Dynamics of Fully Stocked Stands in the Territory of the Former Soviet Union

Shvidenko, A., Venevsky, S.V., Raile, G. and Nilsson, S.

IIASA Working Paper

WP-96-019

February 1996
Dynamics of Fully Stocked Stands
in the Territory of the
Former Soviet Union

Anatoly Shvidenko, Sergey Venevsky,
Gerhard Raile and Sten Nilsson

WP-96-19
February 1996
Dynamics of Fully Stocked Stands in the Territory of the Former Soviet Union

Anatoly Shvidenko, Sergey Venevsky, Gerhard Raile and Sten Nilsson

WP-96-19
February 1996

Professors Shvidenko and Nilsson and Magister Venevsky are from the International Institute for Applied Systems Analysis, Laxenburg, Austria; Mr. Raile is with the North Central Forest Experiment Station of the U.S. Forest Service, St. Paul, Minnesota, USA.

Working Papers are interim reports on work of the International Institute for Applied Systems Analysis and have received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute, its National Member Organizations, or other organizations supporting the work.
FOREWORD

IIASA, the Russian Academy of Sciences, and Russian governmental organizations initiated the Siberian Forest Study in 1992, with the following objectives:

- Identification of possible future sustainable development options for the Siberian forest sector (assess the biosphere role of Siberian forests, and identify suitable strategies for sustainable development of forest resources, industry, infrastructure, and society).

- Identification of policies for various options to be implemented by Russian and international agencies.

The first phase of the study built relevant and consistent databases for the upcoming analyses of the Siberian forest sector (Phase II). Nine cornerstone areas have been identified for the assessment analyses, namely, further development of the databases, greenhouse gas balances, forest resources and forest utilization, biodiversity and landscapes, non-wood functions, environmental status, forest industry and markets, transportation infrastructure, and socioeconomics.

The existing increment estimations in the former USSR and Russia are limited to net increment calculations for periods between inventories which are aggregated for groups of species. Thus, the so called average increment as presented in the Forest State Account is an accumulative characteristic of the growing stock.

The work in this paper presents a system developed to estimate the gross and net increment as well as the natural mortality for major forest species by ecoregion. In this report, the work dealing with evenaged fully stocked stands is presented. In the future, analyses for different stocking densities and different types of age structures will be carried out.

This work on increment and mortality is a crucial step for the further analyses of the greenhouse gas balances, forest resources and forest utilization, and biodiversity and landscapes in cornerstones in phase II.
1. Introduction

Reliable estimates of forest productivity, both inventory and growth, are critical factors for solving various problems in forest ecology and management. Forest productivity is the main criteria for the determination of what constitutes sustainable development of the forest sector. Also, estimates of the major ecological cycles, particularly of carbon cycle, require the calculation of the net primary productivity (NPP) and the net ecosystem productivity (NEP). Gross increment\(^1\) is the largest part of NPP and net increment is the largest part of NEP. Therefore, improvement of forest increment estimates is one of the most important scientific problems with practical applications.

Russia has a long tradition of significant forest growth research, and these results are the basis for the theory of forest stand development dealing with forest stand productivity and forest management (Turskii, 1925; Orlov 1927, 1928; Tovstoles, 1938; Matveev-Motin, 1962; Tjurin, 1945; Naumenko, 1941; Dvoretskij, 1964; Zakharov, 1967; Nikitin, 1966; Antanaitis and Sagreev, 1981). A large number of scientific conferences on forest increment research were held in the 1970s in the former USSR (Anonymous 1967a, Anonymous 1968; Anonymous, 1972; Anonymous, 1975). These meetings maintained Russia’s momentum of forest research in this area.

Unfortunately, neither net nor gross growth indicators have been directly measured by the current forest inventory system. The existing inventory instructions do not require direct growth measurements in the field for each inventoried stand. The increment estimation is limited to net increment calculations for periods between inventories (i.e. the change of growing stock) and are aggregated for all species by Forest Enterprise (Anonymous 1986, 1991; VNIIZlesresurs, 1995). Thus, the so called average increment as presented in the Forest State Account is an accumulative characteristic of the growing stock. The official estimates of net or gross increment for all Russian forests are not reported. Various independent expert estimates for all of Russia vary from 500 to 1500 million m\(^3\) per year for the net growth (stem wood including bark for trees taller than breast height) and from 1400 to 2600 million m\(^3\) per year for gross growth (Sagreev, 1991, Shvidenko, 1991). Official data on the average increment for forested areas, based on the 1993 Forest State Account, under state forest management (705.8 million ha or 92.4% of all Russian forested areas), was reported to be 822.2 million m\(^3\) in 1993 of which 279.1 million m\(^3\) were located in European Russia (VNIIZlesresurs, 1995). Natural mortality is estimated to vary from 400 to 1200 million m\(^3\) per year (Shvidenko, 1991; Sagreev, 1991; Kusmitchev, 1991).

---

\(^1\)Definitions of the terms used are given in Appendix 1.
Recent changes in the Russian forest state inventory system (Strakhov et. al., 1995) will improve the estimation of forest productivity by making direct field crew measurements of growth. However, due to the vast area, the implementation of a new inventory system in Russia will require at least 20 years.

At this stage, it is possible to calculate aggregated estimates of the net and gross increment for the Russian forests using data from the Forest State Account combined with specific information from forest growth models developed in Russia. This publication presents the first stage of such a model approach. The result is a modeling system which uses databases and elements of expert systems for improved increment estimation. Although the proposed system is designed to provide the "big picture" of forest growth, specifically for general ecological evaluation (e.g., for productivity or carbon budget evaluation) it is also obvious that the system can provide useful information necessary to improve the efficiency of the Russian inventory system.

The work presented below was carried out as part of IIASA's Siberian Forest Study, which was initiated by IIASA (International Institute for Applied Systems Analysis, Austria), the Russian government and the Russian Academy of Sciences in 1993.

2. General Structure of the Modeling System

2.1 General Scheme

There are many models, which are used in forest inventory and for different forest management objectives (for individual tree, individual stands, various stand aggregations, large areas of forest cover). At the stand level different types of yield tables are usually used (Kozlovskij and Pavlov, 1967). There are numerous yield tables for different forest types. There are both normal (fully stocked) and modal tables covering both general and regional areas including pure and mixed stands, etc. for the major forest types in Russia. In addition there are a number of different models designed for increment estimation. These tables and models have been approved officially by the former State Committee on Forest in the USSR, and published in special reference books for use by the forest inventory (Sagreev et al., 1992; Moshkaliev, 1984; Shvidenko et al., 1987; Kartashov, 1986; Voinov, 1986; Korjakin, 1990; Gusev, 1993). The basic objective of the results presented here was to generate a computer system which would make it possible to estimate gross and net increment, and natural mortality, by employing the models and tables mentioned above, supplemented and verified with data from the forest inventory and direct measurements on test plots. The modeling system (MS) uses both aggregations of data from individual stands and aggregated data from the forest state account.
The system developed is able to estimate the gross and net increment, as well as the natural mortality for the major forest species, and for ecoregions identified by IIASA’s Siberian Forest Study (Shvidenko et al., 1995). The system can, with the use of additional models, also estimate the Net Ecosystem Productivity (NEP). A simplified diagram of the system structure is shown in Figure 1.

![Figure 1. Block-diagram of the modeling system for evaluation of growth and mortality in Russia.](image-url)
2.2 Basic Principles for the System Development

The modeling system (MS) was developed as a regional system using parameter extension. This means that a) for any type of initial data and for different locations in Russia, the MS can calculate gross increment, net increment, and natural mortality using regional or general models, to minimize errors; and b) the MS can be implemented as a component of a geographical information system (GIS) and forest inventory databases.

The system, which is still under development, includes a hierarchical regionalized set of modules for:

1) Fully stocked stands, both general and aggregated by ecoregion.
2) Productivity under different stocking.
3) Regional models and correction coefficients based on observations in real stands.
4) Auxiliary models of disturbance impact regimes (at this stage we only consider influences of main types of air pollutants).

The modeling activity so far has used available information on species composition, age, site indexes and density. Basic models were developed for evenaged stands. A set of auxiliary models modify the growth of stands for the different types of age structure (TASS) mentioned below. These models employ the aggregation of tree diameter distributions and mortality schemes by species and climatic zones.

