

International Institute for Applied Systems Analysis • A-2361 Laxenburg • Austria Tel: +43 2236 807 • Fax: +43 2236 71313 • E-mail: info@iiasa.ac.at • Web: www.iiasa.ac.at

INTERIM REPORT

IR-97-064/Sept

A Theoretical Consideration on the Land-use Change Model for the Japan Case Study Area

Teitaro Kitamura Masaru Kagatsume Satoshi Hoshino (shosino@ccews2.cc.okayama-u.ac.jp) Hidenori Morita (morita@iiasa.ac.at)

Approved by

Approved by

Günther Fischer (fisher@iiasa.ac.at)
Leader, Modeling Land-Use and Land-Cover Changes
in Europe and Northern Asia Project

Table of Contents

Acknowledgements	iii
1. Introduction	1
2. Background Information on the Japan Model	2
2.1. Land Use Problems in Japan	2
2.2. Unit of Analysis – Importance of Municipality	3
3. Theoretical Framework of the Land-Use Change Model	4
3.1. Basic Structure of the Regional Models	4
3.2. Linkage Model	6
4. Operational Modification for the Pilot Study Area	6
4.1. Modifications for a Simplified Model	6
4.2. Structure of the Kansai Land-Use Change Model	9
5. Procedures and Techniques of the Analysis	10
5.1. Land-Use Analysis	10
5.2. Land-Use Ratio Function f	11
5.3. Driving-Force Prediction Model g	11
5.4. Simulation and Policy Evaluation	12
6. Prospect of This Model Study	12
6.1. Development of the Linkage Model	12
6.2. Estimation of the Explanatory Variables	12
References	13
Appendix	14
A. Canonical Correlation Analysis	14
B. Multinominal Logit Model	14
C. Kane's SIMulation (KSIM) Method	15

Acknowledgments

The authors would like to express their sincere thanks to Dr. Günther Fisher, project leader, Land Use Change project, IIASA, for his encouragement and many valuable comments on earlier drafts of this paper. Furthermore we deeply thank Dr. Kuninori Ootsubo, NIES, the present LU/GEC project leader, for his encouragement extended to us. We deeply thank Ms. Cynthia Enzlberger-Vaughan, LUC project, IIASA, for her kind assistance extended to us.

Authors

Teitaro Kitamura Tokyo University of Agriculture, NODAI Research Institute, Tokyo 156,

Japan;

Masaru Kagatsume Kyoto University, Faculty of Agriculture, Kyoto, 606-01, Japan;

Satoshi Hoshino Okayama University, Faculty of Agriculture, Okayama, 700, Japan, and

alumni, IIASA LUC project, and

Hidenori Morita IIASA LUC project, and Kagawa University, Faculty of Agriculture,

Miki-cho, Kagawa, 761-07, Japan.

1 Introduction

The Japanese case study in the IIASA Land-Use Change (LUC) project aims at investigating local mechanisms of land-use change in the selected case study region and proposing local policy options suitable for local environmental conditions. This paper elaborates on the methodology of a Japanese land-use change model. In Section 2, we first present some background information related to the Japanese model where some premises of the model will be explained. The theoretical framework of the model for projecting major land-use changes is described in Section 3. Next, we discuss the implementation of the model and some modifications required for practical application. Some specific procedures and methodological techniques are also explained here. In the final section, some future tasks will be mentioned.

The Japanese land-use change model discussed here, was initially developed in a research project "Land Use and Global Environment Conservation" (LU/GEC, 1995-1997) sponsored by the National Institute for Environmental Studies, Japan [Kitamura et al., 1996]. This paper is a completely revised version of the original research output. The LU/GEC model is carefully reconsidered and modified for the Japan case study of the IIASA LUC project.

2 Background information on the Japan Model

2.1 Land-use problems in Japan

In general terms, land-use change can be discussed at two levels. One deals with major land-use changes which are related to the competition between broad land-use categories, and the other level concerns competition of land use types within the same major category¹.

The main land-use issues in Japan after World War II to the present are shown in Table 1. The most urgent land-use problems before 1960 were caused by the increase of food production. How to expand farmland and how to raise the agricultural productivity were the main criteria for land-use change because of the food shortages in the postwar period. During the high economic growth period from 1960 to 1975, urbanization and industrialization processes expanded throughout Japan and the population was rapidly concentrating in urban areas.

