codimension-2 bifurcation point. The existence of a codimension-2 homoclinic bifurcation is also ascertained. The bifurcation structure allows one to identify the role played by individual synergism and reactiveness to partner's love and appeal. It also explains why aging has a stabilizing effect on the dynamics of the feelings. All results are in agreement with common wisdom on the argument. Possible extensions are briefly discussed at the end of the paper.

1. Introduction

This paper deals with *love dynamics*, a subject which has received remarkable attention in the last few years. The problem falls in the field of social psychology, where interpersonal relationships are the topic of major concern. Romantic relationships are somehow the most simple case since they involve only two individuals. The most common approach to the problem is rooted in *attachment theory* [Bowlby, 1969, 1973, 1980] which explains why infants become emotionally attached to their primary caregivers and why they often experience emotional distress when physically separated from them. Empirical research has focused on different *attachment styles* (secure, anxious, avoidant, ...) in children [Ainsworth *et al.*, 1978] and adult individuals [Hazan and Shaver, 1987; Collins and Read, 1990; Feeney and Noller, 1990; Bartholomew and Horowitz, 1991; Griffin and Bartholomew, 1994] and several hypotheses have been generated about the nature and emotional quality of romantic relationships possessed by people who exhibit different attachment styles. In this context, the attachment style of an individual is believed to persist relatively unchanged throughout all life, even if a recent study has pointed out some exceptions [Fuller and Fincham, 1995]. In conclusion, one can reasonably argue that the main characteristics of love-stories should be largely dictated by the attachment styles of the individuals involved.

Love-stories are dynamic processes that start from zero (two persons are completely indifferent one to each other when they first meet), develop (more or less quickly) and end up into some sort of regime. Real-life observations tell us that most of the times transients develop very regularly and asymptotic regimes are stationary and associated to positive romantic relationships. But there are also love-stories initially characterized by stormy patterns of the feelings as well as by cyclic regimes, like that identified by Jones [1995] in Petrarch's *Canzoniere*, the most celebrated book of love poems of the Western world. This reminds very much the behavior of dynamical systems tending toward equilibria or limit cycles. But observations also point out the existence of multiple attractors. For example, it is known that steady and high quality romantic relationships can turn into a state of permanent antagonism after a disturbance, for example after a temporary infatuation of one of the two partners for another person. Finally, even bifurcations can be naturally invoked if one looks at the effects of age, which is a slowly varying parameter capable of transforming tempestuous relationships into steady ones.

The above remarks suggest the use of differential equations for modelling the dynamics of the feelings between two individuals. It is therefore not surprising if a few contributions have recently appeared along this line. They are rooted in a one page pioneering paper by Strogatz [1988] entitled "love affairs and differential equations" and are briefly reviewed in the next section. Then, in Sec. 3 the mechanisms generating cyclical love dynamics are investigated through bifurcation analysis. The key point is the detection of a Bogdanov-Takens bifurcation and the result is that the simplest couple with complex dynamics is composed of a secure and synergic individual and a non-secure and non-synergic individual (synergic individuals are those who increase their reactions to partner's love and appeal when they are in love). Interpretation of the results and suggestions for further research conclude the paper.

2. Review of Previous Models

The models proposed up to now are *minimal* models, in the sense that they have the lowest possible number of state variables, namely one for each member of the couple.

Such variables, indicated by x_1 and x_2, are a measure of the love of individual 1 and 2 for the partner. Positive values of x represent positive feelings, ranging from friendship to passion, while negative values are associated with antagonism and disdain. Complete indifference is identified by $x = 0$.

Minimal models are a crude simplification of reality. Firstly, because love is a complex mixture of different feelings (esteem, friendship, sexual satisfaction, ...) and can be hardly captured by a single variable. Secondly, because the tensions and emotions involved in the social life of a person cannot be included in a minimal model. In other words, only the interactions between the two individuals are taken into account, while the rest of the world is kept frozen and does not participate explicitly in the formation of love dynamics. This means that rather than attempting to be complete, the aim is to check which part of the behaviors observed in real life can in theory be explained by the few ingredients included in the model.

Three basic processes, namely *oblivion*, *return* and *instinct*, are assumed to be responsible of love dynamics. More precisely, the instantaneous rate of change $\dot{x}_i$ of individual's i love is assumed to be composed of three terms, *i.e.*,

$$\dot{x}_i = O_i + R_i + I_i$$

where the functions O_i, R_i and I_i have different specifications in each model.

In the first model discussed in the literature [Strogatz, 1988] (see also [Radzicki, 1993; Strogatz, 1994]) oblivion and instinct are neglected, while return R_i, which interprets the reaction of individual i to the partner's love x_j, $j{\neq}i$, is assumed to be proportional to x_j. But the two proportionality coefficients have opposite sign, *i.e.*, one of the two lovers is a bit masochist and hates to be loved and loves to be hated. Thus, the model turns out to be a linear oscillator. Strogatz himself explains why he has made these very extreme assumptions: his goal was to teach harmonic oscillators using "a topic that is already on the minds of many college students: the time evolution of a love-affair".