The basic mathematical principle of the MS is the use of a set of growth functions (Shvidenko and Juditsky, 1983; Kiiviste, 1988) based on various stand characteristics. The growth functions employed are simple and aggregated. These types of functions are only able to describe the growth processes of evenaged stands up to a certain age.

As the first step, the Mitcherlich growth functions were chosen as the basic mathematical functions for implementing experimental data, yield tables, and model outputs into a simple, functional modeling system. The dependent variables height, diameter, basal area, growing stock, total volume and their derivatives, such as annual gross increment, annual net increment and natural mortality, are functions of stand age A and three different coefficients in the Mitcherlich functions \( C_1, C_2, C_3 \). For example, total volume \( TV \) can be calculated as:
\[ TV = c_1 \cdot (1 - \exp(-c_2 \cdot A))^{c_3} \]

The coefficients represent different biological phenomena (\(c_1\) is the maximal value of a growth function (the asymptote), \(c_1 \cdot c_2 \cdot (1 - \frac{1}{c_3})^{(c_3-1)}\) is the maxim current increment value (slope of the curve) and \(\frac{\ln(c_3)}{c_2}\) is the turning point of a growth function).

With the estimates of the coefficients of the Mitcherlich functions, it is possible to calculate annual increment and the percent of annual increment; for gross increment (\(dTV\)) and net increment (\(dGS\)). For example, for the gross increment \(dTV\) and percent of gross increment \(P_{TV}\) we get:

\[ dTV = c_1 \cdot c_2 \cdot c_3 \cdot (1 - \exp(-c_2 \cdot A))^{(c_3-1)} \cdot \exp(-c_2 \cdot A) \]

and

\[ P_{TV} = c_2 \cdot c_3 \cdot \exp(-c_2 \cdot A) / (1 - \exp(-c_2 \cdot A)) \]

The difference between gross increment \(dTV\) and net increment \(dGS\), both calculated as the derivatives from the Mitcherlich functions, gives us the natural mortality \(dM\):

\[ dM = dTV - dGS. \]

The coefficients of the Mitcherlich functions vary by forest species and geographical location.

The fitting of the experimental data was carried out using the following approach:

1. A non-linear regression using the Levenberg-Marquart method was executed to evaluate coefficients of the Mitcherlich functions by age, site index and density. Computer evaluation shows that the estimation of the exponential coefficient is particularly important because all the functions are very sensitive to variation of this coefficient.

2. Quadratic regressions gave us a statistical dependence of the three coefficients by site indexes. This stage allowed us to design a two-dimensional growth function for the stand characteristics by age and site index. It is possible to use other non-linear functions for the regression calculations. However, more sophisticated regressions can generate considerable calculation errors and do not agree with the principles of simplicity and integrity for system development. The analysis so far showed that linear regressions are inaccurate and can not be used. Quadric regressions seem to give adequate results.
3. A three dimensional growth function by age, site index and density was developed using the results obtained by the first two steps. From the experimental data, the density can be calculated by using basal areas from fully-stocked stands evaluated in the previous steps. However, usually the density variations for separate regions are very small and one is forced to find separate individual methods for constructing the final approximate three-dimensional surface. From a mathematical point of view, this is the problem of the multidimensional approximation of a plane with irregular points by an analytical function. Such a problem can only be solved in some cases with a "good" configuration of experimental data sets. Nevertheless, one can try to design such a function by using additional data from other locations and projections from already designed three-dimensional growth planes for similar forest species. Computer analyses for two coniferous species (Larch and Pine) show that such manipulations can be accomplished within in the limits of the accuracy of the function coefficients.

4. Finally all growth functions were evaluated by experts with respect to the stand variables employed in the functions (height, basal area, diameter, growing stock, total volume, gross and net increments, the consequent percents, natural mortality). The iterative procedure described can be restarted if the results are not explicit enough or are in contradiction with results from the forest inventory (taxation).

2.3 The Objects of the Modeling

The basic objects for estimation are sets of stands of dominant forest species by ecoregions. The ecoregions have been defined by using the following criteria:

a) the biospheric features and biogeochemical fluxes and pools should be of comparable orders in the ecoregions,

b) each ecoregion should have a relative climatic and vegetation homogeneity in order to describe the main forest species in terms of average productivity and other average characteristics;

c) mountainous and plain areas should be divided into separate ecoregions;

d) permafrost regions should be treated separately;

e) the level of anthropogenic impact or disturbances should be of similar magnitude in each ecoregion;

f) the division into ecoregions should coincide with the borders of the administrative territories.

According to these criteria, 141 ecoregions have been defined for Russia. The average size of ecoregions are different in European Russia (average total area is 4.4 million ha, forested area
is 1.7 million ha) and in Siberia (average total area is 20.3 million ha, forested area is 9.6 million ha).

### 2.4 Initial Data for Growth Evaluation

The minimum information set of attributes for modeling of the growth at the regional level includes, species, age, site index, density, age structure of stands, regime of forest management (regularly thinned or unmanaged stands), and for specified stands, the radial increment (Nikitin and Shvidenko, 1978; Antanaitis and Sagreev, 1981).

The Forest State Account (FSA) contains the distribution of areas and growing stock by age, site index, and density aggregated by dominant species within a specific utilization category for each Forest Enterprise in Russia.

The FSA does not contain the distribution of forests by stand and type of age structure. However, this information is very important for growth estimation. A unified classification scheme has been developed for type of age structure based on different Russian publications (e.g., Shanin, 1965, 1967; Semetchkin, 1963, 1970):

1. **Evenaged stands (EVA).** Stands in which 70% and more of the growing stock is constituted by trees belonging to one age class (the age classes for coniferous in Russia and for hard deciduous species of seed regeneration is 20 years (for Cedar - 40 years). For the other species with some exceptions it is 10 years). The coefficients of variation for age frequency distribution is $V_a < 8\%$ and for tree diameters is $V_d < 28\%$;

2. **Relative unevenaged stands (RUEA).** Stands in which 70% or more of the growing stock consists of trees belonging to more than one, but within two, age classes ($V_a = 8 \ldots 15\%$, $V_d = 28 \ldots 35\%)$;  

3. **Unevenaged stands (UEV).** The growing stock consists of trees of more than two, and usually of all, age groups (all forests in the Forest State Account are divided into young, middle-aged, premature, mature, and overmature). ($V_a > 15\%$, $V_d > 35$);

4. **Gradually unevenaged stands (GUE).** The growing stock consists of trees of more than one canopy layer in which dominant species belong to different age groups or different types of age structure.

Numerous investigations (e.g. Gusev, 1963; Shanin, 1967; Anonymous 1967b; Verkhunov, 1975; Semetchkin, 1973; Svalov 1963, 1979; Sokolov et al. 1993) report that many parts of
the taiga forests of Russia are unevenaged. Data compiled from available data by ecoregions taken from the "green" (ecoregional) DB of the IIASA Siberian Forest Study, show that in Siberia and the Far East from 40% to 50% of the coniferous forests belong to unevenaged and gradually unevenaged groups (see Tables 1-4).

Table 1. Types of age structure of Siberian and the Far Eastern *middle taiga forests*. Distribution of forested areas of mature and over mature stands in percents.

<table>
<thead>
<tr>
<th>Species</th>
<th>EVA</th>
<th>RUE</th>
<th>UEV</th>
<th>GUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>23</td>
<td>51</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>Spruce</td>
<td>1</td>
<td>41</td>
<td>40</td>
<td>18</td>
</tr>
<tr>
<td>Fir</td>
<td>16</td>
<td>28</td>
<td>14</td>
<td>42</td>
</tr>
<tr>
<td>Larch</td>
<td>10</td>
<td>69</td>
<td>21</td>
<td>0</td>
</tr>
<tr>
<td>Cedar</td>
<td>19</td>
<td>28</td>
<td>28</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 2. Types of age structure of Siberian and the Far Eastern *southern taiga forests*. Distribution of forested areas of mature and over mature stands in percents.