Table 1. Land-use issues in Japan after 1945.

Year	1945-1960	1960-1975	1975-
Period	◆Period of economic recovery	◆Period of high economic growth	◆Period of moderate economic growth
Related social change	Gentle urbanization	 Intensive urbanization Preparation of social infrastructure Development of motorization Establishment of legal land-use control Severe depopulation and overpopulation Development of non-agricultural labor market 	 Mature urbanization Trade liberalization Advancement of social infrastructure Advancement of transportation means Shortage and aging of agricultural labor
Role of rural area expected by the outside	Supply of food Absorption of demobilized labor force	Supply of labor force to non-agri. Sectors Supply of land to non-agricultural sectors	Land conservation Maintenance of environmental quality
Land-use issues	Use competition between farm- and grassland vs. forest land	Use competition between farmland and urban land and forest land Abandonment of farmland Soil deterioration by continuous planting and use of agricultural chemicals	Use competition between farmland and urban land Abandonment of farmland and forest land Reorganization of producing district / production adjustment in condition of excess supply

Also in this period (1960-1975), overpopulation in the metropolitan areas, depopulation in the mountain areas as well as water and atmospheric pollution caused serious social problems. Especially, control of urban expansion and preservation of good farmland were the greatest land-use

2

¹ For example, urban land-use sprawling into farmland is an example of a major land-use change; replacing rice paddies with vegetable production is an example of a minor land-use change.

problems at that time. These problems are still important now. Expansion of abandoned farmland was also actualized, because of the shortage and aging of the agricultural labor force.

At the same time, land-use problems within major land-use categories, such as soil deterioration of agricultural land caused by continuous planting and high input levels of agricultural chemicals, occured in major producing districts. Also, production adjustments of major crops due to excess supply, declining producer prices due to weakening of price-support policies, and the recent trade liberalization, etc. were instrumental in causing changes in agriculture in the post-1975 period.

It is desirable to build a model which is applicable for both types of land-use change, land-cover conversion between major land-use categories and land-cover modification within such classes (Turner et al., 1995). In this study we mainly focus on land conversion, i.e., land-use change between major land use categories which are often irreversible. Therefore, our land-use change model deals with major land-use categories.

2.2 Unit of analysis - importance of municipality

We chose the municipality as the unit of analysis instead of other units based on homogeneity of socio-economic or geographical conditions. The model parameters are estimated statistically from a database organized at municipality level and the model allows to project future land-use for both the whole-region and at municipality level. We consider the municipality to be the most appropriate unit of land use in our research. The land-use change model is expected to provide information on future land-use that is useful for the municipal government. The reasons are as follows.

- The municipality is the finest administrative unit which can integrate individual policy measures: As shown in Figure 1, a great variety of policy measures are provided from the national government through the individual ministries and agencies. The municipality is the "saucer" of the policy measures provided by the government. These measures should be carefully chosen and well integrated according to the actual needs of the local society by the municipality. The municipal government is the lowest administrative body which has a public decision-making ability and can integrate and implement the policy measures.
- Most socio-economic data are available only at municipality level: Most socio-economic statistics are compiled and published by the municipality unit. In our land-use change model, the socio-economic dimension is quite important. Therefore, municipality is suitable from a view point of data availability and handling.

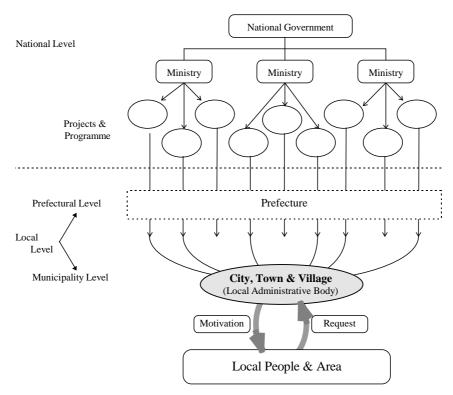


Figure 1. The municipality in the administrative system.

3. Theoretical framework of the land-use change model

We briefly explain the ideal structure of the Japan basic model [Kitamura et al., 1996]. Figure 2 shows an outline of the model. The whole model consists of several *regional models* and *a linkage model*. The purpose for dividing it into some regional models such as urban-region model, agricultural-region model, forestry-region model, etc. is to facilitate building up each model independently.

