Economic Integration, Lobbying by Firms and Workers, and Technological Change

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Contents

1 Introduction ........................................................................................................ 1
2 The setting ........................................................................................................ 3
3 Production and consumption ............................................................................. 4
4 Firms .................................................................................................................. 6
5 Wage bargaining .................................................................................................. 9
6 The common market ............................................................................................. 10
7 The political equilibrium .................................................................................... 13
8 Economic integration .......................................................................................... 15
9 Binding deregulation ........................................................................................... 16
10 Conclusions ........................................................................................................ 18
Appendix ............................................................................................................... 20
References ............................................................................................................ 24
Economic Integration, Lobbying by Firms and Workers, and Technological Change

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Abstract

I examine a common market with the following institutions. Oligopolistic firms improve their productivity by R&D. Wages are determined by union-employer bargaining. Firms and workers lobby the authority that accepts new members and regulates unions’ and firms’ market power. The main findings are as follows. Small common markets have incentives to expand, but large ones are indifferent to new members. With product market deregulation, there is an upper limit for the size of the common market and the growth rate diminishes with integration.

Journal of Economic Literature: F15, J50, O40

Keywords: economic integration, labor unions, market power, endogenous technological change

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1 Introduction

During the latest decades, the European Union (EU) has expanded considerably. This expansion results from a rather complicated process which involves bargaining and lobbying in the EU. On the one hand, the integration has diminished rents which has raised opposition among different interest groups, but, on the other hand, the deregulation of rents has been suggested as the remedy for slow growth and weak international competitiveness of the EU. This paper attempts to model the expansion of a common market as a political process. In order to have interest groups, I introduce wage bargaining, and in order to have a plausible role of rents, I introduce endogenous technological change into the model. This framework can be used as a benchmark for considering the problems of growth and economic integration.

The growth effects of regulation depend decisively on the structure of economy. Where the same technology is used both in production and in R&D, the economy behaves as if the same final good were used both in consumption and in R&D. In that specific case, labor market regulation can be only growth-hampering: an increase in union power decreases profits, incentives to invest in R&D and the growth rate (cf. Peretto 1998). In this study, I assume that there is different technology for production and R&D.\footnote{I take this to the extreme so that R&D employs only labor, for simplicity.} With this specification, there can be a positive dependence union power and technological change through cost-escaping R&D as follows. With higher union wages, firms have more incentives to improve the productivity of labor through R&D. This increases investment in R&D and the growth rate.

There is also some empirical evidence on a positive relationship between R&D and labor market regulation. Caballero (1993) and Hoon and Phelps (1997) show that changes in unemployment and productivity growth are positively associated. Some papers explain R&D by the unionization rate, i.e. the ratio of unionized to all workers,\footnote{Addison and Wagner (1994) found a positive cross-sectional correlation, but Menezes-Filho et.al. (1998) only little correlation in a panel of firms, between R&D and the unionization rate in the UK. Connolly et.al. (1986), Hirsch (1990; 1992), Bronars et.al. (1994) in the USA, and Betts et.al. (2001) in Canada found a negative cross-sectional correlation between these. Hence, the results have been highly institution-specific.} but this is a different issue.\footnote{The unionization rate is not a proper proxy for union power in wage bargaining. In many European countries it tells nothing about union power, because the contract made by}

\begin{eqnarray*}
\end{eqnarray*}
Except cost-escaping, there has been also other attempts to explain a positive wage-growth relationship as follows. Cahuc and Michel (1996) (using an OLG model), as well as Agell and Lommerud (1997) (using an extensive game framework) show that a minimum wage creates an incentive for workers to accumulate human capital. Meckl (2004) extends Aghion and Howitt’s (1998) Schumpeterian growth model so that production employs skilled and unskilled, but R&D only skilled labor. He shows that higher minimum wages for unskilled labor raise employment of skilled labor and the growth rate. Palokangas (1996, 2000) introduces wage bargaining into Romer’s (1990) product-variety model. He shows that if the elasticity of substitution between skilled and unskilled labor is less than one, then an increase in union power raises wages for unskilled workers, reduces the demand for skilled workers in production, and thereby lowers wages for skilled workers. This decreases costs in R&D and promotes growth. Lingens (2003) and Palokangas (2004, 2005) reconstruct the same effect for Schumpeterian and R&D-based growth models. In this study, however, I stick to the assertion of cost-escaping, because it provides a more direct link between rents and technological change.

The prospect of losing rents due to economic integration is also an important issue. Dinopoulos and Zhao (2004) develop a two-sector general equilibrium model of a small open economy unionized and non-unionized firms. To identify economic conditions for anti-globalization arguments, they analyze how economic integration affects the allocation of resources, labor effort and the structure of wages. They, however, postulate a union’s utility as a geometric average of the wage and employment and assume that labor unions ignore the effect of their wages on productivity through R&D. In contrast, I analyze a union of countries, not a small economy, derive a union’s preferences directly from workers’ preferences and assume that unions are aware of how their wage policy affects their members’ employment through R&D.

I organize the remainder of this study as follows. In section 2, I present the institutional setting of the study as an extended game. As a part of the representative union is extended to cover all employers and employees in the industry. In some other countries (e.g. USA, Canada), unions can make agreements only for their members and a unionized worker can be easily replaced by a non-unionized worker. This imposes an additional constraint for the union in wage bargaining, but does not necessarily affect the relative bargaining power of the parties.
this game, I construct specific models for households in section 3, firms in 4, wage bargaining in 5 and for the common market in 6. Finally, I analyze the political equilibrium in section 7 and economic integration in 8 and 9.

2 The setting

I consider a common market that contains a number $J$ of similar regions. A member country of the common market is comprised of a smaller number ($< J$) of regions. Each region contains a fixed amount $L$ of labor, a representative firm and a representative labor union. To examine the political economy of growth and economic integration, the model is then composed as follows:

(i) All firms produce one unit of output from one labor unit. The oligopolistic competition of the firms determines prices in the common market.

(ii) Because the new members have access to the same technology and must adopt the same institutions as the old members, economic integration can be characterized by the increase in the size $J$ of the common market. An expansion of the common market intensifies competition.

(iii) Workers and firms bargain over wages.

(iv) Firms invest in R&D to escape production costs.

(v) I call the decision maker of the common market as the central planner. It regulates the product and labor markets and accepts new members to the common market. The central planner has its own interests and it is lobbied by interest groups that represent workers and firms.

In this model, the common market expands by accepting new regions as members, and any opposition to economic integration manifests itself as an upper limit that the political process sets for the common market. I summarize the institutional structure of the model as follows:

<table>
<thead>
<tr>
<th>Agents</th>
<th>Representatives in wage bargaining</th>
<th>Representatives in lobbying the central planner of the common market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>Labor unions</td>
<td>Worker lobby</td>
</tr>
<tr>
<td>Firms</td>
<td>Employer federation</td>
<td>Employer lobby</td>
</tr>
</tbody>
</table>

4The assumption on similar regions is admittedly strong, but with asymmetric regions there is no analytical result in the model.
I use the common agency model (e.g. Bernheim and Whinston 1986, Grossman and Helpman 1994a, and Dixit, Grossman and Helpman 1997) to establish a political equilibrium with the following sequence of decisions:

1. Worker and employer lobbies make their offers to the central planner (section 7). These offers relate the lobbies’ prospective political contributions to the central planner’s policy.

