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Some Preliminary Experiments with the Financial "Toy-Room"

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Abstract

We describe some preliminary experiments realized with the **Financial "Toy-Room"** (a micro-founded simulation environment for decentralized trade in a homogeneous financial asset). The experiments are aimed at testing the system, and exploring its flexibility in depicting specific contexts as sub-cases. For this purpose, we selected an issue that has been widely investigated in the literature: the existence and characterization of markets in which prices are (or are believed to be) "quality signals" passing information from informed to uninformed traders. In our out-of-equilibrium simulation analysis, we take agents to trade based on a *spread rule*, and introduce *dis-synchronization* in agents' updating processes. Thus, we investigate how dis-synchronization, updating paces and spreads affect persistence of trade and the time-path of prices in extreme regimes (i.e. when all agents are informed, or all agents are uninformed).

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Introduction

In this paper, we describe some preliminary experiments realized with the **Financial "Toy-Room"** (FTR). FTR is a micro-founded simulation environment for decentralized trade in a homogeneous financial asset. We shall not provide a detailed description of the overall model, nor of the code that implements it; the reader is supposed to consider this as the third part of a trilogy whose first part is F. Chiaromonte, G. Dosi (1998), which provides a full account of the model, and whose second part is M. Bertè (1998), which contains all technical details of the code [1].

The aim of these preliminary experiments is to *test-run FTR, and explore its flexibility in depicting specific contexts as sub-cases*. In order to do so, we selected an issue that has been widely investigated in the literature: the existence and characterization of markets in which prices are (or are believed to be) quality signals passing information from informed to uninformed traders.

In Section 1 we give a short sketch of the equilibrium analyses relevant to our experiments. In Sections 2 and 3, we define the setting as a specification of FTR, and identify experimental quantities and mechanisms that are crucial for interpreting simulation outputs. In Section 4 we present our experiments. Section 5 contains some final remarks.

1 Near-perfectly informative markets

Suppose acquiring information on the quality of an asset is costly. Under what conditions can markets in which prices embody quality signals, and hence pass information from informed to uninformed agents, exist as the equilibrium outcome of decentralized optimizations by a group of homogeneous price takers? A large literature has been devoted to this issue. In particular, we will refer to S.J. Grossman, J.E. Stiglitz (1976, 1980). What follows is a loose and incomplete rephrasing of the basic lines of thought of these equilibrium analyses, aimed at capturing the points that will be relevant to our experiments.

Consider an homogeneous asset providing a unitary return Z, and agents that are homogeneous

[1] The code was developed using the LSD platform (M. Valente, 1997).

in terms of preference structure; that is, that weight portfolios of asset and cash [2] through the same utility function. Z is unknown to the agents, but they form expectations, say $\hat{Z}$, about it. $E((\hat{Z}-Z)^2)$ represents the prediction risk. Homogeneity of agents implies, among other things, equal attitudes towards prediction risk.

Assume agents to be aware of the fact that it is impossible to realize gains through the trading process in itself [3], so that they will exchange asset against cash only as long as they maintain different expectations [4].

Assume that the exchange price p is public information, and a noisy version of the return $Z+e$, $e \sim N(0,\sigma^2)$, is available to any agent who is willing to acquire it at a fixed positive cost $c>0$. Agents form their expectations $\hat{Z}$ either observing $Z+e$, or simply observing the price.

Furthermore, assume agents to be aware of the fact that neither the exchange price, nor the share of the population that sustains the cost and observes $Z+e$, are affected by what they themselves do (i.e. both asset exchanges and information acquisitions occur in a perfectly competitive regime).

Finally, assume that a second source of randomness X, independent of the noise e, enters the price formation mechanism.

Loosely speaking, a *near-perfectly informative market* is one in which when an $\alpha \in (0,1]$ share of the population observes $Z+e$, the clearing price $p(\alpha, Z+e, X)$ corresponding to agents optimal trading decisions behaves like $Z+\tilde{e}(\alpha,e,X)$, where the second addend is still a 0-mean normal with

$$\mathrm{var}(\tilde{e}(\alpha,e,X)) = v(\alpha,\sigma^2,\mathrm{var}(X))$$

which decreases in α, increases in σ^2 and $\mathrm{var}(X)$, and coincides with σ^2 if either σ^2 itself or $\mathrm{var}(X)$ are equal to 0. For $\alpha=0$, the clearing price is not related to the return anymore: $p_0(X)$.

In this simplified picture, a near-perfectly informative market is one in which $p(\alpha, Z+e, X)$ and $Z+e$ both convey unbiased information about Z, and only differ in terms of variance (that is, of prediction risk). The two cases $\sigma^2=0$ or $\mathrm{var}(X)=0$ –corresponding to $Z+e$ having infinite precision, or absence of the second source of randomness– are the ones in which the clearing price is exactly equivalent to $Z+e$ in predicting Z.

For given levels of σ^2 and $\mathrm{var}(X)$, the worth of

$$\frac{\sigma^2}{v(\alpha,\sigma^2,\mathrm{var}(X))}$$

under the agents (common) utility decreases in α. On top of the exchange equilibrium, an equilibrium share corresponding to agents optimal information choices is given by an $a(c,\sigma^2,\mathrm{var}(X)) \in (0,1)$ which makes the worth coincide with the cost c, or by 0 (1) if the worth is smaller (larger) than c even at $\alpha=0$ (1). When it exists, the equilibrium share decreases in c, and increases in σ^2 and $\mathrm{var}(X)$.

Again loosely speaking, such an equilibrium exists provided prices are not as good as the costly $Z+e$; that is, provided both σ^2 and $\mathrm{var}(X)$ are strictly positive [5].

[2] The "alternative" asset with known and fixed return.

[3] As for example intermediary gains, or inter-temporal gains, based on buying and selling at different prices.

[4] In other words, if the $\hat{Z}$'s of all agents are all equal, there is no price p at which an agent finds it convenient to buy ($\hat{Z}>p$) and another convenient to sell ($\hat{Z}<p$) due to return expectations, and we assume agents see no further reason to engage in transactions.

[5] More rigorously, if $\sigma^2=0$, no point in [0,1] –including 0 that is approached as σ^2 approaches 0– is an

For given levels $\sigma^2, \mathrm{var}(X) > 0$, one can calculate a finite upper value of the cost, above which the equilibrium share is 0, and a positive lower value, under which the equilibrium share is 1. On the other hand, these equilibria, as well as ones very close to 0 and 1, are *thin trade equilibria*: since all (or almost all) agents' return expectations coincide, there will be no trade, or very little trade. In particular, this will be the case if, regardless of the cost, $a(c, \sigma^2, \mathrm{var}(X))$ approaches 0 because σ^2 or $\mathrm{var}(X)$ approach 0 [6].

Moreover, thin-trade equilibria characterized by higher overall information on the underlying return (lower uncertainty in prediction) –close or coincident with 1– are also characterized by a higher uncertainty in price (*volatility*), and therefore in capital value [7].