3.1 Basic structure of the regional models

Land use is the reflection of various human activities in the region. In other words, the relative intensity of the activities determine the extent of each land use. Each region model is an independent model and consists of several sector models². In these sector models, the relationships between the sectoral activity and a set of explanatory variables are formulated. The area covered by the α -th land use is obtained in multiplying the levels of the sectoral activities estimated in the sectoral models by the respective activity-specific land requirement factors.

_

² For example, in the agricultural-region model agricultural and related sectors are formulated in detail but this model also contains representations of the other sectors.

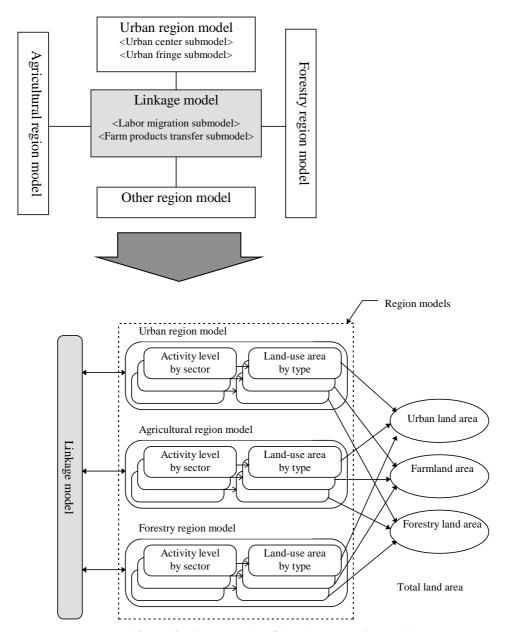


Figure 2. Framework of the Japan basic model.

$$L_{\alpha}^{k}(t) = \sum_{i} L_{i\alpha}^{k}(t) = \sum_{i} e_{i\alpha}^{k}(t) \times y_{i}^{k}(t)$$

Variables:

 $y_i^k(t)$: activity level of the i-th sector at time t, in the k-th region,

 $e_{i_{\alpha}}^{\ \ \ \ \ \ }(t)$: area of the α -th land category required per unit activity of the i-th sector at time t, in the k-th region³.

 $L_{i_{\alpha}}{}^{k}(t)$: extent of the α -th land category due to the activity of the i-th sector at time t, in the k-th region,

 $L_{\alpha}^{k}(t)$: total area of the α -th land category at time t, in the k-th region.

where

i: the i-th sector,

 α : the α -th land category,

k: the k-th region or region model,

t: time.

The sum of major land uses in region k must be equal to the total land area L^k . So, the following equality condition must be satisfied.

$$L^k(t) = \sum_{\alpha} L_{\alpha}^{k}(t)$$

3.2 Linkage model

The interrelations between the regions are reflected through the linkage model. The linkage model describes the migration and transfer of resources such as population, labor force, water, and of various commodities among the region models. The linkage model provides a description of external conditions for each region model and serves to establish consistency among regions.

4. Operational modification for the pilot study area

4.1 Modifications for a simplified model

In the previous section, we briefly mentioned the theoretical framework of the land-use change model. When applying this framework to a case study area, it is inevitable to take into account data availability, local characteristics and the actual land-use situation of the case study area. Operational modifications of the theoretical framework are important to enhance the applicability of the model.

 $^{^3}$ As the first approximation we consider $e_{i_\alpha}\!(t)$ constant over time, i.e., $e_{i_\alpha}\!(t)\cong e_{i_\alpha}\!.$

For a first application the Kansai district was selected as a pilot study area in Japan. The study area comprises of the second largest urban agglomeration in Japan. The detailed results are presented in Hoshino (1996) and Morita et al. (1997). The following points are major modifications of the theoretical framework for this case study area.

- The study area consists of urban centers and urban fringes, though agricultural areas are also included to some extent. In order to simplify the model structure, the region models are integrated into an urban region model only.
- ♦ We simplify the structure of the urban-region model. The land-use ratios are directly estimated from a variety of explanatory variables. A function estimating the percentage of area in each land-use category is defined as *the land-use ratio function: f*.
- ♦ Mutual interactions of the associated factors driving land-use change within the region ought to be reflected in the model. Therefore, we introduce an additional module dealing with mutual interactions between the driving forces, termed *the driving-force prediction model: g.* This module serves the role of a linkage model inside the region model (the internal linkage module). Land-use planning and policy factors are also included in the driving-force prediction model.
- ♦ In the proposed methodology, the linkage model is intended to describe the diverse interactions among the region models. However in this pilot study, we develop only one model, an urban-region model. Therefore we do not require the linkage model for this pilot study. The external conditions of the region model are specified exogenously.
- ♦ Figure 3 shows the operational framework of the Kansai pilot study. The region model is composed of the *prediction model: g* and the *land-use ratio function: f*.