In the two other models oblivion and instinct are also present. Oblivion is specified as

$$O_i = -\alpha_i x_i$$

so that in the extreme case of an individual i who has lost the partner ($R_i, = I_i = 0$), x_i vanishes exponentially at a rate α_i. For this reason, α_i is called *forgetting coefficient*. Instinct I_i describes the reaction of individual i to the partner's appeal A_j. Of course, it must be understood that appeal is not mere physical attractiveness, but more properly and in accordance with evolutionary theory, a suitable combination of different attributes among which age, education, earning potential and social position. Moreover, there might be gender differences in the relative weights of the combination [Feingold, 1990; Sprecher et al., 1994].

In the second model [Rinaldi, 1996a] all processes are assumed to be linear and given by

$$O_i = -\alpha_i x_i \qquad R_i = \beta_i x_j \qquad I_i = \gamma_i A_j$$

with α_i, β_i and γ_i, $i = 1, 2$, positive. Thus, the model turns out to be a positive linear system [Luenberger, 1979; Berman *et al.*, 1989; Rinaldi and Farina, 1995] enjoying, as such, a number of remarkable properties. In particular, if the geometric mean reactiveness to love ($\sqrt{\beta_1\beta_2}$) is smaller than the geometric mean forgetting coefficient ($\sqrt{\alpha_1\alpha_2}$) there is a unique positive and stable equilibrium $E^+=(x_1^+, x_2^+)$, and the two persons, completely indifferent one to each other when they first meet, develop a love story characterized by increasing feelings. Moreover, the quality (x_1^+ and x_2^+) of the romantic relationship at equilibrium improves with the reactiveness to love (β_i) and appeal (γ_i). Finally, an increase of the appeal A_i of individual i gives rise to an increase of the feelings (x_1^+, x_2^+) of both individuals at equilibrium, but the relative improvement is higher for the partner of individual i (in other words, there is a touch of altruism in a woman [man] who tries to improve her [his] appeal). Some of these individual properties can be used to infer properties at the community level: the main result along this line is that a community composed of N linear couples is stable if and only if the partner of the *n-th* most attractive woman of the community is the *n-th* most attractive man (n = 1, 2, ...N). This means that individual appeal is the driving force that creates order in our societies.

In the third model [Rinaldi and Gragnani, 1996] return R_i and instinct I_i are still assumed to depend only upon x_j and A_j, respectively, but the dependence is nonlinear: it takes into account the traits of so-called secure individuals, who are the majority of the individuals in human populations. Secure individuals have positive mental models of themselves and of the others and their romantic relationships are characterized by intimacy, closeness, mutual respect and involvement. They react positively to the partner's love and are not afraid about someone becoming emotionally close to them. In conclusion, their return R_i is an increasing function of x_j. Fig. 1a shows the graph of a typical return function of a secure individual (the same graph characterizes the function $I_i(A_j)$). Note that R_i^+ and R_i^- denote the return for very large positive and negative partner's feelings. Boundedness is a property that holds also for non-secure individuals: it interprets the psycho-physical mechanisms that prevent people from reaching dangerously high stresses. By contrast, the fact that the function increases with x_j is typical of secure individuals, since non-secure individuals react negatively to too high pressures and involvement [Griffin and Bartholomew, 1994], as shown in Fig. 1b. The model is therefore

$$\begin{aligned} \dot{x}_1 &= -\alpha_1 x_1 + R_1^*(x_2) + I_1^*(A_2) \\ \dot{x}_2 &= -\alpha_2 x_2 + R_2^*(x_1) + I_2^*(A_1) \end{aligned} \qquad (1)$$

where the graphs of the functions R^* and I^* are like in Fig. 1a. This model retains many of the properties of the simpler linear model, like the existence of a stable positive equilibrium E^+, and allows one to derive the same conclusions on the role of the appeals at community level. The main difference is that the nonlinearities can give rise to a negative stable equilibrium E^- (generated through a fold bifurcation). Couples with a unique attractor (E^+) are called *robust*, while couples with two attractors (E^+ and E^-) are called *fragile*. Fig. 2 shows the two corresponding state portraits. Note that fragile couples can switch from E^+ to E^- if one of the two individuals has a sudden drop in interest for the partner (see trajectories starting from points 1 and 2 in Fig. 2b).

