GROWTH AND STAGNATION OF ECONOMIES WITH PUBLIC GOODS - A Neoclassical Analysis

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PREFACE

The work in the Regional Development Task is oriented to problems of long-term development of regions and systems of regions. The understanding of long-term regional development problems is closely related to an understanding of the interdependency of factors determining economic growth.

This paper is devoted to growth economics and its application to regional development. Much of my work on growth problems has been oriented to the use of dynamic input-output theory and its application to transportation networks with regions as the nodes of the network. The public goods issues have not been addressed in those studies, because of the fundamental theoretical problems associated with the inclusion of public goods in the input-output framework. This paper presents a first tentative approach to an inclusion of public goods in a growth framework with many regions. A neoclassical economic paradigm is chosen as the starting point and it is shown that an interregional model with public goods possesses equilibrium properties and also relative stability under certain conditions. It is the intention to use the theoretical framework presented in this paper as a starting point for an analysis of technological research and development as an endogenous public good in a regional growth process.

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Population, growth and stagnation - the classical perspective

Classical economists of the 18th and 19th centuries were very much oriented to spatial problems, which is understandable. Production was in those days almost totally oriented to agriculture, fishing, hunting and other land-intensive sectors. Scarcity of land resources was therefore considered to be the major economic problem and colonial policies one reasonable remedy of poverty.

In a situation where land and labor were the only important factors of production it was also reasonable to concentrate analysis to the problem of population growth and its consequences for the standard of living.

I will now look into the question of growth, stagnation and contraction within the framework of a pedagocially formulated version of classical population analysis.

Classical analysis of the stagnation problems

Analysis of stagnation problems goes all the way back to the 18th century. Many economists and foremost Thomas Malthus assumed that growth in the standard of living could only be temporary. A technological innovation could provide a higher standard of living for one generation but future generations would ultimately be drawn back to the minimal consumption standard but at a higher degree of crowding. The approach to the problem can be illustrated in a simple diagram for a closed community.

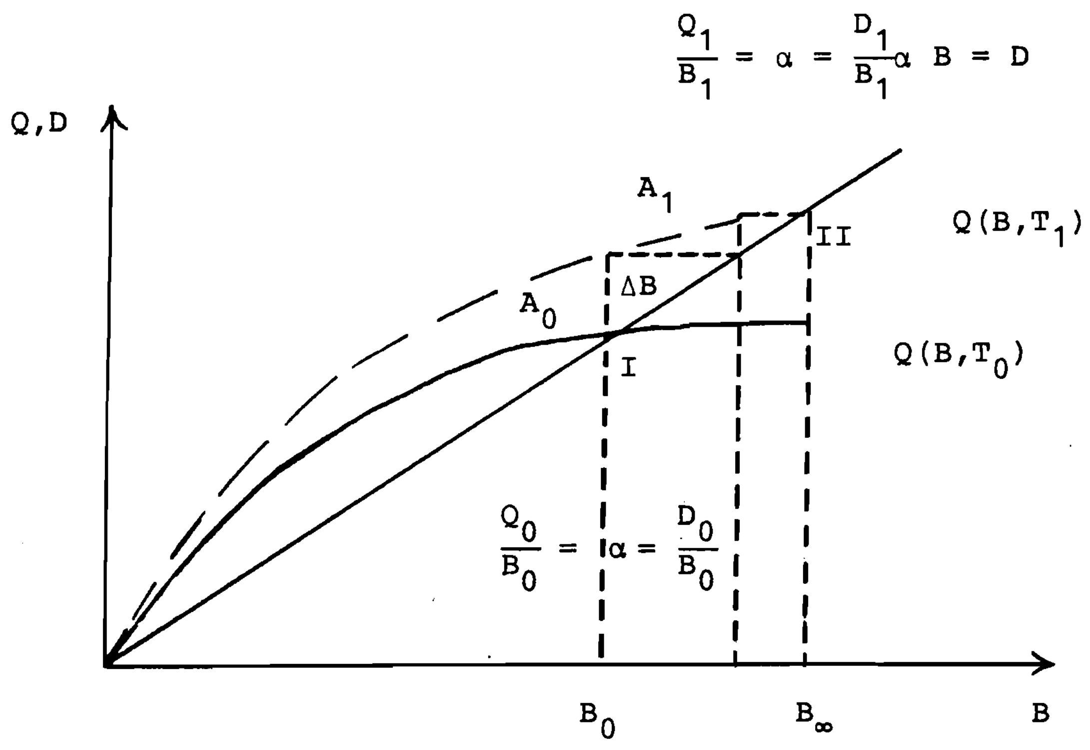

Figure 1: <u>Population dynamics in a pre-feudal peasant economy.</u>

Point I illustrates a first population and production equilibrium point, at which production is equal to demand, assumed to be proportional to total population.

It is then assumed that the technological level rises from T_0 to T_1, which has the effect of raising the production function from $Q(B, T_0)$ to $Q(B, T_1)$.

The classicals assumed that fertility would increase in proportion to excess supply of the commodity (grain), A_0. The result would then be a population increase of ΔB. When ΔB would enter the labor force, there would be an increase of production and a correspondingly smaller excess supply A_1 etc. The final outcome would always be a return to the survival per capita income, α. Technical change could thus never bring about a permanent increase in the standard of living.

Could there ever be a decline in population in this view of development? Not for any economic reasons endogenous to the model. It is, however, possible to see that a political change could trigger off an overpopulation situation. If we assume a shift from a peasant society--in Wolf's terminology (Wolf 1966)--to a society of feudal landowners, such a change would immediately create an excessive farming population. This can be illustrated diagrammatically in Figure 2.

After a feudal optimum is reached the consequence of a technological shift would also in this case be stagnating growth, at least if the feudal rents were used for luxury consumption.

