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A COMPUTER METHOD FOR PROJECTING A POPULATION'S SEX-AGE STRUCTURE

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PREFACE

While simulating the health care system's activity, as well as many other social-economic systems, it proves to be necessary to take into consideration the dynamics of the population sex-age structure.

One of such possible models, which gives us an opportunity to predict the sex-age structure, is presented in this paper.

The model under description should be considered as part of a common health care system's activity model. At present the work connected with the model is being carried out in the IIASA Bio-Medical Project.

ABSTRACT

A computer model which gives an opportunity to predict dynamics of the population sex-age structure is presented in this paper. The system's behavior described depends on such demographic characteristics as birth rate, death rate, and others.

The given model is supposed to be used as part of a general health care system's activity model. In that case it will be possible to investigate the sex-age structure dynamics in connection with the influence of external social-economic subsystems and policy in the health care system area.

The USA demographic statistic data for 1968 have been used for the model's tests, and satisfactory results have been achieved during these tests.

A Computer Method for Projecting a Population's Sex-Age Structure

Alexandre A. Klementiev

This paper proposes a model for investigating the dynamics of a region's sex-age structure (SAS). In the model, indicators for birth rate, death rate and migration are taken into account for each sex-age group.

The need to study population dynamics is apparent in various scientific fields. In the field of health care systems modelling, there are a number of reasons for forecasting the dynamics of the SAS. Among these are:

1. The SAS determines demands made by a population on the resources of the health care system (HCS). This is so because the prevalence of disease in a population is influenced substantially by its SAS.

2. There is also a reverse effect: the HCS influences the birth rate and death rate.

A general approach to HCS modelling will be set forth in a forthcoming paper. In this paper only a demographic model in a form that would facilitate its inclusion in a general HCS dynamic model is presented.

1. THE MODEL'S STRUCTURE AND ITS SYSTEM OF NOTATIONS

The model's structure is presented in Figure 1. Here is used the system of notations employed in the computer language DYNAMO. The entire population is divided into twenty-three sex-age groupings (strata). The population of Stratum I is equal to PN(I), I = $\overline{1,23}$. The sex-age composition of all the strata is given in Table 1.

The number of individuals in each stratum is calculated at every time increment. Each time increment is defined as being one year in length. With this established, the following is taken into account:

- output from each stratum due to mortality,

- output from each stratum due to aging,

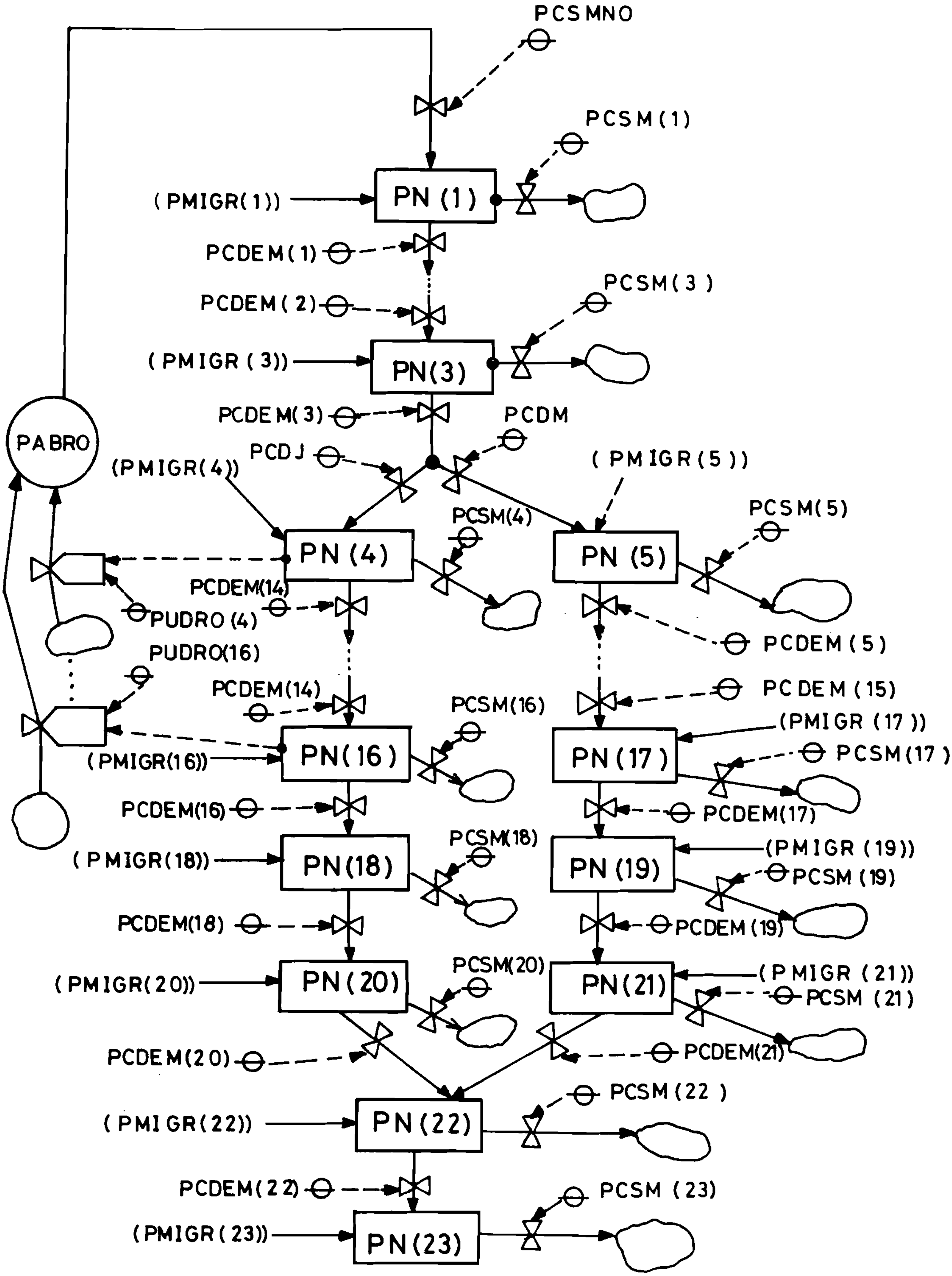
PCSMNO
PCSM(1)
(PMIGR(1))
PN (1)
PCDEM(1)
PCDEM(2)
PCSM(3)
(PMIGR(3))
PN(3)
PCDEM(3)
PCDM
PABRO
PCDJ
(PMIGR(4))
(PMIGR(5))
PCSM(4)
PCSM(5)
PN (4)
PN (5)
PCDEM(14)
PUDRO (4)
PCDEM (5)
PUDRO(16)
PCDEM(14)
PCDEM (15)
PCSM(16)
(PMIGR (17))
PCSM(17)
(PMIGR(16))
PN(16)
PN(17)
PCDEM(16)
PCDEM(17)
PCSM(18)
(PMIGR (19))
(PMIGR(18))
PN(18)
PN(19)
PCSM (19)
PCDEM(18)
PCDEM (19)
PCSM(20)
(PMIGR (21))
(PMIGR(20))
PN(20)
PN(21)
PCSM (21)
PCDEM(20)
PCDEM(21)
PCSM (22)
(PMIGR(22))
PN(22)
PCDEM(22)
PCSM (23)
(PMIGR(23))
PN(23)

FIGURE 1

Table 1

Age-Sex Structure

Stratum I	1	2	3	4	5	6	7	8	9	10	11	12
Sex	♀+♂	♀+♂	♀+♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
Age (years)	0÷4	5÷9	10÷14	15÷19	15÷19	20÷24	20÷24	25÷29	25÷29	30÷34	30÷34	35÷39
Stratum I	13	14	15	16	17	18	19	20	21	22	23	
Sex	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀+♂	♀+♂	
Age (years)	35÷39	40÷44	40÷44	45÷49	45÷49	50÷54	50÷54	55÷59	55÷59	60÷70	70+	

- input into each stratum due to migration*,
- input into each stratum due to aging in a preceding stratum.

Newborns constitute input into the first stratum at a rate equal to PABRO. Here, the perinatal death rate PCSMO is separated from general death rate in the first age group. The birth rate indicators PUDRO(I) are specified for each age group of women of child-bearing age ($I = 2K$, $K = \overline{2,8}$). There is no division according to sex either for children up to 15 years of age or individuals over sixty. Output from Stratum I due to aging (in a downward direction, in Figure 1) is defined by the transition coefficient PCDEM(I), which depends on the death rate in this stratum. Output from the third stratum also depends on the coefficients PCDJ and PCDM, which will be defined below.

