NUMERICAL MODELLING OF THE DYNAMICS OF LAKE BAIKAL

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Introduction

Mathematical modelling of environmental processes has been widely applied. In the Soviet Union, the investigation of almost all of the large basins is being carried out by means of numerical methods. The scientific group of the Siberian Branch of the USSR Academy of Sciences, under the guidance of Academician G.I. Marchuk, has been working on the model of the largest fresh water lake, Lake Baikal.

This report provides a review of the numerical model of lake currents made at the Computing Center. Before presenting a mathematical formulation of the problem, we will describe Lake Baikal, for which a model has been built.

Lake Baikal

Baikal is located between latitude 51°40'-55°45' North and longitude 109°40'-109°55' East. The lake is crescent shaped and 636 km long, the narrowest part being 25 km and the widest part being 75.5 km. The enclosed valley of the lake consists of three deep cavities divided by submarine rifts. Running parallel to Lake Baikal is a high mountain chain cut by several deep valleys.

More than 500 tributaries flow into the lake, of which the biggest are the Selenga, the Upper Angara and the Barguzin. The only river which has its source from Lake Baikal is the Angara river.

Baikal is a fresh water lake, possessing all the characteristics of fresh water. The water density is the most important for dynamics modelling. Since the water density increases with the increase of temperature from 0° to 4° C, a stable stratification of water is possible by an inverse distribution pattern of the temperature.

Mathematical Formulation

Let us use a cylinder system for the convenient description of the geometry of the domain under discussion (see Fig. 1). The following system of equations are valid:

$$\frac{du}{dt} - \ell v = g\frac{\partial \xi}{\partial \gamma} - \frac{g}{\bar{\rho}} \int_0^z \frac{\partial \rho}{\partial \gamma} - D(u) - \frac{1}{\bar{\rho}} \frac{\partial Pa}{\partial \gamma} + \frac{v^2}{\gamma}$$

$$\frac{dv}{dt} - \ell u = \frac{g}{\gamma} \frac{\partial \xi}{\partial \theta} - \frac{g}{\bar{\rho}\gamma} \int_0^z \frac{\partial \rho}{\partial \theta} + D(v) - \frac{1}{\bar{\rho}\gamma} \frac{\partial Pa}{\partial \theta} - \frac{uv}{\gamma}$$

$$\frac{1}{\gamma} \frac{\partial}{\partial \gamma} \gamma u + \frac{1}{\gamma} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0$$

$$\frac{dT}{dt} = Q(T)$$

$$\rho = 1 - \frac{(T - 3.98)^2}{503.57} \cdot \frac{T + 283}{T + 67.26} \cdot 10^{-3}$$

with the boundary conditions at:

$$z = 0: \quad \nu \frac{\partial u}{\partial z} = - \frac{\tau_\gamma}{\bar{\rho}} \quad ; \quad \nu \frac{\partial v}{\partial z} = - \frac{\tau_\theta}{\bar{\rho}} \quad ;$$

$$\nu \frac{\partial T}{\partial z} = -I \quad , \quad w = 0$$

$$z = H: \quad u = v = w = \tau_z$$

at the lateral boundaries $u = v = o$, $\frac{\partial T}{\partial n} = 0$, and under the initial data, $T = T^o$, $u = u^o$, $v = v^o$.

Here, u, v and w are velocity components of the vector u along γ, θ and z directions, respectively. T is temperature; ρ is water density; H is the depth; ξ is the free surface; τ_γ is the wind stress on the surface; ℓ is Coriolis parameter; I is downward flux of heat at the surface; n is normal vector; g is acceleration of gravity; Pa is atmospheric pressure at the surface.