Before moving to the next section, we like to stress that limit cycles can not exist in model (1) even if individuals are non-secure, *i.e.*, even if the functions R^* are like in Fig. 1b. In fact, the divergence of the system (equal to $-(\alpha_1+\alpha_2)$) does not change sign and Bendixon's criterion implies the non-existence of limit cycles. This means that at least one individual i of the couple with cyclical love dynamics must have an instinct function I_i depending also upon x_i or a return function R_i depending upon both state variables. This is related with synergism as discussed in the next section.

3. Synergism and Complex Dynamics

It is known that individual reactions can be enhanced by love. For example, mothers have often a biased view of the beauty of their children. This kind of phenomenon, here called *synergism*, has been empirically observed in a study on perception of physical attractiveness [Simpson *et al.*, 1990] by comparing individuals involved in dating relationships with individuals not involved in them. Although we are not aware of any study pointing out the existence of synergism in the reaction to the partner's love, we can reasonably assume that also return functions can be enhanced by love. Thus, we consider reaction and instinct functions of the form

$$R_i = \left(1 + S_i^R(x_i)\right)R_i^*(x_j) \tag{2}$$

$$I_i = \left(1 + S_i^I(x_i)\right)I_i^*(A_j) \tag{3}$$

where the functions R_i^* and I_i^* are, by definition, the reactions of a completely indifferent individual and the functions $S_i^R(x_i)$ and $S_i^I(x_i)$ are zero for to $x_i \le 0$ and increasing, convex/concave and bounded for $x_i > 0$, as shown in Fig. 3. The upper bounds of the functions S_i^R and S_i^I are indicated by s_i^R and s_i^I and are called synergism coefficients.

We have shown in the preceding section that couples composed of non-synergic individuals, *i.e.*, couples with $s_i^R = s_i^I = 0$, cannot have cyclic behavior. Thus, synergism is necessary for generating complex love dynamics. It can be shown however, that synergism is not sufficient if the couple is composed of secure individuals (*i.e.*, individuals characterized by reaction and instinct functions R^* and I^* like in Fig. 1a). We prove this result by referring to the case of synergic instinct functions (3), *i.e.*, by analyzing the model

$$\begin{aligned}
\dot{x}_1 &= -\alpha_1 x_1 + R_1^*(x_2) + \left(1 + S_1^I(x_1)\right) I_1^*(A_2) \\
\dot{x}_2 &= -\alpha_2 x_2 + R_2^*(x_1) + \left(1 + S_2^I(x_2)\right) I_2^*(A_1)
\end{aligned} \tag{4}$$

A similar proof holds for the case of synergic return functions (2).

The proof of non existence of limit cycles in model (4) is as follows. Note first that the isocline $\dot{x}_1 = 0$ can be given the form $x_2 = x_2(x_1)$, because the function $R_1^*(x_2)$ is invertible in the case of a secure individual (see Fig. 1a). Such isocline is composed by one, two or three different functions defined over disjoint intervals, because the inequality

$$R_1^{*-} \leq \alpha_1 x_1 - \left(1 + S_1^I(x_1)\right) I_1(A_2) \leq R_1^{*+}$$

is satisfied in one, two or three disjoint intervals (note that the function between the two inequality signs is continuous and stationary at most at two points). Obviously, the same properties hold for the isocline $\dot{x}_2 = 0$ which can be given the form $x_1 = x_1(x_2)$. Figure 4 is a sketch of the isoclines in the case the first one ($\dot{x}_1 = 0$) is defined on two intervals and the second ($\dot{x}_2 = 0$) on a single interval. There are seven equilibria, which are either saddles (S) or nodes (N) (a focus cannot exist in system (4) because the product of the elements on the antidiagonal of the Jacobian matrix $\left(dR_1^* / dx_2\right) \cdot \left(dR_2^* / dx_1\right)$ is always positive). The nature of the seven equilibria, can be immediately detected looking at the direction of the trajectories on the isoclines. Also invariant sets delimited by the isoclines can be easily identified, as indicated in Fig. 4 (see shaded regions). Since, by construction, all equilibria are on the boundaries of these invariant sets, cycles can not exist inside these sets. Thus, eventually, they must lie entirely outside, namely in the white regions of Fig. 4. But this is impossible because the union of the invariant sets is connected and expands to infinity (note that isocline $\dot{x}_1 = 0$ [$\dot{x}_2 = 0$] tends to infinity in the x_2 [x_1] direction). The proof of the non existence of limit cycles given here for the case depicted in Fig. 4 (where the first isocline is defined on two intervals and the second on one interval) can be repeated for all other cases of concern. This formally proves that *secure individuals cannot have complex love dynamics even if they are synergic.*

In order to identify a case of cyclic dynamics we now consider couples composed of a secure and synergic individual (1) and a non-secure and non-synergic individual (2). Thus, the model is

$$\begin{aligned} \dot{x}_1 &= -\alpha_1 x_1 + R_1^*(x_2) + \left(1 + S_1^I(x_1)\right) I_1^*(A_2) \\ \dot{x}_2 &= -\alpha_2 x_2 + R_2^*(x_1) + I_2^*(A_1) \end{aligned} \tag{5}$$

where R_1^* and R_2^* are like in Fig. 1a and 1b, respectively, and S_1^I is like in Fig. 3. For $S_1^I(x_1) = 0$ this model degenerates into model (1), which has no cycles. Hence, we should not expect cycles in model (5) for low values of the synergism coefficients.

