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LINKING NATIONAL MODELS OF FOOD AND AGRICULTURE:
An Introduction

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Preface

The food problem is to a large extent a local one; in the long run countries must generate sufficient purchasing power in all income classes to avoid malnutrition. It is, however, also a problem of global interaction, when developed countries attempt to alleviate malnutrition through food aid. It is even more a global concern when economic development is expedited in less developed countries through trade agreements and credit arrangements.

The interactions among nations through trade will be investigated in the international model. An analysis will be conducted of how the national policies of various countries influence other countries.

The development of an international model requires a methodological research concerned with the linkage of national models for food and agriculture. This memorandum is the first of a series on this topic.

Abstract

Part I of this paper deals with methodological questions connected with the linking of national into a model of world trade.

Part II discusses an application with special emphasis on agricultural policies. The national model which is developed is essentially an extension of the model for a market economy in MOIRA: instead of one commodity more commodities are considered and instead of only policies aiming at a certain domestic price, policies which directly limit the trade balance of certain commodities are now also taken into consideration.

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Part I General Introduction

Part II Outlines for National Models

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Part I: General Introduction

1. INTRODUCTION

1.1 International interactions and the "Food and Agriculture"-project at IIASA

Countries can interact in many ways: through trade, capital flows, migration, diplomatic pressure, military pressure, etc. The Food and Agriculture project will concentrate on the interactions through trade. Several approaches are available to model these interactions. They may be classified in two groups.

I) Models in which international trade is studied for its own sake.

II) Models in which international trade is merely a channel through which national models interact.

ad I) In the first class we can distinguish between two kinds of models:

1.2 'Pure' Trade Models

Trade models concentrate on the trade matrix, try to analyze its structure and generate trade matrices in time, given time series for the row and/or column sums. The structure is analyzed by comparing some (theoretically) 'normal' flow with the actual flow between i and j and by computing some parameters (Delta coefficients) which are an indicator of the 'special' nature of the flow. (R.A.S.-technique, Delta coefficients) [3]. In this way one can try to forecast the matrix given the row and column sums.

1.3 Econometric Trade Models

These models (e.g. gravitational models [3]) explain the trade between i and j from the circumstances in i and j respectively and from some distance concepts between i and j. The advantage is that some causal explanation is introduced. Some disadvantages are that the econometric specification of the disturbance term is very difficult, that the time series are often quite short and that some information is lost in estimation through the unexplained component.

Approaches 1.2 and 1.3 have in common that they do not try to link country models--they only generate trade matrices, given certain information about the specific country and given the row and column sums of the trade matrices. The approach is, of course, not fundamentally incompatible with the linking of national models (see Bottomley [3]).

ad II) The IIASA project clearly is more oriented towards the second approach.

This approach may be realized in a more or less formal way.

1.4 Formal linking

What matters here is consistency. The imports of a national model can, for example, be (indirectly) dependent on exports. This can, on a world level, lead to a certain inconsistency between total imports and exports. Some accommodating principle then has to be introduced. One could adapt exports to imports according to some specified rule. This would imply a change in imports through adaptation of the national models. This example makes it clear that formal linking implies an iterative procedure which should converge to an equilibrium, where there are no longer inconsistencies. It considers a closed system so that the outside world cannot perform a slack function thus accommodating for inconsistencies between decisions made within the model. Full equilibrium is aimed at. The national models are considered as being one simultaneous set of equations. The reason for this simultaneity is that the models which have found empirical application are formulated in discrete time with a rather long time-step (e.g. one year). It is very difficult to formulate a theory of trade flows in recursive terms, even with a shorter time period, so that the simultaneous approach will be the most common one. An advantage of this approach is that it makes it possible to take advantage of the theory of general economic equilibrium, thus giving some theoretical foundation to an approach otherwise ensuring only consistency. This will be discussed extensively in the following paragraphs.

The solution of the simultaneous set of nonlinear equations with inequality constraints will be quite complex, and possibly non-unique and unstable.
One can look for simplifications within the formal linking approach (e.g. through partial linearisation) but one can also abandon the equilibrium approach by opening up the system.

1.5 Non-formal linking

This approach introduces a set of slack variables (in general stock mutations or the net supply of an outside actor (country)). The exogenous variables of the national models (for which an equilibrium value was computed in the formal linking approach) adapt in a lagged way to the 'pressures'. In the most simple approaches cross price effects are disregarded. This approach is quite popular because of its computational simplicity. It has however some drawbacks:

1) If one assumes a high absorption capacity of an outside actor (country), why was this actor's (country's) behaviour not considered within the model?
2) If one assumes that the slack function is performed by stocks this role is limited by a nonnegativity constraint;
3) The assumption of a lagged response (in prices) seems incompatible with assumptions of perfect competition. Under perfect competition no price adjustment would be needed once a quantity adjustment has taken place.
   The drawback can be avoided if the response is not lagged. Then the approach may be considered as being identical with a formal linking approach, in which the equilibrium searching algorithm stops after a few iterations, while the

remaining inconsistencies are met by slack variables. Of course all the problems mentioned under 1.4 then come up again.

It is not stated here that non-formal linking is never acceptable. Formal linking may be absolutely impossible or theoretical arguments for a non-formal linking procedure may exist.

One can, for example, incorporate the behavioural assumption that producers raise their prices in relation to a decrease in their stocks but this is a descriptive assumption about imperfect competition which should not be build in as a technicality. The conclusion must be that it is a matter of description of the real world to decide how the national models should react upon their outside world--that is, upon the other countries. Given those models, the consistency at time t must be reached as in a closed system.

1.6 Loose linking

The formal linking approach may be too complex for certain problems. In the 'loose linking' approach, models are run one after the other in their original form given some time path for exogenous variables. The inconsistencies are computed each year and a general proposal for a more consistent run is derived on that basis. This sounds more like an emergency measure--in case the formal linkage happens to fail--than a real alternative.

If the exogenous variables of the national models would always follow some smooth trend then this procedure would be practical, but this is not the case on the world food market! The formal linking approach is pursued in the following pages.