<table>
<thead>
<tr>
<th>Species</th>
<th>EVA</th>
<th>RUE</th>
<th>UEV</th>
<th>GUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>38</td>
<td>49</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Spruce</td>
<td>26</td>
<td>27</td>
<td>25</td>
<td>22</td>
</tr>
<tr>
<td>Fir</td>
<td>27</td>
<td>24</td>
<td>10</td>
<td>39</td>
</tr>
<tr>
<td>Larch</td>
<td>10</td>
<td>74</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Cedar</td>
<td>33</td>
<td>20</td>
<td>22</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3. Types of age structure of Siberian and the Far Eastern *mixed forests*. Distribution of forested areas of mature and over mature stands in percents.

<table>
<thead>
<tr>
<th>Species</th>
<th>EVA</th>
<th>RUE</th>
<th>UEV</th>
<th>GUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>0</td>
<td>70</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>Spruce</td>
<td>10</td>
<td>74</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Fir</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Larch</td>
<td>10</td>
<td>74</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Cedar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Types of age structure of Siberian and the Far Eastern forests in forest steppe. Distribution of forested areas of mature and over mature stands in percents.

<table>
<thead>
<tr>
<th>Species</th>
<th>EVA</th>
<th>RUE</th>
<th>UEV</th>
<th>GUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>80</td>
<td>17</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Spruce</td>
<td>15</td>
<td>30</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>Fir</td>
<td>16</td>
<td>30</td>
<td>10</td>
<td>44</td>
</tr>
<tr>
<td>Larch</td>
<td>18</td>
<td>71</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Cedar</td>
<td>17</td>
<td>29</td>
<td>14</td>
<td>40</td>
</tr>
</tbody>
</table>

The Forest State Account (FSA) data contains information on 29 main forest species, therefore there is a need to have tables and models for all these species. However, such yield tables are either absent or incomplete for the majority of these species.

3. Models of Fully Stocked Stands
3.1 Species and Data

This working paper presents results of the first stage of the MS generation and has two main objectives:

1. to generate models of fully stocked stands of main forest forming species in Russia;
2. to examine the method, proposed for different species under various ecological and growing conditions.

According to the latest Forest State Account (on January 1, 1993), of 705.8 million ha managed by the Russian Forest Service, 5 dominant species cover 82.2% of area. Of those five, pine covers 114.3 million ha or 16.2%; spruce - 75.9 million ha or 10.7%, larch - 229 million ha or 37.7%, cedar - 39.8 mln.ha and 5.6% and birch - 87.7 mln.ha or 12.4%. Stands generated by stone birch (Betula ermanii) and fir (basically Abies sibirica) cover respectively 14.4 million ha (2.0%) respectively 8.3 million ha (1.2%). The genus Pinus is represented by 12 different naturally growing species, but more then 95% of it's area is covered by Pinus sylvestries. Of 11 species of spruce the majority of the area is covered by Picea abies (European part), Picea sibirica (from Ural to Pacific), Picea schrenkiana (Tian Shan mountains) and Picea ajanensis (Far East). Larch is represented by Larix sibirica, Larix sukaszewi, Larix gmelinii and Larix kajanderii (a total of 7 species and many intermediate forms). Russian cedar includes Pinus sibirica (about 90% of the species cover area) and Pinus korajensis. Soft deciduous species of the genus Betula includes more than 70 species in Russia, but the majority of Betula species belongs to Betula pendula and Betula pubescens and numerous similar species.
To develop a method applicable for average estimates of stand dynamics, the concept of general yield tables for fully stocked or normal stands has been suggested by Tjurin et al. (1945). In the 1970s Sagreev suggested a new method for the development of such tables based on the concept of typical and standardized rows for growth models (Sagreev, 1978), as the basis on which general yield tables for fully stocked (normal) stands can be generated (Sagreev et al., 1992).

The tables for pine, spruce, larch, birch, oak, beech, aspen and black alder have been generated by aggregation of data from more than 400 local tables (about 2000 dynamic rows) for evenaged stands growing in different geographical regions and growing conditions. The dynamics of 6 basic indicators (average height, average diameter, basal area, breast height form factor, net and gross increments) was constructed using typical and standardized rows, proposed by Sagreev (1978). The basic approach for the general table design was the estimation of the average height at 100 years stand age, which coincide with the approach of the site index scale developed by Prof. Orlov\(^2\) (1927, 1928).

Both *Pinus sibirica* and *Pinus korajanensis* can generate stands of different age structures. Pure stands are only possible in young ages (Semetchkin et al., 1985), or in some specific parts of the species' ranges. Only one of the yield tables, prepared for the Siberian Forestry Reference book, has data for evenaged Cedar stands.

We used the tables discussed above as the basis for the development of dynamic models. It should be pointed out that the models developed can only be considered as general aggregations for evenaged stands up to the age of natural maturity.

It should also be mentioned, that general tables can not always be applied to territories with severe climatic or edaphic (permafrost) conditions. For example, the modeling of fully stocked Larch stands in the Magadan region (extreme North-East of Russia), having basically relatively unevenaged age stand structure, showed a considerable difference in mortality with the mortality stands presented in the general tables for Larch. Thus, beginning from the age of 50 years, the estimated mortality for Larch stands in the Magadan region is considerably higher than for mortality given by the general tables (see Table 5).

\(^2\)The site index scale developed by Orlov is the site index classification used by Russian foresters since 1911 until the present.
Table 5. Mortality (m$^3$/ha) for Larch stands (M) in the Magadan region compared with the mortality according to the general tables for Larch (G) for different site classes

<table>
<thead>
<tr>
<th>AGE</th>
<th>Mortality for different site indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>M</td>
</tr>
<tr>
<td>50</td>
<td>4.47</td>
</tr>
<tr>
<td>70</td>
<td>3.85</td>
</tr>
<tr>
<td>90</td>
<td>2.77</td>
</tr>
<tr>
<td>110</td>
<td>1.83</td>
</tr>
<tr>
<td>130</td>
<td>1.61</td>
</tr>
<tr>
<td>150</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Therefore, additional mathematical methods are needed in order to provide regional estimates of net and gross growth and mortality.

To validate the estimation methods used for the general tables (Sagreev, et al., 1992) we used the same analysis procedure for the following species for various ecological and growing conditions;
- fir (Abies sibirica)
- hornbeem (Carpinus betulus)
- white acacia (Robinia pseudoacacia)
- gray alder (Alnus cinerea)
- lime (Tilia cordata)
- willow (Genera Salix)

As initial material for the modeling of these species we used local yield tables from central parts of the species range, which are considered by foresters as the most appropriate ones.

It has been recognized that forest stands can not be fully stocked for a long time and yield tables for "normal" stands should be considered to describe some kind of average dynamic for the more productive states of stands. Nevertheless, only such yield tables contain a description of the total productivity dynamics, and they are often the only source for estimation of the interdependence between total productivity and the growing stock in undisturbed stands.

The general tables used for the modeling of the growth of the major forest forming species are of rather good quality.
We used the five biometric indexes [height (H), diameter (D), basal area (BA), growing stock (GS), and total volume (TV)] which are included in most yield tables, for the analyses of the growth process in the fully-stocked stands. The other required indexes can be calculated from these five indicators.

As already illustrated, the growth functions for the five biometric indexes chosen have the following structure:

\[ F = c_1 \cdot (1 - \exp(-c_2 \cdot A))^{c_3}, \]

where \( F \) is the biometric index (H, D, BA, GS or TV), \( A \) is the age of a stand and

\[ c_1 = c_{13} \cdot N^2 + c_{12} \cdot N + c_{11} \]
\[ c_2 = c_{23} \cdot N^2 + c_{22} \cdot N + c_{21} \]
\[ c_3 = c_{33} \cdot N^2 + c_{32} \cdot N + c_{31} \]

where \( N \) is site index number. The site index numbers coincide with the site index scale of Orlov (1927, 1928). Number 1 corresponds to site index 1a in the site index scale of Orlov; number 2 corresponds to site index 1, etc.

### 3.2 Results of Modeling

The final results of the modeling include the following functions:

- net increment \( d\text{GS}(A,N) \);
- gross increment \( d\text{TV}(A,N) \);
- mortality \( M(A,N) = d\text{TV}(A,N) - d\text{GS}(A,N) \);
- percent, net increment \( p\text{GS}(A,N) = (d\text{GS}(A,N)/\text{GS}(A,N)) \times 100 \);
- percent, gross increment \( p\text{TV}(A,N) = (d\text{TV}(A,N)/\text{TV}(A,N)) \times 100 \);
- percent, mortality \( p\text{M}(A,N) = (M/\text{GS}) \times 100 \);
- ratio \( R(A,N) = d\text{TV}/d\text{GS} \).