In order to prove that model (5) can have complex dynamics, we have performed a numerical but rather systematic analysis of its local and global bifurcations. For local bifurcations (Hopf and fold bifurcations) we have used LOCBIF, a professional software package for continuous-time dynamical systems based on a continuation technique [Khibnik *et al.*, 1993]. For global bifurcations (homoclinic bifurcations) we have used AUTO86 and we have followed a two-steps approach. First a homotopic method has been used to generate one point of the bifurcation curve and the corresponding homoclinic orbit. Then, a projection boundary condition method has been used to produce the entire homoclinic bifurcation curve through continuation. Both methods are described in [Champneys and Kuznetsov, 1994]. The analysis has been performed for various functional forms of R_1^*, R_2^* and S_1^I and the results have been consistent. The bifurcation portraits shown in the following make reference to the functional forms

$$
\begin{aligned}
R_1^*(x_2) &= \begin{cases} R_1^{*-} x_2/(x_2-1) & x_2<0 \\ R_1^{*+} x_2/(x_2+1) & x_2 \geq 0 \end{cases} \\
R_2^*(x_1) &= \begin{cases} 2x_1/(1-x_1) & x_1<0 \\ 4[x_1/(2+x_1)][(1-x_1^8)/(1+x_1^8)] & x_1 \geq 0 \end{cases} \\
S_1^I(x_1) &= \begin{cases} 0 & x_1<0 \\ s_1^I x_1^8/(1+x_1^8) & x_1 \geq 0 \end{cases}
\end{aligned}
\tag{6}
$$

Figure 5 shows the bifurcation curves of model (5)-(6) in the two-dimensional space of synergism coefficient (s_1^I) and maximum return (R_1^{*+}) of the first individual (a secure and synergic individual). The organizing center is the Bogdanov-Takens codimension-2 bifurcation point BT in which three bifurcation curves merge: a supercritical Hopf (h), a fold (f) and a homoclinic (p). There are two other codimension-2 points: the cusp C and the degenerate homoclinic bifurcation point D where a fold f and a homoclinic p merge giving rise to a saddle-node homoclinic bifurcation curve q. As a result there are five possible dynamic behaviors identified as 1,2, ..., 5 and described with a sketch of the corresponding state portrait. In regions 1, 2, 3 the attractor is unique (an equilibrium in regions 1 and 2 and a limit cycle in region 3), while in regions 4 and 5 there are alternative attractors. The dashed curve does not involve bifurcations of attractors, while all others do. The diagram confirms our expectations: there are no cycles if the synergism coefficient is low but there are cycles if individual 1 is highly sensitive (high reaction to partner's love R_1^{*+} and high synergism coefficient s_1^I).

The same bifurcation structure has been detected varying other parameters. Figure 6 shows, for example, the bifurcations with respect to the synergism coefficient of the first individual and the appeal of the second. Again the Bogdanov-Takens bifurcation point is the organizing center.

We have also analyzed the case of synergic return functions, namely the model

$$\begin{aligned} \dot{x}_1 &= -\alpha_1 x_1 + \left(1 + S_1^R(x_1)\right) R_1^*(x_2) + I_1^*(A_2) \\ \dot{x}_2 &= -\alpha_2 x_2 + R_2^*(x_1) + I_2^*(A_1) \end{aligned} \tag{7}$$

and we have, once more, obtained the same bifurcation structure. Figure 7 shows, for example, the bifurcation diagram analogous to that of Fig. 5 for the functions $R_1^*(x_2)$, $R_2^*(x_1)$ and $S_1^R(x_1)$ given by Eq. (6) with s_1^I replaced by s_1^R. In conclusion, our bifurcation analysis allows one to state that *a couple composed of a secure and synergic individual and a non-secure and non-synergic individual can have complex (cyclic) dynamics.*

4. Discussion and Conclusion

Dynamics of love between two persons has been investigated in this paper by means of a minimal model composed of two differential equations, one for each individual. Three mechanisms of love growth and decay have been taken into account: the forgetting process, the pleasure of being loved and the reaction to partner's appeal. This has been done by introducing two functions, called return and instinct functions, which differ in the cases of secure and non-secure individuals. The fact, here called synergism, that a woman [man] might react more strongly when she [he] is in love, has also been modelled. As a result, individuals are secure or non-secure, and synergic or non-synergic, for a total of four different classes. Thus, it has been shown that couples composed of non-synergic individuals as well as couples composed of secure individuals can not have complex (*i.e.*, cyclic) dynamics. By contrast, couples composed of a secure and synergic individual and a non-secure and non-synergic individual can have complex dynamics. In other words, the coexistence of synergism and insecurity within the couple is the reason for tempestuous romantic relationships. This result has been proved through the analysis of the local and global bifurcations of the model. Fold, Hopf and homoclinic bifurcations have been detected, in accordance with the existence of a Bogdanov-Takens codimension-2 bifurcation point.