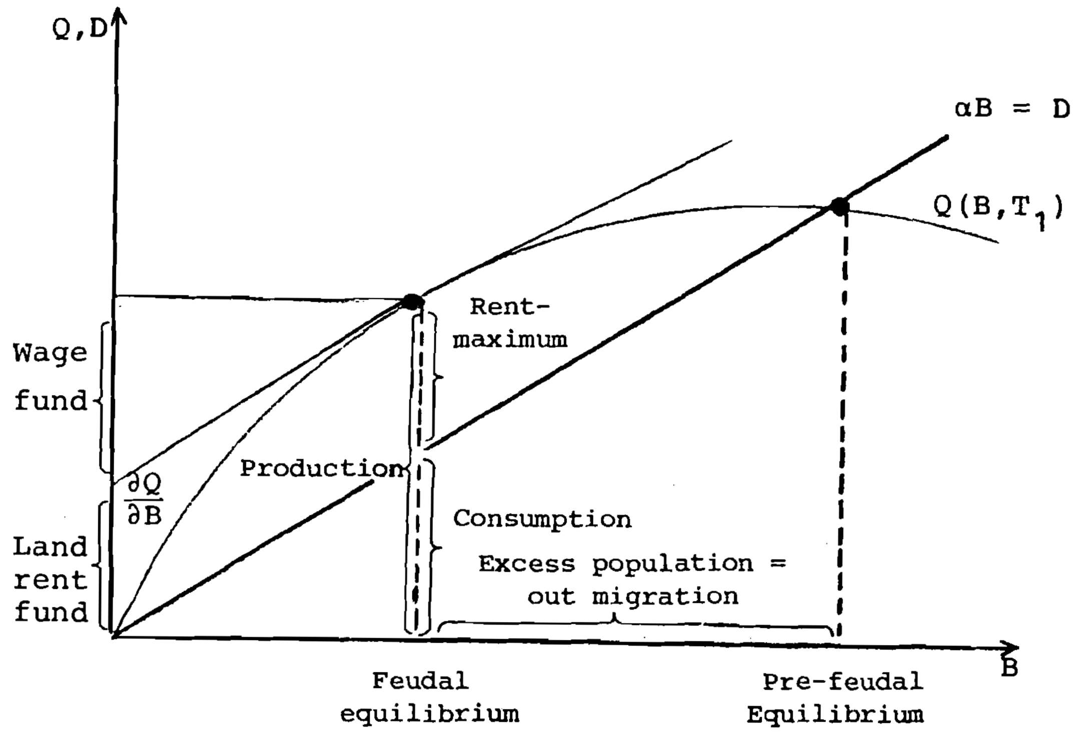

Figure 2: Population Equilibrium in a Feudal Economy

To what extent can this gloomy picture of the development process be modified by an introduction of capital?

This question is addressed in the following section within the framework of neoclassical economics.

Growth and stagnation in the one-sector neoclassical perspective

Neoclassical economics reformulated the development problem in the sense that capital allocation became the focus of attention rather than population development and land use.

Land use was suppressed in early neoclassical economics, and labor and population were assumed to be exogenously determined, i.e. something that could perhaps be predicted but not influenced by economic circumstances.

It is very hard to give any undisputably identifiable neoclassical view of the development process, because so much of its identity has been blurred by Keynesian and semi-Keynesian influences in all the one-sector, one-region growth models of the post-war period. It is, for instance, obvious that Solow's growth model (1956) does not qualify as a purely neoclassical model.

What is then the central aspect of a neoclassical model as opposed to Keynesian growth models? I think that the identity of savers and investors in the neoclassical paradigm and the non-identity of the Keynesian paradigm is the main difference. To a neoclassical economist, there is simply no dichotomy between the decision on saving and investment, while saving is a household decision and investment a decision of a firm in the Keynesian tradition.

The first and fundamental neo-classical assumption is thus:

(A1) The share of production planned to be devoted to investments equals the share of production withdrawn as savings.

The second assumption is:

(A2) Total savings are less than, or equal to, total profits.

Formally this can be expressed as

$$\underbrace{\dot{K}}_{\substack{\text{realized}\\ \text{investments}}} = \underbrace{aQ}_{\substack{\text{planned}\\ \text{investments}}} = \underbrace{sQ}_{\substack{\text{planned}\\ \text{savings}}} \leqq \underbrace{\frac{\partial Q}{\partial K} K}_{\text{profits}} = \underbrace{Q - \frac{\partial Q}{\partial L} \cdot L}_{\substack{\text{production minus}\\ \text{wage-sum}}} , \qquad (1)$$

where: Q = production; K = capital; $\dot{K}$ = investments.

Profits are expressable as simply as this because of the assumption of constant returns to scale at the point of general equilibrium.

It is further assumed that production of the single commodity is determined by a concave, differentiable production function, homogenous of degree one, having malleable capital and labor as arguments:

$$Q = Q\,(K,L) \quad , \qquad \text{where: } L = \text{labor}. \qquad (2)$$

It is almost invariably assumed that labor grows at an exogenously determined rate and not as in the classical models by endogenous economic forces. I will, for convenience, assume that the population is stationary at the level L_0.

Combining (1) and (2) into a differential equation:

$$\dot{K} = s\; Q(K,L_0) \quad , \qquad (3)$$

we now assume that profits are used for savings only:

$$s = \frac{\partial Q}{\partial K}\,\frac{K}{Q} = \varepsilon \quad . \qquad (4)$$

This means that the rate of savings is set equal to the elasticity of production with respect to capital.

Most structural neoclassical production functions have variable elasticities of this kind. An instance is the CES-function:

$$Q = (\alpha K^{-\rho} + (1-\alpha)\ L^{-\rho})^{-1/\rho} \quad . \tag{5}$$

This function has the following elasticity of production with respect to capital:

$$\varepsilon = \frac{1}{1 + (\frac{\alpha}{\beta}\frac{K}{L_0})^{\rho}} \quad . \tag{6}$$

The rate of savings is thus independent of the capital intensity only if $\rho = 0$, which corresponds to the Wicksell/Cobb/ Douglas specification of the production function. With $\rho > 0$ the savings ratio must decline with growing capital intensity.

$\rho \geqq 0$ is commonly looked upon as cases of complementarity of capital and labor. We can consequently claim that the higher the degree of complementarity of the factors of production, the stronger will be the tendency of a falling profit and savings ratio over time.

Substituting (5) and (6) into (3) we get a convenient specification of the neoclassical growth model:

$$\dot{K} = \frac{1}{1 + (cK)^{\rho}} (\alpha K^{-\rho} + (1-\alpha)L_0^{-\rho})^{-1/\rho} \quad , \tag{7}$$

which implies a strong tendency towards zero growth over time for all $\rho > 0$.