1.1 Some remarks

In the equilibrium analysis we have just described, *agents' desires to trade are always satisfied*: the exchange price is a clearing price; that is, a price that renders demand and supply equal to one another, and makes further sales or acquisitions equivalent to any of the homogeneous traders. Moreover, *agents' desires relative to information are always satisfied*: the equilibrium share is one that makes being informed or uninformed perfectly equivalent to any of the homogeneous traders. Finally, satisfaction is due to the fact that *agents' models of the world are always correct*: some traits relative to market functioning are postulated, and agents are taken to be all (equally) aware of them. Usually, the underlying rationale is that such traits have been historically stable enough to allow all agents to learn about them.

In particular, both informed agents and agents that use the price have *rational* (i.e. unbiased) *expectations on the return*, and in choosing whether to get informed agents know $v(\alpha, \sigma^2, \mathrm{var}(X))$, and the values of σ^2 and $\mathrm{var}(X)$.

In one phrase, equilibrium analysis eliminates ex-post/ex-ante distinctions and thereby the role of time.

With our simulation experiments, we will perform an out-of-equilibrium analysis; that is, one in which time is truly oriented and irreversible, and ex-post/ex-ante distinctions are reintroduced.

2 Defining the setting as a special room

Let us put a population of agents $T \in \mathcal{T}$, exchanging the homogeneous asset against cash, into a trading room. The asset return is determined "outside" the room, and unknown to all agents.

equilibrium, while if $\sigma^2 > 0$ but $\mathrm{var}(X) = 0$, the only candidate point is 0, and it is an equilibrium only if c is sufficiently high.

[6] More rigorously, if $V(a(c, \sigma^2, \mathrm{var}(X)), Z + e, X)$ stands for the volume of trade at equilibrium, and c makes the equilibrium share 0 or 1

$$\mathrm{E}(V(0, Z + e, X)) = \mathrm{var}(V(0, Z + e, X)) = 0$$

Moreover, if 0 is approached because σ^2 or $\mathrm{var}(X)$ approach 0

$$\lim\ \mathrm{E}(V(a(c, \sigma^2, \mathrm{var}(X)), Z + e, X)) = \lim\ \mathrm{var}(V(a(c, \sigma^2, \mathrm{var}(X))Z + e, X)) = 0$$

[7] $\mathrm{var}(p(a(c, \sigma^2, \mathrm{var}(X)), Z + e, X))$ is larger when $a(c, \sigma^2, \mathrm{var}(X))$ approaches 1 than when it approaches 0. This might be taken to be undesirable to agents. The reason why this trading-related effect does not affect the decentralized optimization is that agents know the share does not depend on what they themselves do individually.

2.1 Time representation, dis-synchronization and pace of updatings

"Time" [8] in FTR is constructed translating transaction events into minutes by means of a converting sequence $\nu_j, j = 1, 2, \dots$ [9]. Different ν_j's serve the purpose of representing accelerations/decelerations of trade on the minute time-scale. In these experiments, we set $\nu_j = \nu, j = 1, 2, \dots$ (so ν represents the fixed length in transactions of any minute). Once the converting sequence has been specified in this way, "thinning of trade" corresponds to "lingering of time", and *the market breaks down when (event-based) time in it stops running.*

FTR also allows "times" underlying agents' updating processes to have different paces, and be dis-synchronous (among themselves and with respect to overall time), by means of agent-specific converting sequences [10]. **Dis-synchronization and diversified updating paces** might be extremely relevant for persistence of trade. We generate dis-synchronization via a chance element, and allow for parametric diversification, by taking

$$\nu_j[T], j = 1, 2, \dots \ \text{ iid } \ Poiss(\mu[T]) \quad , \quad \text{independent across } \ T$$

2.2 Trade decisions, expectations and spreads

When agents update, they revise the value of a number of decision variables [11].

We assume all agents to limit themselves to transactions that are *spot on both sides.* Whenever agents update, they set

$$D^\sigma_b[T] = D^\sigma_b[T] = D^\alpha_b[T] = D^\alpha_b[T] = \{(0,0)\}$$

where the $D^{(\cdot)}_{(\cdot)}[T]$'s represent sets of allowed posticipations of the actual flows of cash and asset, with respect to the moment in which a transaction is concluded.

We assume all agents to be *always available to trade spot* within the limits of their cash and asset endowments, both as seekers and as acceptors. On the other hand, we assume them to believe it is impossible to realize gains through the trading process in itself, so that they will conclude transactions only as long as they maintain different expectations on the unknown "out-of-the-room" return, and trade will be sustained solely by heterogeneity is such expectations.

Moreover, we assume agents to form an **interval expectation** on the return, using a point-expectation $\hat{Z}[T]$ as if it were "rational" [12], and a guess of its standard deviation $s_{\hat{Z}}[T]$ [13]. Thus, agents trade with a **spread rule** based on their interval expectation. Whenever updating, they

[8] Notice that throughout our description we omit time indexing for all quantities involved in the model, except when referring to their initialization. Only some quantities are indexed by the transaction they are associated with (N), or the minute they refer to (H). The way time is represented and handled in FTR makes this the most obvious and easiest choice. In fact, we deal with 3 distinct "gears" in stepping forward a simulation: trading rounds, transaction (the outcomes of some, but not necessarily all, rounds), and minutes (defined by the stream of transactions via converting sequences).

[9] Minute 1 lasts until the ν_1th transaction is concluded, minute 2 until the $(\nu_1 + \nu_2)$th transaction is concluded, etc.

[10] Agent T always updates his decision variables after concluding a transaction. Moreover, whether he concludes transactions or not, he updates at the beginning of each minute on his watch. The watch is itself brought forward by (overall) transactions, but the sequence is agent-specific: minute 1 lasts until transaction $\nu_1[T]$, minute 2 until transaction $(\nu_1[T] + \nu_2[T])$, etc.

[11] The ones contained in their "trading documents", i.e. the acceptor and seeker sheets.

[12] i.e. as if it were an unbiased estimate of the unknown return.

[13] i.e. an estimate of the standard deviation of the (supposedly) unbiased estimate above.

set [14]

$$f_b^\alpha[T] = f_b^\sigma[T] = 1 \quad , \quad p_b^\alpha[T] = p_b^\sigma[T] = \min\{\hat{Z}[T] - \xi s_{\hat{Z}}[T], m[T]\}$$
$$f_s^\alpha[T] = f_s^\sigma[T] = Ind(q[T] \geq 1) \quad , \quad p_b^\alpha[T] = p_b^\sigma[T] = \hat{Z}[T] + \xi s_{\hat{Z}}[T]$$

where $m[T] \in \mathbb{R}^1_+$ and $q[T] \in \mathbb{N}^1$ represent cash and asset endowments, the $f_{(\cdot)}^{(\cdot)}[T]$'s are $0, 1$ flags indicating availability to buy/sell as acceptor/seeker, and reference prices are cutting points $\in \mathbb{R}^1_+$: buying as acceptor/seeker is feasible at any price $< p_b^{(\cdot)}[T]$, and selling as acceptor/seeker is feasible at any price $> p_s^{(\cdot)}[T]$. ξ counts standard deviations from the center [15].