The symbol $\frac{d}{dt}$ is a total derivative in the cylinder system of coordinates:

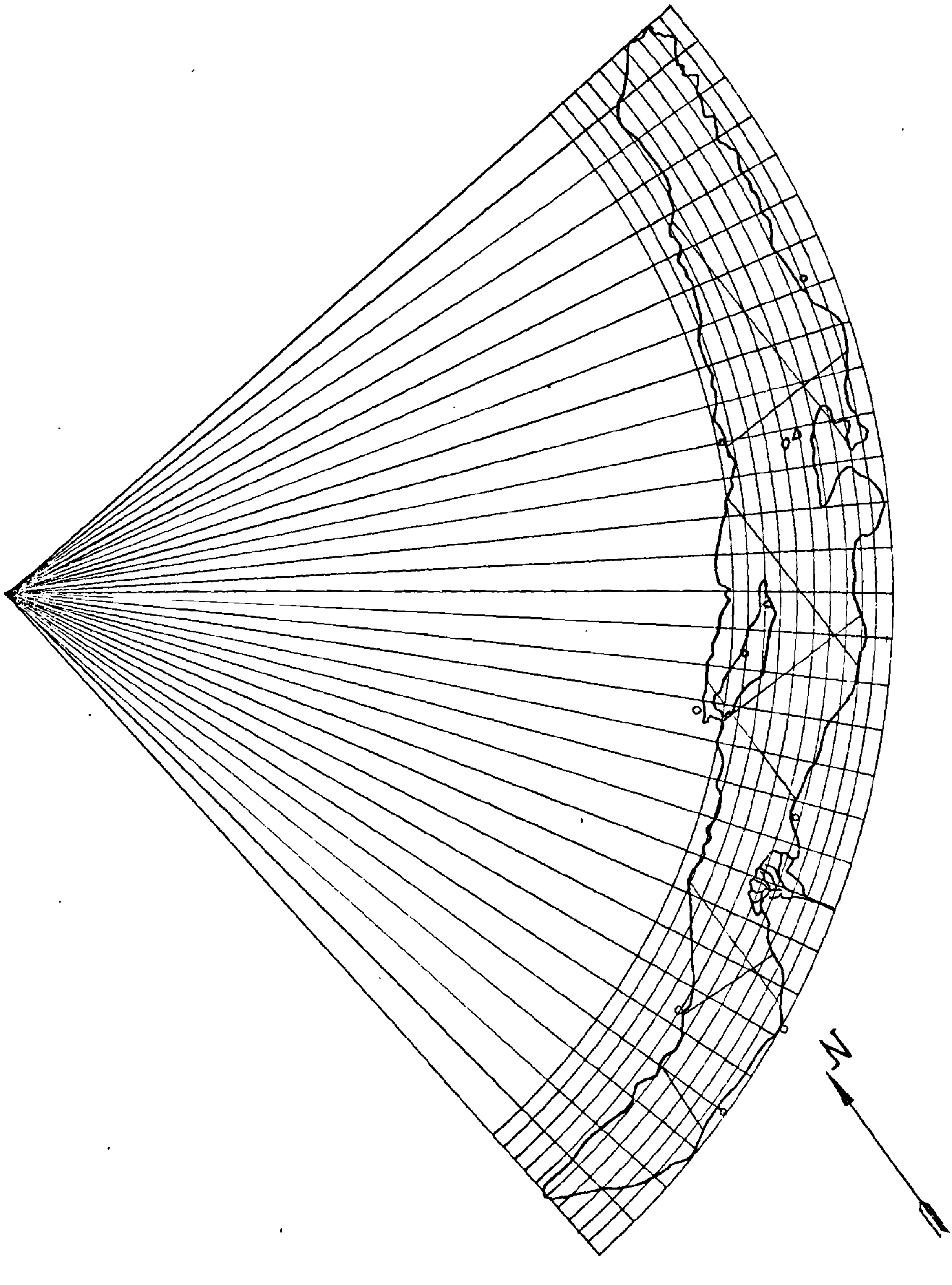

Fig. 1. Grid domain.

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial \gamma} + \frac{v}{\gamma}\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z}$$

The anizotrop form for the turbulent operators was chosen on the basis of the evaluations received by the processing of the measurements.

$$D(\phi) = \frac{A_z}{\gamma}\frac{\partial}{\partial \gamma}\gamma\frac{\partial \phi}{\partial \gamma} + \frac{A_\theta}{\gamma^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial}{\partial z}\nu\frac{\partial \phi}{\partial z}$$

The splitting-up method is used for solving the system of equations. It consists of the solution of some more simple problems instead of a complex one.

