RM-76-46

WATER ECONOMY PROGRAM EVALUATION WITH RESPECT TO TOTAL LOSSES EXPECTED

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May 1976

Preface

A problem of water economy development is considered in this paper. The problem is to define operational control of water economy elements (reservoirs, canals, etc.), taking into account their long-term development dynamics. By long-term development dynamics we mean the creation and expansion of reservoirs, canals and other hydrotechnical structures over a given planning period (about 5-10 years). Also, we want to determine the additional capacities of hydrotechnical structures, schedules for their installation, and their operational control in order to minimize water economy losses expected over the planning period. The supply of resources required for water economy development is a given exogenous variable. The problem is solved numerically under a few assumptions concerned with water distribution over alluvial planes, irrecoverable water, and a mathematical form of the objective function.

Abstract

Given the management goal of minimizing the total losses that occur within the existing construction and design of a water economy system, particular problem of water economy development is stated and solved numerically.

The paper is a continuation of a study in complex water economy planning (RR-75-27). As before, it is assumed that the planning process is consistent with the following procedures: formulation of all development alternatives considered, calculation of all alternative development programs, and water economy damage estimation for each alternative development program.

The problem of total loss minimization is a particular case of a general problem statement. It allows one to reduce the latter two planning procedures to a single one and solve it effectively. With this approach it is possible to simultaneously obtain solutions to both short-term control and long-term development strategy for a given water economy system.

Water Economy Program Evaluation With Respect to Total Losses Expected

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I. Introduction

This paper is a continuation and development of the previous study in water economy planning [1]. The approach under consideration was proposed to us by Professor Z. Kaczmarek in order to illustrate and clarify the general methodology and technique presented in [1]. Here we consider a particular case where all goals (interests) of management are reduced to the minimization of total losses which occur with the existing construction and design of water economy systems.

Thus we consider water economy as an individual branch connected with other branches within the framework of centralized management. We will deal with normative planning of the branch development at high management level.

It is assumed that the planning process is consistent with the following successive procedures:

- formulation of all conceivable development alternatives,
- calculation of all alternative development programs,
- estimation of damage or total losses which can occur within the branch with respect to each given development alternative program.

Since the project evaluation problem is stated in a less general form than in [1], the latter two procedures can be reduced to a single procedure and we can deal with the corresponding model. Under this approach it is possible to simultaneously obtain the solution of the short-term regulation problem and the solution of the long-term development problem.

As before, we assume the existence of certain relations between branches and directive body (center), which provide the information necessary for the procedures considered (a detailed description can be found in [1]). For simplicity we consider a deterministic river basin model.

II. Problem Statement

On the basis of the given initial information, the whole river basin is represented by the oriented network, whose nodes correspond to separate river reaches, canals, hydrotechnical structures, reservoirs, water users (consumers) of different kinds (cities, administrative regions, agricultural areas, etc.). Arcs or arrows which connect nodes show water flow directions.

For every node or for every water economy element we write down a balance equation. All these equations describe river system dynamics. They can be written as follows:

$$W_i(t + 1) = w_i(t) + \sum_{\ell \in \gamma_i^-} F_{\ell i} - \sum_{k \in \gamma_i^+} F_{ik}(t)$$

$$+ F_{oi}(t) - F_{io}(t) + \sum_{m \in \tilde{\gamma}_i^-} E_{mi}(t) \qquad (1)$$

$$- \sum_{n \in \tilde{\gamma}_i^+} E_{in}(t) - E_{io}(t) \quad ,$$

where

$w_i(t)$ = total amount of water in the i-th element at instant t;

$F_{ij}(t)$ = intensity (or rate) of water running from element i (the network node) to element j at instant t;

$F_{io}(t)$ = intensity of water withdrawal in element i at the instant t (intensity of water flow to another sector of the economy);

$F_{oi}(t)$ = intensity of water inflow into element i from without (surface and underground inflow, precipitations);

γ_i^- = set of preceding (upstream) elements (cross-sections, reservoirs, canals);

γ_i^+ = set of elements into which water flows from element i;

$E_{ji}(t)$ = intensity of water inflow into the i-th from the j-th element due to floods;

$E_{ij}(t)$ = intensity of water outflow from the i-th to the j-th element due to floods;

$E_{io}(t)$ = intensity of irreparable losses of water caused by the flood at moment i;

$\tilde{\gamma}_i^+$ = set of elements into which water flows from element i if a flood occurs;

$\tilde{\gamma}_i^-$ = set of elements from which water flows to the i-th element if a flood occurs;

M = total number of elements in the water system $i,j, = 1,\ldots,M$.