2. Some notions of equilibrium

Before taking up the linking discussion again the meaning of some notions of equilibrium will be clarified. This is required as some people seem to fight the concept of equilibrium itself (e.g. J.Kornai: Anti-equilibrium [2]). A short survey of some notions will now be presented.

2.1 Equilibrium condition

When a given description has taken the form of a model then any restriction applied to this model can be called an equilibrium condition: an equilibrium condition is a restriction which selects a certain subset of the feasible set determined by a model.

As this condition is not based on direct description of reality it is occasionally called a non-primitive assumption (see Bliss (2)). It is clear from the definition that, once an equilibrium condition is assumed to hold, the fulfillment of it is just a matter of formal consistency of the model. If the model was already fully determined, the addition of the extra restriction will, in general, make the model overdetermined so that the equilibrium condition cannot be fulfilled. If the original model happened to have one degree of freedom left, then as soon as one assumes that it holds the equilibrium condition simply becomes an equation like any other. One can also derive from the definition that any model which solves (uniquely) for all variables can in several ways be considered as the equilibrium solution of a more general model. To say it differently: a model of full disequilibrium cannot exist (neither does a fully dynamic model, cf.MOIRA, ch. III, [4]).

There are only models in which specific equilibrium restrictions which existed in previous models have been relaxed. In the following some of these specific restrictions will be defined. One should keep in mind that in economic literature no strict agreement exists on these definitions.

2.2 Market equilibrium

A. There is market equilibrium in a static sense when demand for every commodity does not exceed supply, i.e. when the actions of the different decision makers are compatible.

B. There is market equilibrium in a dynamic sense when the price expectations are realized.

2.3 Competitive (market) equilibrium

There is a competitive equilibrium in an economy if there is a market equilibrium and if, while producers maximize profits and consumers maximize utility--subject to the wealth constraint--both accept prices as given.

3. Models of general equilibrium

We are now in a position to discuss general equilibrium models. We shall not describe these models but only list some characteristics of the general equilibrium (G.E.) models, as developed by Arrow, Debreu and others[2]. The original G.E. models describe competitive market equilibrium and have the following characteristics:

a. the models consider an arbitrary given number of commodities, producers and consumers;

b. factor services are treated as commodities, while the factor endowments of consumers and the distribution of profits among consumers are exogenously determined.

c. all consumers and producers take the prevailing prices as given;

d. all commodities and factor services can be traded (without transportation costs);

e. some restrictions on producer technology and consumer preferences are introduced.

The main interest of G.E. Models is to show that prices exist which bring about full compatibility between the decisions of all actors.

We can, of course, locate producers (and/or consumers) in space, thus getting a multiregional or a multicountry model. We may also locate commodities in time, thus introducing an intertemporal aspect (thus introducing capital theory, e.g. investment, credits and debts, into the model).

As all actors take the existing prices as given, it is also easy to introduce distortions, e.g. through producers and/or consumers taxes and subsidies. (The net return from this policy must then be treated as profits and be exogenously redistributed over consumers.) Then the same commodity still has the same producer price in all places--and the same consumer price in all places.

If we introduce tariffs and subsidies on trade, we get different prices between regions (or between consumer, i.e. producer groups).

The tariffs may be discriminating (dependent on origin and destination of the trade flow), the result of this is that one gets different prices for the same commodity. One may introduce transportation costs and untradable resources, thus specializing in spatial economics or in international economics.

4. Linking in a model of competitive general equilibrium:

With the application of general equilibrium theory to the theory of international trade we are finally back to the matter of linking national models. An application will be presented in part II of this paper. The previous paragraph has hopefully made clear that the G.E. framework offers a theory for the linkage of national models. In a G.E. model all producers and consumers are linked together through the market. Governments can build up trade barriers between the nations but essentially all actors are connected to each other. Thus, the world market is not a submodel which shows the interactions between the national models. It is the addition of a world market equilibrium condition to the submodels of consumers and producers in the different nations which yields the world model. That world model is a model in which competitive equilibrium exists in so far as it exists both on the world market and on the domestic markets. Thus it is not sufficient to require that the national models should at every (world market) time-step, for any given vector of world market prices[1], yield a vector of excess demand: at least if one wants to formulate a general equilibrium model. If one only puts that requirement on the national models one considers them as black-boxes, so that approach will be called the black-box approach.

One limitation of the black-box approach is that it does not guarantee the existence let alone the uniqueness or the

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1) This formulation assumes the existence of one world market price.

stability of any equilibrium price vector as nothing is required about the form of the excess demand functions.

But it is obvious on the other hand that the fully competitive G.E. model is not suitable for the IIASA-project because it cannot represent centrally planned economies.

5. <u>The formal linking of black-boxes:</u>

The most simple interpretation of the black-box approach is to assume that governments or traders of all countries "sit around the table" and negotiate with each other until no excess demand exists, that is until equilibrium is reached in the same way as consumers and producers do in a G.E. representation. The simultaneous character of the G.E. solution is preserved. There are however some problems:

1) as mentioned above the existence of an equilibrium is not guaranteed;

2) when the domestic market of a certain country is assumed to be competitive and no sufficient information is available on stock mutations (which is quite often the case) one is forced back to a G.E. model for that country;

The main line of defence of the black-box approach for international interactions is that the reliability of the response of excess demand to price changes on the world market is all that matters. This statement is open to certain objections:

a) The statement is not at all operational: all the problems are hidden within the black-boxes;

b) Although it is not unreasonable to maintain that one needs good national models to get a good world model, the reliability of the national response functions is not a sufficient econometrical condition for a good world model. The world market prices are exogenous variables for the national models; they are however endogenous in the world model. This implies that the 'best' specification and parameter estimates for the world model are not necessarily the best ones of the national models.

It will of course be very difficult to solve all identification problems and to estimate all equations of the world model simultaneously, so the point may seem somewhat theoretical, but it shows that one cannot neglect the inside of the black-boxes altogether.

Before turning to this intricate matter, the requirements which already follow from the formal linking of black boxes will be listed; they will be called external requirements for the national models.

6. External requirements for national models

1) All models must react on world market prices of the commodities as they are classified on the world market;

2) The set of countries and commodities must form a closed system, so that the simultaneous character of interactions is preserved;

3) All national models must be dynamic;

4) All national models must be descriptive and validated as much as possible (reliability!).