It should be noted that such two-dimensional representations of the biometric indexes corresponds well with the existing general yield tables. The following two statistical indicators were used for estimation of the accuracy of the modeling: 1) normalized minimum squared residual \( R \) for the initial non-linear regressions over age by fixed site index numbers for all five biometric indexes

\[ R = 1 - (\text{Residual sum-of-squares}/\text{sum-of-squares}); \]

2) Normalized averaged squared residual \( R \) for the quadratic regression by site index numbers for fifteen coefficients of the Mitcherlich functions for all five biometric indexes.

The results presented in Appendix II have the following structure. All species are grouped into three major groups according to the Russian classification system: Coniferous species,
Hard Deciduous species and Soft Deciduous species. Each of the individual species in a group has:

1. code and corresponding name;
2. basic table source;
3. basic table author;
4. brief description of a table;
5. comments on the working span of the model and special mathematical methods employed (when used);
6. table of nine coefficients for each biometric index $H(10^*m), D(10^*cm), BA(10^*m^2), GS(m^3)$ and $TV(m^3)$;
7. indicators of accuracy

4. Conclusions

The results presented in Appendix II confirmed the ability of the Mitcherlich function to model the regularities of growth dynamics for fully stocked evenaged stands of all the major forest types in Russia.

Many interesting analyses of growth dynamics can be made based on the models presented. The ratios $dTV/dGS$ vary greatly depending on species, level of productivity and age. The comprehensive analysis of the global and local peculiarities in Russia as well as model development for different stocking densities and different types of age structures are goals for future analyses.
References


Sagreev, V. V.: 1978, *Geographical regularities of growth and productivity of stands*. Forest industry, Moscow, 247 pp. (In Russian)

Sagreev, V. V.: 1991, VNIILM, Moscow (personal communication).


APPENDIX 1

Definitions

$\textit{GS}(A)$ - growing stock at the age $A$, the total volume of stem wood overbark of all living trees in a stand, expressed in $\text{m}^3$ per hectare.

$\textit{TV}(A)$ - total volume (total production) by age $A$, total volume produced by all stem wood in a stand up to age $A$, expressed in $\text{m}^3$ per hectare.

$\textit{M}(A)$ - accumulated mortality, the accumulated volume of stem wood of trees which died of natural causes including wood removed under thinning of intensity less than natural mortality up to age $A$, expressed in $\text{m}^3$ per hectare. Hence,

$$\textit{TV}(A) = \textit{GS}(A) + \textit{M}(A).$$

$d\textit{TV}(A)$ - gross increment per year at age $A$, is defined as:

$$d\textit{TV}(A) = \textit{TV}(A) - \textit{TV}(A-1),$$

or

$$d\textit{TV}(A) = f'(A),$$

where $\textit{TV}(A) = f(A)$ is the functional expression for the total volume by age.

$d\textit{GS}(A)$ - net increment per year at age $A$, is defined as:

$$d\textit{GS}(A) = \textit{GS}(A) - \textit{GS}(A-1),$$

or

$$d\textit{GS}(A) = g'(A),$$

where $\textit{GS}(A) = g(A)$ is the functional expression for the growing stock by age.

$d\textit{M}(A)$ - natural mortality per year for age $A$ is the difference between gross and net increment

$$d\textit{M}(A) = d\textit{TV}(A) - d\textit{GS}(A).$$
**Gross increment** per year as a percent of total production to age $A$ can be approximated by:

$$P_{TV}(A) = \frac{dTV(A) \cdot 200}{TV(A) + TV(A - 1)},$$

or explicitly as:

$$P_{TV}(A) = \frac{f'(A) \cdot 100}{f(A)}.$$

**Net increment** per year as a percent of growing stock to age $A$ can be calculated in a similar manner.

The **average increment** $AI$ for a stand is defined as:

$$AI = \frac{GS_A}{A},$$

where $A$-age of a stand; $AI$ is aggregated by dominant species, etc.; calculated as an average weighted by area.

The **average diameter** $D$ is the quadratic mean at breast height (1.3m) of all trees in a stand.

The **average height** $H$ is the regression equation height estimate for a tree of diameter $D$.

**Basal area** $BA$ is expressed in $m^2$ per hectare.

**Density or stocking** is the ratio of a stand’s basal area to the basal area of a fully stocked stand.

**Site indexes** in Russia are determined by the average height at a base index stand age, usually 100 years.

**Type of age structure (TASS)** is a classification system of stands, which reflects the variation of age inside individual stands.

**General tables** are yield tables for fully stocked stands for a whole stand, dominated by a certain species. Apply to all of the former USSR. They are used as a general standard for various comparisons including forest inventories and forest management, if regional tables are not available.
Regional tables are yield tables generated for a region dominated by a certain species. There are different types of regional tables: for normal (or fully stocked stands); for modal (or not fully stocked stands), for different TASS; for specific goals of the forest management (so-called goal programs of forest regeneration); for forest plantations with different initial densities; etc. They are used by the local forest inventory and for forest management.

Growth tables (or models) are regional tables (or models) of dependency of net or/and gross growth for various stand characteristics.

Forested Area is a category of land which was used in the former USSR and is used in the countries of CIS. This is land covered by forests with stocking > 0.3 for young stands and stocking > 0.2 for the rest of the age groups. Stocking 1.0 is defined by yield tables or special "tables of basal area and growing stock under stocking 1.0." Temporary nonstocked forest land is separately accounted for as "unforested area". Lands, covered by shrubs, are included in Forested Area only if closed forests can not grow in this territory due to severe climatic conditions (extreme North, sub alpine mountain zone etc.).
APPENDIX II

RESULTS

1 CONIFEROUS SPECIES

101 PINE (Pinus sp.)

The general yield tables generated by Sagreev were used as input data.

Source: Sagreev et.al. pp 299-304 (1992)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10,160] for site indexes 1b to 3; Age [20,160] for the site indexes 4 to 5; Age [30,160] for site index 5a; Age [50,160] for site index 5b.

The valid range of the model: from 1b to 5b for site indexes and from 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th>Variable</th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>426.79896</td>
<td>-36.72878</td>
<td>-0.85074</td>
<td>0.02153</td>
<td>-0.00134</td>
<td>0.00013</td>
<td>1.244</td>
<td>-0.038</td>
<td>0.015</td>
</tr>
<tr>
<td>D</td>
<td>690.8517</td>
<td>-61.6043</td>
<td>-0.791</td>
<td>0.011865</td>
<td>-0.00047</td>
<td>0.000065</td>
<td>1.18853</td>
<td>0.01435</td>
<td>0.00482</td>
</tr>
<tr>
<td>BA</td>
<td>594.1988</td>
<td>-62.7612</td>
<td>2.162</td>
<td>0.040272</td>
<td>-0.003526</td>
<td>0.000162</td>
<td>1.13309</td>
<td>-0.02862</td>
<td>0.00199</td>
</tr>
<tr>
<td>GS</td>
<td>1019.0606</td>
<td>-162.7926</td>
<td>6.0478</td>
<td>0.027433</td>
<td>-0.002283</td>
<td>0.000154</td>
<td>2.0258</td>
<td>-0.0623</td>
<td>0.0131</td>
</tr>
<tr>
<td>TV</td>
<td>1839.98</td>
<td>-249.891</td>
<td>5.649</td>
<td>0.020669</td>
<td>-0.001531</td>
<td>0.00012</td>
<td>2.0269</td>
<td>-0.1132</td>
<td>0.0239</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

0.993

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

0.985
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY 

$\text{PM}(A,N) = \left( \frac{M}{GS} \right) \times 100$;

RATIO $\text{R}(A,N) = \frac{dTV}{dGS}$. 

Site Index
102 SPRUCE (*Picea* sp.)

The general yield tables generated by Sagreev were used as input data.

Source: Sagreev et.al. pp 304-308 (1992)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10,160] for site indexes 1b to 2; Age [20,160] for site indexes 3 to 5; Age [30,160] for site index 5a; Age [50,160] for site index 5a.

