SPATIAL POPULATION ANALYSIS: METHODS AND COMPUTER PROGRAMS

Frans Willekens

Andrei Rogers

August 1978 WP-78-30

2361
Laxenburg
Austria

International Institute for Applied Systems Analysis

Abstract

This report reviews the integrated methodology for spatial or multiregional demographic analysis, developed at IIASA, and presents the FORTRAN IV codes of the computer programs and a user's manual to implement this methodology. They include: the multiregional life table; multiregional demographic projections; fertility and mobility analyses of life table and stable population analyses; the spatial reproductive value; and the analysis of alternative paths to spatial zero-population-growth. The focus of the report is on the interpretation of the output. The user's manual focuses on the preparation of the data check.

Acknowledgements

The development of computer programs for spatial demographic analysis began at Northwestern University, Evanston, USA, in 1972. A number of former graduate students have collaborated in the project. In particular, we are indebted to Jacques Ledent, Richard Walz, and Richard Raquillet who wrote earlier versions of the programs.

The programs as listed at the end of this report have been written at IIASA. We made intensive use of IIASA's in-house computing facilities, a PDP-11/45, and benefited from some of the nice features of the UNIX time-sharing system. We are most grateful to Computer Services and in particular to Jim Curry and Mark Pearson for their advice and for solving our software problems.

An earlier version of this package of computer programs was published as IIASA Research Memoranda RM-76-58 and RM-77-30. The numerous reactions to these reports were extremely helpful in preparing this volume. In particular we acknowledge the detailed comments of Tom Carroll, Luis Castro, Jacques Ledent, Dimiter Philipov, Richard Raquillet, and Phillip Rees. The computer programs for spatial demographic analysis are also extensively used in the Comparative Migration and Settlement Study, which is being carried out jointly by IIASA's Migration and Settlement Task and scholars in all of the seventeen IIASA member nations. The comments on the Research Memoranda and the suggestions of the contributors to the Comparative Migration and Settlement Study made us restructure the report, add some new subroutines, and revise some of the previously published subroutines completely. Recent methodological innovations have also been introduced.

The manuscript was typed by Margaret Leggett. She performed her task with great skill and managed to keep her good humor even when the final version was not really final.

Contents

PART I: METHODOLOGY

PART II: USER'S MANUAL

Spatial Population Analysis
Methods and Computer Programs

There is a growing awareness among researchers, planners and governments that population growth should be viewed in its spatial dimension. Population declines in major central cities of the more developed world, continuing depopulations of rural areas in the less developed countries, and accelerating suburbanization everywhere have led governments to examine the desirability of population distribution policies.

A fundamental requirement for an effective policy regarding population redistribution is a well-developed understanding of spatial population dynamics. The basic mathematics of spatial demographic growth, recently the subject of study at the International Institute for Applied Systems Analysis (IIASA), has been elaborated as a set of FORTRAN computer programs to provide users with a ready tool for population analysis. These programs are being published in the hope that they may help researchers, students, planners, and policy makers to better understand the dynamic behavior of spatial demographic systems.

Although a number of publications of computer programs for population analysis and for operations research methods have guided us in our work, by far the most influential in this regard has been the book of Keyfitz and Flieger (1971). It has served as our basic reference. Other references were Arriaga (1977), Greenberg, Krueckeberg and Mautner (1973), and Land and Powell (1973).

This report consists of two parts. The first reviews the methodology of multiregional demography that is embodied in the programs. The emphasis, however, is not on methodology but on the interpretation of the output of the computer programs. The output consists of a set of tables, all of which are given and described in this part. The numerical illustrations refer to the same two-region system: Slovenia and the Rest of Yugoslavia. The demographic data on which the computations are based refer to the female population in the year 1961 and are given in Rogers (1975a). Data of a few other multiregional systems are presented in Appendix C.

The second part of this report clarifies our general approach to computer programming, gives a user-oriented description of the various subroutines and of the main program, and explains the format in which the input data must be provided. A glossary of mathematical symbols and FORTRAN names of demographic variables used is given in Appendix A. The FORTRAN listings are presented in Appendix D.

Part I

Methodology

PART I: METHODOLOGY

The dynamics of a multiregional population system are governed by fertility, mortality and migration rates. These fundamental components of demographic analysis determine not only the growth of the population, but also its age composition, spatial distribution, and crude rates.

The observation that a particular combination of age-specific rates results in a unique age and regional composition has induced demographers to read into every population distribution a particular sequence of vital rates. "The demographic history of a population is inscribed in its age distribution" (Keyfitz, et al., 1967, p. 862) For example, an observed population distribution (population pyramid) reflects periods of high fertility (baby boom) and high mortality (wars). A particularly useful way for understanding how the age and regional structure of a population is determined, is to imagine a particular distribution as describing a population that has been subjected to constant fertility, mortality and migration schedules for a prolonged period of time. The population that ultimately develops under such circumstances is called a stable multiregional population.

We also may view sequence of rates prospectively and derive the population distribution that would evolve if the actual observed schedules would remain unchanged for a prolonged period of time. This is the stable population associated with an observed demographic growth regime. The age-specific rates, of course, do not remain constant and therefore the stable population never will be realized. However, the stable population is a concept that enables one to look behind observed rates to explore what may be hidden in current patterns of fertility, mortality, and migration. It shows where the system is heading, in the long run, under current demographic forces. Keyfitz (1972, p. 347) compares stable population analyses to "microscopic examinations", because they magnify the effects of differences in current rates and therefore show more clearly their true meaning. Rogers (1971, p. 426) and Coale (1972, p. 52) compare them to "speedometer readings" to emphasize their monitoring function and hypothetical nature.

In addition to observed and stable population distribution that may be associated with observed fertility, mortality and migration schedules, demographers usually consider a third population distribution, namely, the distribution of the life table population. This stationary, or zero-growth, population describes the mortality and migration experience of a hypothetical population, with an equal number of births and deaths, that is subjected to the observed set of age-specific mortality and migration rates. The demographic picture shown by the life table, therefore, is the outcome of the observed mortality and migration schedules only and is not affected by the age composition and the regional distribution of the observed population. As in stable population theory, life table analysis enables one to separate out the effects of demographic behavior and of age- and regional compositions. The latter act as weights.

The first half of this report consists of eight sections. The first section focuses on the observed population and derives several demographic measures directly from the data. Multiregional life table statistics are then computed. The multiregional demographic growth model and population projection to stability under constant schedules of fertility, mortality and migration are treated next. This produces the stable growth ratio and the age- and region-distribution of the stable population. Stable population analysis is considered further in the following sections and is complemented by additional life table population analysis. In particular, attention is devoted to fertility and mobility analysis in both stable and stationary populations. This brings in the application of the concept of spatial reproductive value, the calculation of the impact of alternative patterns of fertility reduction to replacement level on spatial population characteristics, and the evaluation of the "momentum" of spatial demographic growth.

1. OBSERVED POPULATION CHARACTERISTICS

The first outputs of this collection of computer programs for demographic analysis describe summary characteristics of the observed population. The data inputs are given in Table 1.1. Table 1.2 gives the percentage age distributions of the population, the parents at time of childbearing, deaths, and migrants. The mean age, in each instance, is defined as

$$\bar{m}_i = \sum_x \left(x + \frac{NY}{2}\right) \cdot c_i(x)/100 \qquad (1.1)$$

where $c_i(x)$ is the percentage distribution,

NY is the age interval*, and

$(x + \frac{NY}{2})$ is the average of the interval.

The direct inputs to the life table program consist of observed age-specific rates (Table 1.3). Death rates are computed by dividing the annual number of deaths by the mid-year population in each age group. Fertility and migration rates are derived in a similar fashion. If death, birth, or migration data are not available on an annual basis, but are given for a five-year period, say, then the program reduces the data to an annual basis.** The population must in this case be the population at the mid-period. The sum of the age-specific rates multiplied by the age interval, is called the gross rate. The gross fertility rate (gross rate of reproduction) of Slovenia is 1.1128. The gross migraproduction rate (GRM) is derived in a similar way. The crude rate is the total number of births, deaths or outmigrants divided by the total mid-year population. For example, the crude birth rate of Slovenia is

$$0.017 = 14{,}159/832{,}800.$$

---

*In this report, an age and time interval of 5 years will be assumed.

**Annual data are obtained by dividing five-year data by five. This procedure is not a satisfactory one for migration data and should be used only as a first approximation.

Table 1.1. Observed population characteristics.

region slovenia

age	population	births	deaths	migration from slovenia to slovenia	migration from slovenia to r.yugos.
0	67800.	0.	417.	0.	192.
5	74100.	0.	32.	0.	170.
10	70700.	5.	21.	0.	105.
15	60100.	953.	31.	0.	310.
20	62900.	4444.	47.	0.	451.
25	66500.	4204.	45.	0.	368.
30	67100.	2758.	67.	0.	252.
35	62900.	1438.	77.	0.	111.
40	39500.	308.	76.	0.	40.
45	47900.	34.	171.	0.	26.
50	51300.	15.	268.	0.	34.
55	46100.	0.	369.	0.	29.
60	39600.	0.	513.	0.	35.
65	29500.	0.	763.	0.	28.
70	21700.	0.	1036.	0.	19.
75	14400.	0.	1088.	0.	16.
80	7100.	0.	1041.	0.	5.
85	3600.	0.	733.	0.	4.
total	832800.	14159.	6795.	0.	2195.

region r.yugos.

age	population	births	deaths	migration from r.yugos. to slovenia	migration from r.yugos. to r.yugos.
0	847900.	0.	19051.	231.	0.
5	905200.	0.	606.	150.	0.
10	808100.	54.	386.	127.	0.
15	617400.	16335.	534.	419.	0.
20	725500.	63828.	885.	680.	0.
25	774000.	57477.	1227.	392.	0.
30	728400.	32261.	1277.	255.	0.
35	633300.	14903.	1313.	143.	0.
40	392400.	4729.	1127.	72.	0.
45	437100.	940.	1700.	41.	0.
50	453800.	324.	2896.	59.	0.
55	389300.	0.	3743.	80.	0.
60	325800.	0.	5492.	66.	0.
65	230600.	0.	6407.	36.	0.
70	180000.	0.	8652.	14.	0.
75	120900.	0.	8715.	12.	0.
80	61200.	0.	6843.	12.	0.
85	39300.	0.	5639.	3.	0.
total	8670200.	190851.	76493.	2792.	0.

Table 1.2. Percentage distributions.

region slovenia

age	population	births	deaths	migration from slovenia to slovenia	r.yugos.
0	8.1412	0.0000	6.1369	0.0000	8.7472
5	8.8977	0.0000	0.4709	0.0000	7.7449
10	8.4894	0.0353	0.3091	0.0000	4.7836
15	7.2166	6.7307	0.4562	0.0000	14.1230
20	7.5528	31.3864	0.6917	0.0000	20.5467
25	7.9851	29.6914	0.6623	0.0000	16.7654
30	8.0572	19.4788	0.9860	0.0000	11.4806
35	7.5528	10.1561	1.1332	0.0000	5.0569
40	4.7430	2.1753	1.1185	0.0000	1.8223
45	5.7517	0.2401	2.5166	0.0000	1.1845
50	6.1599	0.1059	3.9441	0.0000	1.5490
55	5.5355	0.0000	5.4305	0.0000	1.3212
60	4.7550	0.0000	7.5497	0.0000	1.5945
65	3.5423	0.0000	11.2288	0.0000	1.2756
70	2.6057	0.0000	15.2465	0.0000	0.8656
75	1.7291	0.0000	16.0118	0.0000	0.7289
80	0.8525	0.0000	15.3201	0.0000	0.2278
85	0.4323	0.0000	10.7873	0.0000	0.1822
total	100.0000	100.0000	100.0000	0.0000	100.0000
m.age	33.3796	27.6427	66.0931	0.0000	25.0376

region r.yugos.

age	population	births	deaths	migration from r.yugos. to slovenia	r.yugos.
0	9.7795	0.0000	24.9055	8.2736	0.0000
5	10.4404	0.0000	0.7922	5.3725	0.0000
10	9.3204	0.0283	0.5046	4.5487	0.0000
15	7.1209	8.5590	0.6981	15.0072	0.0000
20	8.3677	33.4439	1.1570	24.3553	0.0000
25	8.9271	30.1162	1.6041	14.0401	0.0000
30	8.4012	16.9038	1.6694	9.1332	0.0000
35	7.3043	7.8087	1.7165	5.1218	0.0000
40	4.5258	2.4778	1.4733	2.5788	0.0000
45	5.0414	0.4925	2.2224	1.4685	0.0000
50	5.2340	0.1698	3.7860	2.1132	0.0000
55	4.4901	0.0000	4.8933	2.8653	0.0000
60	3.7577	0.0000	7.1797	2.3639	0.0000
65	2.6597	0.0000	8.3759	1.2894	0.0000
70	2.0761	0.0000	11.3108	0.5014	0.0000
75	1.3944	0.0000	11.3932	0.4298	0.0000
80	0.7059	0.0000	8.9459	0.4298	0.0000
85	0.4533	0.0000	7.3719	0.1074	0.0000
total	100.0000	100.0000	100.0000	100.0000	0.0000
m.age	30.6024	27.1063	50.2401	26.0781	0.0000

Table 1.3. Observed rates.

death rates
****

age	slovenia	r.yugos.
0	0.006150	0.022468
5	0.000432	0.000669
10	0.000297	0.000478
15	0.000516	0.000865
20	0.000747	0.001220
25	0.000677	0.001585
30	0.000999	0.001753
35	0.001224	0.002073
40	0.001924	0.002872
45	0.003570	0.003889
50	0.005224	0.006382
55	0.008004	0.009615
60	0.012955	0.016857
65	0.025864	0.027784
70	0.047742	0.048067
75	0.075556	0.072084
80	0.146620	0.111814
85	0.203611	0.143486
gross	2.710558	2.369808
crude	0.008159	0.008823
m.age	79.1635	74.4001

fertility rates
****

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000071	0.000067
15	0.015857	0.026458
20	0.070652	0.087978
25	0.063218	0.074260
30	0.041103	0.044290
35	0.022862	0.023532
40	0.007797	0.012051
45	0.000710	0.002151
50	0.000292	0.000714
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
gross	1.112808	1.357504
crude	0.017002	0.022012
m.age	27.7683	27.4740

Table 1.3. (cont'd)

outmigration rates
****

age	total	migration from slovenia to slovenia	r.yugos.
0	0.002832	0.000000	0.002832
5	0.002294	0.000000	0.002294
10	0.001485	0.000000	0.001485
15	0.005158	0.000000	0.005158
20	0.007170	0.000000	0.007170
25	0.005534	0.000000	0.005534
30	0.003756	0.000000	0.003756
35	0.001765	0.000000	0.001765
40	0.001013	0.000000	0.001013
45	0.000543	0.000000	0.000543
50	0.000663	0.000000	0.000663
55	0.000629	0.000000	0.000629
60	0.000884	0.000000	0.000884
65	0.000949	0.000000	0.000949
70	0.000876	0.000000	0.000876
75	0.001111	0.000000	0.001111
80	0.000704	0.000000	0.000704
85	0.001111	0.000000	0.001111
gross	0.192379	0.000000	0.192379
crude	0.002636	0.000000	0.002636
m.age	31.1589	0.0000	31.1589

age	total	migration from r.yugos. to slovenia	r.yugos.
0	0.000272	0.000272	0.000000
5	0.000166	0.000166	0.000000
10	0.000157	0.000157	0.000000
15	0.000679	0.000679	0.000000
20	0.000937	0.000937	0.000000
25	0.000506	0.000506	0.000000
30	0.000350	0.000350	0.000000
35	0.000226	0.000226	0.000000
40	0.000183	0.000183	0.000000
45	0.000094	0.000094	0.000000
50	0.000130	0.000130	0.000000
55	0.000205	0.000205	0.000000
60	0.000203	0.000203	0.000000
65	0.000156	0.000156	0.000000
70	0.000078	0.000078	0.000000
75	0.000099	0.000099	0.000000
80	0.000196	0.000196	0.000000
85	0.000076	0.000076	0.000000
gross	0.023573	0.023573	0.000000
crude	0.000322	0.000322	0.000000
m.age	34.1303	34.1303	0.0000

The mean age given in this table is the mean age of the schedule. The mean age of the fertility schedule of Slovenia, for example, is

$$\bar{m}_1 = \frac{\sum_x (x + \frac{NY}{2}) \; F_1(x)}{\sum_x F_1(x)} = 27.77$$

where $F_1(x)$ are the age-specific fertility rates of Slovenia and NY is five.

The mean age of the Slovenia to Rest-of-Yugoslavia migration schedule is 31.16 years. The mean age of the migrants is considerably less (25.04 years). This is due to the relatively young age composition of Slovenia's population. The age composition does not affect the migration schedule or its mean age.

Tables 1.4 and 1.5 repeat the basic data for each region, arranged in a different format and give the single-region life table for each region. The gross rates obtained are based on the regional schedules of fertility, mortality and migration only. The life table statistics, in particular the life expectancy at birth [e(0)] only depend on the regional mortality schedule. The life expectancy is therefore the average number of years a person may expect to live if he remains in the region of birth during his whole lifetime (i.e., if the region is closed to outmigration). The net reproduction rate is obtained as follows:

$$NRR_i = \sum_x F_i(x) \; L_i(x) \quad ,$$

where $F_i(x)$ is the regional fertility rate of age group x to x + 5 and $L_i(x)$ is an element of the LL(x)-column of the single-region life table (number of years lived in age group x to x + 5). The net migraproduction rate (NMR) is determined analogously. It is the weighted sum of the age-specific outmigration rates, the weights being the elements of the LL(x)-column of the single-region life table. The national NMR (Table 1.6b) is identical to the Wilber-index (Wilber, 1963; Rogers, 1975b).

Table 1.4a.

slovenia
****

age	population number	population - % -	births number	births - % -	deaths number	deaths - % -	arrivals number	arrivals - % -	departures number	departures - % -	observed rates (x 1000) birth	death	inmig	outmig	net mig
0	67800.	8.14	0.	0.00	417.	6.14	231.	8.27	192.	8.75	0.000	6.150	3.407	2.832	0.575
5	74100.	8.90	0.	0.00	32.	0.47	150.	5.37	170.	7.74	0.000	0.432	2.024	2.294	-0.270
10	70700.	8.49	5.	0.04	21.	0.31	127.	4.55	105.	4.78	0.071	0.297	1.796	1.485	0.311
15	60100.	7.22	953.	6.73	31.	0.46	419.	15.01	310.	14.12	15.857	0.516	6.972	5.158	1.814
20	62900.	7.55	4444.	31.39	47.	0.69	680.	24.36	451.	20.55	70.652	0.747	10.811	7.170	3.641
25	66500.	7.99	4204.	29.69	45.	0.66	392.	14.04	368.	16.77	63.218	0.677	5.895	5.534	0.361
30	67100.	8.06	2758.	19.48	67.	0.99	255.	9.13	252.	11.48	41.103	0.999	3.800	3.756	0.045
35	62900.	7.55	1438.	10.16	77.	1.13	143.	5.12	111.	5.06	22.862	1.224	2.273	1.765	0.509
40	39500.	4.74	308.	2.18	76.	1.12	72.	2.58	40.	1.82	7.797	1.924	1.823	1.013	0.810
45	47900.	5.75	34.	0.24	171.	2.52	41.	1.47	26.	1.18	0.710	3.570	0.856	0.543	0.313
50	51300.	6.16	15.	0.11	268.	3.94	59.	2.11	34.	1.55	0.292	5.224	1.150	0.663	0.437
55	46100.	5.54	0.	0.00	369.	5.43	80.	2.87	29.	1.32	0.000	8.004	1.735	0.629	1.106
60	39600.	4.76	0.	0.00	513.	7.55	66.	2.36	35.	1.59	0.000	12.955	1.667	0.884	0.783
65	29500.	3.54	0.	0.00	763.	11.23	36.	1.29	28.	1.28	0.000	25.864	1.220	0.949	0.271
70	21700.	2.61	0.	0.00	1036.	15.25	14.	0.50	19.	0.87	0.000	47.742	0.645	0.876	-0.230
75	14400.	1.73	0.	0.00	1088.	16.01	12.	0.43	16.	0.73	0.000	75.556	0.833	1.111	-0.278
80	7100.	0.85	0.	0.00	1041.	15.32	12.	0.43	5.	0.23	0.000	146.620	1.690	0.704	0.986
85	3600.	0.43	0.	0.00	733.	10.79	3.	0.11	4.	0.18	0.000	203.611	0.833	1.111	-0.278
tot	832800.	100.00	14159.	100.00	6795.	100.00	2792.	100.00	2195.	100.00					
gross											1.113	2.711	0.247	0.192	
crude(x1000)											17.002	8.159	3.353	2.636	0.717
m.age		33.38		27.64		66.09		26.08		25.04	27.77	79.16	31.78	31.16	
e(0)												72.59			

Table 1.4b.

table - single region life table slovenia mortality level = 72.59

age	p(x)	q(x)	l(x)	d(x)	ll(x)	m(x)	s(x)	t(x)	e(x)
0	0.969714	0.030286	100000.	3029.	4.924284	0.006150	0.983562	72.5850	72.5850
5	0.997843	0.002157	96971.	209.	4.843338	0.000432	0.998179	67.6608	69.7740
10	0.998516	0.001484	96762.	144.	4.834519	0.000297	0.997971	62.8174	64.9194
15	0.997424	0.002576	96619.	249.	4.824708	0.000516	0.996848	57.9829	60.0122
20	0.996271	0.003729	96370.	359.	4.809502	0.000747	0.996446	53.1582	55.1607
25	0.996622	0.003378	96010.	324.	4.792410	0.000677	0.995822	48.3487	50.3578
30	0.995020	0.004980	95686.	477.	4.772389	0.000999	0.994460	43.5563	45.5200
35	0.993898	0.006102	95210.	581.	4.745952	0.001224	0.992167	38.7839	40.7353
40	0.990426	0.009574	94629.	906.	4.708777	0.001924	0.986387	34.0379	35.9701
45	0.982308	0.017692	93723.	1658.	4.644674	0.003570	0.978298	29.3292	31.2936
50	0.974216	0.025784	92064.	2374.	4.543876	0.005224	0.967578	24.6845	26.8122
55	0.960764	0.039236	89691.	3519.	4.396553	0.008004	0.949246	20.1405	22.4557
60	0.937259	0.062741	86171.	5406.	4.173412	0.012955	0.908847	15.7441	18.2706
65	0.878532	0.121468	80765.	9810.	3.792991	0.025864	0.835605	11.5706	14.3263
70	0.786744	0.213256	70955.	15132.	3.169444	0.047742	0.740730	7.7777	10.9614
75	0.682243	0.317757	55823.	17738.	2.347701	0.075556	0.593547	4.6082	8.2550
80	0.463540	0.536460	38085.	20431.	1.393471	0.146620	0.622216	2.2605	5.9354
85	0.325357	0.674643	17654.	11910.	0.867040	0.203611	0.000000	0.8670	4.9113

net reproduction rate 1.065615

net migraproduction rate 0.172110

Table 1.5a.

r.yugos.
****

age	population number	population - % -	births number	births - % -	deaths number	deaths - % -	arrivals number	arrivals - % -	departures number	departures - % -	observed rates (x 1000) birth	death	inmig	outmig	net mig
0	847900.	9.78	0.	0.00	19051.	24.91	192.	8.75	231.	8.27	0.000	22.468	0.226	0.272	-0.046
5	905200.	10.44	0.	0.00	606.	0.79	170.	7.74	150.	5.37	0.000	0.659	0.188	0.166	0.022
10	808100.	9.32	54.	0.03	386.	0.50	105.	4.78	127.	4.55	0.067	0.478	0.130	0.157	-0.027
15	617400.	7.12	16335.	8.56	534.	0.70	310.	14.12	419.	15.01	26.458	0.865	0.502	0.679	-0.177
20	725500.	8.37	63828.	33.44	885.	1.16	451.	20.55	690.	24.36	87.978	1.220	0.622	0.937	-0.316
25	774000.	8.93	57477.	30.12	1227.	1.60	368.	16.77	392.	14.04	74.260	1.585	0.475	0.506	-0.031
30	728400.	8.40	32261.	16.90	1277.	1.67	252.	11.48	255.	9.13	44.290	1.753	0.346	0.350	-0.004
35	633300.	7.30	14903.	7.81	1313.	1.72	111.	5.06	143.	5.12	23.532	2.073	0.175	0.226	-0.051
40	392400.	4.53	4729.	2.48	1127.	1.47	40.	1.82	72.	2.58	12.051	2.872	0.102	0.183	-0.082
45	437100.	5.04	940.	0.49	1700.	2.22	26.	1.18	41.	1.47	2.151	3.889	0.059	0.094	-0.034
50	453800.	5.23	324.	0.17	2896.	3.79	34.	1.55	59.	2.11	0.714	6.382	0.075	0.130	-0.055
55	389300.	4.49	0.	0.00	3743.	4.89	29.	1.32	80.	2.87	0.000	9.615	0.074	0.205	-0.131
60	325800.	3.76	0.	0.00	5492.	7.18	35.	1.59	66.	2.36	0.000	16.857	0.107	0.203	-0.095
65	230600.	2.66	0.	0.00	6407.	8.38	28.	1.28	36.	1.29	0.000	27.784	0.121	0.156	-0.035
70	180000.	2.08	0.	0.00	8652.	11.31	19.	0.87	14.	0.50	0.000	48.067	0.106	0.078	0.028
75	120900.	1.39	0.	0.00	8715.	11.39	16.	0.73	12.	0.43	0.000	72.084	0.132	0.099	0.033
80	61200.	0.71	0.	0.00	6843.	8.95	5.	0.23	12.	0.43	0.000	111.814	0.082	0.196	-0.114
85	39300.	0.45	0.	0.00	5639.	7.37	4.	0.18	3.	0.11	0.000	143.486	0.102	0.076	0.025
tot	8670200.	100.00	190351.	100.00	76493.	100.00	2195.	100.00	2792.	100.00					
gross											1.358	2.370	0.018	0.024	
crude(x1000)											22.012	8.823	0.253	0.322	-0.069
m.age		30.60		27.11		50.24		25.04		26.08	27.47	74.40	33.40	34.13	
e(0)												66.24			

Table 1.5b.

table - single region life table r.yugos. mortality level = 66.24

age	p(x)	q(x)	l(x)	d(x)	ll(x)	m(x)	s(x)	t(x)	e(x)
0	0.893632	0.106368	100000.	10637.	4.734031	0.022468	0.942252	66.2357	66.2357
5	0.996658	0.003342	89363.	299.	4.460697	0.000669	0.997136	61.5016	68.8221
10	0.997615	0.002385	89065.	212.	4.447919	0.000478	0.996651	57.0410	64.0445
15	0.995685	0.004315	88852.	383.	4.433022	0.000865	0.994804	52.5930	59.1916
20	0.993919	0.006081	88469.	538.	4.409988	0.001220	0.993015	48.1600	54.4373
25	0.992105	0.007895	87931.	694.	4.379184	0.001585	0.991690	43.7500	49.7551
30	0.991273	0.008727	87237.	761.	4.342794	0.001753	0.990483	39.3708	45.1311
35	0.989687	0.010313	86475.	892.	4.301466	0.002073	0.987725	35.0280	40.5065
40	0.985742	0.014258	85583.	1220.	4.248664	0.002872	0.983259	30.7266	35.9025
45	0.980741	0.019259	84363.	1625.	4.177539	0.003889	0.974726	26.4779	31.3856
50	0.968593	0.031407	82738.	2599.	4.071956	0.006382	0.960948	22.3004	26.9529
55	0.953055	0.046945	80140.	3762.	3.912937	0.009615	0.936497	18.2284	22.7458
60	0.919124	0.080876	76378.	6177.	3.664454	0.016857	0.895646	14.3155	18.7430
65	0.870102	0.129898	70201.	9119.	3.282053	0.027784	0.830716	10.6510	15.1723
70	0.785449	0.214551	61082.	13105.	2.726453	0.048067	0.745488	7.3690	12.0641
75	0.694612	0.305388	47976.	14651.	2.032538	0.072084	0.640693	4.6425	9.6767
80	0.563069	0.436931	33325.	14561.	1.302234	0.111814	1.004231	2.6100	7.8319
85	0.471979	0.528021	18764.	9908.	1.307744	0.143486	0.000000	1.3077	6.9693

net reproduction rate 1.187424

net migraproduction rate 0.019108

Table 1.6a.

yugoslav
****

age	population number	population - % -	births number	births - % -	deaths number	deaths - % -	arrivals number	arrivals - % -	departures number	departures - % -	observed rates (x 1000) birth	death	inmig	outmig	net mig
0	915700.	9.64	0.	0.00	19468.	23.37	423.	8.48	423.	8.48	0.000	21.260	0.462	0.462	0.000
5	979300.	10.31	0.	0.00	638.	0.77	320.	6.42	320.	6.42	0.000	0.651	0.327	0.327	0.000
10	878800.	9.25	59.	0.03	407.	0.49	232.	4.65	232.	4.65	0.067	0.463	0.264	0.264	0.000
15	677500.	7.13	17288.	8.43	565.	0.68	729.	14.62	729.	14.62	25.517	0.834	1.076	1.076	0.000
20	788400.	8.30	68272.	33.30	932.	1.12	1131.	22.68	1131.	22.68	86.596	1.182	1.435	1.435	0.000
25	840500.	8.84	61681.	30.09	1272.	1.53	760.	15.24	760.	15.24	73.386	1.513	0.904	0.904	0.000
30	795500.	8.37	35019.	17.08	1344.	1.61	507.	10.17	507.	10.17	44.021	1.690	0.637	0.637	0.000
35	696200.	7.33	16341.	7.97	1390.	1.67	254.	5.09	254.	5.09	23.472	1.997	0.365	0.365	0.000
40	431900.	4.54	5037.	2.46	1203.	1.44	112.	2.25	112.	2.25	11.662	2.785	0.259	0.259	0.000
45	485000.	5.10	974.	0.48	1871.	2.25	67.	1.34	67.	1.34	2.008	3.858	0.138	0.138	0.000
50	505100.	5.32	339.	0.17	3164.	3.80	93.	1.86	93.	1.86	0.671	6.264	0.184	0.184	0.000
55	435400.	4.58	0.	0.00	4112.	4.94	109.	2.19	109.	2.19	0.000	9.444	0.250	0.250	0.000
60	365400.	3.85	0.	0.00	6005.	7.21	101.	2.03	101.	2.03	0.000	16.434	0.276	0.276	0.000
65	260100.	2.74	0.	0.00	7170.	8.61	64.	1.28	64.	1.28	0.000	27.566	0.246	0.246	0.000
70	201700.	2.12	0.	0.00	9688.	11.63	33.	0.66	33.	0.66	0.000	48.032	0.164	0.164	0.000
75	135300.	1.42	0.	0.00	9803.	11.77	28.	0.56	28.	0.56	0.000	72.454	0.207	0.207	0.000
80	63300.	0.72	0.	0.00	7884.	9.47	17.	0.34	17.	0.34	0.000	115.432	0.249	0.249	0.000
85	42900.	0.45	0.	0.00	6372.	7.65	7.	0.14	7.	0.14	0.000	148.531	0.163	0.163	0.000
tot	9503000.	100.00	205010.	100.00	83288.	100.00	4987.	100.00	4987.	100.00					
gross											1.337	2.402	0.038	0.038	
crude(x1000)											21.573	8.764	0.525	0.525	0.000
m.age		30.85		27.14		51.53		25.62		25.62	27.49	74.85	33.59	33.59	
e(0)												66.68			

Table 1.6b.

table - single region life table yugoslav mortality level = 66.68

age	p(x)	q(x)	l(x)	d(x)	ll(x)	m(x)	s(x)	t(x)	e(x)
0	0.899064	0.100936	100000.	10094.	4.747659	0.021260	0.945310	66.6828	66.6828
5	0.996748	0.003252	89906.	292.	4.488008	0.000651	0.997217	61.9351	68.8885
10	0.997687	0.002313	89614.	207.	4.475516	0.000463	0.996764	57.4471	64.1051
15	0.995839	0.004161	89407.	372.	4.461034	0.000834	0.994975	52.9716	59.2479
20	0.994107	0.005893	89035.	525.	4.438616	0.001182	0.993287	48.5105	54.4850
25	0.992462	0.007538	88510.	667.	4.408818	0.001513	0.992026	44.0719	49.7932
30	0.991588	0.008412	87843.	739.	4.373664	0.001690	0.990831	39.6631	45.1524
35	0.990067	0.009933	87104.	865.	4.333560	0.001997	0.988128	35.2894	40.5142
40	0.986170	0.013830	86239.	1193.	4.282112	0.002785	0.983551	30.9559	35.8956
45	0.980896	0.019104	85046.	1625.	4.211675	0.003858	0.975085	26.6738	31.3640
50	0.969162	0.030838	83421.	2573.	4.106743	0.006264	0.961635	22.4621	26.9261
55	0.953868	0.046132	80849.	3730.	3.949188	0.009444	0.937858	18.3554	22.7034
60	0.921072	0.078928	77119.	6087.	3.703776	0.016434	0.897091	14.4062	18.6805
65	0.871055	0.128945	71032.	9159.	3.322625	0.027566	0.831266	10.7024	15.0670
70	0.785588	0.214412	61873.	13266.	2.761986	0.048032	0.744979	7.3798	11.9273
75	0.693287	0.306713	48607.	14908.	2.057622	0.072454	0.635479	4.6178	9.5003
80	0.552096	0.447904	33698.	15094.	1.307576	0.115432	0.957939	2.5602	7.5973
85	0.458440	0.541560	18605.	10076.	1.252577	0.148531	0.000000	1.2526	6.7326

net reproduction rate 1.177450

net migraproduction rate 0.031050

2. THE MULTIREGIONAL LIFE TABLE

The multiregional life table is a device for exhibiting the mortality and mobility history of an artificial population, called a cohort. Methods for constructing such a life table are treated in detail in Rogers (1975a, Chapter 3).

The cohort we deal with is a birth cohort, or radix. It represents a group of people born at the same moment in time and in the same region. Their life history is of special interest because it provides the necessary input information for numerical computations with multiregional demographic growth models. In multiregional demography, it is convenient to work with unit radices, i.e., birth cohorts of single persons. This allows a separation of the calculation of life table and other demographic statistics from the radix problem. Unless stated otherwise, the figures presented in this report will be per unit radix.

The computation of the multiregional life table begins with the estimation of age-specific death and outmigration probabilities. The probabilities are derived from observed schedules or rates of mortality and migration. The procedure is described at the end of this section. The probabilities of dying and outmigrating of the female population of the two-region system of Yugoslavia are given in Table 2.1. Note that they differ slightly from the probabilities presented in Rogers (1975a, p. 66), due to a small difference in the estimation method. As a consequence, all life-table statistics deviate slightly from those in Rogers (1975). Probabilities and the two-region life table, consistent with Rogers's are given in Appendix B.

Probabilities of dying and migrating are the inputs for calculating life table statistics. The following statistics are computed by the program and are reviewed in the subsequent sections:

1. life history of a regional birth cohort,

2. number of survivors at exact age x,

Table 2.1. Probabilities of dying and migrating.

region slovenia
****

age	death	migration from slovenia to slovenia	r.yugos.
0	0.030813	0.956084	0.013103
5	0.002164	0.986467	0.011370
10	0.001487	0.991131	0.007381
15	0.002598	0.972070	0.025332
20	0.003770	0.961262	0.034968
25	0.003439	0.969456	0.027105
30	0.005015	0.976525	0.018460
35	0.006121	0.985171	0.008708
40	0.009586	0.985426	0.004988
45	0.017694	0.979646	0.002660
50	0.025793	0.970993	0.003213
55	0.039248	0.957747	0.003005
60	0.062780	0.933124	0.004097
65	0.121486	0.874356	0.004157
70	0.213259	0.783257	0.003484
75	0.317728	0.678322	0.003949
80	0.536332	0.461658	0.002010
85	1.000000	0.000000	0.000000

region r.yugos.
****

age	death	migration from r.yugos. to slovenia	r.yugos.
0	0.106319	0.001261	0.892421
5	0.003341	0.000821	0.995838
10	0.002385	0.000781	0.996834
15	0.004312	0.003333	0.992355
20	0.006075	0.004571	0.989354
25	0.007890	0.002481	0.989630
30	0.008724	0.001721	0.989555
35	0.010310	0.001114	0.988575
40	0.014256	0.000904	0.984840
45	0.019259	0.000460	0.980282
50	0.031406	0.000630	0.967964
55	0.046941	0.000982	0.952077
60	0.080868	0.000939	0.918193
65	0.129894	0.000684	0.869422
70	0.214551	0.000309	0.785139
75	0.305390	0.000353	0.694257
80	0.436969	0.000560	0.562471
85	1.000000	0.000000	0.000000

3. number of years lived between two consecutive ages; or, the age composition of stationary population,
4. number of years lived beyond age x,
5. life expectancies by region of birth,
6. life expectancies by region of residence,
7. survivorship proportions.

2.1 Life Histories

The life histories of the hypothetical population are computed by applying the age-specific probabilities of dying and outmigrating to the regional radices. Any set of birth cohorts may be used. In this section, birth cohorts of 100,000 in each region of Slovenia and the Rest of Yugoslavia, will be used.

We adopt the following notation:*

$q_i(x)$: the probability that a person in region i at exact age x dies before reaching age x + 5.

$p_{ij}(x)$: the probability that a person in region i at exact age x will reside in region j at exact age x + 5.

${}_{j0}\ell_i(x)$: the number of people in region i at exact age x, who are born in region j. Note that the radix or birth cohort of region j may be represented by ${}_{j0}\ell_j(0)$.

${}_{j0}\ell_{i\delta}(x)$: the expected number of people alive in region i at exact age x, born in region j, who will die before reaching x + 5.

${}_{j0}\ell_{ik}(x)$: the expected number of migrants from i to k between ages x and x + 5 among the people living in i at age x and born in j.

---

*A glossary of mathematical symbols and the associated FORTRAN names is given in Appendix A.

The quantities ${}_{j0}\ell(x)$, ${}_{j0}\ell_{i\delta}(x)$ and ${}_{j0}\ell_{ik}(x)$ may also be expressed per unit born, i.e. for a cohort of a single person. They then may be interpreted as probabilities. For instance, ${}_{j0}\hat{\ell}_i(x)$ is the probability that a j-born person is in region i at exact age x, and ${}_{j0}\hat{\ell}_{ik}(x)$ is the probability that a j-born person changes his residence from i to k between ages x and x + 5. The relation between, for example, ${}_{j0}\ell_i(x)$ and ${}_{j0}\hat{\ell}_i(x)$ is straightforward:

$$ {}_{j0}\ell_i(x) = {}_{j0}\hat{\ell}_i(x) \; {}_{j0}\ell_j(0) \quad . \qquad (2.1)$$

The probability-interpretation will be particularly useful in fertility and mobility analyses for stationary and stable populations.

The life history of the cohorts is derived by the consecutive multiplication of the birth cohort by the mortality and migration probabilities. For example, of the 100,000 babies born in Slovenia (region 1), 3081 will die before they reach age 5, i.e.,

$$100{,}000 * 0.030813 = 3081$$

$$ {}_{10}\ell_1(0) * q_1(0) = {}_{10}\ell_{1\delta}(0) \quad ,$$

and 1310 will move to the Rest of Yugoslavia (region 2),

$$100{,}000 * 0.013101 = 1310$$

$$ {}_{10}\ell_1(0) * p_{12}(0) = {}_{10}\ell_{12}(0) \quad .$$

The residual, i.e.,

$$100{,}000 - 3081 - 1310 = 95{,}608$$

or

$$100{,}000 * 0.956084$$

$$_{10}\ell_1(0) * p_{11}(0) = {}_{10}\ell_{11}(0)$$

remain in Slovenia, and are there at exact age 5. Therefore, of the females born in Slovenia, only 95.6% will still be there 5 years later.

Of the 100,000 females born in Slovenia, 96,919 will still be alive at exact age 5. A total of 95,608 will still be in Slovenia and 1,310 will be in the Rest of Yugoslavia. From these 95,608, the number of girls dying before reaching age 10 is

$$95{,}608 * 0.002164 = 207$$

$$_{10}\ell_1(5) * q_1(5) = {}_{10}\ell_{1\delta}(5)$$

and the number migrating to the Rest of Yugoslavia is

$$95{,}608 * 0.011370 = 1087$$

$$_{10}\ell_1(5) * p_{12}(5) = {}_{10}\ell_{12}(5) \quad .$$

The residual is the number of girls, who were in Slovenia at age 5 and are still there at age 10:

$$95{,}608 - 207 - 1087 = 94{,}314$$

or

$$95{,}608 * 0.986467$$

$$_{10}\ell_1(5) * p_{11}(5) = {}_{10}\ell_{11}(5) \quad .$$

Note that ${}_{10}\hat{\ell}_{1\delta}(5) = {}_{10}\ell_{1\delta}(5)/{}_{10}\ell_1(0) = 0.00207$ is the probability that a girl born in Slovenia dies in that region between ages 5 and 10. An analogous interpretation may be given to ${}_{10}\hat{\ell}_{12}(5)$ and ${}_{10}\hat{\ell}_{11}(5)$. Expressing the life histories per unit born yields a set of unconditional probabilities.

What happens to the 1310 migrants born in Slovenia, but who are in the Rest of Yugoslavia at exact age 5? They die, move back to Slovenia or stay in the Rest of Yugoslavia. If one assumes that the mortality and migration behavior depends on the region of residence at the beginning of the interval,* then

$$1310 * 0.003341 = 4$$

$${}_{10}\ell_2(5) * q_2(5) = {}_{10}\ell_{2\delta}(5)$$

girls die before reaching age 10, and

$$1310 * 0.000821 = 1$$

$${}_{10}\ell_2(5) * p_{21}(5) = {}_{10}\ell_{21}(5)$$

move back to Slovenia, while

$$1310 * 0.995838 = 1305$$

remain in the Rest of Yugoslavia.

Pursuing this procedure until the last age group, we obtain a detailed description of the life history of the people born in Slovenia. The last age group is open-ended, therefore all people who reach age 85 are expected to die in that age group, i.e. $q_i(85) = 1.0$, and hence

*This is the Markovian assumption. It is a fundamental hypothesis underlying multiregional and other increment-decrement life tables.

$$ {}_{10}\ell_{1\delta}(85) = {}_{10}\ell_1(85) \quad . \tag{2.2} $$

Note that the total number of deaths is equal to the total number of births. For example, of the 100,000 babies born in Slovenia, 94,721 die in Slovenia and 15,279 die in the Rest of Yugoslavia.

An analogous procedure is followed to derive the life history of the females born in the Rest of Yugoslavia (Table 2.2).

2.2 Expected Number of Survivors at Exact Age x

Table 2.3 is an aggregation of Table 2.2. We noted earlier that of the 100,000 girls born in Slovenia, there are 1310 who at exact age 5 reside in the Rest of Yugoslavia. This number may also be found in Table 2.3. Of the people born in Slovenia and residing in the Rest of Yugoslavia at age 10, for example, some were there already at age 5 and stayed there, while others moved in from Slovenia, i.e.

$$ 2392 = 1305 + 1087 $$

$$ {}_{10}\ell_2(10) = {}_{10}\ell_{22}(5) + {}_{10}\ell_{12}(5) $$

where ${}_{j0}\ell_i(x)$ is the number of people in region i at exact age x, who were born in region j. This expression is equivalent to:

$$ {}_{10}\ell_2(10) = {}_{10}\ell_2(5)p_{22}(5) + {}_{10}\ell_1(5)p_{12}(5) \quad . \tag{2.3} $$

The total of 2392 is given in Table 2.3, its components may be found in Table 2.2

Table 2.3 gives the number of people by place of birth and place of residence. Hence, it measures the age structure of the life table population, although only people at exact ages are considered. A more complete expression of the age structure is given in the next section.

Table 2.2. Life history of initial cohort.

initial region of cohort slovenia
****

1.- region of residence slovenia

age	deaths	migrants to slovenia	migrants to r.yugos.
0	3081.	1310.	1310.
5	207.	1087.	1087.
10	140.	696.	696.
15	243.	2368.	2368.
20	343.	3178.	3178.
25	301.	2369.	2369.
30	425.	1564.	1564.
35	507.	721.	721.
40	782.	407.	407.
45	1422.	214.	214.
50	2031.	253.	253.
55	3001.	230.	230.
60	4599.	300.	300.
65	8305.	284.	284.
70	12749.	208.	208.
75	14879.	185.	185.
80	17038.	64.	64.
85	14669.	0.	0.
total	84721.	1240820.	15438.

2.- region of residence r.yugos.

age	deaths	migrants to slovenia	migrants to r.yugos.
0	0.	0.	0.
5	4.	1305.	1305.
10	6.	2384.	2384.
15	13.	3057.	3057.
20	33.	5367.	5367.
25	67.	8457.	8457.
30	94.	10712.	10712.
35	127.	12136.	12136.
40	183.	12662.	12662.
45	252.	12811.	12811.
50	409.	12608.	12608.
55	604.	12244.	12244.
60	1009.	11454.	11454.
65	1527.	10219.	10219.
70	2253.	8246.	8246.
75	2582.	5870.	5870.
80	2646.	3406.	3406.
85	3469.	0.	0.
total	15279.	159.	132937.

Table 2.2. (cont'd)

initial region of cohort r.yugos.
****

1.- region of residence slovenia

age	deaths	migrants to slovenia	migrants to r.yugos.
0	0.	0.	0.
5	0.	1.	1.
10	0.	1.	1.
15	1.	7.	7.
20	2.	19.	19.
25	3.	25.	25.
30	6.	21.	21.
35	8.	11.	11.
40	13.	7.	7.
45	24.	4.	4.
50	36.	4.	4.
55	55.	4.	4.
60	89.	6.	6.
65	169.	6.	6.
70	270.	4.	4.
75	320.	4.	4.
80	376.	1.	1.
85	342.	0.	0.
total	1713.	14203.	126.

2.- region of residence r.yugos.

age	deaths	migrants to slovenia	migrants to r.yugos.
0	10632.	89242.	89242.
5	298.	88871.	88871.
10	212.	88591.	88591.
15	382.	87915.	87915.
20	534.	86985.	86985.
25	686.	86103.	86103.
30	751.	85228.	85228.
35	879.	84275.	84275.
40	1202.	83008.	83008.
45	1599.	81378.	81378.
50	2556.	78774.	78774.
55	3698.	75003.	75003.
60	6066.	68872.	68872.
65	8947.	59883.	59883.
70	12849.	47021.	47021.
75	14361.	32648.	32648.
80	14268.	18366.	18366.
85	18367.	0.	0.
total	98287.	1839.	1242163.

Table 2.3. Expected number of survivors at exact age x in each region.

age	initial region of cohort	slovenia	
***	***********************************		
	total	slovenia	r.yugos.
0	100000.	100000.	0.
5	96919.	95608.	1310.
10	96707.	94316.	2392.
15	96561.	93481.	3080.
20	96305.	90880.	5425.
25	95930.	87385.	8545.
30	95562.	84737.	10825.
35	95042.	82766.	12276.
40	94409.	81552.	12857.
45	93444.	80376.	13069.
50	91770.	78746.	13025.
55	89330.	76470.	12861.
60	85725.	73251.	12474.
65	80118.	68364.	11754.
70	70286.	59783.	10503.
75	55283.	46828.	8455.
80	37822.	31768.	6055.
85	18139.	14669.	3469.

age	initial region of cohort	r.yugos.	
***	***********************************		
	total	slovenia	r.yugos.
0	100000.	0.	100000.
5	89368.	126.	89242.
10	89070.	198.	88872.
15	88857.	265.	88592.
20	88475.	553.	87922.
25	87938.	934.	87005.
30	87249.	1121.	86128.
35	86492.	1243.	85249.
40	85605.	1319.	84286.
45	84391.	1376.	83015.
50	82768.	1386.	81381.
55	80176.	1398.	78779.
60	76424.	1416.	75008.
65	70269.	1392.	68877.
70	61153.	1264.	59889.
75	48034.	1008.	47026.
80	33353.	701.	32652.
85	18709.	342.	18367.

The computation of the expected number of survivors at exact age x in a multiregional system is more conveniently performed using matrix notation. For our two-region example, let

$$\underset{\sim}{\ell}(x) = \begin{bmatrix} {}_{10}\ell_1(x) & {}_{20}\ell_1(x) \\ {}_{10}\ell_2(x) & {}_{20}\ell_2(x) \end{bmatrix} \tag{2.4}$$

$$\underset{\sim}{P}(x) = \begin{bmatrix} p_{11}(x) & p_{21}(x) \\ p_{12}(x) & p_{22}(x) \end{bmatrix} \tag{2.5}$$

Note that $\underset{\sim}{\ell}(0)$ is a diagonal matrix with the regional radices in the diagonal. The matrix analogue of equation (2.3) is then

$$\underset{\sim}{\ell}(x + 5) = \underset{\sim}{P}(x)\underset{\sim}{\ell}(x) \quad . \tag{2.6}$$

For x = 5, we have

$$\begin{bmatrix} 94,316 & 198 \\ 2,392 & 88,872 \end{bmatrix} = \begin{bmatrix} 0.986467 & 0.000821 \\ 0.011370 & 0.995838 \end{bmatrix} \begin{bmatrix} 95,608 & 126 \\ 1,310 & 89,242 \end{bmatrix}$$

As before, we may express the life history of the hypothetical population interms of unit born. This yields a set of probabilities. For example, the probability that a person, born in region j be in region i, x years later is simply ${}_{j0}\hat{\ell}_i(x) = {}_{j0}\ell_i(x)/{}_{j0}\ell_j(0)$, which is easily derived from Table 2.3. The probability of surviving to age x is the product of conditional probabilities:

$$\hat{\underset{\sim}{\ell}}(x) = \underset{\sim}{P}(x - 5)\ \underset{\sim}{P}(x - 10)\ \ldots\ \underset{\sim}{P}(0) \quad . \tag{2.7}$$

The probability of surviving from x to x + n is also easily computed from Table 2.3. It is equal to the product

$$\underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10)\ \ldots\ \underset{\sim}{P}(x)$$

It follows from (2.6) that

$$\underset{\sim}{\ell}(x + n) = \underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10)\ \ldots\ \underset{\sim}{P}(x)\ \underset{\sim}{\ell}(x) \quad .$$

Hence,

$$\underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10)\ \ldots\ \underset{\sim}{P}(x)$$

$$= \underset{\sim}{\ell}(x + n)\ \underset{\sim}{\ell}^{-1}(x) \quad . \tag{2.8}$$

The probability that an individual in region i at age x will be in j, n years later, is therefore given by

$$\underset{\sim}{\ell}(x + n)\ \underset{\sim}{\ell}^{-1}(x) \text{ or } \hat{\underset{\sim}{\ell}}(x + n)[\hat{\underset{\sim}{\ell}}(x)]^{-1}$$

where the entries of $\underset{\sim}{\ell}(x + n)$ and $\underset{\sim}{\ell}(x)$ are found in Table 2.3 and $\hat{\underset{\sim}{\ell}}(x + n)$ and $\hat{\underset{\sim}{\ell}}(x)$ are the entries divided by the regional radices. For example, if one knows the distribution of people at the time they enter the labor force or marriage, age 20 say, and denote this by $\{\underset{\sim}{w}(20)\}$, then their distribution at retirement age, 60 say, is given by

$$\{\underset{\sim}{w}(60)\} = \underset{\sim}{\ell}(60)\ \underset{\sim}{\ell}^{-1}(20)\{\underset{\sim}{w}(20)\}$$

$$= \begin{bmatrix} 73{,}251 & 1{,}416 \\ 12{,}474 & 75{,}008 \end{bmatrix} \begin{bmatrix} 90{,}880 & 553 \\ 5{,}425 & 87{,}922 \end{bmatrix}^{-1} \{\underset{\sim}{w}(20)\}$$

$$= \begin{bmatrix} 0.805532 & 0.005043 \\ 0.137338 & 0.852580 \end{bmatrix} \{\underset{\sim}{w}(20)\} \quad .$$

The probability that an individual in Slovenia at age 20 will be in the Rest of Yugoslavia at retirement age is quite high, namely 13.7%.

2.3 Duration of Residence and Age Composition of the Life Table Population

The knowledge of the probability that a person, born in a given region, survives to age x and is then in another given region, leads us to ask: how long will the person stay in that region? This duration-of-residence question may be answered for persons born in a given region and for persons living in a specific region at age x.

a. Duration of Residence by Place of Birth

The number of years individuals at age x may expect to live in the next five years, on the average, is

$$\underset{\sim}{L}(x) = \int_0^5 \hat{\underset{\sim}{\ell}}(x + t)dt \tag{2.9}$$

where in the two-region case

$$\underset{\sim}{L}(x) = \begin{bmatrix} {}_{10}L_1(x) & {}_{20}L_1(x) \\ & \\ {}_{10}L_2(x) & {}_{20}L_2(x) \end{bmatrix} \tag{2.10}$$

with ${}_{j0}L_i(x)$ being the expected numbers of person-years lived in region i between x and x + 5, by an individual born in region j. It denotes the average duration of residence in region i by a j-born person and depends on two components: (i) the probability of surviving to age x and (ii) the average time spent in region i in a 5-year interval by a person of age x at the beginning of the interval.

The numerical approximation of (2.9) has given rise to a number of variants of life table construction (Keyfitz, 1968, p. 228). A simple approximation of $\underset{\sim}{L}(x)$ is a linear combination of the probabilities of surviving to exact ages x and x + 5:

$$\underset{\sim}{L}(x) = 5[a \cdot \hat{\underset{\sim}{\ell}}(x) + (1 - a) \cdot \hat{\underset{\sim}{\ell}}(x + 5)] \quad .$$

In the computer program, a is set equal to 0.5. Therefore,

$$\underset{\sim}{L}(x) = \frac{5}{2}[\hat{\underset{\sim}{\ell}}(x) + \hat{\underset{\sim}{\ell}}(x + 5)] \quad . \tag{2.11}$$

For example, $\underset{\sim}{L}(10)$ given in Table 2.4 is computed from Table 2.3 as follows:

$$\underset{\sim}{L}(10) = \frac{5}{2}[\underset{\sim}{\ell}(10) + \underset{\sim}{\ell}(15)]\ \underset{\sim}{\ell}^{-1}(0)$$

$$\begin{bmatrix} 4.69491 & 0.01157 \\ 0.13681 & 4.43660 \end{bmatrix} = \frac{5}{2}\left[\begin{bmatrix} 94,316 & 198 \\ 2,392 & 88,872 \end{bmatrix} + \begin{bmatrix} 93,481 & 265 \\ 3,080 & 88,592 \end{bmatrix}\right]\begin{bmatrix} 100,000 & 0 \\ 0 & 100,000 \end{bmatrix}^{-1}$$

The terminal age interval in a life table is a half-open interval: z years and over. The probability of dying in this interval therefore is unity. Since the length of the interval is infinite, $\hat{\underset{\sim}{\ell}}(z + 5)$ is not available and (2.11) cannot be used to compute $\underset{\sim}{L}(z)$. The number of years lived in the last age group is given by:

$$\underset{\sim}{L}(z) = [\underset{\sim}{M}(z)]^{-1}\hat{\underset{\sim}{\ell}}(z) \tag{2.12}$$

where $\underset{\sim}{M}(z)$ is a matrix with observed regional death and migration rates of the last age group (see Section 2.7).

The duration of residence or person-years-lived interpretation of $\underset{\sim}{L}(x)$ is one of several possible perspectives. It also may be viewed as a measure of the <u>age composition</u> of the multiregional life table population. In this perspective, an element

Table 2.4. Number of years lived in each region by a unit birth cohort.

age	initial region of cohort slovenia		
***	total	slovenia	r.yugos.
0	4.92297	4.89021	0.03276
5	4.84065	4.74810	0.09256
10	4.83172	4.69491	0.13681
15	4.82167	4.60903	0.21264
20	4.80588	4.45662	0.34926
25	4.78729	4.30303	0.48426
30	4.76510	4.18757	0.57753
35	4.73629	4.10796	0.62833
40	4.69634	4.04820	0.64814
45	4.63036	3.97803	0.65234
50	4.52751	3.88038	0.64713
55	4.37639	3.74302	0.63337
60	4.14608	3.54038	0.60569
65	3.76009	3.20367	0.55642
70	3.13922	2.66528	0.47394
75	2.32764	1.96490	0.36273
80	1.39903	1.16092	0.23810
85	0.96385	0.71663	0.24721

age	initial region of cohort r.yugos.		
***	total	slovenia	r.yugos.
0	4.73420	0.00315	4.73105
5	4.46095	0.00809	4.45285
10	4.44818	0.01157	4.43660
15	4.43330	0.02046	4.41284
20	4.41033	0.03717	4.37316
25	4.37968	0.05136	4.32832
30	4.34352	0.05909	4.28442
35	4.30243	0.06406	4.23837
40	4.24991	0.06739	4.18251
45	4.17897	0.06907	4.10990
50	4.07361	0.06960	4.00400
55	3.91500	0.07034	3.84466
60	3.66731	0.07019	3.59712
65	3.28555	0.06639	3.21916
70	2.72968	0.05681	2.67288
75	2.03467	0.04273	1.99194
80	1.30154	0.02606	1.27548
85	1.29669	0.01717	1.27952

${}_{j0}L_i(x)$ denotes the number of j-born people in region i of age x to x + 5, per unit born. The product ${}_{j0}L_i(x)*{}_{j0}\ell_j(0)$ is the total number of j-born people living in region i and x to x + 5 years old. Note that $\underset{\sim}{L}(x)$ represents the relative population distribution by place of residence and place of birth. Instead of being expressed in percentages (fractions of the total), or in some other manner, the population is given in unit births. This is a logical procedure in demography since it separates the fertility component from the survivorship (mortality and migration) component. This will be seen to be a very convenient way of "scaling" in spatial population analysis.

b. Duration of Reference by Place of Residence

As mentioned above, the duration of residence in each region depends on two components: (i) the probability of surviving to age x, and (ii) the average time spent in each region during the 5-year interval by a person of age x at the beginning of the interval. The latter component is the person-years lived between x and x + 5 by region of residence at age x and is equal to

$$\underset{\sim}{L}_r(x) = \underset{\sim}{L}(x)[\hat{\underset{\sim}{\ell}}(x)]^{-1} \quad .$$

Note that $\underset{\sim}{L}_r(x)$ is a conditional measure, since it gives the duration of residence in each region between ages x and x + 5, given that the person reaches age x and is in a specific region at that time. Using the linear approximation of $\underset{\sim}{L}(x)$ we may reduce this expression to

$$\underset{\sim}{L}_r(x) = \frac{5}{2}[\hat{\underset{\sim}{\ell}}(x + 5) + \hat{\underset{\sim}{\ell}}(x)][\hat{\underset{\sim}{\ell}}(x)]^{-1} = \frac{5}{2}[\underset{\sim}{P}(x) + \underset{\sim}{I}] \quad . \quad (2.13)$$

The number of years lived in the last age group is

$$\underset{\sim}{L}_r(z) = \underset{\sim}{L}(z)[\hat{\underset{\sim}{\ell}}(z)]^{-1} \quad ,$$

which is simply

$$\underset{\sim}{L}_r(z) = [\underset{\sim}{M}(z)]^{-1} \quad . \tag{2.14}$$

Numerical values for $L_r(x)$ are given in Table 2.5.