2. The central planner accepts new members to the common market and regulates the product and labor markets (section 6). Product market regulation determines how much firms can coordinate their actions in price settlement, and labor market regulation determines the relative power of the labor unions in wage bargaining.

3. Unions and employers bargain over the wages (section 5).

4. Firms decide how much to invest in R&D (subsection 4.2).

5. Each firm decides on its output given its expectations on the behavior of the other firms (subsection 4.1).

6. The households decide on their consumption (section 3).

This extended game is solved by backward induction. The outcomes of that game are given in section 8 and 9.

3 Production and consumption

In region $j \in \{1, \ldots, J\}$ of the common market, a single firm (hereafter firm $j$) produces good $j$ from labor with technology

$$y_j = B_j n_j,$$

(1)

where $y_j$ output, $n_j$ labor input in production and $B_j$ is the productivity parameter. I assume that all products $j \in \{1, \ldots, J\}$ are perfect substitutes, for simplicity.\(^5\)  The total supply of the composite good in the common market,

\(^5\)With some complication, it is possible to use a CES function here for the same purpose.
$C$, is the sum of regional outputs $y_j$:

$$C = \sum_{j=1}^{J} y_j. \quad (2)$$

The average productivity of the common market is given by

$$B = \frac{1}{J} \sum_{j=1}^{J} B_j. \quad (3)$$

All households in the common market share the same preferences and take income, the prices and the interest rate $r$ as given. Thus, they all behave as if there were a single representative household for the whole common market. The household chooses its flow of consumption $C$ to maximize its utility starting at time $T$,

$$\int_{T}^{\infty} (\log C)e^{-\rho(\theta-T)} d\theta,$$

where $\theta$ is time, $C$ consumption and $\rho > 0$ the constant rate of time preference. Noting (2), this utility maximization leads to the Euler equation$^6$

$$\frac{\dot{E}}{E} = r - \rho \quad \text{with} \quad E = pC = p \sum_{j=1}^{J} y_j, \quad (4)$$

where $p$ the consumption price, $E$ total consumption expenditure, $r$ the interest rate and $\dot{E} = dE/dt$. Because in the model there is no money that would pin down the nominal price level at any time, it is convenient to normalize the households’ total consumption expenditure in the common market, $E$, at the constant number $J$ of regions.$^7$ This and (4) yield

$$E = J, \quad p = E/\sum_{j=1}^{J} y_j = J/\sum_{j=1}^{J} y_j, \quad r = \rho = \text{constant} > 0. \quad (5)$$

Technology (1), (2) and (3) has the useful property that with symmetry throughout the regions, $n_j = n$ for all $j$, total consumption is determined by

$$C|_{n_j=n} = JnB. \quad (6)$$


$^7$With this normalization, the equilibrium price $p$ and the equilibrium wage $w$ are independent of the size of the common market, $J$. 

5
Because consumption per region, $C/J$, is then indifferent of the size $J$ of the common market, there are no scale effects on consumption. In this case, economic integration is motivated only by rents in the goods or labor market.

4 Firms

4.1 Competition in the product market

Following Dixit (1986), I assume that each firm $j$ anticipates the reaction of the other firms $k \neq j$ by

$$
dy_k/dy_j = \varphi y_k/y_j \text{ for } k \neq j,
$$

where $\varphi \in (0, 1)$ is a measure of the firms’ market power. If $\varphi = 0$, the firms behave in Cournot manner, taking each others’ output level as given. The higher $\varphi$, the more the firms can coordinate their actions and the higher price they can charge. The central planner can decrease (increase) $\varphi$ by intensifying (weakening) its competition and anti-trust policies. The product market is fully deregulated for $\varphi = 0$.

I assume, for simplicity, uniform initial productivity in the common market, $B^0_k = B^0$ for all $k$. This implies symmetry $y_k = y$ for all $k$. Noting (5) and (7), the inverse of the anticipated price elasticity of demand for firm $j$ is then given by

$$
\phi(J, \varphi) = -\frac{y_j}{p} \frac{dy_j}{dp} \bigg|_{y_k=y} = \left( \frac{\sum_{k=1}^J y_k}{\sum_{k=1}^J \frac{dy_k}{dy_j}} \right)_{y_k=y} = \frac{1}{J} \left[ \sum_{k=1}^J \frac{dy_k}{dy_j} \right]_{y_k=y}
$$

with $\partial \phi/\partial J = (\varphi - 1)/J^2 < 0$ and $\partial \phi/\partial \varphi = 1 - 1/J > 0$. (8)

Firm $j$ maximizes its profit

$$
\pi_j = py_j - w_j n_j,
$$

where $y_j$ is output, by its labor input $n_j$ holding the wage $w_j$ and productivity $B_j$ constant, given the production function (1) and the price elasticity of the
demand for output, (8). Noting (5), this maximization yields the equilibrium conditions

\[ w_j = \left[ p + y_j \frac{dp}{dy_j} \right] B_j = (1 - \phi) p B_j = \frac{(1 - \phi) J}{\sum_{j=1}^{J} y_j} B_j, \]

\[ \pi_j = p y_j - w_j n_j = p y_j - (1 - \phi) p B_j n_j = \phi p y_j, \quad w_j n_j / \pi_j = 1 / \phi - 1, \]

\[ \sum_{j=1}^{J} w_j n_j = (1 - \phi) p \sum_{j=1}^{J} y_j = (1 - \phi) J, \quad \sum_{j=1}^{J} \pi_j = \phi p \sum_{j=1}^{J} y_j = \phi J. \quad (9) \]

In this setting, the firms and the workers share the value added is fixed proportions \( \phi \) and \((1 - \phi)\), respectively. Given (8), a decrease in firms’ market power \( \phi \) or an increase in the size \( J \) of the common market intensifies competition and decreases the firm’s share \( \phi \).

Results (9) show that labor input in production, \( n_j \), can be constant, provided that the wage \( w_j \) and the profit \( \pi_j \) change in the same proportion. Without this property, there could not be a steady state in the model.

4.2 Research and development (R&D)

Technological change for firm \( j \) is characterized by a Poisson process \( q_j \) as follows. During a short time interval \( d\theta \), there is an innovation \( dq_j = 1 \) with probability \( \Lambda_j d\theta \), and no innovation \( dq_j = 0 \) with probability \( 1 - \Lambda_j d\theta \), where \( \Lambda_j \) is the arrival rate of innovations in the research process. The arrival rate \( \Lambda_j \) is an increasing function of labor devoted to R&D, \( l_j \),

\[ \Lambda_j = \lambda l_j^{1-\nu}, \quad \lambda > 0, \quad \nu \in (0, 1), \quad (10) \]

where \( \lambda \) and \( \nu \) are constants. Decreasing returns to scale \( \nu \in (0, 1) \) in R&D are assumed to ensure the existence of equilibrium. Following Horii and Iwaisako (2007), this can be justified by the possibility of duplication: when two workers innovate in the same industry, they produce very likely less than a double amount of innovations.