Decision variables do not include quantities, because for the time being all transactions in FTR are supposed to concern a single unit of asset [16].

2.3 Fundamentalists and noise traders

We take the current exchange price $p_{(N)}$ (the price of transaction N) to be public information, and assume the existence of a second (non-public) information source which, on call, provides a noisy version of the return $Z_{(H)} + e[T]$, $e[T] \sim N(0, \sigma^2)$ [17]. H indicates the current minute.

We divide agents in two homogeneous groups $\mathcal{T}_{Fu}$ and $\mathcal{T}_{No}$, according to their behavioral state indicator $r[T] = 1, 2$ [18]. A behavioral state entails a given updating pace, a given rule to form the point-expectation, and a given guess of its standard deviation.

A $T \in \mathcal{T}_{Fu}$ ($r[T] = 1$) is an **informed trader (*fundamentalist*)**; that is, an agent with access to the non-public information source. (The inverse of) his pace of updating is $\mu[T] = \mu_{Fu}$. When updating, he forms his center as

$$\hat{Z}[T] = Z_{(H)} + e^\psi[T] \tag{1}$$

Hence, a fundamentalist indeed uses a rational point-expectation. $\psi \in \{0, 1\}$ allows us to further distinguish a case in which the errors all coincide with **one draw**, and a case in which they are **separate and independent draws**, from a $N(0, \sigma^2)$ distribution

$$\psi = 0 \quad : \quad e^0[T] = e_{(H)} \sim N(0, \sigma^2) \ , \ \forall T \in \mathcal{T}_{Fu} \text{ updating in } H$$
$$\psi = 1 \quad : \quad e^1[T] \text{ iid } N(0, \sigma^2) \ , \text{ across } T \in \mathcal{T}_{Fu} \text{ updating in } H$$

A fundamentalist's spread is based on his guess $s_{\hat{Z}}[T] = s_{Fu}$ of σ.

A $T \in \mathcal{T}_{No}$ ($r[T] = 2$) is an **uninformed or *noise trader***; that is, an agent without access to the non-public information source. (The inverse of) his average pace of updating is $\mu[T] = \mu_{No}$. When updating, he forms his center as

$$\hat{Z}[T] = p_{(N)} \tag{2}$$

[14] In the behavioral taxonomy implementation described in M. Bertè (1998), this corresponds to $type[T] = 2$.

[15] An agent is highly confident that the return value falls somewhere in $[\hat{Z}[T] - \xi s_{\hat{Z}}[T], \hat{Z}[T] + \xi s_{\hat{Z}}[T]]$, and is therefore ready to buy at prices on the left of it, and ready to sell at prices on the right.

[16] This, together with the fact that each minute is bound to contain exactly ν transactions, means we are imposing a fixed trading volume per minute.

[17] Normality is the most obvious distributional assumption for noises. On the other hand, changing error distribution would be very easy in FTR; simulation studies are not limited by constraints due to analytical tractability.

[18] In the behavioral taxonomy implementation described in M. Bertè (1998) these are actually state 1, and 3 with a particular twist on parameters.

Hence, a noise trader uses the price as if it were a rational point-expectation; that is, as if

$$p_{(N)} \sim Z_{(H)} + \tilde{e} \ , \ \ \tilde{e} \sim N(0, \tilde{\sigma}^2)$$

This belief on how trade inside the room captures the return through prices, might be right, but also wrong. A noise trader's spread is based on his guess $s_{\hat{Z}}[T] = s_{No}$ of $\tilde{\sigma}$.

In terms of the behavioral taxonomy in F. Chiaromonte, G. Dosi (1998), informed traders are *Strong Fundamentalists* setting $x_o = x = Z_{(H)} + e[T]$, and *forming a spread* with $\gamma[T] = 1$ and $\varepsilon[T] = \xi s_{Fu}$. Uninformed traders are *Noise Traders* setting $x_o = x = p_{(N)}$, and *forming a spread* with $\gamma[T] = 1$ and $\varepsilon[T] = \xi s_{No}$.

Why do agents belong to one or the other category? One can provide explanations of different natures: Maybe fundamentalists *can* access information at no cost, and noise traders simply *can not*. Or maybe fundamentalists are willing to pay for the information, and noise traders are not because they are reasonably confident in the informativeness of prices, and have a *different attitude towards uncertainty on the return*. More generally, any combination of differences in *capabilities* and in *attitudes* could be playing in determining the two groups.

With this heterogeneity-based rationale, we perform the division at the outset, and control the share of fundamentalists

$$\alpha = \frac{no(\mathcal{T}_{Fu})}{no(\mathcal{T}_{Fu}) + no(\mathcal{T}_{No})}$$

as an experimental parameter.

It is important to stress that, unlike equilibrium analysis, we don't need to assume that agents' beliefs or guesses are correct ex-ante. What we will assume is that they are not revised on the time scale of the simulation experiments (agents do not change group, do not evolve different rules to form their point-expectations, and keep their standard deviation guesses).

Notice that whether s_{Fu} is close to σ, and whether s_{No} is larger than σ or s_{Fu} and by how much [19], is controlled when fixing the values of such experimental parameters. Whether prices are indeed rational expectations, and with a variability close to the noise traders' guess s_{No}, on the other hand, is an experimental outcome whose ex-post soundness can be verified under different parameters settings.

2.4 The physics of trading

In a generic *trading round*, an agent is drawn at random to be the seeker. He will have access to $k \in \mathbb{N}^1$ acceptors drawn at random among the remaining agents [20]. Matching his flags and reference prices with the ones of these acceptors, the seeker determines a set of feasible transactions. If the set is empty, the round ends inconclusively. Indicating with T the seeker, and neglecting temporarily the effect of endowments constraints, this occurs if and only if

$$(\hat{Z}[T] - \xi s_{\hat{Z}}[T], \hat{Z}[T] + \xi s_{\hat{Z}}[T]) \ \cap \ (\hat{Z}[T'] - \xi s_{\hat{Z}}[T'], \hat{Z}[T'] + \xi s_{\hat{Z}}[T']) \ \neq \ \emptyset \qquad (3)$$

[19] That is, whether noise traders treat prices as rational expectations on the return, but attribute them a standard deviation larger than the one associated with the non-public information source (or larger than the fundamentalists' guess on it), and maybe whether they make such difference depend on α (or on a guess of it).