If $\rho \to 0$ we get the following differential equation:

$$\dot{K} = \alpha K^{\alpha} L_0^{(1-\alpha)} \Rightarrow g_K = \frac{\dot{K}}{K} = k_0 K^{\alpha-1} \Rightarrow \frac{\partial g_k}{\partial K} < 0 \quad ,$$

where: $k_0 = \alpha L_0^{(1-\alpha)}$.

It is also, in this case, clear that the growth rate of the capital stock will tend towards zero in the long run, although there will be no tendency to have a falling rate of profits.

We can finally observe that the economy will tend towards a positive and steady rate of growth for all $\rho < 0$.

The conclusion is thus: The rate of growth of the neoclassical economy--as specified above--tends towards a positive constant if the factors of production are completely substitutable. Otherwise the neoclassical economy tends towards stagnation.

Population, material, human and infrastructure capital interdependencies in a neoclassical perspective

We have shown above that a neoclassical economy for a single sector employing a homogeneous capital commodity, (K), and some homogeneous labor, (L), will have harmonious properties.

If savings are equal to profits and the wage sum is equal to the production minus profit residual, then the economy will grow in such a way that there is a balance between supply and demand for the product, capital and labor.

The economy might slowly stagnate, if the technology exhibits limited substitutability, but there is no risk of a sudden collapse or even slow decline within this paradigm.

One can argue that this model is of pedagogical value only, because of the limitation to one commodity and only one endogenously variable factor of production. I will thus try to increase its realism by analyzing a dynamic interdependent neoclassical economy in which there are three kinds of capital, material capital, (K), human capital, (H) and infrastructure capital, (G). This means that we must distinguish between the use of physical work by man and the use of knowledge in the production of the single product, (Q), that can be used for consumption or investment.

We will assume that production of the commodity Q is regulated by a conventional neoclassical production function

$$Q = Q(K, L, H, G) \quad , \qquad (8)$$

where

Q = production of the standard commodity;

K = the amount of material capital in use;

L = the employment of labor;

H = the amount of human capital in use; and

G = the amount of infrastructure capital in use.

Equation (8) could have a specification of the CES-type.

$$Q = A(\alpha_1 K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 H^{-\rho} + \alpha_4 G^{-\rho})^{-1/\rho} \quad , \qquad (9)$$

A neoclassical savings/investment assumption gives a capital growth equation of the following type

$$\dot{K} = \varepsilon(x)(1-t)A(\alpha_1 K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 H^{-\rho} + \alpha_4 G^{-\rho})^{-1/\rho} \quad , \qquad (10)$$

where

$s = \varepsilon(x)$ = the elasticity of production with respect to capital;

$x = \{K,L,H,G\}$;

t = the average rate of taxation.

Increases in the supply of labor can be assumed to depend on the consumption standard. This relation can be positive, negative or zero. We will, for the time being, assume that there is a positive relation between total private consumption and labor supply increases. There is no need to assume that the labor supply increase only comes from increasing fertility. The response could come from immigration or increasing employment participation.

The labor supply equation will thus take on the following form:

$$\dot{L} = (1-\varepsilon(x))(1-t)fA(\alpha\ K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 H^{-\rho} + \alpha_4 Q^{-\rho})^{-1/\rho} , \qquad (11)$$

$$f = \frac{\text{labor supply increase}}{\text{disposable consumer income}} .$$

Fertility studies would rather support a hypothesis that population growth is positively related to consumption per capita, but negatively related to human capital per capita. Such a reformulation would not substantially change the conclusions of this section, provided that the human capital effect on fertility is moderate.

Human capital can either be produced by the households individually or in some collective form. Human capital is to a major part produced by the households through their consumption spending habits. It is also obvious that the pattern of consumption is of great importance for the amount of human capital created through consumption. I will here assume that the pattern of consumption is fixed although this will be relaxed at a later stage.

We can now define the share of human capital commodities in the consumer budget to be the scalar h and the productivity of each unit of human capital commodity to be q_H.

The rate of human capital investment can thus be written

$$\dot{H} = (1-\varepsilon(x))(1-t)(h)(q_H)A(\alpha K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 H^{-\rho} + \alpha_4 G^{-\rho})^{-1/\rho} . \quad (12)$$

We finally have to specify a government sector in charge of the creation of infrastructure like roads, ports, property right institutions, etc.

We will in this context simplify the analysis to a case where the public sector produces a continuously variable quantity of infrastructure capital of a fixed structure.

The production function is assumed to be of an exceedingly simple nature. The public sector has a fixed labor force that proportionally transforms the private sector product into infrastructure at the rate q_g. The infrastructure investment function takes on the following form:

$$\dot{G} = tq_g \; A(\alpha_1 K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 H^{-\rho} + \alpha_4 G^{-\rho})^{-1/\rho} \quad . \quad (13)$$

We can rewrite this system--(10)-(13)--as:

$\dot{x} = M(x)$ and assume a solution to be one of proportional growth at the rate $\dot{x} = \lambda x$.

It is possible to employ a theorem due to Nikaido for the analysis of the qualitative behavior of this system:

<u>Theorem</u> (cf. Nikaido, 1968, pp. 105-151; a proof is given on p.152):

Assume the following conditions hold

(a) $M(x) = (M_i(x))$ is defined for all non-negative x in R^n_+, with its values being also on non-negative vectors in R^n_+, $M(x) \geqq 0$.

(b) M(x) is continous as a mapping M: $R^n_+ \to R^n_+$, except possibly at x = 0.

(c) M(x) is positively homogeneous of order m, $1 \geqq m \geqq 0$ in the sense that $M(\alpha x)$ for $\alpha \geqq 0$, $x \geqq 0$.

Let $\Lambda = \{\lambda | M(x) = \lambda x \text{ for some } x \varepsilon P_n\}$, where $P_n = \{x | x \geqq 0, \sum_{i=1}^{n} x_i = 1\}$ is the standard simplex.

Then, Λ contains a maximum which is denoted by $\lambda(M)$. Furthermore, if m = 1, $\lambda(M)$ is the greatest among all the eigenvalues of M.