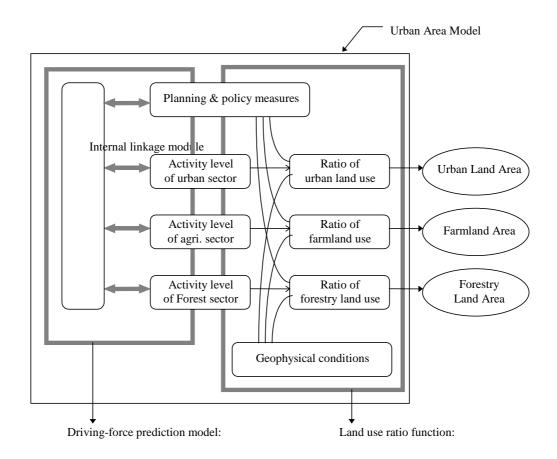


Figure 3. Operational structure of the pilot model.

4.2 Structure of the Kansai land-use change model

With the modifications mentioned above, the operational structure of the basic model consists of the *land-use ratio function* f and the *driving-force prediction model* g. The main relations are represented by the equations below and schematically as shown in Figure 4.

Land-use ratio function $\mathbf{L}(t) = f(\mathbf{S}_t, \, \mathbf{n})$ Driving-force prediction function $\mathbf{S}_t = g(\mathbf{S}_0, \, t)$

Geophysical factors **n** (assumed) constant

- ♦ Land-use ratio function f: The set of relationships which estimate the shares of various land uses from both the driving forces (socio-economic factors, and land-use planning and policy factors) and the geophysical factors. Because the relationships between land-use and the explanatory factors are expected to be stable over a long period, the estimated model is applied over the entire projection period without changing coefficients.
- ♦ **Driving-force prediction model g:** This is a dynamic model predicting future values of the major driving forces. The output values from this model are used as input data to the land-use ratio function f.
- ♦ **Geophysical factors:** The geophysical conditions are assumed constant because changes in these factors are believed to be relatively small over the study period.

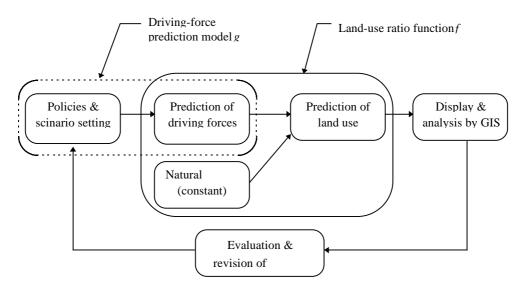


Figure 4. Land-use ratio function and driving-force prediction model.

5. Procedures and techniques of the analysis

Model development and application consists of four steps as shown in Figure 5. The purpose of each step, techniques and their characteristics are as follows.

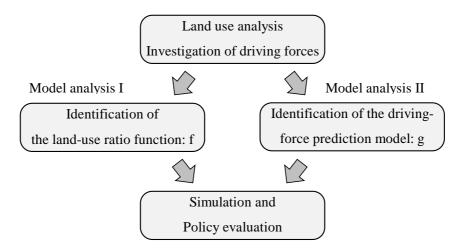


Figure 5. Steps of research works.

5.1 Land-use analysis

This step aims at elaborating the relationships between land-use indicators and their associated factors including socio-economic and geophysical factors. The first task is to test for temporal stability of the land-use structure which is an important prerequisite of the land-use ratio function f to be applicable over time. For this purpose, we apply canonical correlation analysis. Canonical correlation analysis is a multivariate statistical technique that investigates the relationships between two sets of variables, [Okuno et al., 1982]. The predictor set includes various explanatory variables, and the other set, the dependent variables, comprises of criterion measures. Canonical correlation analysis is particularly appropriate when the criterion variables are themselves correlated. In such cases this technique can uncover complex relationships between the predictor and criterion variables. We prepare data sets for two time points each containing the same sets of predictor and criterion variables. We apply the statistical analysis uniformly to these data sets. By comparing the results obtained for the different time points, we can examine the temporal stability of the land-use structure.