2.4 Total Number of Years Lived Beyond Age x

The total number of years newly born babies may expect to live beyond age x, is

$$\underset{\sim}{T}(x) = \sum_{y=x}^{z} \underset{\sim}{L}(y) \tag{2.15}$$

where z is the oldest age group. For example, the value of $\underset{\sim}{T}(10)$ in Table 2.6 is

$$T(10) = \begin{bmatrix} 55.26056 & 0.79946 \\ & \\ 7.45390 & 56.25090 \end{bmatrix} .$$

$$\begin{array}{cc} \overline{62.71446} & \overline{57.05036} \end{array}$$

The number of years that a girl, just born in Slovenia, may expect to live beyond age 10 is 62.71. From this total, 55.26 years are expected to be lived in Slovenia and 7.45 years in the Rest of Yugoslavia. Similarly, a new born baby girl of Slovenia has $_{10}T(60)$ or 15.74 years of retirement to look forward to, 2.48 years of which will be spent in the Rest of Yugoslavia.

2.5 Expectation of Life

The most important life table statistic is the life expectancy. The expectation of life at age x is the number of years an individual may expect to live beyond age x, given that he

Table 2.5. Number of years lived in each region by a person of age x.

age	region of residence at age x slovenia		
***	total	slovenia	r.yugos.
0	4.92297	4.89021	0.03276
5	4.99459	4.96617	0.02842
10	4.99628	4.97783	0.01845
15	4.99351	4.93017	0.06333
20	4.99057	4.90316	0.08742
25	4.99140	4.92364	0.06776
30	4.98746	4.94131	0.04615
35	4.98470	4.96293	0.02177
40	4.97604	4.96356	0.01247
45	4.95577	4.94912	0.00665
50	4.93552	4.92748	0.00803
55	4.90188	4.89437	0.00751
60	4.84305	4.83281	0.01024
65	4.69628	4.68589	0.01039
70	4.46685	4.45814	0.00871
75	4.20568	4.19581	0.00987
80	3.65917	3.65415	0.00503
85	4.92249	4.88468	0.03781

age	region of residence at age x r.yugos.		
***	total	slovenia	r.yugos.
0	4.73420	0.00315	4.73105
5	4.99165	0.00205	4.98959
10	4.99404	0.00195	4.99208
15	4.98922	0.00833	4.98089
20	4.98481	0.01143	4.97338
25	4.98028	0.00620	4.97407
30	4.97819	0.00430	4.97389
35	4.97422	0.00279	4.97144
40	4.96436	0.00226	4.96210
45	4.95185	0.00115	4.95070
50	4.92149	0.00158	4.91991
55	4.88265	0.00245	4.88019
60	4.79783	0.00235	4.79548
65	4.67526	0.00171	4.67355
70	4.46362	0.00077	4.46285
75	4.23652	0.00088	4.23564
80	3.90758	0.00140	3.90618
85	6.96823	0.00260	6.96563

Table 2.6. Total number of years lived beyond age x.

age	initial region of cohort slovenia		
***	************************************		
	total	slovenia	r.yugos.
0	72.47807	64.89886	7.57922
5	67.55511	60.00865	7.54646
10	62.71446	55.26056	7.45390
15	57.88273	50.56564	7.31709
20	53.06106	45.95661	7.10445
25	48.25518	41.49998	6.75520
30	43.46790	37.19696	6.27094
35	38.70280	33.00939	5.69341
40	33.96650	28.90142	5.06508
45	29.27017	24.85322	4.41694
50	24.63980	20.87519	3.76461
55	20.11228	16.99481	3.11747
60	15.73590	13.25179	2.48411
65	11.58982	9.71141	1.87842
70	7.82973	6.50774	1.32200
75	4.69051	3.84246	0.84805
80	2.36288	1.87756	0.48532
85	0.96385	0.71663	0.24721

age	initial region of cohort r.yugos.		
***	************************************		
	total	slovenia	r.yugos.
0	66.24551	0.81071	65.43481
5	61.51131	0.80755	60.70375
10	57.05036	0.79946	56.25090
15	52.60218	0.78789	51.81430
20	48.16888	0.76743	47.40145
25	43.75855	0.73026	43.02829
30	39.37887	0.67889	38.69998
35	35.03535	0.61980	34.41556
40	30.73292	0.55574	30.17718
45	26.48302	0.48835	25.99467
50	22.30404	0.41928	21.88477
55	18.23044	0.34967	17.88077
60	14.31544	0.27934	14.03611
65	10.64813	0.20915	10.43898
70	7.36258	0.14277	7.21982
75	4.63290	0.08596	4.54694
80	2.59823	0.04323	2.55500
85	1.29669	0.01717	1.27952

reaches age x. In multiregional demography, two types of life expectancies may be distinguished: life expectancy by place of residence and life expectancy by place of birth.

a. Life Expectancy by Place of Residence

The life expectancy by place of residence gives the expectation of life at age x of a person residing in a specific region at that age. It is computed as follows:

$$ {}_{x}\underset{\sim}{e}(x) = \underset{\sim}{T}(x)[\hat{\underset{\sim}{\ell}}(x)]^{-1} = [\sum_{y=x}^{z} L(y)][\hat{\underset{\sim}{\ell}}(x)]^{-1} \quad , \qquad (2.16) $$

where

$$ {}_{x}\underset{\sim}{e}(x) = \begin{bmatrix} {}_{1x}e_1(x) & {}_{2x}e_1(x) \\ & \\ {}_{1x}e_2(x) & {}_{2x}e_2(x) \end{bmatrix} \qquad (2.17) $$

and ${}_{ix}e_j(x)$ is the average number of years lived in region j beyond age x by an individual residing in region i and x years of age (whatever the region of birth). The life expectancy at each age except the first is higher than $\underset{\sim}{T}(x)$, since it is a conditional measure. Note that for the last age group $\underset{\sim}{e}(z) = \underset{\sim}{L}_r(z) = [\underset{\sim}{M}(z)]^{-1}$.

The expectations of life by place of residence for 10-year old girls, for example, are (Table 2.7)

$$ \underset{\sim}{e}(10) = \underset{\sim}{T}(10)[\hat{\underset{\sim}{\ell}}(10)]^{-1} \qquad (2.18) $$

$$ \begin{bmatrix} 58.57163 & 0.76931 \\ & \\ 6.29832 & 63.28025 \end{bmatrix} = \begin{bmatrix} 55.26056 & 0.79946 \\ & \\ 7.45390 & 56.25090 \end{bmatrix} \begin{bmatrix} 0.94316 & 0.00198 \\ & \\ 0.02392 & 0.88872 \end{bmatrix}^{-1} $$

$$ \begin{matrix} \overline{64.86995} & \overline{64.04956} \end{matrix} $$

Table 2.7. Expectations of life by place of residence.

age	region of residence at age x		slovenia
***	*****	*****	*****
	total	slovenia	r.yugos.
0	72.47807	64.89886	7.57922
5	69.71486	62.75388	6.96098
10	64.86995	58.57163	6.29832
15	59.96857	54.06794	5.90063
20	55.13594	50.53516	4.60078
25	50.35629	47.45893	2.89736
30	45.53242	43.86935	1.66307
35	40.75395	39.86110	0.89286
40	35.99032	35.42252	0.56781
45	31.31412	30.90905	0.40507
50	26.83282	26.49912	0.33370
55	22.47618	22.21589	0.26030
60	18.29084	18.08561	0.20523
65	14.34499	14.20255	0.14244
70	10.97786	10.88414	0.09371
75	8.26969	8.20417	0.06552
80	5.94569	5.90920	0.03648
85	4.92249	4.88468	0.03781

age	region of residence at age x		r.yugos.
***	*****	*****	*****
	total	slovenia	r.yugos.
0	66.24551	0.81071	65.43480
5	68.82788	0.81626	68.01162
10	64.04956	0.76931	63.28025
15	59.19611	0.72743	58.46869
20	54.43933	0.55490	53.88442
25	49.75403	0.33005	49.42397
30	45.12880	0.21728	44.91151
35	40.50353	0.14591	40.35762
40	35.89935	0.10485	35.79449
45	31.38244	0.07581	31.30663
50	26.94966	0.06373	26.88592
55	22.74258	0.04975	22.69283
60	18.74005	0.03102	18.70902
65	15.16974	0.01671	15.15303
70	12.06200	0.00869	12.05331
75	9.67449	0.00685	9.66764
80	7.82976	0.00559	7.82416
85	6.96823	0.00260	6.96564

A 10-year old girl, living in Slovenia, may expect to live another 64.87 years. Of this, 6.30 years will be spent in the Rest of Yugoslavia, i.e. 10%. A girl of the same age in the Rest of Yugoslavia may expect to spent 0.77 years in Slovenia.

b. Life Expectancy by Place of Birth

This measure gives the expectation of life at age x by region of birth of the person. The region of residence at age x is not taken into account. Define the diagonal matrix $\bar{\underset{\sim}{\ell}}(x)$, with the elements of the vector $\{\underset{\sim}{1}\}'\hat{\underset{\sim}{\ell}}(x)$ in the diagonal ($\{\underset{\sim}{1}\}'$ is a row vector of ones):

$$\bar{\underset{\sim}{\ell}}(x) = \begin{bmatrix} \sum_i {}_{10}\hat{\ell}_i(x) & 0 \\ 0 & \sum_i {}_{20}\hat{\ell}_i(x) \end{bmatrix} \tag{2.19}$$

The matrix of life expectancies by place of birth is obtained as follows:

$$_0\underset{\sim}{e}(x) = \underset{\sim}{T}(x)[\bar{\underset{\sim}{\ell}}(x)]^{-1} \tag{2.20}$$

The life expectancies of 10-year old girls are

$$_0\underset{\sim}{e}(10) = \underset{\sim}{T}(10)[\bar{\underset{\sim}{\ell}}(10)]^{-1}$$

$$\begin{bmatrix} 57.14198 & 0.89757 \\ 7.70768 & 63.15382 \end{bmatrix} = \begin{bmatrix} 55.26056 & 0.79946 \\ 7.45390 & 56.25090 \end{bmatrix} \begin{bmatrix} 0.96707 & 0 \\ 0 & 0.89070 \end{bmatrix}^{-1}$$

$$\begin{matrix} \overline{64.84966} & \overline{64.05138} \end{matrix}$$

A girl born in Slovenia, may expect to live another 64.85 years, <u>when</u> reaching 10 years of age. Of this, 7.70 years will be spent in the Rest of Yugoslavia, i.e. 12%. At age 65, however, 2.34 years of the future lifetime of 14.47 years will be spent in the Rest of Yugoslavia, i.e. 16% (Table 2.8).

It is the special feature of the multiregional life table that the demographic measure of the expectation of life is decomposed according to where that life is spent. It introduces the spatial dimension into classical demographic analysis.

2.6 Survivorship and Outmigration Proportions

A useful application of the multiregional life table is found in multiregional population projection. The assumption is that the survivorship and migration behavior exhibited by the stationary life table population adequately represents the survivorship and migration experience of the empirical population for which the life table was developed.

The necessary information for the projection of age groups beyond the first one is given by age-specific matrices of survivorship proportions. The number of people in age group (x + 5, x + 10) in the stationary population is

$$\underset{\sim}{L}(x + 5) = \underset{\sim}{S}(x)\ \underset{\sim}{L}(x) \tag{2.21}$$

where, in the two-region case,

$$\underset{\sim}{S}(x) = \begin{bmatrix} s_{11}(x) & s_{21}(x) \\ & \\ s_{12}(x) & s_{22}(x) \end{bmatrix} \tag{2.22}$$

with $s_{ij}(x)$ being the proportion of individuals aged x to x + 4 who survive to be x + 5 to x + 9 years old 5 years later, by new places of residence.

Table 2.8. Expectations of life by place of birth.

age	initial region of cohort slovenia		
***	***********************************		
	total	slovenia	r.yugos.
0	72.47807	64.89886	7.57922
5	69.70287	61.91648	7.78638
10	64.84966	57.14198	7.70768
15	59.94392	52.36627	7.57765
20	55.09670	47.71969	7.37701
25	50.30264	43.26082	7.04182
30	45.48670	38.92451	6.56219
35	40.72161	34.73123	5.99039
40	35.97794	30.61291	5.36503
45	31.32368	26.59686	4.72682
50	26.84941	22.74720	4.10220
55	22.51454	19.02471	3.48983
60	18.35621	15.45845	2.89776
65	14.46598	12.12141	2.34457
70	11.13986	9.25898	1.88089
75	8.48453	6.95052	1.53401
80	6.24729	4.96414	1.28315
85	5.31378	3.95087	1.36291

age	initial region of cohort r.yugos.		
***	***********************************		
	total	slovenia	r.yugos.
0	66.24551	0.81071	65.43481
5	68.82913	0.90363	67.92551
10	64.05138	0.89757	63.15382
15	59.19842	0.88669	58.31173
20	54.44369	0.86740	53.57629
25	49.76043	0.83042	48.93001
30	45.13393	0.77811	44.35587
35	40.50714	0.71660	39.79054
40	35.90075	0.64919	35.25156
45	31.38132	0.57867	30.80265
50	26.94770	0.50657	26.44113
55	22.73793	0.43613	22.30180
60	18.73173	0.36551	18.36621
65	15.15340	0.29764	14.85576
70	12.03960	0.23346	11.80614
75	9.64499	0.17895	9.46604
80	7.79018	0.12961	7.66057
85	6.93086	0.09177	6.83909

For example, the number of people in the Rest of Yugoslavia at ages 15 to 19, who were born in Slovenia, per unit radix is (Tables 2.4 and 2.9):

$$ {}_{10}L_2(15) = s_{12}(10)\,{}_{10}L_1(10) + s_{22}(10)\,{}_{10}L_2(10) $$

$$ 0.21264 = 0.01631 * 4.69491 + 0.99460 * 0.13681 \quad . $$

The computation of $\underset{\sim}{L}(x)$ in the life table is not performed using (2.21) but by (2.11). In (2.21), the unknown is $\underset{\sim}{S}(x)$, therefore

$$ \underset{\sim}{S}(x) = \underset{\sim}{L}(x + 5)\ \underset{\sim}{L}^{-1}(x) \quad . \tag{2.23} $$

For x = 10 in the Yugoslavian example, $\underset{\sim}{S}(x)$ is

$$ \begin{bmatrix} 0.98165 & 0.00205 \\ 0.01631 & 0.99460 \end{bmatrix} = \begin{bmatrix} 4.60903 & 0.02046 \\ 0.21264 & 4.41284 \end{bmatrix} \begin{bmatrix} 4.69491 & 0.01157 \\ 0.13681 & 4.43660 \end{bmatrix}^{-1} $$

The number 0.016308, for instance, is the proportion of the girls residing in Slovenia and 10 to 14 years old, that will be alive and in the Rest of Yugoslavia 5 years from now.

With the survivorship proportions, all the life table statistics are derived. They are summarized in Table 2.10.*

---

*A summary table is produced by the computer for a system of <u>two</u> regions only.

Table 2.9. Survivorship proportions.

region slovenia
****

	total	slovenia	r.yugos.
0	0.98355	0.97093	0.01262
5	0.99817	0.98878	0.00939
10	0.99796	0.98165	0.01631
15	0.99681	0.96675	0.03006
20	0.99638	0.96526	0.03112
25	0.99578	0.97293	0.02285
30	0.99443	0.98079	0.01364
35	0.99215	0.98530	0.00685
40	0.98638	0.98256	0.00383
45	0.97829	0.97536	0.00293
50	0.96757	0.96447	0.00310
55	0.94921	0.94570	0.00351
60	0.90883	0.90476	0.00407
65	0.83560	0.83186	0.00374
70	0.74076	0.73717	0.00359
75	0.59374	0.59075	0.00298
80	0.62435	0.61712	0.00723

region r.yugos.
****

	total	slovenia	r.yugos.
0	0.94225	0.00106	0.94119
5	0.99714	0.00080	0.99633
10	0.99665	0.00205	0.99460
15	0.99481	0.00394	0.99087
20	0.99302	0.00354	0.98948
25	0.99169	0.00211	0.98959
30	0.99049	0.00142	0.98906
35	0.98773	0.00101	0.98672
40	0.98326	0.00068	0.98258
45	0.97473	0.00054	0.97418
50	0.96095	0.00080	0.96015
55	0.93651	0.00095	0.93555
60	0.89565	0.00080	0.89485
65	0.83072	0.00049	0.83022
70	0.74549	0.00032	0.74517
75	0.64067	0.00041	0.64025
80	1.00387	0.00085	1.00302

Table 2.10. Multiregional (two-region) life table option 3.

age	q(x,1)	p(x,1,1)	p(x,2,1)	l(x,1,1)	l(x,2,1)	ll(x,1,1)	ll(x,2,1)	m(x,2,1)	md(x,1)	s(x,1,1)	s(x,2,1)	e(x,1,1)	e(x,2,1)
0	0.030813	0.956084	0.013103	100000.	0.	4.89021	0.03276	0.002832	0.006150	0.970932	0.012622	64.90	7.58
5	0.002164	0.986467	0.011370	95503.	1310.	4.74810	0.09256	0.002294	0.000432	0.988783	0.009392	61.92	7.79
10	0.001487	0.991131	0.007381	94316.	2392.	4.69491	0.13681	0.001435	0.000297	0.981648	0.016308	57.14	7.71
15	0.002598	0.972070	0.025332	93481.	3980.	4.60903	0.21264	0.005158	0.000516	0.966751	0.030062	52.37	7.53
20	0.003770	0.961262	0.034958	90330.	5425.	4.45652	0.34926	0.007170	0.000747	0.965259	0.031117	47.72	7.38
25	0.003439	0.969456	0.027105	87385.	8545.	4.30303	0.48426	0.005534	0.000677	0.972930	0.022849	43.26	7.04
30	0.005015	0.976525	0.018460	84737.	10825.	4.18757	0.57753	0.003756	0.000999	0.980794	0.013638	38.92	6.55
35	0.006121	0.985171	0.008708	827[illegible].	12276.	4.10796	0.62833	0.001765	0.001224	0.985297	0.006854	34.73	5.99
40	0.009586	0.985426	0.004988	81552.	12857.	4.04820	0.64814	0.001013	0.001924	0.982557	0.003827	30.61	5.37
45	0.017694	0.979646	0.002660	80376.	13069.	3.97803	0.65234	0.000543	0.003570	0.975364	0.002925	26.60	4.73
50	0.025793	0.970993	0.003213	78746.	13025.	3.88038	0.64713	0.000563	0.005224	0.964468	0.003098	22.75	4.10
55	0.039248	0.957747	0.003005	76470.	12861.	3.74302	0.63337	0.000629	0.008004	0.945701	0.003512	19.02	3.49
60	0.062780	0.933124	0.004097	73251.	12474.	3.54038	0.60569	0.000884	0.012955	0.904756	0.004072	15.46	2.90
65	0.121486	0.874356	0.004157	68364.	11754.	3.20367	0.55642	0.000949	0.025364	0.831860	0.003743	12.12	2.34
70	0.213259	0.783257	0.003484	59783.	10503.	2.66528	0.47394	0.000876	0.047742	0.737167	0.003589	9.26	1.83
75	0.317728	0.678322	0.003949	46828.	8455.	1.96490	0.36273	0.001111	0.075556	0.590754	0.002983	6.95	1.53
80	0.536332	0.461658	0.002010	31768.	6055.	1.16092	0.23810	0.000704	0.146620	0.617122	0.007229	4.95	1.28
85	1.000000	0.000000	0.000000	14669.	3469.	0.71663	0.24721	0.001111	0.203611	0.000000	0.000000	3.95	1.36

age	q(x,2)	p(x,2,2)	p(x,1,2)	l(x,2,2)	l(x,1,2)	ll(x,2,2)	ll(x,1,2)	m(x,1,2)	md(x,2)	s(x,2,2)	s(x,1,2)	e(x,2,2)	e(x,1,2)
0	0.106319	0.892421	0.001261	100000.	0.	4.73105	0.00315	0.000272	0.022468	0.941189	0.001064	65.43	0.81
5	0.003341	0.995838	0.000821	89242.	126.	4.45285	0.00309	0.000166	0.000669	0.996334	0.000802	67.93	0.90
10	0.002385	0.996834	0.000781	88872.	198.	4.43660	0.01157	0.000157	0.000478	0.994601	0.002051	63.15	0.90
15	0.004312	0.992355	0.003333	88592.	265.	4.41284	0.02046	0.000679	0.000865	0.990369	0.003940	58.31	0.89
20	0.006075	0.989354	0.004571	87922.	553.	4.37316	0.03717	0.000937	0.001220	0.989482	0.003541	53.58	0.87
25	0.007889	0.989630	0.002481	87005.	934.	4.32832	0.05136	0.000506	0.001585	0.989587	0.002107	48.93	0.83
30	0.008724	0.989555	0.001721	86128.	1121.	4.28442	0.05909	0.000350	0.001753	0.989063	0.001423	44.36	0.78
35	0.010310	0.988575	0.001114	85249.	1243.	4.23837	0.06406	0.000226	0.002073	0.986718	0.001010	39.79	0.72
40	0.014256	0.984840	0.000904	84286.	1319.	4.18251	0.06739	0.000183	0.002872	0.982578	0.000682	35.25	0.65
45	0.019259	0.980282	0.000460	83015.	1376.	4.10990	0.06907	0.000094	0.003889	0.974184	0.000543	30.80	0.58
50	0.031406	0.967964	0.000630	81381.	1386.	4.00400	0.06960	0.000130	0.006382	0.960150	0.000801	26.44	0.51
55	0.046941	0.952077	0.000982	78779.	1393.	3.84465	0.07034	0.000205	0.009615	0.935551	0.000955	22.30	0.44
60	0.080368	0.918193	0.000939	75008.	1416.	3.59712	0.07019	0.000203	0.016857	0.894848	0.000802	18.37	0.37
65	0.129895	0.869422	0.000684	68977.	1392.	3.21916	0.06639	0.000156	0.027784	0.830224	0.000492	14.86	0.30
70	0.214551	0.785139	0.000309	59889.	1264.	2.67288	0.05631	0.000078	0.048067	0.745168	0.000318	11.81	0.23
75	0.305390	0.694257	0.000353	47026.	1008.	1.99194	0.04273	0.000099	0.072084	0.640255	0.000411	9.47	0.18
80	0.436969	0.562471	0.000560	32652.	701.	1.27548	0.02606	0.000196	0.111814	1.003021	0.000353	7.66	0.13
85	1.000000	0.000000	0.000000	18367.	342.	1.27952	0.01717	0.000076	0.143485	0.000000	0.000000	6.84	0.09

2.7 Estimation of Age-Specific Outmigration and Death Probabilities

Life table probabilities are derived from the observed annual age-specific rates of outmigration and death. The rates are computed by dividing the number of outmigrants and deaths in a certain age group by the mid-year population in that age group. The death and outmigration rates are given in Table 1.3.

Starting from the observed rates, the probabilities of dying and outmigrating may be computed along two lines. The basic difference is the assumption about multiple transitions. Early formulations of the probability estimation procedure permitted no multiple transitions (Rogers, 1975, p. 82)(Option 1). It was assumed that an individual only makes one move during a unit time period, five years say. Later formulations relaxed this assumption (Schoen, 1975; Rogers and Ledent, 1976)(Option 3). Option 2 is treated in Rogers (1975, p. 85), but is not used in this report.

a. Estimation Under Option 3

This formulation begins by arranging the observed outmigration and death rates into the following matrix:

$$\underset{\sim}{M}(x) = \begin{bmatrix} \left[M_{1\delta}(x) + \sum\limits_{j\neq 1} M_{1j}(x)\right] & -M_{21}(x) & \cdots & -M_{n1}(x) \\ -M_{12}(x) & \left[M_{2\delta}(x) + \sum\limits_{j\neq 2} M_{2j}(x)\right] & \cdots & -M_{n2}(x) \\ \vdots & \vdots & & \vdots \\ -M_{1n}(x) & -M_{2n}(x) & \cdots & \left[M_{n\delta}(x) + \sum\limits_{j\neq n} M_{nj}(x)\right] \end{bmatrix}$$

where n is the number of regions;

$M_{i\delta}(x)$ is the age-specific mortality rate in region i;

$M_{ij}(x)$ is the age-specific migration rate from region i to region j.

It can be shown that the probability matrix $\underset{\sim}{P}(x)$ is (Rogers and Ledent, 1976):

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \qquad (2.25)$$

where, for a two-region model, $\underset{\sim}{P}(x)$ is given by (2.5):

$$\underset{\sim}{P}(x) = \begin{bmatrix} p_{11}(x) & p_{21}(x) \\ & \\ p_{12}(x) & p_{22}(x) \end{bmatrix} \qquad (2.5)$$

with $p_{ij}(x)$ being the probability that an individual in region i at exact age x will survive and be in region j five years later. The off-diagonal elements are migration probabilities analogous to transition probabilities in Markov theory. The diagonal element $p_{ii}(x)$ denotes the probability of surviving and remaining in (or returning to) region i. The elements of each column in $\underset{\sim}{P}(x)$ do not sum up to unity since the effects of mortality are included. Rather, $\underset{\sim}{P}(x)$ is analogous to the transition matrix of an absorbing Markov chain. Note than an element $p_{ij}(x)$ does not denote the probability of making a move from i to j by a person living in i at the beginning of the transition period. What it represents is the probability that an individual in region i at the beginning of the time period is in region j at the beginning of the next period. During the period, several moves may have been made.

For example, the matrix of probabilities at age 10 is (Table 2.1):

$$\underset{\sim}{P}(10) = \begin{bmatrix} 0.991131 & 0.000781 \\ & \\ 0.007381 & 0.996834 \end{bmatrix} .$$

$$\begin{array}{cc} \overline{0.998512} & \overline{0.997615} \end{array}$$

The probability that a female in Slovenia at age 10 will survive is 0.998512. The probability she will be in the Rest of Yugoslavia at age 15 is 0.007381.

The probabilities of dying are found by subtraction. The probability that an individual in region i at age x dies before reaching $x + 5$ is

$$q_i(x) = 1 - \sum_{j=1}^{n} p_{ij} \quad . \tag{2.26}$$

The probability of dying in the next five years for a 10 year old in Slovenia is

$$1 - 0.991131 - 0.007381 = 0.001487 \quad .$$

Note that (2.25) is analogous to the single region formula

$$p(x) = [1 + \frac{5}{2}\ (M_\delta(x))]^{-1}[1 - \frac{5}{2}\ (M_\delta(x))]$$

$$= \frac{1 - \frac{5}{2} M_\delta(x)}{1 + \frac{5}{2} M_\delta(x)} \quad . \tag{2.27}$$

Formula (2.27) is equivalent to equation (1.1.9) of Keyfitz (1968, p. 14) and Keyfitz and Flieger (1971, p. 135). The probability of dying is

$$q(x) = 1 - p(x) = \frac{5\ M_\delta(x)}{1 + \frac{5}{2} M_\delta(x)} \tag{2.28}$$

b. Estimation Under Option 1

On the assumption of no multiple transitions, the outmigration probability $p_{ij}(x)$ is given by (Rogers, 1975, p. 82)

$$p_{ij}(x) = \frac{5\ M_{ij}(x)}{1 + \frac{5}{2}\ M_{i\delta}(x) + \frac{5}{2}\ \sum\limits_{j \neq i} M_{ij}(x)} \quad . \tag{2.29}$$

The probability of dying in region i is

$$q_i(x) = \frac{5\ M_{i\delta}(x)}{1 + \frac{5}{2}\ M_{i\delta}(x) + \frac{5}{2}\ \sum\limits_{j \neq i} M_{ij}(x)} \quad . \tag{2.30}$$

The probability of surviving and remaining in the region is found as a residual

$$p_{ii}(x) = 1 - q_i(x) - \sum_{j \neq i} p_{ij}(x) \quad . \tag{2.31}$$

The probabilities computed by this method are given in Appendix B. The matrix probabilities at age 10, for example, is

$$\underset{\sim}{P}(x) = \begin{bmatrix} 0.991089 & 0.000786 \\ & \\ 0.007426 & 0.996826 \end{bmatrix} \quad .$$

For a single region case, $p_{ij}(x) = 0$, and formula (2.25) reduces to (2.27). The distinction between multiple transitions and no multiple transitions is irrelevant in a single-region situation, since one can die only once.

The assumption of multiple versus no multiple transitions affects not only the probabilities directly, but also the person-years lived in the last open-ended age group. Recall (2.12),

$$\underset{\sim}{L}(z) = \underset{\sim}{M}^{-1}(z)\ \hat{\underset{\sim}{\ell}}(z) \quad . \tag{2.12}$$

Under the assumption of no multiple transitions, people cannot migrate and die during the same time-interval. Since all people die in the last age group, the off-diagonal elements of $\underset{\sim}{M}(z)$ are zero and the diagonal consists of regional death rates. Hence ${}_{j0}L_i(z) = {}_{j0}\hat{\ell}_i(z)/M_{i\delta}(z)$ (Rogers, 1975, p. 64).

2.8 Aggregated Life Table Statistics

The life table statistics considered thus far refer to a multiregional system. The life table functions are basically matrix equations and give regional statistics. In order to aggregate the regional measures to yield the life table statistics for the whole system (country), regional weights must be introduced. The weights are the regional radices, specified by the user.

The life table of the aggregate system is a single-region table derived from a set of age-specific mortality rates, which are computed as follows:

$$M_{\cdot\delta}(x) = \frac{{}_{\cdot}\hat{\ell}_{\cdot}(x) - {}_{\cdot}\hat{\ell}_{\cdot}(x+5)}{{}_{\cdot}L_{\cdot}(x)} \tag{2.32}$$

where

$${}_{\cdot}\hat{\ell}_{\cdot}(x) = \sum_j RR_j^{(0)} \sum_i {}_j\hat{\ell}_i(x)$$

and

$${}_{\cdot}L_{\cdot}(x) = \sum_j RR_j^{(0)} \sum_i {}_jL_i(x) \quad ,$$

with $RR_j^{(0)}$ the radix ratio in the life table or stationary population

$$RR_j^{(0)} = \frac{{}_j\ell_j(0)}{\sum_k {}_k\ell_k(0)} \tag{2.33}$$

The death rate of the last age group is

$$M_{\cdot\delta}(z) = \hat{\ell}_{\cdot\cdot}(z) / L_{\cdot\cdot}(z) \quad .$$

Table 2.11 gives the aggregated life-table statistics. Equal radices are specified for both regions. In interpreting the results, one must keep in mind that unless regional radices are set in proportion to an estimate of appropriate life table births, the aggregated life table values will be incorrect. Setting all radices equal implies that regional births in the life table population are all equal in number. If in the observed population they are not, then obviously the life table statistics in the aggregated table are not realistic.

Note the difference between Table 2.11 and Table 1.6b. The latter is derived from a set of average national age-specific death rates. Regional differences are not accounted for and internal migration is not considered. Table 2.11, on the other hand, is aggregated from a multiregional life table, which explicitly considers regionally deviating mortality and migration. If mortality would be the same in all regions, then Table 2.11 and Table 1.6b coincide.

Table 2.11. Single region life table of aggregated system.

age	p(x)	q(x)	l(x)	d(x)	ll(x)	m(x)	s(x)	t(x)	e(x)
0	0.931434	0.068566	100000.	6857.	4.828585	0.014200	0.963181	69.3618	69.3618
5	0.997264	0.002736	93143.	255.	4.650799	0.000548	0.997657	64.5332	69.2837
10	0.998071	0.001929	92889.	179.	4.639950	0.000386	0.997314	59.8824	64.4669
15	0.996555	0.003445	92709.	319.	4.627486	0.000690	0.995811	55.2425	59.5867
20	0.995065	0.004935	92390.	456.	4.608102	0.000989	0.994658	50.6150	54.7840
25	0.994248	0.005752	91934.	529.	4.583485	0.001154	0.993635	46.0069	50.0433
30	0.993018	0.006982	91405.	638.	4.554309	0.001401	0.992326	41.4234	45.3184
35	0.991628	0.008372	90767.	760.	4.519358	0.001681	0.989769	36.8691	40.6194
40	0.987894	0.012106	90007.	1090.	4.473121	0.002436	0.984697	32.3497	35.9412
45	0.981461	0.018539	88918.	1648.	4.404668	0.003743	0.976364	27.8766	31.3510
50	0.971171	0.028829	87269.	2516.	4.300560	0.005850	0.963989	23.4719	26.8960
55	0.956593	0.043407	84753.	3679.	4.145691	0.008874	0.942350	19.1714	22.6202
60	0.927462	0.072538	81074.	5881.	3.906693	0.015054	0.901739	15.0257	18.5332
65	0.874006	0.125994	75193.	9474.	3.522819	0.026893	0.832984	11.1190	14.7872
70	0.786049	0.213951	65719.	14061.	2.934452	0.047916	0.743292	7.5962	11.5535
75	0.688897	0.311103	51659.	16071.	2.181155	0.073682	0.619068	4.6617	9.0241
80	0.517704	0.482296	35588.	17164.	1.350283	0.127112	0.837062	2.4806	6.9703
85	0.420950	0.579050	18424.	10668.	1.130270	0.163004	0.000000	1.1303	6.1348

Table 2.11 (cont'd)

Q(x,i)	= probability that an individual at age x in region i will die before reaching age x + 5.
P(x,j,i)	= probability that an individual at age x in region i will be in region j at age x + 5, i.e. 5 years later.
L(x,j,i)	= number surviving at exact age x in region j, of 100,000 born in region i. This is also the probability that a baby born in region i, will survive and be in region j at exact age x, multiplied by 100,000.
LL(x,j,i)	= total years lived between ages x to x + 5 in region j, per unit born in region i.
M(x,j,i)	= age-specific migration rate from region i to j (equal to observed value).
MD(x,i)	= age-specific death rates in region i (equal to observed value).
S(x,j,i)	= proportion of people in region i and aged x to x + 4 that will survive to be in region j and aged x + 5 to x + 9, five years later.
E(x,j,i)	= part of expectation of life of i-born people at age x, that will be lived in region j, i.e. the average number of years lived in region j by i-born people, subsequent to age x.

3. MULTIREGIONAL POPULATION PROJECTION

The population growth process has been represented by demographers as a matrix multiplication or, equivalently, as a system of linear, first-order, homogeneous difference equations with constant coefficients. This approach was used by Leslie in 1945 to project populations composed of a number of age groups. Rogers (1966) and later Feeney (1970) generalized Leslie's idea to include multiregional population systems.

The general matrix expression of the multiregional growth process is (Rogers, 1975, pp. 122-123):

$$\{\underset{\sim}{K}^{(t+1)}\} = \underset{\sim}{G}\{\underset{\sim}{K}^{(t)}\} \tag{3.1}$$

where $\{\underset{\sim}{K}^{(t)}\}$ is the age and regional distribution of the population at time t,

$\underset{\sim}{G}$ is the multiregional matrix growth operator or generalized Leslie matrix.

The vector $\{\underset{\sim}{K}^{(t)}\}$ is partitioned as follows:

$$\{\underset{\sim}{K}^{(t)}\} = \begin{bmatrix} \{\underset{\sim}{K}^{(t)}(0)\} \\ \{\underset{\sim}{K}^{(t)}(5)\} \\ \vdots \\ \{\underset{\sim}{K}^{(t)}(z)\} \end{bmatrix} \quad \text{and} \quad \{\underset{\sim}{K}^{(t)}(x)\} = \begin{bmatrix} K_1^{(t)}(x) \\ K_2^{(t)}(x) \\ \vdots \\ K_n^{(t)}(x) \end{bmatrix} \tag{3.2}$$

where $K_i^{(t)}(x)$ denotes the population in region i at time t, who are x to x + 4 years of age, and

$\{\underset{\sim}{K}^{(t)}(x)\}$ is the regional distribution of the population in age group x to x + 4.

3.1 The Growth Matrix

The arrangement of the growth matrix $\underset{\sim}{G}$ is:

$$\underset{\sim}{G} = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{B}(\alpha - 5) \ \ldots & \underset{\sim}{B}(\beta - 5) \ \ldots & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{S}(0) & 0 & & & \vdots & \vdots \\ \underset{\sim}{0} & \underset{\sim}{S}(5) & \ddots & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ \underset{\sim}{0} & \underset{\sim}{0} & & \ldots & \underset{\sim}{S}(z - 5) & \underset{\sim}{0} \end{bmatrix}$$

where α and β are the first and last ages of childbearing respectively. The matrix of survivorship proportions $\underset{\sim}{S}(x)$ in a two-region model is

$$\underset{\sim}{S}(x) = \begin{bmatrix} s_{11}(x) & s_{21}(x) \\ s_{12}(x) & s_{22}(x) \end{bmatrix} \qquad (3.4)$$

where $s_{ij}(x)$ is the proportion of x to (x + 4)-year-old residents of region i at time t who are alive and x + 5-to x + 9-years-old in region j five years later at time t + 1. The survivorship matrix is computed as part of the life table equation (2.23);

$$\underset{\sim}{S}(x) = \underset{\sim}{L}(x + 5)[\underset{\sim}{L}(x)]^{-1} \quad ,$$

or in terms of probabilities

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \underset{\sim}{P}(x + 5)]\underset{\sim}{P}(x)[\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} \qquad (3.5)$$

Assuming multiple transitions, it may also be expressed directly in terms of the observed age-specific rates:

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x+5)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \quad . \tag{3.6}$$

The survivorship matrix for the age group z - 5 is, by (2.12),

$$\begin{aligned}\underset{\sim}{S}(z-5) &= \underset{\sim}{L}(z)\,\underset{\sim}{L}^{-1}(z-5)\\ &= \frac{2}{5}\underset{\sim}{M}^{-1}(z)\underset{\sim}{P}(z-5)[\underset{\sim}{I} + \underset{\sim}{P}(z-5)]^{-1}\end{aligned} \tag{3.7}$$

or

$$\underset{\sim}{S}(z-5) = \frac{1}{5}\underset{\sim}{M}^{-1}(z)[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(z-5)] \quad . \tag{3.8}$$

The first row of $\underset{\sim}{G}$ is composed of matrices $\underset{\sim}{B}(x)$:

$$\underset{\sim}{B}(x) = \begin{bmatrix} b_{11}(x) & b_{21}(x) \\ b_{12}(x) & b_{22}(x) \end{bmatrix} \tag{3.9}$$

where $b_{ij}(x)$ is the average number of babies born during the unit time interval <u>and alive</u> in region j at the end of that interval, per x to (x + 4)-year-old resident of region i at the beginning of that interval. The off-diagonal elements of $\underset{\sim}{B}(x)$ are measures of the mobility of children 0 to 4 years old, who were born to x to (x + 4)-year-old parents.

It can be shown that $\underset{\sim}{B}(x)$ obeys the relationship (Rogers, 1975, pp. 120-121):

$$\underset{\sim}{B}(x) = \frac{5}{2}\underset{\sim}{L}(0)[5\hat{\underset{\sim}{\ell}}(0)]^{-1}[\underset{\sim}{F}(x) + \underset{\sim}{F}(x+5)\underset{\sim}{S}(x)]$$

whence

$$\mathbf{B}(x) = \frac{5}{4}\,[\mathbf{P}(0) + \mathbf{I}][\mathbf{F}(x) + \mathbf{F}(x+5)\mathbf{S}(x)] \qquad (3.10)$$

since

$$\mathbf{L}(0) = \frac{5}{2}\,[\hat{\boldsymbol{\ell}}(5) + \hat{\boldsymbol{\ell}}(0)] = \frac{5}{2}\,[\mathbf{P}(0) + \mathbf{I}]\,\hat{\boldsymbol{\ell}}(0) \quad ,$$

where $\mathbf{L}(0)$, $\mathbf{P}(0)$, and $\mathbf{S}(x)$ are life table statistics, and $\hat{\boldsymbol{\ell}}(0)$ is the identity matrix. $\mathbf{F}(x)$ is a diagonal matrix containing the annual regional birthrates of people aged x to x + 4. If the assumption of multiple transitions is met, $[\mathbf{P}(0) + \mathbf{I}]$ may be replaced by $2[\frac{5}{2}\,\mathbf{M}(0) + \mathbf{I}]$ and $\mathbf{B}(x)$ may be computed directly from the observed rates.

The number of births in year t from people aged x to x + 4 at t is $\mathbf{F}(x)\{\mathbf{K}^{(t)}(x)\}$. The number of births during a five year period starting at t, among people aged x to x + 4 at t, is

$$\frac{5}{2}\,[\mathbf{F}(x)\{\mathbf{K}^{(t)}(x)\} + \mathbf{F}(x+5)\{\mathbf{K}^{(t+1)}(x+5)\}]$$

$$= \frac{5}{2}\,[\mathbf{F}(x) + \mathbf{F}(x+5)\mathbf{S}(x)]\{\mathbf{K}^{(t)}(x)\} \quad .$$

Of these births, a proportions $\mathbf{L}(0)[5\hat{\boldsymbol{\ell}}(0)]^{-1}$ will be surviving in the various regions at the end of the time interval. The elements of the matrices $\mathbf{B}(x)$ and $\mathbf{S}(x)$ are given in Table 3.1. For example

$$\mathbf{B}(20) = \frac{5}{4}\,[\mathbf{P}(0) + \mathbf{I}][\mathbf{F}(20) + \mathbf{F}(25)\mathbf{S}(20)]$$

$$\begin{bmatrix} 0.321959 & 0.000802 \\ 0.007623 & 0.381933 \end{bmatrix} = \frac{5}{4} \begin{bmatrix} 1.956084 & 0.001261 \\ 0.013103 & 1.892421 \end{bmatrix} \left[\begin{bmatrix} 0.070652 & 0 \\ 0 & 0.087978 \end{bmatrix} + \begin{bmatrix} 0.063218 & 0 \\ 0 & 0.074260 \end{bmatrix} \begin{bmatrix} 0.965259 & 0.003541 \\ 0.031117 & 0.989482 \end{bmatrix} \right]$$

Table 3.1. The discrete model of multiregional demographic growth.

multiregional projection matrix
****

region slovenia
****

age	first row slovenia	r.yugos.
0	0.000000	0.000000
5	0.000171	0.000003
10	0.038234	0.001277
15	0.205783	0.007635
20	0.321959	0.007623
25	0.252356	0.004084
30	0.155327	0.001800
35	0.074685	0.000696
40	0.020771	0.000159
45	0.002433	0.000021
50	0.000715	0.000005
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

age	survivorship proportions slovenia	r.yugos.
0	0.970932	0.012622
5	0.988783	0.009392
10	0.981648	0.016308
15	0.966750	0.030062
20	0.965259	0.031117
25	0.972930	0.022849
30	0.980794	0.013638
35	0.985297	0.006854
40	0.982557	0.003827
45	0.975364	0.002925
50	0.964468	0.003098
55	0.945701	0.003512
60	0.904756	0.004072
65	0.831860	0.003743
70	0.737167	0.003589
75	0.590754	0.002983
80	0.617122	0.007229

Table 3.1. (cont'd)

region r.yugos.

age	first row slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000157
10	0.000121	0.062407
15	0.000860	0.268805
20	0.000802	0.381933
25	0.000398	0.279343
30	0.000186	0.159828
35	0.000075	0.083796
40	0.000024	0.033507
45	0.000005	0.006732
50	0.000001	0.001689
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

age	survivorship proportions slovenia	r.yugos.
0	0.001064	0.941189
5	0.000802	0.996334
10	0.002051	0.994601
15	0.003940	0.990868
20	0.003541	0.989482
25	0.002107	0.989587
30	0.001423	0.989064
35	0.001010	0.986718
40	0.000682	0.982578
45	0.000543	0.974184
50	0.000801	0.960150
55	0.000955	0.935551
60	0.000802	0.894848
65	0.000492	0.830224
70	0.000318	0.745168
75	0.000411	0.640255
80	0.000853	1.003021

3.2 The Projection Process

The demographic projection model is given by (3.1). Because of the special structure of the generalized Leslie matrix, (3.1) may be expressed in the form of two equation systems:

$$\{\underset{\sim}{K}^{(t+1)}(0)\} = \sum_{\alpha-5}^{\beta-5} \underset{\sim}{B}(x)\{\underset{\sim}{K}^{(t)}(x)\} \tag{3.11}$$

$$\{\underset{\sim}{K}^{(t+1)}(x+5)\} = \underset{\sim}{S}(x)\{\underset{\sim}{K}^{(t)}(x)\} \quad ,$$

$$\text{for } 5 \leq x \leq z-5 \quad . \tag{3.12}$$

The age- and region-specific population is projected forward in time by the equations sytems (3.11) and (3.12) using constant coefficients. The initial population is the observed base-year population. The projections are for unit time intervals of NY years (five, say) that are equal to the age-interval (Table 3.2).

Projection should not be confused with forecasting. Forecasting requires the consideration of the effects that possible future events may have on the demographic parameters. The purpose of projecting the population with a constant growth matrix is to study the future impact of current patterns of behavior.

3.3 The Stable Equivalent Population

In the long run, the age and spatial distribution of a population is independent of the current distribution, and is uniquely determined by the schedules of fertility, mortality and migration represented in the growth matrix. Therefore, if one projects the population with a constant growth matrix for a long enough period of time, then the ultimate (stable) growth ratio and the ultimate (stable) distribution are independent of the current growth rate and population distribution. For constant growth matrices, we may write

$$\{\underset{\sim}{K}^{(t)}\} = \underset{\sim}{G}^{t}\{\underset{\sim}{K}^{(0)}\} \quad . \tag{3.13}$$

Table 3.2. Multiregional population projection.

year 1961
----

population
- - - - -

age	total	slovenia	r.yugos.
0	915700.	67800.	847900.
5	979300.	74100.	905200.
10	878800.	70700.	808100.
15	677500.	60100.	617400.
20	788400.	62900.	725500.
25	840500.	66500.	774000.
30	795500.	67100.	728400.
35	696200.	62900.	633300.
40	431900.	39500.	392400.
45	485000.	47900.	437100.
50	505100.	51300.	453800.
55	435400.	46100.	389300.
60	365400.	39600.	325800.
65	260100.	29500.	230600.
70	201700.	21700.	180000.
75	135300.	14400.	120900.
80	68300.	7100.	61200.
85	42900.	3600.	39300.
total	9503000.	832800.	8670200.

percentage distribution
- - - - - - - - - - - - -

age	total	slovenia	r.yugos.
0	9.6359	8.1412	9.7795
5	10.3052	8.8977	10.4404
10	9.2476	8.4894	9.3204
15	7.1293	7.2166	7.1209
20	8.2963	7.5528	8.3677
25	8.8446	7.9851	8.9271
30	8.3710	8.0572	8.4012
35	7.3261	7.5528	7.3043
40	4.5449	4.7430	4.5258
45	5.1037	5.7517	5.0414
50	5.3152	6.1599	5.2340
55	4.5817	5.5355	4.4901
60	3.8451	4.7550	3.7577
65	2.7370	3.5423	2.6597
70	2.1225	2.6057	2.0761
75	1.4238	1.7291	1.3944
80	0.7187	0.8525	0.7059
85	0.4514	0.4323	0.4533
total	100.0000	100.0000	100.0000
m.ag	30.8458	33.3796	30.6024
sha	100.0000	8.7635	91.2365

Table 3.2 (cont'd)

year 1966

population

age	total	slovenia	r.yugos.
0	967578.	69924.	897654.
5	865621.	66731.	798890.
10	976572.	73995.	902577.
15	875950.	71060.	804890.
20	674103.	60535.	613569.
25	783110.	63284.	719826.
30	833791.	66331.	767460.
35	788197.	66848.	721349.
40	687934.	62615.	625319.
45	424793.	39079.	385715.
50	472913.	46957.	425956.
55	485716.	49841.	435875.
60	408341.	43968.	364372.
65	327792.	36090.	291703.
70	216213.	24653.	191560.
75	150262.	16054.	134208.
80	86006.	8557.	77450.
85	65870.	4434.	61436.
total	10090764.	870954.	9219810.

percentage distribution

age	total	slovenia	r.yugos.
0	9.5887	8.0284	9.7361
5	8.5783	7.6619	8.6649
10	9.6779	8.4959	9.7895
15	8.6807	8.1589	8.7300
20	6.6804	6.9504	6.6549
25	7.7607	7.2660	7.8074
30	8.2629	7.6159	8.3240
35	7.8111	7.6752	7.8239
40	6.8175	7.1892	6.7823
45	4.2097	4.4869	4.1835
50	4.6866	5.3915	4.6200
55	4.8135	5.7225	4.7276
60	4.0467	5.0483	3.9521
65	3.2484	4.1437	3.1639
70	2.1427	2.8306	2.0777
75	1.4891	1.8432	1.4556
80	0.8523	0.9824	0.8400
85	0.6528	0.5091	0.6664
total	100.0000	100.0000	100.0000
m.ag	31.6928	34.1770	31.4581
sha	100.0000	8.6312	91.3688
lam	1.061850	1.045814	1.063391
r	0.012003	0.008959	0.012293

Table 3.2 (cont'd)

year 1971

population

age	total	slovenia	r.yugos.
0	988584.	71442.	917142.
5	914590.	68846.	845745.
10	863211.	66624.	796588.
15	973400.	74488.	898911.
20	871546.	71869.	799676.
25	669603.	60604.	608999.
30	776865.	63088.	713777.
35	826120.	66149.	759971.
40	778819.	66593.	712226.
45	676614.	61949.	614665.
50	414197.	38325.	375872.
55	454757.	45630.	409127.
60	455509.	47550.	407958.
65	366309.	40073.	326237.
70	272479.	30165.	242314.
75	161067.	18235.	142833.
80	95514.	9539.	85975.
85	83092.	5346.	77746.
total	10642277.	906515.	9735762.

percentage distribution

age	total	slovenia	r.yugos.
0	9.2892	7.8809	9.4203
5	8.5939	7.5946	8.6870
10	8.1111	7.3494	8.1821
15	9.1465	8.2170	9.2331
20	8.1895	7.9281	8.2138
25	6.2919	6.6854	6.2553
30	7.2998	6.9593	7.3315
35	7.7626	7.2971	7.8060
40	7.3182	7.3461	7.3156
45	6.3578	6.8337	6.3135
50	3.8920	4.2278	3.8607
55	4.2731	5.0336	4.2023
60	4.2802	5.2454	4.1903
65	3.4420	4.4205	3.3509
70	2.5603	3.3276	2.4889
75	1.5135	2.0115	1.4671
80	0.8975	1.0523	0.8831
85	0.7808	0.5898	0.7986
total	100.0000	100.0000	100.0000
m.ag	32.4294	34.8436	32.2046
sha	100.0000	8.5181	91.4819
lam	1.054655	1.040831	1.055961
r	0.010643	0.008004	0.010890

Table 3.2 (cont'd)

year 1976

population

age	total	slovenia	r.yugos.
0	1049626.	74584.	975042.
5	934446.	70341.	864106.
10	912042.	68752.	843290.
15	860408.	67035.	793373.
20	968496.	75554.	892942.
25	865706.	72204.	793502.
30	664289.	60247.	604042.
35	769723.	62892.	706831.
40	816274.	65944.	750330.
45	765990.	65917.	700072.
50	659735.	60757.	598978.
55	398276.	37265.	361012.
60	426463.	43543.	382920.
65	408603.	43349.	365254.
70	304495.	33496.	270999.
75	202986.	22314.	180673.
80	102335.	10831.	91504.
85	92264.	5960.	86304.
total	11202156.	940982.	10261174.

percentage distribution

age	total	slovenia	r.yugos.
0	9.3699	7.9262	9.5022
5	8.3417	7.4752	8.4211
10	8.1417	7.3064	8.2183
15	7.6807	7.1239	7.7318
20	8.6456	8.0293	8.7021
25	7.7280	7.6733	7.7330
30	5.9300	6.4026	5.8867
35	6.8712	6.6836	6.8884
40	7.2868	7.0080	7.3123
45	6.8379	7.0052	6.8225
50	5.8894	6.4567	5.8373
55	3.5554	3.9602	3.5182
60	3.8070	4.6274	3.7317
65	3.6475	4.6068	3.5596
70	2.7182	3.5596	2.6410
75	1.8120	2.3713	1.7607
80	0.9135	1.1510	0.8917
85	0.8236	0.6334	0.8411
total	100.0000	100.0000	100.0000
m.ag	32.9680	35.3194	32.7524
sha	100.0000	8.4000	91.6000
lam	1.052609	1.038021	1.053967
r	0.010254	0.007463	0.010512

Table 3.2 (cont'd)

year 2001

population

age	total	slovenia	r.yugos.
0	1211954.	81001.	1130953.
5	1105866.	77972.	1027895.
10	1070518.	76680.	993837.
15	1044993.	76582.	968411.
20	1022211.	77190.	945022.
25	974547.	75638.	898909.
30	910599.	72104.	838496.
35	883050.	70384.	812666.
40	825835.	67731.	758104.
45	918860.	74725.	844135.
50	806298.	69342.	736956.
55	599763.	56291.	543472.
60	657486.	56672.	600814.
65	632777.	54501.	578276.
70	501906.	45936.	455970.
75	330384.	31869.	298515.
80	131948.	11951.	119997.
85	145656.	9124.	136532.
total	13774651.	1085692.	12688959.

percentage distribution

age	total	slovenia	r.yugos.
0	8.7984	7.4607	8.9129
5	8.0283	7.1818	8.1007
10	7.7717	7.0628	7.8323
15	7.5863	7.0537	7.6319
20	7.4210	7.1097	7.4476
25	7.0749	6.9668	7.0842
30	6.6107	6.6413	6.6081
35	6.4107	6.4829	6.4045
40	5.9953	6.2385	5.9745
45	6.6707	6.8827	6.6525
50	5.8535	6.3869	5.8079
55	4.3541	5.1848	4.2830
60	4.7732	5.2199	4.7349
65	4.5938	5.0199	4.5573
70	3.6437	4.2310	3.5934
75	2.3985	2.9354	2.3526
80	0.9579	1.1007	0.9457
85	1.0574	0.8404	1.0760
total	100.0000	100.0000	100.0000
m.ag	34.8823	36.8393	34.7148
sha	100.0000	7.8818	92.1182
lam	1.035828	1.024997	1.036766
r	0.007040	0.004938	0.007221

In the limit, we have

$$\{\underset{\sim}{K}^{(\infty)}\} = \lim_{t\to\infty} \underset{\sim}{G}^{t}\{\underset{\sim}{K}^{(0)}\} \quad , \tag{3.14}$$

with $\{\underset{\sim}{K}^{(\infty)}\}$ denoting the ultimate or <u>stable</u> population by age and region. This procedure for computing the stable population is equivalent to the power method for eigenvalue determination.

Once stability is achieved, the age by region composition of the population remains constant. All regions grow at the same constant ratio, λ. (The stable growth ratio is the dominant eigenvalue of $\underset{\sim}{G}$.) The relative stable distribution $\{\underset{\sim}{x}\}$ is the characteristic vector associated with λ, the dominant eigenvalue of $\underset{\sim}{G}$. In other words, $\{\underset{\sim}{x}\}$ is the solution of the following system:

$$\underset{\sim}{G}\{x\} = \lambda\{\underset{\sim}{x}\} \quad . \tag{3.15}$$

The eigenvector $\{\underset{\sim}{x}\}$ is unique up to a scalar; therefore we may choose $\{\underset{\sim}{x}\}$ such that its elements sum up to unity, i.e.

$$\{\underset{\sim}{1}\}'\{\underset{\sim}{x}\} = 1$$

where $\{\underset{\sim}{1}\}$ is a vector of ones.

The ultimate population is, for $\lim_{t\to\infty} = n$,

$$\{\underset{\sim}{K}^{(\infty)}\} = \underset{\sim}{G}^{n}\{\underset{\sim}{K}^{(0)}\} = \underset{\sim}{G}^{n}Y\{\underset{\sim}{x}\}$$

$$= \lambda^{n}Y\{\underset{\sim}{x}\} = \lambda^{n}\{\underset{\sim}{Y}\} \quad . \tag{3.16}$$

The scalar Y is called the <u>stable equivalent</u> of the observed population (Keyfitz, 1969; Rogers, 1975, p. 38). It is the total population which, if distributed as the stable population, would increase at the same rate and lead toward the same population as would, in the long run, the observed population under projection (3.13).

From (3.16) it follows that

$$\{\underset{\sim}{1}\}'\underset{\sim}{G}^n\{\underset{\sim}{K}^{(0)}\} = \lambda^n Y\{\underset{\sim}{1}\}'\{\underset{\sim}{x}\} = \lambda^n Y$$

where

$$Y = \frac{1}{\lambda^n}\{\underset{\sim}{1}\}'\underset{\sim}{G}^n\{\underset{\sim}{K}^{(0)}\} \quad . \tag{3.17}$$

Table 3.3 shows that the stable equivalent population of Yugoslavia is 10.718 million people, 5.58% of which reside in Slovenia and 94.42% in the Rest of Yugoslavia. The age structure of the stable population is considerably older than that of the base year (1961) population. As a consequence, the stable growth rate ($r = 0.006099$) is about half of the average current growth rate.

The stable equivalent population represents the size, age composition and regional distribution of the 1961 population that would be consistent with the observed mortality, fertility and migration schedules. Hence the concept of stable equivalent population enables one to separate the projected population into two components:

i. that part of the change due to the fundamental demographic parameters (schedules), and

ii. that part of the change due to the age and regional structure of the base-year population.