The theorem assures us of the existence and uniqueness of a meaningful general equilibrium solution for the neoclassical economy.

It is thus clear that this system will economically behave in the following way:

a. The neoclassical economy has a unique common growth rate, which is also the maximal one.

b. If this growth rate is achieved, income per capita must stagnate and will remain at this level until the system gets an exogenous shock.

c. With the assumptions made the system will also generate economically feasible values of the variables.

We can get some further insights into the behavior of the system by making some further assumptions about the production function Q.

Let us postulate an economically disputable production function with p = -1. Such a function means that the output is a linear combination of the inputs, an assumption that in reality can only be locally true. The system would now take on the form

$$\begin{aligned}
\dot{K} &= \lambda_{11}K + \lambda_{12}L + \lambda_{13}H + \lambda_{14}G \\
\dot{L} &= \lambda_{21}K + \lambda_{22}L + \lambda_{23}H + \lambda_{24}G \\
\dot{H} &= \lambda_{31}K + \lambda_{32}L + \lambda_{33}H + \lambda_{34}G \\
\dot{G} &= \lambda_{41}K + \lambda_{42}L + \lambda_{43}H + \lambda_{44}G
\end{aligned} \tag{14}$$

where

$$\gamma_{1i} = \bar{\varepsilon}\,(1-t)\,A\alpha_i$$

$$\gamma_{2i} = (1-\bar{\varepsilon})(1-t)\,f A\alpha_i$$

$$\gamma_{3i} = (1-\bar{\varepsilon})(1-t)hq_H\alpha_i$$

$$\gamma_{4i} = tq_g\,A\alpha_i$$

In matrix notation this system can be rewritten as

$$\dot{x} = Mx$$

$$M = \{\gamma_{ji}\} \; ; \qquad x = \{K, L, H, G\}$$

$$\dot{x} = \{\dot{K}, \dot{L}, \dot{H}, \dot{G}\} \; .$$

Assuming proportional growth at the rate $gx = \dot{x}$ where g is some scalar the equilibrium solution should be such that

$$(gI - \underline{M})x = 0 \quad . \tag{15}$$

In difference form the problem would take on the appearance

$$y(t+1) = Ay(t) \quad \text{with} \quad A = \underline{M} + I \quad . \tag{16}$$

I have earlier assumed that the savings parameter is determined by the variables of the system. One could also argue that h and g_H are endogenously determined.

To further simplify the analysis these parameters are for the time being assumed to be exogenously determined and given for the total period under consideration.

This system will only be indecomposable if $0<\varepsilon<1$ and if $0<t<1$. The economy (16) must then necessarily have <u>all</u> $\gamma_{ij}>0$. Such a system will have the following properties:

<u>Theorem</u>: Let an n-th order square matrix $M > 0$ be given and thus $A > 0$ and let $\lambda(A) = \lambda$, $\lambda x = Ax$, $x > 0$.

$$x(t) = \lambda^t x \quad , \quad (t = 0, 1, 2, \ldots) \quad ; \quad \text{(T1)} \quad ,$$

is a special solution of the difference equation

$$y(t+1) = Ay(t) \quad ; \quad \text{(T2)}$$

For the solution $y(t)$ of (T2) that starts from an arbitrary $y(0) \geq 0$ and the balance growth solution $x(t)$, there exist

$$\lim_{t \to \infty} y_i(t)/x_i(t) \quad ; \quad (i = 1, \ldots, n) \quad .$$

These n limits are positive and equal to each other.

<u>Proof</u>: See Nikaido H., Introduction to Sets and Mappings in Modern Economics, North-Holland Publishing Company, Amsterdam 1972, pp. 149-151.

This theorem shows that the linear neoclassical growth economy (14) exhibits <u>relative</u> stability of the balanced growth path.

The meaning of the theorem can be illustrated in a two-factor case with the following diagram.

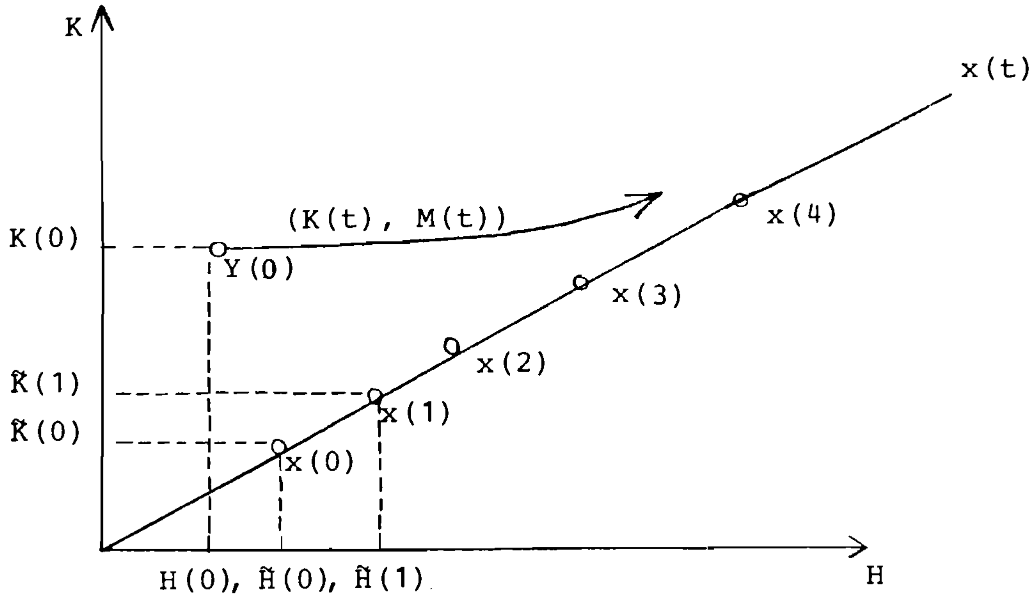

Figure 3: <u>Trajectory from Disequilibrium Position of Sectors</u>

K(t), H(t) indicate simulated values; and

$\hat{K}(t)$, $\hat{H}(t)$ indicate equilibrium values.

We can thus conclude that, as long as a growth process of a region is given by systems (16) or (14), the regional economy will exhibit positive growth of capital, labor, human capital and infrastructure. Such an economy must furthermore be relatively stable in its growth process.