In tests conducted using the model, death rate and birth rate indicators were considered as constants. For the future it is proposed that these indicators be made dependent on HCS activities and on environmental conditions.

The following notations are used in the present work:

I - index of the sex-age group (stratum) (see Table 1);

PN(I) - number of individuals in the I-th stratum (in thousands);

PND - total number of children in strata $I = \overline{1,3}$ (in thousands);

PNJ - total number of women in strata I = 4,6,...,20 (in thousands);

PNM - total number of men in strata I = 5,7,...,21 (in thousands);

PNST - total number of aged individuals in strata I = 22,23 (in thousands);

POPUL - total number of individuals (in thousands):

PCSM(I) - death rate in the I-th stratum; equal to the number of deaths in this stratum per year, per thousand;

PCSMNO - perinatal death rate; equal to the number of newborns that die per year;

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* This may be a negative value if in-migration is less than out-migration.

PUDRO(I) - birthrate in the I-th stratum; equal to the number of births per year per thousand women of child-bearing age of the corresponding age group;

PABRO - absolute birthrate; the number of births per year (in thousands);

PMIGR(I) - population migration into the I-th stratum (in thousands);

PVIH(I) - the number of individuals according to age that depart from the I-th age group;

DT - the time increment;

N - the time period for the forecast;

T - age interval covered by a given stratum.

Using the sex ratio (SERA), the coefficients PCDJ and PCDM are determined from:

$$PCDJ + PCDM = 1$$

$$PCDM/PCDJ = SERA \quad .$$

2. ALGORITHM

Let us consider a case where migration is equal to zero. The dynamics of the SAS are then described in the following way:

$$PN(I)_{t+1} = PN(I)_t + [PVH(I)_t - PVIH(I)_t - PSM(I)_t] * DT$$

$$PVIH(I)_t = PN(I)_t * PCDEM(I)_t$$

$$PSM(I)_t = PN(I)_t * PCSM(I)_t$$

$$PCDEM(I)_t = \begin{cases} \dfrac{(1-PCSM(I)_t * 0.001)^T * PCSM(I)_t * 0.001}{[1-(1-PCSM(I)_t) * 0.001]^T} & , I = \overline{1,22} \\ 0 & , I = 23 \end{cases}$$

$$PVH(I)_t = \begin{cases} PABRO_t * (1 - PCSMNO_t) & , \quad I = 1 \\ PVIH(I-1)_t & , \quad I = 2,3,23 \\ PVIH(3)_t * PCDJ & , \quad I = 4 \\ PVIH(3)_t * PCDM & , \quad I = 5 \\ PVIH(I-2)_t & , \quad I = \overline{6,21} \\ PVIH(20)_t + PVIH(21)_t & , \quad I = 22 \end{cases}$$

$$PABRO_t = 0.001 * \sum_{I=1}^{23} PN(I)_t * PUDRO(I)_t$$

$$PND_t = \sum_{I=1}^{3} PN(I)_t$$

$$PNJ_t = \sum_{K=2}^{10} PN(2K)_t$$

$$PNM_t = \sum_{K=2}^{10} PN(2K+1)_t$$

$$PNST_t = PN(22)_t + PN(23)_t$$

$$POPUL_t = \sum_{I=1}^{23} PN(I)_t$$

The model may also be presented in matrix form. Let the demographic structure at time t be described by the vector

$$\underline{PN}_t = \{PN(I)_t\}_1^{23} \quad ;$$

then

$$\underline{PN}_{t+1} = \underline{PN}_t * \mathcal{M}_t \quad ,$$

where the matrix

$$\mathcal{M}_t = \left[m_t^{ij} \right]$$

is presented in Appendix 1. The elements of the first column m_t^{i1}, i = 4,6,...,16 depend on the specific birth rate. The element m_t^{11} is defined by the death rate among children in the first stratum. Other non-zero elements of the matrix $\mathcal{M}_t$ depend on the death rate in the corresponding strata.

