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Learn-by-doing and Carbon Dioxide Abatement

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Contents

Abstract

There are inherent difficulties in solving LBD (learn-by-doing) models. Basic to such models is the idea that the accumulation of experience leads to a lowering of costs.

This paper is intended to explore some of the algorithmic issues in LBD modeling for carbon dioxide abatement. When using a standard algorithm for nonlinear programming, there is no guarantee that a local LBD optimum will also be a global optimum. Fortunately, despite the absence of guarantees, there is a good chance that one of the standard algorithms will produce a global optimum for models of this type, particularly if there is an artful selection of the starting point or of the terminal conditions. Moreover, there is a new procedure named BARON. In the case of small models, a global optimum can be recognized and guaranteed through BARON.

Eventually, it should be possible for BARON or a similar approach to be extended to large-scale LBD models for climate change. Meanwhile, in order to check for local optima, the most practical course may be to employ several different starting points and terminal conditions.

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About the Authors

Professor Alan S. Manne was born in 1925 in New York. He received his A.B. in 1943 and his Ph.D. in economics in 1950 from Harvard University. Subsequently, he taught at Harvard and Yale, and worked at the RAND Corporation, at the U.S. Agency for International Development in India and at IIASA. During most of the years between 1961 and 1992, he was a professor of operations research at Stanford University. He retired in 1992, and is currently a professor emeritus. He is co-organizer of the International Energy Workshop.

He is the author or coauthor of seven books and 125 published papers. His most recent book, coauthored with Richard Richels, was published in 1992 by the MIT Press. Its title: *Buying Greenhouse Insurance - the Economic Costs of* CO_2 *Emission Limits*.

He is a Fellow of the Econometric Society, Fellow of the American Academy of Arts and Sciences and a Fellow of the National Academy of Engineering. He has received the Lanchester Prize of the Operations Research Society of America, the Larnder Prize of the Canadian Operational Research Society and the Paul Frankel Award of the U.S. Association for Energy Economics. He has received an honorary doctorate from Goteborg University in Sweden and the University of Geneva.

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Learn-by-doing and Carbon Dioxide Abatement

Alan S. Manne and Leonardo Barreto

1. Introduction

There are inherent difficulties in solving LBD (learn-by-doing models. Basic to such models is the idea that the accumulation of experience leads to a lowering of costs. This idea goes back to the model of Arrow (1962) – and even earlier to empirical estimates of airframe production costs. Within the context of global climate change, it has been applied by: Goulder and Mathai (2000), Gritsevskyi and Nakićenović (2000), Kydes (1999), Kypreos *et al*. (2000), Kypreos (2000), Mattsson and Wene (1997), Messner (1997), Seebregts *et al*. (2000), TEEM (1999) and Van der Zwaan *et al*. (1999).

This paper is intended to explore some of the algorithmic issues in LBD modeling for carbon dioxide abatement. When using a standard algorithm for convex nonlinear programming, there is no guarantee that a local LBD optimum will also be a global optimum. Fortunately, despite the absence of guarantees, there is a good chance that one of the standard procedures will produce a global optimum for models of this type – particularly if there is an artful selection of the starting point or of the terminal conditions. Moreover, there is a new algorithm named BARON. In the case of small-size LBD models, a global optimum can be recognized and guaranteed through BARON.

For a general idea of what is involved, see Figure 1. There are just two decision variables, x_1 and x_2. The feasible set consists of a convex polygon: all points within the shaded area ABCD. If the minimand is strictly concave, it can happen that point A is a local optimum. That is, it has lower costs than the adjacent extreme point B, but it has higher costs than the distant extreme point C. This is illustrated by the two-dashed iso-cost lines. The curved line going through A indicates higher system costs than the curved line going through C. In any case, the minimum system cost lies at one of the extreme points (the vertices) – *not* between them. (For a rigorous treatment of this proposition, see Hirsch and Hoffman (1961).) Moreover, there may be only a small difference in costs between the extreme points.