The bifurcation diagrams can be easily interpreted in terms of individual behavior and appeal. They show that, with the exception of a narrow band in parameter space (regions 4 and 5 of Figs. 5-7), the system has a unique global attractor: an equilibrium or a limit cycle. Both attractors are associated to positive feelings, *i.e.*, to a satisfactory quality of the romantic relationship. In general, the attractor is an equilibrium when the individuals are not too sensitive, *i.e.*, when their synergism and reactiveness to partner's love and appeal are, as a whole, low. By contrast, high sensitivity (as well as high attractiveness) implies cyclic behavior, as one would intuitively expect.

Our bifurcation diagrams also show that aging has a stabilizing effect. Indeed, it is generally believed that individual appeal, as well as synergism and reactions to partner's love and appeal, slowly deteriorate with aging. Thus, couples with complex love dynamics (regions 3 and 4 of Figs. 5-7) can slowly vary during their life and finally become stationary by crossing the bifurcation curves h, p or q in Figs. 5-7. The transformation is accompanied by a reduction of the amplitudes of the emotional ups and downs of a cyclic regime (when Hopf bifurcation curve h is approached) or by a reduction of the frequency of these ups and downs (when homoclinic curves p and q are approached).

As for any minimal model, the extensions one could propose are innumerable. In accordance with the most recent developments of attachment theory [Kobak and Hazan, 1991; Sharfe and Bartholomew, 1994], learning and adaptation processes could be taken into account allowing for some behavioral parameters to slowly vary in time. Suitable nonlinearities could be introduced in order to develop theories for particular subclasses of non-secure individuals [Griffin and Bartholomew, 1994]. Men and women could be distinguished by using different state equations [Hendrick and Hendrick, 1995]. The dimension of the model could also be enlarged in order to consider individuals with more complex personalities [Rinaldi, 1996b] or the dynamics of love in larger groups of individuals (*e.g.*, families). Undoubtedly, all these problems deserve further attention.

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Figure Captions

Fig. 1 Return functions $R_i(x_j)$ of secure individuals (a) and non-secure individuals (b).

Fig. 2 State portraits of couples composed of secure individuals: robust couples (a) and fragile couples (b). In (b) the boundary of the two basins of attraction (dashed line) is the stable manifold of a saddle.

Fig. 3 The graph of a typical synergism function.

Fig. 4 The isoclines ($\dot{x}_1 = 0$, $\dot{x}_2 = 0$) and the equilibria of system (4) (N = node, S = saddle). The shaded regions are invariant sets.

Fig. 5 Bifurcation portrait of model (5)-(6) for the following parameter setting: $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, $I_1(A_2) = 0.05$, $I_2(A_1) = 0.05$, $R_1^{*-} = -R_1^{*+}$.

Fig. 6 Bifurcation portrait of model (5)-(6) for the following parameter setting: $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, $I_2(A_1) = 0.05$, $R_1^{*+} = 0.05$, $R_1^{*-} = -0.05$.

Fig. 7 Bifurcation portrait of model (7) for the following parameter setting: $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, $I_1(A_2) = 0.05$, $I_2(A_1) = 0.05$, $R_1^{*-} = -R_1^{*+}$.