We have furthermore shown that there will exist a fixed point solution to the more general system of type (10) to (13) with the properties of uniqueness of the solution.

It has not yet been possible for me to prove analytically that such a more general system is relatively stable. I therefore, had to resort to numerical analysis to reveal its stability properties. We could, however, observe that any linearized versions of the non-linear system (8)-(13) is relatively stable, which implies that the system is always locally, relatively stable.

Numerical Analysis of the Extended Neoclassical Model

It has been shown above that the extended neoclassical economic growth model has a unique common maximal growth rate as its eigenvalue and that this growth rate is compatible with an economically feasible structure of production. It has also been shown that the system is globally relatively stable for all strictly positive matrices in the linear specification of the model.

It is probably possible to show a general stability property of this system by employing the methods of Lyapunov. Such an exercise has not been attempted in this version of the paper.

Strong tendencies to globally stable solutions for all values of $p \geqq -1$ have been found in a set of numerical experiments with the model. Some of these experiments are described in the following section.

The initial experiment concerns an economy with certain standard behavioral properites. The propensity to save is here assumed to be a constant, independent of capital intensity of the economy. Its value has been set equal to 0.2. The rate of taxation, t, is assumed to be a proportional share of production of the private commodity Q and t = 0.3. The labor supply coefficient is assumed to be low but positive, f = 0.01. The share of human capital consumption goods (education, health, literature, etc.) in the household budget, h, is assumed equal to 0.3.

The productivity of human capital commodities is set high at a value of $q_H = 2$, while the infrastructure investment productivity q_G is assumed to be much lower and set equal to 0.6.

The techniques are expressed by $\alpha_1 = 0.25$, $\alpha_2 = 0.35$, $\alpha_3 = 0.20$ and $\alpha_4 = 0.20$. ρ is in this example given a whole set of values ranging from $\rho = -1$ to $\rho = +10$.

It is clear from the parameters given above that the response in accumulation from production differs very much between capital, labor, human capital and infrastructure. This implies that the proportional rate of growth will be highly dependent upon the value of ρ (or the equivalent parameter, elasticity of substitution $\equiv \frac{1}{1 + \rho}$).

It should be suspected that a technology that has large possibilities of substitution should also have great possibilities to have a high rate of growth (if the accumulation parameters are different). A technology with low possibilities of substitution should on the other hand be forced to accept a growth rate close to the lowest of the accumulation parameters of the system. This is also true in the numerical experiments as shown in diagram 1.

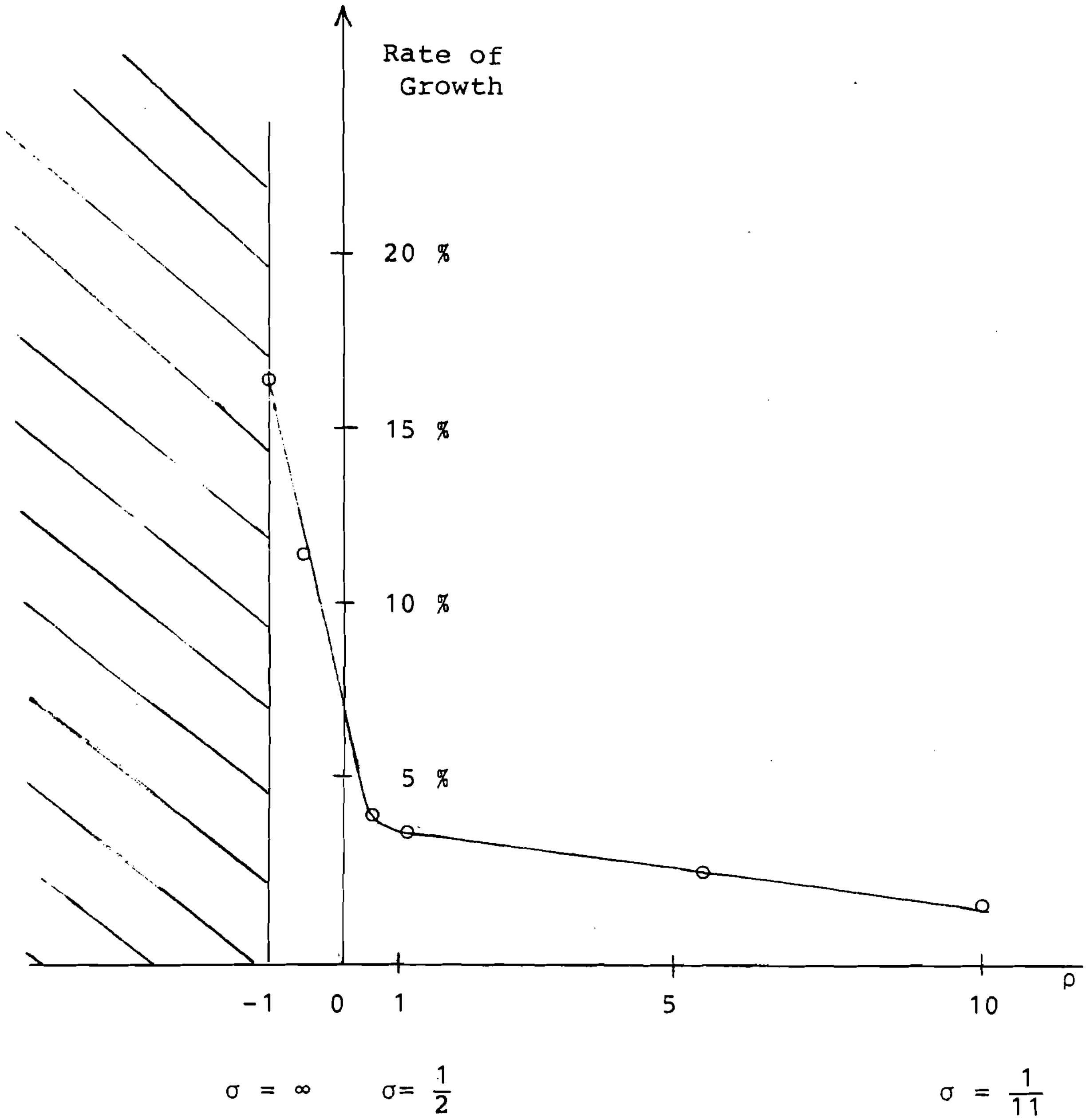

Diagram 1

<u>The income per capita increases rather rapidly in the beginning of the process but converges to a stagnation level.</u>

This implies that a sudden increase in the technological level (for instance by a shift in the parameter $A(t_0)$ to $A(t_1)$) will mean a drastic increase in the growth of income per capita.