3. INITIAL DATA AND RESULTS OF CALCULATIONS

The model was tested using the initial data* in Table 2. The time period for the forecast, N, was set as equal to 100 years. PCSMNO was held to be equal to zero since the perinatal death rate is included in the death rate of the first age group. A forecast was made for each stratum. The results of computations for the amalgamated sex-age groups: CHILDREN (PNO), WOMEN (PNJ), MEN (PNM), AGED (PNST), TOTAL POPULATION (POPUL) are given in Figure 2 and Figure 3. The sex ratio (male/female) was taken as equal to 105/100.

Peculiarities of the model include:

- the separating out of the perinatal death rate;
- division into strata taking into account existing health-care statistics;
- up-dating of the strata according to specific indicators for death rate (PCSM) and according to transition coefficients (PCDEM) in order to better take into account the influence of the HCS on the population SAS.

These peculiarities necessitated a somewhat different structure for this model than that for models found in [2,5,6]. Such a structure is more convenient for the model's inclusion in a general HCS model.

4. CONCLUSION

The model presented here allows one to study the dynamics of a population's sex-age structure while taking into account the influence on these dynamics of:

a. perinatal death rate;

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* These data were kindly supplied by Professor A. Rogers and reflect U.S. demographic statistics for 1968.

Table 2

Initial Data

I	1	2	3	4	5	6	7	8	9	10	11	12
PN(I)	16832.7	19581.1	20399.2	9253.5	9459.3	8289.8	7770.6	6722.5	6497.7	5721.5	5491.7	5584.0
PCSM(I)	5.5	0.5	0.4	0.6	1.5	0.7	2.0	0.8	1.9	1.2	2.2	2.0
PUDRO(I)	-	-	-	63.8	-	157.6	-	134.5	-	73.5	-	63.8
I	13	14	15	16	17	18	19	20	21	22	23	
PN(I)	5313.3	6042.6	5713.3	6143.1	5744.4	5644.5	5251.5	5106.2	4681.6	15322.8	12832.5	
PCSM(I)	3.3	3.0	5.0	4.3	7.7	6.3	12.2	9.2	19.0	24.8	76.5	
PUDRO(I)	-	14.4	-	1.7	-	-	-	-	-	-	-	