I denote the serial number of technology in region \( j \) by \( t_j \) and variables depending on technology \( t_j \) by superscript \( t_j \). The invention of a new technology raises \( t_j \) by one and the level of productivity \( B_j^{t_j} \) by \( a > 1 \). Hence,

\[ B_j^{t_j} = B_j^0 a^{t_j}. \quad (11) \]
During a short time interval $d\theta$, there is a change in technology from $t_j$ to $t_j + 1$ with probability $\Lambda_j d\theta$, and no change with probability $1 - \Lambda_j d\theta$, where $\Lambda_j$ is given by (10).

The average growth rate of the level of productivity (11) in the stationary state is in fixed proportion $(\lambda \log a)$ to $l_j^{1-\nu}$ (cf. Aghion and Howitt 1998, p. 59) and thereby an increasing function of $l_j$. Thus, research input $l_j$ can be used as a proxy of the growth rate in region $j$ and the average research input $\frac{1}{n} \sum_{j=1}^{n} l_j$ as a proxy of the growth rate for the whole common market.

Firm $j$’s dividends are given by

$$\Pi_j = \pi_j - w_j l_j, \quad (12)$$

where $\pi_j$ is profit, $w_j$ the wage in region $j$, $l_j$ labor devoted to R&D and $w_j l_j$ expenditures on R&D. Firm $j$ maximizes the present value of its dividends (12) by its investment in R&D, $l_j$, subject to technological change, given the wage $w_j$. The value of firm $j$’s optimal program at time $T$ is

$$\Omega(t_j, w_j, \pi_j) = \max_{l_j \text{s.t.}(10), (12)} E \int_T^{\infty} \Pi_j e^{-r(\theta-T)} d\theta, \quad (13)$$

where $\theta$ is time, $E$ the expectation operator and $r$ the interest rate. In Appendix A, I show this optimization leads to the two results:

(i) The ratio of dividends to profits, $\Pi_j/\pi_j$, is a decreasing function of labor devoted to R&D, $l_j$ as follows:

$$\frac{\Pi_j}{\pi_j} = c_j = c(l_j) = \frac{r + (1-a)\lambda l_j^{1-\nu}}{r + (1-a)\nu l_j^{1-\nu}}, \quad r + (1-a)\lambda l_j^{1-\nu} > 0,$$

$$c' = \frac{dc_j}{dl_j} = \frac{(\nu - 1)(1-c_j)l_j/l_j}{r + (1-a)\lambda l_j^{1-\nu}} < 0. \quad (14)$$

This can be explained by decreasing returns to scale in R&D.

(ii) The constraint $w_j n_j/\pi_j = 1/\phi - 1$ in (9) can be transformed into the form where labor devoted to production, $n_j$, is a decreasing function of the firms’ share of value added, $\phi$, but total labor input $n_j + l_j$ is an increasing function of labor devoted to R&D, $l_j$:

$$n_j = n(l_j, \phi) = \frac{1}{1-c(l_j)} l_j, \quad \frac{\partial n}{\partial \phi} < 0, \quad n_j + l_j = \frac{1/\phi - c}{1-c} l_j,$$

$$\frac{\partial (n_j + l_j)}{\partial l_j} = \left[ \frac{1}{\phi} - c(l_j) \right] \frac{c(l_j)}{1-c(l_j)} - \frac{c'(l_j)l_j}{1-c(l_j)} > 0. \quad (15)$$
5 Wage bargaining

Because each region $j$ possesses a fixed amount $L$ of labor, its full-employment constraint is given by

$$l_j + n_j \leq L,$$

(16)

where $n_j$ and $l_j$ are labor inputs in production and R&D, respectively.

In each region $j$, the workers’ wage $w_j$ is determined by bargaining between a union representing workers in economy $j$ (hereafter union $j$) and a federation representing the employers of these workers (hereafter employer $j$). I assume, for simplicity, that both parties of bargaining are risk neutral. This allows me to solve the problem as an alternating-offers game. I also assume that the workers have access to perfect unemployment insurance. This ensures that all workers in the same region behave as if there were only one worker in that region.\(^8\) I assume, furthermore, that in the case of a dispute there is no production, and consequently neither labor income nor profits. The reference income is then zero for both the union and the employer.\(^9\)

In wage bargaining, at each time $T$, labor union $j$ maximizes the expected present value of wages,

$$U(l_j, \phi) = E \int_T^\infty (n_j + l_j) w_j e^{-r(\theta-T)} d\theta,$$

(17)

and federation $j$ maximizes the expected present value of dividends $\Pi_j$,

$$F(l_j, \phi) = E \int_T^\infty \Pi_j e^{-r(\theta-T)} d\theta = E \int_T^\infty c(l_j) \pi_j e^{-r(\theta-T)} d\theta,$$

(18)

\(^8\)Otherwise, workers’ income distribution would affect the unions’ behavior and the general equilibrium of the region. Because this would excessively complicate the analysis, I ignore all distributional aspects in this study and leave them for future investigation.

\(^9\)The expected wage outside the firm is commonly assumed to be the union’s reference point, but this is not quite in line with the microfoundations of the alternating offers game. Binmore, Rubinstein and Wolinsky (1986) state (pp. 177, 185-6) that the reference income should not be identified with the outside option point. Rather, despite the availability of these options, it remains appropriate to identify the reference income with the income streams accruing to the parties in the course of the dispute. For example, if the dispute involves a strike, these income streams are the employee’s income from temporary work, union strike funds, and similar sources, while the employer’s income might derive from temporary arrangements that keeps the business running.
subject to the full-employment constraint (16) and the firms’ behavior as a producer and an investor. The outcome of this bargaining can be obtained through maximizing the Generalized Nash Product

\[ U_j^\alpha F_j^{1-\alpha}, \quad \alpha \in [0, 1], \]  

by the wage \( w_j \), where \( \alpha \) is relative union bargaining power. Because \( \alpha \) depends on the regulations of the common market,\(^{10}\) I assume that the central planner of the common market uses \( \alpha \) as the policy instrument. In Appendix B, the maximization of the Generalized Nash Product (19) yields the following equilibrium condition. If there is any unemployment, both the relative union bargaining power \( \alpha \) and the firms’ share of valued added, \( \phi \), promote R&D and growth:

\[ l_j = \ell(\alpha, \phi) \text{ for } l_j + n_j < L, \quad \frac{\partial\ell}{\partial\alpha} > 0, \quad \frac{\partial\ell}{\partial\phi} > 0, \quad \lim_{\alpha \to 0} (L - l_j - n_j) = 0. \]  

(20)

The greater the firm’s share of value added or the higher union wages, more incentives the firm has to increase the productivity of labor through R&D.