The second task of step one (i.e., land-use analysis) is to statistically extract the major driving forces of the observed land-use changes. For this we apply multiple regression analysis. Dependent variables are changes in the share of each land-use category, and explanatory variables are natural conditions, and states and changes of socio-economic conditions. Applying multiple regression analysis, we estimate how much each factor has contributed to the observed land-use changes.

5.2 Land-use ratio function f

The second step is to develop the land-use ratio functions using natural conditions and socio-economic driving forces as explanatory variables. For this purpose, we estimate logit models. Developed in the context of utility theory, the logit model estimates the choice probabilities of each alternative. By interpreting the choice probabilities as the land-use ratios, the logit model is used as the land-use ratio function. The land-use ratio function, using the logit model, has the following characteristics [Kemper, 1985; and Morita et al., 1995].

- ♦ The total sum of all the land-use ratios (i. e., sum of all choice probabilities) is equal to 1. This means that "the competitive relationships among the land-use types under the constant condition of the total land area" is introduced as a constraint condition of the model.
- ♦ We presume that the current land use is the result of land managers acting to maximize their utilities. The logit model can explain these phenomena from the viewpoint of utility theory⁴.
- ♦ In our experience, the logit model produces a better fit of the function using fewer explanatory variables than multiple regression model.

5.3 Driving-force prediction model g

The driving-force prediction model is specified in the third step. The projection results are used as part of the explanatory variables (input data) of the above land-use ratio function f. Kane's Simulation (KSIM) method⁵ is a technique for projecting the values of system variables in time-series, based on a cross-impact matrix approach. An outline of the method is presented in the Appendix. The KSIM method is "one of the techniques pursuing dynamic change of system variables according to the structure of mutual influences of the variables". At first, the system variables are selected from major driving forces that were extracted in the second step. Planning and policy factors should also be included in the system variables. Next, the cross-impact matrix is determined by a consultation of experts. This mutual-influence matrix is a description of relationships among the system variables. [Sawaragi & Kawamura, 1981].

- ♦ Variables which are hard to be quantified can be put in the model.
- ♦ The KSIM method is similar to systems dynamics but mutual interactions among the system variables are much more simplified. Therefore model building is relatively simple and the model is easy to deal with.
- ♦ Model building is an open process to all the participants concerned. Members who do not have

11

⁴ Strictly speaking, this statement is problematic because the unit of sampling is not an individual landowner but an aggregated group of land managers.

⁵ The method is named after its developer, Prof. J. Kane.

- expert knowledge about modeling can also participate in the process.
- ♦ Policy issues can be easily reflected in the system by means of having several policy variables included as system variables, or having the policy reflected in the mutual-influence matrix.

5.4 Simulation and policy evaluation

The final step is model simulation and policy evaluation. At first, we define scenarios and policy options and implement them in the driving-force prediction model (i.e., the KSIM model). We estimate trajectories of the driving forces with the KSIM model. The results are used as input data for the land-use ratio function and to estimate future land-use by types. A variety of policy options can be evaluated in this step.

6. Prospect of this model study

The methodology described in this report for developing a land-use change model has several advantages. (a) the model structure is rather simple. Thus it is relatively easy and inexpensive to apply and to estimate future changes in land use, (b) the model structure is flexible enough to include the specific land-use structure of the pilot area into the model, (c) the driving-force prediction model (the KSIM model) is a participatory modeling technique. Therefore it is open to all the persons concerned.

The following are the major tasks on this modeling research of major land-use changes which should be conducted in near future.

6.1 Development of the linkage model

In this pilot study, we consider only a single region model, an urban-region model. Therefore, the linkage model needed to connect multiple region models was omitted. However, for the model to cover wider regions, including a variety of heterogeneous areas, we have to develop different kinds of region models and also need to develop the linkage model.

6.2 Estimation of the explanatory variables

Because one set of KSIM parameters is required for estimating each set of values of driving forces, it is practically impossible to prepare parameter sets separately for all samples. Thus (1) the municipalities in the study area should be classified into several groups, and one KSIM model should represent one group respectively, or (2) a spatial interaction model disaggregating the total amount of indicator values to each municipality should be introduced.