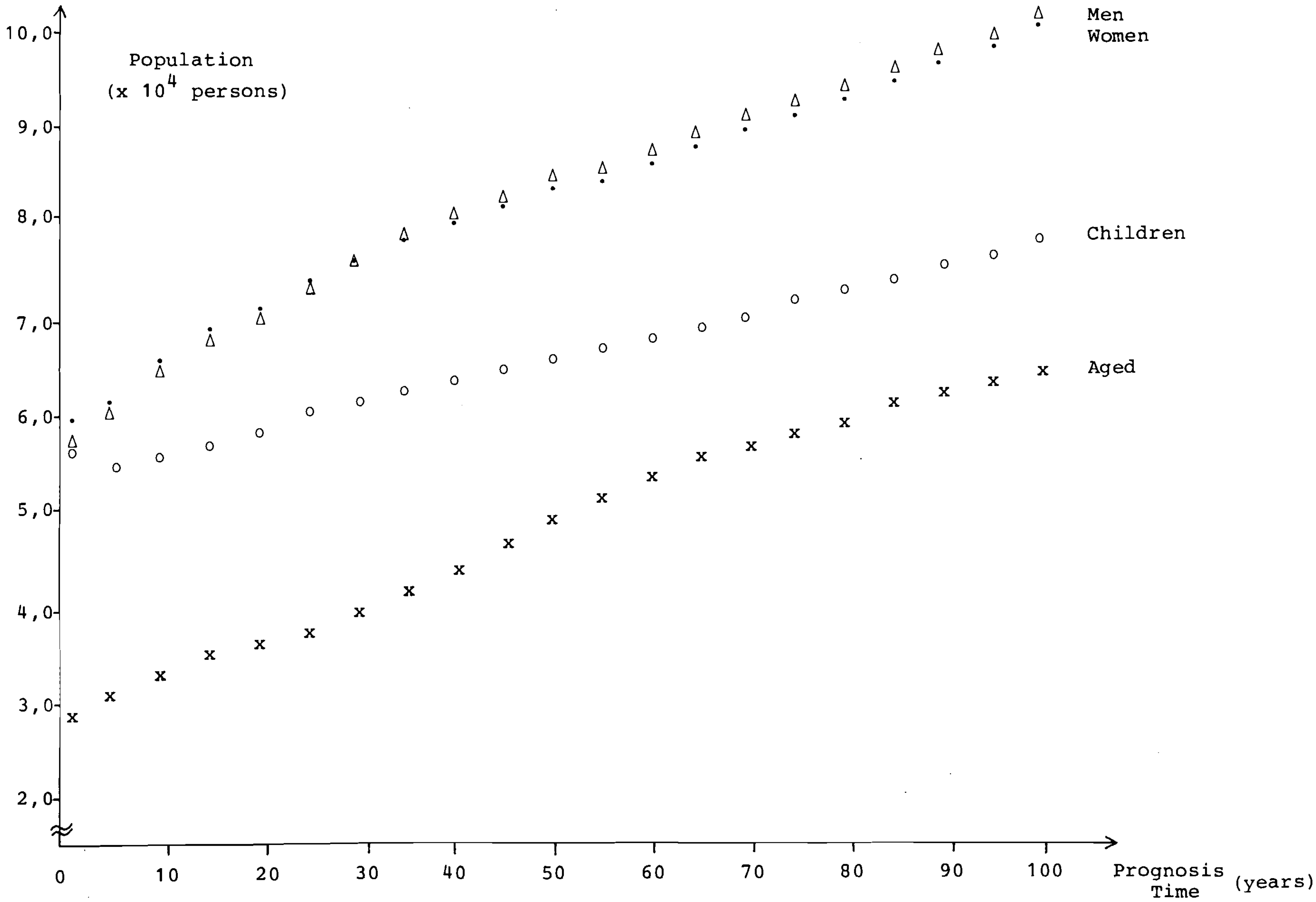

Figure 2. Results of Calculations

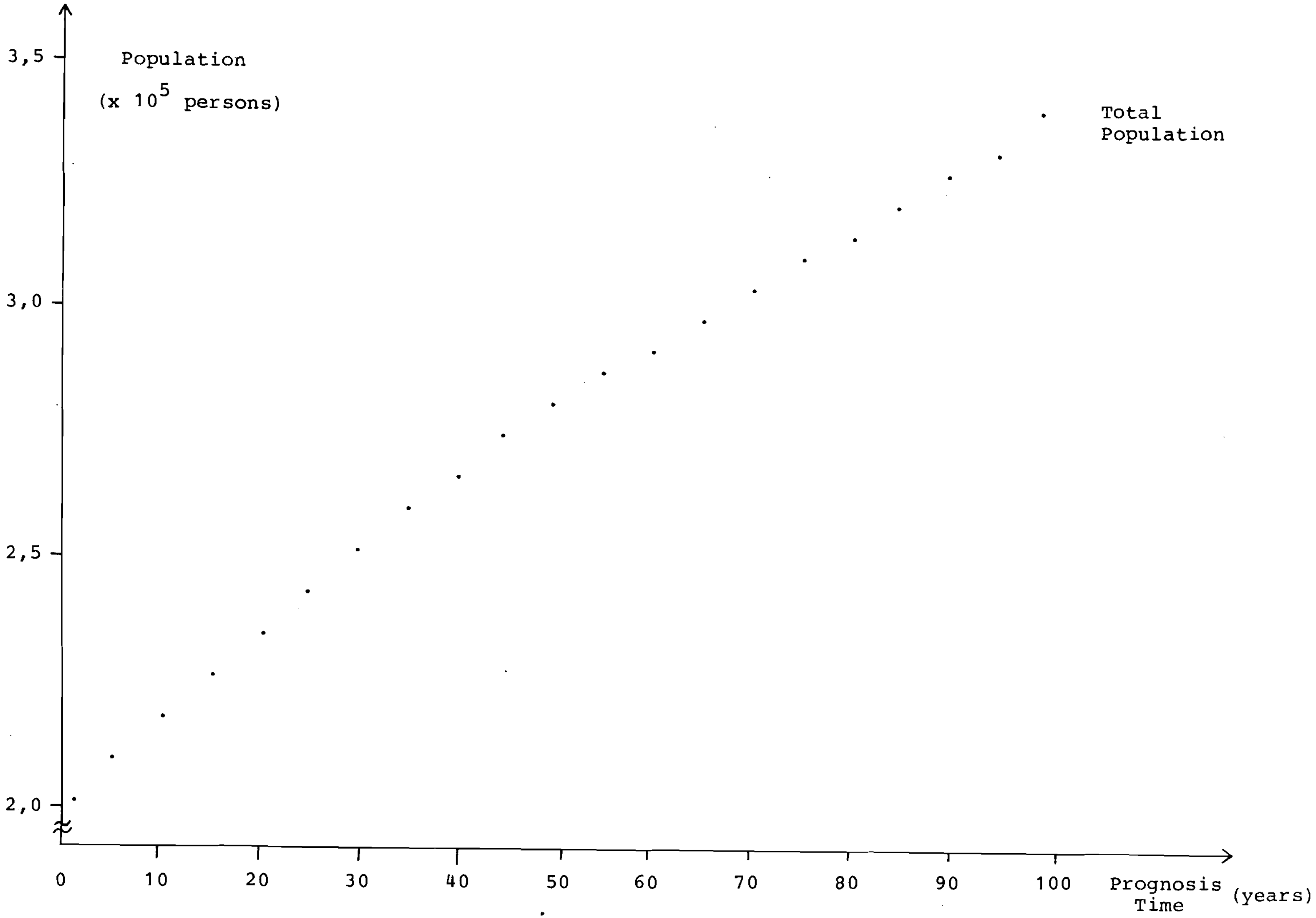

Figure 3. <u>Results of Calculations</u>

b. the death rate in separate sex-age groups;

c. birth rate in corresponding strata for women of child-bearing age.

The model is designed to make demographic forecasts while taking into account the influence on population sex-age structure of environmental factors, the activities of the health care system and also, possibly, of family planning programs.

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APPENDIX 1

Matrix $\mathscr{M}_t$

$$
\mathcal{A}_t = \begin{array}{c}
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5
\end{array} \\
\left[\begin{array}{ccccc}
1 - PCDEM(1)_t - PCSM(1)_t & PCDEM(1)_t & 0 & 0 & 0 \\
0 & 1 - PCDEM(2)_t - PCSM(2)_t & PCDEM(2)_t & 0 & 0 \\
0 & 0 & 1 - PCDEM(3)_t - PCSM(3)_t & PCDEM(3)_t * PCDJ & PCDEM(3)_t * PCDM \\
0.001 * PUDRO(4)_t * (1-PCSMNO_t) & 0 & 0 & 1 - PCDEM(4)_t - PCSM(4)_t & 0 \\
0 & 0 & 0 & 0 & 1 - PCDEM(5)_t - PCSM(5)_t \\