6 The common market

I consider a symmetric equilibrium with \( B_j^0 = B_0 \), in which case

\[ n_j = n, \quad l_j = l, \quad w_j = w \text{ and } \pi_j = \pi. \]

In that equilibrium, noting (15) and (20), the full-employment constraint (16) and the constraint \( \alpha \leq 1 \) can be written as:

\[ L \geq l + n(l, \phi), \quad \ell(1, \phi) \geq \ell(\alpha, \phi) = l. \]  

(21)

\(^{10}\)The microfoundations of relative union bargaining power as follows (cf. Binmore, Rubinstein and Wolinsky 1986): When two players are making alternating offers to each other, they behave so as to maximize a weighed geometric average of their utilities – the Generalized Nash product. The weights of such an average, which reflect the relative bargaining power of the parties, are determined by the parameters of the model. Labor market regulation influences union power through these parameters.
From (9) and (15) it follows that

\[ wn = \frac{1}{J} \sum_{j=1}^{J} w_j n_j = 1 - \phi, \quad \pi = \frac{1}{J} \sum_{j=1}^{J} \pi_j = \phi, \]

\[ (n + l)w = (1 + l/n)wn = (1 - c\phi)wn/(1 - \phi) = 1 - c(l)\phi. \] (22)

By (1), (5), (14) and (15), I define the present value of the expected flow of real income per region, \( y \), as (cf. Aghion and Howitt 1998, p. 61)

\[ \Psi(l, \phi) = E \int_{-\infty}^{\infty} e^{-r(\theta - T)} d\theta = E \int_{-\infty}^{\infty} Bn e^{-r(\theta - T)} d\theta = \frac{B(T)n}{r + (1 - \phi)\lambda l^{1-\nu}}, \]

\[ \psi'(l) = \frac{d\log \psi}{dl} = \frac{1}{l} + \frac{c'}{1 - c} - \frac{(1 - \nu)(1 - a)\lambda l^{1-\nu}}{r + (1 - a)\lambda l^{1-\nu}}, \]

\[ \frac{\psi'}{\psi} > 0. \] (23)

Holding the firms’ share of value added, \( \phi \), constant, a higher level of R&D (i.e. a bigger \( l \)) speeds up growth and increases thereby the present value of the expected flow of real income, \( \Psi \).

The unions and the firms lobby the central planner which decides on the firms’ market power \( \varphi \), the unions’s relative bargaining power \( \alpha \) and new members of the common market (i.e. the size \( J \) of the common market). Following Grossman and Helpman (1994a), I assume that the central planner has its own interests and collects contributions \( R_u \) and \( R_f \) from the union and employer lobbies. A member of the worker lobby earns wages \((n + l)w\) minus political contributions \( R_u \). A member of the employer lobby earn dividends \( \Pi \) minus political contributions \( R_u \). Because the effects through the the price level \( p \) can be internalized at the level of the common market, the worker lobby maximizes the present value \( U \) of the expected flow of a typical worker’s
real income \([(n + l)w - R_u]/p\), and the employer lobby maximizes the present value $F$ of the expected flow of a typical firm’s real dividends $(\Pi - R_f)/p$ at time $T$. Noting (14), (20), (22) and (23), these targets can be defined as follows:

$$U\left(\ell(\alpha, \phi(J, \varphi)), \phi(J, \varphi), R_u\right) = U(l, \phi, R_u) \doteq E \int_T^{\infty} \frac{(n + l)w - R_u}{p} e^{-r(\theta - T)} d\theta = \Psi[(n + l)w - R_u] = \Psi(l, \phi)[1 - c(l)\phi - R_u],$$

$$F\left(\ell(\alpha, \phi(J, \varphi)), \phi(J, \varphi), R_u\right) = F(l, \phi, R_f) \doteq E \int_T^{\infty} \frac{\Pi - R_f}{p} e^{-r(\theta - T)} d\theta = \Psi[\Pi - R_f] = \Psi[c(l)\pi - R_f] = \Psi(l, \phi)[c(l)\phi - R_f],$$

where

$$U(l, \phi, R_f) + F(l, \phi, R_f) = (1 - R_u - R_f)\Psi(l, \phi). \quad (26)$$

Noting (23), the present value the expected flow of the real political contributions at time $T$ is given by

$$E \int_T^{\infty} \frac{R_u + R_f}{p} e^{-r(\theta - T)} d\theta = \Psi(l, \phi)(R_u + R_f). \quad (27)$$

Given this and (26), I specify the central planner’s utility function as follows:

$$G\left(\ell(\alpha, \phi(J, \varphi)), \phi(J, \varphi), R_u, R_f\right) = G(l, \phi, R_u, R_f)
\doteq E \int_T^{\infty} \frac{R_u + R_f}{p} e^{-r(\theta - T)} d\theta + \zeta_w U(l, \phi, R_u) + \zeta_f F(l, \phi, R_f)$$

$$= \Psi(l, \phi)(R_u + R_f) + \zeta_w U(l, \phi, R_u) + \zeta_f F(l, \phi, R_f)$$

$$= \Psi(l, \phi) + (\zeta_w - 1)U(l, \phi, R_u) + (\zeta_f - 1)F(l, \phi, R_f), \quad (28)$$

where constants $\zeta_w \geq 0$ and $\zeta_f \geq 0$ are weights of the worker’s and the firm’s welfare in the government’s preferences, respectively.

Grossman and Helpman’s (1994a) objective function (28) is widely used in models of common agency and it has been justified as follows. The politicians are mainly interested in their own income which consists of the contributions from the public, $R_u + R_f$, but because they must defend their position in general elections, they must sometimes take the utilities of the interest groups $U(l, \phi, R_u)$ and $F(l, \phi, R_f)$ into account directly. The linearity of (28) in $\Psi[R_u + R_f]$ is assumed, for simplicity.
7 The political equilibrium

I assume for a while that the central planner can smoothly regulate unions’ and firms’ market power \((\alpha, \varphi)\). The results can then be extended for the case where the central planner’s choices are more discrete.

Because the functions \(\phi(J, \varphi)\) and \(\ell(\alpha, \phi(J, \varphi))\) in (24), (25) and (28) establish one-to-one correspondence from the central planner’s instruments \((\alpha, \varphi)\) to the vector \((l, \phi)\), one can in the model consider labor devoted to R&D (= the measure of the growth rate, cf. subsection 4.2) \(l\) and the firms’ share of value added, \(\phi\), as the central planner’s policy variables. The unions’ and employers’ lobbies try to affect the central planner by their contributions \(R_u\) and \(R_f\). The contribution schedules are therefore functions of the central planner’s policy variables:

\[
R_u(l, \phi), \quad R_f(l, \phi).
\]

The central planner maximizes its utility function (28) by \((l, \phi)\), given the contribution schedules (29) and the constraints (8) and (21). Following proposition 1 of Dixit, Grossman and Helpman (1997), a subgame perfect Nash equilibrium for this game is a set of contribution schedules \(R_u(l, \phi)\) and \(R_f(l, \phi)\) and policy \((l, \phi)\) such that the following conditions \((i)-(iv)\) hold:

(i) Contributions \(R_u\) and \(R_f\) are non-negative but no more than the contributor’s income.