The working span of the model: from 1b to 5a for site indexes and from 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>449.333</td>
<td>-40.102</td>
<td>-0.545</td>
<td>0.0217</td>
<td>-0.000308</td>
<td>-0.0000089</td>
<td>1.6467</td>
<td>0.0546</td>
<td>0.0159</td>
</tr>
<tr>
<td>D</td>
<td>689.858</td>
<td>-83.128</td>
<td>1.311</td>
<td>0.01971</td>
<td>0.00023</td>
<td>0.00022</td>
<td>2.1328</td>
<td>-0.2074</td>
<td>0.0338</td>
</tr>
<tr>
<td>BA</td>
<td>723.341</td>
<td>-62.014</td>
<td>-0.807</td>
<td>0.03036</td>
<td>-0.003418</td>
<td>0.00033</td>
<td>1.4674</td>
<td>-0.2432</td>
<td>0.0504</td>
</tr>
<tr>
<td>GS</td>
<td>1477.422</td>
<td>-260.036</td>
<td>10.932</td>
<td>0.02493</td>
<td>-0.001106</td>
<td>0.0000032</td>
<td>2.8367</td>
<td>-0.004</td>
<td>0.0133</td>
</tr>
<tr>
<td>TV</td>
<td>2300.477</td>
<td>-295.427</td>
<td>1.866</td>
<td>0.02154</td>
<td>-0.000278</td>
<td>-0.000014</td>
<td>2.8018</td>
<td>0.1009</td>
<td>0.0121</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

$0.992$

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of Mitcherlich functions for all five biometric indexes:

$0.962$
PERCENT OF NET INCREMENT $PGS(A,N)$;

PERCENT OF GROSS INCREMENT $PTV(A,N)$;
PERCENT OF MORTALITY PM(A,N) = \( \frac{M}{GS} \times 100 \);

RATIO R(A,N) = \( \frac{dTV}{dGS} \).
103 FIR (*Abies* sp.)

The yield tables generated by M.I. Chaitovich for *Abies sibirica* in Eastern Kazakhstan were used as input data.


Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [20,120] for site indexes 1 to 4; Age [30,120] for site index 5.

The valid range of the model: from 1 to 5 for site indexes and 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>582.71</td>
<td>-78.118</td>
<td>-0.9305</td>
<td>0.01478</td>
<td>0.00094</td>
<td>0.0004</td>
<td>1.3829</td>
<td>-0.0301</td>
<td>0.0238</td>
</tr>
<tr>
<td>D</td>
<td>-46.108</td>
<td>433.091</td>
<td>-63.551</td>
<td>0.03714</td>
<td>-0.0149</td>
<td>0.00177</td>
<td>2.2591</td>
<td>-0.4463</td>
<td>0.0515</td>
</tr>
<tr>
<td>BA</td>
<td>603.3298</td>
<td>-59.9845</td>
<td>0.7432</td>
<td>0.01778</td>
<td>-0.00004</td>
<td>0.0005</td>
<td>1.1228</td>
<td>-0.2309</td>
<td>0.0655</td>
</tr>
<tr>
<td>GS</td>
<td>1231.437</td>
<td>-213.99</td>
<td>8.222</td>
<td>0.01217</td>
<td>0.00269</td>
<td>-0.00035</td>
<td>1.0777</td>
<td>0.5464</td>
<td>-0.06508</td>
</tr>
<tr>
<td>TV</td>
<td>1867.417</td>
<td>-99.321</td>
<td>-20.7548</td>
<td>0.03352</td>
<td>-0.00944</td>
<td>0.00095</td>
<td>2.72885</td>
<td>-0.1956</td>
<td>0.0123</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

0.989

2) Normalized average square residual (R) for the quadratic regression by site index number for forty five coefficients of Mitcherlich functions for all five biometric indexes:

0.901
PERCENT OF NET INCREMENT PGS(A,N):

PERCENT OF GROSS INCREMENT PTV(A,N):
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100; \)

RATIO \( R(A,N) = dTV/dGS. \)
104 LARCH (Larix sp.)

The general yield tables generated by N.V. Vyvodzev were used as input data.


Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [20,220] for site indexes 1a to 5a.

The valid range of the model: from 1a to 5a for site indexes and 20 to 220 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>450.781</td>
<td>-42.0708</td>
<td>-0.318</td>
<td>0.01707</td>
<td>-0.00058</td>
<td>0.000014</td>
<td>1.0069</td>
<td>-0.02211</td>
<td>0.0091</td>
</tr>
<tr>
<td>D</td>
<td>540.813</td>
<td>-33.845</td>
<td>-1.673</td>
<td>0.01178</td>
<td>-0.00033</td>
<td>0.000071</td>
<td>1.0675</td>
<td>0.0154</td>
<td>0.00085</td>
</tr>
<tr>
<td>BA</td>
<td>529.71</td>
<td>-26.3952</td>
<td>-1.4038</td>
<td>0.05702</td>
<td>-0.01188</td>
<td>0.000785</td>
<td>1.5117</td>
<td>-0.23683</td>
<td>0.02231</td>
</tr>
<tr>
<td>GS</td>
<td>931.045</td>
<td>-122.277</td>
<td>1.88157</td>
<td>0.038593</td>
<td>-0.00666</td>
<td>0.00044</td>
<td>2.68295</td>
<td>-0.43777</td>
<td>0.04684</td>
</tr>
<tr>
<td>TV</td>
<td>1523.999</td>
<td>-242.695</td>
<td>8.16452</td>
<td>0.026492</td>
<td>-0.00191</td>
<td>-0.000024</td>
<td>2.21716</td>
<td>-0.14478</td>
<td>0.01706</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

0.993

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

0.977
PERCENT OF NET INCREMENT PGS(A,N);

Larch General pGS

PERCENT OF GROSS INCREMENT PTV(A,N);

Larch General pTV
PERCENT OF MORTALITY PM(A,N) = (M/GS) * 100;

RATIO R(A,N) = dTV/dGS.
105 CEDAR (Pinus sibirica)

Source and Author: Semtchkin, I.V. (Ed.), 1993, Table 5.37, pp. 275-280.

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [40,240] for site indexes 1a to 5, Age [60,240] for site index 5a, Age [80,240] for site indexes 5b and 5c.

The valid range of the model: from 1a to 5c for site indexes and from 20 to 250 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>474.9722</td>
<td>-42.9302</td>
<td>-0.0595</td>
<td>0.00896</td>
<td>-0.00013</td>
<td>0.000062</td>
<td>1.0454</td>
<td>0.005012</td>
<td>0.01413</td>
</tr>
<tr>
<td>D</td>
<td>1013.006</td>
<td>-160.1949</td>
<td>7.39125</td>
<td>0.010321</td>
<td>-0.000311</td>
<td>0.000118</td>
<td>1.67964</td>
<td>-0.08186</td>
<td>0.05693</td>
</tr>
<tr>
<td>BA</td>
<td>674.1937</td>
<td>-16.64741</td>
<td>-3.1348</td>
<td>0.019271</td>
<td>-0.00171</td>
<td>0.000306</td>
<td>2.18128</td>
<td>-0.71424</td>
<td>0.21457</td>
</tr>
<tr>
<td>GS</td>
<td>1232.217</td>
<td>-131.2741</td>
<td>-0.0788</td>
<td>0.012057</td>
<td>0.000964</td>
<td>0.000021</td>
<td>2.0033</td>
<td>0.37817</td>
<td>0.04183</td>
</tr>
<tr>
<td>TV</td>
<td>2104.71</td>
<td>-237.4296</td>
<td>1.018774</td>
<td>0.006532</td>
<td>-0.0012</td>
<td>-0.000026</td>
<td>1.54121</td>
<td>0.251893</td>
<td>0.03818</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

0.988

2) Normalized average square residual (R) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

0.985
PERCENT OF NET INCREMENT PGS(A,N);

Ceder General pGS

PERCENT OF GROSS INCREMENT PTV(A,N);

Ceder General pTV
PERCENT OF MORTALITY $PM(A,N) = (M/GS) \times 100$;

RATIO $R(A,N) = dTV/dGS$.
2. HARD DECIDUOUS SPECIES

110 OAK OF SEEDS ORIGIN (*Quercus sp.*)

The basic data are from yield tables generated by A.D.Dudarev, designed in line with the Sagreev methodology of growth types for the central part of Russia and Ukraine.