0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0.001 * PUDRO(2k-2)_t * (1-PCSMNO_t) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0.001 * PUDRO(2k)_t * (1-PCSMNO_t) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right.
\end{array}
$$

$k = \overline{2,10}$

$$
\begin{array}{cc}
\begin{array}{cccccc}
\cdots & 2k & 2k+1 & \cdots & 22 & 23
\end{array} & \\
\left.\begin{array}{cccccc}
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & \mathrm{PCDEM}(2k-2)_t & 0 & \cdots & 0 & 0 \\
\cdots & 0 & \mathrm{PCDEM}(2k-1) & \cdots & 0 & 0 \\
\cdots & 1-\mathrm{PCDEM}(2k)_t-\mathrm{PCSM}(2k)_t & 0 & \cdots & 0 & 0 \\
\cdots & 0 & 1-\mathrm{PCDEM}(2k+1)_t-\mathrm{PCSM}(2k+1)_t & \cdots & 0 & 0 \\
\cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 0 & 0 & \cdots & \mathrm{PCDEM}(20)_t & 0 \\
\cdots & 0 & 0 & \cdots & \mathrm{PCDEM}(21)_t & 0 \\
\cdots & 0 & 0 & \cdots & 1-\mathrm{PCDEM}(22)_t-\mathrm{PCSM}(22)_t & \mathrm{PCDEM}(22)_t \\
\cdots & 0 & 0 & \cdots & 0 & 1-\mathrm{PCSM}(23)_t
\end{array}\right] &
\begin{array}{c}
1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \vdots \\ 2k-2 \\ 2k-1 \\ 2k \\ 2k+1 \\ 2k+2 \\ \vdots \\ 20 \\ 21 \\ 22 \\ 23
\end{array}
\end{array}
$$