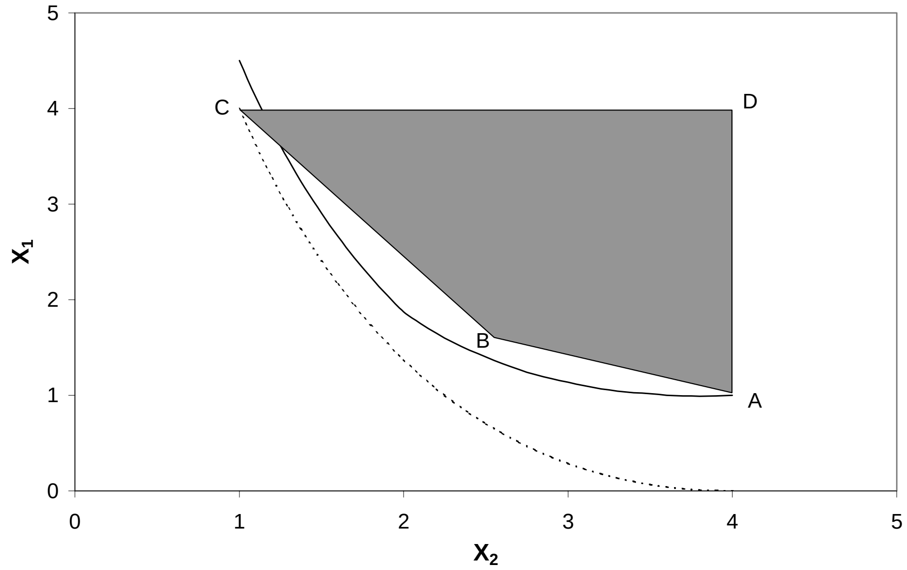

Figure 1: Two-dimensional example of a local optimum.

In connection with the debate over global climate change, a small example of LBD will be examined in this paper. We will show that occasionally one of the standard algorithms fails, but that BARON is successful in producing a global optimum. Simultaneously with this effort, we are applying some of these ideas to a larger, more realistic model known as MERGE. Here the standard algorithms produce plausible solutions, but we have to take any steps that we can to ensure that these represent a global rather than a local optimum.

2. The BARON algorithm

For details on BARON, see Sahinidis (2000) and http://archimedes.scs.uiuc.edu . According to: http://www.gamsworld:

> *BARON is a computational system for solving non-convex optimization problems to global optimality. Purely continuous, purely integer, and mixed-integer nonlinear problems can be solved with the software. The Branch And Reduce Optimization Navigator derives its name from its*

combining interval analysis and duality in its reduce arsenal with enhanced branch and bound concepts as it winds its way through the hills and valleys of complex optimization problems in search of global solutions.

BARON is a tool that allows for the identification of globally optimal solutions. It combines range reduction techniques with an enhanced branch and bound (B&B) algorithm. This combination gives the name to the algorithm: Branch and Reduce. The Branch and Bound algorithm is applied to a (generally convex) relaxation of the original non-convex problem. In each node, a relaxed version of the original problem is solved. If this is a minimization, its solution provides a lower bound for the original non-convex problem. Using this solution as the starting point (or, if available, a better starting point can be used), the original problem is solved and an upper bound for the global optimal solution is found. If the gap between the upper and lower bounds is not small enough, the feasible region is divided in parts. A new relaxed problem is solved for each subdivision and new lower and upper bounds for the global optimum are computed.

The range reduction techniques help to restrict the search space and reduce the relaxation gap. They are applied to every sub-problem of the branch-and-bound search tree in pre- and post-processing steps, helping to improve the performance of the bounding procedure at every node of the tree. Different types of reduction tests can be applied according to the form of the problem. Optimality-based and feasibility-based range reduction tests are possible. Optimality-based range reduction uses the optimal (dual) solution of the relaxed problem to reduce the range of constraints and variables. Feasibility-based range reduction uses heuristic procedures to generate constraints that eliminate infeasible portions of the solution. These constraints approximate the solution of optimization problems that generate improved bounds for the problem variables.

3. The LBD perspective

For an example of LBD, see Figure 2, copied from International Energy Agency, (2000). This reports average unit costs (1990 ECU per kilowatt-hour) for a series of alternative electricity producing technologies in the European Union, 1980-1995. The vertical axis refers to unit costs; the horizontal axis refers to cumulative electricity production at successive dates. With greater “experience” (cumulative production) , there is a pronounced tendency for a decline in the unit costs of novel technologies such as photovoltaics and wind power, but there is no obvious decline in the unit costs of more conventional methods such as supercritical coal and NGCC (natural gas – combined cycle). Significantly enough, nuclear power is *not* plotted on this diagram. If it were plotted, it would almost surely illustrate an increase in unit costs with additional experience – and with additional concerns about reactor safety.