(ii) The policy \((\phi, l)\) maximizes the central planner’s welfare (28) taking the contribution schedules \(R_u\) and \(R_f\) as given,

\[
(l, \phi) \in \arg \max_{(l, \phi) \text{ s.t. } (8) \text{ and } (21)} G(l, \phi, R_u(l, \phi), R_f(l, \phi));
\]

(iii) The worker lobby (employer lobby) cannot have a feasible strategy \(R_u(l, \phi)\) \((R_f(l, \phi))\) that yields it a higher level of utility than in equilibrium, given the central planner’s anticipated decision rule,

\[
(l, \phi) = \arg \max_{(l, \phi) \text{ s.t. } (8) \text{ and } (21)} U(l, \phi, R_u(l, \phi)); \\
(l, \phi) = \arg \max_{(l, \phi) \text{ s.t. } (8) \text{ and } (21)} F(l, \phi, R_f(l, \phi)).
\]

(30)
The worker lobby (employer lobby) provides the central planner at least with the level of utility than in the case it offers nothing \( R_u = 0 \) \( (R_f = 0) \), and the central planner responds optimally given the other lobby’s contribution function,

\[
G(l, \phi, R_u(l, \phi), R_f(l, \phi)) \geq \max_{(l, \phi) \text{ s.t. (8) and (21)}} G(l, \phi, R_u(l, \phi), 0),
\]

\[
G(l, \phi, R_u(l, \phi), R_f(l, \phi)) \geq \max_{(l, \phi) \text{ s.t. (8) and (21)}} G(l, \phi, 0, R_f(l, \phi)).
\]

Noting (29) and (30), the central planner’s utility function (28) changes into

\[
G(l, \phi) = G(l, \phi, R_u(l, \phi), R_f(l, \phi))
\]

\[
= \Psi(l, \phi) + (\zeta_w - 1) \max_{(l, \phi) \text{ s.t. (8) and (21)}} U(l, \phi, R_u(l, \phi))
\]

\[
+ (\zeta_f - 1) \max_{(l, \phi) \text{ s.t. (8) and (21)}} F(l, \phi, R_f(l, \phi)),
\]

\[
\partial G/\partial l = \partial \Psi/\partial l, \quad \partial G/\partial \phi = \partial \Psi/\partial \phi.
\] (31)

The Lagrangean for the maximization of the central planner’s utility function (31) by \((l, \phi)\) subject to the constraints (8) and (21) is given by

\[
\mathcal{H} = G(l, \phi) + \eta[\phi - 1/J] + \varepsilon[L - l - n(l, \phi)] + \vartheta[\ell(1, \phi) - l],
\] (32)

where the multipliers \(\varepsilon\) and \(\vartheta\) are subject to the Kuhn-Tucker conditions

\[
\eta[\phi - 1/J] = 0, \quad \eta \geq 0, \quad \varepsilon[L - l - n(l, \phi)] = 0, \quad \varepsilon \geq 0,
\]

\[
\vartheta[\ell(1, \phi) - l] = 0, \quad \vartheta \geq 0.
\] (33)

Noting (15), (20), (23), (31) and (32), the first-order conditions for the maximization of the central planner’s utility are the following:

\[
\frac{\partial \mathcal{H}}{\partial \phi} = \frac{\partial G}{\partial \phi} + \eta - \varepsilon \frac{\partial n}{\partial \phi} + \vartheta \frac{\partial \ell}{\partial \phi} = \frac{\partial \Psi}{\partial \phi} + \eta - \varepsilon \frac{\partial n}{\partial \phi} + \vartheta \frac{\partial \ell(1, \phi)}{\partial \phi} = 0,
\] (34)

\[
\frac{\partial \mathcal{H}}{\partial l} = \frac{\partial G}{\partial l} - \varepsilon \left[1 + \frac{\partial n}{\partial l}\right] - \vartheta = \frac{\partial \Psi}{\partial l} - \varepsilon \left[1 + \frac{\partial n}{\partial l}\right] - \vartheta = 0.
\] (35)

14
8 Economic integration

I consider now how the structure of the common market depends on economic integration. If the size of the common market is very small, \( J \leq 1/\phi \), then the firms’ share of value added is determined by \( \phi = 1/J \). In that case, by (32) and (33), \( \eta > 0 \) and \( \partial H / \partial J = \eta / J^2 > 0 \) hold true. If the common market is large enough, \( J > 1/\phi \), then, by (32), (33), \( \phi > 1/J, \eta = 0 \) and \( \partial H / \partial J = \eta / J^2 \equiv 0 \) hold true. These results can be rephrased as follows:

Proposition 1 A small common market \( J \leq 1/\phi \) expands (i.e. \( J \) increases), whenever possible. A large common common market \( J > 1/\phi \) is indifferent to new members.

Concerning labor market regulation, there are two possibilities:

(a) If the labor market is deregulated, \( \alpha \to 0 \), then, by (20), there is full employment \( L = l + n(l, \phi) \). In that case, there cannot be monopoly unions \( \alpha = 1 \) that can dictate wages and, by (20), (33) and (34), \( \ell(1, \phi) > l, \vartheta = 0, \varepsilon = \partial \Psi / \partial l / (1 + \partial n / \partial l) > 0 \) hold true.

(b) Otherwise, when \( \alpha \) is sufficiently large, the labor market is regulated, there is unemployment \( L > l + n(l, \phi) \) and, by (20), (33) and (34), \( \varepsilon = 0, \vartheta = \partial \Psi / \partial l > 0, \ell(1, \phi) = l \) and \( \alpha = 1 \) hold true.

Proposition 2 The labor market is either (a) deregulated with full employment or (b) fully regulated with monopoly unions.

The central planner can increase its welfare either (a) by increasing the level of income or (b) by speeding up economic growth. If (a) is more effective than (b), then the central planner eliminates union power altogether to have full employment. On the other hand, if (b) is more effective than (a), then the central planner supports labor unions to promote cost-escaping R&D.

For a small common market with full employment, it is true that \( \phi = 1/J \) and \( L = l + n(l, \phi) = l + n(l, 1/J) \). Differentiating the latter equation totally, and noting (8) and (15), one obtains

\[ \frac{dl}{dJ} = \frac{dn}{d\phi} \left( \frac{1}{J^2} + \left[ \frac{1 + \partial n}{\partial l} \right] \right) > 0. \]
For a small common market with unemployment, it is true that $\phi = 1/J$ and $l = \ell(1, \phi) = \ell(1, 1/J)$. Differentiating the latter equation totally, and noting (20), one obtains

$$\frac{dl}{dJ} = -\frac{d\ell}{d\phi} - \frac{1}{J^2} < 0.$$  

These two results can be rephrased as follows:

**Proposition 3** When the common market is small, $J \leq 1/\phi$, economic integration (i.e. $J$ increases) hampers R&D and growth (i.e. $l$ falls).