For example, the part of the projected 1966 population that is due to the 1961 demographic schedules is $e^{5r}\{\underset{\sim}{Y}\}$ or $\lambda\{\underset{\sim}{Y}\}$, and the part due to the 1961 age and regional composition is

$$\underset{\sim}{G}^5\{\underset{\sim}{K}(0)\} - e^{5r}\{Y\} \quad . \tag{3.18}$$

The percentage distribution is only one of the possible ways to express the age composition of the stable population. Another expression, which is particularly convenient for further analysis, is in terms of unit born, i.e. stable birth cohorts of a single person. This approach is analogous to the one followed in the

Table 3.3. Stable equivalent to original population.

age	total	slovenia	r.yugos.
0	941761.	45091.	896670.
5	862529.	43391.	819138.
10	834269.	42253.	792016.
15	806555.	41807.	764748.
20	778349.	42126.	736223.
25	749839.	41970.	707869.
30	721442.	41054.	680388.
35	693272.	39995.	653277.
40	664367.	38863.	625504.
45	633742.	37452.	596290.
50	599301.	35747.	563554.
55	558830.	33879.	524951.
60	508045.	31563.	476483.
65	441766.	28070.	413697.
70	356092.	22846.	333246.
75	257383.	16438.	240945.
80	159195.	9515.	149680.
85	151509.	5820.	145690.
total	10718246.	597879.	10120367.

percentage distribution

age	total	slovenia	r.yugos.
0	8.7865	7.5418	8.8601
5	8.0473	7.2574	8.0940
10	7.7836	7.0671	7.8260
15	7.5251	6.9926	7.5565
20	7.2619	7.0459	7.2747
25	6.9959	7.0198	6.9945
30	6.7310	6.8666	6.7230
35	6.4681	6.6895	6.4551
40	6.1985	6.5002	6.1806
45	5.9127	6.2642	5.8920
50	5.5914	5.9789	5.5685
55	5.2138	5.6665	5.1871
60	4.7400	5.2791	4.7082
65	4.1216	4.6949	4.0878
70	3.3223	3.8212	3.2928
75	2.4014	2.7494	2.3808
80	1.4853	1.5915	1.4790
85	1.4136	0.9734	1.4396
total	100.0000	100.0000	100.0000
m.ag	35.1368	36.7655	35.0406
sha	100.0000	5.5781	94.4219
lam	1.030967	1.030967	1.030967
r	0.006099	0.006099	0.006099

multiregional life table. Recall that $\hat{\underset{\sim}{\ell}}(x)$ denotes the number of people of exact age x by place of birth and place of residence, and $\underset{\sim}{L}(x)$ is the number of people in age group (x, x + 5) by place of birth and residence. In both measures, the number of people is expressed in unit born. Analogously, we may define matrices $\hat{\underset{\sim}{\ell}}^{(r)}(x)$ and $\underset{\sim}{L}^{(r)}(x)$, representing respectively the number of people of exact age x and in age group (x, x + 5) by place of birth and residence in a situation of stability.

The expression of the stable population in terms of unit born has an additional advantage; namely, its relation to the life table population. The stable population by place of birth and place of residence, per unit born, is given by

$$\hat{\underset{\sim}{\ell}}^{(r)}(x) = e^{-rx}\hat{\underset{\sim}{\ell}}(x) \tag{3.19}$$

and

$$\underset{\sim}{L}^{(r)}(x) = e^{-r(x+2.5)}\underset{\sim}{L}(x) \quad , \tag{3.20}$$

where, for example, in the case of the two-region model,

$$\begin{bmatrix} {}_{10}L_1^{(r)}(x) & {}_{20}L_1^{(r)}(x) \\ {}_{10}L_2^{(r)}(x) & {}_{20}L_2^{(r)}(x) \end{bmatrix} = \begin{bmatrix} e^{-r(x+2.5)}\,{}_{10}L_1(x) & e^{-r(x+2.5)}\,{}_{20}L_1(x) \\ e^{-r(x+2.5)}\,{}_{10}L_2(x) & e^{-r(x+2.5)}\,{}_{20}L_2(x) \end{bmatrix} \tag{3.21}$$

where r is the annual growth rate of the stable population, i.e. the intrinsic growth rate. The rate r only depends on the observed schedules and is independent of the observed population distribution. It is computed as follows:

$$r = \frac{1}{h}\,\ell n\;\lambda$$

with h being the age interval (5 years), and λ the dominant eigenvalue of the population growth matrix. The numerical evaluation of $\underset{\sim}{L}^{(r)}(x)$ for the system Slovenia - Rest of Yugoslavia is given in Table 3.4.

The absolute number of people in each age group by place of residence is

$$\{\underset{\sim}{K}(x)\} = e^{-r(x+2.5)}\underset{\sim}{L}(x)\{\underset{\sim}{Q}\} \quad , \tag{3.22}$$

where $\{\underset{\sim}{Q}\}$ is the stable distribution of births and to be defined in Section 7 of this report. Expression (3.22) is the numerical evaluation of the continuous formula

$$\{\underset{\sim}{K}(x)\} = e^{-rx}\underset{\sim}{\ell}(x)\{\underset{\sim}{Q}\} \quad . \tag{3.23}$$

The numerical values of $\{\underset{\sim}{K}(x)\}$ using (3.23) will be computed in Section 7. The results are comparable to Table 3.3.

At this point it is useful to stress that:

i. The life table population distribution is a special case of (3.19) and (3.20).

ii. Any stationary population, i.e. stable population with zero growth rate, is distributed according to a life table-population. Its relative distribution (in terms of unit births) is therefore independent of how fertility is reduced to replacement level (Table 3.5; Table 2.4).

iii. The column totals in Table 3.5 are the number of people in the life table population, per baby born. Adopting the "person-years lived" interpretation of $\underset{\sim}{L}(x)$, the total would be the life expectancies at birth by place of birth and place of residence

$$\underset{\sim}{e}(0) = \sum_{x} \underset{\sim}{L}(x) \quad . \tag{3.24}$$

For example, the total life expectancy of a baby girl

Table 3.4. Stable population (growth rate = 0.006099).

	initial region of cohort slovenia		
	total	slovenia	r.yugos.
0	4.848468	4.816206	0.032263
5	4.624203	4.535785	0.088418
10	4.477031	4.350264	0.126767
15	4.333519	4.142408	0.191111
20	4.189585	3.885118	0.304468
25	4.048024	3.638548	0.409476
30	3.908241	3.434558	0.473682
35	3.767927	3.268065	0.499862
40	3.623919	3.123785	0.500133
45	3.465690	2.977436	0.488254
50	3.286923	2.817111	0.469811
55	3.081772	2.635767	0.446004
60	2.831896	2.418189	0.413707
65	2.491113	2.122477	0.368636
70	2.017309	1.712745	0.304564
75	1.450846	1.224749	0.226097
80	0.845838	0.701883	0.143955
85	0.565230	0.420256	0.144974
total	57.857544	52.225361	5.632182

	initial region of cohort r.yugos.		
	total	slovenia	r.yugos.
0	4.662560	0.003104	4.659457
5	4.261472	0.007731	4.253741
10	4.121640	0.010724	4.110916
15	3.984470	0.018390	3.966079
20	3.844761	0.032404	3.812358
25	3.703362	0.043433	3.659930
30	3.562462	0.048468	3.513994
35	3.422768	0.050959	3.371809
40	3.279432	0.052004	3.227427
45	3.127837	0.051698	3.076139
50	2.957391	0.050530	2.906861
55	2.756869	0.049529	2.707340
60	2.504885	0.047940	2.456946
65	2.176725	0.043982	2.132743
70	1.754134	0.036506	1.717628
75	1.268236	0.026633	1.241604
80	0.786897	0.015755	0.771142
85	0.760418	0.010069	0.750349
total	52.936321	0.599859	52.336464

Table 3.5. Life table population.

initial region of cohort slovenia

	total	slovenia	r.yugos.
0	4.922967	4.890209	0.032759
5	4.840654	4.748097	0.092557
10	4.831723	4.694913	0.136810
15	4.821670	4.609031	0.212639
20	4.805877	4.456622	0.349255
25	4.787287	4.303031	0.484256
30	4.765104	4.187570	0.577535
35	4.736291	4.107964	0.628327
40	4.696336	4.048200	0.648136
45	4.630364	3.978028	0.652336
50	4.527514	3.880381	0.647134
55	4.376386	3.743020	0.633365
60	4.146075	3.540382	0.605693
65	3.760088	3.203669	0.556420
70	3.139221	2.665276	0.473945
75	2.327638	1.964904	0.362734
80	1.399027	1.160924	0.238103
85	0.963849	0.716635	0.247214
total	72.478081	64.898865	7.579217

initial region of cohort r.yugos.

	total	slovenia	r.yugos.
0	4.734203	0.003152	4.731051
5	4.460945	0.008093	4.452852
10	4.448177	0.011574	4.436603
15	4.433302	0.020462	4.412840
20	4.410329	0.037170	4.373158
25	4.379682	0.051365	4.328318
30	4.343516	0.059095	4.284421
35	4.302426	0.064056	4.238370
40	4.249906	0.067394	4.182512
45	4.178973	0.069071	4.109902
50	4.073606	0.069602	4.004004
55	3.914996	0.070336	3.844660
60	3.667311	0.070187	3.597124
65	3.285550	0.066386	3.219164
70	2.729683	0.056808	2.672875
75	2.034672	0.042728	1.991944
80	1.301538	0.026060	1.275479
85	1.296690	0.017170	1.279520
total	66.245514	0.810706	65.434807

in Slovenia is 72.48 years. A total of 64.90 years are expected to be lived in Slovenia and 7.57 years in the Rest of Yugoslavia.

iv. The column totals in Table 3.4 are the number of people in the stable population per baby born. If the growth rate r is positive, then the stable population is growing and the share of the births in the total population is greater than in the stationary population. Therefore, for r > 0

$$\sum_x e^{-r(x+2.5)} \underset{\sim}{L}(x) < \sum_x \underset{\sim}{L}(x)$$

or

$$\underset{\sim}{e}^{(r)}(0) < \underset{\sim}{e}(0) \quad . \tag{3.25}$$

For example, for each baby born in Slovenia, there are 57.86 persons living in Yugoslavia who were born in Slovenia. Of these, 52.23 are living in Slovenia and 5.63 in the Rest of Yugoslavia. Analogous to the expectation of life at birth-interpretation of $\underset{\sim}{e}(0)$, the matrix $\underset{\sim}{e}^{(r)}(0)$ may be considered as the <u>discounted life expectancy matrix</u>, with r being the rate of discount (Willekens, 1977). The meaning and relevance of this interpretation will be discussed in Section 6.

v. The stable equivalent number of births may be obtained from equation (3.22):

$$\{\underset{\sim}{Q}\} = [\underset{\sim}{L}^{(r)}(x)]^{-1}\{\underset{\sim}{K}(x)\} \quad ,$$

where the quantities $\{\underset{\sim}{K}(x)\}$ and $\underset{\sim}{L}^{(r)}(x)$ are given in Table 3.3 and 3.4 respectively. For example, the relation between the number of births and the number of people in the first age group is $\{\underset{\sim}{Q}\} = [\underset{\sim}{L}^{(r)}(0)]^{-1}\{\underset{\sim}{K}(0)\}$.

Equivalently, $\{\underset{\sim}{Q}\}$ may be derived by means of the following expression:

$$\{\underset{\sim}{Y}\} = \sum_x \{\underset{\sim}{K}(x)\} = \sum_x \underset{\sim}{L}^{(r)}(x)\{\underset{\sim}{Q}\} \quad .$$

Therefore

$$\{\underset{\sim}{Q}\} = [\sum_x L^{(r)}(x)]^{-1}\{\underset{\sim}{Y}\} \quad .$$

$$= [e^{(r)}(0)]^{-1}\{\underset{\sim}{Y}\} \quad .$$

In our two-region illustration, $\{\underset{\sim}{Q}\}$ is

$$\begin{bmatrix} 52.227600 & 0.599632 \\ 5.629005 & 52.336784 \end{bmatrix}^{-1} \begin{bmatrix} 597879 \\ 10120367 \end{bmatrix} = \begin{bmatrix} 9239 \\ 192376 \end{bmatrix}$$

An alternative procedure to compute the stable equivalent number of births will be presented in Section 7 of this report.

The age distribution in terms of unit born is fundamental to further demographic analysis. Fertility analysis is performed by applying age-specific fertility rates to the life table and stable age distributions. For mobility analysis, age-specific outmigration rates are used instead.

4. FERTILITY ANALYSIS

The analysis of fertility begins with the application of the fertility schedule to an age distribution. Let the diagonal matrix $\underset{\sim}{m}(x)$ contain the annual regional fertility rates of the women at exact age x, and let $\underset{\sim}{F}(x)$ be the diagonal matrix of annual regional fertility rates of age group x to x + 4, e.g., in a two-region model,

$$\underset{\sim}{F}(x) = \begin{bmatrix} F_1(x) & 0 \\ 0 & F_2(x) \end{bmatrix} . \qquad (4.1)$$

The integral of the matrices of the age-specific fertility rates over all ages is the gross reproduction rate matrix, i.e.

$$\underset{\sim}{GRR} = \int_0^{\omega} \underset{\sim}{m}(x)dx \doteq 5 \sum_x \underset{\sim}{F}(x) .$$

The GRR-matrix is a diagonal matrix with the regional gross rates of reproduction as its elements. Age-specific fertility rates for Slovenia and the Rest of Yugoslavia are given in Table 1.3 and repeated in Table 4.1. The column totals denote the regional gross reproduction rates.

Regional crude birth rates may be derived by multiplying the age-specific fertility rates by the observed population distribution, in fractions of the total, and summing over all age groups. Denoting the regional distribution of the people aged x to x + 4 by the diagonal matrix $\underset{\sim}{K}(x)$, the regional crude birth rates are given by the vector

$$\{\underset{\sim}{b}^0\} = [\sum_x \underset{\sim}{F}(x)\ \underset{\sim}{K}(x)][\sum_x \underset{\sim}{K}(x)]^{-1}\ \{\underset{\sim}{1}\} . \qquad (4.2)$$

Table 4.1. Age-specific fertility rates.

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000071	0.000067
15	0.015857	0.026458
20	0.070652	0.087978
25	0.063218	0.074260
30	0.041103	0.044290
35	0.022862	0.023532
40	0.007797	0.012051
45	0.000710	0.002151
50	0.000292	0.000714
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
grr	1.112809	1.357504

The product $\underset{\sim}{F}(x)\ \underset{\sim}{K}(x)$ represents the observed regional number of births to a mother aged x to x + 4.

4.1 The Generalized Net Maternity Function

The generalized net maternity (GNM) function is defined as the product (Rogers, 1975a, p. 93)

$$\underset{\sim}{\phi}(x) = \underset{\sim}{m}(x)\ \underset{\sim}{\hat{\ell}}(x) \quad , \tag{4.3}$$

where, for example, in a two-region model,

$$\underset{\sim}{\phi}(x) = \begin{bmatrix} {}_1\phi_1(x) & {}_2\phi_1(x) \\ & \\ {}_1\phi_2(x) & {}_2\phi_2(x) \end{bmatrix}$$

$$= \begin{bmatrix} m_1(x)\ {}_{10}\hat{\ell}_1(x) & m_1(x)\ {}_{20}\hat{\ell}_1(x) \\ & \\ m_2(x)\ {}_{10}\hat{\ell}_2(x) & m_2(x)\ {}_{20}\hat{\ell}_2(x) \end{bmatrix} .$$

An element ${}_i\phi_j(x)$ denotes the expected number of children to be born during a unit time interval in region j to a woman of exact age x, who was born in region i, and who is part of a stationary (life table) population. The fertility rates applied to this stationary population are the observed fertility rates.

Since the actual population data are usually given for five-year age groups, one normally evaluates (4.3) with the numerical approximation.

$$\underset{\sim}{\bar{\phi}}(x) = \underset{\sim}{F}(x)\ \underset{\sim}{L}(x) \tag{4.4}$$

in which the integral $\int_0^5 \underset{\sim}{m}(x + t)\ \hat{\underset{\sim}{\ell}}(x + t)dt$ is replaced by the product of $\underset{\sim}{F}(x)$ and $\underset{\sim}{L}(x)$. The numerical evaluations or the integrals of the generalized net maternity function are given in Table 4.2. They are obtained by multiplying the fertility rates of Table 4.1 by the age composition of the life table population (Table 2.4 or Table 3.5). For example, $\bar{\underset{\sim}{\phi}}(20)$ is:

$$\bar{\underset{\sim}{\phi}}(20) = \begin{bmatrix} 0.070652 & 0 \\ 0 & 0.087978 \end{bmatrix} \begin{bmatrix} 4.456622 & 0.037170 \\ 0.349255 & 4.373158 \end{bmatrix}$$

$$= \begin{bmatrix} 0.314868 & 0.002626 \\ 0.030727 & 0.384741 \end{bmatrix} .$$

The GNM function gives the number of offspring, by age, of a population that is distributed according to the life table (stationary) population, and which is subjected to the observed regional fertility schedules. The total number of offspring per unit birth is

$$\underset{\sim}{NRR} = \sum_x \bar{\underset{\sim}{\phi}}(x) \quad . \tag{4.5}$$

An element

$${}_iNRR_j = \sum_x {}_i\bar{\phi}_j(x)$$

denotes the total number of children expected to be born in region j to a woman who was born in region i, and who is a member of a life table population.* The matrix $\underset{\sim}{NRR}$ is the <u>net reproduction rate matrix</u>, and is the multiregional generalization of the Net Reproduction Rate (NRR) (Rogers, 1975a, p. 106). The elements of $\underset{\sim}{NRR}$ are the totals in Table 4.2.

The matrix $\underset{\sim}{NRR}$ gives the regional distribution of the offspring per unit birth in each region. It has been computed using unit radices. From the discussion of the life table in the previous section it is clear that a birth cohort of $\{\underset{\sim}{Q}_1\}$ would lead to a regional number of offspring, after a generation, of

$$\{\underset{\sim}{Q}_2\} = \underset{\sim}{NRR}\{\underset{\sim}{Q}_1\} \tag{4.6}$$

Note that (4.6) is a growth model of <u>generations</u>.

The GNM function contains additional useful information for fertility analysis. Define the n-th moment of the GNM function (4.3) as

$$\underset{\sim}{R}(n) = \int_{\alpha}^{\beta} x^n \, \underset{\sim}{\phi}(x) \, dx \tag{4.7}$$

where α and β are the lowest and highest reproductive ages respectively, and where, for example, in the two-region case,

$$\underset{\sim}{R}(n) = \begin{bmatrix} {}_1R_1(n) & {}_2R_1(n) \\ \\ {}_1R_2(n) & {}_2R_2(n) \end{bmatrix} .$$

*Recall that a life table population is a stationary population that would result if the mortality and migration schedules were applied to arbitrary regional radices.

Table 4.2. Integrals of generalized net maternity function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000332	0.000009
15	0.073085	0.005626
20	0.314868	0.030727
25	0.272029	0.035961
30	0.172121	0.025579
35	0.093915	0.014786
40	0.031566	0.007811
45	0.002824	0.001403
50	0.001135	0.000462
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.961875	0.122364

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000001	0.000296
15	0.000324	0.116754
20	0.002626	0.384741
25	0.003247	0.321420
30	0.002429	0.189758
35	0.001464	0.099739
40	0.000526	0.050405
45	0.000049	0.008838
50	0.000020	0.002859
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.010687	1.174811

The numerical approximation of (4.7) is

$$\bar{\underset{\sim}{R}}(n) = \sum_{\alpha-5}^{\beta-5} (x + 2.5)^n \bar{\underset{\sim}{\phi}}(x)$$

$$= \sum_{\alpha-5}^{\beta-5} (x + 2.5)^n \underset{\sim}{F}(x) \underset{\sim}{L}(x) \quad . \tag{4.8}$$

Observe that the 0-th moment, $\underset{\sim}{R}(0)$ is identical to $\underset{\sim}{NRR}$.

The 0-th, first and second moments of the GNM function of the two-region system Slovenia-Rest of Yugoslavia are given in Table 4.3. The column totals of $\underset{\sim}{R}(0)$ represent the total number of offspring per woman born in a certain region, e.g.

$${}_iR(0) = \sum_j {}_iR_j(0) \quad . \tag{4.9}$$

The row totals of $\underset{\sim}{R}(0)$ give the total number of children born in a certain region during one generation, per woman born in that region. It is the number of daughters by which a girl baby in a region is replaced. Noting that $\underset{\sim}{R}(0) = \underset{\sim}{NRR}$, the total number of children born in region j during one generation is

$$Q_{2j} = \sum_i {}_iR_j(0)\ Q_{1i} \tag{4.10}$$

and the row total of the j-th region is

$$R_j(0) = \frac{Q_{2j}}{Q_{1j}} = \sum_i \frac{Q_{1i}}{Q_{1j}}\ {}_iR_j(0) \quad . \tag{4.11}$$

The value of $R_j(0)$ depends on the radix ratio Q_{1i}/Q_{1j} of the life table population. Because the radix ratio is not unique, row totals are not given in Table 4.3.

Table 4.3. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.961875	0.010687
r.yugos.	0.122364	1.174811
total	1.084238	1.185497

1 moment

	slovenia	r.yugos.
slovenia	26.499458	0.313662
r.yugos.	3.587497	32.162094
total	30.086954	32.475758

2 moment

	slovenia	r.yugos.
slovenia	767.9428	9.6255
r.yugos.	110.8332	933.2065
total	878.7760	942.8320

Table 4.4 repeats $\underset{\sim}{R}(0)$ or $\underset{\sim}{NRR}$ and gives the dominant eigenvalue and associated eigenvectors of $\underset{\sim}{R}(0)$. The eigenvalue of $\underset{\sim}{R}(0)$, $\lambda_1[R(0)]$, reflects the net reproduction rate of the whole system or country (Rogers and Willekens, 1976c, p. 28). A life table radix ratio that yields a global NRR equal to $\lambda_1[\underset{\sim}{R}(0)]$ is given by the right eigenvector of $\underset{\sim}{R}(0)$.

The net reproduction allocation ${}_i\rho_j$ denotes the fraction of the offspring of the i-born women, born in region j.* For example,

$$ {}_1\rho_2 = \frac{{}_1NRR_2}{{}_1NRR.} = \frac{0.122364}{1.084238} = 0.1129 \quad , $$

i.e. 11.29% of the daughters of Slovenia-born women, are born in the Rest of Yugoslavia.

The moments of the GNM function give rise to other demographically meaningful statistics: the mean and the variance of the GNM function. In the single region case, the mean of the net maternity function is defined as (Keyfitz, 1968, p. 102)

$$ \mu = \frac{\sum\limits_x (x + 2.5)\ F(x)\ L(x)}{\sum\limits_x F(x)\ L(x)} = \frac{\bar{R}(1)}{\bar{R}(0)} \quad . \tag{4.12} $$

It represents the mean age of childbearing of the life table population (given the observed fertility schedule). The variance of the net maternity function is

$$ \sigma^2 = \frac{\sum\limits_x (x + 2.5 - \mu)^2\ F(x)\ L(x)}{\sum\limits_x F(x)\ L(x)} = \frac{R(2)}{R(0)} - \mu^2 \quad , \tag{4.13} $$

and represents the variance of the mean age of childbearing.

---

*The arrangements of the elements in Table 4.4 is the transpose of Table 2 in Rogers (1975b, p. 5).

Table 4.4. Spatial fertility expectancies.

net reproduction rate

	slovenia	r.yugos.
slovenia	0.961875	0.010687
r.yugos.	0.122364	1.174811
total	1.084238	1.185497
eigenvalue	1.180784	
eigenvector		
- right	1.000000	20.483990
- left	1.000000	1.789021

net reproduction allocations

	slovenia	r.yugos.
slovenia	0.887143	0.009015
r.yugos.	0.112857	0.990985
total	1.000000	1.000000

Multiregional generalizations of (4.12) and (4.13) are (Rogers, 1975a, p. 106):

$$
{}_i\mu_j = \frac{\sum_x (x + 2.5)\ F_j(x)\ {}_{i0}L_j(x)}{\sum_x F_j(x)\ {}_{i0}L_j(x)} = \frac{{}_i\bar{R}_j(1)}{{}_i\bar{R}_j(0)} \qquad (4.14)
$$

and

$$
{}_i\sigma_j^{\,2} = \frac{\sum_x (x + 2.5 - {}_i\mu_j)^2\ F_j(x)\ {}_{i0}L_j(x)}{\sum_x F_j(x)\ {}_{i0}L_j(x)} = \frac{{}_i\bar{R}_j(2)}{{}_i\bar{R}_j(0)} - {}_i\mu_j^2 \qquad (4.15)
$$

respectively.

The matrix of mean ages of childbearing of the life table population is given in Table 4.5 as Alternative 1. For example, the mean age of childbearing among Slovenia-born women who are living in the Rest of Yugoslavia is 29.32 years. The mean age for women living in Slovenia is lower, namely 27.55 years. This is consistent with the observation that mothers who have migrated are normally older.

The single-region measures (4.12) and (4.13) may be generalized to a multiregional system in a different way, one which is analogous to the extension of the single-region survivorship proportion to the multiregional survivorship matrix in the life table. The mean age of childbearing in this case is

$$
\begin{aligned}
\underset{\sim}{\mu} &= [\sum_x (x + 2.5)\ \underset{\sim}{F}(x)\ \underset{\sim}{L}(x)][\sum_x \underset{\sim}{F}(x)\ \underset{\sim}{L}(x)]^{-1} \qquad (4.16)\\
&= [\underset{\sim}{\bar{R}}(1)][\underset{\sim}{\bar{R}}(0)]^{-1}
\end{aligned}
$$

and the variance matrix is

$$
\underset{\sim}{\sigma}^2 = [\underset{\sim}{\bar{R}}(2)][\underset{\sim}{R}(0)]^{-1} - \underset{\sim}{\mu}^2 \quad . \qquad (4.17)
$$

Table 4.5. Matrices of mean ages and variances.

** alternative 1 **
****

means
-----

	slovenia	r.yugos.
slovenia	27.549805	29.350246
r.yugos.	29.318336	27.376410
total	28.434071	28.363329

variances
----

	slovenia	r.yugos.
slovenia	39.389587	39.247681
r.yugos.	46.204407	44.878601
total	42.796997	42.063141

** alternative 2 **
****

means
-----

	slovenia	r.yugos.
slovenia	27.547718	0.016397
r.yugos.	0.247326	27.374157
total	27.795044	27.390554

variances
----

	slovenia	r.yugos.
slovenia	39.382019	0.031142
r.yugos.	0.607339	44.868774
total	39.989357	44.899918

These matrices are given in Table 4.5 as Alternative 2. The average age at childbearing of a woman who conceived in Slovenia is 27.795 years. Of this total 27.548 have been lived in Slovenia and 0.247 in the Rest of Yugoslavia.

4.2 The Weighted Generalized Net Maternity Function

Thus far we have limited our discussion to the fertility analysis of a population distributed as in the multiregional life table. It is a stationary population that is generated by observed mortality and migration schedules. The life table population was augmented by observed fertility schedules to give the GNM function and the derived statistics discussed above. We now replace the life table population by the stable population, given in Table 3.4, and perform an analogous analysis. As before we assume unit birth cohorts (now birth cohorts in the stable population).

Computationally, fertility analysis in the stable population is completely analogous to the one described above. The only difference is that $\hat{\underset{\sim}{\ell}}(x)$ now is replaced by

$$\hat{\underset{\sim}{\ell}}^{(r)}(x) = e^{-rx}\,\hat{\underset{\sim}{\ell}}(x) \tag{3.19}$$

and $\underset{\sim}{L}(x)$ by

$$\underset{\sim}{L}^{(r)}(x) = e^{-r(x+2.5)}\,\underset{\sim}{L}(x) \quad . \tag{3.20}$$

Define the <u>Weighted Generalized Net Maternity (WGNM) Function</u> as the product

$$\underset{\sim}{\phi}^{(r)}(x) = \underset{\sim}{m}(x)\,\hat{\underset{\sim}{\ell}}^{(r)}(x) = e^{-rx}\,\underset{\sim}{m}(x)\,\hat{\underset{\sim}{\ell}}(x) \quad . \tag{4.18}$$

The weight applied is e^{-rx}. Since this may be considered as a discounting to birth, with r being the rate of discount, we may denote the WGNM function as a GNM function with discounting. The usefulness of the notion of discounting for demogrraphic analysis becomes clear in the treatment of the reproductive value (Section 6).

An element ${}_i\phi_j^{(r)}(x)$ denotes the expected number of children to be born in region j to an i-born woman who is of exact age x and part of the stable population. It may also be considered as the number of children discounted back to the time of birth of the mother.

The numerical approximation of (4.18) is

$$\bar{\underset{\sim}{\phi}}^{(r)}(x) = \underset{\sim}{F}(x)\ \underset{\sim}{L}^{(r)}(x) \quad , \tag{4.19}$$

and the result is given in Table 4.6. Table 4.6 is obtained by multiplying the fertility rates of Table 4.1 by the age composition of the stable population (Table 3.4). For example,

$$\bar{\underset{\sim}{\phi}}^{(r)}(20) = \begin{bmatrix} 0.070652 & 0 \\ 0 & 0.087978 \end{bmatrix} \begin{bmatrix} 3.885118 & 0.032404 \\ 0.304468 & 3.812358 \end{bmatrix}$$

$$= \begin{bmatrix} 0.274491 & 0.002289 \\ 0.026786 & 0.335403 \end{bmatrix}$$

The WGNM function gives the number of offspring, by age, of a unit birth in the stable population. Summing over all groups we get

$$\underset{\sim}{\Psi}(r) = \sum_x \bar{\underset{\sim}{\phi}}^{(r)}(x) \quad . \tag{4.20}$$

The matrix $\underset{\sim}{\Psi}(r)$ is the characteristic matrix of the multiregional population system (Rogers, 1075a, p. 93). An element ${}_i\Psi_j(r)$ denotes the total number of children expected to be born in region j to a woman born in region i, who is a member of the stable population. The characteristic matrix is the stable analogue of the NRR matrix. It gives the regional distribution of the offspring per unit birth in each region of the stable population. For example, Table 4.6 shows that a woman born in the stable population in Slovenia gives birth to a total of 0.916098 children on the average. Of them, 0.813684 are born in Slovenia and 0.102414 in the Rest of Yugoslavia.

If the stable distribution of births is $\{\underset{\sim}{Q}^s\}$, then the distribution of offspring is also $\{\underset{\sim}{Q}^s\}$ (Rogers, 1975a, p. 93):

$$\{\underset{\sim}{Q}^s\} = \underset{\sim}{\Psi}(r)\{\underset{\sim}{Q}^s\} \quad . \tag{4.21}$$

Equation (4.21) is the multiregional characteristic equation. It can be seen from (4.21) that the relative distribution of births is given by the right eigenvector of $\underset{\sim}{\Psi}(r)$. In our numerical example,

$$\{\underset{\sim}{Q}_1^s\} = \begin{bmatrix} 1 \\ \\ \\ 20.8237 \end{bmatrix} \quad , \tag{4.22}$$

where the subscript denotes "arbitrary scaling". Since the eigenvector of a matrix is fixed up to a scalar, we may choose the scaling of the eigenvector freely. The result (4.22) implies that 4.58% of the births occur in Slovenia and 95.42% in the Rest of Yugoslavia (in the observed population it was 6.91% and 93.09%, respectively).

As with the GNM function, we define the n-th moment of the WGNM function (4.18) as

Table 4.6. Integrals of weighted generalized net maternity function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000308	0.000008
15	0.065686	0.005056
20	0.274491	0.026786
25	0.230022	0.030408
30	0.141170	0.020979
35	0.074713	0.011763
40	0.024358	0.006027
45	0.002113	0.001050
50	0.000824	0.000335
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.813684	0.102414

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000001	0.000275
15	0.000292	0.104933
20	0.002289	0.335403
25	0.002746	0.271785
30	0.001992	0.155636
35	0.001165	0.079346
40	0.000406	0.038895
45	0.000037	0.006615
50	0.000015	0.002075
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.008942	0.994965

$$\underset{\sim}{R}^{(r)}(n) = \int_{\alpha}^{\beta} x^n \underset{\sim}{\phi}^{(r)}(x)\ dx$$

$$= \int_{\alpha}^{\beta} x^n e^{-rx} \underset{\sim}{\phi}(x)\ dx \quad , \qquad (4.23)$$

and evaluate it numerically as

$$\underset{\sim}{\bar{R}}^{(r)}(n) = \sum_{\alpha-5}^{\beta-5} (x + 2.5)^n \underset{\sim}{\bar{\phi}}^{(r)}(x)$$

$$= \sum_{\alpha-5}^{\beta-5} (x + 2.5)^n e^{-r(x+2.5)} \underset{\sim}{F}(x) \underset{\sim}{L}(x) \quad . \qquad (4.24)$$

The moments are given in Table 4.7. Note that the 0-th moment of the WGNM function coincides with $\underset{\sim}{\Psi}(r)$. The column totals of $\underset{\sim}{\Psi}(r)$ represent the total number of offspring in the stable population per woman by her place of birth, e.g.

$$_{i}\Psi(r) = \sum_{j} {_{i}\Psi_{j}}(r) \quad . \qquad (4.25)$$

The row totals give the total number of daughters by which a female baby is replaced in her region of birth in the stable population. It depends, of course, on the stable ratio of births:

$$\Psi_{j}(r) = \sum_{i} \frac{Q^{s}_{1i}}{Q^{s}_{1j}} {_{i}\Psi_{j}}(r) \quad , \qquad (4.26)$$

where Q^{s}_{1i} is an element of the right eigenvector of $\underset{\sim}{\Psi}(r)$.

Table 4.8 repeats the $\underset{\sim}{\Psi}(r)$ matrix. In addition, it shows the net reproduction allocations ${_{i}\rho_{j}}^{(r)}$, with

Table 4.7. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.813684	0.008942
r.yugos.	0.102414	0.994965
total	0.916098	1.003906

1 moment

	slovenia	r.yugos.
slovenia	22.223602	0.260318
r.yugos.	2.974085	26.970327
total	25.197687	27.230646

2 moment

	slovenia	r.yugos.
slovenia	638.2908	7.9234
r.yugos.	90.9879	774.3862
total	729.2787	782.3096

Table 4.8. Spatial fertility expectancies.

net reproduction rate

	slovenia	r.yugos.
slovenia	0.813684	0.008942
r.yugos.	0.102414	0.994965
total	0.916098	1.003906
eigenvalue*	0.999883	
eigenvector		
- right	1.000000	20.823744
- left	1.000000	1.818089

net reproduction allocations

	slovenia	r.yugos.
slovenia	0.888206	0.008907
r.yugos.	0.111794	0.991093
total	1.000000	1.000000

*The eigenvalue should be equal to one. Deviation is due to rounding of the intrinsic growth rate r to six decimal places. The growth rate has been computed by projecting the population growth matrix to stability.

$$ {}_i\rho_j^{(r)} = \frac{{}_i\Psi_j(r)}{{}_i\Psi(r)} \quad . \tag{4.27} $$

For example,

$$ {}_1\rho_2^{(r)} = \frac{{}_1\Psi_2(r)}{{}_1\Psi(r)} = \frac{0.102414}{0.916098} = 0.1118 $$

i.e. 11.18% of the daughters born to Slovenia-born women, are born in the Rest of Yugoslavia.

The mean and the variance of the WGNM function are given in Table 4.9. Once again, two alternative expressions are distinguished.

Alternative 1:

The matrix of mean ages of childbearing in the stable population, $\underset{\sim}{A}$, has elements:

$$ {}_iA_j = \frac{\sum\limits_x (x + 2.5)\ e^{-r(x+2.5)}\ F_j(x)\ {}_{i0}L_j(x)}{\sum\limits_x e^{-r(x+2.5)}\ F_j(x)\ {}_{i0}L_j(x)} = \frac{{}_i\bar{R}_j^{(r)}(1)}{{}_i\bar{R}_j^{(r)}(0)} \quad , \tag{4.28} $$

and the variance $\underset{\sim}{\sigma}^2$ with elements

$$ {}_i\sigma_j^{\,2} = \frac{\sum\limits_x (x + 2.5 - {}_iA_j)^2\ e^{-r(x+2.5)}\ F_j(x)\ {}_{i0}L_j(x)}{\sum\limits_x e^{-r(x+2.5)}\ F_j(x)\ {}_{i0}L_j(x)} $$

$$ = \frac{{}_i\bar{R}_j^{(r)}(2)}{{}_i\bar{R}_j^{(r)}(0)} - {}_iA_j^{\,2} \quad . \tag{4.29} $$

Table 4.9. Matrices of mean ages and variances.

** alternative 1 **

means

	slovenia	r.yugos.
slovenia	27.312317	29.112961
r.yugos.	29.039822	27.106815
total	28.176069	28.109888

variances

	slovenia	r.yugos.
slovenia	38.482666	38.556519
r.yugos.	45.120483	43.525635
total	41.801575	41.041077

** alternative 2 **

means

	slovenia	r.yugos.
slovenia	27.310278	0.016201
r.yugos.	0.243572	27.104626
total	27.553850	27.120827

variances

	slovenia	r.yugos.
slovenia	38.474976	0.033233
r.yugos.	0.622766	43.515686
total	39.097744	43.548920

Alternative 2:

$$\underset{\sim}{A} = [\sum_x (x + 2.5)\ e^{-r(x+2.5)}\ \underset{\sim}{F}(x)\ \underset{\sim}{L}(x)][\sum_x e^{-r(x+2.5)}\ \underset{\sim}{F}(x)\ \underset{\sim}{L}(x)]^{-1}$$

$$= [\underset{\sim}{\bar{R}}^{(r)}(1)][\underset{\sim}{\bar{R}}^{(r)}(0)]^{-1} \qquad (4.30)$$

$$\underset{\sim}{\sigma}^2 = [\underset{\sim}{\bar{R}}^{(r)}(2)][\underset{\sim}{\bar{R}}^{(r)}(0)]^{-1} - \underset{\sim}{A}^2 \quad . \qquad (4.31)$$

5. MOBILITY ANALYSIS

There are two alternative approaches to expressing the level of migration in a multiregional system (Rogers, 1975b). The first expresses the migration level in terms of expected durations, i.e. the fraction of an individual's lifetime that is spent in a particular region. The expectation of life at birth by place of residence is computed in the multiregional life table. The life expectancy matrix

$$\underset{\sim}{e}(0) = \begin{bmatrix} {}_1e_1(0) & {}_2e_1(0) \\ & \\ {}_1e_2(0) & {}_2e_2(0) \end{bmatrix} \qquad (5.1)$$

for the system Slovenia - Rest of Yugoslavia is given in Table 5.1. The total life expectancy of a girl born in Slovenia is 72.48 years, of which 64.90 years are expected to be lived in Slovenia (${}_1e_1(0)$) and 7.58 years in the Rest of Yugoslavia (${}_1e_2(0)$).

Expressing these expectancies as fractions of the total lifetime yields the migration levels ${}_i\theta_j$:

$${}_i\theta_j = {}_ie_j(0)/{}_ie(0) \quad . \qquad (5.2)$$

The second approach adopts a fertility perspective to migration analysis. Unlike death, migration is a recurrent event, analogous to birth. Thus, as in fertility, its level can be measured by counting the events, i.e. the number of moves an average person makes during his lifetime.* Such indices have been developed by Wilber (1963) and Long (1973) for a population

*The number of moves is defined here as the number of times a person is in another region at the end of the unit time interval. Back and forth moves during a unit interval are not counted (a similar assumption has been adopted by Wilber (1963) and Long (1973)).

Table 5.1. Expectations of life at birth.

expectations of life

	slovenia	r.yugos.
slovenia	64.898865	0.810706
r.yugos.	7.579217	65.434807
total	72.478081	66.245514
eigenvalue	67.660088	
eigenvector		
- right	1.000000	3.405951
- left	1.000000	0.364316

migration levels

	slovenia	r.yugos.
slovenia	0.895427	0.012238
r.yugos.	0.104573	0.987762
total	1.000000	1.000000

aggregated at the national level. Rogers (1975b) combines Wilber's and Long's ideas of "expected moves" with the approach generalizing the expected number of children (NRR) to a multiregional system ($\underset{\sim}{\text{NRR}}$).

As before, let $\hat{\underset{\sim}{\ell}}(x)$ be the distribution of the life table population of exact age x, and let $\underset{\sim}{L}(x)$ be the stationary life table population aged x to x + 4, by place of birth and residence. Define $\underset{\sim}{m}^{o}$ as the diagonal matrix of annual regional outmigration rates of people at exact age x, and $\underset{\sim}{M}^{o}(x)$ as the diagonal matrix of outmigration rates of people in age group x to x + 4, e.g. in a two-region system,

$$\underset{\sim}{M}^{o}(x) = \begin{bmatrix} M_1^{o}(x) & 0 \\ & \\ 0 & M_2^{o}(x) \end{bmatrix} \qquad (5.3)$$

with $M_i^{o}(x) = \sum_{j \neq i} M_{ij}(x), M_{ij}(x)$ being the age specific migration rate from region i to region j. Integration of the matrices of age-specific outmigration rates over all ages gives the <u>gross migraproduction rate matrix</u>:

$$\underset{\sim}{\text{GMR}} = \int_0^{\omega} \underset{\sim}{m}^{o}(x)\, dx \doteq 5 \sum_x \underset{\sim}{M}^{o}(x) \quad .$$

The origin-destination migration rates of the two-region system Slovenia - Rest of Yugoslavia are given in Table 1.3. Table 5.2 shows the age-specific regional total outmigration rates (the same in this two-region case). Since the system under consideration contains only two regions, $M_i^{o}(x) = M_{ij}(x)$ for $i \neq j$. The column totals multiplied by five denote the regional gross migraproduction rates.

Table 5.2. Age-specific outmigration rates.

age	slovenia	r.yugos.
0	0.002832	0.000272
5	0.002294	0.000166
10	0.001485	0.000157
15	0.005158	0.000679
20	0.007170	0.000937
25	0.005534	0.000506
30	0.003756	0.000350
35	0.001765	0.000226
40	0.001013	0.000183
45	0.000543	0.000094
50	0.000663	0.000130
55	0.000629	0.000205
60	0.000884	0.000203
65	0.000949	0.000156
70	0.000876	0.000078
75	0.001111	0.000099
80	0.000704	0.000196
85	0.001111	0.000076
gmr	0.192379	0.023573

The application of the age-specific outmigration rates to the life table and to the stable population yields, respectively, the generalized and the weighted generalized net mobility functions.

5.1 The Generalized Net Mobility Function

The <u>generalized net mobility (GM) function</u> is the product

$$\underset{\sim}{\gamma}(x) = \underset{\sim}{m}^{o}(x)\ \hat{\underset{\sim}{\ell}}(x) \quad , \tag{5.4}$$

or, in the case of a two-region system,

$$\begin{bmatrix} {}_1\gamma_1(x) & {}_2\gamma_1(x) \\ \\ {}_1\gamma_2(x) & {}_2\gamma_2(x) \end{bmatrix} = \begin{bmatrix} m_1^o(x)\,{}_{10}\hat{\ell}_1(x) & m_1^o(x)\,{}_{20}\hat{\ell}_1(x) \\ \\ m_2^o(x)\,{}_{10}\hat{\ell}_2(x) & m_2^o(x)\,{}_{20}\hat{\ell}_2(x) \end{bmatrix}$$

The element ${}_i\gamma_j(x)$ denotes the expected number of migrations out of region j, made during a unit time interval following age x, by a woman born in region i. Since the system consists only of two regions, ${}_i\gamma_j(x)$ measures the return migration of the x-year old.

The numerical evaluation of equation (5.4) is

$$\bar{\underset{\sim}{\gamma}}(x) = \underset{\sim}{M}^{o}(x)\underset{\sim}{L}(x) \quad . \tag{5.5}$$

The values of $\bar{\underset{\sim}{\gamma}}(x)$ are given in Table 5.3. The computational procedure is completely analogous to the one used in the fertility analysis. The only difference is that $\underset{\sim}{F}(x)$ of (2.4) is replaced by $\underset{\sim}{M}^{o}(x)$. For example, $\underset{\sim}{\gamma}(20)$ is

Table 5.3. Integrals of generalized net mobility function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.013848	0.000009
5	0.010893	0.000015
10	0.006973	0.000022
15	0.023774	0.000144
20	0.031954	0.000327
25	0.023812	0.000245
30	0.015727	0.000202
35	0.007249	0.000142
40	0.004099	0.000119
45	0.002159	0.000061
50	0.002572	0.000084
55	0.002355	0.000130
60	0.003129	0.000123
65	0.003041	0.000087
70	0.002334	0.000037
75	0.002183	0.000036
80	0.000818	0.000047
85	0.000796	0.000019
total	0.157716	0.001849

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000009	0.001289
5	0.000019	0.000738
10	0.000017	0.000697
15	0.000106	0.002995
20	0.000267	0.004099
25	0.000284	0.002192
30	0.000222	0.001500
35	0.000113	0.000957
40	0.000068	0.000767
45	0.000037	0.000386
50	0.000046	0.000521
55	0.000044	0.000790
60	0.000062	0.000729
65	0.000063	0.000503
70	0.000050	0.000208
75	0.000047	0.000198
80	0.000018	0.000250
85	0.000019	0.000098
total	0.001492	0.018915

$$\bar{\underset{\sim}{\gamma}}(20) = \begin{bmatrix} 0.007170 & 0 \\ 0 & 0.000937 \end{bmatrix} \begin{bmatrix} 4.456622 & 0.037170 \\ 0.349255 & 4.373158 \end{bmatrix}$$

$$= \begin{bmatrix} 0.031954 & 0.000267 \\ 0.000327 & 0.004099 \end{bmatrix}$$

The expected number of migrations an individual makes during his lifetime is given by the summation of $\bar{\underset{\sim}{\gamma}}(x)$ over all x. The result is the net migraproduction matrix (Rogers, 1975b, p. 8):

$$\underset{\sim}{NMR} = \sum_x \bar{\underset{\sim}{\gamma}}(x) \tag{5.6}$$

where, in the case of a two-region system,

$$\underset{\sim}{NMR} = \begin{array}{c} \begin{bmatrix} {}_1NMR_1 & {}_2NMR_1 \\ {}_1NMR_2 & {}_2NMR_2 \end{bmatrix} \\ \begin{array}{cc} \overline{{}_1NMR_{.}} & \overline{{}_2NMR_{.}} \end{array} \end{array}$$

The column sum ${}_iNMR$ denotes the total expected number of migrations to be made by a person born in region i. Some of these, i.e. ${}_iNMR_j$, migrations are made out of region j. In other words, ${}_iNMR_j$ denotes the number of times a person born in region i is expected to leave region j. The total number of migrations expected to be made by the current birth cohorts out of region j is of course

$$E_j = \sum_i {}_i NMR_j Q_{1i}$$

or in matrix notation

$$\{\underset{\sim}{E}\} = \underset{\sim}{NMR} \ \{\underset{\sim}{Q}_1\} \qquad (5.7)$$

The moments of the GM-function are completely analogous to those of the GNM-function. The n-th moment of the GM-function is defined as:

$$\underset{\sim}{D}(n) = \int_0^{\omega} x^n \ \underset{\sim}{\gamma}(x) \ dx \qquad (5.8)$$

where ω is the highest age of the population. The numerical approximation of (5.8) is:

$$\bar{\underset{\sim}{D}}(n) = \sum_{x=0}^{z-5} (x + 2.5)^n \ \bar{\underset{\sim}{Y}}(x)$$

$$= \sum_{x-0}^{z-5} (x + 2.5)^n \ \underset{\sim}{M}^o(x) \ \underset{\sim}{L}(x) \qquad (5.9)$$

with z being the highest age in the discrete case and z-5 the starting age of the highest age group.

The moments of the GM-function are contained in Table 5.4. The zeroth moment, $\bar{\underset{\sim}{D}}(0)$, is identical to the migraproduction matrix, which is given in Table 5.5 together with the migraproduction allocations. The row sums of $\bar{\underset{\sim}{D}}(0)$ represent the elements of $\{\underset{\sim}{E}\}$ for the case of unit regional radices. The net migraproduction allocation ${}_i\varepsilon_j$ denotes the fraction of the migrations made by an i born individual, that are out of region j. For example,

$${}_1\varepsilon_2 = \frac{{}_1NMR_2}{{}_1NMR_.} = \frac{0.001849}{0.159569} = 0.0116 \quad .$$

Table 5.4. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.157716	0.001492
r.yugos.	0.001849	0.018915
total	0.159566	0.020407

1 moment

	slovenia	r.yugos.
slovenia	4.199835	0.055735
r.yugos.	0.073292	0.569751
total	4.273128	0.625486

2 moment

	slovenia	r.yugos.
slovenia	160.327911	2.607642
r.yugos.	3.575156	23.908083
total	163.903061	26.515726

Table 5.5. Spatial migration expectancies.

net migraproduction rate

	slovenia	r.yugos.
slovenia	0.157716	0.001492
r.yugos.	0.001849	0.018915
total	0.159566	0.020407
eigenvalue	0.157736	
eigenvector		
- right	1.000000	0.013320
- left	1.000000	0.010746

net migraproduction allocations

	slovenia	r.yugos.
slovenia	0.988411	0.073101
r.yugos.	0.011589	0.926899
total	1.000000	1.000000

The mean and the variance of the GM-function are given by formulas (4.14) to (4.17) in which $F_j(x)$ is replaced by $M_j^o(x)$ and $\underset{\sim}{F}(x)$ by $\underset{\sim}{M}^o(x)$.

Table 5.6 lists the means and variances of the generalized mobility function.

5.2 The Weighted Generalized Net Mobility Function

Mobility analysis of the stable population leads to the concept of the weighted generalized net mobility (WGM) function. The WGM-function is estimated by replacing the life table population in (5.4) and (5.5) by the stable population.

$$\underset{\sim}{\gamma}^{(r)}(x) = \underset{\sim}{m}^o(x)\; e^{-rx}\; \underset{\sim}{\hat{\ell}}(x) \tag{5.10}$$

and

$$\underset{\sim}{\bar{\gamma}}^{(r)}(x) = \underset{\sim}{M}^o(x)\; e^{-r(x+2.5)}\; \underset{\sim}{L}(x) \quad . \tag{5.11}$$

The weights are e^{-rx} and $e^{-r(x+2.5)}$, respectively. The numerical values of $\underset{\sim}{\bar{\gamma}}^{(r)}(x)$ are given in Table 5.7. Summation of $\underset{\sim}{\bar{\gamma}}^{(r)}(x)$ over all x yields the characteristic mobility matrix $\underset{\sim}{\Gamma}(r)$:

$$\underset{\sim}{\Gamma}(r) = \sum_x \underset{\sim}{\bar{\gamma}}^{(r)}(x) \quad . \tag{5.12}$$

The element ${}_i\Gamma_j(r)$ denotes the average number of migrations out of region j in the stable population that an i-born person is expected to make during his lifetime. The right eigenvector of $\underset{\sim}{\Gamma}(r)$ represents the regional distribution of births that would result in an equal distribution of the outmigrants. In other words, if the births were distributed according to the right eigenvector of $\Gamma(r)$, $\{\underset{\sim}{z}\}$ say, then the relative regional distribution of the migrants and the births are the same. This can easily be seen by writing the characteristic equation

Table 5.6. Matrices of mean ages and variances.

** alternative 1 **

means

	slovenia	r.yugos.
slovenia	26.629034	37.361927
r.yugos.	39.636120	30.121689
total	33.132576	33.741810

variances

	slovenia	r.yugos.
slovenia	307.4528	352.1174
r.yugos.	362.4001	356.6597
total	334.9265	354.3385

** alternative 2 **

means

	slovenia	r.yugos.
slovenia	26.619102	0.847250
r.yugos.	0.111655	30.112883
total	26.730757	30.960133

variances

	slovenia	r.yugos.
slovenia	307.2101	9.6760
r.yugos.	1.5218	356.4759
total	308.7319	366.1519

Table 5.7. Integrals of weighted generalized net mobility function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.013639	0.000009
5	0.010406	0.000015
10	0.006461	0.000020
15	0.021367	0.000130
20	0.027857	0.000285
25	0.020135	0.000207
30	0.012899	0.000166
35	0.005767	0.000113
40	0.003163	0.000092
45	0.001616	0.000046
50	0.001867	0.000061
55	0.001658	0.000092
60	0.002137	0.000084
65	0.002015	0.000058
70	0.001500	0.000024
75	0.001361	0.000022
80	0.000494	0.000028
85	0.000467	0.000011
total	0.134803	0.001462

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000009	0.001269
5	0.000018	0.000705
10	0.000016	0.000646
15	0.000095	0.002692
20	0.000232	0.003573
25	0.000240	0.001854
30	0.000182	0.001230
35	0.000090	0.000761
40	0.000053	0.000592
45	0.000028	0.000289
50	0.000033	0.000378
55	0.000031	0.000556
60	0.000042	0.000498
65	0.000042	0.000333
70	0.000032	0.000134
75	0.000030	0.000123
80	0.000011	0.000151
85	0.000011	0.000057
total	0.001195	0.015841

$$\lambda[\underset{\sim}{\Gamma}(r)]\ \{\underset{\sim}{z}\} = \underset{\sim}{\Gamma}(r)\ \{\underset{\sim}{z}\} \tag{5.13}$$

$$\lambda[\underset{\sim}{\Gamma}(r)]\ \{\underset{\sim}{z}\} = [\sum_x \underset{\sim}{M}^o(x)\ e^{-r(x+2.5)}\ \underset{\sim}{L}(x)]\ \{\underset{\sim}{z}\}$$

$$= \sum \underset{\sim}{M}^o(x)\ \underset{\sim}{L}^{(r)}(x)\ \{\underset{\sim}{z}\}$$

where $\underset{\sim}{L}^{(r)}(x)$ is the distribution of the age group x to x + 4 in the stable population, by place of residence and by place of birth, and $\lambda[\underset{\sim}{\Gamma}(r)]$ is the dominant eigenvalue of $\underset{\sim}{\Gamma}(r)$. In our numerical example, (5.13) is:

$$0.1348\ .\begin{bmatrix} 1.0000 \\ \\ 0.0123 \end{bmatrix} = \begin{bmatrix} 0.1348 & 0.0012 \\ \\ 0.0015 & 0.0158 \end{bmatrix} \begin{bmatrix} 1.0000 \\ \\ 0.0123 \end{bmatrix} .$$

At stability, the migrants have not only the same relative regional distribution as the births, but they also are proportional to the number of births. If the vector of births is $\{\underset{\sim}{Q}^m\}$, with elements proportional to $\{\underset{\sim}{z}\}$, then the vector of migrants $\{\underset{\sim}{Z}\}$ is:

$$\{\underset{\sim}{Z}\} = \underset{\sim}{\Gamma}(r)\{\underset{\sim}{Q}^m\} = \lambda[\underset{\sim}{\Gamma}(r)]\{\underset{\sim}{Q}^m\}$$

For the system Slovenia - Rest of Yugoslavia $\lambda[\underset{\sim}{\Gamma}(r)]$ = 0.134823, i.e. the number of migrants is 13 percent of the number of births. In other words, if the births are distributed according to $\{\underset{\sim}{Q}^m\}$, then the number of people leaving Slovenia during one generation (independent of where they are born) is 13% of the births in Slovenia in the beginning of this generation.

The moments of the WGM-function are defined in a manner analogous to (4.23):

$$\underset{\sim}{D}^{(r)}(n) = \int_0^{\omega} x^n \underset{\sim}{\gamma}^{(r)}(x)\ dx = \int_0^{\omega} x^n e^{-rx} \underset{\sim}{\gamma}(x)\ dx$$

and

$$\bar{\underset{\sim}{D}}^{(r)}(n) = \sum_0^{z-5} x^n \bar{\underset{\sim}{\gamma}}^{(r)}(x) = \sum_0^{z-5} x^n e^{-r(x+2.5)} \bar{\underset{\sim}{\gamma}}(x) \quad .$$

They are given in Table 5.8. The zeroth moment is of course equal to $\underset{\sim}{\Gamma}(r)$, which is repeated in Table 5.9. The mean and variance of the WGM-function are derived in a manner analogous to equations (4.28) to (4.31) and are set out in Table 5.10. Finally, the discounted life expectancy matrix is presented in Table 5.11.

Table 5.8. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.134808	0.001195
r.yugos.	0.001462	0.015841
total	0.136270	0.017037

1 moment

	slovenia	r.yugos.
slovenia	3.352751	0.042231
r.yugos.	0.054825	0.444553
total	3.407576	0.486785

2 moment

	slovenia	r.yugos.
slovenia	119.799019	1.867092
r.yugos.	2.545094	17.526709
total	122.344116	19.393801

Table 5.9. Spatial migration expectancies.

net migraproduction rate

	slovenia	r.yugos.
slovenia	0.134808	0.001195
r.yugos.	0.001462	0.015841
total	0.136270	0.017037
eigenvalue	0.134823	
eigenvector		
- right	1.000000	0.012284
- left	1.000000	0.010046

net migraproduction allocations

	slovenia	r.yugos.
slovenia	0.989274	0.070159
r.yugos.	0.010726	0.929841

Table 5.10. Matrices of mean ages and variances.

** alternative 1 **
****

means

	slovenia	r.yugos.
slovenia	24.870487	35.331787
r.yugos.	37.510452	28.062807
total	31.190470	31.697298

variances

	slovenia	r.yugos.
slovenia	270.1201	313.7201
r.yugos.	334.2782	318.8672
total	302.1992	316.2936

** alternative 2 **
****

means

	slovenia	r.yugos.
slovenia	24.861923	0.789983
r.yugos.	0.102515	28.055071
total	24.964439	28.845053

variances

	slovenia	r.yugos.
slovenia	269.9137	9.0477
r.yugos.	1.4647	318.7004
total	271.3784	327.7481

Table 5.11. Discounted life expectancies at birth.

expectations of life

	slovenia	r.yugos.
slovenia	52.225361	0.599859
r.yugos.	5.632182	52.336464
total	57.857544	52.936321
eigenvalue	54.119823	
eigenvector		
- right	1.000000	3.158183
- left	1.000000	0.336364

migration levels

	slovenia	r.yugos.
slovenia	0.902654	0.011332
r.yugos.	0.097346	0.988668
total	1.000000	1.000000

6. FERTILITY ANALYSIS: CONTINUED

In this section we approach fertility analysis from a somewhat different perspective. Although the starting point is the net reproduction rate matrix (NRR) and the characteristic matrix ($\underset{\sim}{\Psi}(r)$ or $\underset{\sim}{R}^{(r)}(0)$), the interpretation is different. This allows us to derive additional useful information regarding the fertility behavior of the population.