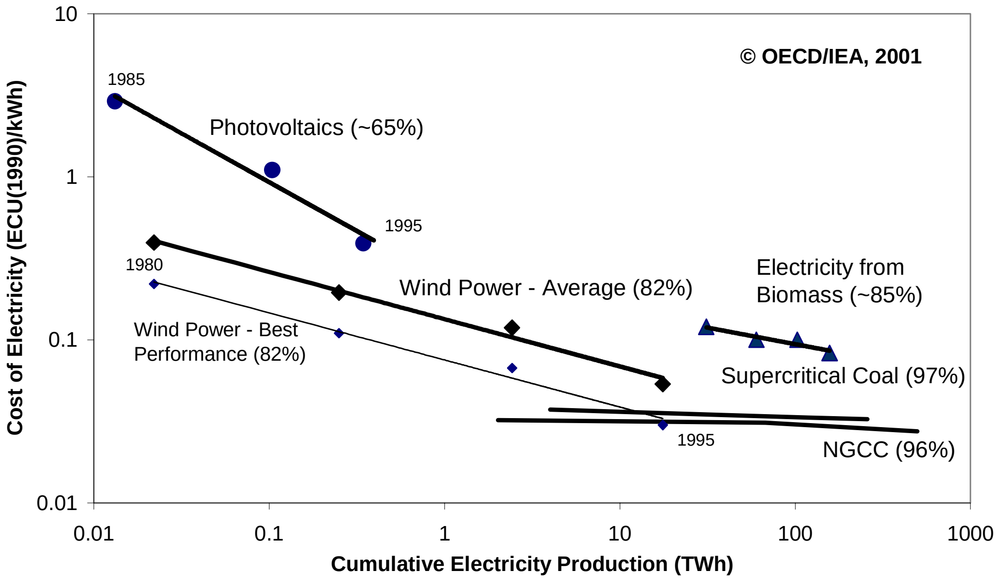

Figure 2: Electric technologies in EU 1980-1995.

In Figure 2, note that the newer technologies tend to be higher in unit costs than the conventional ones. If investors base all their decisions on immediate costs, there would be little tendency to support the newer technologies that are currently more expensive. Their cumulative experience is too small, and they could be “locked out” permanently. This is the rationale for public intervention in the market. LBD entails the acceptance of high near-term costs in return for an expected lowering of future costs. It is an investment choice, and it depends critically upon the rate of discount.

Associated with each technology, Figure 2 shows a “progress ratio” entered in parentheses. This measures the percentage decline in unit costs that is associated with a doubling of experience. In the case of wind, for example, this parameter is shown as

82%. That is, $2^{lrn} = 82\%$. Therefore, the exponent lrn = -.29. This exponent is one of the essential parameters that is entered into an LBD model.

So far, so good. The next analytical issue is the measurement of cumulative experience. Should this be limited to the European Union (as in Figure 2)? Or should it also take account of efforts elsewhere - in Japan, the USA, etc? This is not an easy question to answer. In a global economy, technological experience diffuses widely. It is quite possible that there is a more rapid flow of information between the European and US branches of a given company than between the European branches of different companies.

The geographical range of diffusion is one issue. Another is the measurement of cumulative experience associated with the leftmost point along each curve. If, for example, there is no experience with wind power reported before 1980, how do we measure the cumulative experience at this initial date? The initial cumulative experience is an estimate that must be made thoughtfully, and there are no easy answers. Even in the year 2000, the production of wind and solar electricity provides just a small percentage of the total.

4. A small-scale model of electricity choices

In order to develop a small-scale model of electricity choices, consider the options that are available to the world as a whole. It will be supposed that the world plans to meet the total electricity demands implied by the "reference case" of MERGE. This is a multi-region, multi-technology model for estimating the costs of regional and global greenhouse gas reductions. It is based upon a bottom-up view of energy supplies and a top-down view of energy demands. For details on MERGE, see the website: http://www.stanford.edu/group/MERGE

To meet the reference case demands, suppose that there are just three technologies available:

1. Defender: this is the average type of unit on line in the year 2000; a predominantly fossil mix of technologies, but also includes hydroelectric and nuclear; it is not subject to LBD;

2. Challenger: the initial challenger – the average type of carbon-free technology available in 2000; this is high-cost and subject to learning along the lines of the LBD model; and

3. Advanced: an advanced challenger – the average type of carbon-free technology that might become available in 2050; this is lower-cost and also subject to the endogenous type of learning.

Let the decision variables $X_{j,t}$ denote the quantity of electric energy (trillion kilowatt-hours) produced by technology j in period t (where the time periods refer to successive decades during the 21st century). Together, the three technologies must meet the projected electricity demands. If one is not concerned about carbon accumulation, one could meet these demands solely through technology 1 (the low-cost, predominantly fossil-based option). If one *is* concerned about reducing carbon, there will be a role for the higher-cost carbon-free technologies. The earlier one has the advantage of being available immediately, but the later one has the advantage of being potentially lower in costs. It might, for example, represent nuclear or fusion. Or it might represent advanced developments of wind or of photovoltaic solar – or fossil fuel plants with carbon capture and sequestration.

To express the condition that total demands must be met by a combination of these three technologies, there is first the supply-demand balance constraint:

(1) $$X_{1,t} + X_{2,t} + X_{3,t} \geq E_t$$

Where E_t denotes the demands in decade t.