Source: Sagreev pp 321-325 (1978)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [20, 160] for site indexes 1b and 4, Age [20, 180] for site indexes 1a and 3, Age [20, 200] for site indexes 1 and 2.

The valid range of the model: from 1a to 5 for site indexes and 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>429.5829</td>
<td>-35.14</td>
<td>-1.6982</td>
<td>0.01957</td>
<td>-0.0018</td>
<td>0.00043</td>
<td>1.17594</td>
<td>-0.07154</td>
<td>0.027</td>
</tr>
<tr>
<td>D</td>
<td>1491.06</td>
<td>-377.571</td>
<td>30.0181</td>
<td>0.00414</td>
<td>0.0005</td>
<td>0.00036</td>
<td>1.1226</td>
<td>-0.04584</td>
<td>0.03605</td>
</tr>
<tr>
<td>BA</td>
<td>539.0115</td>
<td>-37.5864</td>
<td>-1.52683</td>
<td>0.01656</td>
<td>-0.001441</td>
<td>0.00044</td>
<td>0.75661</td>
<td>-0.05515</td>
<td>0.01746</td>
</tr>
<tr>
<td>GS</td>
<td>1014.139</td>
<td>-140.12</td>
<td>1.249381</td>
<td>0.01789</td>
<td>-0.00092</td>
<td>0.00026</td>
<td>1.63895</td>
<td>-0.03977</td>
<td>0.0267</td>
</tr>
<tr>
<td>TV</td>
<td>1722.822</td>
<td>-290.004</td>
<td>8.41103</td>
<td>0.01656</td>
<td>-0.00045</td>
<td>0.00014</td>
<td>1.79675</td>
<td>0.004101</td>
<td>0.01287</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes: 0.963

2) Normalized average square residual (R) for the quadratic regression by site index number for forty-five coefficients of the Mitcherlich functions for all five biometric indexes: 0.955
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY $PM(A,N) = (M/GS) * 100$;

RATIO $R(A,N) = dTV/dGS$. 
111 OAK OF VEGETATIVE ORIGIN (Quercus sp.)

The basic data are from yield tables generated by A.D. Dudarev, designed in line with the Sagreev methodology of growth types for the central part of Russia and Ukraine.

Source: Sagreev pp 325-329 (1978)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10, 120] for site indexes 1 and 2, Age [10, 110] for site index 3, Age [10, 100] for site index 4, Age [10, 90] for site index 5.

The valid range of the model: from 1a to 5 for site indexes and 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>317.3517</td>
<td>-40.1756</td>
<td>-0.01128</td>
<td>0.02633</td>
<td>-0.000103</td>
<td>0.00011</td>
<td>1.11336</td>
<td>0.008137</td>
<td>0.02077</td>
</tr>
<tr>
<td>D</td>
<td>464.4327</td>
<td>-53.851</td>
<td>4.98636</td>
<td>0.01825</td>
<td>-0.00025</td>
<td>0.000186</td>
<td>1.23454</td>
<td>0.006646</td>
<td>0.01211</td>
</tr>
<tr>
<td>BA</td>
<td>339.2348</td>
<td>-24.4679</td>
<td>-1.89439</td>
<td>0.02763</td>
<td>-0.00059</td>
<td>0.00062</td>
<td>0.59</td>
<td>-0.01706</td>
<td>0.02565</td>
</tr>
<tr>
<td>GS</td>
<td>694.977</td>
<td>-113.99</td>
<td>4.0308</td>
<td>0.02493</td>
<td>0.000274</td>
<td>0.000039</td>
<td>1.48753</td>
<td>-0.00487</td>
<td>0.01293</td>
</tr>
<tr>
<td>TV</td>
<td>985.228</td>
<td>-201.526</td>
<td>13.3814</td>
<td>0.02076</td>
<td>0.00216</td>
<td>0.000206</td>
<td>1.66343</td>
<td>0.090684</td>
<td>0.0088</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for all five biometric indexes:

\[ R = 0.974 \]

2) Normalized averaged square residual (R) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

\[ R = 0.967 \]
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY PM(A,N)=(M/GS)*100;

RATIO R(A,N)=dTV/dGS.
112 BEECH (*Fagus sp.*)

The general yield tables generated by V.P.Zakutin were used as input data.

Source: Sagreev et.al. pp 329-332 (1992)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10, 160] for site indexes 1b to 4.

The valid range of the model: from 1a to 5 for site indexes and 10 to 200 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>H</strong></td>
<td>437.67</td>
<td>-37.7016</td>
<td>-1.22129</td>
<td>0.02011</td>
<td>-0.0003</td>
<td>0.0000307</td>
<td>1.17055</td>
<td>0.114394</td>
<td>-0.0051</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>611.3511</td>
<td>-57.3567</td>
<td>1.50414</td>
<td>0.01614</td>
<td>-0.00039</td>
<td>0.000071</td>
<td>1.5266</td>
<td>0.11308</td>
<td>-0.0032</td>
</tr>
<tr>
<td><strong>BA</strong></td>
<td>502.631</td>
<td>-63.8714</td>
<td>3.9828</td>
<td>0.031</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8168</td>
<td>0.06546</td>
<td>0.00268</td>
</tr>
<tr>
<td><strong>GS</strong></td>
<td>935.95</td>
<td>-176.806</td>
<td>9.6602</td>
<td>0.02274</td>
<td>-0.00001</td>
<td>0.000016</td>
<td>1.7885</td>
<td>0.2004</td>
<td>-0.00664</td>
</tr>
<tr>
<td><strong>TV</strong></td>
<td>2012.167</td>
<td>-432.796</td>
<td>28.2166</td>
<td>0.01531</td>
<td>0.00049</td>
<td>-0.0000061</td>
<td>1.51631</td>
<td>0.190436</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes: 0.959

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes: 0.881
PERCENT OF NET INCREMENT $\text{PGS}(A,N)$;

Beech General $\text{pGS}$

PERCENT OF GROSS INCREMENT $\text{PTV}(A,N)$;

Beech General $\text{pTV}$
PERCENT OF MORTALITY PM(A,N) = (M/GS) * 100;

Beech General pM

RATIO R(A,N) = dTV/dGS.

Beech General R
113 HORNBEAM (*Carpinus sp.*)

Yield tables designed by K.E. Nikitin for hornbeam stands of seed origin in the Ukraine were used as input data.

Source: Shvidenko et al. (1987)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10, 80] for site indexes 1 to 3.

The valid range of the model: from 1 to 5 for site indexes and 10 to 150 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>341.25</td>
<td>-37.699</td>
<td>0.279</td>
<td>0.0286</td>
<td>-0.00009</td>
<td>0.000275</td>
<td>1.0675</td>
<td>0.0695</td>
<td>0.0055</td>
</tr>
<tr>
<td>D</td>
<td>375.744</td>
<td>-45.118</td>
<td>0.2613</td>
<td>0.01856</td>
<td>0.007235</td>
<td>-0.000625</td>
<td>0.7794</td>
<td>0.3833</td>
<td>0.0005</td>
</tr>
<tr>
<td>BA</td>
<td>458.906</td>
<td>-49.104</td>
<td>0.705</td>
<td>0.0308</td>
<td>0.00274</td>
<td>-0.0002</td>
<td>0.8995</td>
<td>0.1855</td>
<td>-0.0065</td>
</tr>
<tr>
<td>GS</td>
<td>640.941</td>
<td>-88.093</td>
<td>-1.426</td>
<td>0.03026</td>
<td>-0.00168</td>
<td>0.0003</td>
<td>2.326</td>
<td>-0.1034</td>
<td>0.02625</td>
</tr>
<tr>
<td>TV</td>
<td>963.579</td>
<td>-185.508</td>
<td>9.431</td>
<td>0.0266</td>
<td>0.00049</td>
<td>-0.000125</td>
<td>1.921</td>
<td>0.000485</td>
<td>0.000125</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes:

0.9995

2) Normalized average square residual (R) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

0.9991
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY $PM(A,N) = \frac{M}{GS} \times 100$;

$R(A,N) = \frac{dTV}{dGS}$.
121 WHITE ACACIA (*Robinia pseudacacia*)

Yield tables designed by M.V. Davidov for acacia stands of seed origin in the Ukraine were used as input data.