The splitting-up of the physical processes has been chosen to solve the problem, as its advantage is that it is possible to investigate each process separately. New factors can also be taken into account by means of the same procedure. Separating the physical processes in this way is convenient for calculation, and consists of three steps.

The first step is the transference along the trajectory

$$\frac{\partial u}{\partial t} + u^n\frac{\partial u}{\partial \gamma} + \frac{v^n}{\gamma}\frac{\partial u}{\partial \theta} + w^n\frac{\partial u}{\partial z} = 0$$

$$\frac{\partial v}{\partial t} + u^n\frac{\partial v}{\partial \gamma} + \frac{v^n}{\gamma}\frac{\partial v}{\partial \theta} + w^n\frac{\partial v}{\partial z} = 0$$

$$\frac{\partial T}{\partial t} + u^n\frac{\partial T}{\partial \gamma} + \frac{v^n}{\gamma}\frac{\partial T}{\partial \theta} + w^n\frac{\partial T}{\partial z} = 0$$

under conditions div $\bar{u} = 0$ and corresponding to the boundary and initial conditions.

The second step takes the diffusion into account:

$$\frac{du}{dt} = D(u)$$

$$\frac{dv}{dt} = D(v)$$

$$\frac{dT}{dt} = D(T)$$

The solution of the equations of the first step is used as the initial approximation.

The following system of equations is considered in the adaptation step:

$$\frac{\partial u}{\partial t} - \ell v = g \frac{\partial \xi}{\partial \gamma} + F_1$$

$$\frac{\partial v}{\partial t} + \ell u = \frac{g}{\gamma} \frac{\partial \xi}{\partial \theta} + F_2$$

$$\frac{1}{\gamma} \frac{\partial}{\partial \gamma} \int_0^H udz + \frac{1}{\gamma} \frac{\partial}{\partial \theta} \int_0^H vdz = 0$$

where

$$F_1 = - \frac{g}{\bar{\rho}} \int_0^z \frac{\partial \rho}{\partial \gamma} - \frac{1}{\bar{\rho}} \frac{\partial Pa}{\partial \gamma}$$

$$F_2 = - \frac{g}{\bar{\rho}\gamma} \int_0^z \frac{\partial \rho}{\partial \theta} - \frac{1}{\bar{\rho}\gamma} \frac{\partial Pa}{\partial \theta}$$

The solution of this system is reduced to solving the elliptic equation for the stream function.

The numerical procedures applied to solve the system of equations for each step will not be mentioned here. We only point out that the great experience at the Computing Centre on the solution of problems of atmospheric and ocean dynamics is used for the realization of the model.

Much attention was paid to the parametrization of the atmospheric actions while working with the model of the currents. Using the data of measurements for more than 10 years, N.V. Savinova defined the types of wind fields. Six main types of wind were defined and these six types were further divided into subtypes. In all, seventy typical meteorological situations were described. Wind regime characteristics, continuous duration of wind action, type succession, and repetition of each type were also calculated. The catalogue of the wind, pressure fields, and tables of regime characteristics was formed to use this data in the models.

Special Monte-Carlo methods have been supposed to determine the order of succession and the duration of a meteorological situation in the numerical experiments. By these methods, the space structure of the wind and pressure fields is determined from the catalogue and the order of succession is modelled according to the tables of the regime characteristics.

Let us consider a few variations of the model of Lake Baikal and give the results of the numerical experiments.

Model of Wind-Driven Currents

If we assume that water is homogeneous and if we neglect nonlinear terms in the motion equations, we will get the following system of equations:

$$\frac{\partial u}{\partial t} - \ell v = g \frac{\partial \xi}{\partial \gamma} + \nu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + \ell u = \frac{g}{\gamma} \frac{\partial \xi}{\partial \theta} + \nu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial \xi}{\partial t} = \frac{1}{\gamma} \frac{\partial}{\partial \gamma} \gamma \int_0^H u \, dz + \frac{1}{\gamma} \frac{\partial}{\partial \theta} \int_0^H v \, dz$$

under the boundary conditions:

$$z = 0: \quad \nu \frac{\partial u}{\partial z} = - \frac{\tau_\gamma}{\bar{\rho}} \quad , \quad \nu \frac{\partial v}{\partial z} = - \frac{\tau_\theta}{\bar{\rho}}$$

$$z = H: \quad u = v = 0 \quad .$$

It is the most simple version of the model representing baratrop approximation.