In a small common market, economic integration intensifies product market competition, decreases the firms’ share of value added, $\phi$, and increases labor devoted to production. This crowds out labor devoted to R&D (cf. case (a)) and reduces the firms’ incentives to increase the productivity of labor through R&D (cf. case (b)). In both cases, R&D and the growth rate fall.

For a large common market, $J > 1/\phi$, the central planner cannot maintain the firms’s share of value added high enough without letting the firms’ to coordinate their actions (i.e. $\varphi > 0$). Because this makes the firms’ share of value added, $\phi$, the full-employment constraint $L \geq l + n(l, \phi)$ and the equilibrium condition of the labor union, $l = \ell(1, \phi)$, independent of $J$, there is no link from $J$ to the growth rate $l$. I conclude this as follows:

**Proposition 4** In a large common market, $J > 1/\phi$, economic integration (i.e. a bigger $J$) has no effect on the level of R&D, $l$, and the growth rate.

When the central planner can control the intensity of product market competition by its competition policy (i.e. by $\varphi$), it is indifferent to new members.

### 9 Binding deregulation

Where the labor market is deregulated $\alpha \to 0$ by a constitution or an international agreement that binds the central planner, the full employment constraint (21) holds as an equality. In that case, the development patterns are the same as in the full-employment scenario (a) in section 8.
Assume that the product market is deregulated \((\varphi = 0\) and \(\phi = 1/J)\) by a constitution or an international agreement that binds the central planner. To implement the equality constraint \(\phi = 1/J\), I introduce the inequality \(\phi \leq 1/J\), which is opposite to the inequality (8), into the central planner’s maximization problem. The maximization of the Lagrangean (32) by \((\phi, l, \eta, \varepsilon, \vartheta)\) with respect to \(\phi \leq 1/J\) leads to another Lagrangean

\[
\tilde{H} = H + \mu[1/J - \phi],
\]

where the multiplier \(\mu\) satisfies the Kuhn-Tucker conditions

\[
\mu[1/J - \phi] = 0, \quad \mu \geq 0.
\]

The development patterns are the same as with a small common market in the preceding section. Noting (34) and (35), the maximization of the new Langrangean (36) with respect to \((\phi, l)\) yields the first-order conditions

\[
\frac{\partial \tilde{H}}{\partial \phi} = \frac{\partial H}{\partial \phi} - \mu + \eta - \varepsilon \frac{n}{\partial \phi} + \vartheta \frac{\ell(1, \phi)}{\partial \phi} = 0,
\]

\[
\frac{\partial \tilde{H}}{\partial l} = \frac{\partial H}{\partial \phi} + \frac{\partial \Psi}{\partial l} \left(1 + \frac{\partial n}{\partial l}\right) - \vartheta = 0.
\]

Now assume that the common market expands large enough, \(J \to \infty\) and \(\phi = 1/J \to 0\). In that case, by (15) and (33), one obtains

\[
\lim_{\phi \to 0} [L - l - n(l, \phi)] = 0, \quad \lim_{\phi \to 0} \varepsilon > 0, \quad \lim_{\phi \to 0} \vartheta = 0.
\]

Solving for \(\varepsilon\) and \(\mu\) from (38) and (39) and noting (15), (23) and (40) yield

\[
\lim_{\phi \to 0} \varepsilon = \lim_{\phi \to 0} \frac{\partial \Psi}{\partial l} \left[\frac{\partial (n + l)}{\partial l}\right]^{-1} = \lim_{\phi \to 0} \left(\frac{1}{1 - \phi} - 1\right) \psi'(l) \left[\frac{\partial (n + l)}{\partial l}\right]^{-1}
\]

\[
= \lim_{\phi \to 0} \left[\frac{1}{1 - \phi} - 1\right] \psi'(l) \left\{\frac{1}{1 - \phi} - c(l)\right\} \nu \frac{c(l)}{1 - c(l)} - \phi \frac{c'(l)l}{1 - c(l)} \}
\]

\[
= \lim_{\phi \to 0} (1 - \phi) \psi'(l) \left\{[1 - \phi c(l)]\nu \frac{c(l)}{1 - c(l)} - \phi \frac{c'(l)l}{1 - c(l)} \}
\]

\[
= (1/c - 1) \psi'/\nu > 0.
\]
\[
\lim_{\phi \to 0} \mu = \lim_{\phi \to 0} \left[ \frac{\partial \Psi}{\partial \phi} + \eta - \varepsilon \frac{\partial n}{\partial \phi} + \varphi \frac{\partial \ell(1, \phi)}{\partial \phi} \right] 
\geq \lim_{\phi \to 0} \left[ \frac{\partial \Psi}{\partial \phi} - \varepsilon \frac{\partial n}{\partial \phi} \right]
\]

\[
= -\frac{\psi(l)}{\phi^2} + \lim_{\phi \to 0} \frac{\varepsilon}{1 - c(l)} \left[ \frac{l}{\phi^2} - \frac{\psi}{\psi(l)} - 1 \right]
\]

\[
= \frac{\psi}{\phi^2} \left[ l \frac{\psi'(l)}{\psi(l)} - 1 \right] > \frac{\psi}{\phi^2} \left( \frac{1}{c} - 1 \right) > 0.
\]

From this and (36) it follows that a common market which has enough regions (i.e. \( J \) large enough) has no incentives to take in new members:

\[
\lim_{J \to \infty} \frac{\partial \tilde{H}}{\partial J} = \lim_{\phi \to 0} \frac{\partial \tilde{H}}{\partial J} = -\lim_{\phi \to 0} \frac{\mu}{J^2} < 0.
\]

The results can be rephrased as follows:

**Proposition 5** Where product market deregulation is binding, there is an upper limit for economic integration and, in the case of full employment, the growth rate diminishes with the extent of integration.

Where product market is deregulated, economic integration intensifies competition, decreases the firms’ share of value added, \( \phi \), and increases labor devoted to production. This crowds out labor devoted to R&D and decreases the firms’ incentives to increase the productivity of labor through R&D. In both cases, R&D and the growth rate fall. When the growth rate falls low enough, it starts decreasing the central planner’s welfare.

10 Conclusions

This paper examines a common market with a large number of regions, each producing a different good. The market expands by integrating new regions. Firms improve their productivity through investment in R&D. The less there are firms in the common market or the more they can coordinate their actions, the higher their profits. All workers are unionized and their wages depend on relative union bargaining power. If this power is high enough, then there is involuntary unemployment. Both workers and firms lobby the central planner of the common market which affects firms’ and unions’ market power and
decides on new members to the common market. The main findings of the paper can be summarized as follows.

Unions’ and firms’ market power decreases the level of income at each moment of time. On the other hand, the greater the firm’s share of value added or the higher union wages, more incentives the firm has to increase the productivity of labor through R&D. In this respect, there can be an optimal amount of unions’ and firms’ market power in the common market.

The decision on accepting new members depends on institutions. When a common market is small, the small number of competing firms increases a firm’s share of value-added. In that case, it is not in the central planner’s interests to increase the firms’ market power furthermore by letting them to coordinate their actions. A small common market is willing to expand, because this decreases the firms’ market power and intensifies competition. When a common market is large, the number of competing firms is large and their share of value-added is small. In that case, it is in the policy maker’s interests to let the firms coordinate their actions. Because the central planner can fully regulate the intensity of competition by its competition and anti-trust policy, it is indifferent for taking in new members.