Next, there are the constraints that none of these technologies may expand too rapidly. To illustrate this idea concretely, suppose that a new technology cannot supply more than 1 % of the market during the first decade in which it is introduced, and that it cannot expand much more rapidly than a factor of four during subsequent decades. We then have:

(2) $$X_{j,t+1} \leq 0.01 * E_t + 4 * X_{j,t} \qquad \text{(for all j,t)}$$

Similarly, to ensure that technologies are not replaced too rapidly, we impose a maximum annual decline rate of 3% per year. For intervals of a decade, this works out as follows:

(3) $$X_{j,t+1} \geq \left(\frac{1}{1.03}\right)^{10} * X_{j,t} \qquad \text{(for all j, t)}$$

In order to keep track of cumulative carbon emissions from the electric power sector, we take the average of emissions at the beginning and the end of each decade. Cumulative emissions through decade t are represented by the decision variable $CARB_t$. They are proportional to the output of technology 1 (the predominantly fossil fuel defender):

(4) $$CARB_{t+1} = CARB_t + 5 * cec * (X_{1,t} + X_{1,t+1})$$

Where cec represents the average carbon emission coefficient during the year 2000. Under a “business-as-usual” scenario, the cumulative emissions would be roughly 700 billion tons during the 21st century. To illustrate a low-carbon scenario – but one in which there is no immediate need for abatement - we impose an upper bound of 400 billion tons on $CARB_T$.

It is assumed that learning costs depend upon the cumulative production experience for each of the technologies. Let the decision variables $Y_{j,t}$ represent this experience. Basing these variables upon an average of the production at the beginning and end of each decade, we have:

(5) $$Y_{j,t+1} = Y_{j,t} + 5 * (X_{j,t} + X_{j,t+1}) \qquad \text{(for all j, t)}$$

The objective function is expressed as one of minimizing the present value of costs – subject to meeting the supply-demand constraints (1), the expansion and decline constraints (2) and (3), the cumulative carbon constraints (4), the cumulative production experience (5), and both upper and lower bounds on individual variables. In order to employ a market-oriented criterion, we let pv_t (the present value factor for period t) be based upon a 5% real rate of return on capital. This is intended to be net of inflation, and represents a before-tax rate of return. Let the decision variable PVC denote the present value of costs throughout the 21st century. For each time period and each technology, we then have two cost components. The first may be termed “static” and the second “dynamic”.

$$(6) \qquad PVC = \sum_t pv_t \left\{ \sum_j Cost_j X_{j,t} + \sum_j inlc_j X_{j,t} \left[\frac{Y_{j,t}}{acc_j} \right]^{lrn_j} \right\}$$

That is, the static terms are proportional to the cost $_j$ factors. These provide a lower bound on the average cost of each technology. The dynamic terms depend upon the cumulative learning experience. This in turn depends upon the $Y_{j,t}$ decision variables – and also upon three parameters: the initial learning cost coefficient $inlc_j$, the initial experience acc_j, and the learning exponent lrn_j. Table 1 shows illustrative values of these individual parameters. In this case, the costs of the fossil fuel defender remain constant over time – at $40 per thousand kWh. The learning exponent is n.a. (not applicable) in this case. Initially, the first challenger’s costs are twice this level: 30 + 50 = $80 per thousand kWh. These costs decline over time with cumulative experience. The advanced challenger has lower initial costs, but does not become available until 2050.

Table 1: Illustrative values of the cost parameters

Technology j	**1** **Defender**	**2** **Challenger**	**3** **Advanced**
Static cost coefficients, cost $_j$ $ per thousand kWh	40	30	30
Initial learning cost coefficients, inlc $_{j,}$ $ per thousand kWh	0	50	10
Initial accumulated experience, acc $_j$, trillion kWh	1	1	1
Learning exponent, lrn $_j$	n.a.	-.2	-.2

Note that the initial accumulated experience parameters acc $_j$ must be chosen with care. Over time – with cumulative experience – the costs of all three technologies will decline toward the limits imposed by the static cost factors. The parameters acc_j must be checked for their comparability with the values of the cumulative production variables $Y_{j,t}$ during the initial decades of the 21st century.

For further details, see the GAMS code attached as an appendix. This will enable the reader to experiment with other parameters. From earlier work, for example, we know that the optimal solution is highly sensitive to the learning exponent, lrn_j.

Just as in the two-dimensional example (Figure 1), the constraint set of this problem is a convex polyhedron. The minimand is concave. A solution must therefore lie at one or another of the extreme points of the polyhedral constraint set. But it is not sufficient to check *adjacent* extreme points. One must somehow be able to verify that distant extreme points are also handled. This is the role played by BARON.

5. Numerical results from the small-scale model

Figure 3 shows the percentages supplied by each of the three technologies in the global, minimum-cost solution to this problem. Each technology follows a unimodal path. That is, there is at most one maximum point for its deployment. There are distinct phases in which one or another expansion/decline constraint is active. The first challenger is not introduced immediately in 2010. With a cumulative carbon constraint of 400 billion tons, there is enough slack in the system so that the challenger does not need to enter until 2020. Thereafter, it expands at a maximum rate until 2050. The fossil defender begins to decline after 2040. In 2050, the advanced challenger begins to enter at its maximum rate, and after 2070, the earlier challenger begins its decline.