The calculations made by means of this model demonstrate the current dependence on wind. Figure 2 shows the result of the calculation of the currents driven by the southwest and northwest wind types. The experiments have shown that the amount of time necessary to reach the steady state of wind currents is several days and is dependent upon the wind situation.

A Model of the Currents, Induced by the River Inflows and Outflows

Inflows and outflows play an important role in the lake, which is why we had to make special numerical experiments to calculate the currents caused by inflows and outflows alone. In the previous model, the wind influences were eliminated and the outflows of the rivers were given on the basin boundaries. Two cases were considered.

In the first case, the inflow was represented by the three biggest rivers, the Selenga, the Barguzin and the Upper Angara. The value of the Angara inflow was equal to the sum of the outflows of these three rivers. In the second case, besides the three local inflows, the value of the inflow of other rivers was given for the whole coastline. The Angara outflow was chosen in this case in such a way as to compensate the value of the sum inflows. The results of the calculations are shown in Figure 3. The most significant differences occur in Southern

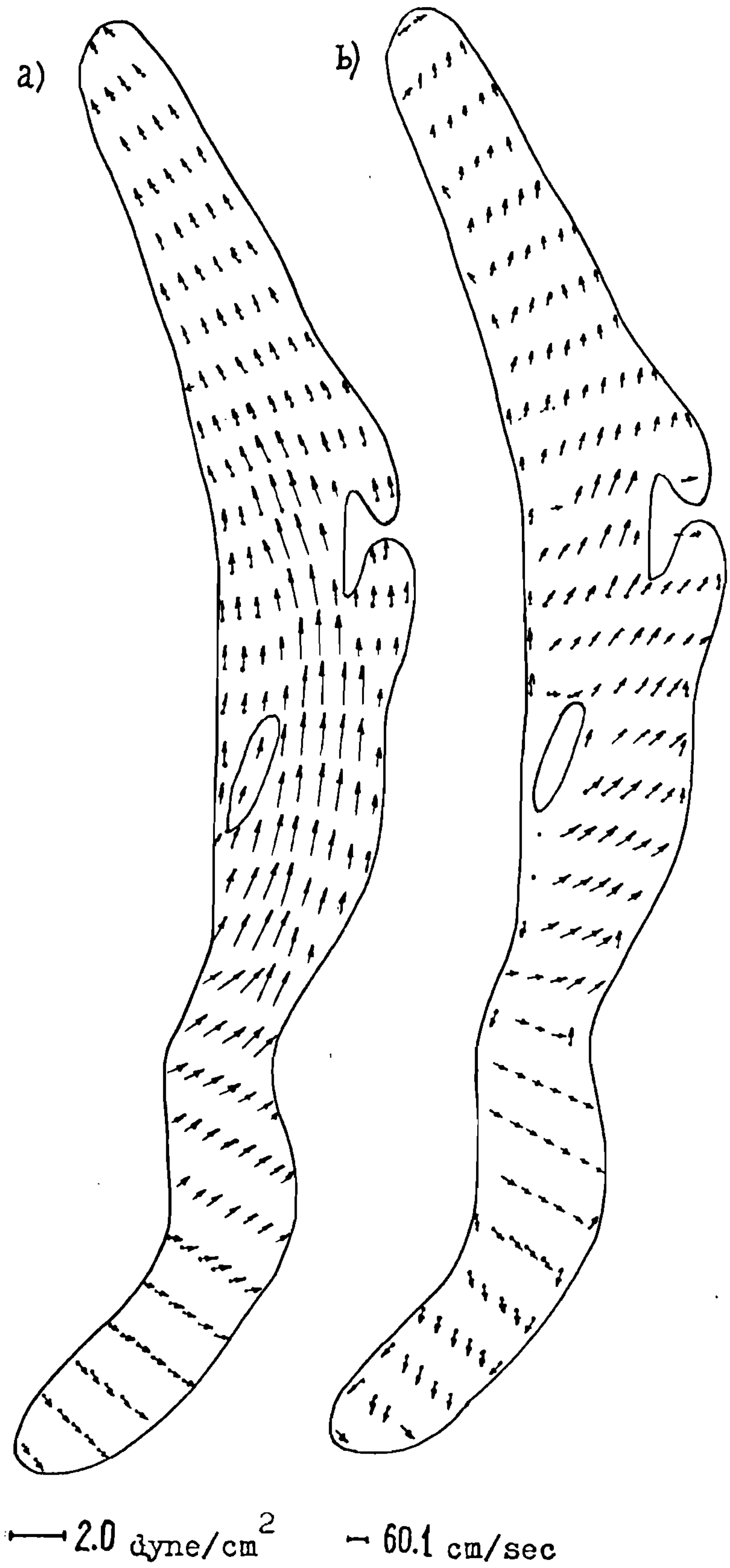

Fig. 2. Wind-induced currents

a) wind stress, b) surface velocitics.