References

- Hoshino, S., 1996. Statistical Analysis of Land-use Change and Driving Forces in the Kansai District, Japan. IIASA working paper WP-96-120, 1-40.
- Kemper, F. J., 1985. Categorical Regression Models for Contextual Analysis. A Comparison of Logit and Linear Probability Models. *In:* Nijkamp, P. et al. (ed.), *Measuring the Unmeasureable*. Martinus Nijhoff Publishers, Dordrecht / Boston / Lancaster, 81-94.
- Kitamura, T., Kagatsume, M., Hoshino, S., Morita, H., Mizuno, K., and Konagaya, K., 1996 (pp 1-49), *in:* Ootsubo, K. (ed.) Long term projection of land use / cover change in Asia and the Pacific Region (1). National Institute for Environment Study, Japan (in Japanese).
- Morita, H., and Kitamura T., 1995. An Empirical Study on the Decision-Making for Moving out of Employees in Samarinda Municipality, the Republic of Indonesia, A Study on the Residential Move of Employees (I). Journal of Rural Planning Association, Vol. 14 No. 2, 9-20 (in Japanese).
- Morita, H., Hoshino, S., Kagasume, M. and Mizuno, K., 1997: A Substantiative Study on the Application of the Land-Use Change Model for the Kansai District, Japan. IIASA, Laxenburg, Interim Report, IR-97-65.
- Okuno, T., 1982. Canonical correlation analysis. (pp. 373-384) *in:* Okuno, T., Kume, H., Haga, T., and Yoshizawa, T., "Multivariate analysis". Nikkagiren, Tokyo (in Japanese).
- Oota, K., 1984. *Theoretical development of disaggregate model*. Textbook for the 15th lecture course on "Doboku-keikakugaku", (pp. 9-24), Doboku-Gakkai (in Japanese).
- Sawaragi, Y. and Kawamura, K., 1981. "Participatory Systems Approach". Chapter 6, *KSIM method and its applications*. 181-203, Nikka-giken, Tokyo (in Japanese).
- Turner, B.L. II, David Skole, Steven Sanderson, Günther Fischer, Louise Fresco and Rik Leemans, 1995. *Land-Use and Land-Cover Change Science/Research Plan*. IGBP Report No. 35, HDP Report No. 7.

Appendix

A. Canonical correlation analysis

The outline of canonical correlation analysis is as follows [Okuno et al., 1982]. Let m be the number of predictors and p be the number of criterion variables, and assume that $m \ge p$. Denote by $x_1, ..., x_m$ the m dimensional vector of the predictor variables and $y_1, ..., y_p$ the p dimensional vector of the criterion measures. The objective of canonical correlation analysis is to find a linear combination of the m predictors (the x's) that maximally correlates with a linear combination of the criterion variables (the y's). We denote the respective linear combinations by

$$X^*_1 = \mathbf{a}, \mathbf{x} = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1m}x_m$$
 and
$$Y^*_1 = \mathbf{b}, \mathbf{y} = b_{11}y_1 + b_{12}y_2 + b_{13}y_3 + \dots + b_{1n}y_n.$$
 (1)

Denote by Σ_{xx} , Σ_{yy} and Σ_{xy} the variance and covariance matrices of x and y. As a function of **a** and **b**, the correlation between X^* and Y^* is given by

$$\rho_{X^*Y^*} = \rho (\mathbf{a}, \mathbf{b}) = \mathbf{a}' \Sigma_{xy} \mathbf{b} / \{ (\mathbf{a}' \Sigma_{xx} \mathbf{a}) (\mathbf{b}' \Sigma_{yy} \mathbf{b}) \}^{1/2}.$$
(2)

The coefficient vectors \mathbf{a} and \mathbf{b} for the first canonical variate are calculated so as to maximize the correlation coefficient ρ (\mathbf{a} , \mathbf{b}). The subsequent pairs of canonical variates are extracted in the same way such that they are uncorrelated with all of the former canonical variates. At most p (where $m \ge p$) canonical variates can be extracted.