7. Some internal requirements for the black boxes

The concept of reliability is very hard to operationalize in any formal way before the models have been developed. In the following, two questions which came up at a taskforce meeting in

July will be discussed. This will illustrate the fact that internal requirements for the national models are needed:

I. Is it possible to have a finer commodity classification within a national model than on the world market?

II. Is it possible to have a shorter time-step in a country than on the world market?

It directly follows from the external requirements that a rougher commodity classification or a longer time-step is not acceptable. How about the opposite situation?

Both questions are quite similar. One commodity at different points in time may be considered as different commodities at the same point in time.

If a price equilibrium is assumed to exist on the world market, all commodities within one class at all points in time within one period are supposed to be traded at the same equilibrium price.

a) If one makes a finer classification within a country or uses a shorter time-step one needs to know the disaggregated domestic prices (p_{ijt} or p_{it_τ}).

b) The class can be considered as a homogenous commodity on the world market only as long as the different commodities within it have the same price.

So, if requirement a) is fulfilled, the conditions which guarantee b) answer simultaneously questions I and II. It may already be stated here that the answers will be affirmative but that the conditions will be quite severe. Before we discuss this matter, some general conditions for the existence of equilibrium between the 'black boxes' have to be listed. After that some limitations will be mentioned and (cf. § 7.2) the question whether an equilibrium with eaual prices exists will be studied.

7.1 Existence of equilibrium between black boxes:

Assumption 1: To any price vector p corresponds, a unique number $z_i(p)$[1], called the aggregate excess demand function for commodity i and so a unique vector of aggregate excess demand functions $Z(p)$.

Assumption 2: $Z(p) = Z(kp)$ for all $p>0$ and $k>0$

Assumption 3: $p\,Z(p) = 0$ (Walras' Law)

Assumption 4: $Z(p)$ continuous[2] over its domain of definition S_n;

$$S_n = \left\{ p \,|\, \Sigma p_i = 1,\ p \geq 0 \right\}$$

a) S_n is by its definition a compact convex set.

b) We can define T(p), a continous mapping of S_n into itself.[3]

Where $T(p) \equiv \frac{p+M(p)}{[p+M(p)]\cdot e}$ where e is the vector: $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$,

$M_i(p)$ is continuous

and $M_i(p) > 0$ if and only if $z_i(p) > 0$

$M_i(p) = 0$ if $z_i(p) = 0$

$p_i + M_i(p) \geq 0$

Note that nothing is assumed about the sign of partial derivatives of $z_i(p)$.

1) This restriction is quite severe and not necessary. If it can be satisfied, the linking is however greatly simplified.

2) This restriction is also severe. It usually is weakened in order to allow for an infinite excess demand when the price is zero. As the proofs do not change essentially but only become somewhat more involved (i.e. by applying Kakutani's instead of Brouwer's fixed point theorem), we shall keep to 4 for expository purposes.

3) cf. Arrow and Hahn [2].

c) Using a) and b) we may by Brouwer's fixed point theorem state that there is some p^* for which $p^* = T(p^*)$. The construction of $M_i(p)$ in turn implies that $Z(p) \leqq 0$.

Note that assumptions 1-4 are internal requirements for the black boxes. The proof presented here will be repeatedly referred to in the following pages.

Stock mutations

As mentioned before an equilibrium condition is a non-primitive assumption. When we use a G.E. model to describe the world food scene, we assume that the condition of market equilibrium is a realistic one. Note that from the market equilibrium condition -- $z(p^*) \leq 0$ together with the so called "Walras Law" $pZ(p) = 0$ -- it follows that in equilibrium a commodity which is in excess supply will have a zero price and that no commodities are in excess demand; stock mutations of positively priced commodities do not arise. This is clearly not always realistic as stocks may well perform an adjustment function and necessarily do so if no price equilibrium would exist otherwise. The stock mutations mentioned here are undesired and unforeseen by the actors. Desired stock mutations are of course included in $z(p)$.

Risk: This leads us to a conceptual problem. General equilibrium models are deterministic. They can be so as they assume perfect foresight of prices. If undesired stock mutations exist there is clearly no perfect foresight. But even without these mutations the assumption of perfect foresight is somewhat strange in a non-stationary state. Uncertainty will always be present especially in the interrelations between 'black-box' nations. This should be taken into account in the formulation of the national models.

Oligopoly

The decisions of large nations more strongly influence the world market price than those of small nations and the nations may be aware of this when determining their agricultural policies. This should also be reflected within the national models. The world market may then become a market of monopolistic competition. Though this need not influence the computation of prices, as the problem of market equilibrium remains, the conditions for existence and especially for stability of the equilibrium will be different than under perfect competition. (See some remarks on oligopolistic equilibrium, IIASA internal paper.)

7.2 Equilibrium with equal prices

We now want to derive conditions under which two prices, say p_1 and p_2, will always be equal in equilibrium, independently from variations in their excess demands Z_1, Z_2. As long as this is the case, commodities 1 and 2 may be considered as fully substitutable when they are traded between countries.

This implies, at least in a competitive economy, that a certain degree of indifference on the consumer's side and/or on the side of producers must exist, because equal prices imply unit slopes for isoquants or for isoutility functions. To formulate it more precisely:

On the producers side the indifference can only be realized if,

1) the commodities can, at zero cost, be transformed into each other after production.*

* When considering more time-steps, transformation through stock formation is possible but only in one direction!

2) a) The commodities are fully substitutable as intermediate inputs.

b) There is no lag in production and both products are produced in identical production processes.

3) On the consumer's side the indifference implies that the ratio of marginal utilities is one, for all positive values of the consumption of both commodities.
If we only look at excess demand functions, we need not be specific on the source of indifference.

Define:

z_1, z_2 net demand for commodity 1, 2 in country group A

z_3 net demand vector for all other commodities in country group A

$z_o = z_1 + z_2$

z_1^*, z_2^* net demand for commodities 1, 2 in country group B

z_3^* net demand vectors for all other commodities in country group B.