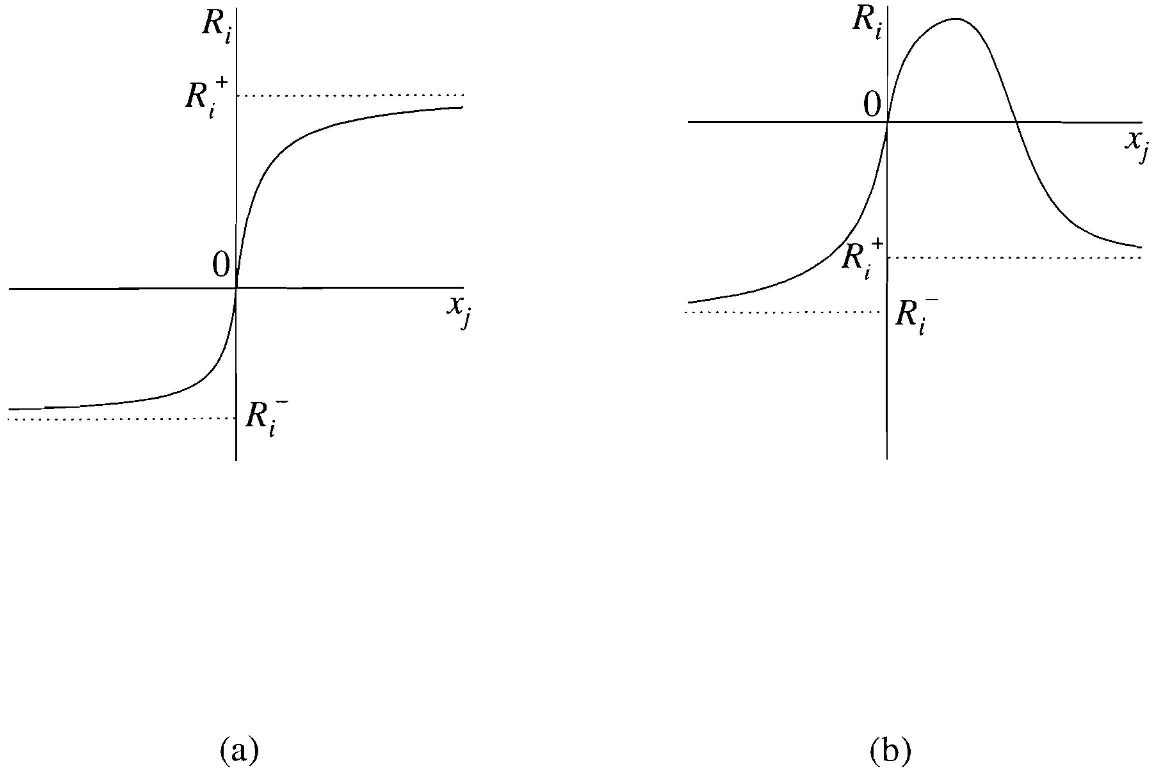
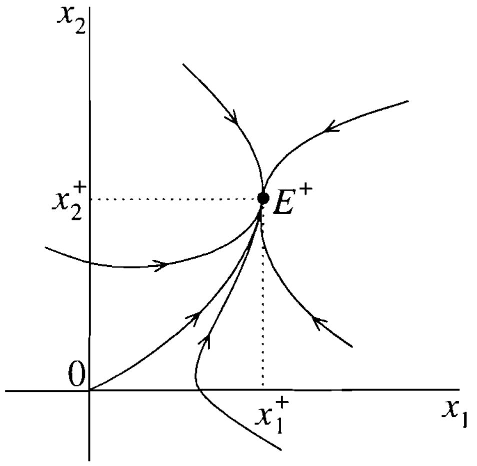
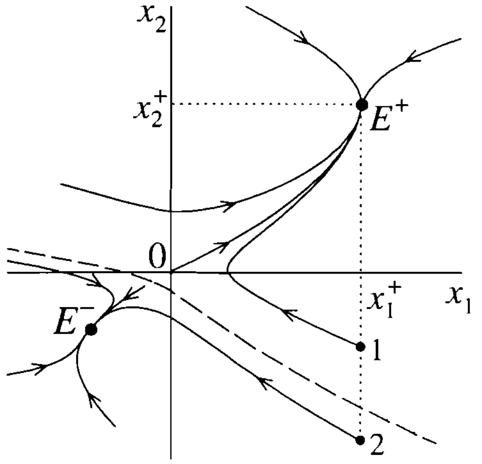


Figure 1



(a)



(b)