B. Multinominal logit model

The outline of the logit model is as follows [Oota, 1984]. Denote by A the set of alternatives j (j=1, ..., J), by U_j the utility from alternative j. Probability P_i , that an individual chooses the i thalternative, is expressed by

$$P_i = P \{ U_i \ge \max (U_i; j \ne i, j \in A) \}.$$
 (3)

Suppose that U_i can be separated into two parts, a stochastic disturbance term ϵ_i and a deterministic term V_i .

$$U_i = V_i + \varepsilon_i \tag{4}$$

Various models can be developed by assuming different distribution functions for ε_i . When adopting the Gumbel distribution, it becomes easy to derive the model specification from equation (3). The following equation is the general form of the density function of the Gumbel distribution.

$$f(\varepsilon) = \omega \exp\{-\omega (\varepsilon - \eta)\} \exp[-\omega \exp\{-\omega (\varepsilon - \eta)\}]$$
 (5)

It is known that the difference of density function (5) of Gumbel distribution becomes *logistic* distribution. Therefore the equation (3) is expressed by the following equation.

$$P_{i} = \exp(V_{i}) / \sum_{j} \exp(V_{j})$$
 (6)

The Gumbel distribution resembles normal distribution in shape, and the difference between both distributions can be practically ignored. Assume that the deterministic term V_i of the utility function U_i is a linear function, then the utility V_i for alternative i can be written as:

$$V_{i} = \theta_{i1} X_{i1} + \theta_{i2} X_{i2} + ... + \theta_{iK} X_{iK} + C_{i} = \sum_{k} \theta_{ik} X_{ik} + C_{i}$$
(7)

 $X_{i1}, X_{i2}, ..., X_{iK}$ are the attributes (explanatory variables) of the alternative i, and $\theta_{i1}, \theta_{i2}, ..., \theta_{iK}$ are their parameters, respectively. From equation (6) and (7), the form of logit model is expressed as the following equation. These parameters are estimated by the maximum likelihood method.

$$P_{i} = \left\{ exp\left(\sum_{k} \theta_{ik} X_{ik}\right) + C_{i} \right\} / \sum_{j} exp\left\{\left(\sum_{k} \theta_{jk} X_{jk}\right) + C_{j} \right\}$$
(8)

C. Kane's SIMulation (KSIM) method

The procedure of the KSIM method is as follows [Sawaragi and Kawamura, 1981].

- 1. Denote x_i , (i =1,..., n) as the system variables. The system structure is defined by a cross impact matrix $A=(a_{ij})$ which describes interactions between each pair of system variables.
- 2. The system variables are bounded. Maximum and minimum values for each original variable (X_i^{max}, X_i^{min}) respectively) must be specified. Initial values for the system variable x_i^0 (i = 1, ..., n) are calculated by equation (9) where X_i^0 shows the initial value of the original variable.

$$\mathbf{x}_{i}^{0} = \{\mathbf{X}_{i}^{0} - \mathbf{X}_{i}^{\min}\} / \{\mathbf{X}_{i}^{\max} - \mathbf{X}_{i}^{\min}\}$$
(9)

$$0 \le x_i(t) \le 1; \ \forall \ i, t \ge 0 \tag{10}$$

- 3. The cross impact matrix $A=(a_{ij})$ of (n,n) dimension is determined in consultation with experts of the respective fields. The value of element a_{ij} represents the intensity of influence that variable x_j exerts on variable x_i .
- 4. When the j-th system variable brings about a positive or negative effect on the i-th variable, then a_{ii} takes a positive or negative value.
- 5. By repeated accounting of the following formula, 1 time-series prediction can be traced.

$$x_{i}(t+\delta t) = x_{i}(t)^{Pi(t)}; \forall x_{i},$$

$$(11)$$

where t represents time, and δt represents a time increment. The exponentiation part, $P_i(t)$, is as follows.

$$P_{i}(t) = \{ 1 + (\delta t/2) \sum_{j} (|a_{ij}| - a_{ij}) x_{j} \} / \{ 1 + (\delta t/2) \sum_{j} (|a_{ij}| + a_{ij}) x_{j} \}$$
(12)

6. The differential form of equation (11) is shown in equation (13)

$$d x_{i}(t) / dt = - \{x_{i}(t) \times \log_{e} x_{i}(t)\} \sum_{j} a_{ij} x_{j}$$
(13)

By this equation, following points are understood. The change of the system variable is determined by both the total influence from the other variables and the level of the system variable itself. When the value is close to either 1 or 0 of the scale, the change of the variable becomes small. If the value of one variable increases, influence of that variable on the other variables also increases.