Source: Shvidenko et al. (1987)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [5, 50] for site index 1c, Age [5, 40] for site indexes 1b and 1a, Age [10, 30] for site index 1c.

The valid range of the model: from 1c to 2 for site indexes 5 to 80 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>266.96</td>
<td>-35.258</td>
<td>10.158</td>
<td>0.04239</td>
<td>0.003285</td>
<td>-0.002325</td>
<td>1.199</td>
<td>0.04554</td>
<td>0.0451</td>
</tr>
<tr>
<td>D</td>
<td>470.828</td>
<td>-106.58</td>
<td>-0.834</td>
<td>0.0268</td>
<td>0.00525</td>
<td>0.00325</td>
<td>1.2983</td>
<td>0.1803</td>
<td>0.0845</td>
</tr>
<tr>
<td>BA</td>
<td>311.643</td>
<td>-15.7181</td>
<td>1.7665</td>
<td>0.07393</td>
<td>0.001332</td>
<td>-0.002912</td>
<td>1.8554</td>
<td>0.5554</td>
<td>-0.03775</td>
</tr>
<tr>
<td>GS</td>
<td>304.4913</td>
<td>-37.793</td>
<td>-1.415</td>
<td>0.05862</td>
<td>-0.00251</td>
<td>0.00102</td>
<td>2.4805</td>
<td>0.3865</td>
<td>0.12752</td>
</tr>
<tr>
<td>TV</td>
<td>507.412</td>
<td>-48.598</td>
<td>-18.3</td>
<td>0.05043</td>
<td>-0.00768</td>
<td>0.00374</td>
<td>2.3859</td>
<td>-0.0659</td>
<td>0.2825</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions by ages with fixed site index numbers from all of the five biometric indexes

R = 0.998

2) Normalized squared residual (R) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for all five biometric indexes:

R = 0.985
PERCENT OF NET_INCREMENT PGS(A,N):

PERCENT OF GROSS_INCREMENT PTV(A,N):
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100 \);

RATIO \( R(A,N) = \frac{dT}{dGS} \).
3. SOFT DECIDUOUS SPECIES

124 BIRCH (Betula sp.)

The general yield tables generated by N.I. Salikov were used as input data.

Source: Sagreev et.al. pp 315-318 (1992)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [10, 100] for site indexes 1a to 5.

The valid range of the model: from 1a to 5 for site indexes and 10 to 105 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>359.422</td>
<td>-39.6944</td>
<td>-0.03371</td>
<td>0.02522</td>
<td>-0.000147</td>
<td>0.000019</td>
<td>0.8714</td>
<td>0.16905</td>
<td>0.00009</td>
</tr>
<tr>
<td>D</td>
<td>410.3517</td>
<td>-53.5466</td>
<td>0.4193</td>
<td>0.02467</td>
<td>-0.000014</td>
<td>0.00014</td>
<td>1.3822</td>
<td>0.08775</td>
<td>0.01165</td>
</tr>
<tr>
<td>BA</td>
<td>389.213</td>
<td>-38.203</td>
<td>-1.2226</td>
<td>0.02469</td>
<td>-0.00181</td>
<td>0.00124</td>
<td>0.7029</td>
<td>-0.03655</td>
<td>0.05309</td>
</tr>
<tr>
<td>GS</td>
<td>585.333</td>
<td>-117.458</td>
<td>5.4448</td>
<td>0.02497</td>
<td>-0.00016</td>
<td>0.000061</td>
<td>1.4184</td>
<td>0.18967</td>
<td>0.00473</td>
</tr>
<tr>
<td>TV</td>
<td>900.72</td>
<td>-172.228</td>
<td>7.4892</td>
<td>0.02218</td>
<td>0.000024</td>
<td>1.5663</td>
<td>0.21614</td>
<td>-0.00022</td>
<td></td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual \( R \) for the initial non-linear regressions over age with fixed site index numbers for five biometric indexes:

\[ R = 0.936 \]

2) Normalized average square residual \( R \) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes:

\[ R = 0.909 \]
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY PM(A,N) = (M/GS) * 100;

RATIO R(A,N) = \( \frac{dTV}{dGS} \).
125 ASPEN (Populus tremula)

The general yield tables generated by V.S. Cherniavskii were used as input data.

Source: Sagreev et.al. pp 319-321 (1992)

Description of table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [5, 100] for the site indexes 1a to 5.

The valid range of the model: from 1a to 5 for site indexes and 10 to 105 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>376.251</td>
<td>-33.6703</td>
<td>-0.024</td>
<td>0.02392</td>
<td>-0.001158</td>
<td>0.0000006</td>
<td>0.8668</td>
<td>0.1667</td>
<td>0.002854</td>
</tr>
<tr>
<td>D</td>
<td>580.127</td>
<td>-63.115</td>
<td>-1.6158</td>
<td>0.01287</td>
<td>-0.000086</td>
<td>-0.000015</td>
<td>1.1177</td>
<td>0.01989</td>
<td>-0.0026</td>
</tr>
<tr>
<td>BA</td>
<td>442.096</td>
<td>-38.071</td>
<td>0.3362</td>
<td>0.0308</td>
<td>-0.001605</td>
<td>0.00012</td>
<td>0.7406</td>
<td>0.00552</td>
<td>0.00077</td>
</tr>
<tr>
<td>GS</td>
<td>673.6207</td>
<td>-95.4629</td>
<td>0.9502</td>
<td>0.029</td>
<td>-0.003675</td>
<td>-0.0004</td>
<td>1.60536</td>
<td>-0.0621</td>
<td>0.015284</td>
</tr>
<tr>
<td>TV</td>
<td>989.97</td>
<td>-203.766</td>
<td>13.2536</td>
<td>0.03097</td>
<td>-0.000938</td>
<td>0.000056</td>
<td>1.91102</td>
<td>-0.03408</td>
<td>0.009124</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (R) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes:

   0.979

2) Normalized average square residual (R) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes:

   0.963
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100; \)

RATIO \( R(A,N) = \frac{dT_V}{dGS} \)

Aspen General \( pM \)

Aspen General \( R \)
126 GRAY ALDER (*Alnus cinerea*)

Yield tables designed by I.D. Iurkevich and V.I. Porfenov for gray alder stands of seeds origin in Belorussia were used as input data.

Source: Zacharov et al., p 122 (1967)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [5, 50] for site indexes 1 to 3.

The valid range of the model: from 1a to 4 for site indexes and 5 to 80 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>299.085</td>
<td>-43.5615</td>
<td>2.5473</td>
<td>0.02726</td>
<td>0.012145</td>
<td>-0.00013</td>
<td>0.73245</td>
<td>0.05405</td>
<td>0.03375</td>
</tr>
<tr>
<td>D</td>
<td>223.114</td>
<td>-8.493</td>
<td>-1.104</td>
<td>0.1148</td>
<td>-0.01895</td>
<td>0.00165</td>
<td>2.6088</td>
<td>-0.4543</td>
<td>0.0745</td>
</tr>
<tr>
<td>BA</td>
<td>319.33</td>
<td>-5.0605</td>
<td>0.3425</td>
<td>0.2287</td>
<td>-0.07812</td>
<td>0.00808</td>
<td>1.7648</td>
<td>-0.3898</td>
<td>0.0395</td>
</tr>
<tr>
<td>GS</td>
<td>420.085</td>
<td>-63.677</td>
<td>3.732</td>
<td>0.0589</td>
<td>0.000745</td>
<td>0.000175</td>
<td>1.6321</td>
<td>-0.2491</td>
<td>0.0865</td>
</tr>
<tr>
<td>TV</td>
<td>493.12</td>
<td>-45.679</td>
<td>-0.5845</td>
<td>0.08137</td>
<td>-0.02417</td>
<td>0.00425</td>
<td>1.99</td>
<td>-0.5399</td>
<td>0.1285</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes:

$$0.998$$

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes:

$$0.997$$
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100; \)

\[ \begin{array}{c|c|c|c|c|c|c|c|c|c} \hline Site & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\ \hline Index & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline \end{array} \]

RATIO \( R(A,N) = \frac{dT_V}{dGS}. \)
126 BLACK ALDER (*Alnus nigra*)

The general yield tables generated by A.V. Tiurin were used as input data.