Recall that both NRR and $\underset{\sim}{\Psi}(r)$ represent the regional distribution of the offspring by place of birth of the mother. The matrix NRR refers to a life table population and $\underset{\sim}{\Psi}(r)$ to a stable population. The intrinsic or stable growth rate is r. In equation (4.18), the rate r also may be considered to be a rate of discount. Introducing the notion of discounting, and hence a time preference to the fact of having children, adds an interesting new dimension to fertility analysis.

The central concept here is the reproductive value. It has been developed by Fisher (1929), and studied by Goodman (1967, 1971), Keyfitz (1975) and others. For a reformulation of the concept and a generalization to multiregional demographic systems, see Rogers (1975), Rogers and Willekens (1976b), and Willekens (1977). In this study we highlight only a few important elements of the theory of spatial reproductive value (Section 6.1), and then focus on computational algorithms (Section 6.2).

6.1 The Theory of the Spatial Reproductive Value

Fisher (1929) looks at life as a debt one has incurred at birth, and at the offspring of a child as the repayment of this debt. Let the debt or loan incurred at birth be equal to unity. At stability, the present value of the subsequent repayment must equal the debt, i.e.,

$$1 = \int_0^\infty e^{-ra}\, m(a)\hat{\ell}(a)da = \Psi(r) \tag{6.1}$$

where $m(a)\hat{\ell}(a)da$ is the expected number of children to be born between ages a and a + da to a baby born in a life table population and following the observed fertility schedule, and r is the rate of discount. Equation (6.1) is of course identical to the characteristic equation of a single-region population system.

The multiregional counterpart of (6.1) is

$$\{\underset{\sim}{Q}^{s}\} = \underset{\sim}{\Psi}(r)\ \{\underset{\sim}{Q}^{s}\} \quad , \tag{6.2}$$

where $\{\underset{\sim}{Q}^{s}\}$ is the right eigenvector associated with the dominant eigenvalue of $\underset{\sim}{\Psi}(r)$. An alternative generalization of (6.1) is

$$\{\underset{\sim}{\nu}(0)\}' = \{\underset{\sim}{\nu}(0)\}'\underset{\sim}{\Psi}(r) \tag{6.3}$$

where $\{\underset{\sim}{\nu}(0)\}'$ is the corresponding left eigenvector of $\underset{\sim}{\Psi}(r)$ and where the prime denotes transposition.

Both formulations, (6.2) and (6.3), have their demographic significance. Equation (6.2) has already been considered in Section 2 of this paper. The eigenvector $\{\underset{\sim}{Q}^{s}\}$ gives the regional distribution of births in the stable population. Following the investment approach to life and childbearing, $\{\underset{\sim}{Q}^{s}\}$ denotes that spatial distribution of the investments (or births) which makes the intrinsic rate of return of each investment equal to r, the equilibrium rate of return.

Whereas $\{\underset{\sim}{Q}^{s}\}$ denotes the <u>number</u> of births, the left eigenvector $\{\underset{\sim}{\nu}(0)\}'$ represents the marginal <u>value</u> of a 0-year old girl. The value is measured in terms of a contribution to the ultimate population of the demographic system. It reflects the capacity to produce new life. (Note that, since the model we consider is linear, the marginal value of one birth is equal to its average value.)

Exploring the investment approach to fertility analysis a little further, we note that if the regional distribution of births is $\{\underset{\sim}{Q}^s\}$, then the present value of the offspring must also equal $\{\underset{\sim}{Q}^s\}$ (equation (6.2)). This implies that

$$Q_i^s = \sum_j {}_j\Psi_i(r)\ Q_j^s \quad . \tag{6.4}$$

In each region, the discounted number of offspring must be equal to the current number of births. In other words, each region must pay back the debt it has incurred by receiving Q_i^s births. A part of this debt is paid back by people born in another region. People born in region j, for example, contribute a total of ${}_jNNR_i\ Q_j^s$ to region i, which has a discounted value of ${}_j\Psi_i\ Q_j^s$. Recall that in the numerical illustration of Slovenia - Rest of Yugoslavia,

$$\underset{\sim}{NRR} = \begin{bmatrix} 0.9619 & 0.0107 \\ & \\ 0.1224 & 1.1748 \end{bmatrix}$$

Equation (6.2) is

$$\begin{bmatrix} 1.0000 \\ \\ 20.8237 \end{bmatrix} = \begin{bmatrix} 0.8137 & 0.0089 \\ & \\ 0.1024 & 0,9950 \end{bmatrix} \begin{bmatrix} 1.0000 \\ \\ 20.8237 \end{bmatrix}$$

Therefore, a baby born in Slovenia is replaced by an average of

$$0.9619 *1.0000 + 0.0107 * 20.8237 = 1.1844$$

babies born in the stable population. An average of 0.9619 babies will be born to mothers who are born in Slovenia themselves, and 0.2225 will be born to mothers born in the Rest of Yugoslavia. The present value of 0.9619 babies is 0.8137 and of 0.2225 babies is

0.0089 * 20.8237 = 0.1862. Hence the average present value of a baby born in Slovenia to a Slovenian-born woman is

$$\frac{0.8137}{0.9619} = 0.8459 \quad ,$$

while that of a baby born in Slovenia to a Rest of Yugoslavia-born woman is

$$\frac{0.9950}{1.1748} = 0.8469 \quad .$$

The deviation is explained by the difference in mean ages at childbearing in the stable population and in the stationary population.

Equation(6.2) expresses births in one generation as a function of the number of births in the previous generation. It denotes the number of daughters by which a woman is replaced in the stable population, or, alternatively, the present value of the daughters replacing a woman, under the mortality and migration regime given by the life table. The regional distribution of births is consistent with the given fertility, mortality and migration schedules and with the growth rate or rate of discount, r. Since these schedules differ from on region to another, whereas r is unique, a birth in a less fertile region contributes less to the sustainment of the overall r than a birth in a highly fertile area. The value of a birth for sustaining r depends on the capacity of the 0-year old to produce new lives. This capacity is measured by the <u>reproductive value</u>.

The vector $\{\underset{\sim}{\nu}(0)\}'$ denotes the reproductive value of a baby or a 0-year old girl, by region of birth. If the reproductive value of a 0-year old in region i is $\nu_i(0)$, then the <u>value</u> of the discounted number of offspring must also be $\nu_i(0)$, which, for a two-region system, gives:

$$\nu_i(0) = {}_i(0)\ {}_i\Psi_i(r) + \nu_j(0)\ {}_i\Psi_j(r) \quad ,$$

or

$$\nu_i(0) = \sum_x e^{-r(x+2.5)} \nu_i(0)\ F_i(x)\ {}_iL_i(x) +$$

$$\sum_x e^{-r(x+2.5)} \nu_j(0)\ F_j(x)\ {}_iL_j(x) \quad , \tag{6.5}$$

Equation (6.5) suggests an equivalent formulation: the present worth of the reproductive value of the offspring must equal the reproductive value of the 0-year old. If $\nu_i(0)$ represents the value (cost) of the life invested in an individual, then that individual must pay back the value of this investment. Since $\nu_i(0) \neq \nu_j(0)$, $\sum_j {}_i\Psi_j(r) \neq 1$, which means that the discounted number of offspring of an individual does not have to be exactly unity.

Consider the Slovenia - Rest of Yugoslavia example. The matrix $\underset{\sim}{\Psi}(r)$ is given in Table 4.8. The left eigenvector is

$$\{\underset{\sim}{\nu}(0)\} = \begin{bmatrix} 1.0000 \\ \\ 1.8181 \end{bmatrix} \tag{6.6}$$

and equation (6.3) becomes

$$[1.0000\ \ 1.8181] = [1.0000\ \ 1.8181] \begin{bmatrix} 0.8137 & 0.0089 \\ \\ 0.1024 & 0.9950 \end{bmatrix}$$

If the reproductive value of a 0-year old in Slovenia is unity, then the reproductive value of the baby in the Rest of Yugoslavia is 1.818. Any other scaling may be used since the eigenvector is fixed up to a scalar. Throughout this paper, the regional reproductive values are scaled such that $\nu_1(0) = 1$.

Note that the discounted number of daughters of a Slovenian-born girl is 0.916098, i.e. less than unity. Therefore, she does not replace herself by one child (discounted). The value of the offspring, however, is equal to her reproductive value at birth:

$$\nu_1(0) = 1.000 = 1.000 * 0.814 + 1.818 * 0.102 \quad .$$

6.2 The Computation of the Spatial Reproductive Value

The above interpretation of (6.3) suggests the question: what is the productive capacity for a girl aged x? The answer is the expected number of subsequent children discounted back to age x and weighted for the region of birth. The vector of reproductive values of x-year old women, differentiated by region of residence, is

$$\{\underset{\sim}{\nu}(x)\}' = \{\underset{\sim}{\nu}(0)\}' \int_x^{\infty} e^{-r(a-x)} \underset{\sim}{m}(a) \; \hat{\underset{\sim}{\ell}}(a) \, da \; [\hat{\underset{\sim}{\ell}}(x)]^{-1}$$

$$= \{\underset{\sim}{\nu}(0)\}' \underset{\sim}{n}(x), \text{ say.} \qquad (6.7)$$

For example, in a two-region case, the matrix

$$\underset{\sim}{n}(x) = \begin{bmatrix} n_{11}(x) & n_{21}(x) \\ & \\ n_{12}(x) & n_{22}(x) \end{bmatrix} \qquad (6.8)$$

represents the expected total number of female offspring per woman at age x, discounted back to age x. The element $n_{ij}(x)$ gives the discounted number of daughters to be born in region j to a woman now x years of age and a resident of region i.

There exist two approaches to evaluate (6.2) and (6.7) numerically. The first evaluates the reproductive values at exact age x:

$$\{\underset{\sim}{\nu}(x)\}' = \{\underset{\sim}{\nu}(0)\}' \sum_{a=x}^{\beta-5} [e^{-r(a+2.5-x)} \underset{\sim}{F}(a)\underset{\sim}{L}(a)][\hat{\underset{\sim}{\ell}}(x)]^{-1} \tag{6.9}$$

$$= \{\underset{\sim}{\nu}(0)\}' \bar{\underset{\sim}{n}}_x \quad \text{, say} \tag{6.10}$$

Both $\bar{\underset{\sim}{n}}_x$ and $\{\underset{\sim}{\nu}(x)\}$ refer to exact age x. The values of $\bar{\underset{\sim}{n}}_x$ for Slovenia - Rest of Yugoslavia are given in Table 6.1. For example, the discounted number of female descendants of a woman living in Slovenia and 10 years old is 1.0020. A total of 0.9168 are expected to be born in Slovenia and 0.0852 in the Rest of Yugoslavia. On the other hand, a woman of the same age in the Rest of Yugoslavia has an expected discounted number of daughters of 1.1984. Because of the low migration level out of the Rest of Yugoslavia and the relatively low fertility in Slovenia, an average of only 0.0087 daughters will be born to these women in Slovenia.

Reproductive values by age, $\{\underset{\sim}{\nu}(x)\}$ are presented in Table 6.2. For example, the reproductive value of 10-year old girls is

$$\{\underset{\sim}{\nu}(10)\}' = \{\underset{\sim}{\nu}(0)\}' \bar{\underset{\sim}{n}}_{10} \quad ,$$

or

$$[1.0717 \;\; 2.1718] = [1.000 \;\; 1.8181] \begin{bmatrix} 0.9168 & 0.0087 \\ 0.0852 & 1.1898 \end{bmatrix}$$

Note that $\bar{\underset{\sim}{n}}_o$ is identical to the characteristic matrix $\underset{\sim}{\Psi}(r)$.

Table 6.1. Discounted number of offspring per person of exact age x.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	0.916098	0.813684	0.102414
5	0.971974	0.877290	0.094684
10	1.002008	0.916766	0.085242
15	1.032697	0.953191	0.079506
20	0.981935	0.929147	0.052788
25	0.652050	0.630546	0.021505
30	0.351121	0.344535	0.006587
35	0.153904	0.152560	0.001344
40	0.042955	0.042713	0.000242
45	0.004837	0.004808	0.000028
50	0.001425	0.001419	0.000006
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.003906	0.008942	0.994965
5	1.158388	0.009091	1.149298
10	1.198426	0.008655	1.189770
15	1.238313	0.008205	1.230108
20	1.148223	0.005268	1.142955
25	0.743625	0.001748	0.741878
30	0.394425	0.000555	0.393871
35	0.184442	0.000131	0.184311
40	0.072078	0.000023	0.072054
45	0.013777	0.000002	0.013775
50	0.003460	0.000000	0.003460
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 6.2. Spatial reproductive value per person of exact age x.

	slovenia	r.yugos.
0	1.000000	1.818089
5	1.049434	2.098617
10	1.071744	2.171764
15	1.097740	2.244651
20	1.025120	2.083262
25	0.669643	1.350547
30	0.356510	0.716647
35	0.155004	0.335225
40	0.043153	0.131025
45	0.004860	0.025046
50	0.001429	0.006290
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

The second approach computes the average reproductive value for each age group x to x + 4. Denoting this by $\{{}_5\underset{\sim}{V}_x\}$, we have thus

$$\{{}_5\underset{\sim}{V}_x\}' = \{\underset{\sim}{\nu}(0)\}' \frac{5}{2} \sum_{a=x}^{\beta-5} [e^{-r(a-x)} \underset{\sim}{F}(a) \underset{\sim}{L}(a)$$

$$+ e^{-r(a+5-x)} \underset{\sim}{F}(a+5) \underset{\sim}{L}(a+5)][\underset{\sim}{L}(x)]^{-1}$$

$$= \{\underset{\sim}{\nu}(0)\}' \frac{5}{2} \sum_{a=x}^{\beta-5} [\underset{\sim}{F}(a) + e^{-5r} \underset{\sim}{F}(a+5) \underset{\sim}{S}(a)]$$

$$e^{-r(a-x)} \underset{\sim}{L}(a)[\underset{\sim}{L}(x)]^{-1} \tag{6.11}$$

$$= \{\underset{\sim}{\nu}(0)\}' \; {}_5\underset{\sim}{N}_x \quad \text{, say.} \tag{6.12}$$

The matrix ${}_5\underset{\sim}{N}_x$ gives the discounted number of offspring per person <u>in age group x to x + 4</u>, and not the number per person at exact age x (Table 6.3). It has been shown by Willekens (1977, p.14) that ${}_5\underset{\sim}{N}_x$ may be expressed in terms of ${}_5\underset{\sim}{N}_{x+5}$:

$${}_5\underset{\sim}{N}_x = \frac{5}{2} \underset{\sim}{F}(x) + [\frac{5}{2} \underset{\sim}{F}(x+5) + {}_5\underset{\sim}{N}_{x+5}] \; e^{-5r} \; \underset{\sim}{S}(x) \quad . \tag{6.13}$$

The associated average reproductive values by age group are listed in Table 6.4.

The discounted number of offspring and the reproductive value in (6.12) and (6.13) are expressed per person in age group x to x + 4 of the life table population. To obtain an estimate of the discounted number of offspring and the reproductive value of the total observed population, we multiply ${}_5\underset{\sim}{N}_x$ and $\{{}_5\underset{\sim}{V}_x\}$ by the observed population distribution and sum over all age groups:

$$\underset{\sim}{NK} = \sum_{x=0}^{z-5} {}_5\underset{\sim}{N}_x \; \underset{\sim}{K}(x) \tag{6.14}$$

Table 6.3. Discounted number of offspring per person in age group x to x + 4.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	0.943846	0.844675	0.099171
5	0.986889	0.896788	0.090101
10	1.017243	0.934790	0.082452
15	1.007583	0.941816	0.065767
20	0.819229	0.785372	0.033857
25	0.503858	0.492105	0.011753
30	0.254236	0.251224	0.003012
35	0.099456	0.098907	0.000549
40	0.024256	0.024194	0.000062
45	0.003168	0.003158	0.000010
50	0.000731	0.000731	0.000000
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.076650	0.009032	1.067617
5	1.178234	0.008881	1.169354
10	1.218204	0.008436	1.209768
15	1.194268	0.006721	1.187548
20	0.949872	0.003167	0.946705
25	0.572512	0.000973	0.571539
30	0.291554	0.000260	0.291294
35	0.129434	0.000047	0.129387
40	0.043596	0.000004	0.043592
45	0.008750	0.000001	0.008750
50	0.001785	0.000000	0.001785
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 6.4. Spatial reproductive value per person in age group x to x + 4.

	slovenia	r.yugos.
0	1.024976	1.950056
5	1.060599	2.134870
10	1.084696	2.207902
15	1.061386	2.165788
20	0.846927	1.724361
25	0.513473	1.040082
30	0.256701	0.529859
35	0.099906	0.235284
40	0.024307	0.079257
45	0.003176	0.015908
50	0.000731	0.003245
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

and

$$\{\underset{\sim}{V}\} = \{\underset{\sim}{\nu}(0)\}' \; {}_5\underset{\sim}{N}_x \; \underset{\sim}{K}(x) = \{\underset{\sim}{\nu}(0)\}' \; \underset{\sim}{N}K \quad , \tag{6.15}$$

where $\underset{\sim}{K}(x)$ is the diagonal matrix containing the regional populations aged x to x + 4.

The value of $\underset{\sim}{N}K$ is given in Table 6.5. Under the 1961 regime of fertility, mortality and migration, the total discounted number of female offspring of Yugoslavia is 5,528,633. Of them, 382,697 or 6.92% will be born in Slovenia. However, the female residents of Slovenia will account for only 379,094 or 6.68% of the total discounted number of births. Of the ultimate discounted 382,697 female children born in Slovenia, 29,936 can be attributed to women now residing in the Rest of Yugoslavia. On the other hand, of the discounted 379,094 daughters born to the female population of Slovenia, 26,333 will be born in the Rest of Yugoslavia, and 352,761 in Slovenia.

The reproductive value of the total female population by place of residence is obtained by weighting the discounted number of offspring for the region of birth, as in (6.15). If we attach to a birth in Slovenia the reproductive value of unity, then a birth in the Rest of Yugoslavia has a reproductive value of 1.818. Adopting this scaling, the total reproductive value by region of residence is:

$$[1.0000 \quad 1.8181] \cdot \begin{bmatrix} 352,761 & 29,936 \\ 26,333 & 5,119,602 \end{bmatrix} = \begin{bmatrix} 400,637 \\ 9,337,829 \end{bmatrix}$$

The total reproductive value for the whole of Yugoslavia is (Table 6.6):

$$V = 400,637 + 9,337,829 = 9,738,466 \quad .$$

Table 6.5. Total discounted number of offspring of observed population.

	total	slovenia	r.yugos.
slovenia	382697.	352761.	29936.
r.yugos.	5145935.	26333.	5119602.
total	5528633.	379094.	5149538.

Table 6.6. Reproductive value of the total population.

	total	percentage
slovenia	400637.	4.11
r.yugos.	9337829.	95.89
total	9738466.	100.00

Note that the unit in which V is measured is the reproductive value of a birth or a 0-year old in Slovenia. The choice of the unit is arbitrary, since its only function is that of a "numeraire".

7. FURTHER STABLE POPULATION ANALYSIS

In Sections 4 and 5 of this report, we performed some introductory analyses of the fertility and migration characteristics of stationary populations. In this section, stable population analysis is advanced by means of the notions of spatial reproductive value that was developed in the previous section.

If age-specific birth, death and migration rates remain fixed, then a population exposed to these rates ultimately will evolve into a stable population whose principal characteristics are: unchanging regional age compositions and regional shares; constant regional annual rates of birth, death, and migration; and a fixed multiregional annual rate of growth that also is the annual rate of population growth in each and every region (Rogers and Willekens, 1976c, p. 12). The constant growth rate implies that births and population increase at the same rate and follow an exponential growth path. This trajectory may be expressed in terms of observed population characteristics. This is the topic of the first part of this section. The second part focuses on the calculation of the intrinsic rates of birth, death, out- and inmigration.

7.1 The Ultimate Trajectory of Births and Population

When a multiregional population system has been projected to stability, its births grow exponentially and their regional distribution remains constant. The ultimate birth trajectory then is (Willekens, 1977, p. 29):*

$$\{\underset{\sim}{Q}^{(t)}\} = e^{rt} \frac{V}{\{\underset{\sim}{\nu}(0)\}'\underset{\sim}{\kappa}\{\underset{\sim}{Q}_1\}} \{\underset{\sim}{Q}_1\} \quad , \tag{7.1}$$

where r is the stable growth rate, V is the total reproductive value of the whole population system, $\{\underset{\sim}{\nu}(0)\}'$ and $\{\underset{\sim}{Q}_1\}$ are, respectively, the left and right eigenvectors of $\underset{\sim}{\Psi}(r)$, associated

---

*The superscript of $\{\underset{\sim}{Q}^s\}$ is dropped for convenience.

with the dominant eigenvalue, and $\underset{\sim}{\kappa}$ is the matrix of mean ages of childbearing in the stable population, defined in (4.30) as:

$$\underset{\sim}{\kappa} = [\underset{\sim}{R}^{(r)}(1)][\underset{\sim}{R}^{(r)}(0)]^{-1} \quad . \tag{7.2}$$

The expression $\{\underset{\sim}{\nu}(0)\}'\underset{\sim}{\kappa}\{\underset{\sim}{Q}_1\}$ is a normalizing factor. Writing

$$\kappa = \{\underset{\sim}{\nu}(0)\}'\underset{\sim}{\kappa}\{\underset{\sim}{Q}_1\} \quad , \tag{7.3}$$

yields the simple expression for the ultimate birth trajectory:

$$\{\underset{\sim}{Q}^{(t)}\} = e^{rt} \frac{V}{\kappa} \{\underset{\sim}{Q}_1\} \quad . \tag{7.4}$$

If $\{\underset{\sim}{Q}_1\}$ is chosen such that its elements sum up to unity, then the ultimate total number of births is proportional to the total reproductive value. The total number of births is then allocated to the different regions according to $\{\underset{\sim}{Q}_1\}$.

Substituting V in (7.4) shows that the stable number of births in each region $\{\underset{\sim}{Q}^{(t)}\}$ also is a linear combination of the discounted number of offspring by region of birth (see Willekens, 1977, pp. 32-33). The stable equivalent of births is:

$$\{\underset{\sim}{Q}^{(0)}\} = \{\underset{\sim}{Q}\} = \frac{V}{\kappa} \{\underset{\sim}{Q}_1\} \quad . \tag{7.5}$$

Recall our numerical illustration. The matrix of mean ages of childbearing is given in Table 11. Since the grwoth rate r is 0.006099, the normalizing factor, (7.3), is 1054.256 (Table 7.2). The total reproductive value V has been computed to be 9,738,466; hence the stable equivalent of births is by (7.5):

$$\{\underset{\sim}{Q}\} = \frac{9,738,466}{1054.256} \begin{bmatrix} 1.0000 \\ \\ 20.8237 \end{bmatrix} = \begin{bmatrix} 9,237 \\ \\ 192,355 \end{bmatrix} \quad . \tag{7.6}$$

The total number of births is 201,592. Of this number of babies 4.58% will be born in Slovenia and 95.42% in the Rest of Yugoslavia.*

The stable equivalent population in each age group x to x + 4 is easily obtained by the formula (3.22):

$$\{\underset{\sim}{K}(x)\} = e^{-r(x+2.5)}\ \underset{\sim}{L}(x)\{\underset{\sim}{Q}\} \quad . \tag{3.22}$$

The stable equivalent of the total population is:

$$\{\underset{\sim}{Y}\} = \sum_x \{\underset{\sim}{K}(x)\} = [\sum_x e^{-r(x+2.5)}\ \underset{\sim}{L}(x)]\{\underset{\sim}{Q}\} \quad . \tag{7.7}$$

Defining

$$\sum_x e^{-r(x+2.5)}\ \underset{\sim}{L}(x) = \underset{\sim}{e}^{(r)}(x) \tag{7.8}$$

as the matrix of discounted life expectancies at birth, reduces equation (7.7) to

$$\{\underset{\sim}{Y}\} = \underset{\sim}{e}^{(r)}(0)\{\underset{\sim}{Q}\} \quad . \tag{7.9}$$

The numerical values of the stable equivalent population are given in Table 7.1. Note that those values are very close to the ones given in Table 3.3, which was obtained by projecting the observed population.**

Equations (3.22) and (7.7) demonstrate that for population analysis it is more convenient to express the relative age composition of the population in unit births instead of in fractions or percentages of the total population. The values of

*Compare this allocation with the observed number of births (205,010) and the regional distribution: 6.90% in Slovenia vs. 93.10% in the Rest of Yugoslavia.

**Minor deviations are due to rounding error.

Table 7.1. Stable equivalent of total population.

	total	slovenia	r.yugos.
0	941653.	45036.	896568.
5	862430.	43385.	819045.
10	834173.	42247.	791926.
15	806462.	41802.	764660.
20	778259.	42121.	736138.
25	749753.	41965.	707788.
30	721359.	41049.	680310.
35	693192.	39990.	653201.
40	664290.	38859.	625431.
45	633668.	37448.	596221.
50	599231.	35742.	563489.
55	558765.	33875.	524890.
60	507986.	31559.	476427.
65	441715.	28066.	413649.
70	356051.	22843.	333208.
75	257353.	16436.	240917.
80	159177.	9514.	149663.
85	151491.	5819.	145673.
total	10717010.	597806.	10119204.

percentage distribution

	total	slovenia	r.yugos.
0	8.787	7.542	8.860
5	8.047	7.257	8.094
10	7.784	7.067	7.826
15	7.525	6.993	7.557
20	7.262	7.046	7.275
25	6.996	7.020	6.995
30	6.731	6.867	6.723
35	6.468	6.690	6.455
40	6.198	6.500	6.181
45	5.913	6.264	5.892
50	5.591	5.979	5.569
55	5.214	5.666	5.187
60	4.740	5.279	4.708
65	4.122	4.695	4.088
70	3.322	3.821	3.293
75	2.401	2.749	2.381
80	1.485	1.592	1.479
85	1.414	0.973	1.440
total	100.000	100.000	100.000
share	100.000	5.578	94.422

$$e^{-r(x+2.5)} \underset{\sim}{L}(x)$$

are given in Table 3.4.

7.2 Stable Equivalents and Intrinsic Rates

The fertility, mortality and migration characteristics of a stable population may be described by a small number of parameters, namely the intrinsic rates (Rogers, 1975a, pp. 109-115). The intrinsic rates are directly related to the stable equivalents of births, deaths, and migrants. Therefore, we treat both simultaneously.

Applying the fixed age-specific schedules of fertility, mortality and migration to the stable equivalent of the population gives the stable equivalent of births, deaths and migrants. The stable equivalent of births has already been computed. Applying the fertility schedule to the population distribution of (3.22) and summing over all age groups yields, of course, the characteristic equation:

$$\{\underset{\sim}{Q}\} = \sum_x \underset{\sim}{F}(x)\{\underset{\sim}{K}(x)\}$$

$$= [\sum_x \underset{\sim}{F}(x)\ e^{-r(x+2.5)}\ \underset{\sim}{L}(x)]\{\underset{\sim}{Q}\} = \underset{\sim}{\bar{\Psi}}\{\underset{\sim}{Q}\} \quad .$$

The intrinsic birth rate of region i is the ratio between Q_i and the stable equivalent population Y_i, which may be written as (Rogers, 1975a, p. 115):

$$b_i = \frac{Q_i}{Y_i} = \frac{Q_i}{\sum_x e^{-r(x+2.5)} \sum_j {}_jL_i(x)\ Q_j}$$

$$= \frac{1}{\sum_x e^{-r(x+2.5)} \sum_j \frac{Q_j}{Q_i}\ {}_jL_i(x)} \quad .$$

The vector of intrinsic birth rates is:

$$\{\underset{\sim}{b}\} = \underset{\sim}{Y}^{-1} \{\underset{\sim}{Q}\} \quad , \tag{7.10}$$

where $\underset{\sim}{Y}$ is the diagonal matrix of stable equivalents of total populations, i.e.

$$\underset{\sim}{Y}\{\underset{\sim}{1}\} = \{\underset{\sim}{Y}\} \quad .$$

The vector $\{\underset{\sim}{b}\}$ also may be expressed as

$$\{\underset{\sim}{b}\} = \sum_{x} \underset{\sim}{F}(x)\{\underset{\sim}{C}(x)\} \quad , \tag{7.11}$$

where $\{\underset{\sim}{C}(x)\}$ denotes the age composition of the population as fractions of the total, i.e.

$$\{\underset{\sim}{C}(x)\} = \underset{\sim}{Y}^{-1}\{\underset{\sim}{K}(x)\} \quad . \tag{7.12}$$

The proportion of the regional population, which is aged x to x + 4, may be expressed as

$$\{\underset{\sim}{C}(x)\} = \underset{\sim}{Q}^{-1} \underset{\sim}{b}\ e^{-r(x+2.5)} \underset{\sim}{L}(x)\{\underset{\sim}{Q}\} \quad , \tag{7.13}$$

since by (7.10) Y^{-1} is equal to $\underset{\sim}{Q}^{-1}\underset{\sim}{b}$, where both $\underset{\sim}{Q}$ and $\underset{\sim}{b}$ are diagonal matrices. Defining $\underset{\sim}{C}(x)$ as

$$\underset{\sim}{C}(x) = \underset{\sim}{b}\ e^{-r(x+2.5)} \underset{\sim}{L}(x) \quad , \tag{7.14}$$

gives

$$\{\underset{\sim}{C}(x)\} = \underset{\sim}{Q}^{-1} \underset{\sim}{C}(x)\{\underset{\sim}{Q}\} \quad . \tag{7.15}$$

To compute the stable equivalents of deaths, outmigrants and inmigrants, we must reconsider the age-specific death and migration rates (Ledent, 1977). The deaths and outmigrants in age group x to x + 4 in a life table population are given by (Rogers and Ledent, 1976, p. 289).

$$\hat{\underset{\sim}{\ell}}(x) - \hat{\underset{\sim}{\ell}}(x + 5) = \underset{\sim}{M}(x)\ \underset{\sim}{L}(x) \quad , \tag{7.16}$$

where $\hat{\underset{\sim}{\ell}}(x)$ represents the distribution of the life table population at exact age x by place of birth and place of residence (in terms of unit born),

$\underset{\sim}{L}(x)$ is given in (2.10) and represents the distribution of the life table population aged x to x + 4 by place of birth and place of residence (in terms of unit born), and

$\underset{\sim}{M}(x)$ is the matrix (2.24).

Equation (7.16) is the discrete approximation of the continuous relation

$$\hat{\underset{\sim}{\ell}}(x) - \hat{\underset{\sim}{\ell}}(x + 5) = \int_0^5 \underset{\sim}{\mu}(x + t)\hat{\underset{\sim}{\ell}}(x + t)dt \quad , \tag{7.17}$$

where $\underset{\sim}{\mu}(x)$ is a matrix of the format $\underset{\sim}{M}(x)$. Its elements are the age-specific forces of mortality $\mu_{i\delta}(x)$ and of migration $\mu_{ij}(x)$, i.e.

$$\underset{\sim}{\mu}(x) = -\frac{1}{dx}\,[d\hat{\underset{\sim}{\ell}}(x)]\,[\hat{\underset{\sim}{\ell}}(x)]^{-1}$$

$$= -\frac{d\ \ell n\ \hat{\ell}\ (x)}{dx}$$

Equation (7.7) represents the decrements due to death and outmigrations in a stationary population. To derive the decrements in a population growing at rate r, we write

$$\hat{\underset{\sim}{\ell}}^{(r)}(x) - \hat{\underset{\sim}{\ell}}^{(r)}(x + 5) = \int_0^5 \underset{\sim}{\mu}(x + t)\ \hat{\underset{\sim}{\ell}}^{(r)}(x + t)dt \quad ,$$

with $\hat{\underset{\sim}{\ell}}^{(r)}(x) = e^{-rx}\hat{\underset{\sim}{\ell}}(x)$. Hence

$$\hat{\underset{\sim}{\ell}}^{(r)}(x) - \hat{\underset{\sim}{\ell}}^{(r)}(x+5) = \int_0^5 \underset{\sim}{\mu}(x+t)\; e^{-r(x+t)}\hat{\underset{\sim}{\ell}}(x+t)dt \quad ,$$

$$= \int_0^5 e^{-r(x+t)} d\;\hat{\underset{\sim}{\ell}}(x+t) \quad . \qquad (7.18)$$

Integration by parts yields

$$\hat{\underset{\sim}{\ell}}^{(r)}(x) - \hat{\underset{\sim}{\ell}}^{(r)}(x+5) = e^{-rx}\hat{\underset{\sim}{\ell}}(x) - e^{-r(x+5)}\hat{\underset{\sim}{\ell}}(x+5)$$

$$- r\int_0^5 e^{-r(x+t)}\hat{\underset{\sim}{\ell}}(x+t)dt \quad .$$

$$= e^{-rx}\hat{\underset{\sim}{\ell}}(x) - e^{-r(x+5)}\hat{\underset{\sim}{\ell}}(x+5)$$

$$- r\,\underset{\sim}{L}^{(r)}(x) \quad .$$

The age-specific death and outmigration rates in the stable population are given by the matrix

$$\underset{\sim}{M}^{(r)}(x) = [\hat{\underset{\sim}{\ell}}^{(r)}(x) - \hat{\underset{\sim}{\ell}}^{(r)}(x+5)][\underset{\sim}{L}^{(r)}(x)]^{-1}$$

$$= [e^{-rx}\hat{\underset{\sim}{\ell}}(x) - e^{-r(x+5)}\hat{\underset{\sim}{\ell}}(x+5) - r\underset{\sim}{L}^{(r)}(x)][\underset{\sim}{L}^{(r)}(x)]^{-1}$$

$$= [e^{-rx}\hat{\underset{\sim}{\ell}}(x) - e^{-r(x+5)}\hat{\underset{\sim}{\ell}}(x+5)][e^{-r(x+2.5)}L(x)]^{-1} - r\underset{\sim}{I} \quad ,$$

which after substitution yields*

$$\underset{\sim}{M}^{(r)}(x) = \frac{2}{5}\, e^{2.5r}[\underset{\sim}{I} - e^{-5r}\underset{\sim}{P}(x)][\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} - r\underset{\sim}{I} \quad . \quad (7.19)$$

For the last age group z, the rates are:

$$\underset{\sim}{M}^{(r)}(z) = \hat{\underset{\sim}{\ell}}^{(r)}(z)[\underset{\sim}{L}^{(r)}(z)]^{-1} - r\underset{\sim}{I}$$

$$= e^{2.5r}\hat{\underset{\sim}{\ell}}(z)[\underset{\sim}{L}(z)]^{-1} - r\underset{\sim}{I}$$

$$= e^{2.5r}\underset{\sim}{M}(z) - r\underset{\sim}{I} \quad . \quad (7.20)$$

The outmigration rates $M_{ij}^{(r)}(x)$ are contained in the off-diagonal elements of $\underset{\sim}{M}^{(r)}(x)$. The death rates $M_{i\delta}^{(r)}(x)$ are equal to the diagonal elements minus the outmigration rates, i.e. plus the off-diagonal elements in the same column.

To facilitate further analysis, define the diagonal matrix ${}^{\delta}\underset{\sim}{M}^{(r)}(x)$ of regional death rates, and the diagonal matrix ${}^{o}\underset{\sim}{M}^{(r)}(x)$ of <u>total</u> regional outmigration rates, i.e.

$${}^{o}M_{ii}^{(r)}(x) = \sum_{j \neq i} M_{ij}^{(r)}(x) \quad . \quad (7.22)$$

Let ${}^{oo}\underset{\sim}{M}^{(r)}(x)$ be the matrix of outmigration rates, i.e.

---

*Compare this with the formula for the life table (= observed) rates. Solving equation (2.25)

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)]$$

for $\underset{\sim}{M}(x)$ yields

$$\underset{\sim}{M}(x) = \frac{2}{5}[\underset{\sim}{I} - \underset{\sim}{P}(x)][\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} \quad . \quad (7.21)$$

$$
{}^{oo}\underset{\sim}{M}^{(r)} = \begin{bmatrix} 0 & M_{21}^{(r)}(x) & \dots & M_{n1}^{(r)}(x) \\ M_{12}^{(r)} & 0 & \dots & M_{n2}^{(r)}(x) \\ \vdots & & & \vdots \\ M_{1n}^{(r)} & M_{2n}^{(r)} & \dots, & 0 \end{bmatrix}
$$

(7.23)

Once consistent age-specific death and migration rates are derived, we may proceed with the computation of the stable equivalents of deaths and out- and inmigrants, and of the associated intrinsic rates. The stable equivalent of deaths is:

$$
\{\underset{\sim}{D}\} = \sum_x {}^{\delta}\underset{\sim}{M}^{(r)}(x)\{\underset{\sim}{K}(x)\}
$$

$$
= [\sum_x {}^{\delta}\underset{\sim}{M}^{(r)}(x)\ e^{-r(x+2.5)}\underset{\sim}{L}(x)]\{\underset{\sim}{Q}\} \quad . \tag{7.24}
$$

The intrinsic death rates follow immediately:

$$
\{\underset{\sim}{d}\} = \underset{\sim}{Y}^{-1}\{\underset{\sim}{D}\} \tag{7.25}
$$

or

$$
\{\underset{\sim}{d}\} = \sum_x {}^{\delta}\underset{\sim}{M}^{(r)}(x)\{\underset{\sim}{C}(x)\} \quad . \tag{7.26}
$$

The stable equivalent of the outmigrants from region i to region j is:

$$
O_{ij} = \sum_x M_{ij}^{(r)}(x)\ K_i(x) \tag{7.27}
$$

where $M_{ij}^{(r)}(x)$ is the age-specific migration rate and $K_i(x)$ is the stable population of region i aged x to x + 4. In general, we may write the origin destination flow of stable equivalent migrations as

$$\underset{\sim}{O} = \sum_x {}^{oo}\underset{\sim}{M}^{(r)}(x)\ \underset{\sim}{K}(x) \tag{7.28}$$

where ${}^{oo}\underset{\sim}{M}^{(r)}(x)$ is defined in (7.23) and $\underset{\sim}{K}(x)$ is a diagonal matrix of stable regional populations of ages x to x + 4. The outmigration rates are simply:

$$\underset{\sim}{o} = \underset{\sim}{O}\ \underset{\sim}{Y}^{-1} \tag{7.29}$$

or

$$\underset{\sim}{o} = \sum_x {}^{oo}\underset{\sim}{M}^{(r)}(x)\ \underset{\sim}{C}(x) \tag{7.30}$$

where $\underset{\sim}{C}(x) = \underset{\sim}{K}(x)\ \underset{\sim}{Y}^{-1}$, i.e. $\underset{\sim}{C}(x)\{\underset{\sim}{1}\} = \{\underset{\sim}{C}(x)\}$.

The stable equivalent of the total number of outmigrants is:

$$\{\underset{\sim}{O}\}' = \{\underset{\sim}{1}\}'\ \underset{\sim}{O} \tag{7.31}$$

and the total outmigration rates are:

$$\{\underset{\sim}{o}\}' = \{\underset{\sim}{1}\}'\ o = \{\underset{\sim}{1}\}'\ \underset{\sim}{O}\ \underset{\sim}{Y}^{-1} = \{\underset{\sim}{O}\}'\underset{\sim}{Y}^{-1} \quad . \tag{7.32}$$

An equivalent expression for (7.31) is:

$$\begin{aligned} \{\underset{\sim}{O}\}' &= \sum_x \{\underset{\sim}{1}\}'\, {}^{oo}\underset{\sim}{M}^{(r)}(x)\ \underset{\sim}{K}(x) \\ &= \sum_x \{{}^{o}\underset{\sim}{M}^{(r)}(x)\}'\underset{\sim}{K}(x) \quad , \end{aligned} \tag{7.33}$$

where $\{{}^{o}\underset{\sim}{M}^{(r)}(x)\}$ is the vector of total outmigration rates, defined in (7.22).

The stable equivalent of the total number of inmigrants by region is:

$$\{\underset{\sim}{I}\} = \underset{\sim}{O}\{\underset{\sim}{1}\} \quad . \tag{7.34}$$

and the inmigration rates are:

$$\begin{aligned}\{\underset{\sim}{i}\} &= \underset{\sim}{Y}^{-1}\{\underset{\sim}{I}\} \\ &= \underset{\sim}{Y}^{-1}\ \underset{\sim}{O}\ \{\underset{\sim}{1}\} \quad .\end{aligned} \tag{7.35}$$

The matrix $\underset{\sim}{h} = \underset{\sim}{Y}^{-1}\ \underset{\sim}{O}$ contains inmigration rates by region of origin and region of destination. An element h_{ij} describes the migrants from region i to j as a fraction of the population in j.

There exists a unique relationship between inmigration rates and outmigration rates. Since by (7.29)

$$\underset{\sim}{O} = \underset{\sim}{o}\ \underset{\sim}{Y} \quad ,$$

we have

$$\underset{\sim}{i} = \underset{\sim}{Y}^{-1}\ \underset{\sim}{o}\ \underset{\sim}{Y} \quad , \tag{7.36}$$

and the total inmigration rates

$$\{\underset{\sim}{i}\} = \underset{\sim}{i}\ \{\underset{\sim}{1}\} = \underset{\sim}{Y}^{-1}\ \underset{\sim}{o}\ \underset{\sim}{Y}\ \{\underset{\sim}{1}\} \quad . \tag{7.37}$$

The stable equivalents of births, deaths and outmigrants and inmigrants are given in Table 7.2, together with the intrinsic rates. Note that the intrinsic rates obey the following definitional relationship:

$$r = b_i - d_i - o_i + i_i \quad . \tag{7.38}$$

Thus equation (7.38) provides an independent check of the results.

Table 7.2. Stable equivalents and intrinsic rates.

	births		deaths		outmigration		immigration	
	number	rate	number	rate	number	rate	number	rate
slovenia	9237.	0.015452	7177.	0.012005	1469.	0.002457	3050.	0.005102
r.yugos.	192355.	0.019009	129048.	0.012753	3050.	0.000301	1469.	0.000145
total	201592.	0.018810	136224.	0.012711	4519.	0.000422	4519.	0.000422

stable growth rate	0.006099
normalizing factor	1054.2560

8. SPATIAL ZERO POPULATION GROWTH

The demographic system we have considered thus far is one that is characterized by constant fertility, mortality and migration schedules. The ultimate population evolving under these conditions is a stable population, with the following features: fixed age and regional structures, unchanging regional birth, death, and migration rates, and a unique and constant growth rate.

The growing public concern about rapid population increase has generated a vast literature on the social and economic impacts of high fertility and has focused attention on fertility decline as a means for relieving socio-economic problems. An immediate drop of fertility to replacement level, however, would not stop population growth. Children outnumber parents in a growing population. Consequently, the number of potential parents in the next generation will be larger than at present. This built-in tendency for continued growth causes the number of people to increase for some time before the population becomes stationary (i.e. stable, but with zero growth). The ratio by which the ultimate stationary population exceeds a current population undisturbed by migration has been studied by Keyfitz (1971).

Although population growth is an important concern, where people choose to live in the future presents issues and problems that are potentially as serious as those posed by the number of children they choose to have. A drop in fertility, for example, not only causes the population to continue to grow for a while, but, in the presence of migration, also affects the regional distribution of this population. The spatial impact of fertility reduction has been studied by Rogers and Willekens (1976a, 1976b).

The spatial momentum of zero population growth may be computed numerically and, if the initial population is stable, analytically. In the first section, the numerical approach is discussed. The analytical approach is examined in the following section.

8.1 The Numerical Approach

The numerical approach to spatial zero population growth analysis substitutes fertility schedules representing fertility at replacement level in place of observed fertility schedules. All the computations for population projection and stable population analysis are done over, and the new results are then compared with the results obtained using the original fertility schedules.

Many alternative fertility reduction schemes are possible. Some age groups may have a proportionally greater decline than others because of difference in birth control practices, or because of shifts in the patterns of marriage and divorce. Alternatively, the decline may be age-independent, i.e. the proportional decline of age-specific fertility rates is the same at all ages. Keyfitz (1971) considers a fertility drop that is age-independent. Most demographers have followed this practice and it is also adopted in the discussion that follows.

Regional differences in fertility decline are introduced through two alternative schemes:

Alternative 1: the cohort replacement alternative: in each region, the fertility of each female cohort is reduced to bare replacement level, i.e. to a level of one daughter (net) per woman born there.

Alternative 2: the proportional reduction alternative: every regional fertility schedule is reduced by the same proportion at all ages.

To derive mathematical expressions for both alternatives, recall (4.6), which may be written as:

$$\{\underset{\sim}{Q}_2\} = \underset{\sim}{R}(0)\{\underset{\sim}{Q}_1\} = [\sum_x \underset{\sim}{F}(x)\ \underset{\sim}{L}(x)]\{\underset{\sim}{Q}_1\} \quad , \qquad (8.1)$$

where $\{\underset{\sim}{Q}_1\}$ is the vector of births and $\{\underset{\sim}{Q}_2\}$ the vector of their offspring, i.e. births in the next generation. Equation (8.1) expresses the births in one generation as a function of births in the previous generation.

A multiregional population system that is growing exhibits a net reproduction matrix $\tilde{R}(0)$ with a dominant characteristic root $\lambda_1[R(0)]$ that is greater than unity. The total number of offspring per woman born in a certain region is given by the column totals of $\tilde{R}(0)$, i.e.:

$$ {}_iR(0) = \sum_j {}_iR_j(0) \quad . \tag{8.9}$$

If fertility is reduced according to the <u>cohort replacement alternative</u>, then

$$ {}_i\hat{R}(0) = 1 \qquad \text{for all } i \quad ;$$

or, in matrix form,

$$ \tilde{\hat{R}}(0)'\{\tilde{1}\} = \{\tilde{1}\} \quad . \tag{8.2}$$

This means that every woman would have a net reproduction rate of unity. The problem is now to determine by how much the observed age-specific fertility rates must be altered for this to occur.

Let γ_i be the required fertility adjustment factor for region i, i.e.

$$ {}_i\hat{R}(0) = 1 = \gamma_i \, {}_iR(0) \quad .$$

In general, we have

$$ \tilde{\hat{R}}(0) = \tilde{\gamma}\tilde{R}(0) \quad , \tag{8.3}$$

where $\tilde{\gamma}$ is a diagonal matrix of regional fertility adjustment factors. Substituting (8.3) into (8.2) gives

$$ \tilde{R}(0)'\tilde{\gamma}\{\tilde{1}\} = \{\tilde{1}\} \quad ,$$

whence

$$\{\underset{\sim}{\gamma}\} = [\underset{\sim}{R}(0)']^{-1}\{\underset{\sim}{1}\} \quad . \tag{8.4}$$

Therefore, the cohort replacement alternative yields the replacement fertility rates $\hat{\underset{\sim}{F}}(x)$;

$$\hat{\underset{\sim}{F}}(x) = \underset{\sim}{\gamma}\underset{\sim}{F}(x) \quad , \tag{8.5}$$

where $\underset{\sim}{F}(x)$ is the diagonal matrix of observed regional fertility rates of age group x to x + 4, and $\underset{\sim}{\gamma}$ is the diagonal matrix with the elements of $\{\underset{\sim}{\gamma}\}$ in the diagonal.

Recall our numerical illustration: the two-region system of Slovenia and the Rest of Yugoslavia. The matrix of fertility adjustment factors is given in Table 8.1. Since the women of both regions originally had a net reproduction rate greater than unity, the fertility adjustment factors are less than one, causing a fertility drop in both regions. In Slovenia, fertility rates drop to 93.24% of their original values, whereas in the Rest of Yugoslavia they decline to 84.27% of their previous levels. The difference is caused by differences in the intial fertility levels. The new fertility rates $\hat{\underset{\sim}{F}}(x)$ are also given in Table 8.1. Note that the gross rates of reproduction drop in the same proportion as the age-specific fertility rates.

With these new rates, fertility analysis is performed as before (see Sections 4, 6 and 7). The results are listed in Tables 8.2 to 8.13. A comparison of these results with Tables 4.2 to 4.9 reveals the impact of the fertility drop to replacement level.

In the <u>proportional reduction alternative</u>, the age-specific fertility rates of each region are reduced in the same proportion. The fertility adjustment factor is identical for each region and is equal to

$$\gamma_i = \gamma_j = \gamma = \frac{1}{\lambda_1[\underset{\sim}{R}(0)]} \quad . \tag{8.6}$$

Table 8.1. Zero population growth alternative 1.

matrix of fertility adjustment factors

	slovenia	r.yugos.
slovenia	0.932431	0.000000
r.yugos.	0.000000	0.842719
total	0.932431	0.842719

fertility analysis

age-specific rates

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000066	0.000056
15	0.014785	0.022296
20	0.065878	0.074141
25	0.058946	0.062580
30	0.038326	0.037324
35	0.021317	0.019831
40	0.007271	0.010156
45	0.000662	0.001812
50	0.000273	0.000602
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
grr	1.037617	1.143994

Table 8.2. Integrals of generalized net maternity function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000310	0.000008
15	0.068147	0.004741
20	0.293593	0.025894
25	0.253648	0.030305
30	0.160491	0.021556
35	0.087569	0.012460
40	0.029433	0.006582
45	0.002633	0.001182
50	0.001058	0.000389
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.896882	0.103118

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000001	0.000250
15	0.000303	0.098391
20	0.002449	0.324229
25	0.003028	0.270866
30	0.002265	0.159913
35	0.001365	0.084052
40	0.000490	0.042478
45	0.000046	0.007448
50	0.000019	0.002409
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.009965	0.990035

Table 8.3. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.896882	0.009965
r.yugos.	0.103118	0.990035
total	1.000000	1.000000

1 moment

	slovenia	r.yugos.
slovenia	24.708921	0.292468
r.yugos.	3.023252	27.103607
total	27.732174	27.396076

2 moment

	slovenia	r.yugos.
slovenia	716.0538	8.9751
r.yugos.	93.4012	786.4307
total	809.4550	795.4058

Table 8.4. Spatial fertility expectancies.

net reproduction rate

	slovenia	r.yugos.
slovenia	0.896882	0.009965
r.yugos.	0.103118	0.990035
total	1.000000	1.000000
eigenvalue	1.000000	
eigenvector		
- right	1.000000	10.348280
- left	1.000000	1.000005

net reproduction allocations

	slovenia	r.yugos.
slovenia	0.896882	0.009965
r.yugos.	0.103118	0.990035
total	1.000000	1.000000

Table 8.5. Matrices of mean ages and variances.

** alternative 1 **
****

means

	slovenia	r.yugos.
slovenia	27.549809	29.350248
r.yugos.	29.318338	27.376410
total	28.434074	28.363329

variances

	slovenia	r.yugos.
slovenia	39.389526	39.247437
r.yugos.	46.204285	44.878540
total	42.796906	42.062988

** alternative 2 **
****

means

	slovenia	r.yugos.
slovenia	27.547722	0.018143
r.yugos.	0.223530	27.374157
total	27.771252	27.392300

variances

	slovenia	r.yugos.
slovenia	39.381897	0.034458
r.yugos.	0.548902	44.868652
total	39.930798	44.903111

Table 8.6. Discounted number of offspring per person of exact age x.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.000000	0.896882	0.103118
5	1.030596	0.937944	0.092652
10	1.031793	0.950707	0.081086
15	1.032312	0.958820	0.073492
20	0.958980	0.911243	0.047737
25	0.631089	0.611876	0.019213
30	0.337591	0.331763	0.005827
35	0.146985	0.145805	0.001179
40	0.040823	0.040613	0.000210
45	0.004616	0.004592	0.000025
50	0.001348	0.001343	0.000005
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.000000	0.009965	0.990035
5	1.119092	0.009841	1.109251
10	1.122919	0.009098	1.113820
15	1.125394	0.008376	1.117019
20	1.018808	0.005255	1.013552
25	0.653396	0.001724	0.651672
30	0.344488	0.000541	0.343947
35	0.160096	0.000127	0.159969
40	0.062111	0.000022	0.062089
45	0.011876	0.000002	0.011874
50	0.002961	0.000000	0.002960
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 8.7. Spatial reproductive value per person of exact age x.

	slovenia	r.yugos.
0	1.000000	1.000005
5	1.030597	1.119098
10	1.031793	1.122925
15	1.032312	1.125400
20	0.958981	1.018813
25	0.631089	0.653400
30	0.337591	0.344490
35	0.146985	0.160097
40	0.040823	0.062112
45	0.004616	0.011876
50	0.001348	0.002961
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

Table 8.8. Discounted number of offspring per person in age group x to x + 4.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.015376	0.916952	0.098425
5	1.031201	0.944280	0.086921
10	1.032055	0.954744	0.077312
15	0.995479	0.935349	0.060130
20	0.795004	0.764491	0.030514
25	0.484481	0.473986	0.010495
30	0.242555	0.239887	0.002667
35	0.094084	0.093602	0.000483
40	0.022789	0.022734	0.000055
45	0.002993	0.002984	0.000009
50	0.000682	0.000682	0.000000
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.056171	0.009920	1.046251
5	1.121002	0.009473	1.111529
10	1.124155	0.008739	1.115416
15	1.072245	0.006781	1.065464
20	0.836703	0.003149	0.833555
25	0.499564	0.000958	0.498606
30	0.252693	0.000254	0.252440
35	0.111355	0.000045	0.111310
40	0.037177	0.000004	0.037173
45	0.007462	0.000001	0.007461
50	0.001504	0.000000	0.001504
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 8.9. Spatial reproductive value per person in age group x to x + 4.

	slovenia	r.yugos.
0	1.015377	1.056177
5	1.031202	1.121008
10	1.032056	1.124161
15	0.995479	1.072251
20	0.795004	0.836708
25	0.484481	0.499567
30	0.242555	0.252695
35	0.094084	0.111356
40	0.022789	0.037177
45	0.002993	0.007462
50	0.000682	0.001504
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

Table 8.10. Total discounted number of offspring of observed population.

	total	slovenia	r.yugos.
slovenia	389998.	358522.	31477.
r.yugos.	4741045.	25023.	4716022.
total	5131043.	383545.	4747499.

Table 8.11. Reproductive value of the total population.

	total	percentage
slovenia	383545.	7.47
r.yugos.	4747525.	92.53
total*	5131070.	100.00

*The small deviation from the total discounted number of offspring of the observed population is due to rounding error.

Table 8.12. Stable equivalent of total population.

	total	slovenia	r.yugos.
0	888829.	81158.	807671.
5	840853.	79658.	761195.
10	838528.	79375.	759152.
15	835824.	79476.	756348.
20	831645.	79814.	751831.
25	826110.	79703.	746407.
30	819574.	79118.	740456.
35	812089.	78652.	733437.
40	802470.	78237.	724234.
45	789281.	77366.	711915.
50	769610.	75847.	693763.
55	740059.	73707.	666352.
60	694006.	70341.	623665.
65	622513.	64142.	558372.
70	517445.	53632.	463814.
75	385494.	39683.	345811.
80	245111.	23585.	221526.
85	237109.	14744.	222365.
total	12496551.	1208238.	11288313.

percentage distribution

	total	slovenia	r.yugos.
0	7.113	6.717	7.155
5	6.729	6.593	6.743
10	6.710	6.570	6.725
15	6.688	6.578	6.700
20	6.655	6.606	6.660
25	6.611	6.597	6.612
30	6.558	6.548	6.559
35	6.499	6.510	6.497
40	6.422	6.475	6.416
45	6.316	6.403	6.307
50	6.159	6.277	6.146
55	5.922	6.100	5.903
60	5.554	5.822	5.525
65	4.981	5.309	4.946
70	4.141	4.439	4.109
75	3.085	3.284	3.063
80	1.961	1.952	1.962
85	1.897	1.220	1.970
total	100.000	100.000	100.000
share	100.000	9.669	90.331

Table 8.13. Stable equivalents and intrinsic rates.

	births		deaths		outmigration		inmigration	
	number	rate	number	rate	number	rate	number	rate
slovenia	16486.	0.013645	16889.	0.013978	2838.	0.002349	3240.	0.002682
r.yugos.	170603.	0.015113	170200.	0.015078	3240.	0.000287	2838.	0.000251
total	187089.	0.014971	187089.	0.014971	6079.	0.000486	6079.	0.000486

stable growth rate	0.000000
normalizing factor	311.2360

The matrix of fertility adjustment factors is given in Table 8.14 together with the new fertility rates:

$$\hat{\hat{\underset{\sim}{F}}}(x) = \underset{\sim}{\gamma}\underset{\sim}{F}(x) \quad , \tag{8.7}$$

where $\underset{\sim}{\gamma} = \gamma\underset{\sim}{I}$.

This reduction scheme produces a different stationary population. A baby girl born in Slovenia is replaced by only 0.918 daughters on the average, while a girl born in the Rest of Yugoslavia replaces herself with 1.004 daughters. Further results of this replacement alternative are given in Tables 8.15 to 8.26.

8.2 The Analytical Approach

If the initial population is stable, the momentum of spatial zero population growth may be expressed as a simple analytical formula. The ultimate number of stationary equivalent births is by (7.1):

$$\{\hat{\underset{\sim}{Q}}\} = \frac{1}{\{\hat{\underset{\sim}{v}}(0)\}'\hat{\underset{\sim}{\kappa}}\{\hat{\underset{\sim}{Q}}_1\}} \int_0^{\omega} \{\hat{\underset{\sim}{v}}(x)\}'\{\underset{\sim}{k}(x)\}dx \ \{\hat{\underset{\sim}{Q}}_1\} \tag{7.1}$$

where the caret designates a stationary population. The total reproductive value $\hat{V}$ is

$$\hat{V} = \int_0^{\omega} \{\hat{\underset{\sim}{v}}(x)\}'\{\underset{\sim}{k}(x)\}dx \quad ,$$

with $\{\underset{\sim}{k}(x)\}$ being the vector defining the regional distribution of people at exact age x. If the distribution $\{\underset{\sim}{k}(x)\}$ is stable, then by (3.23)

$$\{\underset{\sim}{k}(x)\} = e^{-rx}\ \hat{\underset{\sim}{\ell}}(x)\ \{\underset{\sim}{Q}\} \quad , \tag{8.8}$$

Table 8.14. Zero population growth alternative 2.

matrix of fertility adjustment factors

	slovenia	r.yugos.
slovenia	0.846895	0.000000
r.yugos.	0.000000	0.846895
total	0.846895	0.846895

fertility analysis

age-specific rates

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000060	0.000057
15	0.013429	0.022407
20	0.059835	0.074508
25	0.053539	0.062890
30	0.034810	0.037509
35	0.019361	0.019929
40	0.006604	0.010206
45	0.000601	0.001821
50	0.000248	0.000605
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
grr	0.942432	1.149663

Table 8.15. Integrals of generalized net maternity function.

initial region of cohort slovenia

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000281	0.000008
15	0.061895	0.004765
20	0.266660	0.026022
25	0.230380	0.030455
30	0.145768	0.021663
35	0.079536	0.012522
40	0.026733	0.006615
45	0.002391	0.001188
50	0.000961	0.000391
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.814607	0.103629

initial region of cohort r.yugos.