The assumption of a positive labor supply response to production can be disputed on the basis of some empirical evidence. (See however Schultz, 1975).

A negative labor supply coefficient f leads, however, to severe problems with the long-run behavior of the model economy. Assuming an $f = -0.1$ indicates the nature of this adverse reaction. The economy would then increase its rate of growth to some high value and grow rapidly for a finite number of periods with a labor force declining towards zero. There would not necessary be any clear sign of a tendency to collapse until the economy would go through some minimal threshold.

Public Goods and Economic Development

The former section has been devoted to the issue of interactions between the public sector, industry and households.

We have, however, not looked into the matter of the public character of most infrastructure capital created by the public sector.

The following slightly sectorized model can be used to highlight some of the qualitative aspects of public goods in a growth process.

$$\dot{K}_1 = \varepsilon_1(x)(1-t)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} + $$
$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} \quad .$$

$$\dot{K}_2 = \varepsilon_2(x)(t-1)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} + \alpha_{23}H_2^{-\rho_2} + $$
$$+ \alpha_{24}G^{-\rho_2})^{-1/\rho_1} \quad . \tag{17}$$

$$\dot{L}_1 = m(1-t)f((1-\varepsilon_1)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} + $$
$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} + (1-\varepsilon_2)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} + $$
$$+ \alpha_{23}H_2^{-\rho_2} + \alpha_{24}G^{-\rho_2})^{-1/\rho_2} \quad .$$

$$\dot{L}_2 = (1-m)(1-t)f \quad ((1-\varepsilon_1)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} +$$

$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} + (1-\varepsilon_2)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} +$$

$$+ \alpha_{23}H_2^{-\rho_2} + \alpha_{24}G^{-\rho_2})^{-1/\rho_2} \quad .$$

$$\dot{H}_1 = n(1-t)hq_H \quad ((1-\varepsilon_1)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} +$$

$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} + (1-\varepsilon_2)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} +$$

$$+ \alpha_{23}H_2^{-\rho_2} + \alpha_{24}G^{-\rho_2})^{-1/\rho_2} \quad .$$

(17)

$$\dot{H}_2 = (1-n)(1-t)hq_H((1-\varepsilon_1)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} +$$

$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} + (1-\varepsilon_2)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} +$$

$$+ \alpha_{23}H_2^{-\rho_2} + \alpha_{24}G^{-\rho_2})^{-1/\rho_2} \quad .$$

$$\dot{G} = t \quad q_G \quad ((1-\varepsilon_1)A_1(\alpha_{11}K_1^{-\rho_1} + \alpha_{12}L_1^{-\rho_1} + \alpha_{13}H_1^{-\rho_1} +$$

$$+ \alpha_{14}G^{-\rho_1})^{-1/\rho_1} + (1-\varepsilon_2)A_2(\alpha_{21}K_2^{-\rho_2} + \alpha_{22}L_2^{-\rho_2} +$$

$$+ \alpha_{23}H_2^{-\rho_2} + \alpha_{24}G^{-\rho_2})^{-1/\rho_2} \quad .$$

In this model version the public character of the infrastructure has been spelled out. Capital, labor and human capital are all of private nature, which is indicated by subscript $i = 1, 2$ for the first and second sectors, respectively. This implies that total labor and human capital has to be allocated between the two sectors in accordance with the fixed parameters m, $1-m$ and n, $1-n$ where $0 \leqq m, n \leqq 1$.

<u>This situtation is completely different for the public good G. This is treated as an input that enters the production function of both sector 1 and 2.</u>

The difference in treatment of the factors can be conveniently shown in the linearized version of the model (ass. $\rho = -1$).

The model then takes on the following appearance:

$$\begin{bmatrix} \dot{K}_1 \\ \dot{K}_2 \\ \dot{L}_1 \\ \dot{L}_2 \\ \dot{H}_1 \\ \dot{H}_2 \\ \hline \dot{G} \end{bmatrix} = \left[\begin{array}{cccccc:c} \gamma_{11} & 0 & \gamma_{12} & 0 & \gamma_{13} & 0 & \gamma_{14} \\ 0 & \gamma_{21} & 0 & \gamma_{22} & 0 & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} \\ \gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} \\ \hdashline \gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} \end{array}\right] \begin{bmatrix} K_1 \\ K_2 \\ L_1 \\ L_2 \\ H_1 \\ H_2 \\ G \end{bmatrix} \qquad (18)$$

The only column with all $\gamma_{ij} > 0$ is now the column of interactions with the public commodity G. The matrix will have some other interesting properties with respect to the public commodity.

The "public parameter" is $\gamma_{37} = m(1-t)f(1-\varepsilon_1)A_1\alpha_{14} + (1-\varepsilon_2)A_2\alpha_{24})$ which can be compared with the "private parameter"

$$\gamma_{36} = m(1-t)f((1-\varepsilon_2)A_2\alpha_{23}) \ .$$

If the public good and the private good are equally productive in the sense that $A_2\alpha_{24} = A_2\alpha_{23} = A_1\alpha_{11}$ and if $\varepsilon_1 = \varepsilon_2$, then $\gamma_{37} = 2\gamma_{36}$, which means that <u>the public sector contributes more to growth than the private sector</u> (see Frobenius theorem on the influence of individual parameters on the maximal eigen-value).

This difference between elements of the public sector column (7) is true for all its elements compared to the elements of other columns.

It is as true in this growth model as it was in the paper on public goods in the classical paper by Samuelson (1954) that negotiation on the proper size of G with individual micro agents tend to make G inoptimally small from a macro point of view.

Regional Growth and Public Goods in a Neoclassical Framework

The spatial dimension of a growth process has up to this point been kept implicit. Introduction of public goods makes a regionalization highly warranted. Concepts like accessibility agglomeration and urbanization economies are spatial in nature and at the same time closely related to public phenomena.