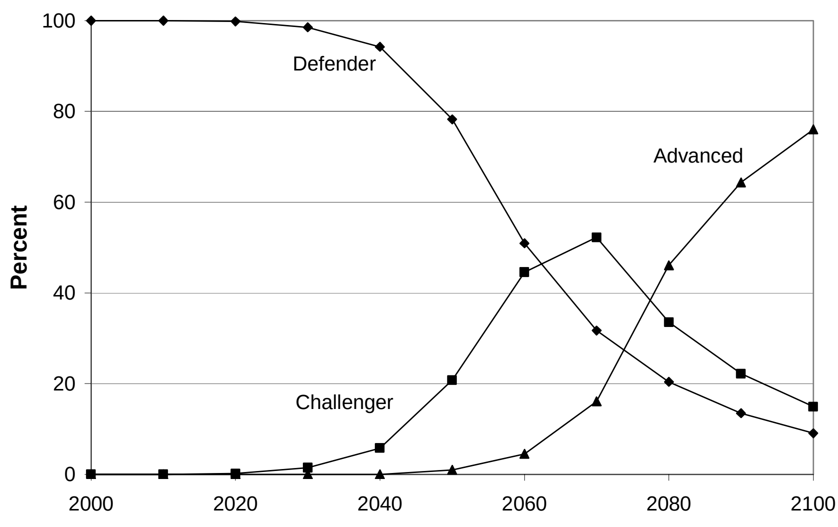

Figure 3: Percentages of demand supplied by alternative technologies.

Along with these introduction patterns, there is a distinct pattern of learning costs. To see how the average unit learning costs change with cumulative experience, see Figure 4. There, results are reported for two alternative values of the accumulated initial learning experience parameter. When acc_j = 1.0, we obtain the upper experience curve. In this case, it is optimal to wait until 2020 before introducing the challenger. Alternatively, if acc_j = 0.1, this provides a more attractive initial point for the challenger. The same unit costs are attained with less experience. In turn, this creates an incentive for more rapid deployment of the challenger – and an earlier date at which costs begin to be lowered.

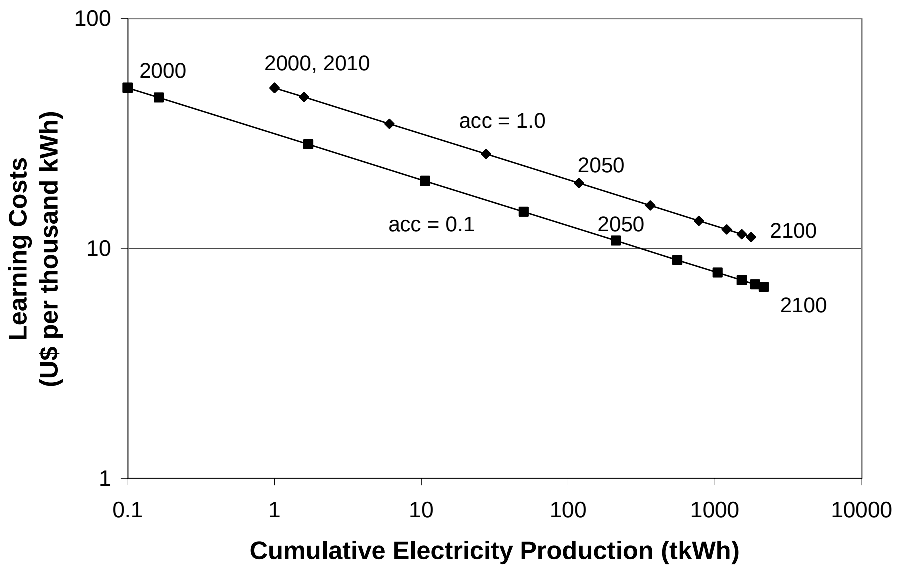

Figure 4: Experience curves for challenger – alternative values of initial experience, acc.

Each of the experience curves is determined by the three dynamic LBD parameters listed in Table 1. The vertical distance of the 2000 value is the initial learning cost coefficient, $inlc_{j.}$ The horizontal distance of the 2000 value is the initial accumulated experience, acc_j. And the slope of the experience curves (on a log-log scale) is given by the learning exponent, $lrn_{j.}$ The rate at which we progress down the experience curve is determined by the endogenous learning process. The less expensive the challenger, the more rapidly it is deployed.

To put things into perspective, it is useful to examine Figure 5. This presents the total of the static plus the dynamic learning costs for the challenger. Both cost curves begin at the same point in 2000 – at twice the level of the defender technology - but they diverge thereafter. Throughout the 21st century, there is no date at which the upper curve lies below the costs of the defender ($40 per thousand kWh). Without a carbon constraint, there would be no rationale to introduce the challenger. With the lower curve, however, the challenger's costs lie below those of the defender from 2050 onward. Under these circumstances, the challenger is introduced at the maximum rate from the earliest date that it becomes available, and the 400 billion ton carbon constraint becomes inactive.