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000001	0.000251
15	0.000275	0.098878
20	0.002224	0.325836
25	0.002750	0.272209
30	0.002057	0.160705
35	0.001240	0.084468
40	0.000445	0.042688
45	0.000042	0.007485
50	0.000017	0.002421
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
total	0.009051	0.994941

Table 8.16. Moments of integral function.

0 moment

	slovenia	r.yugos.
slovenia	0.814607	0.009051
r.yugos.	0.103629	0.994941
total	0.918236	1.003992

1 moment

	slovenia	r.yugos.
slovenia	22.442253	0.265639
r.yugos.	3.038233	27.237909
total	25.480486	27.503548

2 moment

	slovenia	r.yugos.
slovenia	650.3669	8.1518
r.yugos.	93.8640	790.3277
total	744.2309	798.4795

Table 8.17. Spatial fertility expectancies.

net reproduction rate

	slovenia	r.yugos.
slovenia	0.814607	0.009051
r.yugos.	0.103629	0.994941
total	0.918236	1.003992
eigenvalue	1.000000	
eigenvector		
- right	1.000000	20.483990
- left	1.000000	1.789017

net reproduction allocations

	slovenia	r.yugos.
slovenia	0.887143	0.009015
r.yugos.	0.112857	0.990985
total	1.000000	1.000000

Table 8.18. Matrices of mean ages and variances.

** alternative 1 **
****

means

	slovenia	r.yugos.
slovenia	27.549803	29.350245
r.yugos.	29.318340	27.376406
total	28.434072	28.363325

variances

	slovenia	r.yugos.
slovenia	39.389832	39.247742
r.yugos.	46.204163	44.878723
total	42.796997	42.063232

** alternative 2 **
****

means

	slovenia	r.yugos.
slovenia	27.547716	0.016397
r.yugos.	0.247327	27.374157
total	27.795044	27.390554

variances

	slovenia	r.yugos.
slovenia	39.382263	0.031142
r.yugos.	0.607282	44.868713
total	39.989544	44.899857

Table 8.19. Discounted number of offspring per person of exact age x.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	0.918236	0.814607	0.103629
5	0.945013	0.851902	0.093111
10	0.944982	0.863494	0.081488
15	0.944719	0.870863	0.073856
20	0.875624	0.827651	0.047974
25	0.575054	0.555746	0.019308
30	0.307186	0.301329	0.005856
35	0.133615	0.132430	0.001185
40	0.037098	0.036887	0.000211
45	0.004195	0.004170	0.000025
50	0.001225	0.001220	0.000005
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.003992	0.009051	0.994941
5	1.123686	0.008938	1.114747
10	1.127603	0.008264	1.119340
15	1.130161	0.007607	1.122554
20	1.023348	0.004773	1.018575
25	0.656467	0.001566	0.654901
30	0.346143	0.000492	0.345651
35	0.160877	0.000115	0.160762
40	0.062417	0.000020	0.062397
45	0.011934	0.000002	0.011933
50	0.002975	0.000000	0.002975
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 8.20. Spatial reproductive value per person of exact age x.

	slovenia	r.yugos.
0	1.000000	1.789017
5	1.018480	2.003240
10	1.009277	2.010781
15	1.002992	2.015875
20	0.913476	1.827021
25	0.590288	1.173196
30	0.311806	0.618867
35	0.134550	0.287721
40	0.037265	0.111649
45	0.004215	0.021350
50	0.001229	0.005322
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

Table 8.21. Discounted number of offspring per person in age group x to x + 4.

region of residence slovenia

	region of birth of offspring		
	total	slovenia	r.yugos.
0	0.931748	0.832835	0.098912
5	0.945009	0.857657	0.087352
10	0.944855	0.867161	0.077695
15	0.909973	0.849545	0.060428
20	0.725025	0.694360	0.030665
25	0.441052	0.430505	0.010547
30	0.220562	0.217881	0.002681
35	0.085500	0.085015	0.000485
40	0.020704	0.020649	0.000055
45	0.002719	0.002710	0.000009
50	0.000619	0.000619	0.000000
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

region of residence r.yugos.

	region of birth of offspring		
	total	slovenia	r.yugos.
0	1.060445	0.009010	1.051435
5	1.125641	0.008604	1.117037
10	1.128880	0.007937	1.120943
15	1.076902	0.006159	1.070743
20	0.8[illegible]0545	0.002860	0.837685
25	0.501947	0.000870	0.501076
30	0.253921	0.000230	0.253691
35	0.111903	0.000041	0.111862
40	0.037361	0.000004	0.037358
45	0.007499	0.000001	0.007498
50	0.001512	0.000000	0.001512
55	0.000000	0.000000	0.000000
60	0.000000	0.000000	0.000000
65	0.000000	0.000000	0.000000
70	0.000000	0.000000	0.000000
75	0.000000	0.000000	0.000000
80	0.000000	0.000000	0.000000

Table 8.22. Spatial reproductive value per person in age group x to x + 4.

	slovenia	r.yugos.
0	1.009791	1.890046
5	1.013931	2.007002
10	1.006158	2.013324
15	0.957651	1.921737
20	0.749220	1.501493
25	0.449374	0.897305
30	0.222677	0.454087
35	0.085883	0.200164
40	0.020747	0.066837
45	0.002726	0.013416
50	0.000619	0.002704
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

Table 8.23. Total discounted number of offspring of observed population.

	total	slovenia	r.yugos.
slovenia	354222.	325633.	28589.
r.yugos.	4764538.	25147.	4739390.
total	5118760.	350780.	4767980.

Table 8.24. Reproductive value of the total population.

	total	percentage
slovenia	370621.	4.17
r.yugos.	8507440.	95.83
total	8878061.	100.00

Table 8.25. Stable equivalent of total population.

	total	slovenia	r.yugos.
0	877046.	42646.	834400.
5	828160.	42294.	785866.
10	825832.	42450.	783382.
15	823123.	43278.	779845.
20	818937.	44912.	774025.
25	813373.	46092.	767281.
30	806806.	46462.	760345.
35	799314.	46651.	752663.
40	789710.	46725.	742985.
45	776636.	46417.	730219.
50	757174.	45670.	711504.
55	727909.	44617.	683292.
60	682258.	42847.	639412.
65	611629.	39279.	572350.
70	508282.	32956.	475326.
75	378761.	24445.	354316.
80	241512.	14587.	226925.
85	236912.	9195.	227716.
total	12303373.	701522.	11601851.

percentage distribution

	total	slovenia	r.yugos.
0	7.128	6.079	7.192
5	6.731	6.029	6.774
10	6.712	6.051	6.752
15	6.690	6.169	6.722
20	6.656	6.402	6.672
25	6.611	6.570	6.613
30	6.558	6.623	6.554
35	6.497	6.650	6.487
40	6.419	6.661	6.404
45	6.312	6.617	6.294
50	6.154	6.510	6.133
55	5.916	6.360	5.890
60	5.545	6.108	5.511
65	4.971	5.599	4.933
70	4.131	4.698	4.097
75	3.079	3.485	3.054
80	1.963	2.079	1.956
85	1.926	1.311	1.963
total	100.000	100.000	100.000
share	100.000	5.702	94.298

Table 8.26. Stable equivalents and intrinsic rates.

	births		deaths		outmigration		inmigration	
	number	rate	number	rate	number	rate	number	rate
slovenia	8607.	0.012269	10337.	0.014736	1610.	0.002295	3333.	0.004752
r.yugos.	176307.	0.015196	174577.	0.015047	3333.	0.000287	1610.	0.000139
total	184914.	0.015030	184914.	0.015030	4944.	0.000402	4944.	0.000402

stable growth rate	0.000000
normalizing factor	1031.4951

where $\underset{\sim}{\{Q\}}$ represents the regional distribution of births before the drop in fertility. Substituting $\{\underset{\sim}{k}(x)\}$ in (7.1) into (8.8) and simplifying gives (Rogers and Willekens, 1976b, p. 22):

$$\{\hat{\underset{\sim}{Q}}\} = \frac{1}{\mu r} [\{\hat{\underset{\sim}{v}}(0)\}'\underset{\sim}{\gamma}[\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)] \{\underset{\sim}{Q}\}] \{\hat{\underset{\sim}{Q}}_1\} \quad , \qquad (8.9)$$

where $\mu = \{\hat{\underset{\sim}{v}}(0)\}' \hat{\underset{\sim}{\kappa}} \{\hat{\underset{\sim}{Q}}_1\}$, with $\hat{\underset{\sim}{\kappa}} = \underset{\sim}{\mu} = \underset{\sim}{\gamma} \underset{\sim}{R}(1) \underset{\sim}{R}^{-1}(0) \underset{\sim}{\gamma}^{-1}$ being the matrix of mean ages of childbearing in the stationary population after the decline in fertility. The matrices $\underset{\sim}{R}(0)$ and $\underset{\sim}{\Psi}(r)$ and the vector of stable equivalent births refer to the stable population before the drop in fertility. The matrix of fertility adjustment factors is $\underset{\sim}{\gamma}$.

It can be shown that equation (8.9) is equivalent to

$$\{\hat{\underset{\sim}{Q}}\} = \frac{1}{r} \underset{\sim}{R}(1)^{-1} [\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)] \{\underset{\sim}{Q}\} \qquad (8.10)$$

$$= \underset{\sim}{S}^{o} \{\underset{\sim}{Q}\} \quad , \text{ say.}$$

The stationary births are therefore a linear combination of the stable births, before the drop in fertility. The conversion matrix is $\underset{\sim}{S}^{o}$. The numerical evaluation is given in Table 8.27.

The ultimate stationary population is

$$\{\hat{\underset{\sim}{Y}}\} = [\int_0^{\omega} \hat{\underset{\sim}{\ell}}(x)dx] \{\hat{\underset{\sim}{Q}}\} = \underset{\sim}{e}(0) \{\hat{\underset{\sim}{Q}}\} \qquad (8.11)$$

and the total reproductive value is

$$\hat{V} = \{\hat{\underset{\sim}{v}}(0)\} \frac{1}{r} \underset{\sim}{\gamma}[\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)] \{\underset{\sim}{Q}\} \quad . \qquad (8.12)$$

Let $\underset{\sim}{Y}$ be the diagonal matrix of the total observed population, then

$$\underset{\sim}{Y} \{\underset{\sim}{1}\} = [\int_0^{\omega} e^{-rx} \hat{\underset{\sim}{\ell}}(x)dx] \{\underset{\sim}{Q}\} = \underset{\sim}{e}^{(r)}(0) \{\underset{\sim}{Q}\} \quad , \qquad (8.13)$$

where $\underset{\sim}{e}^{(r)}(0)$ has been labeled the matrix of discounted life expectancies. Recalling the characteristic equation, (8.13) also may be written as

$$\{\underset{\sim}{Y}\} = \underset{\sim}{e}^{(r)}(0)\ [\underset{\sim}{\Psi}(r)]^{-1}\ \{\underset{\sim}{Q}\}\quad , \tag{8.14}$$

whence

$$\{\underset{\sim}{Q}\} = \underset{\sim}{\Psi}(r)\ [e^{(r)}(0)]^{-1}\ \{\underset{\sim}{Y}\} = \underset{\sim}{b}\ \{\underset{\sim}{Y}\} \tag{8.15}$$

The spatial momentum of zero population growth is

$$\underset{\sim}{Y}^{-1}\ \{\hat{\underset{\sim}{Y}}\} = \frac{1}{\mu r}\ [\{\hat{\underset{\sim}{\nu}}(0)\}'\underset{\sim}{\gamma}[\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)]\ \{\underset{\sim}{Q}\}]\ \underset{\sim}{Y}^{-1}\ \underset{\sim}{e}(0)\ \{\hat{\underset{\sim}{Q}}_1\} \tag{8.16}$$

$$= \frac{1}{\mu r}\ [\{\hat{\underset{\sim}{\nu}}(0)\}'\underset{\sim}{\gamma}[\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)]\ \{\underset{\sim}{b}\}]\ \underset{\sim}{e}(0)\ \{\hat{\underset{\sim}{Q}}_1\} \tag{8.17}$$

where $\{\underset{\sim}{b}\}$ is the vector of regional intrinsic birth rates before the drop in fertility. Applying (8.10) the momentum becomes

$$\underset{\sim}{Y}^{-1}\ \{\hat{\underset{\sim}{Y}}\} = e(0)\frac{1}{r}\ \underset{\sim}{R}(1)^{-1}\ [\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)]\{\underset{\sim}{b}\}\quad . \tag{8.18}$$

Introducing (8.15) into (8.16) gives yet another expression for the momentum

$$\underset{\sim}{Y}^{-1}\ \{\hat{\underset{\sim}{Y}}\} = \frac{1}{\mu r}\ [\{\hat{\underset{\sim}{\nu}}(0)'\ \underset{\sim}{\gamma}[\underset{\sim}{R}(0) - \underset{\sim}{\Psi}(r)]\ \underset{\sim}{b}\ \{\underset{\sim}{1}\}]\ \underset{\sim}{e}(0)\{\hat{\underset{\sim}{Q}}_1\}\quad . \tag{8.19}$$

The analytical approach is illustrated in Table 8.27. It is assumed that the initial population coincides with the stable equivalent population of Slovenia and the Rest of Yugoslavia. Hence the regional births are contained in the vector

$$\{\underset{\sim}{Q}\} = \begin{bmatrix} 9,237 \\ \\ \\ 192,355 \end{bmatrix} ,$$

and the population by age-group and region is given in Table 7.1.

Table 8.27 reveals that, given a population of 597,806 in Slovenia and 10,119,204 in the Rest of Yugoslavia, an immediate drop of fertility to replacement level would result in an ultimate population increase of 15.74% in Slovenia and of 14.66% in the Rest of Yugoslavia. The momentum is a consequence of the growth potential in the initial age and regional distribution of the population.*

---

*Note that the stationary population distribution in unit births was given in Tables 2.4 and 3.5.

Table 8.27. Spatial momentum of zero population growth.

matrix converting stable to stationary births

	slovenia	r.yugos.
slovenia	0.916844	-0.000054
r.yugos.	-0.000574	0.916786
total	0.916270	0.916732

stable and stationary equivalents

	births		population		population
	stable	stationary	stable	stationary	momentum
slovenia	9237.	8459.	597806.	691925.	1.1574
r.yugos.	192355.	176343.	10119204.	11603081.	1.1465

Part II

User's Manual

PART II. USER'S MANUAL

9. PROGRAM DESCRIPTION

The concept underlying the programs is that of a modular system. It consists of a set of subroutines, each of which performs a specific task, such as matrix inversion, calculating the dominant eigenvalue and associated eigenvectors, computing the integral functions and their moments, and so on. The main program is kept very short; it coordinates the computations through CALL statements. Information is transmitted from one subroutine to another as follows:

- arrays: labeled COMMON statements,
- parameters: argument string in the CALL statement.

No computations are performed in the main program.

The subroutines consist of the frequently used general purpose subroutines and special purpose subroutines:

i. <u>General purpose subroutines</u>:

MULTIP: matrix multiplication.

INVERT: matrix inversion.

EIGEN: computation of dominant eigenvalue and associated right and left eigenvectors.

ii. <u>Special purpose subroutines</u>:

DATAS: reads and prints the data as they are read in; computes the observed rates.

PRELIM: performs a preliminary analysis with the data.

TOTSY: computes and prints demographic features of a single region; computes and prints the single region life table.

PROBR: computes and prints the probabilities of dying and outmigrating, following Option 1.

PROBSC: computes and prints the probabilities of dying and outmigrating, following Option 3.

HIST: computes and prints the complete life histories of the regional birth cohorts.

LIFE: computes and prints the multiregional life table.

PRLIF2: prints the summary life table for a two-region system.

WHOLE: computes aggregate age-specific death rates that are consistent with the aggregation of the multiregional life table.

GROWTH: computes and prints the generalized Leslie matrix.

PROJEC: projects the population until stability is reached.

LMAT: computes the age composition of the stationary (life table) population in terms of unit born (the $\underset{\sim}{L}(x)$ matrices).

RELAM: reads in the stable growth ratio.

FERMOB: performs the fertility and mobility analysis; computes the integral functions, i.e., the (weighted) generalized maternity and mobility functions, and their zero-th, first and second moments. In addition, it calculates the matrices of mean ages at childbearing and mobility and the matrices of the variances of the ages at childbearing and mobility

AGEDIS: generates the stationary and stable population distributions by age and region and in terms of unit radices.

RVALUE: computes the discounted number of offspring and computes the spatial reproductive values by age and region.

RINTR: computes the stable equivalents of births, deaths, outmigrants and inmigrants, and the intrinsic rates.

MOMENT: computes the spatial momentum of zero population growth (analytical approach).

ZERO: replaces the observed regional fertility schedules with fertility schedules at replacement level. Two alternative fertility reduction schemes are possible.

The purpose of separating each major task into subroutines is to keep the whole structure of the programs very clear and to enable the user to change parts of the programs according to his needs. Clarity and flexibility are major objectives which we tried to keep in mind while writing the programs. Computational efficiency was of secondary importance. In a rapidly growing field such as multiregional demographic analysis, computer programs must be flexible and easy to adapt to new theoretical or methodological developments. The computer programs published here are not final fixed products; they are working tools to produce useful numerical demographic results. The user is urged to adapt them to fit his own needs in order to get the most out of them.

Each subroutine that performs a major computational task is covered in detail. In Part I, we focused on the clarification of the output. For a detailed mathematical treatment of the various topics, the user is referred to Rogers (1975) and to the papers of the dynamics subtask of IIASA's Migration and Settlement Task (see list at the back of this report).

9.1 The General Purpose Subroutines

a. MULTIP:

SUBROUTINE MULTIP (N,K,L)

Task: multiplication of two matrices A1 and B.
C = A1 * B.

Parameters:
N: number of rows of A1.
K: number of columns of A1 (and consequently, number of rows of B).
L: number of columns of B.

Input:
- parameters in the CALL statement.
- matrices A1 and B in a labeled COMMON:
  COMMON/CMUL/A1(N,K), B(K,L), C(N,L).

Output: the result of the matrix multiplication is stored in the N x L matrix C.

Printing: none.

b. INVERT:

SUBROUTINE INVERT (NR)

Task: inversion of the matrix CC.

Parameters: NR: rank of CC.

Input:
- parameter NR in the CALL statement. The subroutine assumes that CC is nonsingular, and that all the diagonal elements are nonzero.
- matrix CC in labeled COMMON:
  COMMON/CINV/CC(NR,NR).

Output: the original matrix CC is replaced by the inverted matrix.

Printing: none.

c. EIGEN:

SUBROUTINE EIGEN (NR, NP, NEIG)

Task: calculation of the dominant eigenvalue of the matrix CE and of the associated right and left eigenvectors. EIGEN may also be used to compute row and column totals and to print a matrix.

Parameters: NR: dimension of the matrix.

NEIG: parameter related to the computation.

NEIG = 1: the complete computation procedure is performed: row and column totals, dominant eigenvalue and associated eigenvectors.

NEIG = 0: only the column sums are computed and printed. By using this option, EIGEN may be used to print a matrix.

NP: parameter related to printing.

NP = 1: EIGEN prints the original matrix and its column sums. The dominant eigenvalue and its right and left eigenvectors are printed if NEIG = 1.

NP = 0: nothing is printed.

NP = 2: row and column sums are printed together with the matrix.

Input: - parameters in CALL statement.

- the matrix CE in labeled COMMON:

COMMON/CEIGEN/CE(NR,NR),ROOT , VECT(NR),VECTL(NR).

Output: the dominant eigenvalue ROOT, the right eigenvector VECT(I) and the left eigenvector VECTL(I) are stored in labeled COMMON.

Printing: according to the specification of the parameter NP.

Algorithm: let the original matrix be $\underset{\sim}{A}$:

$$\underset{\sim}{A} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix}$$

The dominant eigenvalue of $\underset{\sim}{A}$ is obtained by the power method (for details see Rogers 1971, Chapter 7):

$$\lambda^{(n)} = \frac{a_{11}^{(n+1)}}{a_{11}^{(n)}} ,$$

where the superscript denotes the iteration and $a_{11}^{(n)}$ is the first element of the matrix $\underset{\sim}{A}^n$. As n becomes large, $\lambda^{(n)}$ converges to the true eigenvalue. The iteration terminates when

$$-\varepsilon < \left[\frac{a_{12}^{(n+1)}}{a_{11}^{(n+1)}} - \frac{a_{12}^{(n)}}{a_{11}^{(n)}} \right] < \varepsilon ,$$

with $\varepsilon = 0.000001$.

The right eigenvector $\{\underset{\sim}{\xi}\}$ associated with λ is proportional to any column of $\underset{\sim}{A}^n$ for n large. In the program, $\{\underset{\sim}{\xi}\}$ is taken to be the first column of $\underset{\sim}{A}^n$,--scaled such that $\xi_1 = 1$. The scaling selected is arbitrary, since an eigenvector is constant up to a scalar. For convenience, we have retained the scaling $\xi_1 = 1$, i.e. the first element of $\{\underset{\sim}{\xi}\}$ is unity.

The right eigenvector is the solution to the system

$$\{\underset{\sim}{\nu}\}' \ [\underset{\sim}{A} - \lambda \underset{\sim}{I}] = \{\underset{\sim}{0}\}'$$

As before, we take ν_1 to be unity. Therefore

$$\begin{bmatrix} 1 \\ \nu_2 \\ \vdots \\ \nu_n \end{bmatrix}' \begin{bmatrix} a_{11}-\lambda & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22}-\lambda & \cdots & a_{n2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn}-\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} \nu_2 \\ \vdots \\ \nu_n \end{bmatrix} = \begin{bmatrix} a_{21} \\ \vdots \\ a_{n1} \end{bmatrix}' \begin{bmatrix} a_{22}-\lambda & \cdots & a_{n2} \\ \vdots & & \vdots \\ a_{2n} & \cdots & a_{nn}-\lambda \end{bmatrix}^{-1}$$

9.2 The Special Purpose Subroutines

a. DATAS:

SUBROUTINE DATAS (NPR,NA,NY,ZFNY,NR,XZB,XZD,
XZO,IPROB,INIT,NHORIZ,INTV,ITOLX,NTOLL,NEIG)

Task: - reads data and prints them as they are read in (for details, see Section 10 on Preparation of Data deck).

- computes observed rates.

Parameters: see Section 10.

Input: see Section 10.

Output: data as they are read in. The data are stored in labeled COMMON.

b. PRELIM:

SUBROUTINE PRELIM (NA,NY,ZNFY,NR,XZB,XZD,XZO)

Task: performs a preliminary analysis with the data.

a. computes and prints age compositions, rates, mean ages, etc.;

b. calls TOTSY for each region separately and for the country.

Parameters: see Section 10.

Input: DATAS must precede PRELIM.

Output: Tables of Section 1.

c. TOTSY:

SUBROUTINE TOTSY (NA,ZNFY,XZB,XZD,XZO,NWHOL, REGL)

Task: single region analysis: computes and prints demographic features of each region, including a single region life table.

The subroutine TOTSY is called at two occasions:

- by PRELIM to compute life tables for each region separately and for the country;
- by WHOLE to compute the life table for the aggregate (sum of regions) system.

Parameters: NA,ZFNY,XZB,XZD,XZO: see Section 10.

NWHOL = 0: aggregate system (aggregate death rates are derived from multiregional life table). The table with basic demographic features is not printed. Only the life table of the aggregate (sum of regions) system is computed and printed. (NWHOL = 0 if TOTSY is called from WHOLE).

NWHOL = 1: both basic features - table and life table are printed (NWHOL = 1 if TOTSY is called from PRELIM).

Output: Tables 1.4 to 1.6.
Table 2.11.

d. PROBSC:

SUBROUTINE PROBSC (NA,ZFNY,NR,IPROB)

Task: computes and prints the probabilities of dying and outmigrating following Option 3.

Parameters: NA,ZNFY,NR,IPROB: see Section 10.
IPROB must be 3.

Input: DATAS must precede PROBSC.

Output: Table 2.1.

Algorithm: see Section 2.7.

e. PROBR:

SUBROUTINE PROBR (NA,ZFNY,NR,IPROB)

Task: computes and prints the probabilities of dying and outmigrating following Option 1.

Parameters: NA,ZNFY,NR,IPROB: see Section 10.
IPROB must be 1.

Input: DATAS must precede PROBR.

Output: Table B1 of Appendix B.

Algorithm: see Section 2.7.

f. HIST:

SUBROUTINE HIST (NA,NR,IHIST)

Task: computes and prints the complete life histories of each regional birth cohort.

Parameters: NA,NR: see Section 10.
IHIST: parameter indicating that life histories are computed. If subroutine HIST is called, IHIST takes the value of one and the computation of the matrices $\underset{\sim}{\ell}(x)$ is skipped in the subroutine LIFE.
If HIST is not called before LIFE, then IHIST is zero and $\underset{\sim}{\ell}(x)$ is computed in LIFE.

Input: PROBSC or PROBR must precede HIST.

Output: Table 2.2.

Algorithm: see Section 2.1.

g. LIFE:

SUBROUTINE LIFE (NA,ZFNY,NR,IPROB,IHIST,ILIF)

Task: computes and prints the multiregional life table.

Parameters: NA, ZFNY,NR,IPROB: see Section 10.

IHIST: see HIST

ILIF: parameter indicating that life table is computed.
If subroutine LIFE is called, ILIF takes the value of one and the computation of $\underset{\sim}{L}(x)$ and $\underset{\sim}{S}(x)$ (needed later) is skipped in subroutines following LIFE.
If LIFE is not called, ILIF is zero and $\underset{\sim}{L}(x)$ is computed in a special subroutine LMAT while $\underset{\sim}{S}(x)$ is computed in GROWTH.

Input: PROBSC or PROBR must precede LIFE.
HIST may precede LIFE but it is not necessary.

Output: multiregional life table (Tables 2.3 to 2.9).

Algorithm: see Sections 2.2 to 2.6.

h. PRLIF2:

SUBROUTINE PRLIF2 (NA,NY,IPROB)

Task: prints the summary life table for a two-region system.

Parameters: NA,NY,IPROB: see Section 10.

Input: subroutine LIFE must precede PRLIF2.

Output: Table 2.10.

i. WHOLE:

SUBROUTINE WHOLE (NA,ZFNY,NR)

Task: computes aggregate age-specific death rates from multiregional life table statistics, applying the radix ratio provided by the user. These death rates are transmitted to TOTSY which computes and prints the single region life table of the aggregated system.

The columns of this table are identical to the weighted sums of the results of the multiregional life table, the weights being the regional radices given by the user.

Parameters: NA,ZNFY,NR: see Section 10.

Input: LIFE must precede WHOLE.

Output: TOTSY, called by WHOLE, produces Table 2.11.

Algorithm: see Section 2.8.

j. GROWTH:

SUBROUTINE GROWTH (NA,ZFNY,NR,ILIF)

Task: computes and prints the generalized Leslie matrix.

Parameters: NA,ZFNY,NR: see Section 10.
The value of ILIF depends on whether LIFE is called or not: if LIFE precedes GROWTH, ILIF = 1; if LIFE does not precede GROWTH, ILIF = 0 and $\underset{\sim}{S}(x)$ is computed in GROWTH.

Input: PROBSC or PROBR must precede GROWTH.

Output: the nonzero submatrices of the generalized Leslie matrix (Table 3.1).

Algorithm: see Section 3.1.

k. PROJEC:

SUBROUTINE PROJEC (NA,NY,ZFNY,NR,INIT, NHORIZ,INTV,ITOLX,NTOLL,ZLAMDK,IPROJ)

Task: projects the population over an interval of NY years (usually 5). The projection interval is the same as the age interval.

Parameters: NA,NY,ZFNY,NR,INIT,NHORIZ,INTV,ITOLX,NTOLL: see Section 10.

ZLAMDK: the stable growth <u>ratio</u> λ. To derive the stable growth <u>rate</u> r, apply the formula $r = [\ell n\lambda]/NY$.

IPROJ: parameter indicating that PROJEC is called and stable growth ratio is computed. If PROJEC is called, IPROJ = 1.

Input: GROWTH must precede PROJEC.

Output: population projections by age and region (Tables 3.2 and 3.3).

For pragmatic reasons, a distinction is made between <u>short-term</u> and <u>long-term</u> projections. Short-term projection outputs are given for every NY years, whereas long-term projection outputs are listed for every NHORIZ years (100 or 200, say). The limit between short and long-term projections is specified by the user. The purpose of the long-term projection is to identify the stable characteristics of the population system.

In addition to the regional and total population in each age group and the age composition, the output contains the mean age (M.AGE) of the population, the regional shares (SHA) of the total population, and the growth ratio (LAM) of the previous period, i.e., from (t - 1) to t, and the average annual growth rate $r = \frac{\ell n \text{ LAM}}{\text{NY}}$.

Algorithm: see Sections 3.2 and 3.3.

The stable population is characterized by a constant growth rate and age-by-region distribution. This feature underlies the stopping criterion for the projection process (or iteration in the power method). The user may choose between two options:

a. $\lambda_1(t) - \lambda_1(t-1) \leq \text{TOLX}$ (ITOLX = 3)

b. $\lambda_{NR}(t) - \lambda_1(t) \leq \text{TOLX}$ (ITOLX = 2)

where NR is the number of regions,

$\lambda_1(t)$ is the growth ratio of region 1 in the period from (t - 1) to t, TOLX is the tolerance level for the eigenvalue computation.

The user makes his choice by specifying the parameter ITOLX (default value ITOLX = 2). The tolerance level TOLX is

$$TOLX = 10^{-NTOLL}$$

where NTOLL is given by the user (default value NTOLL = 7).

l. LMAT:

SUBROUTINE LMAT (NA,NR,ZFNY,ILIF)

Task: computes the life table (stationary) population distribution by region of birth and region of residence, in terms of unit birth cohorts [$\underset{\sim}{L}(x)$].

Parameters: NA,NR,ZFNY: see Section 10.

ILIF: see description of LIFE. If life table is computed (ILIF = 1), LMAT is not called.

Input: PROBSC or PROBR must precede LMAT.

Output: nothing printed.

Algorithm: see life table, Sections 2.2 and 2.3.

m. RELAM:

SUBROUTINE RELAM (ZFNY,ZLAMDK,RSTAB)

Task: reads the stable population growth ratio; computes the stable growth rate and prints both.

Parameters: ZFNY: see Section 10.

ZLAMDK: stable population growth ratio (over period NY).

RSTAB: stable population growth rate (annual).

Input: ZLAMDK, added to the input data set (see Section 10).

RELAM is called only if IPROJ = 0, i.e., if PROJEC is not called.

Output: RSTAB.

Algorithm: RSTAB = [ℓn ZLAMDK]/ZFNY.

n. FERMOB:

SUBROUTINE FERMOB (NA,ZFNY,NR,KK,NEIG,RG)

Task: performs fertility and mobility analyses. It computes and prints the integral functions (generalized maternity and mobility function and the weighted functions), and their moments. From this information it derives the matrices of mean ages at childbearing and mobility and the variances of the ages at childbearing and mobility.

Parameters: NA,ZFNY,NR: see Section 10.

KK: parameter indicating the type of population analyzed. This parameter is already defined in the argument string.

KK = 1: life table population.

KK = 2: stationary (ZPG) population.

KK = 3: stable population.

NEIG: see description of EIGEN.

RG: relevant annual growth rate:

- life table and ZPG analysis:
  RG = 0.
- stable population analysis:
  RG = RSTAB, i.e., the stable growth rate.

Input:
- LIFE or LMAT must precede FERMOB.
- if KK = 3 (stable population), the stable growth rate RSTAB must be given.

NOTE: RSTAB is derived from the stable growth ratio λ or ZLAMDK.

The value of ZLAMDK is given by subroutine PROJEC. If PROJEC does not precede FERMOB (IPROJ = 0), ZLAMDK must be read in by RELAM.

Output: tables of fertility and mobility analyses (Tables of Sections 4 and 5 and equivalent tables of Section 8.1).

Algorithm: see Sections 4 and 5.
The integral functions are computed by (4.4),(4.19),(5.5) and (5.11). In the program, it reduces to a single expression:

PATFUN = HU(X)*L(X,I,J)*ZGRAL(X,J),

where HU(X) is the weighting factor: HU(X) = 1 in the life table and ZPG population, and is equal to EX(X) in the stable population, L(X,I,J,) is the number of people in region J aged X to X+NY, who are born in region I, ZGRAL(X,J) contains the age and region-specific rates applied to the population distribution to give the integral function. In the case of the maternity function, ZGRAL(X,J) = RATF(X,J), i.e., the regional age-specific fertility rates. In the mobility analysis, ZGRAL(X,J) = $\sum_I$ RATM(X,I,J), i.e., the regional age-specific total outmigration rates.

o. AGEDIS:

SUBROUTINE AGEDIS (NA,ZFNY,NR,RSTAB)

Task: computation and printing of the three types of population distribution: observed population, life table population and stable population.

Parameters: see Section 10.
RSTAB: stable annual growth rate.

Input:
- LIFE or LMAT must precede AGEDIS.
- PROJEC or RELAM must precede AGEDIS

Output: Tables 3.4 and 3.5.

Algorithm: the observed population is printed as read in. The life table population is computed in LIFE or LMAT. The stable population is computed by equation (3.20) of the text:

$$\underset{\sim}{L}^{(r)}(x) = e^{-(x+\frac{NY}{2})r} \underset{\sim}{L}(x) \quad ,$$

or, in FORTRAN:

EX(X) * L(X,I,J) ,

where

EX(X) = EXP(-Z * RSTAB) ,

with

Z = FLOAT(NAGE(X)) + ZFNY * 0.5 .

p. RVALUE:

SUBROUTINE RVALUE (NA,ZFNY,NR,RG,ZVT)

Task: computation and printing of the discounted number of offspring and the reproductive values by age and region.

Parameters: NA,ZFNY,NR: see Section 10.

RG: the ultimate growth rate of the population under consideration:

RG = 0: in the ZPG population.

RG = RSTAB in the stable population.

ZVT: total reproductive value of whole system (all regions), (e.g., column total of Table 6.6).

Input: LIFE or LMAT must precede RVALUE.

- in stable population analysis RG = RSTAB, which must be known through PROJEC or RELAM.
- in stationary (ZPG) population analysis, RG = 0.

Output: tables of Section 6 and equivalent tables of Section 8.1.

Algorithm: see Section 6.

q. RINTR:

SUBROUTINE RINTR (NA,ZFNY,NR,RG,ZVT)

Task: computes and prints the stable equivalents and the intrinsic rates.

Parameters: NA,ZFNY,NR: see Section 10.
RG and ZVT: see description of RVALUE.

Input: RVALUE must precede RINTR.

Output: Tables 7.1 and 7.2. The stable equivalents of births and total population are saved for later use and stored in the arrays QQ(I) and YY(I) respectively. The stable equivalent population by age and region is contained in POPST(X,I).

Algorithm: see Section 7.

r. MOMENT:

SUBROUTINE MOMENT (NA,ZFNY,NR,RSTAB)

Task: spatial ZPG-analysis following the analytical approach.

Parameters: NA,ZFNY,NR: see Section 10.
RSTAB: stable growth rate.

Input:
- it is assumed that the initial population is stable and that the births are given by the vector QQ(I).
- subroutine RINTR must precede MOMENT.

Output: table 8.27.

Algorithm: see Section 8.2.

s. ZERO:

SUBROUTINE ZERO (NA,NR,NZERO,RONRR)

Task: ZERO replaces the observed regional fertility schedules with fertility schedules at replacement level. The new fertility rates are computed according to two alternative fertility reduction schemes described in Section 8.1.

Parameters: NA,NR: see Section 10.

NZERO: denotes the alternative fertility reduction scheme

NZERO = 1: the cohort-replacement alternative.

NZERO = 2: the proportional reduction alternative.

RONRR: dominant eigenvalue of the net reproduction rate matrix (NRR). It is computed when ZERO is called for the first time (NZERO = 1) and is used to compute the matrix of fertility adjustment factors in the proportional reduction alternative (NZERO = 2).

Input: LIFE or LMAT must precede ZERO.

Output:
- diagonal matrix of fertility adjustment factors. The diagonal elements are stored in the vector VI(I) and printed (Tables 8.1 and 8.14).
- new fertility rates at replacement level: RATF(X,I). (Tables 8.1 and 8.14).

Algorithm: see Section 8.1.

9.3 Main Program

The main program is kept very short. Its function is to co-ordinate the calculations, and it therefore consists merely of CALL statements.

The subroutines do not have to be called in sequence. Multiregional life table computation, population projection, fertility-mobility analysis, ZPG-analysis (numerical approach) and the computation of reproductive values all may be performed independently, starting from the raw input data. For instance, the user may want the fertility-mobility analysis of a stationary population without first calculating the whole life table. This is easily done by combining the subroutines DATAS, PROBSC (or PROBR), LMAT and FERMOB. Other possible combinations are given in Table 9.1. The subroutines in parentheses are not called by the main program but by other subroutines. They are mostly general purpose subroutines that must be linked with the special purpose routines.

10. PREPARATION OF THE DATA DECK

There is a single data deck for the computation of all programs. All data are read in at the beginning of the set of programs by the subroutine DATAS. The advantage of concentrating all the READ statements in a single subroutine is that it enables the user to easily change the FORMAT statements to fit particular data sets.

The data are read in fixed format from unit 5 (the conventional unit for cards in most computers).

The data deck used to produce the tables in this paper is given in Table 10.1. (See also Figure 10.1). The card sequence is as follows:

1. Identification card
2. Parameter card
3. Title cards
4. Names of the regions
5. Regional radices
6. For each region:
   a. population
   b. births
   c. deaths
   d. migrants
7. Stable growth ratio (optional)
8. The last card of the deck is a blank card. It may be a colored card to identify the end of the deck to the user.

Table 9.1 Alternative combinations of subroutines for spatial population analysis.

F U N C T I O N	SUBROUTINES REQUIRED
Preliminary Analysis	DATAS, PRELIM, (TOTSY)
Complete Life Histories of Birth Cohorts	DATAS, PROBSC or PROBR, HIST, (MULTIP, INVERT)
Life Table	DATAS, PROBSC or PROBR, LIFE, WHOLE, (TOTSY, MULTIP, INVERT)
- in case of two regions:	(optional) PRLIF2
Projection	DATAS, PROBSC or PROBR, GROWTH, PROJEC (MULTIP, INVERT)
Fertility-Mobility Analysis	DATAS, PROBSC or PROBR, LMAT, FERMOB (MULTIP, INVERT, EIGEN)
- for stable population:	+ RELAM
Age and Regional Distribution of Observed Life Table and Stable Population	DATAS, PROBSC or PROBR, LMAT, RELAM, AGEDIS, (MULTIP, INVERT)
Reproductive Value	DATAS, PROBSC or PROBR, LMAT, RELAM, RVALUE (MULTIP, INVERT, EIGEN)
Further Stable Population Analysis	DATAS, PROBSC (not PROBR), LMAT, RELAM, RVALUE, RINTR (MULTIP, INVERT, EIGEN)
Zero Population Growth Analysis: Analytical Approach (Momentum)	DATAS, PROBSC (not PROBR), LMAT, RELAM, RVALUE, RINTR, MOMENT, (MULTIP, INVERT, EIGEN)
Zero Population Growth Analysis: Numerical Approach (Two Alternatives)	DATAS, PROBSC or PROBR, LMAT, ZERO, FERMOB, (MULTIP, INVERT, EIGEN)
	or
	DATAS, PROBSC, LMAT, ZERO, [FERMOB], RVALUE, RINTR (MULTIP, INVERT, EIGEN)

Table 10.1.

```
yugoslavia 1961 females  two regions  : data set
18 2 5 3 1 1 1 319612001 500 2 7
    multiregional life table
     yugoslavia : two regions 1961
     slovenia  - rest yugoslavia
sloveniar.yugos.yugoslav
 100000. 100000.
   67800.    74100.    70700.    60100.    62900.    66500.    67100.    62900.  ] Population of Slovenia
   39500.    47900.    51300.    46100.    39600.    29500.    21700.    14400.  ]
    7100.     3600.                                                            ]
       .         .        5.      953.     4444.     4204.     2758.     1438.      303.       34.  ] Births in Slovenia
     15.         .         .         .         .         .         .         .         .          ]
    417.       32.       21.       31.       47.       45.       67.       77.       76.      171.  ] Deaths in Slovenia
    268.      369.      513.      763.     1036.     1038.     1041.      733.                      ]
       .         .         .         .         .         .         .         .         .         .  ] Migrants from Slovenia to Slovenia
       .         .         .         .         .         .         .         .                    ]
    192.      170.      105.      310.      451.      358.      252.      111.       40.       25.  ] Migrants from Slovenia to Rest
     34.       29.       35.       23.       19.       16.        5.        4.                     ]  of Yugoslavia
  847900.   905200.   803100.   617400.   725500.   774000.   728400.   633300.  ] Population of Rest of Yugoslavia
  392400.   437100.   453800.   339300.   325800.   230600.   180000.   120900.  ]
   61200.    39300.                                                            ]
       .         .       54.    16335.    63828.    57477.    32261.    14903.     4729.      940.  ] Births in Rest of Yugoslavia
    324.         .         .         .         .         .         .         .         .          ]
   19051.      606.      386.      534.      885.     1227.     1277.     1313.     1127.     1700.  ] Deaths in Rest of Yugoslavia
    2896.     3743.     5492.     6407.     8652.     8715.     6843.     5639.                    ]
    231.      150.      127.      419.      680.      392.      255.      143.       72.       41.  ] Migrants from Rest of Yugoslavia
     59.       80.       66.       36.       14.       12.       12.        3.                     ]  to Slovenia
       .         .         .         .         .         .         .         .         .         .  ] Migrants from Rest of Yugoslavia
       .         .         .         .         .         .         .         .                    ]  to Rest of Yugoslavia
1.030967                                                                                           ] = Stable Growth Ratio
                                                                                                   ] -- Blank Card
```

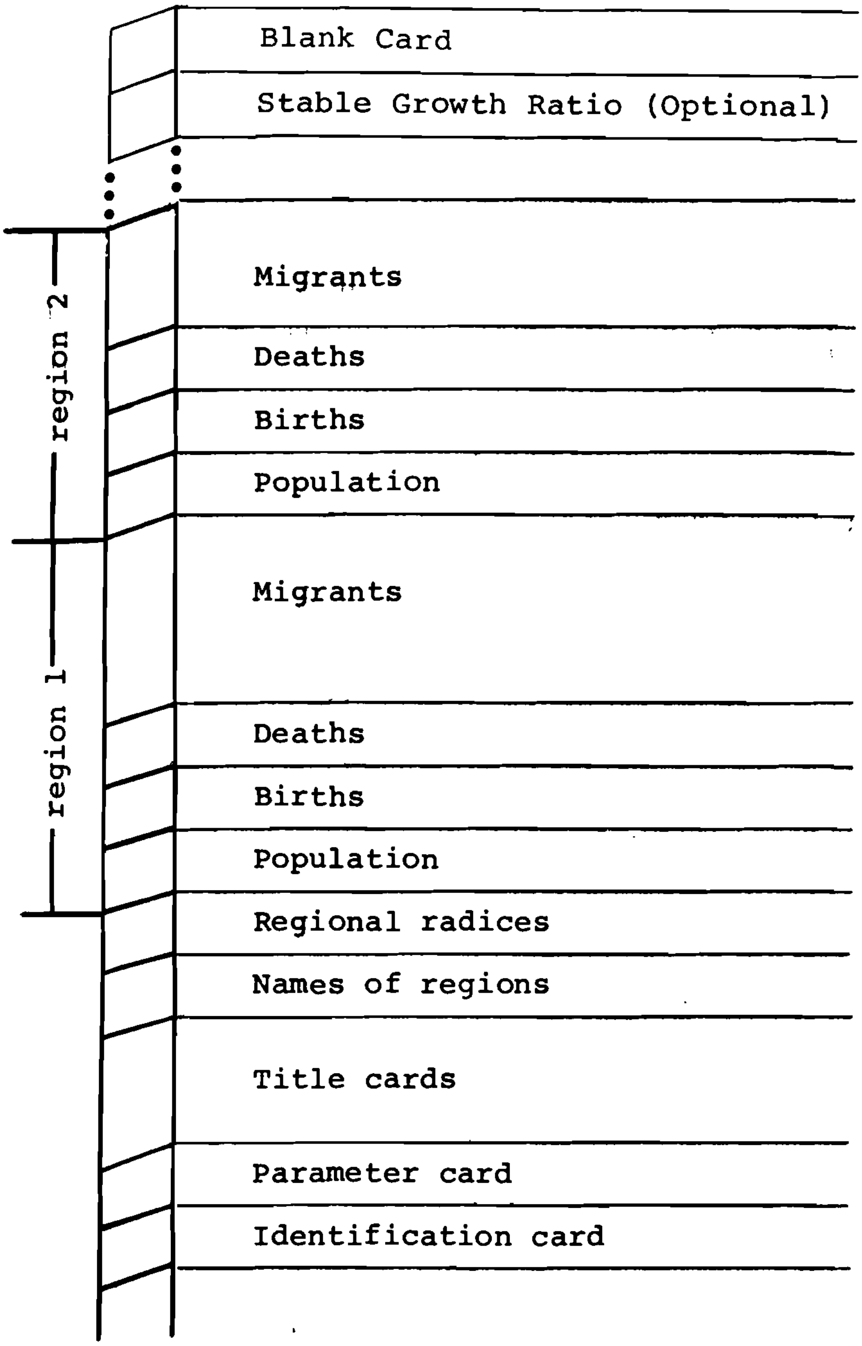

Figure 10.1

Data Deck for Multiregional Population Model.

1. Identification card

   The first card of the deck is an identification card. It may contain any information for the user. The identification card is not read in. Its only purpose is to simplify the handling of the data deck. Therefore, it is advisable to use a card of a specific color as the first card of the deck.

2. Parameter card

   The parameter card contains instructions to the program concerning some general characteristics of the data and concerning the desired computations. The parameter card is composed of 14 "fields", each requiring a specific piece of information. The parameter names, their interpretation, required format and default values are given in Table 10.2.

3. Title cards

   There are NU title cards. There is no limit on NU, as long as it is greater than zero. Each title card is printed out as it is read in. All 80 columns of the card may be used. The title is not stored.

4. Names of the regions

   In listing the output, each region is identified by its name. Each name consists of a maximum of eight characters. Any character can be used. The names appear in sequence on the same card. The last name in the sequence is this of the country or total system.

COLS	FORMAT	VAR. NAME
1-80	10A8	REG(I),I = 1,NR1

   where NR1 = NR + 1. The name of the country is contained in REG(NR1).

5. Regional radices

   The radices appear in sequence on one card.

COLS	FORMAT	VAR. NAME
1-80	10F8.0	RADIX(I),I = 1,NR

Table 10.2. Parameter specification.

COLUMNS	FORMAT	NAME	INTERPRETATION	DEFAULT VALUE
1-2	I2	NA	Number of age groups	---
3-4	I2	NR	Number of regions. The upper limit is 12 (for output reasons)	---
5-6	I2	NY	Width of age group (e.g., 5 years) ZFNY = real value of NY: ZFNY = FLOAT(NY)	---
7-8	I2	NU	Number of title cards	---
9-10	I2	NZB	Time interval for which birth data are given, e.g., NZB = 1: one-year birth data NZB = 5: five-year birth data	1
11-12	I2	NZD	Time interval of death data	1
13-14	I2	NZO	Time interval of migration data	1
15-16	I2	IPROB	Option for estimating death and outmigration probabilities IPROB = 1: Option 1 method (PROBR) IPROB = 3: Option 3 method (PROBSC)	3
17-20	I4	INIT	Base (initial) year	---
21-24	I4	NHORIZ	Time horizon of short term projections: year until which the detailed projection output is given at each time period	INIT
25-28	I4	INTV	Time interval in years, for long-term projection output, e.g., INTV = 100 - detailed projection output is given every 100 years (starting from base year)	200
29-30	I2	ITOLX	Choice of stopping criterion for stable population analysis ITOLX = 2: criterion is difference in growth ratio between the first and the last region ITOLX = 3: criterion is difference in growth ratio of the first region between the actual and the previous time period	2
31-32	I2	NTOLL	Tolerance level for stopping criterion Tolerance level = 10^{-NTOLL}	7
33-34	I2	NEIG	Option to compute in FERMOB the dominant eigenvalue and associated right and left eigenvectors of <u>all</u> the moments of the (weighted) generalized maternity and mobility functions and of the matrices of mean ages and variances. NEIG = 0: no computation (generally NEIG = 0) NEIG = 1: computation	

6. Population data, births, deaths and migrants

The data related to each region are given sequentially, i.e.,

- observations for region 1
- observations for region 2

.
.
.

- observations for region NR

The age structure is contained on one card followed by continuation cards. If no observations on births are available or required (as for life table computations), the number of births by age of mother is replaced by zero (i.e., 0.).

Intra-regional migration data are read in and must be replaced by 0's if not available.

The sequence of cards and the formats are as follows (in the case of 18 age groups):

Observations for Region I:

	CARD #	COLS	FORMAT	VAR. NAME
	1a	1-80	8F10.0	POP(X,I),X = 1,8
	1b	1-80	8F10.0	POP(X,I),X = 9,16
	1c	1-20	2F10.0	POP(X,I),X = 17,18
	2a	1-80	10F8.0	BIRTH(X,I),X = 1,10
	2b	1-64	8F8.0	BIRTH(X,I),X = 11,18
	3a	1-80	10F8.0	DEATH(X,I),X = 1,10
	3b	1-64	8F8.0	DEATH(X,I),X = 11,18
J = 1,NR	4a	1-80	10F8.0	OMIG(X,J,I),X = 1,10
	4b	1-64	8F8.0	OMIG(X,J,I),X = 11,18
	. . . .			
	3+2(NR-1)a	1-80	10F8.0	OMIG(X,NR,I),X = 1,10
	3+2(NR-1)b	1-64	8F8.0	OMIG(X,NR,I),X = 11,18

REFERENCES

Arriaga, P. (1975), *Computer Programs for Demographic Analysis*, U.S. Bureau of the Census, Washington, DC.

Coale, A.J. (1972), *The Growth and Structure of Human Populations: A Mathematical Investigation*, Princeton University Press, Princeton, N.J.

Feeney, G.M. (1970), Stable Age by Region Distributions, *Demography*, 6, 341-348.

Fisher, R.A. (1929), *The Genetical Theory of Natural Selection*, 2nd rev. ed., Dover, New York.

Goodman, L.A. (1967), On the Reconciliation of Mathematical Theories of Population Growth, *Journal of the Royal Statistical Society*, A-130, 541-553.

Goodman, L.A. (1971), On the Sensitivity of the Intrinsic Growth Rates to Changes in the Age-Specific Birth and Death Rates, *Theoretical Population Biology*, 2, 339-354.

Greenberg, M., D. Krueckeberg, and R. Mautner (1973), *Long-Range Population Projections for Minor Civil Divisions: Computer Programs and User's Manual*, Rutgers University Center for Urban Policy Research, New Brunswick, N.J.

Keyfitz, N. (1968), *Introduction to the Mathematics of Population*, Addison-Wesley, Reading, Mass.

Keyfitz, N. (1969), Age Distribution and Stable Equivalent, *Demography*, 6, 261-269.

Keyfitz, N. (1971), On the Momentum of Population Growth, *Demography*, 7, 71-80.

Keyfitz, N. (1972), On Future Population, *Journal of the American Statistical Association*, 67, 347-363.

Keyfitz, N. (1975), Reproductive Value: With Applications to Migration, Contraception, and Zero Population Growth, in H. Blalock, et al., eds., *Quantitative Sociology*, Academic Press, New York, 587-612.

Keyfitz, N., and W. Flieger (1971), *Population: Facts and Methods of Demography*, Freeman, San Francisco.

Keyfitz, N., D. Nagnur, and D. Sharma (1967), On the Interpretation of Age Distributions, *Journal of the American Statistical Association*, 62, 862-874.

Land, A., and S. Powell (1973), *FORTRAN Codes for Mathematical Programming: Linear, Quadratic and Discrete*, John Wiley and Sons, London.

Long, L.H. (1973), New Estimates of Migration Expectancies in the United States, *Journal of the American Statistical Association*, 68, 37-43.

Rogers, A. (1966), The Multiregional Growth Operator and the Stable Interregional Age Structure, *Demography*, 3, 537-544.

Rogers, A. (1968), *Matrix Analysis of Interregional Population Growth and Distribution*, University of California Press, Berkeley.

Rogers, A. (1971), *Matrix Methods in Urban and Regional Analysis*, Holden-Day, San Francisco.

Rogers, A. (1975a), *Introduction to Multiregional Mathematical Demography*, John Wiley and Sons, New York.

Rogers, A. (1975b), *Spatial Migration Expectancies*, RM-75-57, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rogers, A., and J. Ledent (1976), Increment-Decrement Life Tables: A Comment, *Demography*, 13, 287-290.

Rogers, A., and F. Willekens (1976a), *Spatial Zero Population Growth*, RM-76-25, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rogers, A., and F. Willekens (1976b), *The Spatial Reproductive Value and the Spatial Momentum of Zero Population Growth*, RM-76-81, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rogers, A., and F. Willekens (1976c), Spatial Population Dynamics, *Papers of the Regional Science Association*, 36, 3-34.

Schoen, R. (1975), Constructing Increment-Decrement Life Tables, *Demography*, 12, 313-324.

Wilber, G.W. (1963), Migration Expectancy in the United States, *Journal of the American Statistical Association*, 58, 444-453.

Willekens, F. (1977), *The Spatial Reproductive Value: Theory and Applications*, RM-77-9, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Appendix A

Glossary of Mathematical Symbols and FORTRAN Names of Demographic Variables

Glossary of mathematical symbols and FORTRAN names of demographic variables.

Symbol	Page in Rogers (1975)	FORTRAN Name	Description
			Subscripts
		X	age group (1,2,3,...)
		IO(or I)	region of birth
		I	region of residence
		J	region of destination (in case of migration)
			Data and Preliminary Analysis
$K_i(x)$	82	POP(X,I)	population by age and region
		BIRTH(X,I)	births " " " "
$D_i(x)$	82	DEATH(X,I)	deaths " " " "
$K_{ij}(x)$	82	OMIG(X,J,I)	migrants from I to J by age and region
$\ell_i(0)$	73	RADIX(I)	radix of region I
		REG(I)	name of region I
$M_{i\delta}(x)$	82	RATD(X,I)	age-specific death rates of region I
		RATF(X,I)	age-specific fertility rates of region I
$M_{ij}(x)$	83	RATM(X,J,I)	age-specific migration rates from I to J

		RATDT(X)	age-specific death rates for whole system
		RATFT(X)	age-specific fertility rates for whole system
		RATMT(X)	age-specific migration rates for whole system
		POPC(X)	age distribution of population for total system

Life Table

$q_i(x)$	60	Q(X,I)	probability of dying in I between ages X and X + h.
$p_{ij}(x)$	60	P(X,J,I)	probability of being in J at age X + h, while in I at X
${}_{i0}\ell_j(x)$	60	CL(X,I,J)	probability that an individual born in I will be in J at age X
${}_{i0}L_j(x)$	61	L(X,I,J)	number of years lived in J between ages X and X + h by an individual born in I
${}_{i0}e_j(x)$	63	E(X,I,J)	expected number of years remained to be lived in region J by an individual born in I, and now of age X
$s_{ij}(x)$	79	SU(X,I,J)	proportion of people aged X to X + h in region I, surviving to be in region J and X + h to X + 2h years old h years later

Population Projection

$s_{ij}(x)$	118	SU,(X,I,J)	see life table
$b_{ij}(x)$	118	BR(X,J,I)	average number of babies born during the unit time interval and alive in region J at end of that interval, per X to (X + h) year old residents of region I at beginning of that interval

Fertility and Mobility Analysis

		ZMOM(P,J,I)	P-th moment of the integral function
		HU(X)	weighting factor for generalized net maternity and mobility functions HU(X) = 1 for stationary populations HU(X) = $e^{-r(x+NY*0.5)}$ for stable populations
		ZGRAL(X,J)	the age- and region-specific rates entering the integral function

Spatial Reproductive Value and Further Stable Population Analysis

$\bar{\underset{\sim}{n}}_x$		V(X,J,I)	discounted number of offspring in region J per person residing in region I at age X, respectively in age group X to X + 4
$\bar{\underset{\sim}{\Psi}}(r)$		PSI(J,I)	zero-th moment of the weighted generalized net maternity function
		ROOTPSI	dominant eigenvalue of $\bar{\underset{\sim}{\Psi}}(r)$
$\{\underset{\sim}{Q}_1^s\}$	93	VRPSI(J)	right eigenvector of $\bar{\underset{\sim}{\Psi}}(r)$, associated with ROOTPSI
$\{\underset{\sim}{v}(0)\}$		VLPSI(I)	left eigenvector of $\bar{\underset{\sim}{\Psi}}(r)$, associated with ROOTPSI
$\{\underset{\sim}{Q}\}$	105	QQ(I)	stable equivalent of regional births
$\{\underset{\sim}{Y}\}$	112	YY(I)	stable equivalent of the regional total population
$\{\underset{\sim}{K}(x)\}$		POPST(X,I)	stable equivalent of the population by age and region

		Momentum of Spatial Zero Population Growth	
$\underset{\sim}{e}(o)$	74	EO(J,I)	matrix of life expectancies at birth: average lifetime an I-born person may expect to live in region J.
$\underset{\sim}{R}(0)$	106	RO(J,I)	NRR - matrix: the average number of children an I-born person may expect to have in region J.
$\underset{\sim}{R}(1)$	106	R1(J,I)	first moment of the generalized net maternity function
$\{\underset{\sim}{\hat{Q}}\}$		QQZP(I)	regional allocation of births in ZPG-population
$\{\underset{\sim}{\hat{Y}}\}$		YYZP(I)	regional allocation of the total population in ZPG-population
		Zero Population Growth Analysis	
		VI(I)	vector containing the diagonal elements of the matrix of fertility adjustment factors.