[20] Notice that, under whatever endowment constraint, all agents will always have at least $f_b^{\alpha}[T] = f_b^{\sigma}[T] = 1$, and hence be ready to enter the round as both seekers and acceptors (the cash constraint affecting their buying reference prices).

for all accessible acceptors T'. If the set of feasible transactions is not empty (i.e. if the intersection above is empty for at least one T'), the seeker selects among feasible transactions the one that is most convenient in terms of price. The transaction is then concluded [21] at a price given by a convex combination of the two reference prices, with weight $\lambda \in [0,1]$ for the seeker, and $(1-\lambda)$ for the acceptor. Again neglecting endowment constraints, and assuming for instance the most convenient transaction to involve the seeker as a buyer

$$p_{(N)} \;=\; \lambda(\hat{Z}[T] - \xi s_{\hat{Z}}[T]) \;+\; (1-\lambda)(\hat{Z}[T'] + \xi s_{\hat{Z}}[T']) \tag{4}$$

2.5 Some remarks on possibilities

We restrict to *spot trading*, but studying persistence when agents engage in "short" or "forward" trading (see F. Chiaromonte, G. Dosi 1998) could be quite interesting, and certainly constitutes an avenue for further simulation experiments.

Moreover, we *do not introduce a birth process*, and *maintain the underlying return constant throughout*: $Z_{(H)} = Z$, $H = 1, 2, \ldots$ with no intrinsic random fluctuation [22]. Clearly, a properly specified birth process would provide means for exogenously sustaining heterogeneity in return expectations, and thereby trade [23]. So would a properly specified time-path for the return. These, too, are avenues for further simulation experiments.

FTR permits to introduce non-spot trading and births, and obviously to specify a non-trivial dynamics for $Z_{(H)}$. We decided to leave these elements out not because of technical difficulties, but rather to limit the set of experimental determinants in these preliminary simulations. In a sense, since our fundamental aim was to test the system, we wanted to maintain the special room simple enough as to be able to "run simulations in our heads".

3 Frame quantities, the experimental space, and some reading keys

With our first round of experiments, we perform an out-of-equilibrium investigation of how, given the price formation mechanism implicit in the special room we just described, **persistence of trade** and the **time-path of prices** are affected by

- dis-synchronization and pace of agents updating processes
- the fact that agents use a spread rule, with a spread parameter based on their standard deviation guess, and
- single versus iid error draws for fundamentalists' updatings

in the two extreme regimes; that is when all agents are noise traders, or all agents are fundamentalists ($\alpha = 0$ or 1). Regarding the path of prices, we focus in particular on its relation to the underlying return value, and volatility.

[21] And immediately completed with delivery and payment, since trading is spot on both sides.

[22] In terms of the description in F. Chiaromonte, G. Dosi (1998), we take $z_{(H)} = Z$ (constant) and $\mathrm{var}(\zeta_{(H)}) = 0$, but we allow for "reading errors".

[23] The premise for this is obviously that newcomers' expectations are not cloned from expectations of incumbents –possibly in the process of converging, but generated as to maintain heterogeneity.

We label **frame quantities** those parameters and variables initializations that need to be provided as input for each simulation experiment, but are not immediately relevant for the analysis to be performed. Their values are kept constant across experiments [24]:

Frame Quantities		
$no(\mathcal{T})$	(overall) no. of agents	50
ν	length (in trans.'s) of each minute	10
ξ	no. of st. dev.'s for spreads	1
$Z_0 = Z$	(const.) value of the return	200
σ	st. dev. of $e[T]$'s	25
k	no. of accessible acceptors	$no(\mathcal{T})/2 = 25$
λ	seeker's power in pricing	0
$\hat{Z}_0[T]$	initialization of centers	$no(\mathcal{T})/2 = 25$ at each $100 + / - 50$
$m_0[T], T \in \mathcal{T}$	initialization of cash endowments	$m_0[T] = 10Z = 2000, \forall T \in \mathcal{T}$
$q_0[T], T \in \mathcal{T}$	initialization of asset endowments	$q_0[T] = 10, \forall T \in \mathcal{T}$

On a technical note, we define trade to have vanished after $(no(\mathcal{T})/(k+1))\,\rho$ inconclusive trading rounds. The choice is obviously arbitrary, but the idea is to keep trying long enough as to be reasonably confident that the lack of conclusion is not merely due to chance in drawing seeker and acceptors [25]. ρ represents an additional frame quantity, that we fix at $\rho = k+1 = 26$. Thus, we will declare trade to have vanished after $no(\mathcal{T}) = 50$ attempts.

Finally, if trade does not vanish before, a simulation experiment runs until 500 transactions are concluded, i.e. for 50 minutes.

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[24] This doesn't necessarily mean that those quantities do no affect the phenomena under consideration, but rather that (if they do) it isn't their effect we intend to investigate. Maintaining their values fixed across experiments means we are performing a conditional investigation on experimental quantities. Rigorously speaking, this would be satisfactory only under the assumption that frame and experimental quantities have independent effects on the phenomena. More loosely, it would still be acceptable if frame quantities were fixed at "neutral levels" (i.e. levels conditioning on which one reproduces the marginal effect of experimental quantities), or at levels which are taken to be the ones of real interest (e.g. because they are derived from empirical evidence).

[25] Given the structure of a trading round, the probability of a generic trader T being involved (as seeker or acceptor) is easily shown to be

$$\frac{1}{no(\mathcal{T})} + \left(1 - \frac{1}{no(\mathcal{T})}\right) \frac{\binom{no(\mathcal{T})-1}{k}}{\binom{no(\mathcal{T})-2}{k-1}} = \frac{k+1}{no(\mathcal{T})}$$

Trading rounds are independent. What is the probability that $\exists$ an agent which is not involved in R successive rounds? This is the probability that agent 1 is not involved, *or* agent 2 is not involved, etc. As such, it is bound from above by the sum of the probabilities that each single agent is not involved, which are all equal. The generic one is given by the product of the probabilities of an agent not being involved in each single round. Those are all equal as well. In summary

$$P(\exists T \text{ not involved in } R \text{ successive rounds}) \leq no(\mathcal{T}) \left(1 - \frac{k+1}{no(\mathcal{T})}\right)^R$$

The expression we consider is

$$no(\mathcal{T}) \left(1 - \frac{k+1}{no(\mathcal{T})}\right)^{\frac{no(\mathcal{T})}{k+1}\rho}$$

When $no(\mathcal{T})$ is fairly large, $k = no(\mathcal{T})/2$ and $\rho = k+1$ (so that the exponent is $no(\mathcal{T})$), the above is approximately $no(\mathcal{T})/2^{no(\mathcal{T})}$, which is indeed extremely small. Hence, with $no(\mathcal{T})$ attempts the probability that there is an agent which is never involved is negligible. As a consequence, so is the probability that the lack of trade is merely due to chance in drawing seeker and acceptors.