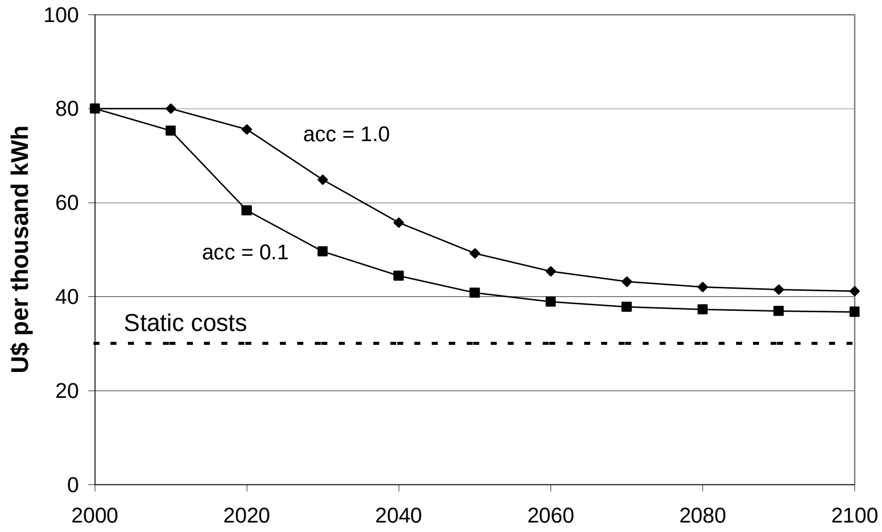

Figure 5: Static and learning costs for challenger.

These projections should not be taken literally, but they do indicate that this type of LBD model has a tendency toward "bang-bang" behavior. That is, a technology may not be introduced immediately when it becomes available. When it is introduced, however, it tends to enter at a maximum growth rate, and eventually to be phased out at a maximum decline rate.

When acc_j = 1.0, the results of BARON are duplicated by two standard convex nonlinear programming algorithms: CONOPT2 and MINOS5. There is a coincidence between the local and the global optimum. How often does this occur? Not always. For example, when we take the same model but eliminate the carbon constraint, we obtain two different solutions. CONOPT2 generates the same global optimum as BARON, but MINOS5 generates a very different local optimum.

Without a carbon constraint, the global optimum is one in which the defender supplies all of the demands through 2040. From 2050 onward, the advanced low-cost challenger then expands at the maximum rate. The locally optimal solution is one in which the defender supplies the world's demands throughout the entire horizon. The other two technologies are both locked out. With a different starting point, MINOS5 produces still a different solution, but again one that is not a global optimum. In all the sensitivity analyses that we have conducted, CONOPT2 has duplicated the same globally optimal solution as BARON, but MINOS5 has produced a number of local optima.

Caveat: these experiments are not conclusive. To our knowledge, there is no theoretical reason for the superiority of one or another of these standard methods when the minimand is concave. Eventually, it should be possible for BARON or a similar approach to be extended to large-scale LBD models. Meanwhile, in order to check for local optima, the most practical course is to apply several different nonlinear programming algorithms – and several different starting solutions with each of them.

6. An alternative approach – terminal conditions

Another possibility is to experiment with alternative terminal conditions. This is the approach that has been applied at a large scale in connection with the MERGE model. To see how this works, consider Figure 6. This is based on the small scale numerical model described in this paper, but the inlc parameter for the advanced technology has been increased to \$20 \$/MWh to provide a clearer example.

For the initial challenger, the cumulative experience through 2100 is shown on the vertical axis, Y(chl, 2100). The cumulative experience for the advanced challenger is shown on the horizontal axis, Y(adv, 2100). Both of these variables are expressed in trillion kilowatt-hours. The feasible combinations of these two variables are shown within the shaded polygon. The lower edge of this area is the 45° line determined by the cumulative carbon emissions constraint. The upper edge is implied by equalities in the supply-demand balances, constraints (1). Again this is a 45° line. The leftmost edge is governed by the lower bound constraint on Y(adv,2100), and the rightmost edge by the combined effect of the expansion and decline constraints.

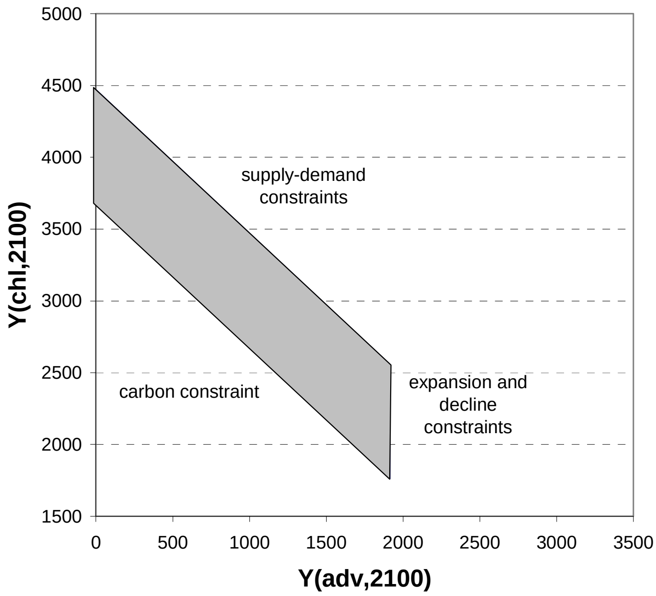

Figure 6: Feasible combinations of the two cumulative experience variables.