Source: Sagreev et.al. pp 333-334 (1992)

Description of table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [5, 60] for site index 1a, Age [5, 80] for site index 1 and 2, Age [5, 60] for site index 3.

The valid range of the model: from 1a to 3 for site indexes and 10 to 105 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>279.17</td>
<td>1.4425</td>
<td>-9.236</td>
<td>0.0433</td>
<td>-0.0158</td>
<td>0.00416</td>
<td>1.0445</td>
<td>-0.1788</td>
<td>0.0644</td>
</tr>
<tr>
<td>D</td>
<td>519.618</td>
<td>-191.858</td>
<td>25.64</td>
<td>0.01632</td>
<td>-0.0063</td>
<td>0.00012</td>
<td>1.0713</td>
<td>-0.02213</td>
<td>0.030112</td>
</tr>
<tr>
<td>BA</td>
<td>423.596</td>
<td>30.602</td>
<td>-25.054</td>
<td>0.04032</td>
<td>-0.019442</td>
<td>0.00545</td>
<td>1.0884</td>
<td>-0.2328</td>
<td>0.078107</td>
</tr>
<tr>
<td>GS</td>
<td>631.01</td>
<td>102.81</td>
<td>-59.51</td>
<td>0.03463</td>
<td>0.04887</td>
<td>0.0265</td>
<td>1.8142</td>
<td>-1.2002</td>
<td>-0.654</td>
</tr>
<tr>
<td>TV</td>
<td>957.005</td>
<td>126.7357</td>
<td>-93.555</td>
<td>0.04907</td>
<td>-0.023047</td>
<td>0.00389</td>
<td>1.7635</td>
<td>-0.3642</td>
<td>0.1</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes:

0.978

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes:

0.977
PERCENT OF NET INCREMENT PGS(A,N):

![Net Increment Graph](image1)

PERCENT OF GROSS INCREMENT PTV(A,N):

![Gross Increment Graph](image2)
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100; \)

RATIO \( R(A,N) = dTV/dGS. \)
130  LIME (Tilia sp.)

Two yield tables were used for the design of the growth functions. Table 1 was prepared by V.B.Kozlovskii and V.M. Pavlov for lime stands of seed origin in the Tula region. Table 2 was generated by E.S. Murtachtanov for lime stands in the Middle Volga region.

Source: Gusev, pp 292-297 (1993)

Comment: The general table was constructed basically from two tables called Table 1 and Table 2. Table 1 covered four site indexes but only for total volume. Table 2 contained total and net volume only for two site indexes. The following method was used to construct net volume estimates for all four site indexes that are consistent with the total volumes given in Table 1. Regression analysis was used to estimate Mitcherlich coefficients of TV and GS, for Table 2. Then the ratio \( R = \frac{dTV}{dGS} \) was constructed for two site indexes from Table 2, using the coefficients of the Mitcherlich functions. The ratio \( R \) was estimated for neighboring site indexes, using simple relations of distances between functions of \( R \) for the 4 site indexes. After, multivariate regression estimate of \( dGS \) for four site indexes of Table 1, it was possible to construct table values of \( dTV \) for these site indexes, by employing the hypothesis that \( R \) values are the same as in the Table 2. These table values of \( dTV \) functions for different site indexes were analyzed using non-linear regressions. Coefficients of the Mitcherlich function for the total volume for Table 1 were estimated. The new ratios \( R \) for Table 1 were used to construct new TV of Table 2 in order to correct recurrent relations between \( R \) in Table 2. Such iterative procedure gave us satisfactory results for both tables. Table 1 was used as a basis for the general table.

Description of input Table 1: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS); Age [20, 200] for site indexes 1 to 4.

Description of input Table 2: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume TV; Age [10, 150] for site indexes 2 and 3.

The valid range of the model: from 1a to 4 for site indexes and age 10 to 250 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>455.615</td>
<td>-44.609</td>
<td>-0.1425</td>
<td>0.0219</td>
<td>0.0</td>
<td>0.0</td>
<td>1.503</td>
<td>0.032</td>
<td>-0.004</td>
</tr>
<tr>
<td>D</td>
<td>777.971</td>
<td>-65.851</td>
<td>-2.013</td>
<td>0.0117</td>
<td>0.0</td>
<td>0.0</td>
<td>1.462</td>
<td>-0.023</td>
<td>0.004</td>
</tr>
<tr>
<td>BA</td>
<td>655.685</td>
<td>-68.752</td>
<td>2.165</td>
<td>0.0182</td>
<td>-0.001945</td>
<td>0.000535</td>
<td>0.9286</td>
<td>-0.13805</td>
<td>0.02005</td>
</tr>
<tr>
<td>GS</td>
<td>1239.918</td>
<td>-204.054</td>
<td>7.867</td>
<td>0.0194</td>
<td>-0.001871</td>
<td>0.000271</td>
<td>1.9602</td>
<td>-0.1412</td>
<td>0.00975</td>
</tr>
<tr>
<td>TV</td>
<td>1322</td>
<td>-100</td>
<td>0.0</td>
<td>0.02147</td>
<td>-0.00274</td>
<td>0.0000911</td>
<td>1.9</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>
1) Normalized minimum square residual ($R$) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes:

$0.992$

2) Normalized average square residual ($R$) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes:

$0.995$
PERCENT OF NET INCREMENT PGS(A,N);

PERCENT OF GROSS INCREMENT PTV(A,N);
PERCENT OF MORTALITY \( PM(A,N) = (M/GS) \times 100; \)

RATIO \( R(A,N) = dTV/dGS. \)
132 WILLOW (*Salix sp.*)

Yield tables designed by M.V. Davidov for willow stands of seed origin in the Ukraine, were used as input data.

Source: Shvidenko et. al., pp. 276-279 (1987)

Description of input table: Characteristics: height (H), diameter (D), basal area (BA), growing stock (GS), total volume (TV); Age [5, 30] for site indexes 1e to 1c, Age [5, 40] for site indexes 1b and 1a.

The valid range of the model: from 1e to 2 for site indexes and 10 to 70 years for age.

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>c11</th>
<th>c12</th>
<th>c13</th>
<th>c21</th>
<th>c22</th>
<th>c23</th>
<th>c31</th>
<th>c32</th>
<th>c33</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>286.66</td>
<td>-21.692</td>
<td>-1.873</td>
<td>0.03164</td>
<td>-0.000865</td>
<td>0.001225</td>
<td>1.0506</td>
<td>0.101</td>
<td>-0.00043</td>
</tr>
<tr>
<td>D</td>
<td>1069.348</td>
<td>-225.265</td>
<td>-32.887</td>
<td>0.01184</td>
<td>0.00399</td>
<td>0.00125</td>
<td>1.2087</td>
<td>0.1452</td>
<td>0.028</td>
</tr>
<tr>
<td>BA</td>
<td>451.556</td>
<td>-32.105</td>
<td>-3.97</td>
<td>0.04976</td>
<td>0.00216</td>
<td>0.0002</td>
<td>1.3359</td>
<td>0.2374</td>
<td>0.02825</td>
</tr>
<tr>
<td>GS</td>
<td>504.206</td>
<td>-18.5505</td>
<td>2.975</td>
<td>0.04332</td>
<td>-0.005189</td>
<td>-0.00098</td>
<td>2.164</td>
<td>0.104</td>
<td>-0.0528</td>
</tr>
<tr>
<td>TV</td>
<td>754.906</td>
<td>-47.038</td>
<td>-12.652</td>
<td>0.037</td>
<td>-0.0037</td>
<td>0.00258</td>
<td>2.0823</td>
<td>0.2363</td>
<td>0.0798</td>
</tr>
</tbody>
</table>

1) Normalized minimum square residual (*R*) for the initial non-linear regressions over age with fixed site index numbers for the five biometric indexes: 0.999

2) Normalized average square residual (*R*) for the quadratic regression by site index number for forty five coefficients of the Mitcherlich functions for the five biometric indexes: 0.997
PERCENT OF NET INCREMENT PGS(A,N);

![Graph of Willow General PGS](image)

PERCENT OF GROSS INCREMENT PTV(A,N);

![Graph of Willow General PTV](image)
PERCENT OF MORTALITY PM(A,N) = (M/GS) * 100;

RATIO R(A,N) = dTV/dGS.