The **experimental space** is given by the cross product of the experimental quantities ranges:

Experimental Quantities		
α	share of fundamentalists in the room	$\in [0,1]$
μ_{Fu}	(inverse of) av. pace of updating for fundamentalists	$\in \mathbb{N}^1$
μ_{No}	(inverse of) av. pace of updating for noise traders	$\in \mathbb{N}^1$
ψ	single vs iid error draws for fundamentalists	$\in \{0,1\}$
s_{Fu}	guess of error st. dev. by fundamentalists	$\in \mathbb{R}^1_+$
s_{No}	guess of st. dev. by noise traders	$\in \mathbb{R}^1_+$

Obviously whether $s_{No} > s_{Fu}$ is relevant only for simulation experiments in which both parameters are involved (which will not be the case in our first round). Given our centers initialization, we will also require $s_{Fu}, s_{No} \leq 50$; this guarantees that an agent whose initial center is 50 still starts with a non-negative buying reference price [26].

Before passing to the description of our experiments, let us list some general **reading keys** that we will use in interpreting outcomes.

- **K1**: $\mu_{(\cdot)}$ is both the mean and the variance of a Poisson. Consequently, μ_{Fu} is the (inverse of) the average pace of updatings for fundamentalists, but also a measure of the average degree of dis-synchronization among them [27]. The same holds for noise traders.
- **K2**: Fast updating paces (small $\mu_{(\cdot)}$), ceteris paribus, increase the mobility of centers in the directions dictated by their updating functions given in Equation 1 and Equation 2.
- **K3**: As can be seen through Equation 3, large spreads ($s_{(\cdot)}$) have the effect of reducing the overall number of feasible transactions (a transaction is feasible only between agents whose centers have a distance larger than the sum of their spreads). Thus, ceteris paribus, large spreads tend to thin trade.
- **K4**: On the other hand, as can be seen through Equation 4, large spreads allow for wide gaps between centers and the prices of concluded transactions (since we fixed $\lambda = 0$, the price will be one spread away from the center of the acceptor).
- **K5**: Fast updating paces can always be seen as corresponding to good ex-ante stands on the side of the agents.
- **K6**: A guess s_{Fu} close to σ can be seen as a good ex-ante stand on the side of fundamentalists. This concept can be loosely paralleled for noise traders: First, noise traders can't be sure that prices are indeed rational point-expectations. Second, even if they are reasonably confident in unbiasedness, they might take the standard deviation to depend inversely on α. Thus, especially if α is small, a high s_{No} can be seen as a good ex-ante stand on the side of noise traders.

[26] As a mater of fact, we take both guesses to be smaller than $\sigma = 25$.

[27] In the sense that a large μ_{Fu} makes the $\nu_j[T]$'s of a single fundamentalist T, as well as the $\nu_j[T]$'s and $\nu_j[T']$'s of two distinct fundamentalists T, T', more different on average.

4 A first round of simulation experiments

4.1 Experiment 1

In Experiment 1, we assume that all agents in the room are noise traders, i.e. that $\alpha = 0$. If all agents updated at all times, the system would die immediately: All point-expectations on the return would be set to $p_{(1)}$ right after the first transaction is concluded, eliminating the initial heterogeneity. Centers would all coincide, and no further trading would be possible [28]. The simulation would still run the canonical number of attempts, and then terminate.

Can dis-synchronization of the updating processes counteract convergence and make trade persist, at least temporarily? And how would prices behave in the meanwhile?

We wanted to explore how duration of trade (measured by H_{quit}; the minute in which time stops), and the path of prices, depend on the (common) updating pace and spread of the noise traders. Thus, we performed simulations on a grid of values for μ_{No} and s_{No} [29]. Results can be summarized as follows:

Persistence:

Duration of Trade: H_{quit}			
	$s_{No} = 5$	$s_{No} = 10$	$s_{No} = 15$
$\mu_{No} = 2$	5	3	2
$\mu_{No} = 10$	11	6	4
$\mu_{No} = 20$	18	9	7
$\mu_{No} = 30$	21	10	7

Trade duration is directly linked to μ_{No}, and inversely linked to s_{No}. The system never reaches the 50 minutes limit, and achieves non-negligible duration only for large enough μ_{No} and small enough s_{No}.

Here return expectations are based on prices, and hence there is a circular mechanism (centers determine prices, that in turn determine centers, etc) [30]. As a consequence, both K2 and K4 translate into the centers moving faster towards each other, the smaller μ_{No} and the larger s_{No}. Moreover, the inverse effect of s_{No} on duration is strengthened by $K3$. Recall that fast updating paces and large spreads ought to characterize agents with better ex-ante stands (K5 and K6).

The path of prices:

In all cases, along the temporary persistence phase prices spiral towards the middle of the initial centers range (which is not related to the underlying Z), with a progressively smaller volatility (i.e. within minute variability). Fig. 1 shows the path of prices for the longest simulation ($\mu_{No} = 30$, $s_{No} = 5$).

4.2 Experiment 2A

In Experiment 2A, we assume that all agents in the room are fundamentalists, i.e. that $\alpha = 1$. We further assume that agents updating in a given minute observe Z plus a single error draw

[28] Incidentally, since the spreads are all the same (all agents belong to the same group) this makes coincidence of centers equivalent to that of the whole intervals.

[29] μ_{Fu}, ψ,σ and s_{Fu} are irrelevant, as they are not used by the simulations relative to Experiment 1.