The initial conditions are determined by the state of the basin at the initial moment of the planning period:

$$w_i(t_o) = w_i^o \quad . \tag{2}$$

The values of $w_i(t)$, F_{ij}, are limited by the maximum $(\bar{w}_i(t),\bar{F}_{ij})$ and minimum $(\underline{w}_i(t),0)$ feasible capacity of water reservoirs and canals:

$$\underline{w}_i(t) \le w_i(t) \le \bar{w}_i(t) \quad ; \tag{3}$$

$$0 \le F_{ij}(t) \le \bar{F}_{ij}(t) \quad . \tag{4}$$

If element i corresponds to a reach of a river or canal, then

$$\bar{w}_i(t) = 0 \quad , \tag{5}$$

as there is no accumulation of water in such reaches. $F_{oi}(t)$ are given functions of time. $E_{ij}(t)$, $E_{io}(t)$ are certain given functions of all other variables and are determined by the amount of water and by the relief in the vicinity of a given reach.

Flood water flows are subject to the following natural constraints:

$$E_{ij}(t) \geq 0 \quad , \quad E_{jo}(t) \geq 0 \quad .$$

We have to make a few assumptions with respect to processes which take place at each element when floods occur.

The value E_{io} is a total sum of such components of water balance as filtration, evaporation and accumulation of water in the vicinity of a given river reach. Defining this value we assume the following hypotheses to hold:

Hypothesis 1

$$E_{io}(t) = \alpha_i(t)\left(\sum_{m \in \tilde{\gamma}_i^-} E_{mi}(t) + \sum_{n \in \tilde{\gamma}_i^+} E_{in}(t) \right) \quad , \tag{6}$$

$$0 \leq \alpha_i(t) \leq 1 \quad .$$

This means that the total water amount which leads to the formation of swamps in the neighborhood of the i-th element is a certain portion of the water which goes through the i-th element from various sources and from the i-th element to other elements. This sum does not include water which moves within the river bed and within the dike area in particular. Thus under the hypothesis we implicitly take into consideration space distribution of river basin elements (Fig. 1).

Then we assume that flood outflow is distributed between elements of the set $\tilde{\gamma}_i^+$ in proportions which are dependent on a landscape of the given regions. Thus we have the following:

Hypothesis 2

$$E_{in}(t) = \beta_{in}\left(\sum_{\ell \in \gamma_i^-} F_{\ell i}(t) - \sum_{k \in \gamma_i^-} F_{ik}(t) + F_{oi}(t) \right. \tag{7}$$

$$\left. - F_{io}(t) + (1-\alpha m(t)) \sum_{m \in \tilde{\gamma}_i^-} E_{mi}(t) + W_i(t) - \bar{W}_i(t) \right)_+ \quad ,$$

where

$$(x)_+ = \begin{cases} x \quad , & \text{if} \quad x \geq 0 \\ 0 \quad , & \text{if} \quad x < 0 \end{cases}$$