Figure 2

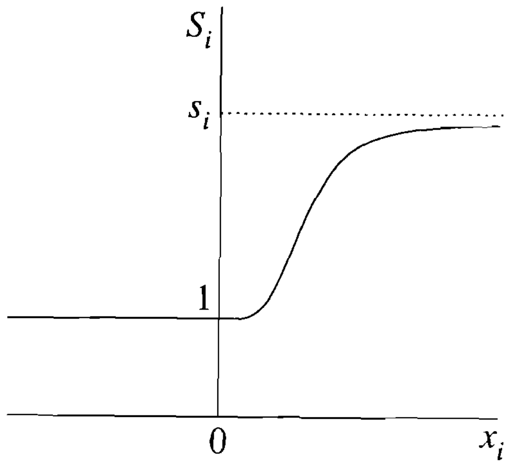


Figure 3

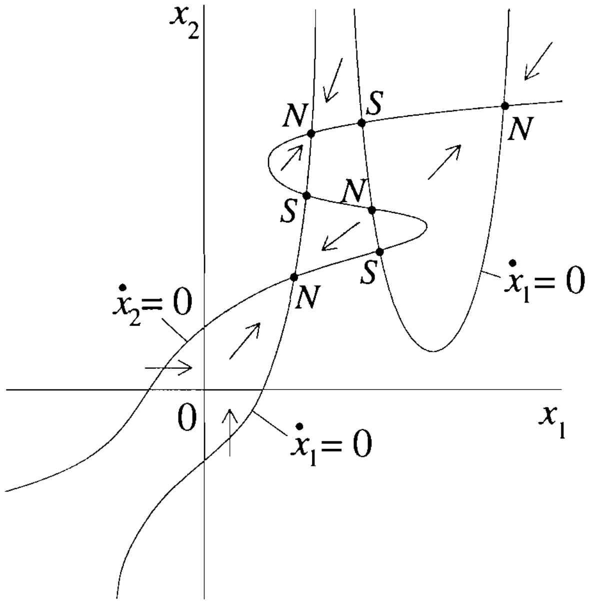
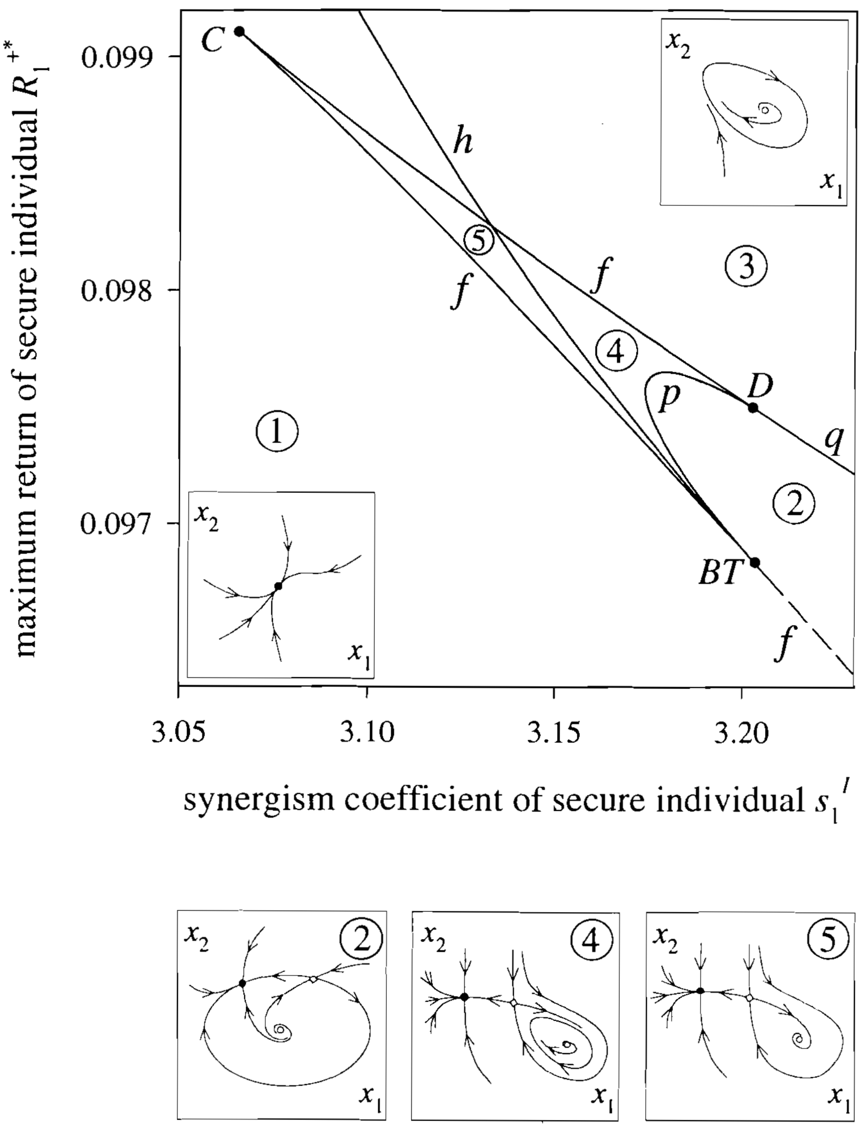
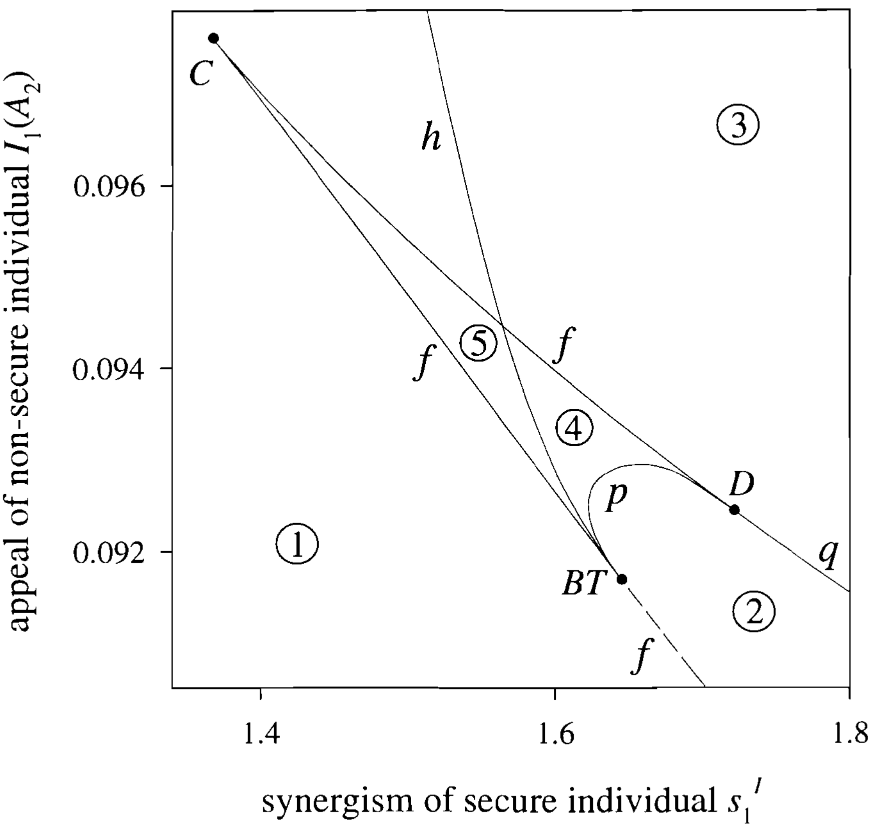