Appendix B

Multiregional Life Table: Slovenia - Rest of Yugoslavia

Option 1

table - multiregional (two-region) life table option 1

age	q(x,1)	p(x,1,1)	p(x,2,1)	l(x,1,1)	l(x,2,1)	ll(x,1,1)	ll(x,2,1)	m(x,2,1)	md(x,1)	s(x,1,1)	s(x,2,1)	e(x,1,1)	e(x,2,1)
0	0.030077	0.956075	0.013848	100000.	0.	4.89019	0.03462	0.002332	0.006150	0.970926	0.013035	54.90	7.67
5	0.002145	0.936462	0.011393	95607.	1385.	4.74804	0.09633	0.002294	0.000432	0.988779	0.009410	61.87	7.87
10	0.001479	0.991129	0.007393	94314.	2468.	4.69485	0.14065	0.001485	0.000297	0.981625	0.016363	57.09	7.79
15	0.002543	0.972027	0.025430	93480.	3158.	4.60887	0.21671	0.005158	0.000516	0.966690	0.030203	52.32	7.66
20	0.003664	0.961182	0.035155	90875.	5511.	4.45622	0.35392	0.007170	0.000747	0.965201	0.031282	47.63	7.45
25	0.003332	0.969422	0.027246	87373.	8646.	4.30242	0.48958	0.005534	0.000677	0.972905	0.022969	43.22	7.11
30	0.004934	0.976509	0.018557	84724.	10937.	4.18689	0.58329	0.003756	0.000999	0.980783	0.013711	38.88	6.63
35	0.006075	0.985166	0.008758	82752.	12395.	4.10727	0.63431	0.001765	0.001224	0.935293	0.006898	34.69	6.05
40	0.009550	0.985423	0.005026	81539.	12978.	4.04752	0.65422	0.001013	0.001924	0.982555	0.003859	30.58	5.42
45	0.017658	0.979646	0.002686	80362.	13191.	3.97736	0.65844	0.000543	0.003570	0.975363	0.002965	26.57	4.77
50	0.025742	0.970992	0.003266	78732.	13147.	3.87973	0.65323	0.000663	0.005224	0.964466	0.003160	22.72	4.14
55	0.039176	0.957745	0.003079	76457.	12933.	3.74240	0.63946	0.000629	0.008004	0.945699	0.003635	19.00	3.53
60	0.062607	0.933122	0.004271	73239.	12596.	3.53982	0.61185	0.000884	0.012955	0.904755	0.004300	15.44	2.93
65	0.121193	0.874355	0.004448	68353.	11878.	3.20318	0.56273	0.000949	0.025864	0.831858	0.004087	12.11	2.37
70	0.212840	0.783256	0.003903	59774.	10631.	2.66489	0.48028	0.000976	0.047742	0.737165	0.004125	9.25	1.90
75	0.317016	0.678322	0.004662	46822.	8580.	1.96464	0.36889	0.001111	0.075556	0.590754	0.003602	6.94	1.55
80	0.535769	0.461657	0.002573	31764.	6175.	1.16082	0.24326	0.000704	0.146620	0.620486	0.003141	4.96	1.29
85	1.000000	0.000000	0.000000	14669.	3555.	0.72043	0.24777	0.001111	0.203611	0.000000	0.000000	3.95	1.36

age	q(x,2)	p(x,2,2)	p(x,1,2)	l(x,2,2)	l(x,1,2)	ll(x,2,2)	ll(x,1,2)	m(x,1,2)	md(x,2)	s(x,2,2)	s(x,1,2)	e(x,2,2)	e(x,1,2)
0	0.106299	0.892412	0.001289	100000.	0.	4.73103	0.00322	0.000272	0.022468	0.941182	0.001082	65.43	0.82
5	0.003340	0.995833	0.000827	89241.	129.	4.45280	0.00825	0.000166	0.000669	0.996330	0.000807	67.92	0.92
10	0.002385	0.996831	0.000785	88871.	201.	4.43654	0.01175	0.000157	0.000478	0.994578	0.002077	63.14	0.91
15	0.004308	0.992312	0.003380	88591.	269.	4.41268	0.02074	0.000679	0.000865	0.990806	0.004009	58.30	0.90
20	0.006066	0.989272	0.004661	87916.	561.	4.37273	0.03774	0.000937	0.001220	0.989424	0.003606	53.57	0.88
25	0.007885	0.989596	0.002519	86993.	949.	4.32767	0.05220	0.000506	0.001585	0.989562	0.002137	48.92	0.84
30	0.008720	0.989539	0.001741	86114.	1139.	4.28369	0.06003	0.000350	0.001753	0.989053	0.001437	44.35	0.79
35	0.010307	0.988570	0.001123	85234.	1262.	4.23762	0.06503	0.000226	0.002073	0.936714	0.001017	39.78	0.73
40	0.014251	0.984838	0.000910	84271.	1339.	4.18177	0.06839	0.000183	0.002872	0.982576	0.000688	35.24	0.66
45	0.019255	0.980281	0.000464	83000.	1396.	4.10917	0.07007	0.000094	0.003889	0.974183	0.000550	30.80	0.59
50	0.031397	0.967963	0.000640	81367.	1406.	4.00329	0.07060	0.000130	0.006382	0.960149	0.000816	26.44	0.52
55	0.046921	0.952076	0.001003	78765.	1418.	3.84398	0.07136	0.000205	0.009615	0.935550	0.000931	22.30	0.44
60	0.080837	0.918191	0.000971	74994.	1437.	3.59649	0.07126	0.000203	0.016857	0.894846	0.000840	18.36	0.37
65	0.129850	0.869420	0.000730	68865.	1414.	3.21861	0.06749	0.000156	0.027784	0.830223	0.000533	14.85	0.30
70	0.214514	0.785139	0.000347	59879.	1286.	2.67244	0.05786	0.000078	0.048067	0.745167	0.000368	11.80	0.24
75	0.305323	0.694256	0.000420	47019.	1028.	1.99166	0.04364	0.000099	0.072084	0.640254	0.000530	9.46	0.18
80	0.436764	0.562470	0.000766	32648.	717.	1.27532	0.02683	0.000196	0.111814	1.003547	0.000659	7.66	0.13
85	1.000000	0.000000	0.000000	18365.	356.	1.27993	0.01749	0.000076	0.143486	0.000000	0.000000	6.84	0.09

probabilities of dying and migrating
************* option 1 *************
****

region slovenia
****

age	death	migration from slovenia to slovenia	r.yugos.
0	0.030077	0.956075	0.013848
5	0.002145	0.986462	0.011393
10	0.001479	0.991129	0.007393
15	0.002543	0.972027	0.025430
20	0.003664	0.961182	0.035155
25	0.003332	0.969422	0.027246
30	0.004934	0.976509	0.018557
35	0.006075	0.985166	0.008758
40	0.009550	0.985423	0.005026
45	0.017668	0.979646	0.002686
50	0.025742	0.970992	0.003266
55	0.039176	0.957745	0.003079
60	0.062607	0.933122	0.004271
65	0.121198	0.874355	0.004448
70	0.212840	0.783256	0.003903
75	0.317016	0.678322	0.004662
80	0.535769	0.461657	0.002573
85	1.000000	0.000000	0.000000

region r.yugos.
****

age	death	migration from r.yugos. to slovenia	r.yugos.
0	0.106299	0.001289	0.892412
5	0.003340	0.000827	0.995833
10	0.002385	0.000785	0.996831
15	0.004308	0.003380	0.992312
20	0.006067	0.004661	0.989272
25	0.007885	0.002519	0.989596
30	0.008720	0.001741	0.989539
35	0.010307	0.001123	0.988570
40	0.014251	0.000910	0.984838
45	0.019255	0.000464	0.980281
50	0.031397	0.000640	0.967963
55	0.046922	0.001003	0.952076
60	0.080837	0.000971	0.918191
65	0.129850	0.000730	0.869420
70	0.214514	0.000347	0.785139
75	0.305323	0.000420	0.694256
80	0.436764	0.000766	0.562470
85	1.000000	0.000000	0.000000

Appendix C

Data for Multiregional Demographic Analysis*

C1. Great Britain, 1970**

C2. Hungary, 1974***

*As part of the Comparative Migration and Settlement Study, IIASA is applying the computer programs presented in this report to analyze patterns of multiregional demographic change in the IIASA National Member Organization countries. This analysis is carried out jointly with national scholars. The data given in this Appendix provide a sample of input data.

**The data for Great Britain have been provided by Prof. P. Rees of the University of Leeds.

***The data for Hungary have been provided by Dr. K. Tekse and Dr. K. Bies of the Demographic Research Institute, Budapest.

Observed population characteristics: Great Britain

region north

age	population	births	deaths	migration from north north	yorkshir	north to n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	263500.	0.	1125.	0.	945.	803.	451.	337.	294.	1292.	513.	212.	701.
5	299600.	0.	104.	0.	757.	667.	318.	365.	224.	1054.	278.	39.	355.
10	247500.	0.	77.	0.	542.	457.	216.	256.	149.	827.	211.	53.	247.
15	237600.	5693.	142.	0.	1097.	725.	330.	437.	191.	2199.	503.	81.	449.
20	244300.	20203.	162.	0.	2051.	1352.	754.	750.	378.	3633.	688.	261.	949.
25	213000.	14813.	154.	0.	1108.	967.	591.	454.	343.	1946.	475.	191.	652.
30	206300.	7487.	192.	0.	626.	571.	322.	388.	181.	1067.	324.	105.	467.
35	203000.	3149.	320.	0.	485.	471.	247.	180.	104.	747.	175.	56.	254.
40	211900.	852.	612.	0.	361.	344.	182.	131.	72.	552.	129.	42.	186.
45	227800.	49.	1155.	0.	361.	234.	135.	103.	59.	454.	119.	44.	162.
50	187300.	0.	1553.	0.	309.	199.	112.	83.	52.	391.	101.	37.	139.
55	203900.	0.	2614.	0.	221.	154.	87.	62.	37.	280.	80.	20.	102.
60	193000.	0.	3965.	0.	166.	200.	88.	31.	40.	203.	116.	10.	90.
65	161200.	0.	5395.	0.	129.	109.	63.	34.	35.	184.	75.	10.	85.
70	115500.	0.	5992.	0.	78.	67.	38.	21.	21.	111.	48.	6.	52.
75	74260.	0.	5959.	0.	75.	66.	39.	20.	19.	112.	45.	6.	47.
80	42068.	0.	5299.	0.	47.	42.	26.	13.	13.	72.	28.	5.	30.
85	22872.	0.	5347.	0.	28.	25.	19.	9.	9.	43.	16.	2.	17.
total	3359700.	52246.	40167.	0.	9386.	7453.	3929.	3730.	2221.	15182.	3924.	1243.	4984.

region yorkshir

age	population	births	deaths	migration from yorkshir to north	yorkshir	n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	404000.	0.	1920.	1619.	0.	1411.	1614.	762.	645.	2143.	684.	290.	461.
5	408500.	0.	142.	1163.	0.	1125.	1123.	410.	475.	1782.	410.	184.	320.
10	353200.	0.	109.	782.	0.	740.	728.	295.	299.	1252.	273.	132.	202.
15	331600.	9391.	221.	1306.	0.	1183.	1171.	748.	369.	2838.	518.	239.	279.
20	358100.	31340.	264.	2416.	0.	2563.	2778.	1371.	714.	5623.	1100.	532.	531.
25	309200.	23140.	226.	1342.	0.	1713.	1712.	644.	627.	2415.	793.	355.	433.
30	288300.	11104.	263.	896.	0.	1078.	1136.	494.	323.	1605.	442.	221.	278.
35	280400.	4936.	412.	753.	0.	770.	634.	435.	253.	1165.	281.	135.	259.
40	294400.	1384.	803.	555.	0.	571.	474.	304.	184.	845.	208.	98.	189.
45	319500.	96.	1464.	450.	0.	489.	428.	125.	128.	498.	172.	74.	140.
50	279600.	0.	2039.	385.	0.	422.	367.	107.	110.	431.	144.	63.	121.
55	297800.	0.	3720.	299.	0.	318.	231.	82.	85.	308.	125.	53.	95.
60	276300.	0.	5633.	294.	0.	292.	290.	72.	59.	185.	179.	69.	96.
65	232000.	0.	7814.	312.	0.	306.	276.	83.	51.	394.	164.	53.	53.
70	168800.	0.	8782.	178.	0.	181.	160.	51.	30.	187.	88.	24.	29.
75	111523.	0.	8696.	110.	0.	113.	99.	32.	19.	115.	55.	16.	16.
80	63054.	0.	7671.	63.	0.	64.	56.	19.	10.	67.	30.	10.	10.
85	35623.	0.	8032.	34.	0.	35.	30.	11.	5.	35.	16.	6.	6.
total	4811900.	81391.	58271.	12957.	0.	13374.	13357.	6059.	4336.	21798.	5682.	2601.	3518.

region n.west

age	population	births	deaths	migration from north	yorkshir	n.west to n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	572800.	0.	2718.	715.	1094.	0.	637.	1316.	273.	2752.	829.	1977.	788.
5	591300.	0.	221.	489.	891.	0.	494.	641.	249.	2233.	837.	836.	441.
10	506900.	0.	171.	350.	697.	0.	342.	453.	184.	1625.	585.	562.	291.
15	454100.	12598.	296.	700.	939.	0.	574.	869.	405.	3422.	949.	765.	353.
20	590800.	42037.	360.	1517.	2106.	0.	1369.	2198.	603.	6836.	1518.	1729.	985.
25	430000.	32687.	311.	750.	1320.	0.	864.	1239.	391.	3278.	1036.	1068.	741.
30	399300.	16255.	372.	546.	1003.	0.	437.	827.	210.	2307.	600.	836.	536.
35	387700.	7322.	610.	323.	564.	0.	312.	425.	127.	1784.	499.	514.	224.
40	406300.	1995.	1202.	264.	456.	0.	252.	352.	102.	1390.	403.	433.	183.
45	448000.	136.	2327.	281.	455.	0.	249.	400.	106.	941.	409.	575.	189.
50	383700.	0.	3292.	188.	301.	0.	163.	263.	69.	621.	271.	332.	126.
55	429300.	0.	5939.	180.	267.	0.	153.	245.	71.	574.	270.	405.	120.
60	397300.	0.	8783.	187.	129.	0.	117.	191.	69.	442.	320.	674.	142.
65	333500.	0.	11626.	152.	286.	0.	172.	186.	60.	525.	220.	255.	64.
70	243200.	0.	13135.	98.	191.	0.	113.	127.	37.	343.	144.	167.	44.
75	158469.	0.	12666.	67.	125.	0.	73.	84.	27.	227.	94.	103.	29.
80	90917.	0.	11335.	38.	72.	0.	43.	48.	13.	127.	52.	62.	17.
85	49114.	0.	11442.	19.	36.	0.	21.	25.	7.	63.	27.	30.	9.
total	6788700.	113930.	86856.	6364.	10752.	0.	6385.	9889.	3008.	29490.	9933.	10480.	5282.

region e.midl.

age	population	births	deaths	migration from north	yorkshir	e.midl. to n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	234700.	0.	1152.	571.	1625.	613.	0.	1203.	721.	1927.	788.	262.	597.
5	286300.	0.	93.	393.	1133.	488.	0.	893.	647.	1543.	647.	224.	327.
10	248700.	0.	72.	248.	679.	296.	0.	546.	402.	1025.	407.	132.	191.
15	226700.	5982.	141.	477.	1117.	499.	0.	931.	695.	2524.	759.	219.	257.
20	263100.	20942.	180.	865.	2121.	931.	0.	1786.	896.	4399.	1050.	458.	575.
25	217600.	17111.	160.	533.	1615.	820.	0.	1264.	662.	2234.	777.	256.	420.
30	204500.	7693.	190.	366.	1001.	524.	0.	915.	489.	1418.	468.	203.	231.
35	203300.	3097.	230.	234.	721.	230.	0.	670.	381.	1005.	349.	95.	142.
40	210300.	862.	540.	163.	514.	199.	0.	469.	269.	730.	245.	72.	101.
45	224600.	68.	975.	101.	379.	172.	0.	276.	185.	675.	204.	76.	77.
50	191200.	0.	1356.	80.	303.	139.	0.	217.	149.	543.	167.	62.	61.
55	199200.	0.	2321.	66.	244.	111.	0.	167.	122.	416.	142.	49.	51.
60	185600.	0.	3549.	76.	255.	99.	0.	85.	151.	237.	222.	54.	66.
65	156500.	0.	5037.	39.	293.	87.	0.	129.	154.	284.	235.	78.	30.
70	115000.	0.	5650.	22.	169.	50.	0.	83.	88.	168.	133.	45.	17.
75	75798.	0.	5624.	12.	93.	27.	0.	47.	48.	93.	72.	24.	9.
80	43814.	0.	5081.	7.	54.	17.	0.	30.	28.	57.	42.	15.	6.
85	25388.	0.	5556.	6.	31.	10.	0.	18.	16.	23.	23.	10.	5.
total	3362800.	55755.	37957.	4259.	12347.	5362.	0.	9734.	6103.	19306.	6760.	2350.	3163.

region w.midl.

age	population	births	deaths	migration from w.midl. to north	yorkshir	n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	453100.	0.	1959.	463.	565.	1225.	1680.	0.	269.	2112.	1068.	637.	271.
5	440200.	0.	167.	339.	456.	755.	1008.	0.	308.	1931.	973.	493.	328.
10	376700.	0.	129.	256.	351.	582.	760.	0.	239.	1555.	748.	367.	248.
15	358000.	9462.	230.	334.	594.	933.	1171.	0.	402.	3178.	1189.	473.	328.
20	421800.	31682.	299.	622.	1036.	1659.	2456.	0.	495.	5848.	1836.	1020.	571.
25	355000.	27196.	248.	437.	636.	1393.	1723.	0.	343.	3174.	1221.	807.	463.
30	324500.	13196.	291.	230.	567.	686.	1079.	0.	187.	1905.	855.	494.	219.
35	321300.	6010.	444.	228.	270.	558.	709.	0.	160.	1482.	710.	348.	213.
40	327500.	1656.	843.	187.	231.	473.	598.	0.	135.	1254.	610.	304.	178.
45	356400.	138.	1594.	76.	181.	361.	369.	0.	112.	841.	508.	270.	125.
50	298700.	0.	2198.	59.	139.	278.	284.	0.	88.	650.	399.	212.	96.
55	302800.	0.	3668.	57.	119.	253.	246.	0.	78.	566.	393.	208.	91.
60	274700.	0.	5471.	95.	67.	276.	162.	0.	76.	409.	742.	352.	133.
65	219200.	0.	7150.	49.	36.	137.	82.	0.	40.	207.	378.	187.	67.
70	154500.	0.	7778.	18.	14.	55.	34.	0.	15.	83.	150.	72.	27.
75	101518.	0.	7674.	15.	9.	44.	24.	0.	10.	62.	112.	50.	20.
80	58390.	0.	6915.	7.	5.	24.	15.	0.	6.	36.	64.	27.	13.
85	33692.	0.	7424.	6.	2.	15.	6.	0.	5.	20.	36.	15.	6.
total	5173000.	89340.	54482.	3478.	5278.	9707.	12406.	0.	2964.	25313.	11997.	6336.	3397.

region e.anglia

age	population	births	deaths	migration from e.anglia to north	yorkshir	n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	128700.	0.	489.	85.	348.	272.	537.	359.	0.	2361.	464.	187.	311.
5	134700.	0.	50.	60.	207.	223.	394.	223.	0.	1710.	449.	72.	174.
10	113900.	0.	38.	41.	132.	132.	246.	153.	0.	1121.	290.	49.	105.
15	114200.	2695.	83.	92.	230.	119.	372.	319.	0.	2134.	479.	91.	100.
20	137700.	10058.	113.	276.	478.	370.	899.	475.	0.	4185.	620.	185.	279.
25	108600.	8029.	57.	143.	277.	276.	595.	333.	0.	2748.	337.	141.	258.
30	103100.	3475.	69.	81.	242.	229.	361.	257.	0.	1524.	220.	141.	144.
35	101300.	1400.	123.	81.	167.	125.	265.	130.	0.	1156.	309.	53.	90.
40	97800.	337.	221.	55.	104.	78.	173.	80.	0.	744.	183.	37.	55.
45	109300.	24.	397.	51.	82.	59.	168.	36.	0.	702.	102.	64.	39.
50	92400.	0.	545.	49.	80.	57.	164.	34.	0.	683.	93.	63.	38.
55	100800.	0.	935.	26.	46.	29.	87.	21.	0.	374.	53.	34.	21.
60	95800.	0.	1539.	13.	57.	10.	61.	49.	0.	381.	52.	38.	20.
65	84700.	0.	2393.	36.	33.	6.	74.	39.	0.	308.	69.	13.	16.
70	51400.	0.	2684.	35.	30.	6.	72.	38.	0.	393.	67.	12.	15.
75	43795.	0.	2940.	17.	17.	3.	38.	20.	0.	154.	33.	6.	8.
80	27672.	0.	2899.	11.	11.	2.	23.	11.	0.	101.	22.	4.	5.
85	17633.	0.	3396.	7.	6.	2.	16.	8.	0.	64.	13.	3.	4.
total	1673500.	26018.	19021.	1159.	2547.	1938.	4546.	2576.	0.	20753.	3867.	1193.	1682.

region s.east

age	population	births	deaths	migration from north	yorkshir	s.east to n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	1392600.	0.	5192.	1339.	2051.	2298.	3247.	2375.	4603.	0.	6146.	982.	1840.
5	1366500.	0.	417.	866.	1217.	1756.	2125.	1529.	2729.	0.	4796.	778.	1359.
10	1197800.	0.	330.	612.	849.	1198.	1453.	1039.	1868.	0.	3408.	590.	956.
15	1139500.	24194.	725.	1058.	1400.	1774.	2104.	1995.	2564.	0.	5690.	1233.	1597.
20	1382200.	94958.	972.	2573.	3690.	5197.	5660.	4797.	5692.	0.	10495.	2434.	3610.
25	1152200.	85789.	805.	1725.	2327.	3135.	3929.	2730.	4201.	0.	7342.	1535.	2438.
30	1059300.	41336.	949.	1084.	1348.	1787.	2049.	1905.	2783.	0.	5034.	885.	1546.
35	1027800.	16793.	1261.	564.	849.	1365.	1355.	1192.	1919.	0.	3433.	654.	945.
40	1031000.	4171.	2363.	409.	618.	995.	977.	858.	1440.	0.	2601.	473.	683.
45	1151100.	281.	4463.	402.	544.	797.	734.	817.	1465.	0.	3125.	520.	638.
50	1019400.	0.	6455.	338.	458.	671.	618.	610.	1242.	0.	2664.	441.	541.
55	1097800.	0.	11738.	331.	431.	633.	590.	463.	1292.	0.	2751.	435.	519.
60	998300.	0.	17427.	320.	287.	562.	581.	342.	1628.	0.	3356.	391.	338.
65	824600.	0.	23619.	297.	337.	469.	627.	420.	1191.	0.	3066.	368.	387.
70	614500.	0.	27208.	136.	163.	225.	295.	202.	556.	0.	4453.	177.	181.
75	410253.	0.	27512.	79.	105.	141.	184.	130.	336.	0.	905.	107.	110.
80	268275.	0.	28533.	49.	68.	90.	117.	84.	213.	0.	573.	67.	69.
85	181372.	0.	36202.	27.	42.	56.	69.	59.	128.	0.	349.	41.	42.
total	17315502.	267522.	196171.	12209.	16784.	23155.	26726.	21657.	35830.	0.	67198.	12751.	17799.

region s.west

age	population	births	deaths	migration from north	yorkshir	s.west to n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	295400.	0.	1126.	345.	642.	395.	521.	994.	390.	5132.	0.	567.	546.
5	309700.	0.	93.	292.	434.	453.	465.	583.	283.	3647.	0.	357.	476.
10	265700.	0.	70.	191.	275.	292.	297.	422.	196.	2598.	0.	247.	311.
15	254400.	5967.	155.	266.	318.	393.	426.	963.	332.	5551.	0.	477.	442.
20	284400.	21739.	190.	573.	937.	928.	655.	1888.	678.	10603.	0.	365.	1053.
25	228600.	17291.	150.	410.	661.	703.	598.	1024.	377.	6283.	0.	620.	695.
30	217200.	7729.	183.	399.	387.	496.	376.	693.	254.	3516.	0.	419.	453.
35	212100.	3085.	241.	207.	258.	355.	262.	516.	201.	2594.	0.	219.	292.
40	220200.	820.	466.	139.	176.	241.	175.	364.	140.	1326.	0.	168.	197.
45	242800.	66.	946.	67.	102.	137.	123.	321.	110.	1649.	0.	197.	94.
50	213700.	0.	1365.	59.	89.	122.	107.	284.	97.	1456.	0.	173.	84.
55	233500.	0.	2578.	44.	62.	35.	71.	189.	72.	1040.	0.	123.	59.
60	226900.	0.	4081.	64.	46.	72.	34.	82.	98.	1114.	0.	127.	58.
65	201800.	0.	5912.	14.	91.	88.	74.	186.	82.	1018.	0.	92.	39.
70	151800.	0.	6906.	10.	57.	59.	45.	117.	50.	642.	0.	59.	28.
75	103238.	0.	7496.	7.	42.	41.	33.	85.	36.	453.	0.	40.	19.
80	62969.	0.	7117.	5.	24.	26.	20.	53.	22.	278.	0.	24.	13.
85	39293.	0.	8196.	3.	16.	17.	11.	30.	12.	163.	0.	15.	7.
total	3763700.	56697.	47271.	3905.	4567.	4899.	4283.	8784.	3430.	49563.	0.	4789.	4856.

region wales

age	population	births	deaths	migration from wales to north	yorkshir	n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	212900.	0.	912.	176.	292.	935.	302.	599.	178.	1072.	715.	0.	217.
5	229300.	0.	69.	175.	104.	450.	195.	459.	85.	899.	595.	0.	145.
10	195100.	0.	52.	110.	71.	319.	130.	318.	53.	646.	399.	0.	90.
15	191300.	4933.	126.	155.	197.	711.	271.	690.	140.	1824.	823.	0.	103.
20	200900.	16230.	147.	275.	498.	1213.	454.	1100.	227.	3777.	1330.	0.	318.
25	166700.	12172.	120.	143.	204.	757.	380.	700.	190.	1519.	764.	0.	364.
30	158300.	5811.	146.	164.	146.	450.	194.	339.	93.	859.	510.	0.	127.
35	157200.	2531.	226.	107.	105.	350.	137.	318.	86.	570.	317.	0.	94.
40	170500.	679.	460.	84.	91.	295.	117.	263.	68.	433.	276.	0.	83.
45	187300.	51.	959.	17.	76.	206.	85.	143.	23.	395.	258.	0.	39.
50	157600.	0.	1321.	11.	49.	130.	54.	93.	14.	248.	162.	0.	24.
55	170100.	0.	2192.	12.	54.	136.	54.	108.	14.	274.	189.	0.	23.
60	164700.	0.	3473.	18.	50.	131.	11.	176.	30.	357.	329.	0.	2.
65	145000.	0.	4840.	7.	47.	134.	20.	133.	25.	150.	141.	0.	15.
70	103000.	0.	5335.	6.	39.	117.	20.	118.	26.	135.	117.	0.	14.
75	60174.	0.	5263.	4.	26.	83.	15.	79.	18.	96.	79.	0.	9.
80	37183.	0.	4611.	2.	16.	47.	8.	47.	12.	56.	46.	0.	6.
85	20633.	0.	4746.	1.	7.	25.	5.	24.	6.	28.	23.	0.	4.
total	2733900.	42487.	34998.	1467.	2072.	6489.	2452.	5712.	1299.	13369.	7073.	0.	1674.

region scotland

age	population	births	deaths	migration from scotland to north	yorkshir	n.west	e.midl.	w.midl.	e.anglia	s.east	s.west	wales	scotland
0	449900.	0.	2029.	791.	773.	932.	789.	703.	212.	2532.	734.	161.	0.
5	475400.	0.	207.	556.	606.	730.	676.	471.	203.	1944.	565.	134.	0.
10	436300.	0.	140.	334.	370.	437.	395.	299.	133.	1314.	355.	89.	0.
15	387500.	9038.	238.	442.	525.	602.	361.	547.	284.	3132.	643.	191.	0.
20	404600.	31492.	324.	918.	734.	1235.	727.	927.	388.	5756.	1075.	283.	0.
25	322600.	25885.	273.	937.	565.	1069.	672.	535.	183.	3174.	641.	252.	0.
30	292600.	13406.	374.	452.	338.	764.	407.	461.	170.	2043.	370.	91.	0.
35	289700.	5875.	573.	371.	314.	517.	286.	197.	163.	1227.	280.	101.	0.
40	302000.	1537.	949.	266.	229.	365.	210.	146.	116.	881.	206.	80.	0.
45	322900.	102.	1838.	149.	148.	138.	152.	126.	60.	522.	138.	116.	0.
50	284100.	0.	2570.	108.	107.	102.	110.	91.	44.	379.	100.	85.	0.
55	313400.	0.	4413.	73.	75.	75.	78.	64.	33.	276.	74.	51.	0.
60	288800.	0.	6618.	54.	43.	99.	49.	40.	30.	302.	61.	28.	0.
65	243300.	0.	8443.	77.	24.	91.	34.	39.	29.	227.	70.	18.	0.
70	173800.	0.	9353.	39.	14.	59.	21.	23.	16.	130.	37.	11.	0.
75	115000.	0.	9406.	31.	10.	54.	17.	19.	12.	111.	30.	11.	0.
80	63600.	0.	8018.	16.	6.	30.	9.	10.	7.	64.	17.	6.	0.
85	33600.	0.	7869.	8.	3.	17.	5.	4.	3.	35.	9.	3.	0.
total	5199100.	87335.	63640.	5622.	4899.	7316.	4939.	4702.	2091.	24049.	5405.	1716.	0.

Observed population characteristics: Hungary

region central

age	population	births	deaths	migration from central to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	183240.	0.	2099.	5553.	826.	1547.	874.	1052.	648.
5	157052.	0.	45.	2520.	334.	649.	391.	451.	274.
10	177336.	0.	61.	1878.	244.	415.	265.	391.	225.
15	241620.	6153.	150.	7932.	5305.	11190.	3543.	3312.	2237.
20	273523.	19484.	226.	12985.	6329.	11590.	4355.	6138.	3820.
25	246690.	14442.	229.	9491.	3903.	5455.	2479.	3114.	1999.
30	215011.	6503.	329.	5434.	1729.	2851.	1332.	1453.	931.
35	184038.	1759.	367.	3025.	1106.	1928.	734.	833.	569.
40	200093.	363.	685.	2543.	935.	1635.	720.	696.	451.
45	210055.	0.	1136.	2116.	851.	1393.	554.	599.	397.
50	210403.	0.	1723.	1751.	707.	1109.	597.	577.	420.
55	138638.	0.	1731.	992.	409.	551.	325.	383.	271.
60	139406.	0.	3754.	1093.	473.	667.	456.	484.	332.
65	145450.	0.	4632.	934.	278.	374.	237.	358.	228.
70	112054.	0.	5938.	814.	207.	265.	268.	321.	187.
75	72390.	0.	5995.	555.	104.	162.	200.	212.	115.
80	34329.	0.	4617.	356.	50.	105.	108.	132.	76.
85	16784.	0.	3832.	196.	32.	47.	65.	76.	29.
total	2968109.	48804.	37540.	60168.	22966.	41851.	17974.	20602.	13259.

region n.hung.

age	population	births	deaths	migration from n.hung. to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	127636.	0.	1112.	807.	5671.	539.	130.	259.	93.
5	120278.	0.	56.	428.	2852.	319.	78.	131.	69.
10	121525.	0.	44.	702.	2947.	251.	78.	206.	46.
15	133739.	4992.	110.	6595.	12300.	1313.	493.	738.	304.
20	124525.	13422.	133.	6452.	13186.	2333.	693.	1040.	411.
25	104499.	7546.	137.	3022.	7653.	1181.	332.	512.	224.
30	97776.	3483.	172.	1806.	3783.	630.	167.	267.	110.
35	96851.	1352.	226.	1184.	2292.	379.	95.	175.	65.
40	101370.	413.	355.	1092.	1892.	365.	107.	184.	63.
45	101299.	0.	535.	906.	1402.	275.	99.	101.	43.
50	95104.	0.	746.	816.	1261.	240.	89.	99.	40.
55	53748.	0.	625.	434.	568.	104.	44.	50.	23.
60	84102.	0.	1470.	420.	665.	142.	56.	52.	26.
65	68217.	0.	2078.	323.	570.	89.	37.	44.	31.
70	53391.	0.	2618.	249.	486.	76.	17.	33.	18.
75	34235.	0.	2901.	146.	327.	46.	13.	13.	10.
80	16981.	0.	2293.	79.	221.	25.	2.	11.	9.
85	8339.	0.	1936.	35.	99.	8.	1.	5.	4.
total	1543604.	31213.	17552.	25405.	58120.	8992.	2567.	3937.	1589.

region n.plain

age	population	births	deaths	migration from n.plain to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	138718.	0.	1026.	1550.	635.	5119.	445.	422.	177.
5	126733.	0.	43.	789.	331.	2449.	217.	239.	89.
10	127686.	0.	44.	1309.	283.	2929.	312.	206.	62.
15	166736.	4840.	103.	13673.	1895.	13039.	1428.	956.	279.
20	154039.	15336.	161.	11876.	2474.	13651.	1651.	1305.	513.
25	132471.	8920.	136.	5399.	1271.	6593.	695.	740.	251.
30	126539.	3841.	181.	2686.	607.	3932.	352.	421.	140.
35	117949.	1197.	226.	2030.	413.	1871.	139.	289.	91.
40	125200.	288.	379.	1777.	335.	1374.	145.	183.	52.
45	122769.	0.	618.	1459.	291.	1051.	126.	125.	55.
50	115946.	0.	949.	1152.	222.	868.	94.	117.	42.
55	67616.	0.	771.	526.	127.	442.	61.	59.	25.
60	97443.	0.	1848.	572.	108.	665.	59.	80.	33.
65	79169.	0.	2544.	465.	95.	602.	54.	62.	31.
70	61798.	0.	3313.	360.	83.	538.	54.	49.	24.
75	37919.	0.	3231.	225.	57.	413.	39.	33.	14.
80	17144.	0.	2394.	134.	40.	236.	20.	12.	10.
85	7869.	0.	1823.	71.	14.	97.	9.	12.	1.
total	1823844.	34422.	19792.	46305.	9336.	55070.	5912.	5316.	1889.

region s.plain

age	population	births	deaths	migration from s.plain to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	100335.	0.	992.	511.	112.	152.	275.	682.	6078.
5	96908.	0.	40.	275.	68.	71.	147.	359.	3472.
10	93719.	0.	35.	393.	36.	61.	160.	507.	3579.
15	113429.	4046.	94.	2946.	289.	394.	799.	2099.	13166.
20	106537.	10402.	101.	3781.	382.	490.	1016.	2565.	13953.
25	92435.	5962.	102.	1876.	212.	242.	524.	1215.	6016.
30	93441.	2739.	147.	984.	99.	126.	249.	614.	4031.
35	94550.	951.	239.	578.	59.	93.	155.	356.	2546.
40	97890.	223.	401.	468.	73.	63.	141.	305.	2248.
45	92897.	0.	491.	410.	41.	39.	116.	244.	1732.
50	90522.	0.	713.	395.	35.	36.	81.	201.	1529.
55	53031.	0.	575.	278.	18.	24.	52.	138.	803.
60	74958.	0.	1507.	339.	35.	33.	62.	165.	1031.
65	58422.	0.	1849.	274.	26.	23.	61.	146.	1099.
70	44892.	0.	2346.	269.	16.	22.	39.	122.	921.
75	26256.	0.	2433.	157.	13.	11.	46.	90.	637.
80	13967.	0.	1956.	104.	4.	7.	19.	46.	316.
85	6672.	0.	1593.	40.	4.	4.	10.	22.	137.
total	1357973.	24326.	15524.	14065.	1522.	1311.	3943.	9367.	65258.

region n.t-danu

age	population	births	deaths	migration from n.t-danu to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	90952.	0.	892.	978.	222.	287.	282.	7140.	618.
5	65466.	0.	33.	524.	137.	163.	148.	3235.	283.
10	88303.	0.	28.	684.	114.	131.	117.	4597.	293.
15	111394.	4019.	76.	4620.	649.	840.	860.	17359.	2097.
20	102138.	10003.	104.	5980.	806.	1135.	1090.	18300.	2476.
25	87665.	5506.	95.	3019.	475.	672.	551.	10153.	1159.
30	84549.	2213.	128.	1523.	258.	381.	329.	5023.	544.
35	83107.	702.	175.	789.	173.	259.	207.	2723.	324.
40	91754.	177.	316.	829.	147.	174.	160.	2299.	309.
45	92269.	0.	478.	682.	92.	143.	108.	1576.	218.
50	89518.	0.	689.	620.	91.	104.	103.	1466.	221.
55	50084.	0.	543.	376.	41.	47.	61.	759.	129.
60	77334.	0.	1417.	432.	45.	60.	74.	911.	149.
65	65918.	0.	2148.	400.	29.	46.	53.	892.	110.
70	51978.	0.	2777.	350.	29.	38.	47.	831.	95.
75	30893.	0.	2828.	255.	21.	22.	26.	569.	68.
80	13936.	0.	2070.	150.	14.	13.	10.	319.	32.
85	6436.	0.	1570.	74.	6.	10.	7.	175.	12.
total	1303694.	22620.	16373.	22285.	3349.	4516.	4233.	78318.	9137.

region s.t-danu

age	population	births	deaths	migration from s.t-danu to central	n.hung.	n.plain	s.plain	n.t-danu	s.t-danu
0	93825.	0.	838.	828.	183.	431.	4421.	323.	313.
5	91603.	0.	43.	463.	103.	186.	2380.	172.	152.
10	91179.	0.	34.	637.	68.	196.	3033.	195.	141.
15	120017.	4007.	102.	4092.	415.	1187.	12604.	734.	679.
20	114405.	10361.	149.	4947.	765.	1676.	12913.	1326.	1111.
25	101807.	6495.	120.	2440.	305.	671.	6875.	659.	547.
30	96688.	2856.	191.	1391.	170.	354.	3364.	370.	270.
35	91564.	933.	217.	806.	109.	196.	2070.	221.	172.
40	96128.	211.	365.	778.	131.	169.	1546.	149.	150.
45	100163.	0.	561.	625.	71.	110.	1320.	128.	102.
50	97377.	0.	773.	560.	65.	91.	1148.	127.	83.
55	55703.	0.	740.	302.	34.	46.	563.	58.	59.
60	83829.	0.	1779.	402.	51.	53.	826.	91.	63.
65	76144.	0.	2287.	378.	45.	50.	849.	78.	57.
70	59444.	0.	3023.	346.	16.	51.	710.	72.	47.
75	36736.	0.	3066.	241.	21.	40.	509.	49.	51.
80	17767.	0.	2434.	131.	12.	14.	325.	29.	32.
85	8880.	0.	2169.	69.	3.	11.	153.	11.	12.
total	1451260.	24364.	18891.	19436.	2568.	5523.	55609.	4824.	4011.

Appendix D

FORTRAN Listing of Computer Programs

D1. General Purpose Subroutines

```
      SUBROUTINE EIGEN (NR,NP,NEIG)
      DIMENSION ZMOMT(7),HU(7)
      COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
      COMMON /CINV/ CC(7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CREG/ REG(13)
       DOUBLE PRECISION REG
C ------------------------------------------------------------
C  NR : NUMBER OF ROWS (COLUMNS)
C   NP = 0 : DOMINANT EIGENVALUE AND EIGENVECTORS COMPUTED BUT NOT PRINTED
C   NP = 1 : DOMINANT EIGENVALUE AND EIGENVECTORS COMPUTED AND PRINTED
C NEIG = 0 :DOMINANT EIGENVALUE AND EIGENVECTORS NOT COMPUTED,
C          AND MATRIX ONLY IS PRINTED
C ------------------------------------------------------------
C COMPUTE DOMINANT EIGENVALUE BY POWER METHOD
C ------------------------------------------------------------
      IF (NEIG.EQ.0) GO TO 820
      DO 21 I=1,NR
      DO 21 J=1,NR
      A1(J,I)=CE(J,I)
      B(J,I)=CE(J,I)
   21 CONTINUE
      CALL MULTIP (NR,NR,NR)
      Z4=10000.
   23 CONTINUE
      Z5=Z4
       Z=C(1,1)
      DO 22 I=1,NR
      DO 22 J=1,NR
      C(J,I)=C(J,I)/Z
      A1(J,I)=C(J,I)
   22 B(J,I)=C(J,I)
      CALL MULTIP (NR,NR,NR)
      Z4=C(2,1)/C(1,1)
      Z3=Z4-Z5
      TOLEIG=0.000001
      T2=-TOLEIG
      IF ((Z3.LT.T2).OR.(Z3.GT.TOLEIG)) GO TO 23
      DO 24 I=1,NR
      DO 24 J=1,NR
      A1(J,I)=C(J,I)
   24 B(J,I)=CE(J,I)
      CALL MULTIP (NR,NR,NR)
      ROOT=C(1,1)/A1(1,1)
C ------------------------------------------------------------
C COMPUTE RIGHT AND LEFT EIGENVECTOR (VECT AND VECTL)
C ------------------------------------------------------------
      DO 25 J=1,NR
   25 VECT(J)=C(J,1)/C(1,1)
   26 CONTINUE
      NR1=NR-1
```

```
      DO 11 I=1,NR1
      DO 11 J=1,NR1
      I1=I+1
      J1=J+1
      IF (I.EQ.J) CC(I,J)=CE(J1,I1)-ROOT
      IF (I.NE.J) CC(I,J)=CE(J1,I1)
   11 CONTINUE
      CALL INVERT (NR1)
      DO 12 I=1,NR1
      I1=I+1
      B(I,1)=-CE(1,I1)
      DO 12 J=1,NR1
   12 A1(I,J)=CC(I,J)
      CALL MULTIP (NR1,NR1,1)
      VECTL(1)=1.
      DO 13 I=1,NR1
      I1=I+1
   13 VECTL(I1)=C(I,1)
C ------------------------------------------------------------
C PRINT MATRIX, EIGENVALUE AND EIGENVECTORS
C ------------------------------------------------------------
      IF (NP.EQ.0) GO TO 30
  820 CONTINUE
      ZTOTT=0.
      DO 6 J=1,NR
      ZMOMT(J)=0.
      DO 8 I=1,NR
    8 ZMOMT(J)=ZMOMT(J)+CE(I,J)
      IF (NP.EQ.2) ZMOMT(J)=ZMOMT(J)/FLOAT(NR)
      ZTOTT=ZTOTT+ZMOMT(J)
    6 CONTINUE
      PRINT 62, (REG(J),J=1,NR)
   62 FORMAT (/11X,12(2X,A8))
      PRINT 64
   64 FORMAT (1X)
      DO 5 I=1,NR
      IF (ZTOTT.LT.200.) PRINT 91, REG(I),(CE(I,J),J=1,NR)
   91 FORMAT (1X,A8,2X,12F10.6)
    5 IF (ZTOTT.GE.200.) PRINT 80, REG(I),(CE(I,J),J=1,NR)
   80 FORMAT (1X,A8,2X,12F10.4)
      IF (ZTOTT.LT.200.) PRINT 7, (ZMOMT(J),J=1,NR)
    7 FORMAT (/4X,5HTOTAL,2X,12F10.6)
      IF (ZTOTT.GE.200.) PRINT 81, (ZMOMT(J),J=1,NR)
   81 FORMAT (/4X,5HTOTAL,2X,12F10.4)
      IF (NEIG.EQ.0) GO TO 30
      PRINT 64
      PRINT 93, ROOT
   93 FORMAT (1X,10HEIGENVALUE,5X,F11.6)
      PRINT 94
   94 FORMAT (1X,11HEIGENVECTOR)
      PRINT 95, (VECT(J),J=1,NR)
   95 FORMAT (4X,7H- RIGHT ,12F10.6)
      PRINT 96, (VECTL(J),J=1,NR)
   96 FORMAT (4X,7H-  LEFT,12F10.6)
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MULTIP (N,K,L)
C     A1 * B = C
C N : NUMBER OF ROWS OF A1
C K : NUMBER OF COLUMNS OF A1 = NUMBER OF ROWS OF B
C L : NUMBER OF COLUMNS OF B
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      DO 3 I=1,N
      DO 3 J=1,L
      C(I,J)=0.
      DO 3 JJ=1,K
      C(I,J)=C(I,J)+A1(I,JJ)*B(JJ,J)
    3 CONTINUE
       RETURN
      END
```

```
      SUBROUTINE INVERT (NR)
C   NR : DIMENSION OF MATRIX CC TO BE INVERTED
      DIMENSION PIVOT(7)
      COMMON /CINV/ CC(7,7)
      DO 606 I=1,NR
      PIVOT(I)=CC(I,I)
      CC(I,I)=1.0
      DO 607 J=1,NR
      CC(I,J)=CC(I,J)/PIVOT(I)
  607 CONTINUE
      IF (NR.EQ.1) GO TO 10
      DO 608 K=1,NR
      IF (K.EQ.I) GO TO 608
      H=CC(K,I)
      CC(K,I)=0.
      DO 609 L=1,NR
      CC(K,L)=CC(K,L)-CC(I,L)*H
  609 CONTINUE
  608 CONTINUE
  606 CONTINUE
  10  CONTINUE
      RETURN
       END
```