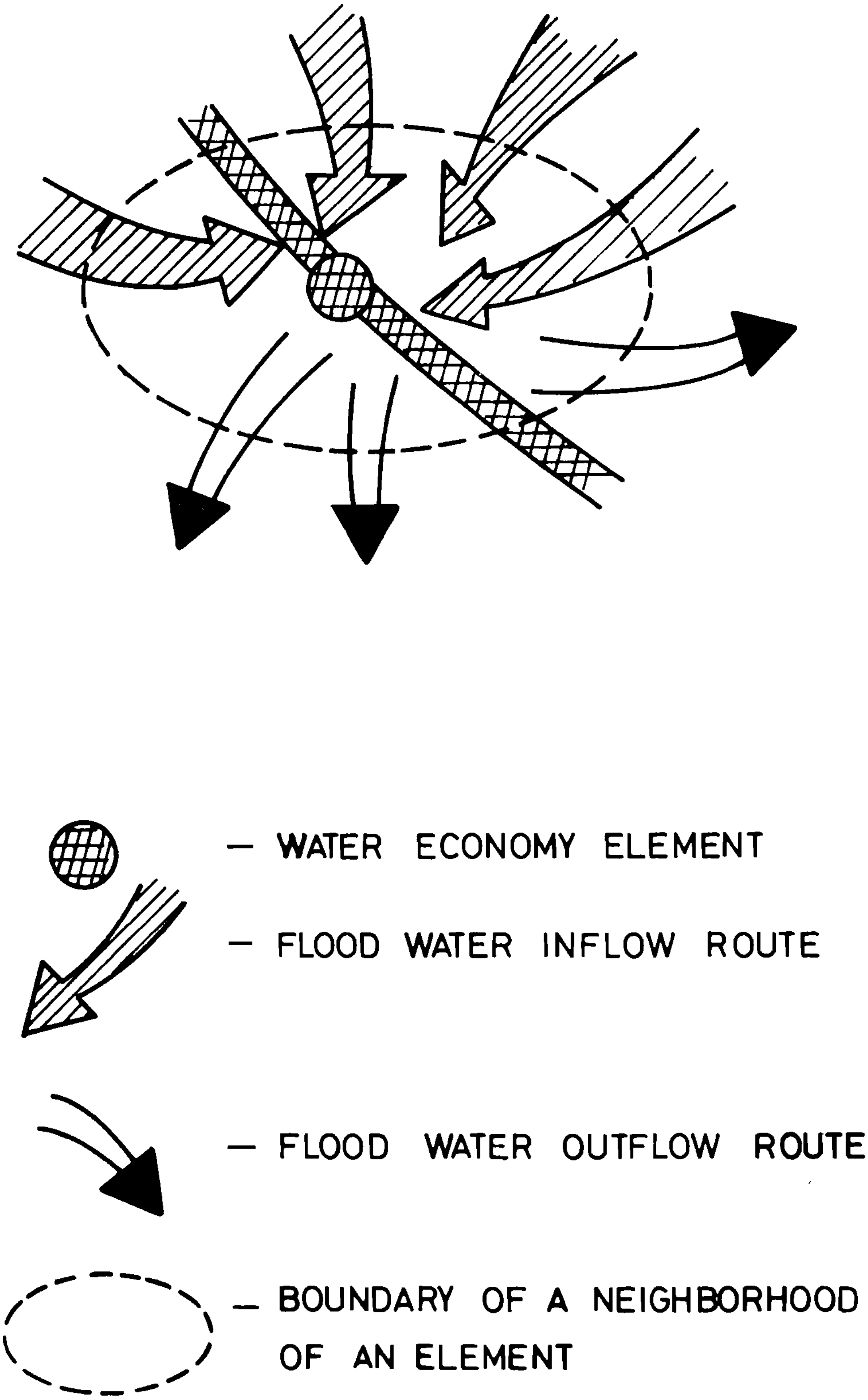


Figure 1 . Space distribution of river basin flows

and

$$\sum_{n \in \tilde{\gamma}_i^+} \beta_{in} = 1 \quad , \quad 0 \leq \beta_{in} \leq 1 \quad .$$

This means that the amount of water which goes from the i-th to the n-th element when a flood occurs is a portion of the water amount which cannot be kept within the river-bed part of the river. A typical situation is shown in Figure 2. Note that the problem does not become more complex if we assume coefficients α and β to be dependent on time. It allows one to simulate, for instance, whether to eliminate changes over time. Determination of these coefficients can be done, on the one hand, by retrospective expert estimation; on the other hand, specific mathematical models are able to provide information required for their evaluation.

Our third assumption concerns water consumption.