[30] Which gives the convergence process a *self-reinforcing nature.*

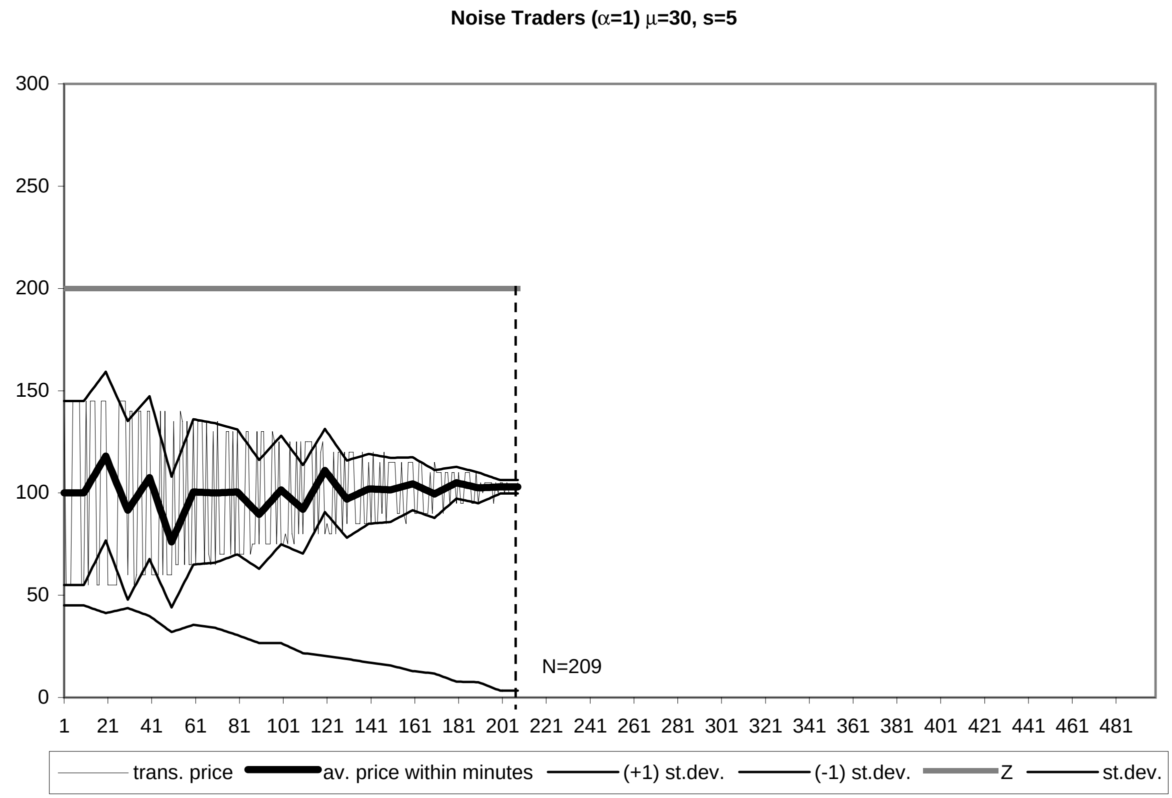

Figure 1: An instance of the path of prices for Experiment 1

($\psi = 0$), with $\sigma = 25$. If all agents updated at all times, the system would die immediately: All point-expectations on the return would be set to $Z + e_{(1)}$ right after the first transaction is concluded, eliminating the initial heterogeneity. Centers would coincide and no further trading would be possible.

Repeating the questions in Experiment 1 and exploring the same grid of values, this time for μ_{Fu} and s_{Fu} [31], we obtained the following results [32]:

Persistence:

Duration of Trade: H_{quit}			
	$s_{Fu} = 5$	$s_{Fu} = 10$	$s_{Fu} = 15$
$\mu_{Fu} = 2$	max	11	2
$\mu_{Fu} = 10$	max	max	14
$\mu_{Fu} = 20$	max	max	25
$\mu_{Fu} = 30$	max	max	max

The system reaches the 50 minutes limit in most cases. It still terminates for small enough μ_{Fu} and large enough s_{Fu}, although trade duration is larger than that of the corresponding cases in Experiment 1, except for $\mu_{Fu} = 2$, $s_{Fu} = 15$ (i.e. the best ex-ante stand).

Since return expectations are *not* based on prices, K2 and K4 do not induce self-reinforcement in centers convergence. Although all agents updating in a given minute observe the same $Z + e_{(H)}$, persistence is achieved unless their ex-ante stands are "too good", and in particular, unless s_{Fu}

[31] μ_{No} and s_{No} are irrelevant, as they are not used by the simulations relative to Experiment 2A.
[32] "max" means the simulation reached the 50 minutes limit.

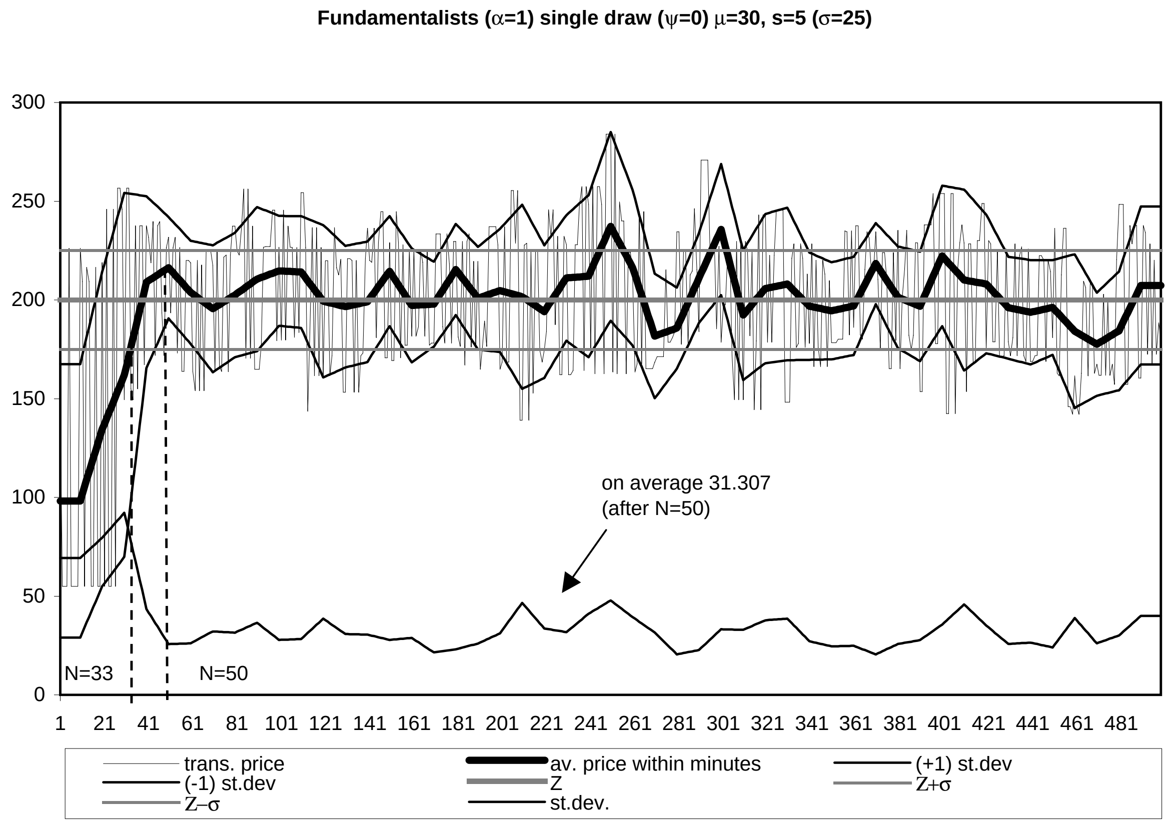

Figure 2: An instance of the path of prices for Experiment 2A

is as large as to paralyze trade because of K3.

The path of prices:

In the first phase of each simulation, prices undergo a *traverse*, moving from values around the middle of the initial centers range towards values around the underlying Z. During this phase volatility (i.e. within minutes variability) decreases. After the traverse is concluded, prices remain around Z with a fairly constant volatility. As an instance, Fig. 2 shows the path of prices for $\mu_{Fu} = 30$, $s_{Fu} = 5$ (it can be compared with Fig. 1 for Experiment 1).

One might then ask how duration of the traverse, and volatility thereafter, depend on μ_{Fu} and s_{Fu}. As a first approximation, we considered the traverse to be concluded [33] somewhere in between (a) the first transaction at which (the running calculation of) the average price within minutes crosses $Z - \sigma$ from below, and (b) the first transaction at which (the running calculation of) the average price within minutes has a maximum, after (a). We then indicated with H_{trav} the minute in which this mid-point falls. As a measurement of the (constant) volatility after the traverse, we took the average of (the running calculation of) the price standard deviation within minutes, from the peak in (b) to the end of the simulation [34].