D2. Special Purpose Subroutines

```
      SUBROUTINE DATAS (NPR,NA,NY,ZFNY,NR,XZB,XZD,XZO,IPROB,
     1INIT,NHORIZ,INTV,ITOLX,NTOLL,NEIG)
       DIMENSION TITLE(10)
      COMMON /C1/ POP(18,7)
      COMMON /CBIR/ BIRTH(18,7),DEATH(18,7),OMIG(18,7,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
       DOUBLE PRECISION TITLE
      INTEGER X
C -------------------------------------------------------------
C SKIP FIRST CARD
C READ PARAMETER CARD
C -------------------------------------------------------------
      READ (5,222)
 222  FORMAT (1X)
      READ (5,2) NA,NR,NY,NU,NZB,NZD,NZO,IPROB,INIT,NHORIZ,
     1INTV,ITOLX,NTOLL,NEIG
   2  FORMAT (8I2,3I4,3I2)
C -----------------------------------------------------------
C DEFINE DEFAULT VALUES FOR PARAMETERS
C -----------------------------------------------------------
      DO 6 X=1,NA
   6  NAGE(X)=(X-1)*NY
       IF (NZB.EQ.0) NZB=1
       IF (NZD.EQ.0) NZD=1
       IF (NZO.EQ.0) NZO=1
      IF (NHORIZ.LE.INIT) NHORIZ=INIT
      IF (ITOLX.EQ.0) ITOLX=2
      IF (NTOLL.EQ.0) NTOLL=7
      IF (INTV.EQ.0) INTV=200
      XZB=FLOAT(NZB)
      XZD=FLOAT(NZD)
      XZO=FLOAT(NZO)
      ZFNY=FLOAT(NY)
      DO 45 I=1,NR
  45  IF (RADIX(I).EQ.0.) RADIX(I)=100000.
C -------------------------------------------------------------
C PRINT TITLE
C -------------------------------------------------------------
      IF (NPR.EQ.0) GO TO 51
  17  FORMAT (1H1,1X)
      DO 15 J=1,3
  15  PRINT 16
  16  FORMAT  (//1X)
      PRINT 22
  22  FORMAT (1X/4(20X,90(1H*)/),20X,5(1H*),80X,5(1H*))
  51  CONTINUE
      DO 4 I=1,NU
      READ (5,14) (TITLE(J),J=1,10)
  14  FORMAT (10A8)
      IF (NPR.NE.0) PRINT 5, (TITLE(J),J=1,10)
   4  CONTINUE
```

```
    5  FORMAT (20X,5(1H*),10A8,5(1H*))
       IF (NPR.EQ.0) GO TO 52
       PRINT 24
   24  FORMAT (20X,5(1H*),80X,5(1H*)/4(20X,90(1H*)/))
C -----------------------------------------------------------------------
C LIST PARAMETERS
C READ AND LIST OTHER INPUT DATA
C -----------------------------------------------------------------------
       PRINT 37
   37  FORMAT (1H1,10X,10HINPUT DATA/11X,10(1H*)//)
       PRINT 8, NA,NR,NY,NU,NZB,NZD,NZO,IPROB,INIT,NHORIZ,
      1INTV,ITOLX,NTOLL,NEIG
    8  FORMAT (10X,8I2,3I4,3I2)
       DO 41 I=1,NU
   41  PRINT 42
   42  FORMAT (10X,5HTITLE)
C READ AND LIST NAMES OF REGIONS REG(J)
   52  CONTINUE
       NR1=NR+1
       READ (5,14) (REG(J),J=1,NR1)
       IF (NPR.NE.0) PRINT 144, (REG(J),J=1,NR)
  144  FORMAT (10X,10A8)
C READ AND LIST RADICES  RADIX(J)
       READ (5,13) (RADIX(J),J=1,NR)
       IF (NPR.NE.0) PRINT 133, (RADIX(J),J=1,NR)
   13  FORMAT (10F8.0)
  133  FORMAT (10X,10F8.0)
       RADIXT=0.
       DO 66 I=1,NR
   66  RADIXT=RADIXT+RADIX(I)
C READ AND LIST POPULATION, BIRTHS, DEATHS, MIGRANTS
       DO 10 I=1,NR
       READ (5,3) (POP(X,I),X=1,NA)
    3  FORMAT (8F10.0)
       IF (NPR.NE.0) PRINT 38, (POP(X,I),X=1,NA)
   38  FORMAT (10X,8F10.0/10X,8F10.0/10X,8F10.0)
       READ (5,31) (BIRTH(X,I),X=1,NA)
   31  FORMAT (10F8.0)
       IF (NPR.NE.0) PRINT 36, (BIRTH(X,I),X=1,NA)
   36  FORMAT (10X,10F8.0/10X,10F8.0)
       READ (5,31) (DEATH(X,I),X=1,NA)
       IF (NPR.NE.0) PRINT 36, (DEATH(X,I),X=1,NA)
       DO 32 J=1,NR
       READ (5,31) (OMIG(X,J,I),X=1,NA)
       IF (NPR.NE.0) PRINT 36, (OMIG(X,J,I),X=1,NA)
   32  CONTINUE
   33  CONTINUE
   10  CONTINUE
C ---------------------------------------------------------------------
C PRINT THE LIST OF PARAMETERS
C ---------------------------------------------------------------------
       IF (NPR.EQ.0) GO TO 54
       PRINT 17
       DO 18 J=1,3
   18  PRINT 16
```

```
      PRINT 19
   19 FORMAT (20X,18HLIST OF PARAMETERS/20X,18(1H*)/)
      PRINT 20, NA,NY,NR
   20 FORMAT (/10X,2HNA,4X,1H=,I5,8X,2HNY,4X,1H=,I5,8X,2HNR,4X,1H=,I5)
      PRINT 21, NZB,NZD,NZO
   21 FORMAT (/10X,3HNZB,3X,1H=,I5,8X,3HNZD,3X,1H=,I5,8X,3HNZO,3X,1H=,
     1I5)
      PRINT 25, NU,IPROB,NEIG
   25 FORMAT (/10X,2HNU,4X,1H=,I5,8X,5HIPROB,1X,1H=,I5,8X,4HNEIG,2X,
     11H=,I5)
      PRINT 28, INIT,NHORIZ,INTV
   28 FORMAT (/10X,4HINIT,2X,1H=,I5,8X,6HNHORIZ,1H=,I5,
     18X,4HINTV,2X,1H=,I5)
      PRINT 29, ITOLX,NTOLL
   29 FORMAT (/10X,5HITOLX,1X,1H=,I5,8X,5HNTOLL,1X,1H=,I5)
   54 CONTINUE
C -----------------------------------------------------------------
C COMPUTE THE OBSERVED MORTALITY, FERTILITY AND MIGRATION RATES
C -----------------------------------------------------------------
      DO 35 I=1,NR
      DO 35 X=1,NA
      RATD(X,I)=DEATH(X,I)/(POP(X,I)*XZD)
      RATF(X,I)=BIRTH(X,I)/(POP(X,I)*XZB)
      DO 35 J=1,NR
   35 RATM(X,J,I)=OMIG(X,J,I)/(POP(X,I)*XZO)
      RETURN
      END
```

```
      SUBROUTINE PRELIM(NA,NY,ZFNY,NR,XZB,XZD,XZO)
      DIMENSION HU(7),HUU(7)
      DIMENSION POPT(7),DEATHT(7),BIRTHT(7),OMIGT(7,7)
      DIMENSION POPC2(18),DETHC2(18),BIRC2(18),ZMIGC2(18)
      DIMENSION GRD(7),GRR(7),GRO(7,7),GROT(7)
      DIMENSION CRUDD(7),CRUDF(7),CRUDO(7,7),CRUDOT(7)
      DIMENSION AGEP(7),AGED(7),AGEF(7),AGEO(7,7),AGEOT(7)
      COMMON /C1/ POP(18,7)
      COMMON /CBIR/ BIRTH(18,7),DEATH(18,7),OMIG(18,7,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      COMMON /CTOT/ BIRTHC(18),DEATHC(18),CMIGC(18),CMIGA(18)
      COMMON /CTOTRAT/ POPC(18),RATDT(18),RATFT(18),RATMT(18)
      DOUBLE PRECISION REG,REGL
      INTEGER X,XX
        REAL L
      NAA=NA-1
      PRINT 65
   65 FORMAT (1H1,5X,35HOBSERVED POPULATION CHARACTERISTICS/6X,
     135(1H=)/)
C ----------------------------------------------------
C PRINT NUMBER OF PEOPLE, BIRTHS,DEATHS AND MIGRANTS ;
C ----------------------------------------------------
      ISKIP=3
      DO 6 I=1,NR
       IF (ISKIP.NE.I) GO TO 164
       PRINT 165
  165 FORMAT (1H1/1X)
      ISKIP=ISKIP+2
  164 CONTINUE
      PRINT 15, REG(I)
   15 FORMAT (//5X,6HREGION,3X,A8/5X,17(1H-))
      PRINT 16,REG(I)
   16 FORMAT (3X,3HAGE,1X,10HPOPULATION,4X,6HBIRTHS,4X,6HDEATHS,5X,
     114HMIGRATION FROM,1X,A8,1X,2HTO)
      IF (NR.LE.10) PRINT 17,(REG(J),J=1,NR)
      IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
   17 FORMAT (37X,10(1X,A8))
   80 FORMAT (36X,12A8)
      PRINT 66
   66 FORMAT (1X)
      DO 14 X=1,NA
      IF (NR.LE.10) PRINT 8, NAGE(X),POP(X,I),BIRTH(X,I),DEATH(X,I),
     1(OMIG(X,J,I),J=1,NR)
   14 IF (NR.GT.10) PRINT 81, NAGE(X),POP(X,I),BIRTH(X,I),DEATH(X,I),
     1(OMIG(X,J,I),J=1,NR)
    8 FORMAT (3X,I3,1X,3F10.0,10F9.0)
   81 FORMAT (2X,I3,1X,3F10.0,12F8.0)
      POPT(I)=0.
      DEATHT(I)=0.
      BIRTHT(I)=0.
      DO 41 J=1,NR
   41 OMIGT(J,I)=0.
```

```
      D  42 X=1,NA
      POPT(I)=POPT(I)+POP(X,I)
      DEATHT(I)=DEATHT(I)+DEATH(X,I)
      BIRTHT(I)=BIRTHT(I)+BIRTH(X,I)
      DO 42 J=1,NR
      OMIGT(J,I)=OMIGT(J,I)+OMIG(X,J,I)
   42 CONTINUE
      IF (NR.LE.10) PRINT 40, POPT(I),BIRTHT(I),DEATHT(I),
     1(OMIGT(J,I),J=1,NR)
      IF (NR.GT.10) PRINT 82, POPT(I),BIRTHT(I),DEATHT(I),
     1(OMIGT(J,I),J=1,NR)
   40 FORMAT (/1X,5HTOTAL,1X,3F10.0,10F9.0)
   82 FORMAT (/1X,5HTOTAL,3F10.0,12F8.0)
    6 CONTINUE
C ---------------------------------------------------------
C COMPUTE AND PRINT PERCENTAGE DISTRIBUTION
C COMPUTE AND PRINT MEAN AGES
C ---------------------------------------------------------
      PRINT 44
   44 FORMAT (1H1,10X,24HPERCENTAGE DISTRIBUTIONS/11X,24(1H*)/)
      ISKIP=3
      DO 45 I=1,NR
      IF (ISKIP.NE.I) GO TO 166
      PRINT 165
      ISKIP=ISKIP+2
  166 CONTINUE
      PRINT 15, REG(I)
      PRINT 16, REG(I)
      IF (NR.LE.10) PRINT 17, (REG(J),J=1,NR)
      IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
       PRINT 66
      ZP=0.
      ZB=0.
      ZD=0.
      DO 700 J=1,NR
  700  HU(J)=0.
       AGEP(I)=0.
      AGED(I)=0.
      AGEF(I)=0.
       DO 68 J=1,NR
   68 AGEO(J,I)=0.
      DO 46 X=1,NA
      Z2=0.
      Z3=0.
      Z1=100.*POP(X,I)/POPT(I)
      IF (DEATHT(I).NE.0.) Z3=100.*DEATH(X,I)/DEATHT(I)
      IF (BIRTHT(I).NE.0.) Z2=100.*BIRTH(X,I)/BIRTHT(I)
      ZP=ZP+Z1
      ZB=ZB+Z2
      ZD=ZD+Z3
      DO 148 J=1,NR
      HUJ(J)=0.
      IF (OMIGT(J,I).EQ.0.) GO TO 148
      HUJ(J)=100.*OMIG(X,J,I)/OMIGT(J,I)
      HU(J)=HU(J)+HUJ(J)
  148 CONTINUE
```

```
      IF (NR.LE.10) PRINT 47, NAGE(X),Z1,Z2,Z3,(HUU(J),J=1,NR)
      IF (NR.GT.10) PRINT 84, NAGE(X),Z1,Z2,Z3,(HUU(J),J=1,NR)
   47 FORMAT (3X,I3,1X,3F10.4,10F9.4)
   84 FORMAT (2X,I3,1X,3F10.4,12F8.4)
      Z=FLOAT(NAGE(X))+FLOAT(NY)*0.5
      Z=Z/100.
       AGEP(I)=AGEP(I)+Z*Z1
      AGED(I)=AGED(I)+Z*Z3
      AGEF(I)=AGEF(I)+Z*Z2
      DO 67 J=1,NR
   67 AGEO(J,I)=AGEO(J,I)+Z*HUU(J)
   46 CONTINUE
      IF (NR.LE.10) PRINT 147, ZP,ZB,ZD,(HU(J),J=1,NR)
      IF (NR.GT.10) PRINT 85, ZP,ZB,ZD,(HU(J),J=1,NR)
  147 FORMAT (/1X,5HTOTAL,1X,3F10.4,10F9.4)
   85 FORMAT (/1X,5HTOTAL,3F10.4,12F8.4)
      IF (NR.LE.10) PRINT 38, AGEP(I),AGEF(I),AGED(I),(AGEO(J,I),J=1,NR)
      IF (NR.GT.10) PRINT 86, AGEP(I),AGEF(I),AGED(I),(AGEO(J,I),J=1,NR)
   38 FORMAT (1X,5HM.AGE,1X,3F10.4,10F9.4)
   86 FORMAT (1X,5HM.AGE,3F10.4,12F8.4)
   45 CONTINUE
C -----------------------------------------------------
C COMPUTE AND PRINT OBSERVED RATES, GROSS RATES, MEAN AGES OF SCHEDULES
C COMPUTE AND PRINT CRUDE RATES
C -----------------------------------------------------
      DO 635 I=1,NR
      CRUDD(I)=DEATHT(I)/(POPT(I)*XZD)
      CRUDF(I)=BIRTHT(I)/(POPT(I)*XZB)
      Z=0.
      DO 69 J=1,NR
      Z=Z+OMIGT(J,I)
   69 CRUDO(J,I)=OMIGT(J,I)/(POPT(I)*XZO)
      CRUDOT(I)=Z/(POPT(I)*XZO)
  635 CONTINUE
      DO 5  I=1,NR
      DO 5 X=1,NA
      RATD(X,I)=DEATH(X,I)/(POP(X,I)*XZD)
      RATF(X,I)=BIRTH(X,I)/(POP(X,I)*XZB)
      DO 21 J=1,NR
   21 RATM(X,J,I)=OMIG(X,J,I)/(POP(X,I)*XZO)
    5 CONTINUE
      DO 35 I=1,NR
      GRD(I)=0.
      GRR(I)=0.
      DO 36 J=1,NR
   36 GRO(J,I)=0.
      DO 35 X=1,NA
      GRD(I)=GRD(I)+RATD(X,I)
      GRR(I)=GRR(I)+RATF(X,I)
      DO 35 J=1,NR
      GRO(J,I)=GRO(J,I)+RATM(X,J,I)
   35 CONTINUE
      PRINT 20
   20 FORMAT (1H1,5X,14HOBSERVED RATES/6X,14(1H*))
```

```
      DO 33 I=1,NR
      GROT(I)=0.
      DO 78 J=1,NR
      GROT(I)=GROT(I)+GRO(J,I)
 78   CONTINUE
      AGED(I)=0.
      AGEF(I)=0.
      DO  30 J=1,NR
 30   AGEO(J,I)=0.
      DO 48 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      IF (GRD(I).GT.0.) AGED(I)=AGED(I)+Z*RATD(X,I)/GRD(I)
      IF (GRR(I).GT.0.) AGEF(I)=AGEF(I)+Z*RATF(X,I)/GRR(I)
      DO 48 J=1,NR
 48   IF (GRO(J,I).GT.0.) AGEO(J,I)=AGEO(J,I)+Z*RATM(X,J,I)/GRO(J,I)
 33   CONTINUE
      PRINT 31
 31   FORMAT (/20X,11HDEATH RATES/20X,11(1H*)/)
      PRINT 32, (REG(J),J=1,NR)
 32   FORMAT (3X,3HAGE,5X,12(1X,A8))
      PRINT 66
      DO 18 X=1,NA
 18   PRINT 19, NAGE(X),(RATD(X,J),J=1,NR)
 19   FORMAT (3X,I3,5X,13F9.6)
      DO 11 J=1,NR
 11   HU(J)=GRD(J)*ZFNY
      PRINT 37, (HU(J),J=1,NR)
 37   FORMAT (/1X,5HGROSS,5X,F9.6,12F9.6)
      PRINT 39, (CRUDD(J),J=1,NR)
 39   FORMAT (1X,5HCRUDE,5X,F9.6,12F9.6)
      PRINT 49, (AGED(J),J=1,NR)
 49   FORMAT (1X,5HM.AGE,5X,F9.4,12F9.4)
      PRINT 171
171   FORMAT (//20X,15HFERTILITY RATES/20X,15(1H*)/)
      PRINT 32, (REG(J),J=1,NR)
      PRINT 66
      DO 72 X=1,NA
 72   PRINT 19, NAGE(X),(RATF(X,J),J=1,NR)
      DO 12 J=1,NR
 12   HU(J)=GRR(J)*ZFNY
      PRINT 37, (HU(J),J=1,NR)
      PRINT 39, (CRUDF(J),J=1,NR)
      PRINT 49, (AGEF(J),J=1,NR)
      PRINT 73
 73   FORMAT (1H1,19X,18HOUTMIGRATION RATES/20X,18(1H*))
       ISKIP=3
      DO 79 I=1,NR
      AGEOT(I)=0.
       IF (ISKIP.NE.I) GO TO 167
       PRINT 165
       ISKIP=ISKIP+2
167   CONTINUE
      PRINT 74, REG(I)
 74   FORMAT (/20X,14HMIGRATION FROM,1X,A8,1X,2HTO)
      PRINT 75, (REG(J),J=1,NR)
 75   FORMAT (3X,3HAGE,9X,5HTOTAL,12(1X,A8))
```

```
      PRINT 65
      DO 76 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      ZZ=0.
      DO 77 J=1,NR
      ZZ=ZZ+RATM(X,J,I)
   77 CONTINUE
      IF (GROT(I).GT.0.) AGEOT(I)=AGEOT(I)+Z*ZZ/GROT(I)
   76 PRINT 19, NAGE(X),ZZ,(RATM(X,J,I),J=1,NR)
      HHU=GROT(I)*ZFNY
      DO 13 J=1,NR
   13 HU(J)=GRO(J,I)*ZFNY
      PRINT 37, HHU,(HU(J),J=1,NR)
      PRINT 39, CRUDOT(I),(CRUDO(J,I),J=1,NR)
      PRINT 49, AGEOT(I),(AGEO(J,I),J=1,NR)
      PRINT 66
   79 CONTINUE
C -----------------------------------------------------------
C LIFE TABLE FOR EACH REGION SEPARATELY
C -----------------------------------------------------------
      DO 22 I=1,NR
      DO 23 X=1,NA
      POPC(X)=POP(X,I)
      DEATHC(X)=DEATH(X,I)
      BIRTHC(X)=BIRTH(X,I)
      CMIGC(X)=0.
      CMIGA(X)=0.
      DO 23 J=1,NR
      CMIGA(X)=CMIGA(X)+OMIG(X,I,J)
   23 CMIGC(X)=CMIGC(X)+OMIG(X,J,I)
      REGL=REG(I)
      CALL TOTSY (NA,ZFNY,XZB,XZD,XZO,1,REGL)
   22 CONTINUE
C -----------------------------------------------------------
C LIFE TABLE FOR COUNTRY
C -----------------------------------------------------------
      DO 53 X=1,NA
      POPC(X)=0.
      DEATHC(X)=0.
      BIRTHC(X)=0.
      CMIGC(X)=0.
      CMIGA(X)=0.
      DO 53 I=1,NR
      POPC(X)=POPC(X)+POP(X,I)
      DEATHC(X)=DEATHC(X)+DEATH(X,I)
      BIRTHC(X)=BIRTHC(X)+BIRTH(X,I)
      DO 53 J=1,NR
      CMIGC(X)=CMIGC(X)+OMIG(X,J,I)
      CMIGA(X)=CMIGA(X)+OMIG(X,I,J)
   53 CONTINUE
      NR1=NR+1
      REGL=REG(NR1)
      CALL TOTSY (NA,ZFNY,XZB,XZD,XZO,1,REGL)
      RETURN
      END
```

```
      SUBROUTINE TOTSY(NA,ZFNY,XZB,XZD,XZO,NWHOL,REGL)
C -----------------------------------------------------------------
C   NWHOL = 0 AGGREGATE SYSTEM (TOTAL POPULATION FEATURES ARE NOT PRINTED)
C   REGL  = NAME OF REGION CONSIDERED
C -----------------------------------------------------------------
      DIMENSION ZMIGA(18)
      DIMENSION POPC2(18),DETHC2(18),BIRC2(18),ZMIGC2(18)
      DIMENSION P(18),Q(18),CL(18),CLL(18),T(18),SU(18),E(18)
      COMMON /CNAG/ NAGE(18)
      COMMON /CTOT/ BIRTHC(18),DEATHC(18),CMIGC(18),CMIGA(18)
      COMMON /CTOTRAT/ POPC(18),RATDT(18),RATFT(18),RATMT(18)
      DOUBLE PRECISION REGL
      INTEGER X,XX
        REAL L
      NAA=NA-1
      ZFNY2=ZFNY*0.5
C ------------------------------------------------------------
C COMPUTE AND PRINT CHARACTERISTICS OF TOTAL POPULATION SYSTEM
C -----------------------------------------------------------
      IF (NWHOL.EQ.0) GO TO 555
      PRINT 51,REGL
  51  FORMAT (1H1,30X,A8/31X,8(1H*)//)
  102  CONTINUE
      PRINT 52
  52  FORMAT (1X,3HAGE,6X,10HPOPULATION,8X,6HBIRTHS,10X,6HDEATHS,5X,
     14X,8HARRIVALS,7X,10HDEPARTURES,13X,25HOBSERVED RATES ( X 1000 ))
      PRINT 559
 559  FORMAT (5X,5(3X,6HNUMBER,2X,5H- % -),5X,5HBIRTH,4X,5HDEATH,
     14X,5HINMIG,3X,6HOUTMIG,2X,7HNET MIG/)
C  TOTAL POPULATION, BIRTHS, DEATHS, MIGRANTS
      PP4=0.
      BB4=0.
      DD4=0.
      ZMM4=0.
      ZMI=0.
      DO 54 X=1,NA
      PP4=PP4+POPC(X)
      DD4=DD4+DEATHC(X)
      BB4=BB4+BIRTHC(X)
      ZMM4=ZMM4+CMIGC(X)
      ZMI=ZMI+CMIGA(X)
  54  CONTINUE
C PERCENTAGE DISTRIBUTION
      ZP=0.
      ZB=0.
      ZD=0.
      ZM=0.
      ZI=0.
      DO 53 X=1,NA
      POPC2(X)=100.*POPC(X)/PP4
      IF (DD4.NE.0.) DETHC2(X)=100.*DEATHC(X)/DD4
      IF (BB4.NE.0.) BIRC2(X)=100.*BIRTHC(X)/BB4
      IF (ZMM4.NE.0.) ZMIGC2(X)=100.*CMIGC(X)/ZMM4
      IF (ZMI.NE.0.) ZMIGA(X)=100.*CMIGA(X)/ZMI
```

```
      ZP=ZP+POPC2(X)
      ZB=ZB+BIRC2(X)
      ZD=ZD+DETHC2(X)
      ZM=ZM+ZMIGC2(X)
      ZI=ZI+ZMIGA(X)
   53 CONTINUE
C   RATES
      DO 58 X=1,NA
      RATDT(X)=DEATHC(X)/(POPC(X)*XZD)
      RATFT(X)=BIRTHC(X)/(POPC(X)*XZB)
      RATMT(X)=CMIGC(X)/(POPC(X)*XZO)
   58 CONTINUE
C  PRINT OBSERVED POPULATION CHARACTERISTICS
      RAMI=0.
      DO 59 X=1,NA
      RIT=CMIGA(X)/(POPC(X)*XZO)
      RAMI=RAMI+RIT
      Z=1000.
      RZD=RATDT(X)*Z
      RZF=RATFT(X)*Z
      RZM=RATMT(X)*Z
      RZI=RIT*Z
      RNT=RZI-RZM
      PRINT 63, NAGE(X),POPC(X),POPC2(X),BIRTHC(X),BIRC2(X),DEATHC(X),
     1DETHC2(X),CMIGA(X),ZMIGA(X),CMIGC(X),ZMIGC2(X),RZF,
     1RZD,RZI,RZM,RNT
   63 FORMAT (1X,I3,F10.0,F7.2,4(F9.0,F7.2),1X,5F9.3)
   59 CONTINUE
      PRINT 64, PP4,ZP,BB4,ZB,DD4,ZD,ZMI,ZI,ZMM4,ZM
   64 FORMAT (/1X,3HTOT,F10.0,F7.2,4(F9.0,F7.2))
C GROSS RATES
      RADD4=0.
      RAFF4=0.
      RAMM4=0.
      DO 31 X=1,NA
      RADD4=RADD4+RATDT(X)
      RAFF4=RAFF4+RATFT(X)
      RAMM4=RAMM4+RATMT(X)
   31 CONTINUE
      RZF=RAFF4*ZFNY
      RZD=RADD4*ZFNY
      RZI=RAMI*ZFNY
      RZM=RAMM4*ZFNY
      PRINT 18, RZF,RZD,RZI,RZM
   18 FORMAT (1X,5HGROSS,80X,5F9.3)
C  CRUDE RATES
      Z=1000.
      RAFC4=Z*BB4/(PP4*XZB)
      RADC4=Z*DD4/(PP4*XZD)
      RAMC4=Z*ZMM4/(PP4*XZO)
      RIMC4=Z*ZMI/(PP4*XZO)
      RNET=RIMC4-RAMC4
      PRINT 71, RAFC4,RADC4,RIMC4,RAMC4,RNET
   71 FORMAT (1X,5HCRUDE,7H(X1000),73X,5F9.3)
```

```
C  MEAN AGE
      AGEPC4=0.
      AGEFC4=0.
       AGEDC4=0.
      AGEMC4=0.
      AGIMC4=0.
      AGEFR4=0.
      AGEDR4=0.
      AGEMR4=0.
      AGIMR4=0.
      DO 81 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      Z1=Z/100.
      AGEPC4=AGEPC4+Z1*POPC2(X)
      AGEFC4=AGEFC4+Z1*BIRC2(X)
       AGEDC4=AGEDC4+Z1*DETHC2(X)
      AGEMC4=AGEMC4+Z1*ZMIGC2(X)
      AGIMC4=AGIMC4+Z1*ZMIGA(X)
      IF (RAFF4.GT.0.) AGEFR4=AGEFR4+Z*RATFT(X)/RAFF4
       IF (RADD4.GT.0.) AGEDR4=AGEDR4+Z*RATDT(X)/RADD4
      IF (RAMM4.GT.0.) AGEMR4=AGEMR4+Z*RATMT(X)/RAMM4
      IF (RAMI.GT.0.) AGIMR4=AGIMR4+Z*CMIGA(X)/(POPC(X)*XZO*RAMI)
   81 CONTINUE
      PRINT 82, AGEPC4,AGEFC4,AGEDC4,AGIMC4,AGEMC4,AGEFR4,AGEDR4,
     1AGIMR4,AGEMR4
   82 FORMAT (1X,5HM.AGE,8X,F7.2,4(9X,F7.2),1X,5F9.2)
  555 CONTINUE
C ------------------------------------------------------------
C SINGLE REGION LIFE TABLE OF TOTAL POPULATION SYSTEM
C ------------------------------------------------------------
      DO 10 X=1,NA
      P(X)=(1.-ZFNY2*RATDT(X))/(1.+ZFNY2*RATDT(X))
   10 Q(X)=1.-P(X)
      CL(1)=1.
      DO 11 X=1,NAA
      XX=X+1
      CL(XX)=CL(X)*P(X)
   11 CLL(X)=ZFNY2*(CL(X)+CL(XX))
      CLL(NA)=CL(NA)/RATDT(NA)
      T(NA)=CLL(NA)
      E(NA)=T(NA)/CL(NA)
      DO 12 X=1,NAA
      IX=NA-X
      T(IX)=T(IX+1)+CLL(IX)
   12 E(IX)=T(IX)/CL(IX)
      IF (NWHOL.EQ.1) PRINT 17, E(1)
   17 FORMAT (1X,4HE(0),90X,F9.2)
      DO 13 X=1,NAA
      XX=X+1
   13 SU(X)=CLL(XX)/CLL(X)
      IF (NWHOL.NE.0) PRINT 30
   30 FORMAT (1H1/)
      PRINT 5060, REGL,E(1)
 5060 FORMAT (1H0,//10X,5HTABLE,4X,26H- SINGLE REGION LIFE TABLE,3X
     1A8,5X,17HMORTALITY LEVEL =,F6.2/11X,74(1H-)//)
```

```
      PRINT 5062
 5062 FORMAT (1X,3HAGE,7X,4HP(X),7X,4HQ(X),7X,4HL(X),7X,4HD(X),6X,
     15HLL(X),7X,4HM(X),7X,4HS(X),6X,4HT(X),6X,4HE(X)/)
      DO 5063 X=1,NA
      Z1=CL(X)*100000.
      Z2=100000.*CL(X)*Q(X)
 5063 PRINT 5064, NAGE(X),P(X),Q(X),Z1,Z2,CLL(X),RATDT(X),SU(X),
     1T(X),E(X)
 5064 FORMAT (1X,I3,2F11.6,2F11.0,3F11.6,2F10.4)
C COMPUTE AND PRINT NRR AND NMR
      IF (NWHOL.EQ.0) GO TO 556
      RNM=0.
      RNR=0.
      DO 14 X=1,NA
      RNR=RNR+CLL(X)*RATFT(X)
   14 RNM=RNM+CLL(X)*RATMT(X)
      PRINT 66
   66 FORMAT (//1X)
      PRINT 15, RNR
   15 FORMAT (//30X,21HNET REPRODUCTION RATE,4X,F10.6)
      PRINT 16,RNM
   16 FORMAT (/30X,24HNET MIGRAPRODUCTION RATE,1X,F10.6)
  556 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PROBSC (NA,ZFNY,NR,IPROB)
      DIMENSION RM(7,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CINV/ CC(7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      COMMON /CRMLA/ RMLA(7,7)
      DOUBLE PRECISION REG
      INTEGER X
C ---------------------------------------
C   MATRIX OF OBSERVED RATES  M(X)
C ---------------------------------------
      NAA=NA-1
      ZZZ=ZFNY*0.5
      DO 100 X=1,NA
      DO 5 I=1,NR
      Z=RATD(X,I)
      DO 4 J=1,NR
      IF (I.EQ.J) GO TO 4
      Z=Z+RATM(X,J,I)
    4 CONTINUE
      RM(I,I)=Z
      DO 6 J=1,NR
      IF (J.EQ.I) GO TO 6
      RM(J,I)=-RATM(X,J,I)
    6 CONTINUE
    5 CONTINUE
      IF (X.NE.NA) GO TO 13
      DO 14 I=1,NR
      DO 14 J=1,NR
   14 RMLA(J,I)=RM(J,I)
      GO TO 100
   13 CONTINUE
C ---------------------------------------------
C  PROBABILITY MATRICES
C ---------------------------------------------
      DO 7 I=1,NR
      DO 7 J=1,NR
      IF (I.EQ.J) CC(J,I)=1.+ZZZ*RM(J,I)
    7 IF (I.NE.J) CC(J,I)=ZZZ*RM(J,I)
      CALL INVERT (NR)
      DO 8 I=1,NR
      DO 8 J=1,NR
       A1(J,I)=CC(J,I)
      IF (J.EQ.I) B(J,I)=1.-ZZZ*RM(J,I)
    8 IF (J.NE.I) B(J,I)=-ZZZ*RM(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 9 I=1,NR
      DO 9 J=1,NR
    9 P(X,J,I)=C(J,I)
  100 CONTINUE
```

```
      DO 10 I=1,NR
      DO 10 J=1,NR
   10 P(NA,J,I)=0.
C -----------------------------------------------
C PRINT PROBABILITIES
C -----------------------------------------------
 9999 FORMAT (1H1//1X)
      PRINT 9999
      PRINT 4500, IPROB
 4500 FORMAT (20X,36HPROBABILITIES OF DYING AND MIGRATING/
     120X,13(1H*),7H OPTION,I2,1X,13(1H*)/33X,10(1H*))
       ISKIP=3
      DO 726 I=1,NR
      IF (ISKIP.NE.I) GO TO 121
      PRINT 9999
       ISKIP=ISKIP+2
  121 CONTINUE
      PRINT 9001, REG(I)
 9001 FORMAT (//20X,6HREGION,2X,A8,1X/20X,16(1H*)/)
      PRINT 9011, REG(I)
 9011 FORMAT (5X,3HAGE,5X,5HDEATH,5X,14HMIGRATION FROM,1X,A8,1X,2HTO)
      PRINT 9020, (REG(J),J=1,NR)
 9020 FORMAT (18X,12(1X,A8))
      PRINT 66
   66 FORMAT (1X)
      DO 726 X=1,NA
      ZZ=0.
      DO 11 J=1,NR
      ZZ=ZZ+P(X,J,I)
   11 CONTINUE
      ZQ=1.-ZZ
      PRINT 9103, NAGE(X),ZQ,(P(X,J,I),J=1,NR)
 9103 FORMAT (5X,I3,1X,13F9.6)
  726 CONTINUE
       RETURN
      END
```

```
      subroutine probr (na,zfny,nr,iprob)
      dimension q(18,7)
      common /cnag/ nage(18)
      common /cpq/ p(18,7,7)
      common /crate/ ratd(18,7),ratm(18,7,7),ratf(18,7)
      common /creg/ reg(13)
      common /crmla/ rmla(7,7)
      double precision reg
      integer x
      naa=na-1
      zfny2=zfny*0.5
c -----------------------------------------------------
c   compute probabilities
c -----------------------------------------------------
      do 8 i=1,nr
      do 8 x=1,na
      z2=0.
      do 6 j=1,nr
      if (i.eq.j) go to 6
      z2=z2+ratm(x,j,i)
    6 continue
      z=1.+zfny2*ratd(x,i)+zfny2*z2
      q(x,i)=zfny*ratd(x,i)/z
      do 7 j=1,nr
      if (i.eq.j) go to 7
      p(x,j,i)=zfny*ratm(x,j,i)/z
    7 continue
    8 continue
      do 10 x=1,naa
      do 10 i=1,nr
      pmigt=0.
      do 9 j=1,nr
      if (i.eq.j) go to 9
      pmigt=pmigt+p(x,j,i)
    9 continue
      p(x,i,i)=1.-q(x,i)-pmigt
   10 continue
      do 11 i=1,nr
      q(na,i)=1.
      do 11 j=1,nr
      p(na,j,i)=0.
   11 continue
      do 12 i=1,nr
      do 12 j=1,nr
      if (i.eq.j) rmla(i,i)=ratd(na,i)
      if (i.ne.j) rmla(j,i)=0.
   12 continue
```

```
c -------------------------------------------------
c    print probabilities
c -------------------------------------------------
 9999 format (1h1//1x)
      print 9999
      print 4500, iprob
 4500 format (20x,36hprobabilities of dying and migrating/
     120x,13(1h*),7h option,i2,1x,13(1h*)/33x,10(1h*))
       iskip=3
      do 726 i=1,nr
      if (iskip.ne.i) go to 121
      print 9999
       iskip=iskip+2
 121  continue
      print 9001, reg(i)
 9001 format (//20x,6hregion,2x,a8,1x/20x,16(1h*)/)
      print 9011, reg(i)
 9011 format (5x,3hage,5x,5hdeath,5x,14hmigration from,1x,a8,1x,2hto)
      print 9020, (reg(j),j=1,nr)
 9020 format (18x,10(1x,a8))
      print 66
   66 format (1x)
      do 726 x=1,na
      print 9103, nage(x),q(x,i),(p(x,j,i),j=1,nr)
 9103 format (5x,i3,1x,11f9.6)
  726 continue
      return
      end
```

```
      SUBROUTINE HIST (NA,NR,IHIST)
      DIMENSION HULP(7),CM(7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CCL/ CL(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      REAL L
      INTEGER X,XX
   66 FORMAT (1X)
      IHIST=1
C -------------------------------------------------------------
C COMPUTE THE NUMBER OF SURVIVORS AT EXACT AGE X
C -------------------------------------------------------------
      DO 5 I=1,NR
      CL(1,I,I)=1.
      DO 5 J=1,NR
      IF (I.NE.J) CL(1,I,J)=0.
    5 CONTINUE
      NAA=NA-1
      DO 14 X=1,NAA
      XX=X+1
      DO 15 I=1,NR
      DO 15 J=1,NR
      A1(J,I)=P(X,J,I)
   15 B(J,I)=CL(X,I,J)
      CALL MULTIP (NR,NR,NR)
      DO 16 I=1,NR
      DO 16 J=1,NR
   16 CL(XX,I,J)=C(J,I)
   14 CONTINUE
C -----------------------------------------------------------
C COMPUTE AND PRINT THE LIFE HISTORY OF THE INITIAL COHORT
C -----------------------------------------------------------
      PRINT 9201
 9201  FORMAT (1H1,/20X,30HLIFE HISTORY OF INITIAL COHORT/20X,30(1H*))
      DO 250 IO=1,NR
       IF (IO.NE.1) PRINT 9211
 9211 FORMAT (1H1,1X)
      PRINT 9202, REG(IO)
 9202 FORMAT (1H0,20X,24HINITIAL REGION OF COHORT,2X,A8/21X,
     134(1H*)/)
      ISLIP=1
      DO 20 I=1,NR
      IF (ISKIP.NE.I) GO TO 29
      PRINT 9211
      ISKIP=ISKIP+2
   29 CONTINUE
      PRINT 21, I,REG(I)
   21 FORMAT (10X,I2,2H.-,1X,19HREGION OF RESIDENCE,2X,A8/)
      PRINT 22
   22 FORMAT (9X,6HDEATHS,5X,11HMIGRANTS TO)
```

```
      PRINT 23, (REG(J),J=1,NR)
   23 FORMAT (1X,3HAGE,11X,12(1X,A8))
      PRINT 66
      CDRT=0.
      DO 6 J=1,NR
    6 HULP(J)=0.
      DO 230 X=1,NA
      ZZ=0.
      DO 119 J=1,NR
  119 ZZ=ZZ+P(X,J,I)
      ZQ=1.-ZZ
      ZZ=CL(X,IO,I)*ZQ
      CDR=ZZ*RADIX(IO)
      CDRT=CDRT+CDR
      DO 24 J=1,NR
      ZZ=CL(X,IO,I)*P(X,J,I)
      RM(J)=ZZ*RADIX(IO)
   24 HULP(J)=HULP(J)+RM(J)
      PRINT 25, NAGE(X),CDR,(RM(J),J=1,NR)
   25 FORMAT (1X,I3,2X,13F9.0)
  230 CONTINUE
      PRINT 26, CDRT,(HULP(J),J=1,NR)
   26 FORMAT (/1X,5HTOTAL,13F9.0)
      PRINT 66
      PRINT 66
   20 CONTINUE
  250 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LIFE (NA,ZFNY,NR,IPROB,IHIST,ILIF)
      DIMENSION CM(7)
      DIMENSION E(18,7,7),T(7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CCL/ CL(18,7,7)
      COMMON /CINV/ CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CREG/ REG(13)
      COMMON /CRMLA/ RMLA(7,7)
      COMMON /CSU/ SU(18,7,7)
      DOUBLE PRECISION REG
      REAL L
      INTEGER X,XX,XY,XZ1
      ILIF=1
      NAA=NA-1
      IPREX=1
   66 FORMAT (1X)
 9103 FORMAT (1X,I3,1X,F10.5,12F9.5)
 9020 FORMAT (15X,12(1X,A8))
 9001 FORMAT (//20X,6HREGION,2X,A8/20X,16(1H*)/)
 9999 FORMAT (1H1//1X)
 9011 FORMAT (1X,3HAGE,6X,5HDEATH,5X,14HMIGRATION FROM,1X,A8,1X,2HTO)
C ----------------------------------------------------
C EXPECTED NUMBER OF SURVIVORS AT EXACT AGE X
C ----------------------------------------------------
C COMPUTE NUMBER OF SURVIVORS
      IF (IHIST.NE.0) GO TO 5
      DO 76 I=1,NR
      CL(1,I,I)=1.
      DO 76 J=1,NR
      IF (I.NE.J) CL(1,I,J)=0.
   76 CONTINUE
      DO 77 X=1,NAA
      XX=X+1
      DO 6 I=1,NR
      DO 6 J=1,NR
      A1(J,I)=P(X,J,I)
    6 B(J,I)=CL(X,I,J)
      CALL MULTIP (NR,NR,NR)
      DO 7 I=1,NR
      DO 7 J=1,NR
    7 CL(XX,I,J)=C(J,I)
   77 CONTINUE
    5 CONTINUE
C PRINT NUMBER OF SURVIVORS
      PRINT 9999
      PRINT 4831
 4831 FORMAT (20X,51HEXPECTED NUMBER OF SURVIVORS AT EXACT AGE X IN EACH
     17H REGION,/20X,58(1H*))
      ISKIP=3
```

```
      DO 60 IO=1,NR
      IF (ISKIP.NE.IO) GO TO 123
      PRINT 9999
      ISKIP=ISKIP+2
 123  CONTINUE
      PRINT 9502,REG(IO)
 9502 FORMAT (//1X,3HAGE,6X,24HINITIAL REGION OF COHORT,2X,A8/1X,
     13(1H*),6X,34(1H*)/)
      PRINT 9100, (REG(J),J=1,NR)
 9100  FORMAT (10X,5HTOTAL,10(1X,A8))
       PRINT 66
      DO 60 X=1,NA
      CLT=0.
      DO 8 J=1,NR
      CM(J)=CL(X,IO,J)*RADIX(IO)
   8  CLT=CLT+CM(J)
      PRINT 9101, NAGE(X),CLT,(CM(J),J=1,NR)
 9101 FORMAT (1X,I3,1X,F10.0,12F9.0)
  60  CONTINUE
C ---------------------------------------------------
C NUMBER OF YEARS LIVED BETWEEN X AND X+NY
C ---------------------------------------------------
C          - BY REGION OF BIRTH
C ---------------------------------------------------
      DO 10 IO=1,NR
      DO 10 X=1,NAA
      ZZ=ZFNY*0.5
      XX=X+1
      DO 9 I=1,NR
   9  L(X,IO,I)=ZZ*(CL(X,IO,I)+CL(XX,IO,I))
  10  CONTINUE
  18  CONTINUE
C NUMBER OF YEARS LIVED IN LAST AGE GROUP
      DO 2 I=1,NR
      DO 2 J=1,NR
   2  CC(J,I)=RMLA(J,I)
      CALL INVERT (NR)
      DO 3 I=1,NR
      DO 3 J=1,NR
      A1(J,I)=CC(J,I)
   3  B(J,I)=CL(NA,I,J)
      CALL MULTIP (NR,NR,NR)
      DO 4 I=1,NR
      DO 4 J=1,NR
   4  L(NA,I,J)=C(J,I)
 120  CONTINUE
 50   CONTINUE
      PRINT 9999
      PRINT 4832
 4832 FORMAT (10X,39HNUMBER OF YEARS LIVED IN EACH REGION BY
     1,20H A UNIT BIRTH COHORT/10X,59(1H*))
       ISKIP=3
      DO 34 IO=1,NR
      IF (IO.NE.ISKIP) GO TO 124
```

```
      PRINT 9999
      ISKIP=ISKIP+2
 124  CONTINUE
      PRINT 9502, REG(IO)
      PRINT 9100, (REG(J),J=1,NR)
      PRINT 66
      DO 58 X=1,NA
      CLLT=0.
      DO 11 J=1,NR
  11  CLLT=CLLT+L(X,IO,J)
      PRINT 9103, NAGE(X),CLLT,(L(X,IO,J),J=1,NR)
  58  CONTINUE
  34  CONTINUE
C ----------------------------------------------------
C          - BY REGION OF RESIDENCE AT AGE X
C ----------------------------------------------------
       ZFNY2=ZFNY*0.5
       PRINT 9999
      PRINT 80
  80  FORMAT (10X,39HNUMBER OF YEARS LIVED IN EACH REGION BY
     1,16H PERSON OF AGE X/10X,55(1H*))
      ISKIP=3
      DO 85 I=1,NR
      IF (I.NE.ISKIP) GO TO 82
      PRINT 9999
      ISKIP=ISKIP+2
  82  CONTINUE
      PRINT 83, REG(I)
  83  FORMAT (//1X,3HAGE,6X,28HREGION OF RESIDENCE AT AGE X,
     12X,A8/1X,3(1H*),6X,38(1H*)/)
      PRINT 9100, (REG(J),J=1,NR)
      PRINT 66
      DO 81 X=1,NA
      CMT=0.
      DO 84 J=1,NR
      IF (I.EQ.J) CM(J)=ZFNY2*(1.+P(X,J,I))
      IF (I.NE.J) CM(J)=ZFNY2*P(X,J,I)
      IF (X.EQ.NA) CM(J)=CC(J,I)
      CMT=CMT+CM(J)
  84  CONTINUE
      PRINT 9103, NAGE(X),CMT,(CM(J),J=1,NR)
  81  CONTINUE
  85  CONTINUE
C -----------------------------------------------------
C LIFE TABLE MORTALITY AND MIGRATION RATES (NO MULTIPLE TRANSITION)
C ----------------------------------------------------
      IF (IPROB.EQ.3) GO TO 159
      PRINT 9999
      PRINT 4833
4833  FORMAT (20X,30HDEATH RATE AND MIGRATION RATES/20X,30(1H*))
      ISKIP=3
      DO 59 I=1,NR
      IF (ISKIP.NE.I) GO TO 122
      PRINT 9999
      ISKIP=ISKIP+2
 122  CONTINUE
```

```
      PRINT 9001, REG(I)
      PRINT 9011, REG(I)
      PRINT 9020, (REG(J),J=1,NR)
      PRINT 66
      DO 59 X=1,NA
      Z1=ZFNY2*(1.+P(X,I,I))
      PMIGT=0.
      DO 35 J=1,NR
   35 PMIGT=PMIGT+P(X,J,I)
      ZQ=1.-PMIGT
      RATDV=ZQ/Z1
      DO 36 J=1,NR
   36 CM(J)=P(X,J,I)/Z1
      IF (X.NE.NA) GO TO 37
      RATDV=RATD(NA,I)
      DO 38 J=1,NR
   38 CM(J)=0.
   37 CONTINUE
      PRINT 9103, NAGE(X),RATDV,(CM(J),J=1,NR)
   59 CONTINUE
  159 CONTINUE
C -------------------------------------------------
C SURVIVORSHIP PROPORTIONS
C -------------------------------------------------
      DO 61 X=1,NAA
      XX=X+1
      DO 74 IO=1,NR
      DO 74 J=1,NR
   74 CC(IO,J)=L(X,IO,J)
      CALL INVERT (NR)
      DO 75 IO=1,NR
      DO 75 J=1,NR
      SU(X,IO,J)=0.
      DO 75 JJ=1,NR
      SU(X,IO,J)=SU(X,IO,J)+CC(IO,JJ)*L(XX,JJ,J)
   75 CONTINUE
   61 CONTINUE
      PRINT 9999
      PRINT 4834
 4834 FORMAT (30X,24HSURVIVORSHIP PROPORTIONS/30X,24(1H*))
       ISKIP=3
      DO 64 I=1,NR
       IF (ISKIP.NE.I) GO TO 125
       PRINT 9999
       ISKIP=ISKIP+2
  125 CONTINUE
      PRINT 9001, REG(I)
      PRINT 9100, (REG(J),J=1,NR)
      NAA=NA-1
       PRINT 66
      DO 63 X=1,NAA
      SSU=0.
      DO 62 J=1,NR
   62 SSU=SSU+SU(X,I,J)
   63 PRINT 9103, NAGE(X),SSU,(SU(X,I,J),J=1,NR)
   64 CONTINUE
```

```
C -----------------------------------------------------------
C  NUMBER OF YEARS LIVED BEYOND AGE X AND LIFE EXPECTANCY BY PLACE OF BIRTH
C -------------------------------------------------------------
      PRINT 9999
      PRINT 4835
 4835 FORMAT (10X,40HTOTAL NUMBER OF YEARS LIVED BEYOND AGE X/
     110X,40(1H*))
       ISKIP=3
      DO 51 IO=1,NR
        IF (ISKIP.NE.IO) GO TO 126
         PRINT 9999
         ISKIP=ISKIP+2
  126  CONTINUE
       PRINT 9502, REG(IO)
      PRINT 9100, (REG(J),J=1,NR)
      PRINT 66
      DO 14 X=1,NA
      TT=0.
      DO 17 I=1,NR
      T(I)=0.
      DO 12 XY=X,NA
  12  T(I)=T(I)+L(XY,IO,I)
  17  TT=TT+T(I)
      PRINT 9103, NAGE(X),TT,(T(J),J=1,NR)
      CLT=0.
      DO 333 J=1,NR
 333  CLT=CLT+CL(X,IO,J)
      DO 13 J=1,NR
      E(X,IO,J)=0.
      IF (CLT.EQ.0.) GO TO 13
      E(X,IO,J)=T(J)/CLT
  13  CONTINUE
  14  CONTINUE
  51  CONTINUE
C  PRINT LIFE EXPECTANCY
      PRINT 9999
      PRINT 4830
 4830 FORMAT (30X,38HEXPECTATIONS OF LIFE BY PLACE OF BIRTH/
     130X,38(1H*))
 876  ISKIP=3
      DO 65 IO=1,NR
        IF (ISKIP.NE.IO) GO TO 127
        PRINT 9999
       ISKIP=ISKIP+2
  127 CONTINUE
      IF (IPREX.EQ.1) PRINT 9502, REG(IO)
      IF (IPREX.EQ.25) PRINT 83, REG(IO)
      PRINT 9100, (REG(J),J=1,NR)
      PRINT 66
      DO 65 X=1,NA
      EE=0.
      DO 15 J=1,NR
  15  EE=EE+E(X,IO,J)
      PRINT 9103, NAGE(X),EE,(E(X,IO,J),J=1,NR)
  65  CONTINUE
```

```
      IF (IPREX.EQ.25) GO TO 877
C ------------------------------------------------------------
C LIFE EXPECTANCY BY PLACE OF RESIDENCE
C -------------------------------------------------------
      PRINT 9999
      PRINT 56
   56 FORMAT (30X,42HEXPECTATIONS OF LIFE BY PLACE OF RESIDENCE/
     130X,42(1H*))
      DO 49 I=1,NR
      DO 49 J=1,NR
   49 A1(J,I)=0.
      NA1=NA+1
      DO 57 IX=1,NA
      X=NA1-IX
      DO 52 I=1,NR
      DO 52 J=1,NR
   52 CC(J,I)=CL(X,I,J)
      CALL INVERT (NR)
      DO 54 I=1,NR
      DO 54 J=1,NR
      A1(J,I)=A1(J,I)+L(X,I,J)
   54 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 55 I=1,NR
      DO 55 J=1,NR
   55 E(X,I,J)=C(J,I)
   57 CONTINUE
      IPREX=25
      GO TO 876
  877 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PRLIF2 (NA,NY,IPROB)
      DIMENSION ZEX(2)
      COMMON /CCL/ CL(18,7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CSU/ SU(18,7,7)
      REAL L
      INTEGER X,XY
      PRINT 9999
 9999 FORMAT (1H1//1X)
      PRINT 5054, IPROB
 5054 FORMAT (7X,5HTABLE,4X,39H- MULTIREGIONAL (TWO-REGION) LIFE TABLE,
     11X,6HOPTION,I2/7X,5(1H-),4X,48(1H-)/)
      I=1
      J=2
      PRINT 5059, I,((I,I,J,I),K=1,3),J,I,I,((I,I,J,I),K=1,2)
 5059 FORMAT (//1X,3HAGE,4X,4HQ(X,,I1,1H),2(2X,4HP(X,,I1,1H,,I1,1H)),
     12(1X,4HL(X,,I1,1H,,I1,1H)),2(1X,5HLL(X,,I1,1H,,I1,1H)),
     12X,4HM(X,,I1,1H,,I1,1H),3X,5HMD(X,,I1,1H),2(2X,4HS(X,,I1,
     11H,,I1,1H)),2(1X,4HE(X,,I1,1H,,I1,1H))/)
      DO 5057 X=1,NA
      ZZ=1.-P(X,1,1)-P(X,2,1)
      ZCL1=CL(X,1,1)*RADIX(1)
      ZCL2=CL(X,1,2)*RADIX(1)
      CLT=0.
      DO 13 J=1,2
   13 CLT=CLT+CL(X,1,J)
      DO 15 J=1,2
      ZEX(J)=0.
      IF (CLT.EQ.0.) GO TO 15
      TT=0.
      DO 14 XY=X,NA
   14 TT=TT+L(XY,1,J)
      ZEX(J)=TT/CLT
   15 CONTINUE
      PRINT 5056, NAGE(X),ZZ,P(X,1,1),P(X,2,1),ZCL1,ZCL2,
     1L(X,1,1),L(X,1,2),RATM(X,2,1),RATD(X,1),SU(X,1,1),SU(X,1,2),
     1(ZEX(J),J=1,2)
 5057 CONTINUE
 5056 FORMAT (1X,I3,3F10.6,2F9.0,2F10.5,4F10.6,2F9.2)
      PRINT 66
   66 FORMAT (1X)
      I=1
      J=2
      PRINT 5059, J,((J,J,I,J),K=1,3),I,J,J,((J,J,I,J),K=1,2)
      DO 5058 X=1,NA
      ZZ=1.-P(X,2,2)-P(X,1,2)
      ZCL1=CL(X,2,2)*RADIX(2)
      ZCL2=CL(X,2,1)*RADIX(2)
      CLT=0.
      DO 16 J=1,2
   16 CLT=CLT+CL(X,2,J)
```

```
      DO 18 J=1,2
      ZEX(J)=0.
      IF (CLT.EQ.0.) GO TO 18
      TT=0.
      DO 17 XY=X,NA
   17 TT=TT+L(XY,2,J)
      ZEX(J)=TT/CLT
   18 CONTINUE
 5058 PRINT 5056, NAGE(X),ZZ,P(X,2,2),P(X,1,2),ZCL1,ZCL2
     1,L(X,2,2),L(X,2,1),RATM(X,1,2),RATD(X,2),SU(X,2,2),SU(X,2,1),
     1ZEX(2),ZEX(1)
      RETURN
      END
```

```
      SUBROUTINE WHOLE (NA,ZFNY,NR)
      DIMENSION CLLTOT(18),CLTOT(18)
      COMMON /CCL/ CL(18,7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CRAD/ RADIX(7),RADIXT
      COMMON /CREG/ REG(13)
      COMMON /CTOTRAT/ POPC(18),RATDT(18),RATFT(18),RATMT(18)
      DOUBLE PRECISION REG,REGL
      REAL L
      INTEGER X,XX
      DATA REGL/8H AGGREG./
      DO 10 X=1,NA
      CLTOT(X)=0.
      CLLTOT(X)=0.
      DO 10 I=1,NR
      ZCLT=0.
      ZCLLT=0.
      DO 11 J=1,NR
      ZCLT=ZCLT+CL(X,I,J)
      ZCLLT=ZCLLT+L(X,I,J)
   11 CONTINUE
      CLTOT(X)=CLTOT(X)+ZCLT*RADIX(I)/RADIXT
      CLLTOT(X)=CLLTOT(X)+ZCLLT*RADIX(I)/RADIXT
   10 CONTINUE
C -------------------------------------------------------
C COMPUTE LIFE TABLE DEATH RATES ASSOCIATED WITH AGGREGATED SYSTEM
C -------------------------------------------------------
      NAA=NA-1
      DO 12 X=1,NAA
      XX=X+1
   12 RATDT(X)=(CLTOT(X)-CLTOT(XX))/CLLTOT(X)
      RATDT(NA)=CLTOT(NA)/CLLTOT(NA)
      PRINT 13
   13 FORMAT (1H1,10X,31HLIFE TABLE OF AGGREGATED SYSTEM/
     111X,31(1H*)/)
      PRINT 14, (REG(J),J=1,NR)
   14 FORMAT (6X,12(2X,A8))
      PRINT 15
   15 FORMAT (1X)
      PRINT 16, (RADIX(J),J=1,NR)
   16 FORMAT (6X,12F10.0)
      PRINT 15
      PRINT 15
      PRINT 15
      CALL TOTSY (NA,ZFNY,1.,1.,1.,0,REGL)
      RETURN
      END
```

```
      SUBROUTINE LMAT (NA,NR,ZFNY,ILIF)
       COMMON /CINV/ CC(7,7)
       COMMON /CL/ L(18,7,7)
       COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
       COMMON /CPQ/ P(18,7,7)
       COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
       COMMON /CRMLA/ RMLA(7,7)
       REAL L
       INTEGER X
C
      NAA=NA-1
      ILIF=1
      DO 23 I=1,NR
      DO 23 J=1,NR
      IF (I.EQ.J) C(J,I)=1.
      IF (I.NE.J) C(J,I)=0.
   23 CONTINUE
C ---------------------------------------------------
C NUMBER OF YEARS LIVED BETWEEN X AND X+H
C ---------------------------------------------------
      DO 333 X=1,NAA
      DO 24 I=1,NR
      DO 24 J=1,NR
      IF (I.EQ.J) A1(J,I)=(P(X,J,I)+1.)*ZFNY*0.5
      IF (I.NE.J) A1(J,I)=P(X,J,I)*ZFNY*0.5
   24 B(J,I)=C(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 25 I=1,NR
      DO 25 J=1,NR
      L(X,I,J)=C(J,I)
   25 A1(J,I)=P(X,J,I)
      CALL MULTIP (NR,NR,NR)
  333  CONTINUE
C ------------------------------------------------------
C NUMBER OF YEARS LIVED IN LAST AGE GROUP
C ------------------------------------------------------
      DO 2 I=1,NR
      DO 2 J=1,NR
      IF (I.EQ.J) C(J,I)=1.
    2 IF (I.NE.J) C(J,I)=0.
      DO 4 X=1,NAA
      DO 3 I=1,NR
      DO 3 J=1,NR
      A1(J,I)=P(X,J,I)
    3 B(J,I)=C(J,I)
      CALL MULTIP (NR,NR,NR)
    4 CONTINUE
      DO 5 I=1,NR
      DO 5 J=1,NR
    5 CC(J,I)=RMLA(J,I)
      CALL INVERT (NR)
      DO 6 I=1,NR
      DO 6 J=1,NR
      A1(J,I)=CC(J,I)
    6 B(J,I)=C(J,I)
```

```
      CALL MULTIP (NR,NR,NR)
      DO 27 I=1,NR
      DO 27 J=1,NR
   27 L(NA,I,J)=C(J,I)
      RETURN
      END
```

```
      SUBROUTINE RELAM (ZFNY,ZLAMDK,RSTAB)
      READ (5,3) ZLAMDK
    3 FORMAT (F12.8)
      RSTAB=ALOG(ZLAMDK)/ZFNY
      RETURN
      END
```

```
      SUBROUTINE GROWTH (NA,ZFNY,NR,ILIF)
      COMMON /CNAG/ NAGE(18)
      COMMON /CGROW/ BR(18,7,7),POPR(18,7)
      COMMON /CINV/ CC(7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      COMMON /CRMLA/ RMLA(7,7)
      COMMON /CSU/ SU(18,7,7)
      DOUBLE PRECISION REG
      INTEGER X,XX
      REAL L
      NAA=NA-1
      ZZ=ZFNY*0.25
      ZFNY2=ZFNY*0.5
C ---------------------------------------------------------
C COMPUTE SURVIVORSHIP PROPORTIONS IF ILIF=0
C ---------------------------------------------------------
      IF (ILIF.NE.0) GO TO 50
      DO 30 X=1,NAA
      XX=X+1
      DO 21 I=1,NR
      CC(I,I)=1.+P(X,I,I)
      DO 21 J=1,NR
      IF (I.NE.J) CC(J,I)=P(X,J,I)
   21 CONTINUE
      CALL INVERT (NR)
      DO 22 I=1,NR
      DO 22 J=1,NR
      A1(J,I)=P(X,J,I)
   22 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
      IF (X.EQ.NAA) GO TO 44
      DO 23 I=1,NR
      A1(I,I)=1.+P(XX,I,I)
      DO 23 J=1,NR
      IF (I.NE.J) A1(J,I)=P(XX,J,I)
   23 B(J,I)=C(J,I)
      CALL MULTIP (NR,NR,NR)
      GO TO 25
   44 DO 26 I=1,NR
      DO 26 J=1,NR
   26 CC(J,I)=RMLA(J,I)
      CALL INVERT (NR)
      DO 27 I=1,NR
      DO 27 J=1,NR
      A1(J,I)=CC(J,I)/ZFNY2
   27 B(J,I)=C(J,I)
      CALL MULTIP (NR,NR,NR)
   25 DO 28 I=1,NR
      DO 28 J=1,NR
   28 SU(X,I,J)=C(J,I)
   30 CONTINUE
   50 CONTINUE
```

```
C ------------------------------------------------------------
C COMPUTE FIRST ROW OF GENERALIZED LESLIE MATRIX
C ------------------------------------------------------------
      DO 4 X=1,NAA
      XX=X+1
      DO 3 I=1,NR
      DO 3 J=1,NR
      IF(I.EQ.J) A1(J,I)=RATF(XX,I)
      IF(I.NE.J) A1(J,I)=0.
    3 B(J,I)=SU(X,I,J)
      CALL MULTIP (NR,NR,NR)
      DO 5 I=1,NR
      DO 5 J=1,NR
      IF (I.EQ.J) B(J,I)=RATF(X,I)+C(J,I)
    5 IF (I.NE.J) B(J,I)=C(J,I)
      DO 7 I=1,NR
      DO 7 J=1,NR
      IF (I.EQ.J) A1(J,I)=ZZ*(P(1,J,I)+1.)
      IF (I.NE.J) A1(J,I)=ZZ*P(1,J,I)
    7 CONTINUE
      CALL MULTIP (NR,NR,NR)
      DO 8 I=1,NR
      DO 8 J=1,NR
    8 BR(X,J,I)=C(J,I)
    4 CONTINUE
C ---------------------------------------------------------------
C  PRINT GROWTH MATRIX (FIRST ROW AND SUBDIAGONAL ELEMENTS)
C ----------------------------------------------------------------
      PRINT 10
   10 FORMAT (1H1,5X,48HTHE DISCRETE MODEL OF MULTIREGIONAL DEMOGRAPHIC
     1,6HGROWTH/6X,54(1H*)/6X,54(1H*)/)
      PRINT 11
   11 FORMAT (/5X,31HMULTIREGIONAL PROJECTION MATRIX/5X,31(1H*))
      DO 20 I=1,NR
       IF (I.NE.1) PRINT 120
  120 FORMAT (1H1,1X)
      PRINT 12, REG(I)
   12 FORMAT (//20X,6HREGION,2X,A8/20X,16(1H*))
      PRINT 13
   13 FORMAT (/5X,3HAGE,8X,9HFIRST ROW)
      PRINT 14, (REG(J),J=1,NR)
   14 FORMAT (11X,12(2X,A8))
      PRINT 15
   15 FORMAT (1X)
      DO 16 X=1,NAA
   16 PRINT 17, NAGE(X),(BR(X,J,I),J=1,NR)
   17 FORMAT (5X,I3,3X,12F10.6)
      PRINT 18
   18 FORMAT (/5X,3HAGE,8X,24HSURVIVORSHIP PROPORTIONS)
      PRINT 14, (REG(J),J=1,NR)
      PRINT 15
      DO 19 X=1,NAA
   19 PRINT 17, NAGE(X),(SU(X,I,J),J=1,NR)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PROJEC (NA,NY,ZFNY,NR,INIT,NHORIZ,INTV,
     1ITOLX,NTOLL,ZLAMDA,IPROJ)
      DIMENSION ZMIN1(7),HUP(7),ZLAMB(7),AGEM(7),ZR(7)
      DIMENSION PREC(7),HU(7)
      DIMENSION POPTOT(7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CGROW/ BR(18,7,7),POPR(18,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      COMMON /CSU/ SU(18,7,7)
      COMMON /CTOTRAT/ PCT(18),RATDT(18),RATFT(18),RATMT(18)
      DOUBLE PRECISION REG
      INTEGER X,X1,X2
      DATA ZDAT1/5HM.AGE/,ZDAT2/5HSHA  /,ZDAT3/5HLAM  /,ZDAT4/5H  R  /
      IPROJ=1
      JGO=0
      IPROJ=0
      Z11=(-1)*NTOLL
      TOLX=10.**(Z11)
      NAA=NA-1
       ZLAM1=10.
      NYEAR1=INIT
      NYEAPR=INIT+INTV
      PRINT 1
    1 FORMAT (1H1,5X,35HMULTIREGIONAL POPULATION PROJECTION/6X,
     135(1H*)/)
      GO TO 509
C -----------------------------------------------------------------
C PROJECT POPULATION NY YEARS
C ------------------------------------------------------------------
 500  CONTINUE
C IPROJ = ITERATION NUMBER
C NYEAR1=PROJECTION YEAR (=INIT + IPROJ*NY )
C ZMIN1(I) = POPULATION OF REGION I AT TIME T-1
C ZMINT    = POPULATION OF TOTAL SYSTEM AT TIME T-1
      IPROJ=IPROJ+1
      NYEAR1=NYEAR1+NY
      DO 3 I=1,NR
      HUP(I)=0.
    3 ZMIN1(I)=POPTOT(I)
      ZMINT=PTOTA
C FIRST AGE GROUP
      DO 2 X=1,NA
      DO 4 J=1,NR
      B(J,1)=POPR(X,J)
      DO 4 I=1,NR
    4 A1(J,I)=BR(X,J,I)
      CALL MULTIP (NR,NR,1)
      DO 5 J=1,NR
    5 HUP(J)=HUP(J)+C(J,1)
    2 CONTINUE
```

```
C OTHER AGE GROUPS
      DO 6 X=1,NAA
      X1=NA-X
      X2=X1+1
      DO 7 J=1,NR
      B(J,1)=POPR(X1,J)
      DO 7 I=1,NR
    7 A1(J,I)=SU(X1,I,J)
      CALL MULTIP (NR,NR,1)
      DO 8 J=1,NR
    8 POPR(X2,J)=C(J,1)
    6 CONTINUE
      DO 9 J=1,NR
    9 POPR(1,J)=HUP(J)
  509 CONTINUE
C   COMPUTE TOTAL POPUL ATION
      DO 11 X=1,NA
      PCT(X)=0.
      DO 11 J=1,NR
   11 PCT(X)=PCT(X)+POPR(X,J)
      DO 13 J=1,NR
      POPTOT(J)=0.
      DO 13 X=1,NA
   13 POPTOT(J)=POPTOT(J)+POPR(X,J)
      PTOTA=0.
      DO 17 J=1,NR
   17 PTOTA=PTOTA+POPTOT(J)
C -------------------------------------------------------------
C CHECK WHETHER OUTPUT MUST BE PRINTED
C --------------------------------------------------------------
      IF ((NYEAR1.GT.NHORIZ).AND.(NYEAR1.NE.NYEAPR)) GO TO 501
C --------------------------------------------------------------
C PRINT PROJECTED POPULATION
C ---------------------------------------------------------------
      IF (IPROJ.GT.0) PRINT 51
   51 FORMAT (1H1,1X)
      PRINT 52, NYEAR1
   52 FORMAT (5X,4HYEAR,1X,I5/5X,10(1H-)/)
       PRINT 253
  253 FORMAT (10X,10HPOPULATION/10X,5(2H- )/)
  578 IF (NR.LE.10) PRINT 53, (REG(J),J=1,NR)
   53 FORMAT (1X,3HAGE,2X,6X,5HTOTAL,10(3X,A8))
      IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
   80 FORMAT (1X,3HAGE,2X,6X,5HTOTAL,12(1X,A8))
      PRINT 54
   54 FORMAT (1X)
      DO 55 X=1,NA
      IF (NR.LE.10) PRINT 56, NAGE(X),PCT(X),(POPR(X,J),J=1,NR)
   56 FORMAT (1X,I3,2X,11F11.0)
   55 IF (NR.GT.10) PRINT 81, NAGE(X),PCT(X),(POPR(X,J),J=1,NR)
   81 FORMAT (1X,I3,2X,F11.0,12F9.0)
       PRINT 54
      IF (NR.LE.10) PRINT 57, PTOTA,(POPTOT(J),J=1,NR)
   57 FORMAT (1X,5HTOTAL,11F11.0)
      IF (NR.GT.10) PRINT 82, PTOTA,(POPTOT(J),J=1,NR)
   82 FORMAT (1X,5HTOTAL,F11.0,12F9.0)
```

```
C  PERCENTAGE DISTRIBUTION
      PRINT 58
   58 FORMAT (//10X,23HPERCENTAGE DISTRIBUTION/10X,12(2H- )/)
      IF (NR.LE.10) PRINT 53, (REG(J),J=1,NR)
      IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
      PRINT 54
      ZHU=0.
      DO 23 J=1,NR
   23 HU(J)=0.
      DO 59 X=1,NA
      PRCT=100.*PCT(X)/PTOTA
      ZHU=ZHU+PRCT
      DO 14 J=1,NR
      PERC(J)=100.*POPR(X,J)/POPTOT(J)
   14 HU(J)=HU(J)+PERC(J)
      IF (NR.LE.10) PRINT 60, NAGE(X),PRCT,(PERC(J),J=1,NR)
   60 FORMAT (1X,I3,2X,11F11.4)
   59 IF (NR.GT.10) PRINT 84, NAGE(X),PRCT,(PERC(J),J=1,NR)
   84 FORMAT (1X,I3,2X,F11.2,12F9.2)
      IF (NR.LE.10) PRINT 761, ZHU, (HU(J),J=1,NR)
  761 FORMAT (/1X,5HTOTAL,11F11.4)
      IF (NR.GT.10) PRINT 85, ZHU,(HU(J),J=1,NR)
   85 FORMAT (/1X,5HTOTAL,F11.2,12F9.2)
C  MEAN AGE
      AGEMT=0.
       DO 21 J=1,NR
   21 AGEM(J)=0.
      DO 20 X=1,NA
      N9=NAGE(X)
      A9=FLOAT(N9)+ZFNY*0.5
      AGEMT=AGEMT+A9*PCT(X)/PTOTA
      DO 20 J=1,NR
   20 AGEM(J)=AGEM(J)+A9*POPR(X,J)/POPTOT(J)
      IF (NR.LE.10) PRINT 22, ZDAT1,AGEMT,(AGEM(J),J=1,NR)
   22 FORMAT (1X,A5,11F11.4)
      IF (NR.GT.10) PRINT 86, ZDAT1,AGEMT,(AGEM(J),J=1,NR)
   86 FORMAT (1X,A5,F11.4,12F9.4)
C  REGIONAL SHARE
      Z=0.
      DO 16 J=1,NR
      HUP(J)=(POPTOT(J)/PTOTA)*100.
   16 Z=Z+HUP(J)
      IF (NR.LE.10) PRINT 22, ZDAT2,Z,(HUP(J),J=1,NR)
      IF (NR.GT.10) PRINT 86, ZDAT2,Z,(HUP(J),J=1,NR)
  501 CONTINUE
C GROWTH RATIO  (LAM)
      IF (IPROJ.EQ.0) GO TO 500
      IF (JGO.GE.1) GO TO 505
      DO 62 J=1,NR
   62 ZLAMB(J)=POPTOT(J)/ZMIN1(J)
       ZZ=PTOTA/ZMINT
      IF ((NYEAR1.GT.NHORIZ).AND.(NYEAR1.NE.NYEAPR)) GO TO 502
      IF (NYEAR1.GT.NHORIZ) NYEAPR=NYEAPR+INTV
  505 CONTINUE
```

```
      IF (NR.LE.10) PRINT 64, ZDAT3,ZZ,(ZLAMB(J),J=1,NR)
   64 FORMAT (1X,A5,11F11.6)
      IF (NR.GT.10) PRINT 88, ZDAT3,ZZ,(ZLAMB(J),J=1,NR)
   88 FORMAT (1X,A5,F11.6,12F9.6)
C ANNUAL GROWTH RATE
      RSTAB=ALOG(ZZ)/ZFNY
      DO 27 J=1,NR
   27 HUP(J)=ALOG(ZLAMB(J))/ZFNY
      IF (NR.LE.10) PRINT 64, ZDAT4,RSTAB,(HUP(J),J=1,NR)
      IF (NR.GT.10) PRINT 88, ZDAT4,RSTAB,(HUP(J),J=1,NR)
  502 CONTINUE
      IF (JGO.GE.1) GO TO 504
C --------------------------------------------------------------
C COMPARE GROWTH RATIO WITH TOLERANCE LEVEL
C --------------------------------------------------------------
      IF (ITOLX.EQ.3) ZTOLX=ZLAMB(1)-ZLAM1
      IF (ITOLX.EQ.3) ZLAM1=ZLAMB(1)
      IF (ITOLX.EQ.2) ZTOLX=ZLAMB(NR)-ZLAMB(1)
      TTOLX=-TOLX
      IF ((ZTOLX.GT.TOLX).OR.(ZTOLX.LT.TTOLX)) GO TO 500
      JGO=JGO+1
C ZLAMDA = STABLE GROWTH RATIO
      ZLAMDA=ZZ
      PRINT 18, TOLX
   18 FORMAT (1H0,1X,30HTOLERANCE LEVEL FOR EIGENVALUE,E14.4)
      PRINT 65, IPROJ
   65 FORMAT (2X,39HNUMBER OF ITERATIONS TO REACH STABILITY,I7)
C -------------------------------------------------------------
C  STABLE EQUIVALENT
C -------------------------------------------------------------
      ZS=ZLAMDA**IPROJ
      DO 66 J=1,NR
      POPTOT(J)=POPTOT(J)/ZS
      DO 66 X=1,NA
   66 POPR(X,J)=POPR(X,J)/ZS
      DO 68 X=1,NA
   68 PCT(X)=PCT(X)/ZS
      PTOTA=PTOTA/ZS
      PRINT 69
   69 FORMAT (1H1,1X,40HSTABLE EQUIVALENT TO ORIGINAL POPULATION/2X,
     140(1H*)/)
      GO TO 578
  504 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AGEDIS (NA,ZFNY,NR,R)
      DIMENSION HULP(7),HU1(7)
      COMMON /C1/ POP(18,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CEX/ EX(18)
      COMMON /CL/ L(18,7,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      INTEGER X
      REAL L
      DO 3 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      Z=-Z*R
      EX(X)=EXP(Z)
    3 CONTINUE
      PRINT 5
    5 FORMAT (1H1,9X,41HPOPULATION DISTRIBUTION BY AGE AND REGION/
     110X,41(1H*)/)
C ------------------------------------------------------------
C PRINT OBSERVED POPULATION DISTRIBUTION
C ------------------------------------------------------------
      PRINT 6
    6 FORMAT (/10X,43HOBSERVED POPULATION (BY PLACE OF RESIDENCE)/
     110X,43(1H=))
      PRINT 7, (REG(J),J=1,NR)
    7 FORMAT (/6X,12(2X,A8))
      PRINT 64
   64 FORMAT (1X)
      DO 25 J=1,NR
      HULP(J)=0.
      DO 25 X=1,NA
   25 HULP(J)=HULP(J)+POP(X,J)
      DO 8 X=1,NA
    8 PRINT 9, NAGE(X),(POP(X,J),J=1,NR)
    9 FORMAT (1X,I3,2X,12F10.0)
      PRINT 15, (HULP(J),J=1,NR)
   15 FORMAT (/1X,5HTOTAL,10F10.0)
C ------------------------------------------------------------
C COMPUTE AND PRINT LIFE TABLE AND STABLE POPULATION DISTRIBUTION
C ITER = 1 : LIFE TABLE
C ITER = 2 : STABLE POPULATION
C -------------------------------------------------------------
      DO 21 ITER=1,2
   10 FORMAT (1H1,9X,21HLIFE TABLE POPULATION/10X,21(1H=))
   11 FORMAT (//10X,24HINITIAL REGION OF COHORT,2X,A8/
     110X,34(1H-)/)
   20 FORMAT (1H1,9X,33HSTABLE POPULATION  (GROWTH RATE =,F10.6,
     12H )/10X,45(1H=))
      IF (ITER.EQ.1) PRINT 10
      IF (ITER.EQ.2) PRINT 20,R
      ISKIP=3
      DO 21 I=1,NR
      IF (ISKIP.NE.I) GO TO 40
      PRINT 41
   41 FORMAT (1H1/1X)
```

```
      ISKIP=ISKIP+2
40    CONTINUE
      PRINT 11, REG(I)
      IF (NR.LE.10) PRINT 14, (REG(J),J=1,NR)
14    FORMAT (12X,5HTOTAL,10(3X,A8))
      IF (NR.GT.10) PRINT 80, (REG(J),J=1,NR)
80    FORMAT (11X,5HTOTAL,12(1X,A8))
      PRINT 64
       ZH=0.
      DO 24 J=1,NR
24     HULP(J)=0.
      DO 22 X=1,NA
      Z=0.
      DO 23 J=1,NR
      IF (ITER.EQ.1) HU1(J)=L(X,I,J)
      IF (ITER.EQ.2) HU1(J)=EX(X)*L(X,I,J)
      Z=Z+HU1(J)
23    HULP(J)=HULP(J)+HU1(J)
      IF (NR.LE.10) PRINT 13, NAGE(X),Z,(HU1(J),J=1,NR)
13    FORMAT (1X,I3,2X,11F11.6)
22    IF (NR.GT.10) PRINT 81, NAGE(X),Z,(HU1(J),J=1,NR)
81    FORMAT (1X,I3,2X,13F9.5)
      DO 26 J=1,NR
26    ZH=ZH+HULP(J)
      IF (NR.LE.10) PRINT 18, ZH,(HULP(J),J=1,NR)
18    FORMAT (/1X,5HTOTAL,11F11.6)
      IF (NR.GT.10) PRINT 82, ZH,(HULP(J),J=1,NR)
82    FORMAT (/1X,5HTOTAL,13F9.5)
21    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FERMOB (NA,ZFNY,NR,NOPMOB,NEIG,R)
      DIMENSION ZMOMT(7)
      DIMENSION HULP(7,7),HULP2(7,7),ZGRR(7),ZMOM(3,7,7)
      DIMENSION HULP7(7),HU(18)
      DIMENSION HULP5(7),ZGRAL(18,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
      COMMON /CINV/ CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      INTEGER X
      REAL L
   64 FORMAT (1X)
C ------------------------------------------------------------
C NOPMOB = 1 LIFE TABLE ANALYSIS
C NOPMOB = 2 STATIONARY (ZPG) POPULATION ANALYSIS
C NOPMOB = 3 STABLE POPULATION ANALYSIS
C ------------------------------------------------------------
      PRINT 222
  222 FORMAT (1H1,1X)
      PRINT 223
  223 FORMAT (10X,41(1H*)/10X,41(1H*)/10X,3H* *,35X,3H* *)
      IF (NOPMOB.EQ.1) PRINT 224
  224 FORMAT (10X,3H* *,1X,33HANALYSIS OF LIFE TABLE POPULATION,
     11X,3H* *)
      IF (NOPMOB.EQ.2) PRINT 229
  229 FORMAT (10X,3H* *,1X,33HANALYSIS OF STATIONARY POPULATION,
     11X,3H* *)
      IF (NOPMOB.EQ.3) PRINT 226
  226 FORMAT (10X,3H* *,1X,29HANALYSIS OF STABLE POPULATION,5X,
     13H* *)
      PRINT 227
  227 FORMAT (10X,3H* *,35X,3H* */10X,41(1H*)/10X,41(1H*)/)
      DO 228 X=1,NA
      HU(X)=1.
C COMPUTE WEIGHTS FOR STABLE AGE COMPOSITION
      IF (NOPMOB.NE.3) GO TO 228
      DO 2 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      Z1=-Z*R
      HU(X)=EXP(Z1)
    2 CONTINUE
  228 CONTINUE
C ---------------------------------------------------------
C FERTILITY ANALYSIS (INTEGR=1) AND MOBILITY ANALYSIS (INTEGR=2)
C ---------------------------------------------------------
      DO 580 INTEGR=1,2
       IF ((INTEGR.EQ.2).AND.(NOPMOB.EQ.2)) GO TO 580
      DO 121 I=1,NR
      DO 121 X=1,NA
      IF (INTEGR.NE.2) GO TO 24
```

```
      Z=0.
      DO 23 J=1,NR
   23 Z=Z+RATM(X,J,I)
      ZGRAL(X,I)=Z
   24 CONTINUE
      IF (INTEGR.EQ.1) ZGRAL(X,I =RATF(X,I)
  121 CONTINUE
      IF (INTEGR.EQ.1) PRINT 36
   36 FORMAT (1H0,9X,18HFERTILITY ANALYSIS/10X,18(1H*))
      IF (INTEGR.EQ.2) PRINT 37
   37 FORMAT (1H1,9X,18HMIGRATION ANALYSIS/10X,18(1H*))
C ----------------------------------------------------------------
C   PRINT RATES
C ----------------------------------------------------------------
      PRINT 5
    5 FORMAT (//10X,18HAGE-SPECIFIC RATES/10X,18(1H=))
      PRINT 7, (REG(J),J=1,NR)
    7 FORMAT (/1X,3HAGE,2X,12(2X,A8))
      PRINT 64
      DO 687 X=1,NA
  687 PRINT 686, NAGE(X),(ZGRAL(X,J),J=1,NR)
  686 FORMAT (1X,I3,2X,12F10.6)
      DO 684 J=1,NR
      ZGRR(J)=0.
      DO 684 X=1,NA
  684 ZGRR(J)=ZGRR(J)+ZGRAL(X,J)*ZFNY
      IF (INTEGR.EQ.1) PRINT 685, (ZGRR(J),J=1,NR)
  685 FORMAT (/1X,3HGRR,2X,12F10.6)
      IF (INTEGR.EQ.2) PRINT 688, (ZGRR(J),J=1,NR)
  688 FORMAT (/1X,3HGMR,2X,12F10.6)
C ----------------------------------------------------------------
C INTEGRALS OF NET MATERNITY AND MOBILITY FUNCTIONS
C ----------------------------------------------------------------
      IF ((INTEGR.EQ.1).AND.(NOPMOB.LE.2)) PRINT 120
  120 FORMAT (1H1,9X,47HINTEGRALS OF GENERALIZED NET MATERNITY FUNCTION/
     110X,47(1H=))
      IF ((INTEGR.EQ.1).AND.(NOPMOB.EQ.3)) PRINT 124
  124 FORMAT (1H1,9X,38HINTEGRALS OF WEIGHTED GENERALIZED NET ,
     118HMATERNITY FUNCTION/10X,56(1H=))
      IF ((INTEGR.EQ.2).AND.(NOPMOB.LE.2)) PRINT 122
  122 FORMAT (1H1,9X,46HINTEGRALS OF GENERALIZED NET MOBILITY FUNCTION/
     110X,46(1H=))
      IF ((INTEGR.EQ.2).AND.(NOPMOB.EQ.3)) PRINT 123
  123 FORMAT (1H1,9X,38HINTEGRALS OF WEIGHTED GENERALIZED NET ,
     117HMOBILITY FUNCTION/10X,55(1H=))
      DO 110 I=1,NR
      DO 110 J=1,NR
  110  HULP(J,I)=0.
      ISKIP=3
      DO 3 I=1,NR
      IF (ISKIP.NE.I) GO TO 40
      PRINT 222
      ISKIP=ISKIP+2
   40 CONTINUE
```

```
      PRINT 4, REG(I)
    4 FORMAT (//10X,24HINITIAL REGION OF COHORT,2X,A8/
     110X,34(1H-)/)
      PRINT 7, (REG(J),J=1,NR)
      DO 8 X=1,NA
       DO 109 J=1,NR
      HULP4(J)=HU(X)*ZGRAL(X,J)*L(X,I,J)
  109 HULP(J,I)=HULP(J,I)+HULP4(J)
    8 PRINT 9, NAGE(X),(HULP4(J),J=1,NR)
    9 FORMAT (1X,I3,2X,12F10.6)
      PRINT 108, (HULP(J,I),J=1,NR)
  108 FORMAT (/1X,5HTOTAL,12F10.6)
    3 CONTINUE
C ---------------------------------------------
C MOMENTS OF INTEGRAL FUNCTIONS
C ---------------------------------------------
      PRINT 33
   33 FORMAT (1H1,9X,28HMOMENTS OF INTEGRAL FUNCTION/10X,28(1H=))
      NMOMEN=2
      NMOM=NMOMEN+1
      DO 13 IMOM=1,NMOM
      IN8=IMOM-1
      DO 12 I=1,NR
      DO 12 J=1,NR
      ZMOM(IMOM,J,I)=0.
      DO 12 X=1,NA
      PATFU=HU(X)*ZGRAL(X,J)*L(X,I,J)
      Z=1.
      IF (IN8.EQ.0) GO TO 12
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      Z=Z**IN8
   12 ZMOM(IMOM,J,I)=ZMOM(IMOM,J,I)+Z*PATFU
      PRINT 61, IN8
   61 FORMAT (//9X,I2,1X,6HMOMENT/9X,9(1H-))
      DO 90 I=1,NR
      DO 90 J=1,NR
   90 CE(J,I)=ZMOM(IMOM,J,I)
      CALL EIGEN(NR,1,NEIG)
   13 CONTINUE
C -----------------------------------------------------
C MATRICES OF MEAN AGES
C -----------------------------------------------------
      PRINT 167
  167 FORMAT (1H1,9X,35HMATRICES OF MEAN AGES AND VARIANCES/10X,35(1H=))
      PRINT 125
  125 FORMAT (//1X)
      IK=1
      PRINT 723, IK
  723 FORMAT (3X,2H**,1X,11HALTERNATIVE,I2,1X,2H**/3X,19(1H*))
      PRINT 67
   67 FORMAT (/9X,5HMEANS/9X,5(1H-))
      DO 19 I=1,NR
      DO 19 J=1,NR
   19 CE(J,I)=ZMOM(2,J,I)/ZMOM(1,J,I)
      CALL EIGEN(NR,2,NEIG)
```

```
      PRINT 68
   68 FORMAT (//9X,9HVARIANCES/9X,9(1H-))
      DO 21 I=1,NR
      DO 21 J=1,NR
      Z=CE(J,I)*CE(J,I)
   21 CE(J,I)=ZMOM(3,J,I)/ZMOM(1,J,I)-Z
      CALL EIGEN (NR,2,NEIG)
      PRINT 125
      PRINT 64
      IK=2
      PRINT 723, IK
      PRINT 67
      DO 14 I=1,NR
      DO 14 J=1,NR
   14 CC(J,I)=ZMOM(1,J,I)
      CALL INVERT (NR)
      DO 17 I=1,NR
      DO 17 J=1,NR
      A1(J,I)=ZMOM(2,J,I)
   17 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 91 I=1,NR
      DO 91 J=1,NR
   91 CE(J,I)=C(J,I)
      CALL EIGEN (NR,1,NEIG)
      PRINT 68
       DO 93 I=1,NR
      DO 93 J=1,NR
      A1(J,I)=ZMOM(3,J,I)
   93 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 94 I=1,NR
      DO 94 J=1,NR
      HULP(J,I)=C(J,I)
      A1(J,I)=CE(J,I)
   94 B(J,I)=CE(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 95 I=1,NR
      DO 95 J=1,NR
   95 CE(J,I)=HULP(J,I)-C(J,I)
      CALL EIGEN (NR,1,0)
C ------------------------------------------------------------
C EXPECTANCIES AND ALLOCATIONS
C ------------------------------------------------------------
      IF (INTEGR.EQ.1) GO TO 579
        PRINT 777
  777 FORMAT (1H1,9X,30HSPATIAL MIGRATION EXPECTANCIES/10X,30(1H*)/)
      DO 771 I=1,NR
      DO 771 J=1,NR
      CE(J,I)=0.
      DO 771 X=1,NA
  771 CE(J,I)=CE(J,I)+HU(X)*L(X,I,J)
        PRINT 772
  772 FORMAT (//10X,20HEXPECTATIONS OF LIFE/10X,20(1H-)/)
       CALL EIGEN (NR,1,1)
```

```
      PRINT 774
774   FORMAT (//10X,16HMIGRATION LEVELS/10X,16(1H-)/)
      DO 775 I=1,NR
      Z=0.
      DO 776 J=1,NR
776   Z=Z+CE(J,I)
      DO 775 J=1,NR
775   CE(J,I)=CE(J,I)/Z
      CALL EIGEN (NR,1,0)
579   CONTINUE
      IF (INTEGR.EQ.2) PRINT 777
      IF (INTEGR.EQ.1) PRINT 882
882   FORMAT (1H1,9X,30HSPATIAL FERTILITY EXPECTANCIES/10X,30(1H*)/)
      IF (INTEGR.EQ.2) PRINT 886
886   FORMAT (//10X,24HNET MIGRAPRODUCTION RATE  /10X,24(1H-)/)
      IF (INTEGR.EQ.1) PRINT 887
887   FORMAT (//10X,21HNET REPRODUCTION RATE/10X,21(1H-)/)
      DO 888 I=1,NR
      DO 888 J=1,NR
888   CE(J,I)=ZMOM(1,J,I)
      CALL EIGEN (NR,1,1)
      PRINT 64
      IF (INTEGR.EQ.2) PRINT 891
891   FORMAT (//10X,31HNET MIGRAPRODUCTION ALLOCATIONS/10X,31(1H-)/)
      IF (INTEGR.EQ.1) PRINT 892
892   FORMAT (//10X,28HNET REPRODUCTION ALLOCATIONS/10X,28(1H-)/)
      DO 889 I=1,NR
      Z=0.
      DO 890 J=1,NR
890   Z=Z+CE(J,I)
      DO 889 J=1,NR
889   CE(J,I)=CE(J,I)/Z
      CALL EIGEN (NR,1,0)
580   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RVALUE (NA,ZFNY,NR,R,ZVT)
      DIMENSION V(18,7,7),HU(18)
      COMMON /C1/ POP(18,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
      COMMON /CINV/ CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CPSI/ PSI(7,7),VRPSI(7),VLPSI(7),ROPSI
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      INTEGER X,X1,XX
      REAL L
      NAA=NA-1
   64 FORMAT (1X)
   78 FORMAT (1H1,1X)
C ---------------------------------------------------------------
C COMPUTE LEFT EIGENVECTOR OF CHARACTERISTIC MATRIX : VLPSI(J)
C ---------------------------------------------------------------
      PRINT 67
   67 FORMAT (1H1/1X)
      DO 10 X=1,NA
      HU(X)=1.
      IF (R.EQ.0.) GO TO 10
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      Z=-Z*R
      EX(X)=EXP(Z)
      HU(X)=EX(X)
   10 CONTINUE
      DO 11 I=1,NR
      DO 11 J=1,NR
      CE(J,I)=0.
      DO 11 X=1,NA
   11 CE(J,I)=CE(J,I)+HU(X)*RATF(X,J)*L(X,I,J)
      CALL EIGEN (NR,1,1)
      ROPSI=ROOT
      DO 12 I=1,NR
      VLPSI(I)=VECTL(I)
      VRPSI(I)=VECT(I)
      DO 12 J=1,NR
   12 PSI(J,I)=CE(J,I)
C ---------------------------------------------------------------
C RESULTS FOR PEOPLE AT EXACT AGE X (IRES=1) AND IN AGE GROUP
C X TO X+H (IRES=2)
C ---------------------------------------------------------------
      DO 251 IRES=1,2
      IF (IRES.EQ.1) PRINT 250
  250 FORMAT (1H1,1X,32HTHE SPATIAL REPRODUCTIVE VALUE :,1X,
     133HRESULTS FOR PEOPLE AT EXACT AGE X /2X,30(1H*),3X,
     133(1H*)/2X,30(1H*),3X,33(1H*))
      IF (IRES.EQ.2) PRINT 258
  258 FORMAT (1H1,34X,33HRESULTS FOR PEOPLE IN AGE GROUP X /
     135X,33(1H*)/35X,33(1H*))
```

```
      PRINT 51
   51 FORMAT (//10X,41HDISCOUNTED NUMBER OF OFFSPRING PER PERSON,
     1/10X,41(1H*)/)
C -------------------------------------------------------------
C COMPUTE DISCOUNTED NUMBER OF OFFSPRING PER PERSON OF EXACT AGE X
C -------------------------------------------------------------
      IF (IRES.EQ.2) GO TO 257
      Z=-ZFNY*0.5*R
      ZZ=EXP(Z)
      ZZZ=ZFNY*0.5*ZZ
      DO 252 I=1,NR
      DO 252 J=1,NR
  252 V(NA,J,I)=RATF(NA,J)*ZZZ
      DO 255 X=1,NAA
      XX=NA-X
      X1=XX+1
      DO 254 I=1,NR
      DO 254 J=1,NR
      IF (I.EQ.J) A1(J,I)=ZZZ*RATF(XX,J)+ZZ*ZZ*V(X1,J,I)
      IF (I.NE.J) A1(J,I)=ZZ*ZZ* V(X1,J,I)
  254 B(J,I)=P(XX,J,I)
      CALL MULTIP (NR,NR,NR)
      DO 255 I=1,NR
      DO 255 J=1,NR
      IF (I.EQ.J) V(XX,J,I)=ZZZ*RATF(XX,J)+C(J,I)
      IF (I.NE.J) V(XX,J,I)=C(J,I)
  255 CONTINUE
      DO 268 I=1,NR
      DO 268 J=1,NR
  268 V(1,J,I)=PSI(J,I)
      GO TO 730
C --------------------------------------------------------------
DISCOUNTED NUMBER OF OFFSPRING PER PERSON IN AGE GROUP X TO X+H
C --------------------------------------------------------------
  257 CONTINUE
C COMPUTE MATRIX OF SURVIVORSHIP PROPORTIONS
      DO 53 I=1,NR
      DO 53 J=1,NR
   53 V(NA,J,I)=RATF(NA,J)*ZFNY*0.5
      Z=-ZFNY*R
      ZZ=EXP(Z)
      DO 52 X=1,NAA
      XX=NA-X
      X1=XX+1
      DO 21 I=1,NR
      DO 21 J=1,NR
   21 CC(J,I)=L(XX,I,J)
      CALL INVERT (NR)
      DO 22 I=1,NR
      DO 22 J=1,NR
      A1(J,I)=L(X1,I,J)
   22 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
```

```
C COMPUTE DISCOUNTED NUMBER OF OFFSPRING
      DO 54 I=1,NR
      DO 54 J=1,NR
      IF (I.EQ.J) A1(J,I)=ZFNY*0.5*RATF(X1,J)+V(X1,J,I)
      IF (I.NE.J) A1(J,I)=V(X1,J,I)
   54 B(J,I)=C(J,I)*ZZ
      CALL MULTIP (NR,NR,NR)
      DO 55 I=1,NR
      DO 55 J=1,NR
      IF (I.EQ.J) V(XX,J,I)=ZFNY*0.5*RATF(XX,J)+C(J,I)
   55  IF (I.NE.J) V(XX,J,I)=C(J,I)
   52 CONTINUE
  730 CONTINUE
C ------------------------------------------------------------
C PRINT RESULTS
C ------------------------------------------------------------
      ISKIP=3
      DO 58 I=1,NR
      IF (ISKIP.NE.I) GO TO 888
      PRINT 67
      ISKIP=ISKIP+2
  888 CONTINUE
      PRINT 56, REG(I)
   56 FORMAT (//10X,19HREGION OF RESIDENCE,2X,A8/10X,29(1H-)/)
      PRINT 57
   57 FORMAT (10X,28HREGION OF BIRTH OF OFFSPRING)
      IF (NR.LE.10) PRINT 80, (REG(J),J=1,NR)
   80 FORMAT (10X,5HTOTAL,10(2X,A8))
      IF (NR.GT.10) PRINT 81, (REG(J),J=1,NR)
   81 FORMAT (9X,5HTOTAL,12(1X,A8))
      PRINT 64
      DO 58 X=1,NAA
      Z=0.
      DO 60 J=1,NR
   60 Z=Z+V(X,J,I)
      IF (NR.LE.10) PRINT 59, NAGE(X),Z,(V(X,J,I),J=1,NR)
   59 FORMAT (1X,I3,1X,F10.6,10F10.6)
      IF (NR.GT.10) PRINT 83, NAGE(X),Z,(V(X,J,I),J=1,NR)
   83 FORMAT (1X,I3,1X,13F9.6)
   58 CONTINUE
C ------------------------------------------------------------
C COMPUTE AND PRINT SPATIAL REPRODUCTIVE VALUES
C ------------------------------------------------------------
      PRINT 61
   61 FORMAT (1H1,10X,37HSPATIAL REPRODUCTIVE VALUE PER PERSON /
     111X,37(1H*)/)
      PRINT 65, (REG(J),J=1,NR)
   65 FORMAT (5X,12(2X,A8))
      PRINT 64
      DO 62 X=1,NAA
      DO 63 J=1,NR
      A1(1,J)=VLPSI(J)
      DO 63 I=1,NR
   63 B(J,I)=V(X,J,I)
      CALL MULTIP (1,NR,NR)
```

```
      IF (IRES.NE.1) GO TO 264
      IF (X.NE.1) GO TO 264
      DO 263 J=1,NR
  263 C(1,J)=VLPSI(J)
  264 CONTINUE
      PRINT 66, NAGE(X),(C(1,J),J=1,NR)
   66 FORMAT (1X,I3,1X,12F10.6)
   62 CONTINUE
  251 CONTINUE
C -------------------------------------------------------------
C RESULTS FOR TOTAL POPULATION
C -------------------------------------------------------------
      PRINT 71
   71 FORMAT (1H1,36HTOTAL DISCOUNTED NUMBER OF OFFSPRING ,
     123H OF OBSERVED POPULATION /1X,58(1H*)/)
      DO 72 I=1,NR
      DO 72 J=1,NR
      CE(J,I)=0.
      DO 72 X=1,NAA
   72 CE(J,I)=CE(J,I)+V(X,J,I)*POP(X,I)
      ICHE=0
      ZZTOT=0.
      DO 89 J=1,NR
      HU(J)=0.
      DO 88 I=1,NR
   88 HU(J)=HU(J)+CE(I,J)
   89 ZZTOT=ZZTOT+HU(J)
      IF (ZZTOT.LT.10000000.) GO TO 23
      ICHE=1
      PRINT 24
   24 FORMAT (25X,10HIN 100,000/25X,10(1H*)/)
      ZZTOT=ZZTOT*0.00001
      DO 25 I=1,NR
      HU(I)=HU(I)*0.00001
      DO 25 J=1,NR
   25 CE(I,J)=CE(I,J)*0.00001
   23 CONTINUE
      IF (NR.LE.10) PRINT 84, (REG(J),J=1,NR)
   84 FORMAT (16X,5HTOTAL,10(2X,A8))
      IF (NR.GT.10) PRINT 259, (REG(J),J=1,NR)
  259 FORMAT (15X,5HTOTAL,12(1X,A8))
      PRINT 64
      DO 87 I=1,NR
      ZZT=0.
      DO 86 J=1,NR
   86 ZZT=ZZT+CE(I,J)
      IF (NR.LE.10) PRINT 85, REG(I),ZZT,(CE(I,J),J=1,NR)
   85 FORMAT (1X,A8,2X,11F10.0)
      IF (NR.GT.10) PRINT 26, REG(I),ZZT,(CE(I,J),J=1,NR)
   26 FORMAT (1X,A8,2X,13F9.0)
   87 CONTINUE
      IF (NR.LE.10) PRINT 28, ZZTOT,(HU(J),J=1,NR)
   28 FORMAT (/4X,5HTOTAL,2X,11F10.0)
      IF (NR.GT.10) PRINT 29, ZZTOT,(HU(J),J=1,NR)
   29 FORMAT (/4X,5HTOTAL,2X,13F9.0)
```

```
      PRINT   265
265   FORMAT (//1X,42HREPRODUCTIVE VALUE OF THE TOTAL POPULATION
     1/1X,42(1H*)/)
      IF (ICHE.EQ.1) PRINT 27
 27   FORMAT (15X,10HIN 100,000/15X,10(1H*)/)
      PRINT 887
887   FORMAT (17X,5HTOTAL,1X,10HPERCENTAGE/)
      DO 260 J=1,NR
      A1(1,J)=VLPSI(J)
      DO 260 I=1,NR
260   B(J,I)=CE(J,I)
      CALL MULTIP (1,NR,NR)
      ZVT=0.
      DO 92 I=1,NR
 92   ZVT=ZVT+C(1,I)
      ZZ=0.
      DO 261 I=1,NR
      Z=100.*C(1,I)/ZVT
      ZZ=ZZ+Z
261   PRINT 262, REG(I),C(1,I),Z
262   FORMAT (1X,A8,2X,F11.0,F11.2)
      PRINT 93, ZVT,ZZ
 93   FORMAT (/4X,5HTOTAL,2X,F11.0,F11.2)
      IF (ICHEC.EQ.1) ZVT=ZVT*100000.
      RETURN
      END
```

```
      SUBROUTINE RINTR (NA,ZFNY,NR,R,ZVT)
      DIMENSION DISV(7),HULP(13,8),HZ(18),HM(7,7)
      DIMENSION HU(18)
      COMMON /CAGEM/ AGEM(7,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CEIGEN/ CE(7,7),ROOT(7),VECT(7),VECTL(7)
      COMMON /CINV/CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPQ/ P(18,7,7)
      COMMON /CPSI/ PSI(7,7),VRPSI(7),VLPSI(7),ROPSI
      COMMON /CQQ/ QQ(7),POPST(18,7),YY(7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      INTEGER X,X1,XX
      REAL L
      NAA=NA-1
      DO 16 X=1,NA
      HU(X)=1.
      IF (R.EQ.0.) GO TO 16
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      ZZ=-Z*R
      HU(X)=EXP(ZZ)
   16 CONTINUE
C -----------------------------------------------------
C   COMPUTE NORMALIZING FACTOR
C -----------------------------------------------------
      DO 10 I=1,NR
      DO 10 J=1,NR
      CC(J,I)=0.
      A1(J,I)=0.
      DO 10 X=1,NA
      Z=FLOAT(NAGE(X))+ZFNY*0.5
      PATFU=HU(X)*RATF(X,J)*L(X,I,J)
      A1(J,I)=A1(J,I)+Z*PATFU
   10 CC(J,I)=CC(J,I)+PATFU
      CALL INVERT (NR)
      DO 11 I=1,NR
      DO 11 J=1,NR
   11 B(J,I)=CC(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 12 I=1,NR
      DO 12 J=1,NR
   12 AGEM(J,I)=C(J,I)
      DO 85 I=1,NR
      B(I,1)=VRPSI(I)
      DO 85 J=1,NR
   85 A1(J,I)=AGEM(J,I)
      CALL MULTIP (NR,NR,1)
      ZNORM=0.
      DO 86 I=1,NR
   86 ZNORM=ZNORM+C(I,1)*VLPSI(I)
      VKNORM=ZVT/ZNORM
```

```
C -----------------------------------------------------------------
C STABLE EQUIVALENTS OF BIRTHS
C -----------------------------------------------------------------
       DO 94 I=1,NR
      QQ(I)=VRPSI(I)*VKNORM
  94  CONTINUE
  64  FORMAT (1X)
C -----------------------------------------------------------------
C STABLE EQUIVALENT OF TOTAL POPULATION
C -----------------------------------------------------------------
      PRINT 96
  96  FORMAT (1H1,10X,37HSTABLE EQUIVALENT OF TOTAL POPULATION/
     111X,37(1H*)/)
      DO 121 X=1,NA
      DO 121 J=1,NR
      HULP(J,1)=0.
      DO 120 I=1,NR
 120  HULP(J,1)=HULP(J,1)+L(X,I,J)*QQ(I)
 121  POPST(X,J)=HU(X)*HULP(J,1)
      YT=0.
      DO 122 I=1,NR
      YY(I)=0.
      DO 134 X=1,NA
 134  YY(I)=YY(I)+POPST(X,I)
 122  YT=YT+YY(I)
      IF (NR.LE.10) PRINT 133, (REG(J),J=1,NR)
 133  FORMAT (11X,5HTOTAL,10(2X,A8))
      IF (NR.GT.10) PRINT 33, (REG(J),J=1,NR)
  33  FORMAT (11X,5HTOTAL,12(1X,A8))
      PRINT 64
      DO 123 X=1,NA
      HZ(X)=0.
      DO 132 J=1,NR
 132  HZ(X)=HZ(X)+POPST(X,J)
      IF (NR.LE.10) PRINT 124, NAGE(X),HZ(X),(POPST(X,J),J=1,NR)
 124  FORMAT (1X,I3,2X,11F10.0)
      IF (NR.GT.10) PRINT 34, NAGE(X),HZ(X),(POPST(X,J),J=1,NR)
  34  FORMAT (1X,I3,2X,F10.0,12F9.0)
 123  CONTINUE
      IF (NR.LE.10) PRINT 125, YT, (YY(J),J=1,NR)
 125  FORMAT (/1X,5HTOTAL,11F10.0)
      IF (NR.GT.10) PRINT 35, YT,(YY(J),J=1,NR)
  35  FORMAT (/1X,5HTOTAL,F10.0,12F9.0)
      PRINT 31
  31  FORMAT (//10X,23HPERCENTAGE DISTRIBUTION /10X,23(1H-))
      IF (NR.LE.10) PRINT 133, (REG(J),J=1,NR)
      IF (NR.GT.10) PRINT 33, (REG(J),J=1,NR)
      PRINT 64
      DO 126 X=1,NA
      HZ(X)=100.*HZ(X)/YT
      DO 127 J=1,NR
 127  DISV(J)=100.*POPST(X,J)/YY(J)
      IF (NR.LE.10) PRINT 135, NAGE(X),HZ(X),(DISV(J),J=1,NR)
 135  FORMAT (1X,I3,2X,11F10.3)
 128  FORMAT (1X,I3,2X,F10.3,12F9.3)
```

```
 126  IF (NR.GT.10) PRINT 128, NAGE(X),HZ(X),(DISV(J),J=1,NR)
      PRINT 64
      Z=100.
      IF (NR.LE.10) PRINT 141, Z,(Z,J=1,NR)
 141  FORMAT (1X,5HTOTAL,11F10.3)
      IF (NR.GT.10) PRINT 142, Z,(Z,J=1,NR)
 142  FORMAT (1X,5HTOTAL,F10.3,12F9.3)
      DO 138 J=1,NR
 138  HZ(J)=100.*YY(J)/YT
      IF (NR.LE.10) PRINT 139, Z,(HZ(J),J=1,NR)
 139  FORMAT (1X,5HSHARE,11F10.3)
 140  FORMAT (1X,5HSHARE,F10.3,12F9.3)
      IF (NR.GT.10) PRINT 140, Z,(HZ(J),J=1,NR)
C ---------------------------------------------------
C STABLE EQUIVALENTS AND INTRINSIC RATES OF DEATH AND MIGRATION
C ---------------------------------------------------
      DO 43 I=1,NR
      HULP(I,1)=QQ(I)
  43  CONTINUE
      PRINT 131
 131  FORMAT (1H1,10X,38HSTABLE EQUIVALENTS AND INTRINSIC RATES/
     111X,38(1H*)/)
      PRINT 49
  49  FORMAT (1H0,18X,10X,6HBIRTHS,21X,6HDEATHS,18X,12HOUTMIGRATION,
     116X,11HINMIGRATION/)
      PRINT 115
 115 FORMAT (13X,4(3X,6X,6HNUMBER,8X,4HRATE)/)
      DO 25 X=1,NAA
      DO 21 I=1,NR
      DO 21 J=1,NR
      IF (I.EQ.J) CC(J,I)=P(X,J,I)+1.
  21  IF (I.NE.J) CC(J,I)=P(X,J,I)
      CALL INVERT (NR)
      Z2=0.5*ZFNY
      Z1=Z2*R
      Z=-ZFNY*R
      Z8=EXP(Z)
      Z9=EXP(Z1)/Z2
      DO 22 I=1,NR
      DO 22 J=1,NR
       B(J,I)=CC(J,I)
      IF (I.EQ.J) A1(J,I)=1.-P(X,J,I)*Z8
  22  IF (I.NE.J) A1(J,I)=-P(X,J,I)*Z8
      CALL MULTIP (NR,NR,NR)
      DO 23 I=1,NR
      DO 23 J=1,NR
      IF (I.EQ.J) HM(J,I)=Z9*C(J,I)-R
  23  IF (I.NE.J) HM(J,I)=Z9*C(J,I)
      DO 25 I=1,NR
      Z7=0.
      DO 24 J=1,NR
      IF (I.EQ.J) GO TO 24
      Z7=Z7+HM(J,I)
  24  CONTINUE
      RATD(X,I)=HM(I,I)+Z7
```

```
      DO 25 J=1,NR
      RATM(X,J,I)=-HM(J,I)
 25   CONTINUE
      DO 27 I=1,NR
      DO 26 J=1,NR
 26    RATM(NA,J,I)=0.
 27   RATD(NA,I)=EXP(Z1)*RATD(NA,I)-R
      DO 111 I=1,NR
      DO 118 J=2,8
118   HULP(I,J)=0.
      DO 111 X=1,NA
      HULP(I,3)=HULP(I,3)+POPST(X,I)*RATD(X,I)
      DO 119 J=1,NR
      IF (I.EQ.J) GO TO 119
      HULP(I,5)=HULP(I,5)+POPST(X,I)*RATM(X,J,I)
      HULP(I,7)=HULP(I,7)+POPST(X,J)*RATM(X,I,J)
119   CONTINUE
111   CONTINUE
      NR1=NR+1
      DO 20 J=1,8
 20   HULP(NR1,J)=0.
      DO 113 I=1,NR
      DO 113 J=1,4
      JK=J*2-1
      JJ=JK+1
      HULP(I,JJ)=HULP(I,JK)/YY(I)
      HULP(NR1,JK)=HULP(NR1,JK)+HULP(I,JK)
113   CONTINUE
      DO 177 I=1,NR
177   PRINT 114, REG(I),(HULP(I,J),J=1,8)
114   FORMAT (5X,A8,4(3X,F12.0,F12.6))
      DO 130 J=1,4
      JJ=J*2
130   HULP(NR1,JJ)=HULP(NR1,JJ-1)/YT
      PRINT 116, (HULP(NR1,J),J=1,8)
116   FORMAT (/8X,5HTOTAL,4(3X,F12.0,F12.6))
      PRINT 178, R
178   FORMAT (//10X,18HSTABLE GROWTH RATE,4X,F10.6)
      PRINT 179, ZNORM
179   FORMAT (/10X,18HNORMALIZING FACTOR,2X,F12.4)
      RETURN
      END
```

```
      SUBROUTINE MOMENT (NA,ZFNY,NR,R)
      DIMENSION RO(7,7),R1(7,7),HU(7,7),QQZP(7),EO(7,7),
     1YYZP(7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
      COMMON /CINV/ CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CMUL/ A1(7,7),B(7,7),C(7,7)
      COMMON /CPSI/ PSI(7,7),VRPSI(7),VLPSI(7),ROOTPSI
      COMMON /CQQ/ QQ(7),POPST(18,7),YY(7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      COMMON /CREG/ REG(13)
      DOUBLE PRECISION REG
      INTEGER X
      REAL L
      PRINT 50
   50 FORMAT (1H1,10X,42HSPATIAL MOMENTUM OF ZERO POPULATION GROWTH/
     111X,42(1H*)/11X,42(1H*)/)
C ----------------------------------------------------------
C COMPUTE AND PRINT MATRIX CONVERTINGS STABLE TO STATIONARY BIRTHS
C ----------------------------------------------------------
      DO 3 I=1,NR
      DO 3 J=1,NR
      EO(J,I)=0.
      RO(J,I)=0.
      R1(J,I)=0.
      DO 3 X=1,NA
      EO(J,I)=EO(J,I)+L(X,I,J)
      IZ=NAGE(X)
      Z1=FLOAT(IZ)+ZFNY*0.5
      Z=RATF(X,J)*L(X,I,J)
      RO(J,I)=RO(J,I)+Z
    3 R1(J,I)=R1(J,I)+Z1*Z
      DO 4 I=1,NR
      DO 4 J=1,NR
      HU(J,I)=RO(J,I)-PSI(J,I)
    4 CC(J,I)=R1(J,I)
      CALL INVERT (NR)
       DO 5 I=1,NR
      DO 5 J=1,NR
      A1(J,I)=CC(J,I)
    5 B(J,I)=HU(J,I)
      CALL MULTIP (NR,NR,NR)
      DO 6 I=1,NR
      B(I,1)=QQ(I)
      DO 6 J=1,NR
    6 A1(I,J)=C(I,J)/R
      CALL MULTIP (NR,NR,1)
      DO 7 I=1,NR
    7 QQZP(I)=C(I,1)
      PRINT 11
   11 FORMAT (//10X,45HMATRIX CONVERTING STABLE TO STATIONARY BIRTHS/
     110X,45(1H-))
```

```
      DO 10 I=1,NR
      DO 10 J=1,NR
   10 CE(J,I)=A1(J,I)
      CALL EIGEN (NR,1,0)
C ---------------------------------------------------------------
C COMPUTE AND PRINT STABLE AND STATIONARY EQUIVALENTS AND MOMENTUM
C ---------------------------------------------------------------
      DO 8 I=1,NR
      B(I,1)=QQZP(I)
      DO 8 J=1,NR
    8 A1(J,I)=EO(J,I)
      CALL MULTIP (NR,NR,1)
      DO 9 I=1,NR
    9 YYZP(I)=C(I,1)
      PRINT 12
   12 FORMAT (//10X,33HSTABLE AND STATIONARY EQUIVALENTS/
     110X,33(1H-))
      PRINT 13
   13 FORMAT (/21X,6HBIRTHS,16X,10HPOPULATION,9X,10HPOPULATION)
      PRINT 14
   14 FORMAT (/13X,2(5X,6HSTABLE,1X,10HSTATIONARY,2X),3X,8HMOMENTUM)
      PRINT 64
   64 FORMAT (1X)
      DO 15 I=1,NR
      IF (YY(I).NE.0.) Z=YYZP(I)/YY(I)
   15 PRINT 16, REG(I),QQ(I),QQZP(I),YY(I),YYZP(I),Z
   16 FORMAT (3X,A8,2X,2(2F11.0,2X),F11.4)
      DO 17 I=1,NR
   17 HU(I,1)=0.
      DO 18 I=1,NR
      HU(1,1)=HU(1,1)+QQ(I)
      HU(2,1)=HU(2,1)+QQZP(I)
      HU(3,1)=HU(3,1)+YY(I)
   18 HU(4,1)=HU(4,1)+YYZP(I)
      Z=HU(4,1)/HU(3,1)
      PRINT 19, (HU(I,1),I=1,4),Z
   19 FORMAT (/6X,5HTOTAL,2X,2(2F11.0,2X),F11.4)
      RETURN
      END
```

```
      SUBROUTINE ZERO (NA,NR,NZERO,RONRR)
C RONRR = DOMINANT EIGENVALUE OF NET REPRODUCTION RATE MATRIX
C NZERO=1 COHORT REPLACEMENT ALTERNATIVE
C NZERO=2 PROPORTIONAL REDUCTION ALTERNATIVE
      DIMENSION VI(7)
      COMMON /CEIGEN/ CE(7,7),ROOT,VECT(7),VECTL(7)
      COMMON /CINV/ CC(7,7)
      COMMON /CL/ L(18,7,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      REAL L
      INTEGER X
   64 FORMAT (1X)
      PRINT 29
   29 FORMAT (1H1,5X,70(1H*)/6X,70(1H*))
      PRINT 30, NZERO
   30 FORMAT (1H0,10X,22HZERO POPULATION GROWTH,5X,11HALTERNATIVE
     1,I2/11X,22(1H*),5X,13(1H*)/11X,22(1H*),5X,13(1H*)/)
C -----------------------------------------------------
C COMPUTE NET RATE OF REPRODUCTION MATRIX
C -----------------------------------------------------
      IF (NZERO.NE.1) GO TO 41
      PRINT 8
    8 FORMAT (1H0,10X,32HNET REPRODUCTION RATE MATRIX NRR/11X,
     132(1H-)/)
      DO 6 I=1,NR
      DO 6 J=1,NR
      CE(J,I)=0.
      DO 6 X=1,NA
      CE(J,I)=CE(J,I)+RATF(X,J)*L(X,I,J)
    6 CONTINUE
      CALL EIGEN (NR,1,1)
      RONRR=ROOT
C -----------------------------------------------------
C COMPUTE FERTILITY ADJUSTMENT FACTORS (ALTERNATIVE 1)
C -----------------------------------------------------
      DO 32 I=1,NR
      DO 32 J=1,NR
      CC(J,I)=CE(I,J)
   32 CONTINUE
      PRINT 64
      PRINT 64
      PRINT 64
      PRINT 34
   34 FORMAT (10X,25HINVERSE OF TRANSPOSE NRR' /10X,25(1H-))
      CALL INVERT (NR)
      DO 33 I=1,NR
      DO 33 J=1,NR
   33 CE(J,I)=CC(J,I)
      CALL EIGEN (NR,5,0)
      DO 37 I=1,NR
      VI(I)=0.
      DO 37 J=1,NR
   37 VI(I)=VI(I)+CC(I,J)
      GO TO 42
   41 CONTINUE
```

```
C -------------------------------------------------------
C COMPUTE FERTILITY ADJUSTMENT FACTORS (ALTERNATIVE 2)
C -------------------------------------------------------
      DO 38 I=1,NR
      VI(I)=1./RONRR
   38 CONTINUE
   42 CONTINUE
C -------------------------------------------------------
C PRINT MATRIX OF FERTILITY ADJUSTMENT FACTORS
C -------------------------------------------------------
      PRINT 64
      PRINT 64
      PRINT 64
      PRINT 35
   35 FORMAT (10X,39HMATRIX OF FERTILITY ADJUSTMENT FACTORS /
     110X,38(1H-))
      DO 36 I=1,NR
      DO 36 J=1,NR
      CE(J,I)=0.
   36 IF (I.EQ.J) CE(J,I)=VI(I)
      CALL EIGEN (NR,5,0)
C -------------------------------------------------------
C REPLACE FERTILITY RATES BY RATES AT REPLACEMENT LEVEL
C -------------------------------------------------------
      DO 40 I=1,NR
      DO 40 X=1,NA
   40 RATF(X,I)=RATF(X,I)*VI(I)
      PRINT 164
  164 FORMAT (1H1,1X)
      RETURN
      END
```