<u>Hypothesis 3</u>

$$w_i(t) \leq d_i^w(t) \quad ,$$

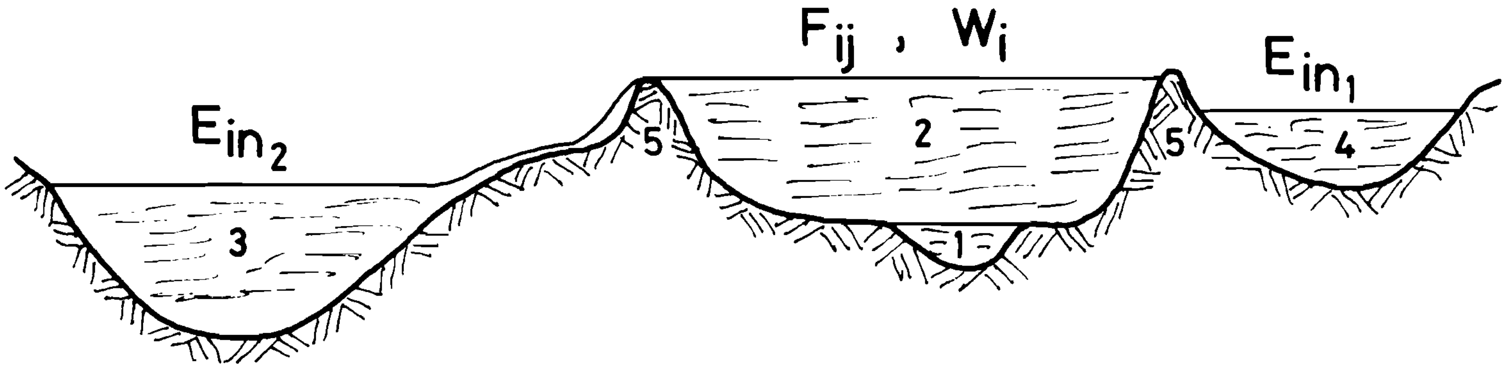
$$F_{io}(t) \leq d_i^f(t) \quad . \tag{8}$$

This means that supply should not exceed water requirements (demands) of consumers at each water economy element. Here d_i^f is water demand of consumers who are exploiting water in a given river or canal area, and d_i^w is the demand of consumers exploiting water in the i-th reservoir. We consider the functions $d_i^f(t)$ and $d_i^w(t)$ to be defined over all planning period T.

We assume that the criterion for losses is additive with respect to consumers and is a sum of penalties (expressed in monetary units) for water shortages of consumers and losses caused by floods.

<u>Hypothesis 4</u>

The objective function is as follows:



1 - RIVER BED CROSS SECTION,

2 - CROSS SECTION WITHIN DIKE AREA,

3,4 - CROSS SECTION OF NEIGHBORING AREA,

5 - DIKE CROSS SECTION

Figure 2 : River reach cross – section

$$J = \sum_{t=t_o}^{T} \sum_{i=1}^{N} c_i^W\left(d_i^W(t) - w_i(t)\right) + c_i^F\left(d_i^F(t) - F_{io}(t)\right)$$

$$+ \sum_{n\in\gamma+} c_{in}^F\left(d_{in}^F(t) - F_{in}(t)\right) + c_i^F\left(\sum_{n\in\tilde{\gamma}_i} E_{nj} + \sum_{m\in\tilde{\gamma}_i^+} E_{im}\right)$$

$$+ c_i^O E_{io}(t) \quad , \tag{9}$$

where

c_i^W = penalty for insufficient water amount in the i-th reservoir (navigation, environment);

c_i^F = penalty for insufficient water supply of the i-th element (city, industry, agriculture, etc.);

c_{in}^F = penalty for insufficient water flow in the in-th river bed or canal (navigation, hydropower stations, etc.);

c_i^E = penalty for damage caused by floods in the neighborhood of the i-th element;

c^O = penalty for water losses and corresponding consequences (e.g. swamping of agricultural areas).

The problem is to find short-term regulation rules (controls $F_{ij}(t)$, F_{io}) and long-term development control (expansion and reconstruction schedules of existing water economy structures) (controls $u^k(t)$) which minimize total losses J over the planning period.

It easily can be seen that taking into account (8) and (9), the problem can be reformulated as follows:

$$\mathfrak{J} = \sum_{t=t_o}^{T} \sum_{i=1}^{N} c_i^E\left(\sum_{n\in\tilde{\gamma}_i^-} E_{ni}(t) + \sum_{m\in\tilde{\gamma}_i^+} E_{im}(t)\right) + c_i^O E_{io}(t)$$

$$- c_i^W w_i(t) - c_i^F F_{io}(t) - \sum_{n\in\gamma_i^+} c_{in}^F F_{in}(t) \to \mathrm{Min} \tag{10}$$

subject to

$$w_i(t+1) = w_i(t) + \sum_{\ell \in \gamma_j^-} F_{\ell i}(t) - \sum_{k \in \gamma_i^+} F_{jk}(t)$$

$$+ F_{oi}(t) - F_{io}(t) + \left(1 - \alpha_i(t)\right) \sum_{m \in \gamma_{\bar{i}}} E_{mi}(t)$$