Now turn to Figure 7. This shows how the minimum present value of costs varies when we alter the terminal value of the cumulative experience with the advanced technology. It looks as though there is no local minimum. This appears to be a monotone decreasing function, and the global minimum occurs when the advanced technology is brought in at a maximum level.

However, when we take a closer look at the left-hand portion of this diagram (the stacked column graph shown in Figure 8), there is a second local minimum. This occurs at the lowest admissible value of Y(adv,2100). To avoid this local minimum, all that we need to do is to introduce an arbitrary lower bound on this decision variable. E.g., with a lower bound of 20, we rule out the local solution at 1, and the nonlinear solver CONOPT3 proceeds directly to the global optimum at the maximum value of 1912. (Again see Figure 7.)

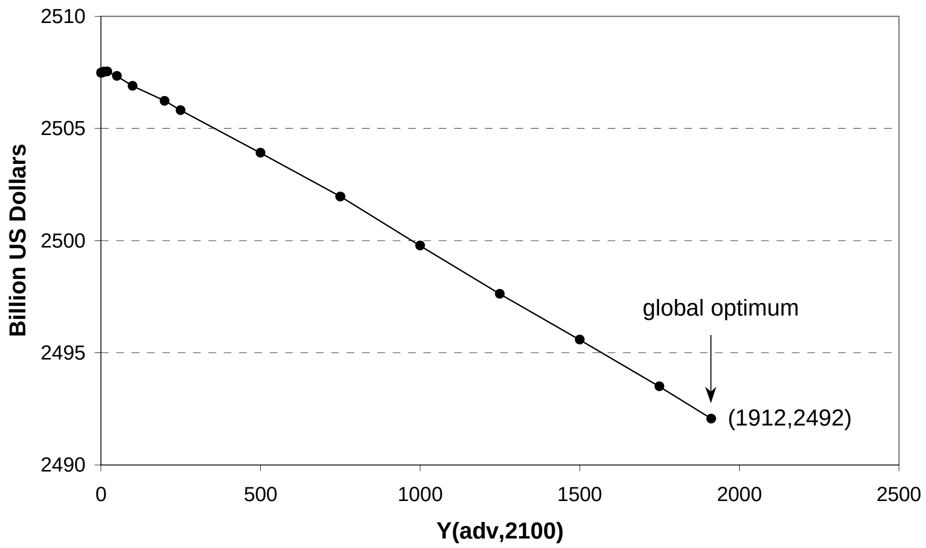

Figure 7: Present value of costs (inlc(adv) increased to 20 $/MWh).

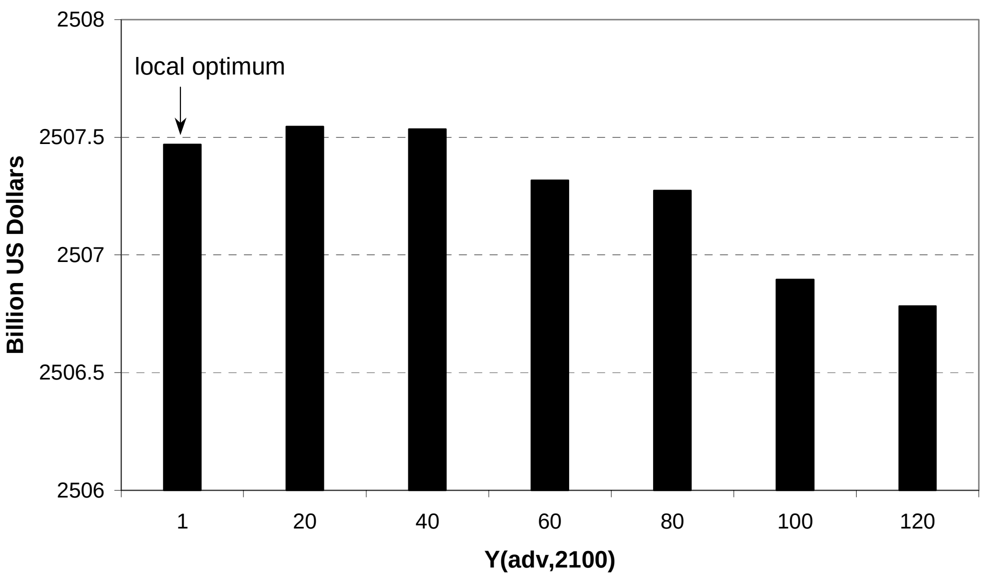

Figure 8: Present value of costs – A closer look (inlc(adv) increased to 20 $/MWh).