Concerning the regulation of relative union bargaining power, the central planner can increase its welfare either (a) by increasing the level of income or (b) by speeding up economic growth. If (a) is more effective than (b), then the central planner eliminates union power altogether to have full employment. On the other hand, if (b) is more effective than (a), then the central planner supports labor unions to promote cost-escaping R&D.

When the labor market is deregulated either by constitution or an international agreement that binds the central planner, the development patterns are the same as in the case of full employment above. When the product market is regulated either by constitution or an international agreement that binds the central planner, the firms’ market power and share of value-added is determined wholly by the size of the common market. Because there is an optimal level for the firms’ value-added, there must be an upper limit for economic integration. When the common market expands, the firms’ share of value-added decreases. In that case, the firms’ have less incentives for R&D, and, ultimately, the growth rate decreases.
Appendix

A. The functions (14) and (15)

From (9) and (11) it follows that
\[
\pi_j^{t_j+1}/\pi_j^{t_j} = B_j^{t_j+1}/B_j^{t_j} = a. \tag{41}
\]

The Bellman equation corresponding to (13) is given by
\[
r \Omega(t_j, w_j) = \max_{l_j}\left\{ \Pi_j + \Lambda_j \left[ \Omega(t_j + 1, w_j, \pi_j) - \Omega(t_j, w_j, \pi_j) \right] \right\} = \max_{l_j}\left\{ \pi_j - w_j l_j + \lambda l_j^{1-\nu} \left[ \Omega(t_j + 1, w_j, \pi_j) - \Omega(t_j, w_j, \pi_j) \right] \right\}. \tag{42}
\]

The first-order condition corresponding to this is given by
\[
(1 - \nu) \lambda l_j^{1-\nu} \left[ \Omega(t_j + 1, w_j, \pi_j) - \Omega(t_j, w_j, \pi_j) \right] = w_j. \tag{43}
\]

I try the solution
\[
\Pi_j = c_j \pi_j, \quad c_j \in (0, 1), \quad \Omega = \Pi_j / \delta_j, \tag{44}
\]
in which dividends $\Pi_j$ is in fixed proportion $c_j$ to profits $\pi_j$, and the subjective discount factor $\delta_j > 0$ is independent of income $\pi_j$. Given (41) and (44), one obtains
\[
\tilde{\Omega} = \Omega(t_j + 1, w_j, \pi_j) = c_j \pi_j^{t_j+1} / \delta_j = ac_j \pi_j^{t_j} / \delta_j = a \Omega(t_j, w_j, \pi_j). \tag{45}
\]
Inserting this and (44) into (42), one obtains $r = \Pi_j / \Omega + \lambda l_j^{1-\nu} (\tilde{\Omega} / \Omega - 1) = \delta_j + (a - 1) \lambda l_j^{1-\nu}$ and
\[
\delta_j = r + (1 - a) \lambda l_j^{1-\nu} > 0. \tag{46}
\]

From (44) and (12) it follows that
\[
w_j l_j = \pi_j - \Pi_j = (1/c_j - 1) \Pi_j = (1 - c_j) \pi_j. \tag{47}
\]
Inserting (44), (45), (46) and (47) into (43), one obtains
\[
(a - 1)(1 - \nu) \lambda = (1 - \nu) \lambda \left( \frac{\tilde{\Omega}}{\Omega} - 1 \right) = \frac{w_j l_j^{\nu}}{\Omega} = \frac{w_j \delta_j l_j^{\nu}}{\Pi_j} = \frac{\delta_j \left( \frac{1}{c_j} - 1 \right) l_j^{\nu}}{\Pi_j} = \left( \frac{1}{c_j} - 1 \right) \delta_j l_j^{\nu-1} = \left[ r l_j^{\nu-1} + (1 - a) \lambda \right] \frac{1 - c_j}{c_j}. \tag{48}
\]

\[^{11}\text{cf. Dixit and Pindyck (1994), Wälde (1999).}\]
Differentiating the logarithm of this equation totally yields
\[
\frac{(\nu - 1)r^{\nu-2} l_j}{l_j^{\nu-1} + (1-a)\lambda} \frac{dl_j}{l_j} = \left( \frac{1}{1-c_j} + \frac{1}{c_j} \right) dc_j = \frac{dc_j}{(1-c_j)c_j}.
\]

Noting (10), (44), (46), this equation defines the function
\[
\frac{\Pi_j}{\pi_j} = c_j = c(l_j) = \frac{r + (1-a)\lambda l_j^{1-\nu}}{r + (1-a)\lambda \nu l_j^{1-\nu}} > 0,
\]

\[
1 - c_j = \frac{(1-\nu)(a-1)\lambda l_j^{1-\nu}}{r + (1-a)\lambda \nu l_j^{1-\nu}}.
\]

From (48) it follows that
\[
\frac{dc_j}{dl_j} = \frac{(\nu - 1)r^{\nu-2}(1-c_j)c_j}{l_j^{\nu-1} + (1-a)\lambda} = \frac{\nu(l_j - c_j)}{r + (1-a)\lambda l_j^{1-\nu}} < 0.
\] (48)

From (48) it follows that
\[
d \left[ \frac{l_j}{1-c(l_j)} \right] = \frac{1}{1-c_j} + \frac{l_j c'}{1-c_j} = \frac{1}{1-c_j} + \frac{(\nu - 1)r c_j}{r + (1-a)\lambda l_j^{1-\nu}}
\]
\[
= \frac{1}{1-c_j} \left[ 1 + \frac{(\nu - 1)r c_j}{r + (1-a)\lambda l_j^{1-\nu}} \right] = \frac{1}{1-c_j} \left[ 1 + \frac{(\nu - 1)r}{r + (1-a)\lambda l_j^{1-\nu}} \right]
\]
\[
= \frac{\nu}{1-c_j} \frac{r + (1-a)\lambda l_j^{1-\nu}}{r + (1-a)\lambda l_j^{1-\nu}} = \frac{\nu c_j}{1-c_j} > 0.
\] (49)

Finally, noting (9), (47), (48) and (49), one obtains
\[
n_j = \left( \frac{1}{\phi} - 1 \right) \frac{\pi_j}{w_j} = \left( \frac{1}{\phi} - 1 \right) \frac{l_j}{1-c_j} = \left( \frac{1}{\phi} - 1 \right) l_j = n(l_j, \phi),
\]
\[
n_j + l_j = \frac{1}{\phi} - c_j l_j,
\]
\[
\frac{\partial (n_j + l_j)}{\partial l_j} = \frac{1}{\phi} - c_j \frac{d}{dl_j} \left[ \frac{l_j}{1-c(l_j)} \right] = \frac{c(l_j)}{1-c(l_j)} - \frac{c(l_j)}{1-c(l_j)} > 0.
\] (50)

Results (48)-(50) imply and (14) and (15).