[33] An accurate definition is not trivial, as what one needs to identify is when exactly the path of prices enters a regime of equally sized fluctuations about the underlying Z.

[34] The cell for $\mu_{Fu} = 2$, $s_{Fu} = 15$ is left empty because the simulation was too short to meaningfully distinguish a traverse and an after-traverse in it.

Duration of Traverse: H_{trav}				Volatility after the Traverse			
	$s_{Fu} = 5$	$s_{Fu} = 10$	$s_{Fu} = 15$		$s_{Fu} = 5$	$s_{Fu} = 10$	$s_{Fu} = 15$
$\mu_{Fu} = 2$	2	2		$\mu_{Fu} = 2$	24.834	24.442	
$\mu_{Fu} = 10$	3	3	3	$\mu_{Fu} = 10$	32.039	27.995	19.662
$\mu_{Fu} = 20$	4	4	3	$\mu_{Fu} = 20$	31.253	25.624	19.667
$\mu_{Fu} = 30$	4	4	4	$\mu_{Fu} = 30$	31.307	25.710	24.911

Traverse duration increases as μ_{Fu} increases. The link between traverse duration and pace of updating resembles that between trade duration and pace of updating in Experiment 1. On the other hand, s_{Fu} has no significant effect on traverse duration.

Volatility after the traverse decreases as s_{Fu} increases. This is due to K4: while centers remain apart enough for trade to persists, and as long as the effect of K3 does not enter the picture, a larger s_{Fu} allows prices to be closer to each other.

The link between μ_{Fu} and volatility is ambiguous, and can be interpreted through K1. On one side, one has an inverse effect due to pace of updating: a larger μ_{Fu} (smaller pace of updating) reduces the frequency with which the noise distribution is sampled, and thereby centers heterogeneity and ultimately price variability. On the other side, one has a direct effect due to degree of dis-synchronization: since all agents updating in the same H observe the same draw, a smaller μ_{Fu} (bringing closer to synchronicity) reduces centers heterogeneity and ultimately price variability.

4.3 Experiment 2B

In Experiment 2B, we again take all agents in the room to be fundamentalists ($\alpha = 1$), but we assume that agents updating in a given minute observe Z plus iid error draws ($\psi = 1$), with $\sigma = 25$. Here, even if all agents updated synchronously, one would expect trade to persist, due to the intrinsic heterogeneity in point-expectations (centers) generated by the separate draws.

We performed simulations in which all agents update synchronously every M_{Fu} transactions [35], on the usual grid of values, this time for M_{Fu} and s_{Fu} [36].

Persistence:

As expected, the system reaches the 50 minutes limit at any level of M_{Fu} and s_{Fu}. Persistence is achieved even when agents have the best ex-ante stands.

The path of prices:

Paths are qualitatively similar to those for Experiment 2A: In a first phase prices undergo a traverse towards Z, with decreasing volatility. In a second phase they remain about Z, with a fairly constant volatility. Here though, the traverse can be completely eliminated by fast updating paces (see below). Fig. 3 shows the path of prices for $M_{Fu} = 30$, $s_{Fu} = 5$ (it can be compared with Fig. 1 for Experiment 1 and Fig. 2 for Experiment 2A). In contrast, Fig. 4 shows the path of prices for $M_{Fu} = 2$, $s_{Fu} = 15$; that is, the best ex-ante stand case.

Regarding duration of the traverse, and volatility thereafter, one has [37]:

[35] Notice this is *not* a special case of the Poisson updating scheme we described in Section 2: to reintroduce synchronization we have to eliminate chance in the updating scheduling.

[36] Again, μ_{No} and s_{No} are irrelevant, as they are not used by the simulations relative to Experiment 2B.

[37] "none" means (the running calculation of) the average price within minutes is already over $Z - \sigma$ at the very