4

D3. Main Program

```
C ************************************************************************
C
C  MAIN PROGRAM FOR SPATIAL POPULATION ANALYSIS
C
C  INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS  (IIASA)
C
C  ATTENTION    ZERO = O
C
C ************************************************************************
C
      DIMENSION RATFZE(18,7)
      COMMON /C1/ POP(18,7)
      COMMON /CNAG/ NAGE(18)
      COMMON /CGROW/ BR(18,7,7),POPR(18,7)
      COMMON /CRATE/ RATD(18,7),RATM(18,7,7),RATF(18,7)
      INTEGER X
      NPR=1
      IHIST=0
      ILIF=0
      IPROJ=0
      CALL DATAS (NPR,NA,NY,ZFNY,NR,XZB,XZD,XZO,IPROB,
     1INIT,NHORIZ,INTV,ITOLX,NTOLL,NEIG)
C -------------------------------------------------------------
C PRELIMINARY ANALYSIS
C -------------------------------------------------------------
      CALL PRELIM (NA,NY,ZFNY,NR,XZB,XZD,XZO)
C -------------------------------------------------------------
C COMPUTE PROBABILITIES
C -------------------------------------------------------------
      CALL PROBSC (NA,ZFNY,NR,IPROB)
C -------------------------------------------------------------
C MULTIREGIONAL LIFE TABLE
C -------------------------------------------------------------
      CALL HIST (NA,NR,IHIST)
      CALL LIFE(NA,ZFNY,NR,IPROB,IHIST,ILIF)
      CALL WHOLE(NA,ZFNY,NR)
      IF (NR.EQ.2) CALL PRLIF2(NA,NY,IPROB)
C -------------------------------------------------------------
C MULTIREGIONAL DEMOGRAPHIC PROJECTION
C -------------------------------------------------------------
      CALL GROWTH (NA,ZFNY,NR,ILIF)
      DO 10 I=1,NR
      DO 10 X=1,NA
   10 POPR(X,I)=POP(X,I)
      CALL PROJEC (NA,NY,ZFNY,NR,INIT,NHORIZ,INTV,ITOLX,NTOLL,
     1ZLAMDK,IPROJ)
C -------------------------------------------------------------
C MULTIREGIONAL LIFE TABLE ANALYSIS
C   FERTILITY AND MOBILITY ANALYSIS
C -------------------------------------------------------------
      IF (ILIF.NE.1) CALL LMAT (NA,NR,ZFNY,ILIF)
      CALL FERMOB (NA,ZFNY,NR,1,NEIG,0.)
```

```
C -----------------------------------------------------------
C SPATIAL STABLE POPULATION ANALYSIS
C -----------------------------------------------------------
      IF (ILIF.NE.1) CALL LMAT(NA,NR,ZFNY,ILIF)
      IF (IPROJ.NE.1) CALL RELAM(ZFNY,ZLAMDK,RSTAB)
      CALL AGEDIS (NA,ZFNY,NR,RSTAB)
      CALL FERMOB (NA,ZFNY,NR,3,NEIG,RSTAB)
      CALL RVALUE (NA,ZFNY,NR,RSTAB,ZVT)
      CALL RINTR (NA,ZFNY,NR,RSTAB,ZVT)
C SPATIAL ZPG ANALYSIS : ANALYTICAL APPROACH (MOMENTUM)
       CALL MOMENT (NA,ZFNY,NR,RSTAB)
C -----------------------------------------------------------
C SPATIAL ZERO POPULATION GROWTH ANALYSIS
C -----------------------------------------------------------
      IF (ILIF.NE.1) CALL LMAT(NA,NR,ZFNY,ILIF)
      DO 30 I=1,NR
      DO 30 X=1,NA
   30 RATFZE(X,I)=RATF(X,I)
C NUMERICAL APPROACH : TWO ALTERNATIVE FERTILITY REDUCTION SCHEMES
      NNZERO=2
      DO 33 NZERO=1,NNZERO
      CALL ZERO (NA,NR,NZERO,RONRR)
      CALL FERMOB (NA,ZFNY,NR,2,NEIG,0.)
      CALL RVALUE (NA,ZFNY,NR,0.,ZVT)
      CALL RINTR (NA,ZFNY,NR,0.,ZVT)
      DO 31 I=1,NR
      DO 31 X=1,NA
   31 RATF(X,I)=RATFZE(X,I)
   33 CONTINUE
C -----------------------------------------------------------
      STOP
      END
```

PAPERS OF THE MIGRATION AND SETTLEMENT STUDY

A. Papers in the Dynamics Series

1. Andrei Rogers and Frans Willekens, Spatial Population Dynamics, RR-75-24, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975, published in Papers, Regional Science Association, 36 (1976), 3-34.

2. Andrei Rogers and Jacques Ledent, Multiregional Population Projection, internal working paper, International Institute for Applied Systems Analysis, Laxenburg, Austria, published by Jean Cea, ed., Optimization Techniques: Modelling and Optimization in the Service of Man, Part 1, Springer Verlag, Berlin, 1976, pp. 31-58.

3. Andrei Rogers and Jacques Ledent, Increment-Decrement Life Tables: A Comment, internal working paper, International Institute for Applied Systems Analysis, Laxenburg, Austria, published in Demography, 13 (1976), 287-290.

4. Andrei Rogers, Spatial Migration Expectancies, RM-75-57, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

5. Andrei Rogers, Aggregation and Decomposition in Population Projection, RM-76-11, International Institute for Applied Systems Analysis, Laxenburg, Austria, published in revised form in Environment and Planning A, 8 (1976), 515-541.

6. Andrei Rogers and Luis J. Castro, Model Multiregional Life Tables and Stable Populations, RR-76-09, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

7. Andrei Rogers and Frans Willekens, Spatial Zero Population Growth, RM-76-25, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

8. Frans Willekens, Sensitivity Analysis, RM-76-49, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976, published in revised form in Environment and Planning A, 9 (1977), 653-674.

9. Andrei Rogers and Frans Willekens, The Spatial Reproductive Value and the Spatial Momentum of Zero Population Growth, RM-76-81, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

10. Frans Willekens, The Spatial Reproductive Value: Theory and Applications, RM-77-09, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

11. Michael A. Stoto, On the Relationship of Childhood to Labor Force Migration Rates, RM-77-55, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

12. Jacques Ledent, Intrinsic Rates and Stable Age-Specific Mortality (and Migration) Rates of the Growth Matrix Operator in the Single Region (Multiregion) Population Model, RM-77-37, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

13. Andrei Rogers, Richard Raquillet and Luis J. Castro, Model Migration Schedules and their Applications, RM-77-57, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

14. Andrei Rogers, The Formal Demography of Migration and Redistribution: Measurement and Dynamics, RM-78-15, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

15. Jacques Ledent, Some Methodological and Empirical Considerations in the Construction of Increment-Decrement Life Tables, RM-78-25, International Institute for Applied Systems Analysis, Laxenburg, Austria.

16. Jacques Ledent, Stable Growth in the Nonlinear Components-of-Change Model of Interregional Population Growth and Distribution, RM-78-28, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

B. Papers in the Demometrics Series

1. Andrei Rogers, Demometrics of Migration and Settlement, RM-76-68, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976, forthcoming in Papers of the Regional Science Association, British Section, Pion Ltd., London.

2. John Miron, Job-Search, Migration and Metropolitan Growth, RM-77-03, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

3. Jacques Ledent, Regional Multiplier Analysis: A Demometric Approach, RR-78-03, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

C. Papers in the Policy Analysis Series

1. Yuri Evtushenko and Ross D. MacKinnon, Non-Linear Programming Approaches to National Settlement System Planning, RR-77-26, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975, published in revised form in Environment and Planning A, 8 (1976), 637-653.

2. R.K. Mehra, An Optimal Control Approach to National Settlement System Planning, RM-75-58, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

3. Frans Willekens, Optimal Migration Policies, RM-76-50, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

4. Anatoli Propoi and Frans Willekens, A Dynamic Linear Programming Approach to National Settlement System Planning, RM-77-08, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

5. Frans Willekens and Andrei Rogers, Normative Modeling in Demo-Economics, RR-77-23, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

D. Papers in the Comparative Study Series

1. Ross. D. MacKinnon and Anna Maria Skarke, Exploratory Analyses of the 1966-1971 Austrian Migration Table, RR-75-31, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975, published in revised form in Regional Studies, 11 (1977), 99-111.

2. Galina Kiseleva, The Influence of Urbanization on the Birthrate and Mortality Rate for Major Cities in the USSR, RM-75-68, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

3. George Demko, Soviet Population Policy, RM-75-74, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

4. Andrei Rogers, The Comparative Migration and Settlement Study: A Summary of Workshop Proceedings and Conclusions, RM-76-01, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

5. Andrei Rogers, Two Methodological Notes on Spatial Population Dynamics in the Soviet Union, RM-76-48, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

6. Frans Willekens and Andrei Rogers, Computer Programs for Spatial Demographic Analysis, RM-76-58, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

7. Frans Willekens and Andrei Rogers, More Computer Programs for Spatial Demographic Analysis, RM-77-30, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

8. Frans Willekens, The Recovery of Detailed Migration Patterns from Aggregate Data: An Entropy Maximizing Approach, RM-77-58, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

E. Related Papers of the Population, Resources and Growth Study

1. Nathan Keyfitz, Understanding World Models, RM-77-18, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

2. Andrei Rogers, Migration, Urbanization, Resources and Development, RR-77-41, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

3. Roman Kulikowski, Optimization of Rural-Urban Development and Migration, RM-77-41, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

4. Frans Willekens, Spatial Population Growth in Developing Countries: With a Special Emphasis on the Impact of Agriculture, International Institute for Applied Systems Analysis, Laxenburg, Austria, (forthcoming).

5. Henry Rempel, The Role of Rural-Urban Migration in the Urbanization and Economic Development Occurring in Kenya, RM-78-12, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

6. Allen C. Kelley and C. Swartz, The Impact of Family Structure on Microeconomic Decision Making in Developing Countries: Kenya, International Institute for Applied Systems Analysis, Laxenburg, Austria, (forthcoming).