Case 1: <u>Country group A has a full indifference (substitutability) of the first type between commodity 1 and 2.</u>

Consider now the condition of a non-positive excess demand vector on the world market for commodities 0,3.

$$\longrightarrow Z_o = z_o + z_1^* + z_2^* \leqq 0$$
$$Z_3 = z_3 + z_3^* \leqq 0$$

On the basis of the proof given in § 7.1 one may state that an equilibrium price vector will exist. Assuming that this vector has a nonzero price for z_o and that the possible equilibria are distinct, then the only requirement which still needs to be met is:

$$z_1 + z_1^* \leqq 0$$

$$z_2 + z_2^* \leqq 0$$

which may be written as

$$z_1 + z_1^* \leqq 0$$

$$z_o - z_1 + z_2^* \leqq 0$$

$$\longrightarrow \quad z_2^* + z_o \leqq z_1 \leqq z_1^*$$

Clearly in equilibrium at a positive price we may write:

$$z_1 = -z_1^* = z_2^* + z_o$$

so that this condition can be met as long as z_1, z_2 are not restricted.

Case 2: <u>Restrictions on z_1 and z_2</u>

If the substitutability is not of type 1 then there are constraints on z_1 and z_2.

We define:

$$z_o \equiv x_o - y_o$$

$$z_1 \equiv x_1 - y_1$$

$$z_2 \equiv x_2 - y_2$$

Where x stands for demand and y for supply. For any given equilibrium price vector with $p_1 = p_2$, the variables z_1^*, z_2^* and z_o are known to be uniquely determined and to add up to zero.

Moreover for equilibrium on the markets of commodity 1 and 2 it is required that

$$z_1 = -z_1^*$$

$$z_2 = z_o + z_1^* \quad .$$

These equalities can always hold as long as z_1, z_2 are not restricted. In the present case the following restrictions must also be satisfied (bars indicate that the variable has a given equilibrium value).

Case 2.1 Consumers and producers (in country group A) are indifferent (type 2 and 3)

In this case the total consumption and the total production is given so that the total excess demand is restricted as the country cannot export more than it produces or import more than it consumes of the aggregate commodity:

$$-\bar{Y}_o \leqq z_1 \leqq \bar{X}_o$$

$$-\bar{Y}_o \leqq z_2 \leqq \bar{X}_o$$

Case 2.2 Only producers are indifferent

$$-\bar{Y}_o \leqq z_1 \leqq \bar{X}_1$$

$$-\bar{Y}_o \leqq z_2 \leqq \bar{X}_2$$

Case 2.3 Only consumers are indifferent

$$-\bar{Y}_1 \leqq z_1 \leqq \bar{X}_o$$

$$-\bar{Y}_2 \leqq z_2 \leqq \bar{X}_o$$

In the cases described here the size of the country group A is very important. A small country group A is very unlikely to be able to accommodate for the excess demand of country group B.

It has thus been shown that under certain conditions an equilibrium with equal prices can exist. The uniqueness, stability of that equilibrium was not discussed. Neither was it stated whether an equilibrium with unequal prices would also exist under the given circumstances. But clearly if no restrictions are imposed on z_1, z_2 and if within group A there is one country with a competitive market economy then no equilibrium can exist at unequal prices (e.g. $p_1 > p_2$) as this would lead to $z_1 \longrightarrow +\infty$ and $z_2 \longrightarrow -\infty$.

8. Conclusion

The requirements presented above are quite severe and it seems highly advisable to keep a common commodity classification and a common time step for all countries.

It is not stated that it is always unrealistic to introduce differences in time step. It seems for example quite clear that the supply for agricultural non-animal products forms a rather discrete process while demand and supply of animal products is more of a continuous type. This poses interesting dynamical problems of stockholding and price formation in terms of difference-differential equation comparable to those of Kalecki's model of the trade cycle (see Allen, Math. economics, p. 242). However it is well known that general equilibrium theory becomes quite unrealistic when one takes a very short time step.

Some further requirements can be derived (e.g. with regard to the stability of world market equilibrium) but it is of course impossible to derive a full list which would guarantee the reliability of the national models.

Every new requirement introduces one more feature which all the national models must have in common. However the absence of a full list of requirements does not allow the conclusion that the wider the variety is in the structure of country models, the better the world model will be. Though an impression of richness and realism may be conveyed by it, it is rather a sign of weakness as differentiation between structures is only needed when the various phenomena cannot be explained in terms of one theory, with the different values of variables reflecting the variety.

Part II: Outlines for national models

1. Introduction

The second part of the paper presents outlines for a model of a country with a market economy. This can be considered as an extension of the MOIRA-model in the following respects:

1. n commodities will be considered instead of 1;
2. Apart from a policy which fixes domestic prices, a policy which limits the balance of trade in the various commodities will be introduced;
3. the non-agricultural sector and the relations between agricultural and non-agricultural sector will be treated more explicitly.

At all stages, existence of a market equilibrium will be investigated. The net supply of food from the agricultural sector is predetermined in MOIRA, so that changes in demand (and stocks) are the only adjustment mechanisms on the world market. We shall maintain this assumption here and even disregard stock mutations. This implies that in equilibrium all supply has to equal demand through an adjustment in demand induced by price changes: Hence, special attention is required for the demand equations.

2. Static demand equations:

As more commodities are now involved, a theoretically justifiable set of demand equations has to be set up. The neoclassical demand theory seems to yield a fruitful starting point.

The following assumptions are made:

2.1 <u>All prices are equilibrium prices except when stated otherwise.</u>

2.2 <u>All commodities considered are final products.</u>

2.3 <u>All commodities are consumed in positive amounts.</u>

2.4 <u>We disregard the inequality of income distribution.</u>

Each of these assumptions will have to be relaxed later on, but for the moment the problem considered is:

$$\max u = u(x_1,\dots,x_n)$$

$$\text{S.T.} \quad \Sigma p_i x_i = M \quad ,$$

where

x_i = quantity consumed of the i^{th} commodity/caput

p_i = price of the i^{th} commodity

M = monetary income (to be fully spent on consumption)/caput.

The non-agricultural product is represented as the n^{th} commodity. The utility function is assumed to be continuous, concave and to have continuous first and second derivatives. The first derivative is assumed to be positive.

The first order conditions (F.O.C.) are:

$$\frac{\partial u}{\partial x_i} = \lambda p_i$$

$$\sum p_i x_i = M \quad .$$

From this one can derive some well-known restrictions.

(1)	Homogeneity	$\sum e_{ij} + e_{iM} = 0$
(2)	Engel aggregation	$\sum w_i \; e_{iM} = 1$
(3)	Cournot aggregation	$\sum w_i \; e_{ij} = -w_j$
(4)	Slutsky condition	$e_{ji} = e_{ij}(w_i/w_j) + w_i(e_{iM}-e_{jM})$
(5)	Substitution restriction	$e_{ii} < 0$ (uncomp.subst.)
		$e_{ii} + w_i \; e_{iM} < 0$ (comp. subst.)
(6)	Logical restriction	$\sum_i w_i = 1$

where

$$e_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i} \quad ; \quad e_{iM} = \frac{\partial x_i}{\partial M} \frac{M}{x_i} \quad ; \quad w_i = \frac{p_i x_i}{\Sigma p_j x_j} \quad .$$

If we consider the F.O.C. we find that the demand functions should be obtained by solving for the quantities in terms of prices and income. As this solution may be quite complex the econometrical estimation of the parameters of the utility function may be difficult. It will be shown in Appendix I that more convenient approaches are available which make use of the possible linear homogeneity of the utility function and of the fact that we are really only interested in the parameters of the demand functions which have to be such that the above mentioned requirements are satisfied.

The demand model may be thought of as being applicable to the nation as a whole, where all consumers have the same taste and the same income.

- assuming such a model for all nations,
- taking the supply (Y) of all commodities as given,
- assuming no trade barriers or transportation costs,
- assuming that the incomes of all nations are measured in a common unit and equal to the value of the supply, then

an equilibrium world market price exists. The proof of this statement proceeds exactly along the same lines as the one given in § 7 of part I.

3. Policies

3.1 Domestic price policy

The government of the country may want to fix domestic prices. As a result of this a difference will occur between world market prices (p^w) and domestic prices (p). This price difference accrues to the country or has to be paid by it; from now on vector notation will be used. The budget equation becomes:

$$px = (p^w - p)(y - x) + M$$

$$\text{with } M = py$$

Clearly this is equivalent with $p^w x = p^w y$, implying that under the given assumptions (equal income distribution within the nation, given supply) the domestic price policy has no effect on demand, because we have assumed here that the consumer knows the definition of the tariff receipts. Otherwise there would be sensitivity to domestic prices. It will be shown later that it is precisely on the supply and on the income distribution that the domestic price policy has an effect.

3.2 Self-sufficiency targets, quota:

Self-sufficiency targets may be formulated as

$$L \leqq y - x \leqq R$$

where L, R are target vectors.

Normally the targets will be such that

$L \leqq 0$ (maximum net import)

$R \geqq 0$ (maximum net export)

The selfsufficiency targets may be realized through the imposition of quota on international trade. A tariff on international trade only generates a difference between domestic and world market prices. Under the present assumptions, it does not influence demand so that self-sufficiency targets cannot be realized.[1)]

If the restrictions are ineffective one is back to the model of the previous paragraph. If, however, a restriction is effective the domestic consumption of a commodity is given by the constraint.

3.3 Equilibrium with quota

The introduction of quota sets bounds on the excess demand functions. This restriction may however be inconsistent with Walras' Law.

If $L > 0$ then $p^w L > 0$
but $p^w(y-x) = 0$

both requirements are inconsistent as $p^w \neq 0$.[2)]

[1)] Even when the tariff receipts are not a side condition of the utility maximization, tariffs are not a very adequate means for realizing selfsufficiency targets. This can be seen, if one writes out the first order conditions.

[2)] On the international level $\Sigma L_c \leq 0$ and $\Sigma R_c \geqq 0$ must be satisfied (c is the country index)

The constraints have to satisfy at the national level the restriction $p^w L \leq 0$, $p^w R \geq 0$. This is clearly the case if, for example: $L \leq 0$, $R \geq 0$.

4. Endogenous income distribution between the agricultural and non-agricultural sector: general formulation

Up to this point only the consumer has been considered in each country. This was based on the assumption of a completely equal income distribution. We now relax this assumption and consider m income classes and n products: •The agricultural sector produces n-1 final products and the non-agricultural sector produces the n^{th} product, which is also a final product. There are no intermediate commodities. The model is thus a model of a pure exchange economy.

- The income of every income class is equal to the value of the output.
- Income is equally distributed within income classes.

4.1 Free trade

The demand model under free trade now becomes

$$\max \quad u^j(x^j) \quad ,$$

$$\text{S.T.} \ p^w x^j = p^w y^j \quad .$$

4.2 Domestic price policy

The introduction of a difference between the domestic and world market price causes a complication, as to how the revenue from this difference has to be distributed over income groups.

The budget equations may, in a general way, be written as

$$px^j = \alpha_j(p^w-p)(y-x) + py^j$$

where α_j is a distribution variable: $0 \leq \alpha_j \leq 1$, $\Sigma\alpha_j = 1$.
Clearly this distribution variable must be such that:

$$py^j + \alpha_j(p^w-p)(y-x) > 0 \quad .$$

It will be assumed in this paper that this requirement is fulfilled and α_j will be treated as parameter. A more formal treatment will be presented in a separate paper.

4.2.1 Solution for the demand equations

The excess demand functions now result from the simultaneous solution of the several utility maximization problems:

$$\max u^j(x^j)$$

$$\text{S.T. } px^j = \alpha_j TR + py^j \quad .$$

Given TR a unique x^j is determined. Clearly x^j is a continuous function of TR.

From the summation of the budget constraints, we get

$$px = TR + py \quad .$$

Is there an equilibrium value of TR such that

$$TR^* = (p^w-p)(y-x) \quad ?$$

As can be seen from substitution, the problem is equivalent to the question whether

$p^w(y - x(TR)) = 0$ has a solution, TR*.

From the budget equation we know that px does not reach any saturation level as TR increases for any value of p.

If we assume that

$$\frac{dp^w x}{dp^y} > 0$$

then we know that

- $$\frac{dp^w x}{dTR} > 0$$

- $$\lim_{TR \nearrow \infty} p^w x = +\infty$$

$$\lim_{TR \nearrow py} p^w x = 0$$

So we may conclude that the set of demand equations has a unique solution under the given assumption.

4.2.2 Equilibrium solution on the world market

On the national level Walras' Law still holds as can be seen from the summation of the budget equations.
The imposition of a tariff has an effect on income distribution and by that, an effect on demand. It may also have a long run effect on production but this is not relevant under the present assumptions.
The imposition of a tariff does not however introduce any discontinuity in the excess demand function, so that an equilibrium still can be shown to exist on the world market.

4.3 Quota

4.3.1 Domestic price equilibrium with quota

When a constraint on international trade becomes effective, say for the i^{th} commodity, then its total domestic consumption is given.

However, by the same token its domestic price becomes variable because the world market price (minus possible tariffs) becomes irrelevant for determining domestic prices. This is so because arbitrage in that commodity becomes impossible. We thus get the equations:

$$\mu_i (y_i - x_i - L_i) = 0 \quad ,$$

$$\nu_i (y_i - x_i - R_i) = 0 \quad ,$$

$$p_i = \bar{p}_i + \mu_i - \nu_i \quad ,$$

$$L_i \leqq (y_i - x_i) \leqq R_i \quad ,$$

$$i = 1,\ldots,n \quad ,$$

$$\mu_i, \nu_i \geqq 0 \quad .$$

Where $\bar{p}_i$ is the desired domestic price and p_i the domestic price realized in equilibrium. The national model is obtained by adding this set of equations to the previously derived behavioural model.

Is it now possible for any given set of world market prices to calculate domestic consumption and domestic equilibrium prices? This will be investigated formally in a separate paper. We now only superficially glance at the problem.

1) $p(x^j - y^j) = \alpha_j (p^w - p)(y - x)$

2) $\frac{\partial u^j}{\partial x_j^i} = \lambda_j p_i$

3) $\mu_i (y_i - x_i - L_i) = 0$ $\qquad i = 1, \ldots, n$

4) $\nu_i (y_i - x_i - R_i) = 0$

5) $L_i \leqq y_i - x_i \leqq R_i$

6) $p_i = \bar{p}_i + \mu_i - \nu_i$

7) $\Sigma p_i^w (y_i - x_i) = 0$.

We define:

$$Z = \Sigma z^j$$

for given values of p^w and $\bar{p}$, the national excess demand equation may be written as

$$Z = Z(\rho) \quad \text{where } \rho = (\mu, -\nu)$$

now define

$$q = \begin{pmatrix} q1 \\ q2 \end{pmatrix} = \begin{bmatrix} Z(\rho) + L \\ -Z(\rho) - R \end{bmatrix} .$$

Thus, $q = q(\rho)$.

The quota restrictions imply $q \leqq 0$

and the complementarity equations imply: $\rho . q = 0$.

The existence proof for domestic equilibrium is quite complex for two reasons:

1) the excess demand functions $q = q(\rho)$ are not homogeneous to the degree zero in ρ;

2) The complementary conditions $\rho \cdot q = 0$ are not automatically implied by the demand equations.

For proving the existence of domestic equilibrium we must formulate a more general model, which in equilibrium can be shown to satisfy the complementarity conditions.
This will be done in a separate paper. This present model is still indeterminate as the taxation policy (determination of α_j) is not discussed.
Moreover, it needs some generalization as the export constraint implies

$$x \geqq y-R \quad ,$$

which might be inconsistent with nonnegative prices.

* Note that if no quota constraints are effective, the domestic prices are $\bar{p}_i$ and the demand equations imply by themselves balance of trade equilibrium so that (7) can be omitted. (This by the way illustrates the fact that Walras' Law - at the national level - can be an equilibrium condition instead of an identity even under barter trade.)

A classical proof of the existence of equilibrium would show that there exists a price vector $w = [p^w, p_1 \ldots, p_m]$, (m is the number of countries), such that all the markets are cleared. Here we want to proceed by showing that for any p^w, and under the assumption that the national excess demands could be satisfied, $p_1 \ldots p_m$ exist. If this equilibrium is unique and if the excess demand functions are continuous in world market prices, world market equilibrium can be proved to exist. The existence of equilibrium will then be proved in a hierarchical way. This will be useful when investigating algorithms for computation.

5. The assumptions in MOIRA

1. No tariff receipts for the agricultural sector

In MOIRA the assumption was made that the non-agricultural sector gets the tariff receipts because government and traders are part of the non-agricultural sector and because in countries where the agricultural sector is important, direct taxation (subsidy) of the farmers is not usual practice. The income distribution between the agricultural and non-agricultural sector is in these countries mainly realized through the prices which the farmer gets for his product. So if we disregard all other taxes and only consider tariffs it is not unreasonable to assume that on the one hand the non-agricultural sector earns the price differential while on the other hand, agricultural prices are set in order to reach a certain income parity ratio between sectors so that an increase in tariff receipts is transmitted (with a lag) to the agricultural sector through a rise in the domestic food price. In terms of our present model the distribution parameter α_j is set to zero in the agricultural sector.

2. Only one agricultural commodity and one non-agricultural commodity are considered.

3. The agricultural consumer owns all the factors of production in agriculture, so that he gets the total value added of the sector as income. The same assumption is made for the non-agricultural consumer.

4. The agricultural sector is assumed to produce its own food separately so that its food consumption plan is made up together with its food production plan with a one period lag. Thus the net food supply to the non-agricultural sector is predetermined: $y_f \equiv y_f - x_f^a$. The subscript f indicates food, nf non-food, hyperscript n indicates non-agricultural sector and a the agricultural sector.

5. There is assumed to be no difference between the world market price and the domestic price of the non-food commodity. The budget equations can now be written as:

a) agricultural sector:

$$p_{nf}\, x_{nf}^a = p_f\, y_f^n \;;$$

b) non-agricultural sector:

$$p_f\, x_f^n + p_{nf}\, x_{nf}^n = p_{nf}\, y_{nf} + (p_f^w - p_f)(y_f^n - x_f^n) \;.$$

ad a) It directly follows that:

$$x_{nf}^a = \frac{p_f}{p_{nf}^w}\, y_f^n \;.$$

ad b) This is equivalent to:

$$\frac{p_f^w}{p_{nf}^w}\, x_f^n + x_{nf}^n = y_{nf} + \frac{(p_f^w - p_f)}{p_{nf}^w} \cdot y_f^n \;.$$

The non-agricultural commodity may now be taken as numéraire. So we set $p^w_{nf} = 1$.

Summation of both equations yields

$$p^w_f \; x^n_f + (x^a_{nf} + x^n_{nf}) = y_{nf} + p^w_f \; y^n_f \quad .$$

Thus $p^w_f \; z^n_f + z_n = 0$ which means that Walras Law is satisfied on the country level.

The equilibrium can now be shown to exist in the same way this was done before. However, it must again be stressed that this equilibrium need not be unique, stable, or with reasonable bounds for the price vector.

As a matter of fact in the MOIRA runs some limits were set on price mutations. When no equilibrium could be found a situation of excess demand/supply was assumed to exist thus forcing the countries to adapt domestic prices or to allow stock mutations. These matters will not be discussed further, although deeper investigation is needed in this field.

6. Some extensions

6.1 The budget constraint as an equilibrium condition

Up to this point we worked with the static budget constraint as if it was a structural equation of the model. It indeed provided us with a sufficient condition for Walras' Law.

$$p^w \; z^j = 0 \longrightarrow \Sigma p^w \; z^j = 0$$

This condition is however not necessary and it will in general not be fulfilled in reality. It may be considered as an equilibrium condition for the balance of trade. This condition may be replaced by a more general one which would allow for debts:

$$p^w \; z^j + k^j = 0 \quad .$$

The most easy way would be to keep k^j exogenous or at least predetermined for all countries with the restriction that $\sum_j k^j = 0$, this would avoid going into the pricing of debts which would introduce many complications, such as future markets. The approach sketched here could thus take into account dis-equilibrium of the balance of trade, but it would still be an equilibrium condition as far as it states that there is monetary equilibrium. This provides some illustration of the statement that every model has its equilibrium condition(s). Once we assume that it holds this condition becomes a balance equation which closes the system, so that a simultaneous approach may become relevant.

As long as the budget constraint is the only constraint it implies that money can buy anything, anywhere, as no limited con-vertibility between the national currencies is introduced. Important parts of the national income cannot, however, in reality be traded internationally. This may be reflected by the imposition of quota.

6.2 Intermediate demand and current supply adaptation

Although the introduction of intermediate demand and current supply adaptation does not essentially change the framework developed here, it causes serious complications of a computational nature. The full consequences of the extension will not be investigated here. A very general kind of neo-classical model will be sketched and some specific assumptions will be added.

1. Neoclassical assumptions:

   - producers are assumed to maximize net revenue at given prices, within their technological constraints;

   - consumers maximize utility given the prices and the budget constraints.

2. Specific assumption:

   Net sectoral revenue is distributed to the consumer of the sector. This can be motivated by saying that if one assumes that the sector owns all the factors then it will also get the profit.

3. The equations are:

   a) agricultural consumer

max. $u^a(x^a)$

S.T. $p_f^w\, x_f^a + p_{nf}\, x_{nf}^a = p_f\, y_f - p_{nf}\, v_{nf}^a$

   b) non-agricultural consumer

max. $u^n\, (x^n)$

S.T. $p_f\, x_f^n + p_{nf}\, x_{nf}^n = p_{nf}\, y_{nf} - p_{nf}\, v_f^n$

   c) agricultural producer

max. $p_f\, y_f - p_{nf}\, v_{nf}^a$

S.T. $\begin{bmatrix} y_f \\ v_{nf}^a \end{bmatrix} \varepsilon\ T^a$ (agricultural technology) .

d) non-agricultural producer

max. $p_{nf}\ y_{nf} - p_f\ v_f^n$

S.T. $\begin{bmatrix} y_{nf} \\ v_f^n \end{bmatrix} \varepsilon\ T^n$ (non-agricultural technology)

The price sensitivity of supply will not be discussed now. It will however be intuitively clear that no fundamental changes occur by the introduction of producer behaviour. Moreover, the international trade policy of tariffs, subsidies and quota mainly introduces complications in consumer behaviour as long as the producer is assumed to face domestic prices. Therefore the matter will not be investigated further in this paper.

6.3 Producer and consumer tax/subsidy

There may be more than one domestic price: producers or consumers may get a subsidy/tax.
If producers pay a tax (i.e. get a subsidy) while there is a tariff on international trade and the income class considered gets a share a_j of the total (taxes and tariffs) the budget equation becomes (assuming balance of trade equilibrium):

$$p^c x^j = \alpha_j[(p^c-p)y + (p^w-p^c)(y-x)] + py^j$$

where p^c is the consumer price and p the producer price.
If the consumer pays a tax/subsidy, the equation becomes:

$$p^c x^j = \alpha_j [(p^c - p)x + (p^w - p)(y-x)] + py^j$$

$$(p^c - p)z + (p^w - p^c)(y-x) = (p^w - p)y - (p^w - p^c)x$$

$$(p^c - p)z + (p^w - p)(y-x) = (p^w - p)y - (p^w - p^c)x \quad .$$

So both formulations are fully equivalent.

Again on the national level, Walras' Law is satisfied so that no specific complications arise.

One should notice, however, that the money amounts involved in reaching a certain price differential will always be much lower when one is using a tariff than when one is using a tax, and that the distribution rule is again not investigated.

(Cf. "General equilibrium with taxes: a computational exercise and an existence proof", Shoven, Whalley), [2].

6.4 More than one world market price:

The agricultural products which are traded internationally can in general be considered as perfectly homogeneous products in the sense that the physical characteristics of the commodity are not (strongly) dependent upon the country of origin. Therefore arbitrage between markets is possible. So that the prices paid internationally will tend to be equalized.

There are, however, some possible counteracting factors.