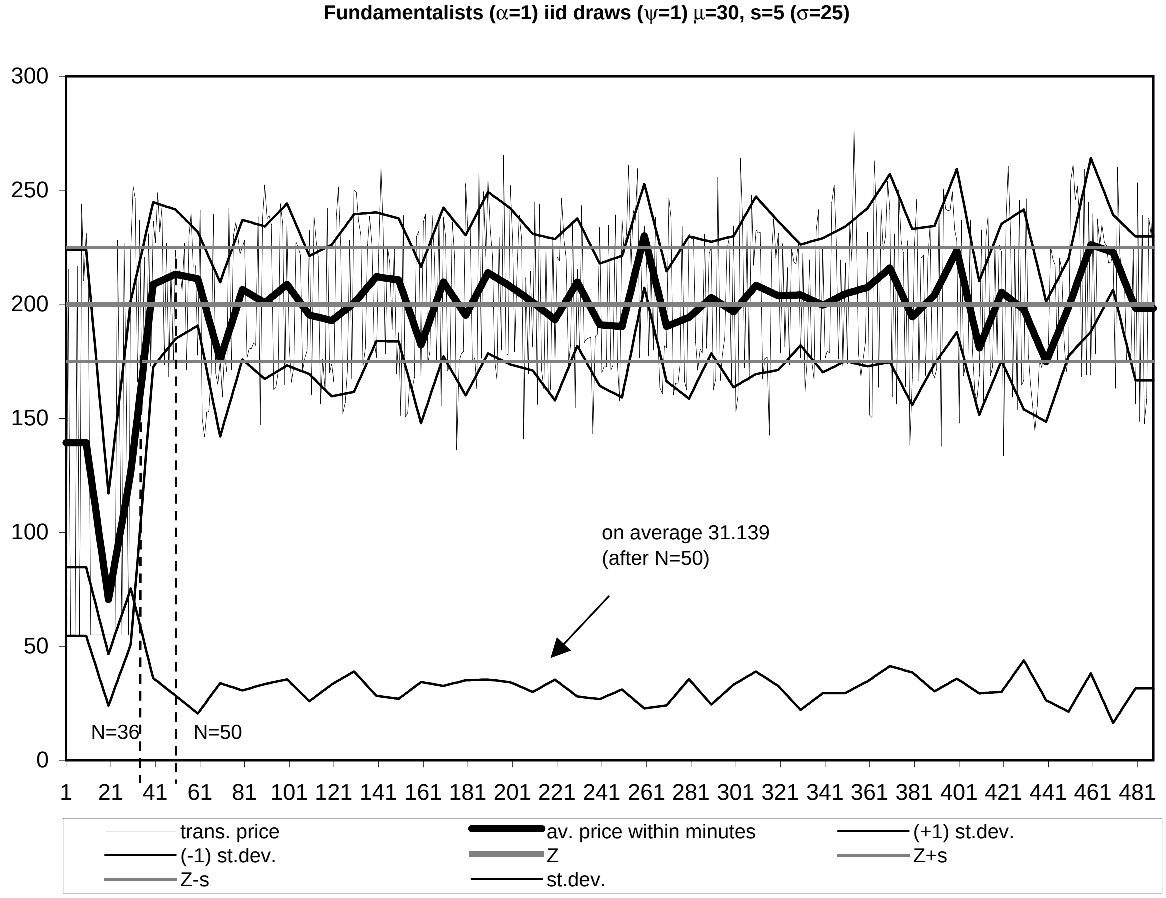

Figure 3: An instance of the path of prices for Experiment 2B

Duration of Traverse: H_{trav}				**Volatility after the Traverse**			
	$s_{Fu} = 5$	$s_{Fu} = 10$	$s_{Fu} = 15$		$s_{Fu} = 5$	$s_{Fu} = 10$	$s_{Fu} = 15$
$M_{Fu} = 2$	none	none	none	$M_{Fu} = 2$	45.874	40.152	36.710
$M_{Fu} = 10$	2	2	2	$M_{Fu} = 10$	37.397	33.012	29.303
$M_{Fu} = 20$	3	3	3	$M_{Fu} = 20$	34.481	29.472	25.765
$M_{Fu} = 30$	4	4	4	$M_{Fu} = 30$	31.139	26.765	23.336

As in Experiment 2A, traverse duration increases as M_{Fu} increases (in particular, for $M_{Fu} = 2$ there is no traverse at all), and s_{Fu} has no significant effect.

Again as in Experiment 2A, volatility after the traverse decreases as s_{Fu} increases, due to K4. Unlike Experiment 2A though, the link between M_{Fu} and volatility is unambiguously an inverse one. In fact, having imposed synchronicity, one has only the pace of updating effect: a larger M_{Fu} reduces the frequency with which the noise distribution is sampled (iid, by all agents at the same time). Smaller centers heterogeneity and price variability follow.

When comparing corresponding simulations in Experiments 2A and 2B, one finds that the latter has shorter traverses and higher volatilities in all cases, except those with M_{Fu} (μ_{Fu}) equal to 30. When the (average or deterministic) sampling frequency is very low, the highly dis-synchronous single-sampling regime in Experiment 2A differs very little from the synchronous iid-sampling regime in Experiment 2B.

beginning.

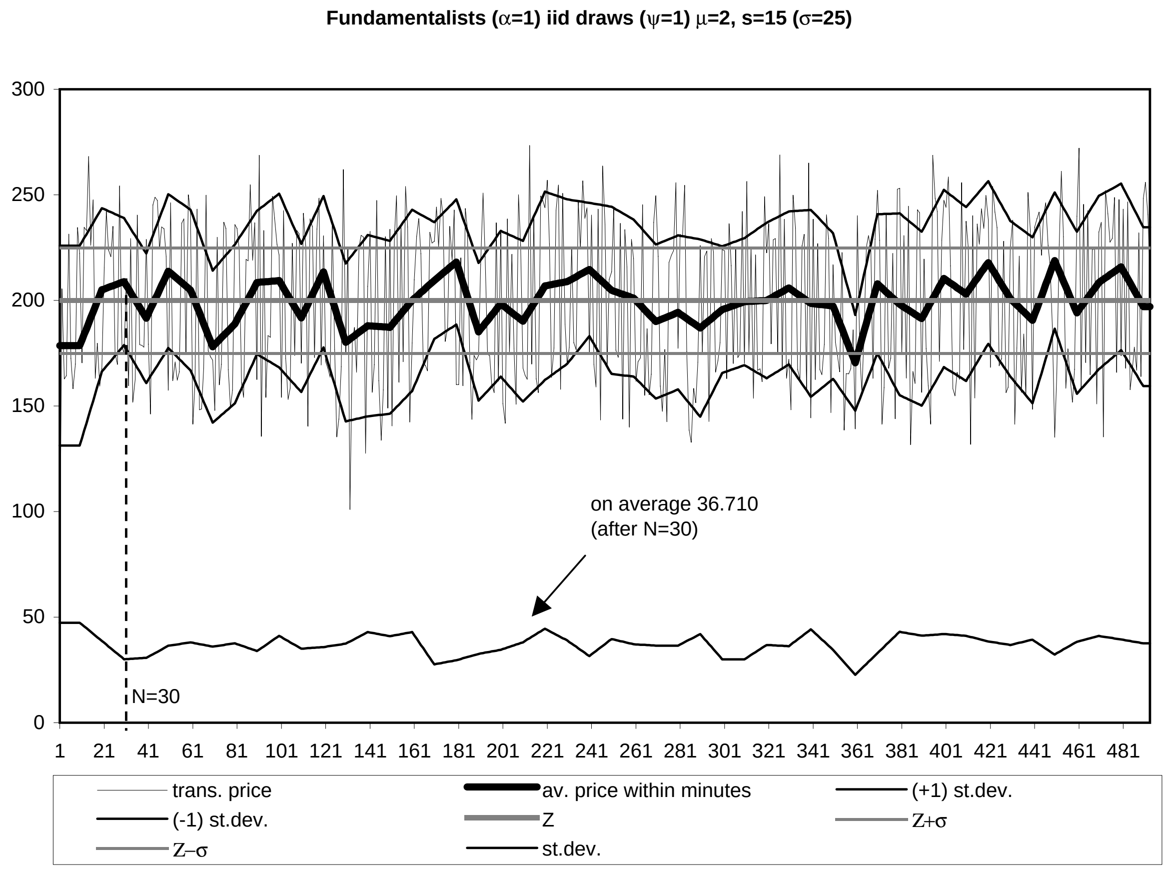

Figure 4: Another instance of the path of prices for Experiment 2B

5 Final remarks

The experiments described in this paper served the purpose of test-running FTR and exploring its flexibility in depicting specific contexts as sub-cases. Thus, we did not pursue novel economic content by addressing scarcely investigated questions or devising sophisticated specifications of our Toy-Room. Rather, we selected a widely investigated issue, and maintained the specification and the experiment-plan as simple as possible, to be able to "benchmark" and readily check the consistency of our simulation outcomes.

On the other hand, even this preliminary set-up is rich enough to permit further inquiries. First, one could perform a second round of experiments in which the room is inhabited by both fundamentalists and noise traders at the same time: for values of α inside $(0, 1)$, one could investigate how persistence of trade and the time-path of prices are affected by

- the share of fundamentalists in the room
- the ratio between updating paces of fundamentalists and noise traders
- the ratio between standard deviation guesses of fundamentalists and noise traders.

Relation to the underlying return value and volatility of the path of prices would allow us to asses their *informational content*, and hence the *ex-post soundness of traders' behaviors*, under different parameters settings.

Moreover, as mentioned in Section 2.5, the set-up could be easily enlarged to encompass non-spot trading, an entry process, and a non-trivial dynamics for the external return. Those aspects could all affect persistence and the time-path of prices in a substantial way.

Last, these experiments represent instances of a much broader class of investigations concerning the analysis of "ecologies" composed by two or more trader-types, in terms of aggregate dynamics and performance of different types [38].

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[38] Some of these investigations are currently being carried out by some members of the T.E.D. project at I.I.A.S.A.