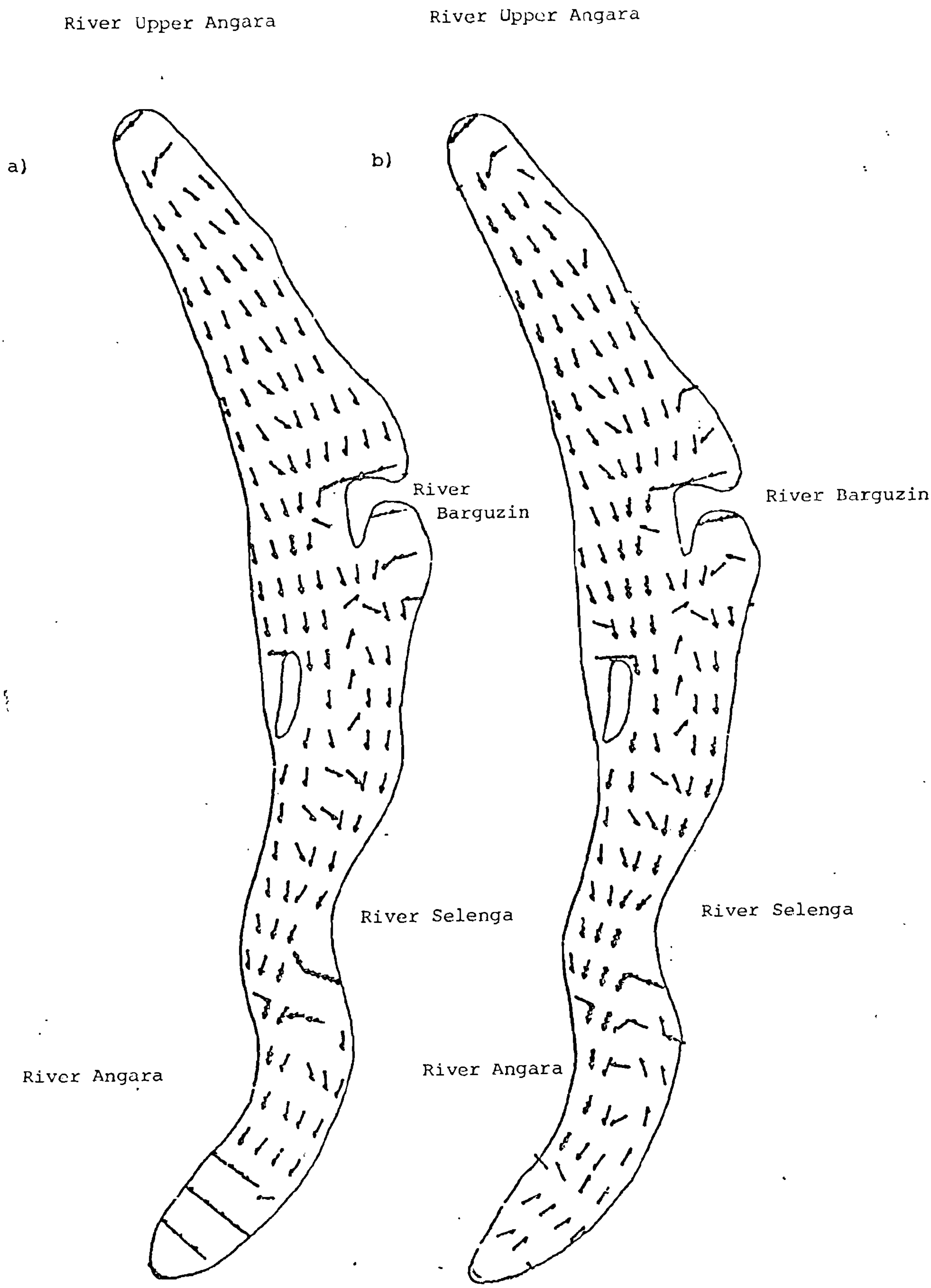

Fig. 3. Currents induced by river inflows and outflows

a) only three biggest inflows considered

b) all rivers considered.

Baikal, and that is natural, as the only outflow and 57 per cent of the inflow are located here. However, it is necessary to point out that the currents induced by the rivers are small. The average velocity is only a few millimeters per second. The conclusion is that the currents caused by inflows and outflows are small compared to the currents induced by wind.

A Baroclinic Model

This model is represented by the whole system of equations and reflects mutual adaptations of the currents and temperature patterns.