1) Transportation costs:

This yields the so called Samuelson equilibrium conditions:

$$p_j \leqq p_i + T_{ij}$$

$$Q_{ij}[p_j - (p_i + T_{ij})] = 0$$

where T_{ij} is the cost of transporting one unit from region i to j and Q_{ij} is the quantity transported from region i to j and p_i is the price in region i.

2) Customs unions:

Some countries may organize free trade with each other but may restrict trade with outside nations (by tariffs and quota). When import and export quota for certain commodities are set at zero level, the case becomes equivalent to a situation with prohibitive transportation costs. The problem with a customs union is that one has to devise a rule for distributing the tariff receipts.

3) Discriminating tariffs:

It is possible, in the same way as customs unions discriminate between members and non-members, that a country would levy different quota or tariffs on trade flows depending on or destination. Whether different world market prices will arise depends on the indifference of other countries (see part I, § 7).

6.5 Stock mutations:

We now introduce stock mutations and sketch the consequences in the model of § 4.

Stocks can be owned at different levels.

1. Producer level:

Consequence: Adaptation of direct supply, m, such that the new stock $k_t^j = k_{t-1}^j + m$ satisfies $0 \leq k_t^j \leq k_u^j$ (k_u^j is the storage capacity). If the producer only stores 'as a producer' he only stores his own product q_j, thus $m_j \leq q_j$.

2. Consumer level:

The consumer may hold stocks of commodities he does not produce. The consequence of this is that he demands commodities for two purposes: storage and consumption.
This must be reflected in his utility function. The commodities which have been stored appear next year as a net supply so that the consumer behaves 'as if' he was a producer.

3. National level:

The stock formation is done collectively by the government so that the government has to pay it out of the budget.
Assuming that taxes and tariffs are distributed in the same way the only change in the demand model is a redefinition of government receipts (TR):

$$TR = (p^{w} - p)(y - x) - p^{w}I \quad ,$$

where I is restricted by storage capacity k^{u} .
(Note that the definition may be extended for all kinds of government consumption and investment.)

4. World level:

National governments pay to finance regional or world stocks:

$$I_c = I_c^{Dom.} + I_c^{World}$$

$$I^{World} = \sum_c I_c^{World}$$

with the budgetary consequences of 3.

7. Conclusion

In the present paper a framework has been developed for modelling international trade policies in a country with competitive domestic markets. It has been shown that the country models may be linked through a competitive world market. However, a more formal treatment still has to be presented. The supply side of the market economy was not depicted and government policies have not yet been endogenized.

APPENDIX I

Consumer Optimization, Tariffs and the Theory of Duality

A simple procedure will be presented here for solving demand functions from a utility maximization. Consider

$$\max.\ u(x)$$
$$\text{S.T. } px = M$$

This yields as F.O.C.:

$$\frac{\partial u}{\partial x_i} = \nu p_i \qquad i = 1,\ldots,n$$
$$px = M \quad .$$

This simultaneous set of equations may be very hard to solve (and to estimate econometrically!). Consider, however, a linear homogenous utility function. From the F.O.C., we may derive

$$u = \nu M \quad .$$

ν may be considered as the reverse of an aggregate price index function. Maximization of u is equivalent to minimization of $\mu = 1/\nu$. So we consider a specified expenditure function instead of a utility function. We can see that

$$C \equiv u \cdot \mu = \Sigma p_i x_i, (= M) \rightarrow x_i = \frac{\partial C}{\partial p_i} \quad .$$

This is a very easy solution. It means that

$$x_i = u \cdot \frac{\partial \mu}{\partial p_i}{}^{(1)} \quad ,$$

(1) The functions $\nu(p)$, $\mu(p)$ must exist, be jointly continuous, concave and nondecreasing in p and positively linear homogenous.

which again implies that quantity proportions are:

$$\frac{x_i}{x_j} = \frac{\frac{\partial \mu}{\partial p_i}}{\frac{\partial \mu}{\partial p_j}} \qquad \left(\text{cf. } \frac{p_i}{p_j} = \frac{\frac{\partial u}{\partial x_i}}{\frac{\partial u}{\partial x_j}} \right) .$$

The expenditure function $C = \mu(p)u$ is a special case of the form known as the Gorman polar form.

$$C = \mu(p) \cdot u + \lambda(p)$$

where

$\lambda(p)$ is an expenditure which has to take place in any case, independently from tne utility level reached.

Differentiating this function to p_i yields x_i . If $\mu(p)$ is a Cobb-Douglas and $\lambda(p)$ a linear function, we are in the case of the L.E.S. system.

We shall not discuss all the kinds of possible expenditure functions which correspond to given utility functions as this is done extensively in the papers by Keller and by Norman. What we want to find out here, is whether and how instruments of agricultural policies may be built into a "dual" formulation.

Case 1 -- Income Distribution

Each consumer class has its own behaviour:

$$\max. \ u\ (x_j)$$

$$\text{S.T. } px_j = M_j + \alpha_j (p^W - p)(y-x)$$

For the individual consumer class the expenditure function is again

$$C_j = \mu(p) \cdot u(x_j)$$

so that $\frac{x_{i_j}}{x_{h_j}}$ may be derived from:

$$\frac{\partial C_j}{\partial p_i} = \frac{\partial \mu(p)}{\partial p_i} \cdot u(x_j) = x_{i_j}$$

$$\text{thus} \quad \frac{x_{i_j}}{x_{h_j}} = \frac{\dfrac{\partial \mu(p)}{\partial p_i}}{\dfrac{\partial \mu(p)}{\partial p_h}}$$

Now the simultaneous system becomes very simple. For every consumer we have given proportions:

$$\frac{x_{i_j}}{x_{h_j}} = a_{i_j} \,, \qquad i = 1,\ldots,h,\ldots,n$$

while the budget equations are:

$$\sum_i p_i \, x_{ij} = [M_j + \alpha_j \, \Sigma(p_i^W - p_i) \, Y_i] - \alpha_j \sum_i \left((p_i^W - p_i) \sum_j x_{ij}\right).$$

This is clearly a linear set of equations which is very easy to solve.

Case 3 -- Quota

This case is more complex. Here the quantity demanded is given in some equations while the prices are not given. If the quota are effective in all commodities the solution from the primal F.O.C. is available. In mixed cases, however, no such simple solution seems to be available.

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