Spatial analysis has in later years tended to favor the concept of accessibility as a representation of locational quality.

In its most general form accessibility is any spatial measure fulfilling the requirements of the following definition. A structural measure consistent with this definition is

$$a_r = \sum_s f(d_{rs}) G_s \quad , \qquad (19)$$

a_r = accessibility to public goods from region r;

$f(d_{rs})$ = a strictly monotonous declining function of distance (d) between region r and region s; and

G_s = amount of public good G in region s.

Each one of the production sectors is now assumed to have the same production functions as in equation (17), but with accessibility to public goods instead of the public good itself as an argument. Thus:

$$\dot{K}_{ij} = \varepsilon_{ij}(1-t)A_{ij}(\alpha_{1ij}K_{ij}^{-\rho_{ij}} + \alpha_{2ij}L_{ij}^{-\rho_{ij}} + \alpha_{3ij}H_{ij}^{-\rho_{ij}} + \alpha_{4ij}a_{ij}^{-\rho_{ij}})^{-1/\rho_{ij}} \quad ; \quad \{ {i = 1,\ 2 \atop j = 1,\ 2} \} \quad . \qquad (20)$$

A linearized version of this model would have the following accumulation equation corresponding to equation (20):

$$\dot{K}_{ij} = \varepsilon_{ij}(1-t)A_{ij}(\gamma_{1ij}K_{ij} + \gamma_{2ij}L_{ij} + \gamma_{3ij}H_{ij} +$$

$$\gamma_{4ij}f(d_{j1})G_1 + \gamma_{4ij}f(d_{j2})G_2) \quad .$$

The linear system for a 2x2-case could thus be written as:

$$\begin{bmatrix} \dot{K}_{11} \\ \dot{K}_{12} \\ \dot{K}_{21} \\ \dot{K}_{22} \\ \dot{L}_{11} \\ \dot{L}_{12} \\ \dot{L}_{21} \\ \dot{L}_{22} \\ \dot{H}_{11} \\ \dot{H}_{12} \\ \dot{H}_{21} \\ \dot{H}_{22} \\ \dot{G}_1 \\ \dot{G}_2 \end{bmatrix} = \left[\begin{array}{cccc|cccc|cccc|cc}
\gamma_{11} & 0 & 0 & 0 & \gamma_{15} & 0 & 0 & 0 & \gamma_{19} & 0 & 0 & 0 & \gamma_{1,13}(d_{11}) & \gamma_{1,14}(d_{12}) \\
0 & \gamma_{22} & 0 & 0 & 0 & \gamma_{26} & 0 & 0 & 0 & \gamma_{2,10} & 0 & 0 & \gamma_{2,13}(d_{21}) & \gamma_{2,14}(d_{22}) \\
0 & 0 & \gamma_{33} & 0 & 0 & 0 & \gamma_{37} & 0 & 0 & 0 & \gamma_{3,11} & 0 & \gamma_{3,13}(d_{11}) & \gamma_{3,14}(d_{21}) \\
0 & 0 & 0 & \gamma_{44} & 0 & 0 & 0 & \gamma_{48} & 0 & 0 & 0 & \gamma_{4,12} & \gamma_{4,13}(d_{21}) & \gamma_{4,14}(d_{22}) \\
\hline
\gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} & \gamma_{58} & \gamma_{59} & \gamma_{5,10} & \gamma_{5,11} & \gamma_{5,12} & \gamma_{5,13}(d_{11}) & \gamma_{5,14}(d_{12}) \\
\gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} & \gamma_{68} & \gamma_{69} & \gamma_{6,10} & \gamma_{6,11} & \gamma_{6,12} & \gamma_{6,13}(d_{21}) & \gamma_{6,14}(d_{22}) \\
\gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} & \gamma_{78} & \gamma_{79} & \gamma_{7,10} & \gamma_{7,11} & \gamma_{7,12} & \gamma_{7,13}(d_{11}) & \gamma_{7,14}(d_{21}) \\
\gamma_{81} & \gamma_{82} & \gamma_{83} & \gamma_{84} & \gamma_{85} & \gamma_{86} & \gamma_{87} & \gamma_{88} & \gamma_{89} & \gamma_{8,10} & \gamma_{8,11} & \gamma_{8,12} & \gamma_{8,13}(d_{21}) & \gamma_{8,14}(d_{22}) \\
\hline
\gamma_{91} & \gamma_{92} & \gamma_{93} & \gamma_{94} & \gamma_{95} & \gamma_{96} & \gamma_{97} & \gamma_{98} & \gamma_{99} & \gamma_{9,10} & \gamma_{9,11} & \gamma_{9,12} & \gamma_{9,13}(d_{11}) & \gamma_{9,14}(d_{12}) \\
\gamma_{10,1} & \gamma_{10,2} & \gamma_{10,3} & \gamma_{10,4} & \gamma_{10,5} & \gamma_{10,6} & \gamma_{10,7} & \gamma_{10,8} & \gamma_{10,9} & \gamma_{10,10} & \gamma_{10,11} & \gamma_{10,12} & \gamma_{10,13}(d_{21}) & \gamma_{10,14}(d_{22}) \\
\gamma_{11,1} & \gamma_{11,2} & \gamma_{11,3} & \gamma_{11,4} & \gamma_{11,5} & \gamma_{11,6} & \gamma_{11,7} & \gamma_{11,8} & \gamma_{11,9} & \gamma_{11,10} & \gamma_{11,11} & \gamma_{11,12} & \gamma_{11,13}(d_{11}) & \gamma_{11,14}(d_{21}) \\
\gamma_{12,1} & \gamma_{12,2} & \gamma_{12,3} & \gamma_{12,4} & \gamma_{12,5} & \gamma_{12,6} & \gamma_{12,7} & \gamma_{12,8} & \gamma_{12,9} & \gamma_{12,10} & \gamma_{12,11} & \gamma_{12,12} & \gamma_{12,13}(d_{21}) & \gamma_{12,14}(d_{22}) \\
\hline
\gamma_{13,1} & \gamma_{13,2} & \gamma_{13,3} & \gamma_{13,4} & \gamma_{13,5} & \gamma_{13,6} & \gamma_{13,7} & \gamma_{13,8} & \gamma_{13,9} & \gamma_{13,10} & \gamma_{13,11} & \gamma_{13,12} & \gamma_{13,13}(d_{11}) & \gamma_{13,14}(d_{12}) \\
\gamma_{14,1} & \gamma_{14,2} & \gamma_{14,3} & \gamma_{14,4} & \gamma_{14,5} & \gamma_{14,6} & \gamma_{14,7} & \gamma_{14,8} & \gamma_{14,9} & \gamma_{14,10} & \gamma_{14,11} & \gamma_{14,12} & \gamma_{14,13}(d_{21}) & \gamma_{14,14}(d_{22})
\end{array}\right] \begin{bmatrix} K_{11} \\ K_{12} \\ K_{21} \\ K_{22} \\ L_{11} \\ L_{12} \\ L_{21} \\ L_{22} \\ H_{11} \\ H_{12} \\ H_{21} \\ H_{22} \\ G_1 \\ G_2 \end{bmatrix}$$