Figure 4 shows the surface currents calculated for the end of November. The analysis of the current pattern shows that the counterclockwise motion is characteristic for the upper layers as a whole.

The pictures of the calculated currents are in agreement with the pictures of the currents drawn after the processing of the measured data.

The above-mentioned models allow one to solve some problems directed towards the improvement and precision of the model itself. In the future, it will be possible to solve the problems closely connected with the planning of the activities on Lake Baikal.

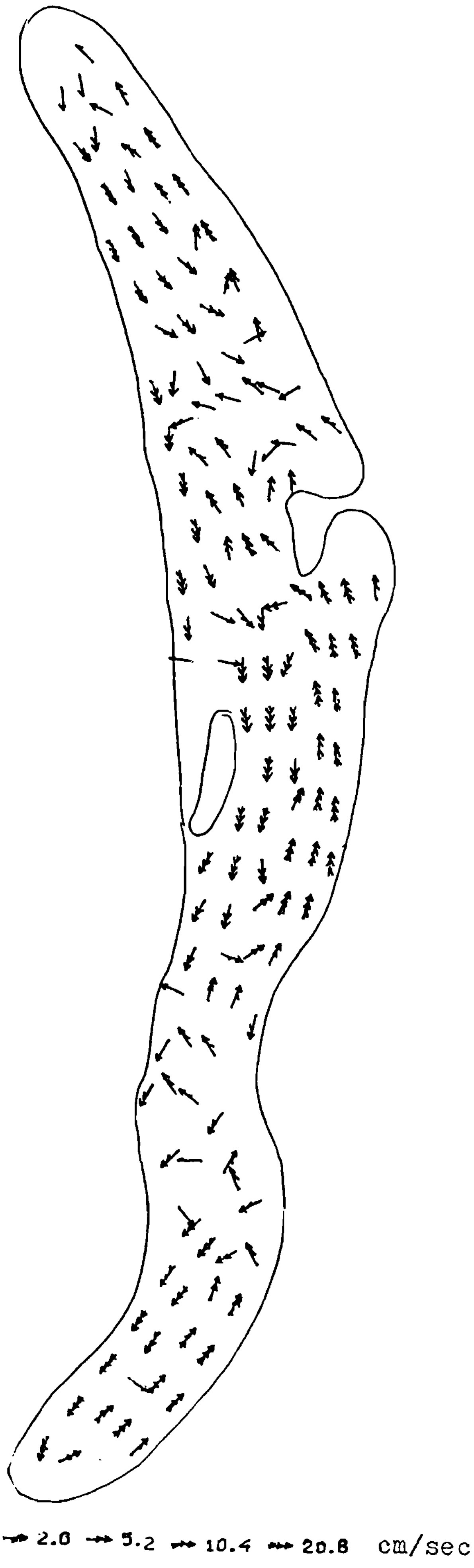

Fig. 4. Calculated surface velocities, end of November.