$$- \sum_{k \in \gamma_i^+} E_{in}(t) \quad , \tag{11}$$

$$E_{in}(t) = \beta_{in} \Bigg(\sum_{\ell \in \tilde{\gamma}_i^-} F_{\ell i}(t) + \sum_{k \in \tilde{\gamma}_i^+} F_{ik}(t) + F_{oi}(t)$$

$$- F_{io}(t) + (1-\alpha m(t)) \sum_{m \in \tilde{\gamma}_{\bar{i}}} E_{mi}(t) + W_i(t) - \dot{W}(t) + \tag{12}$$

$$\sum_{n \, \gamma_i^+} \beta_{in} = 1 \tag{13}$$

$$0 \le \beta_{in} \le 1 \quad n \in \tilde{\gamma} \quad , \tag{14}$$

$$w_i(t) \le d_i^W(t) \quad , \tag{15}$$

$$F_{io}(t) \le d_i^F(t) \quad , \tag{16}$$

$$0 \le \bar{F}_{ij}(t_o) + \sum_{\xi \in G_{ij}} f_{ij}^{\xi} z^{\xi}(t) - F_{ij}(t) \tag{17}$$

$$F_{ij}(t) \ge 0 \quad , \tag{18}$$

$$w_i(t) \le w_i(t_o) + \sum_{\eta \in V_i} \omega_i^{\eta} z^{\eta}(t) \quad , \tag{19}$$

$$w_i(t) \ge \underline{w}_i \quad , \tag{20}$$

$$z^j(t+1) = z^j(t) + u^j(t) \prod_{\ell \in \Gamma_{\bar{j}}} \Theta(z^\ell - 1)\Theta(1 - z^j(t)) \quad (21)$$

$$z^j(t_o) = z_o^j \quad , \quad (22)$$

$$z^j(t) \leq 1 \quad , \qquad j = 1,2,\ldots,M \quad (23)$$

$$\sum_{j=1}^{M} c_{ij} u^j(t) \leq c^i(t) \quad (24)$$

where

G_{ij} = set of activities whose completion will lead to putting additional canal or river capacities into operation;*

V_i = set of activities which provides the coming into operation of additional reservoir capacities in the i-th element;

ω_i^η = magnitude (or portion) of the useful capacity of the i-th reservoir which can be loaded after completion of the η-th building activity;

c_{ij} = the nominal i-th resource requirement for the j-th activity;

$c^i(t)$ = the i-th resource supply for water economy development within the year t.

For the building dynamics (Eq. 20), see also [2]. Equation (24) merely describes resource supply constraints.

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* We define additional canal capacity (f_{ij}^ξ) as a capacity which can be used by water consumers after completion of the ξ-th building stage of a given water economy structure. For example, canal capacity can be increased by creating additional dikes or dams.

III. Solution of the Problem

Let us introduce the following notation:

Δ^1_{in} = left hand side in (17),

Δ^2_{ij} = right hand side in (19),

$\tau^w_{i_\eta}$ = time moment when right hand side equality holds in (19),

τ^{in} = time moment when equality holds in (15),

$\bar{\tau}^{ij}$ = time moment when equality holds in (17),

τ^i = time moment when equality holds in (20),

τ^j_f = time moment when equality holds in (23).

We modify the problem by introducing discontinuous penalties for violation phase constraints (13), (15), (17), (19), (20) into the objective function:

$$I = \mathfrak{J} + \gamma \sum_{t=t_o}^{T} \sum_{i=1}^{N} \Theta_-\left(w_i(t) - d_i^w(t)\right)$$

$$\Theta\left(\underline{w}_i - w_i(t)\right) + \Theta\left(-\Delta_i^2\right)$$

$$+ \sum_{j\in\tilde{\gamma}_i^+} \Theta\left(-\Delta^1_{ij}\right) \quad ,$$

where

$$\gamma = \text{large positive number},$$

$$\Theta(x) = \begin{cases} 1 & , \text{ if } x > 0 \\ 0 & , \text{ if } x \le 0 \end{cases} .$$

Thus we reduce the solution of the initial problem to the following one:

$$I \to \text{Min}$$
$$\text{s.t. } (11),\ (12),\ (16),\ (18),\ (21),\ (22),\ (24)\ . \tag{25}$$