Figure 4



maximum return of secure individual R_1^{+*}
0.099
0.098
0.097
3.05
3.10
3.15
3.20
synergism coefficient of secure individual s_1^I
C
h
f
f
f
p
D
q
BT
1
2
3
4
5
x_1
x_2

Figure 5



C
h
3
5
f
f
4
D
p
1
q
BT
2
f
0.096
0.094
0.092
1.4
1.6
1.8
appeal of non-secure individual $I_1(A_2)$
synergism of secure individual $s_1{}^I$

Figure 6

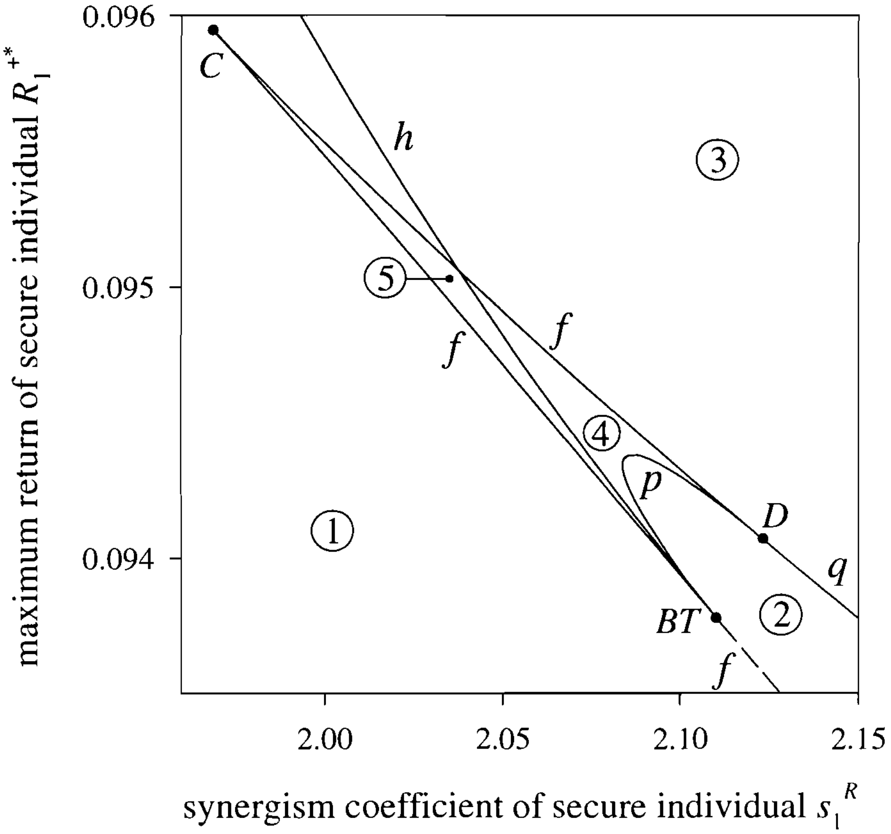

0.096
0.095
0.094
maximum return of secure individual R_1^{+*}
2.00
2.05
2.10
2.15
synergism coefficient of secure individual s_1^R
C
h
3
5
f
f
4
p
D
1
q
BT
2
f

Figure 7