A similar procedure has been applied to MERGE, and it seems to work well. Caveat: in MERGE, there is only one LBD technology for the electric sector and one for the nonelectric sector. With several LBD technologies, these arbitrary bounds would have to be selected with greater care. There would then be considerable value in developing an algorithm such as BARON – one which is guaranteed to find a global optimum. Until such a procedure is developed, it will be useful to employ the terminal conditions heuristic.

APPENDIX

A Small-Scale LBD Model for the Electricity Sector

```
Title: Learn-by-doing

option limrow = 100;
option limcol = 100;
option nlp    = conopt2;

SETS

    t    time periods

       /2000, 2010, 2020, 2030, 2040,
        2050, 2060, 2070, 2080, 2090, 2100/

    j   technologies
       /def, chl, adv/

PARAMETER elecdem(t)   World electricity demand - tkwh - reference
case

*  Based on M4-3dist.zip, 06-02-01.

/2000    12.735
 2010    18.523
 2020    24.420
 2030    30.729
 2040    41.698
 2050    52.802
 2060    65.155
 2070    81.675
 2080    98.667
 2090   115.501
 2100   133.561 /

PARAMETER tax(t)       Global carbon tax - dollars per ton;

        tax(t) = 0;

PARAMETER cost(j)   Asymptotic unit cost - $ per thousand kwh

    / def        40
      chl        30
      adv        30    /


PARAMETER inlc(j)   Initial learning cost - $ per thousand kwh

    / def         0
      chl        50
      adv        10    /


PARAMETER acc(j)    Initial cumulative production - tkwh
```

```
* Following parameter must be chosen thoughtfully.  Compare with Y(t)
results.

    / def         1
      chl         1
      adv         1    /

PARAMETER lrn(j)    Learning parameter

    / def         -.2
      chl         -.2
      adv         -.2  /

SCALAR
  cec  carbon emissions coefficient in 2000;
  cec  = 2.166/12.735;

PARAMETERS
  decf    maximum decline factor - defender
  pv(t)   present value factor;

  decf    =  (1/1.03)**10;
  pv(t)   =  (1/1.05)**(10*(ord(t) - 1));

display decf, pv;

POSITIVE VARIABLES
   X(j,t)  electricity supplied by technology - tkwh
   Y(j,t)  accumulated electricity supplied by technology - tkwh
   CARB(t) accumulated carbon supplied by fossil fuel defender -
billion tons

VARIABLES
   PVC     present value of costs - billion dollars

EQUATIONS
   DEM(t)       supply-demand balances - tkwh

   DEC(j,t)     decline constraints - tkwh

   EXP(j,t)     expansion constraints - tkwh

   YDF(j,t)     definition of accumulated electricity supplies - tkwh
   CARBDF(t)    definition of accumulated carbon emissions - billion
tons
   PVCDF        definition of present value of costs - billion
dollars;

*   Supply-demand balances.
DEM(t)..        sum(j, X(j,t))   =g=  elecdem(t);

*   Decline constraints.
DEC(j,t+1)..           X(j,t+1)  =g=  decf *  X(j,t);

*   Expansion constraints.
EXP(j,t+1)..           X(j,t+1)  =l=  .01 * elecdem(t+1)
                                   +    4 * X(j,t);

*   Accumulated electricity supplies.
```

```
YDF(j,t+1)..      Y(j,t+1)       =e=    Y(j,t)  + 5 * (X(j,t)  +
X(j,t+1));

*   Accumulated carbon emissions.
CARBDF(t+1)..      CARB(t+1)     =e=    CARB(t)
        +  5 * cec * (X("def",t) + X("def",t+1));

*   Present value of costs..

PVCDF..         PVC           =e=  sum (t, pv(t)* (

*   Static costs
                  sum(j, cost(j) * X(j,t))

*   Dynamic costs

              +  sum(j, inlc(j)  * X(j,t)  * (  Y(j,t) /acc(j)
)**lrn(j)   )

*   Carbon taxes
               +  tax(t)  *  cec * X("def",t)/elecdem("2000")

                                                    ));

*   Avoid nasty program calls.
    Y.LO(j, t+1)  =   acc(j);

*   Limit on accumulated carbon.
    CARB.UP("2100")       =   400;

*   Initial conditions
    X.FX("def","2000")     =   elecdem("2000");
    Y.FX(j    ,"2000")     =   acc(j);

    CARB.FX("2000")  =   0;

*   Limits on advanced challenger
    X.UP("adv", "2010")    =   0;
    X.UP("adv", "2020")    =   0;
    X.UP("adv", "2030")    =   0;
    X.UP("adv", "2040")    =   0;

MODEL  LBD   /all/;

LBD.OPTFILE  = 1;

SOLVE LBD minimizing PVC using nlp;

PARAMETERS
  lrng(j,t)      learning costs - mills per kwh
  crtx(t)        carbon taxes - dollars per ton;

  lrng(j,t)   =  inlc(j) * (Y.L(j,t)/acc(j))**lrn(j);
  crtx(t+1)   =  carbdf.m(t+1)/pv(t+1);

  display lrng, crtx;
```

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