This is an optimal control theory problem; for its numerical solution, the method developed in [2] can be used. In this case the Hamiltonian is as follows:

$$\begin{aligned} H(u,F,E) = {} & \sum_{j=1}^{M} p_j(t+1)\Theta_-\left(1 - z^j(t)\right) \prod_{\ell\, \Gamma_j} \Theta_+(z^\ell(t) - 1)\, u^j \\ & + \sum_{i=1}^{N} \sum_{k\, \gamma_i^+} \left(\lambda_i(t+1) + c_{ik}^F\right) F_{ik} \\ & - \sum_{i=1} \lambda_i(t+1) \sum_{\ell\, \gamma_i^-} F_{\ell i} \\ & - \sum_{i=1}^{N} \left(\lambda_i(t+1)\, \alpha_i^1 + c_i^1\right) \sum_{m\, \tilde{\gamma}_i^-} E_{mi} \\ & + \lambda_i\alpha_i + c_i \sum_{k\, \tilde{\gamma}_i^+} E_{ik} + c_i^F F_{io}\ , \end{aligned} \tag{26}$$

where

$$\alpha_i^1 = 1 - \alpha_i\ , \quad c_i^1 = c_i^F + \alpha_i c_i^o\ ;$$

and $p_j(t)$, $\lambda_i(t)$ are Lagrange multipliers which satisfy the following dynamical equations:

$$p_j(t) = p_j(t+1)\ ,$$

$$p_j(T) = 0\ ,$$

$$\lambda_i(t) = \lambda_i(t+1) + c_i^w \quad ,$$

$$\lambda_i(T) = 0 \quad ,$$

and jump conditions

$$p_j(t_f^j - 1) = p_j(t_f^j) + \frac{1}{u^j(t_f^j - 1)} \sum_{i \varepsilon \Gamma_j^+} p_i(t_f^j)\, u^i(t_f^j)$$

$$p_j(t_{j\eta}^w - 1) = p_j(t_{j\eta}^w) + \frac{\gamma \omega_\eta^j}{u^j(t_{j\eta}^w - 1)} \quad , \qquad \eta \;\varepsilon\; V_j^-$$

(27)

$$p_j(t^{\xi\eta} - 1) = p_j(t^{\xi\eta}) + \frac{\gamma f_{\xi\eta}^j}{u^j(t^{\xi\eta} - 1)} \quad , \quad \xi\eta \;\varepsilon\; G_j^- \quad ,$$

$$\lambda_i(\tau^i - 1) = \lambda_i(\tau^i) + \frac{\gamma}{\overset{o}{w}_i(\tau^i - 1)}$$

$$\lambda_i(t^i - 1) = \lambda_i(t^i) - \frac{\gamma}{\overset{o}{w}_i(t^i - 1)}$$

(28)

$$\lambda_i(t_{i_\eta}^w - 1) = \lambda_i(t_{i_\eta}^w) + \frac{\gamma}{\overset{o}{w}_i(t_{i_\eta}^w - 1)}$$

$$\lambda_i(\tau^{in} - 1) = \lambda_i(\tau^{in}) + \frac{\gamma}{\overset{o}{w}_i(\tau^{in} - 1)} \quad , \qquad n \;\varepsilon\; \tilde{Y}_i^+$$

where $\overset{o}{w}_i$ = right hand side of the i-th equation in (1).
The algorithm works as follows:

1. Find initial control approximation $\left\{F_{ij}^{(o)}(t), u^{j(o)}(t)\right\}$. We can also take existing water regulation rules without considering construction of new water economy structures ($u^{j(o)} \equiv 0$) as an initial approximation.

2. Integrate equations (1), (21) under given initial conditions (2), (22). During integration we obtain values τ^{in}, τ^{i}, t^{ij}, t_i^w, t_f^j, z, w.

3. Under given z, w, u and **F** integrate systems (27), (28) from t = T to t = t.

4. Find controls which give a maximum to Hamiltonian (26) under given p and λ.

5. Repeat 2-4 until a remarkable decrease of the functional takes place.

Finally we would like to note that the problem presented can be generalized in different ways. For example, nonlinear (convex) penalties can be introduced into the objective function and floods can be considered as floods within dike areas and outside dikes. The latter consideration is especially important when distribution of pollution over water economy elements is taken into consideration in the model.

References

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[2] Zimin, I.N. (1975), *Optimal Control Theory Problems with Network Constraints and Their Applications*, RM-76-13, International Institute for Applied Systems Analysis, Laxenburg, Austria