Matrix $\mathcal{A}_t$

APPENDIX 2

JAN 22 05:27 1976 BIM1.F PAGE 1

```
C PROGRAM BIM1
        DIMENSION PN(23),PCSM(23),PUDRO(23),PMIGR(23),PVH(23),PVIH(23),
&       PSM(23),PCOEM(23)
C INPUT FORMAT
1       FORMAT (2F6.4,7F6.2,3F8.2,/,10F8.2,/,10F8.2,/,16F5.2,/,7F5.2,2F5.1,
&               2F6.4,F4.2,I3)
C PRINT FORMAT
2       FORMAT (10X,'PCSMNO=',F5.3,10X,'T1=',F4.0,10X,'T2=',F4.0/
&       'PCDJ=',F5.3,10X,'PCDM=',F5.3,4X,'DT=',F3.1)
3       FORMAT (5X,'I',T13,'PN(I)',T30,'PCSM(I)', T45,'PMIGR(I)',T60,'I',
&         T75,'PN(I)',T90,'PCSM(I)',T105,'PMIGR(I)'//5X,I2,T13,F6.0,T30,F5.0,
&       T45,F6.0,T60,I2,T75,F6.0,T90,F5.1,T105,F6.0)

C COMMAND TO INFORM USER THE PROGRAM IS REDY TO READ DATA
        WRITE (6,50)
50      FORMAT (5X,'PLEASE,INPUT DATA WITH 1FORMAT')

        READ(5,1) PCSMNO,PCOWF,(PUDRO(I),I=4,16,2),(PN(I),I=1,23),(PCSM(I),
&       I=1,23),T1,T2,PCDJ,PCDM,DT,N
        DO 5 I=1,23
5       PMIGR(I)=0
C PRINTING OF INPUT MASSIVE
        WRITE(6,2) PCSMNO,T1,T2,PCDJ,PCDM,DT
C (I,PN(I),PCSM(I),PMIGR(I),I=1,23)
C VNESHNII CYKL
    DO 10 J=1,N


    PABRO=0
C NEWBORNS
     DO 100 I=4,16,2
100  PABRO=PABRO+PUDRO(I)*PN(I)*0.001
C LIVE NEWBORNS
     PVH(1)=PABRO*(1.-PCSMNO)
C STRATA VOLUME
        DO 17 I=1,23
        PCOEM(I)=(1.-PCSM(I)*0.001)**T1*PCSM(I)*0.001/(1.0-(1.-PCSM(I)
&       *0.001)**T1)
        PVIH(I)=PN(I)*PCOEM(I)
        PSM(I)=PN(I)*PCSM(I)*0.001
        IF (I-22) 14,15,16
15      PVH(22)=PVIH(20)+PVIH(21)
        PCOEM(22)=(1.-PCSM(22)*0.001)**T2*PCSM(22)*0.001/(1.-(1.-PCSM(22)
&       *0.001)**T2)
        PVIH(22)=PN(22)*PCOEM(22)
        GO TO 17
16      PVH(23)=PVIH(22)
        PVIH(23)=0
        GO TO 17
14      IF(I-3) 11,12,13
11      K=I+1
        PVH(K)=PVIH(I)
        GO TO 17
12      PVH(4)=PVIH(3)*PCDJ
        PVH(5)=PVIH(3)*PCDM
        GO TO 17
```

```
13       K=I+2
         PVH(K)=PVIH(I)
         GO TO 17
17       PN(I)=PN(I)+(PVH(I)+PMIGR(I)-PVIH(I)-PSM(I))*DT


     PND=PN(1)+PN(2)+PN(3)
     PNJ=0
      DO 27 I=4,20,2
27    PNJ=PNJ+PN(I)
     PNM=0
      DO 21 I=5,22,2
21    PNM=PNM+PN(I)
     PNST=PN(22)+PN(23)
     POPUL=PND+PNJ+PNM+PNST
         IF (J-Z) 30,30,40
30       WRITE(6,20) J,(PN(I),I=1,23),PND,PNJ,PNM,PNST,POPUL
20       FORMAT (5X,I3,5X,10F10.1/15X,10F10.1/15X,3F10.1/15X,5F10.1//)
         GO TO 10
40       L=J/10
         Z=L
         Y=J
         X=Y/10

         IF(X-10) 30,30,10
10       CONTINUE
         STOP
         END
```