Figure 4: Linear System of Private and Public Good Interactions in Space.

A monotonously falling distance decay function $\gamma_{ij}(d_{i1})$ commonly used in accessibility studies is

$$\gamma_{ij}(d_{j1}) = n_{ij}e^{\mp\gamma_i d_{j1}} ,$$

with the properties $\gamma_{ij} \to n_{ij}$ when $d_{jl} \to 0$ and $\gamma_{ij} \to 0$ when $d_{j1} \to \infty$.

This means that there are finite upper and lower limits to the effect of the public good, when location is varied.

It has been shown above that all $\gamma_{ij} \geqq 0$. The "new" parameters $\gamma_{ij}(d_{j1}) > 0$. All of the results shown above thus hold also for the spatially extended economy depicted by system (21).

We thus know that the system has a maximum, unique equilibrium rate of growth with a viable sectoral and spatial structure of production of private and public goods.

Applying Perron-Frobenius' theorem, we can furthermore ascertain that a reduction of any of the distances between the nodes of the network must increase the equilibrium rate of growth. The equilibrium structure of production will also change with any change in an individual distance, d_{j1}.

A decrease in a distance d_{j1} can thus cause a region to get a slower rate of growth of its production sectors in the short run, when the economy traverses from an old to a new turnpike structure. This is illustrated in Figure 5.

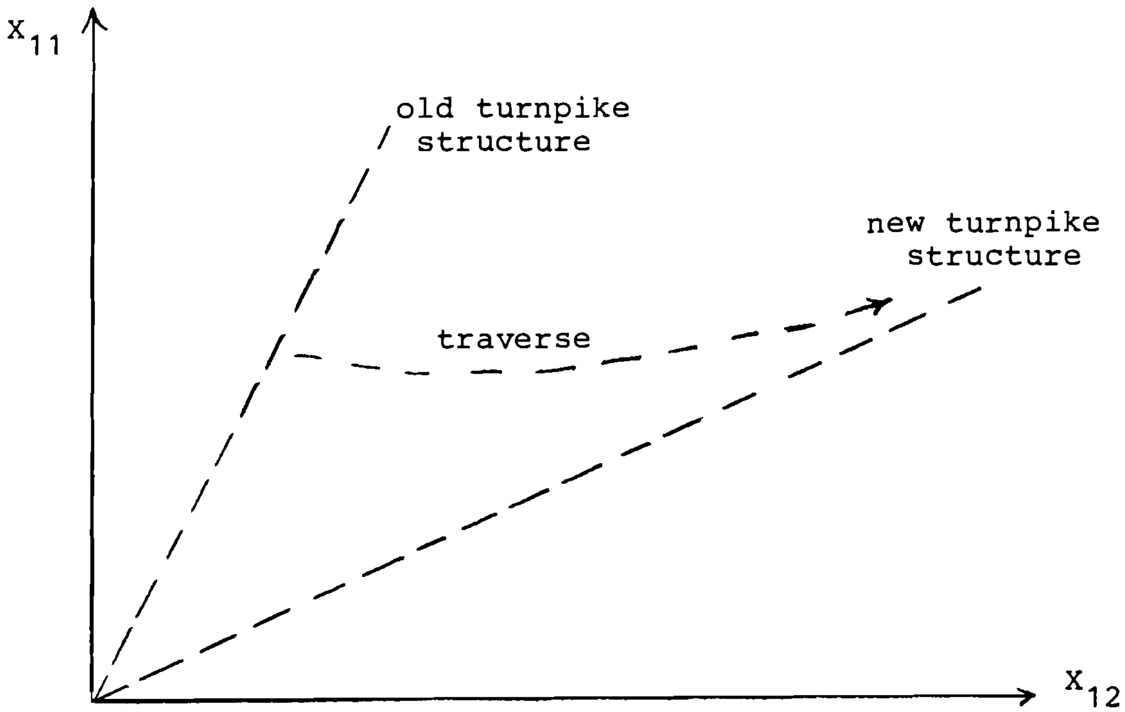

Figure 5: Illustration of Traverse of Regional Production Structure from an Old to a New Equilibrium as a Consequence of Transportation Network Investment.

Figure 5 illustrates how a decrease of one of the distances can lead to a new equilibrium sectorial and spatial structure (and a faster rate of growth). This figure further illustrates that region 2 gets almost all the growth in the beginning phases with almost stagnation accruing to sector 7 of region 1 in the initial phases of the traverse movement. Regional stagnation is thus a possibly persistent but never permanent feature of this development process.

Conclusions

A neoclassical framework for analysis of public goods within a growth process has been presented in this paper. It is shown that there exists a stable share of public goods in a growing economy with a certain class of well behaved production functions. It is also shown that there must exist some positive non-confiscatory rate of taxation that maximizes the rate of equilibrium growth. A regionalization adds an important element to the analysis. The concept of accessibility is used as a tool for the introduction of public goods into the regionalized growth analysis.

Some important conclusions can be drawn:

a) A reduction of communication distance between any two regions will always increase the rate of equilibrium growth and the relative importance of the public sector.

b) A reduction of communication distance leads to changes in the relative share of production of all regions.

This implies that a communication reform can lead to stagnation of certain regions in the short run with proportional and faster growth of all regions in the long run.