B. The function (20)

Because there is one-to-one correspondence between the wage $w_j$ and labor input in R&D, $l_j$, through (9) and (15), in the maximization of the Generalized Nash Product $U_j^\alpha F_j^{1-\alpha}$ can be maximized by $l_j$. Noting (9), I
obtain that both \((n_j + l_j)w_j\) and \(\Pi_j = c_j\pi_j\) grow at the same rate as \(B_j\). The parties’ expected utilities (17) and (18) can then be transformed into the following form (cf. Aghion and Howitt 1998, p. 61)

\[
U(l_j, \phi) = \frac{B_j(T)(n_j + l_j)w_j}{B_j[r + (1 - a)\lambda l_j^{1-\nu}]} , \quad F(l_j, \phi) = \frac{B_j(T)c(l_j)\pi_j}{B_j[r + (1 - a)\lambda l_j^{1-\nu}]} .
\] (51)

Given (9), (14), (15) and (51), the outcome of bargaining is obtained through maximizing by \(l_j\) the following increasing transformation of the Generalized Nash product \(U_j^\alpha F_j^{1-\alpha}\):

\[
\Gamma_j(l_j, C, \alpha) = \log \left[ U_j^\alpha F_j^{1-\alpha} \right] = \alpha \log U_j + (1 - \alpha) \log F_j
\]

\[
= \alpha \log \left[ (n_j + l_j)w_jB_j^{-1} \right] + (1 - \alpha) \log \left[ c(l_j)\pi_jB_j^{-1} \right]
- \log \left[ r + (1 - a)\lambda l_j^{1-\nu} \right] + \Delta
\]

\[
= \alpha \log(1 + l_j/n_j) + \alpha \log \left[ w_jn_jB_j^{-1} \right] + (1 - \alpha) \log \left[ w_jn_jB_j^{-1} \right]
(1 - \alpha) \log c(l_j) - \log \left[ r + (1 - a)\lambda l_j^{1-\nu} \right] + \Delta
\]

\[
= \alpha \log \left[ (1 + l_j/n_j) \log \left[ w_jn_jB_j^{-1} \right] + (1 - \alpha) \log c(l_j)
- \log \left[ r + (1 - a)\lambda l_j^{1-\nu} \right] + \Delta
\]

\[
= \alpha \log \left[ 1 + \frac{1 - c(l_j)}{1/\phi - 1} \right] + \log l_j - \log \left[ 1 - c(l_j) \right]
+ (1 - \alpha) \log c(l_j) - \log \left[ r + (1 - a)\lambda l_j^{1-\nu} \right] + \Delta
\]

\[
= \alpha \log \left[ 1/\phi - c(l_j) \right] + \log l_j - \log \left[ 1 - c(l_j) \right] + (1 - \alpha) \log c(l_j)
- \log \left[ r + (1 - a)\lambda l_j^{1-\nu} \right] + \Delta \quad \text{with} \quad r + (1 - a)\lambda l_j^{1-\nu} > 0 ,
\] (52)

where \(\Delta\) denotes terms that are independent of \(l_j\), subject to the constraint \(l_j + n(l_j, \phi) \leq L\). The Lagrangean of this problem is given by

\[
\mathcal{L}_j = \Gamma_j(l_j, C, \alpha) + \beta [L - l_j - n(l_j, \phi)] ,
\] (53)

where the multiplier \(\beta\) satisfies the Kuhn-Tucker conditions

\[
\beta [L - l_j - n(l_j, \phi)] = 0 , \quad \beta \geq 0 .
\] (54)

Noting (52), (53) and (54), one obtains the first-order condition

\[
\frac{\partial \mathcal{L}_j}{\partial l_j} = \frac{\partial \Gamma_j}{\partial l_j} - \beta \left[ 1 + \frac{\partial n}{\partial l_j} \right] = 0 ,
\] (55)
where
\[
\frac{\partial \Gamma_j}{\partial l_j} = \frac{1}{l_j} + \frac{c'(l_j)}{1-c(l_j)} + (1-\alpha)\frac{c'(l_j)}{c(l_j)} \frac{\alpha c'(l_j)}{1/\phi - c(l_j)} + \frac{(1-\nu)(a-1)\lambda l_j^{-\nu}}{r + (1-a)\lambda l_j^{1-\nu}}.
\]
(56)

which defines the function
\[
l_j = \ell(\alpha, \phi) \text{ for } l_j + n_j < L.
\]
(57)

Noting
\[
\frac{\partial^2 \Gamma_j}{\partial l_j \partial \alpha} = -\frac{c'}{c} - \frac{c'}{1/\phi - c} > 0, \quad \frac{\partial^2 \Gamma_j}{\partial l_j \partial \phi} = -\frac{\alpha c'}{(1-\phi c)^2} > 0,
\]
and the second-order condition \(\partial^2 \Gamma_j/\partial l_j^2 < 0\) for \(l_j + n_j < L\), one obtains
\[
\frac{\partial \ell}{\partial \alpha} = -\frac{\partial^2 \Gamma_j}{\partial l_j \partial \alpha} / \frac{\partial^2 \Gamma_j}{\partial l_j^2} > 0 \quad \text{and} \quad \frac{\partial \ell}{\partial \phi} = -\frac{\partial^2 \Gamma_j}{\partial l_j \partial \phi} / \frac{\partial^2 \Gamma_j}{\partial l_j^2} > 0 \quad \text{for } l_j + n_j < L.
\]
(58)

From (14), (53), (54), (55) and (56) it follows that
\[
\lim_{\alpha \to 0} \frac{\partial \Gamma_j}{\partial l_j} = \frac{1}{l_j} + \frac{c'}{1-c} + \frac{c'}{c} + \frac{(1-\nu)(a-1)\lambda l_j^{-\nu}}{r + (1-a)\lambda l_j^{1-\nu}}
\]
\[
= \frac{1}{l_j} + \frac{c'/c}{1-c} + \frac{(1-\nu)(a-1)\lambda l_j^{-\nu}}{r + (1-a)\lambda l_j^{1-\nu}} = \frac{c'/c}{1-c} + \frac{1}{l_j} \frac{r + (1-a)\nu \lambda l_j^{1-\nu}}{r + (1-a)\lambda l_j^{1-\nu}}
\]
\[
= \frac{(\nu-1) r/l_j}{r + (1-a)\lambda l_j^{1-\nu}} + \frac{1}{l_j} \frac{r + (1-a)\nu \lambda l_j^{1-\nu}}{r + (1-a)\lambda l_j^{1-\nu}} = \frac{\nu}{l_j} > 0,
\]
\[
\lim_{\alpha \to 0} \beta \lim_{\alpha \to 0} \left(1 + \frac{\partial n}{\partial l_j}\right) = \lim_{\alpha \to 0} \frac{\partial \Gamma_j}{\partial l_j} > 0, \quad \lim_{\alpha \to 0} \beta > 0, \quad \lim_{\alpha \to 0} (L - l_j - n_j) = 0.
\]
(59)

Results (57), (58) and (59) imply (20).
References:


