

# A Model of Endogenous Technological Change Through Uncertain Returns on Learning (R&D and Investments)

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## Abstract

A model is presented that endogenizes the two most important sources of technological change – *uncertainty*, and *technological learning* through research and development (R&D) and learning by doing (investments)– into an intertemporal optimization framework. Mathematically, the resulting problem is one of non-convex, non-smooth, stochastic optimization. The simple, stylized sectoral (energy) model includes one demand and one resource category. The model selects from three competing technologies, which differ in their current costs and in their (uncertain) potentials for future cost reductions through learning. The resulting model fully endogenizes the process of technological change, which is driven by expected, but uncertain, returns from investments into R&D and niche-market applications. These in turn can render new technologies increasingly competitive, ultimately leading to pervasive diffusion. The model, while definitely oversimplified, nevertheless allows several robust conclusions. First, it was possible to find an operable analytical solution for an optimization problem that simultaneously involves stochasticity (uncertainty) as well as non-convexity (increasing returns through technological learning). Second, the S-shaped patterns of technological entry and diffusion endogenously generated by the model are consistent with those observed historically and in the empirical literature on technological diffusion. Third, the model illustrates the possibility of wide-ranging technological outcomes resulting from even small differences in initial conditions and the (uncertain) rates of technological learning. Fourth, the resulting diffusion of new technologies of our model can yield pronounced discontinuities in the environmental performance of technologies. For instance, future emissions could decline radically even in absence of environmental constraints.

Fifth, and perhaps most importantly, the model demonstrates an entirely endogenous mechanism of technological change in which technologies that appear to be extremely economically unattractive from today's perspective (e.g. a factor 40 more expensive) can diffuse into the market under both criteria of uncertainty and intertemporal optimization (cost minimization), *if* upfront investments into R&D and niche market applications are made. These are shown also to constitute an optimal contingency policy *vis à vis* uncertainty in future energy demand and possible (uncertain) emergence of environmental constraints (e.g., a tax on carbon emissions).

## 1 Introduction

It is an often stated truism that new technologies do not fall like “manna from heaven.” Technological change is both costly and uncertain; it requires dedicated efforts in form of research and development (R&D) and demonstration projects (application in niche markets), subsumed under RD&D, and finally also (initially risky) commercial *investments*. In turn these efforts can reduce uncertainty and affect also other characteristics of new technologies, such as performance, efficiency, productivity, and of course costs. These potential future benefits and resulting economic returns provide the rationale why private firms and society at large invest in the pursuit of new technologies (e.g., Mansfield *et al.*, 1971 and 1977) In short, as Joseph A. Schumpeter observed long ago (in 1934), technological change arises from “*within*” the economic system, and is central to its growth.

That technological change is the most important single source of long-run productivity and economic growth is confirmed by theory (for a review see e.g., Metcalfe, 1987; and Freeman, 1994), historical evidence (e.g., Madison, 1991, 1995; and Mokyr, 1990), and calculations performed within (neoclassical) growth models ever since the first contributions of Tinbergen (1942) and Solow (1957, cf. Griliches, 1996). In fact, its importance may even be understated in growth accounting models that assume independence between factors of production (capital, labor) on one side and technological change on the other (Abramovitz, 1993). For instance, there is a evident relationship between technological change and capital. Embodied technological change requires investment, i.e. capital. In turn capital productivity increases through technological change, e.g. through cost reductions. This relationship in form of technological learning is at the core of the model of

endogenous technological change presented here.

Yet, for all these arguments and evidence technological change has largely been treated as exogenous in existing models. This is true of models developed within the tradition of growth theory and associated production function models (so-called “top-down,” models), as well as those developed within a systems engineering perspective (e.g., detailed sectoral “bottom-up” optimization models). In both modeling traditions, technological change is either reduced to an aggregate exogenous trend parameter (the “residual” of the growth accounts), or introduced in form of numerous (exogenous) assumptions on costs and performance of future technologies. Common to both modeling traditions is that the only endogenous mechanism of technological change is that of progressive resource depletion and resulting cost increases. Such constraints which are at odds with historical experience (cf. Barnett and Morse, 1967) trigger both substitution of factor inputs as well as the penetration of otherwise uneconomic technologies. These are either represented generically as aggregates in form of so-called “backstops” (a term coined by Nordhaus, 1973), or through detailed assumptions on numerous technologies individually.

Perhaps one of the reasons for this apparent impasse is that both modeling traditions usually operate within an optimization framework. However, in reality, future characteristics of technologies are not known *ex ante*, but result from the (uncertain) results of intervening actions (R&D and investments), i.e. *technological learning*. Endogenization of uncertainty and of R&D and technological learning into models is therefore mathematically cumbersome involving stochasticity and recursive formulations.

Technological learning is a classical example of increasing returns, i.e. the more learning takes place, the better a technology’s performance. It is the technology counterpart of the increasing returns resulting from the accumulation of knowledge or increases in human capital that are the focus of endogenous growth theory (e.g. Romer, 1986, and 1990; or Grossman and Helpman, 1991) and as discussed increasingly also in the technology domain (cf. Arthur, 1983, and 1989).

Our paper and model aims to make a contribution in the domain of endogenizing technological change as arising from *uncertain technological learning*. As such, the model formulation complements more traditional approaches of induced technical change focusing on relative resource and factor endowments

and prices (e.g. Hyami and Ruttan, 1985).<sup>1</sup>

We do not claim to develop any form of a “realistic” model in the sense of technological or sectoral detail. Instead, our model is deliberately highly stylized and simple. Our main objective is to demonstrate the feasibility of a mathematical formulation that simultaneously involves technological uncertainty and increasing returns (learning). Formally, this means dealing with a non-convex, non-smooth, stochastic optimization problem. We will also show that, despite its simplicity, the model yields interesting insights into dynamics and patterns of endogenized technological change. The model reveals both patterns of possible technological bifurcations as well as diffusion patterns that are consistent with historical observations. Perhaps the overall most significant result is that the model (given certain plausible assumptions on parameter distributions) can endogenously generate radical departures from existing technological practices. New technologies can penetrate the market even if they are initially by a factor of 40 (or more) more expensive than the existing dominant technology. Moreover such a strategy is both “rational” and “optimal” given risk diversification considerations and the potential returns from R&D and upfront investments that both enable technological learning.

The plan for the remainder of the paper is as follows. Section 2 summarizes the major sources of technological change considered in the model: Uncertainty and learning (R&D, demonstration, and investments). A distinguishing feature of our model is that all components are integrated. Section 3 gives a brief overview of the model structure and solution algorithms that are deployed. (More detail is given in the Mathematical Appendix to the paper). Section 4 presents some illustrative simulations performed with the model and the insights that these provide. Section 5 extends the simulations to perform sensitivity analyses of four important model parameters: the distribution of uncertainty of future learning rates, uncertainty in demand, variations in the discount rate, and an uncertain emission tax. Finally, Section 6 provides a concluding discussion of the results obtained as well as a (tentative) outlook on future research directions.

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<sup>1</sup>For an overview see e.g. Binswanger, 1978; Jorgenson and Fraumeni, 1981; and Ruttan, 1996. Our model of course also includes prices (in form of rising resource costs), but in our discussion we focus on uncertainty and technological learning as drivers of change.

## 2 Sources of Technological Change

In this Section we discuss two (interrelated) sources of technological change: Uncertainty and learning. Uncertainty, its associated economic risks and opportunities, as well as the strategies adopted in face of both risks and opportunities are a main driver of technological evolution. Learning itself is seen as a result of both the classical “learning by doing” (read: commercial investments) as well as of research and development (R&D), and demonstration efforts (in niche markets), subsumed here under the heading of RD&D. All these components are interrelated and have to be considered holistically as has been repeatedly argued by critics of “linear” models of innovation (cf. OECD, 1992), a point which we take up in our model formulation, where both R&D and investments taken together are modeled as a single learning process whose actual outcome is however subject to uncertainty. Learning is not only the main endogenous mechanism for reducing uncertainty, but is also a means of improving technical, economic, sometimes even social, characteristics of new technologies that are the main drivers for their widespread diffusion.

### 2.1 Uncertainty

There is only one certainty related to technological change: the technology of tomorrow will be different from that of today. But to what extent, and by which concrete configurations? The importance of technological uncertainty has been recognized and explored ever since the earliest days of global environmental modeling (e.g., Nordhaus, 1973; Starr and Rudman, 1973). Different approaches have been followed for analyzing the impacts of technological uncertainty including the formulation of alternative scenarios (e.g., IIASA-WEC, 1995); model sensitivity analysis (e.g., Nordhaus, 1973, 1979); and sensitivity analysis based on expert polls or Delphi-type methods (e.g., Manne and Richels, 1994).

In each of these types of analysis the subjective choice of the technological uncertainty range investigated is made either by the modelers themselves in the sensitivity analysis, or by the experts polled. Also, whereas scenarios or sensitivity analyses yield insights into the variations in model outcomes that result from changes in input assumptions, technological uncertainty is not endogenized into the decision rules (usually based on some optimization criterion) that have been employed in the models. In other words, although

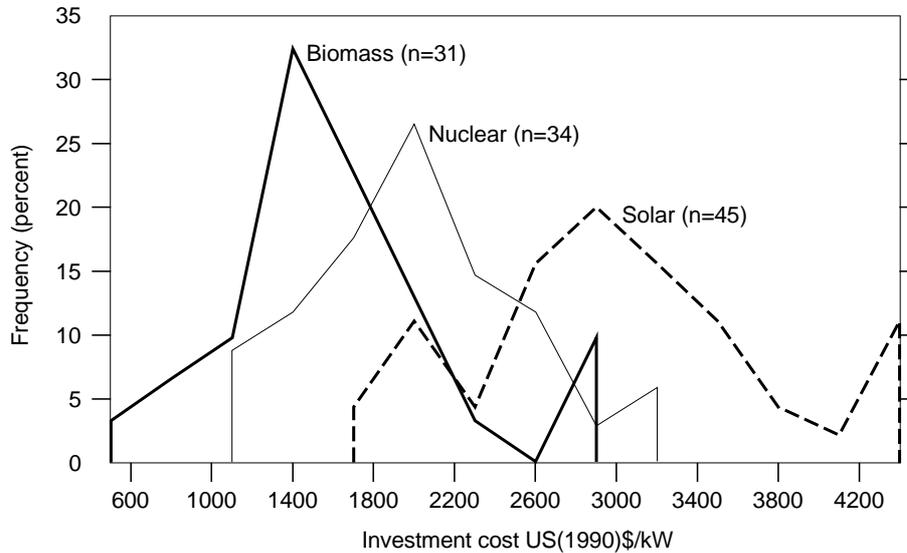


Figure 1: Range of investment cost distributions from CO2DB technology inventory for biomass, nuclear, and solar electricity generation. Source: IIASA-WEC, 1995.

we know of different future outcomes depending on when, how, and in what direction uncertainty is resolved, largely we remain ignorant about robust (or even “optimal”) strategies in the face of uncertainty. In the model of endogenous technological change proposed here, uncertainty translates into both economic risks and opportunities (benefits), and both are directly endogenized into the model’s decision rules and the resulting technology strategies.

A typical example of the range of “technological expectations” (Rosenberg, 1982) of near-term investment costs for biomass, nuclear and solar electricity generating technologies derived from numerous engineering studies is given in *Figure 1*. The resulting frequency distribution curves around the mean can serve as a convenient, empirically derived measure of technological uncertainty. Such distributions have for instance been used to introduce explicitly uncertainty into optimization models (cf. Golodnikov *et al.*, 1995). Note in particular, the two-sided “tails” in the cost distribution of future technology costs for solar technology, which reflects comparatively higher frequencies of optimistic and pessimistic technological expectations (a point to which we shall return briefly below).

A general methodology for endogenizing uncertainty in optimization problems is described in Ermoliev and Wets (1988) and an improved algorithm was suggested by Ermoliev (1995). It was applied in a sectoral (energy) model using stochastic optimization by Golodnikov *et al.* (1995) and Messner *et al.* (1996).<sup>2</sup> (See also Grübler and Messner, 1996.) There, the subjective nature of defining technological uncertainty ranges is replaced by an empirical approach, that draws on a detailed statistical analysis (Strubegger and Reitgruber, 1995) of investment costs of current and future energy technologies derived from engineering studies. The resulting empirically derived uncertainty distributions are incorporated directly into the optimization algorithm of the model –i.e., into its underlying decision-making rule. This is done through adding a risk term<sup>3</sup> in the objective function that integrates (weighted by probabilities) stochastically drawn data samples into the final solution. The algorithm assures short computation time and full endogenization of uncertainty in the model solution<sup>4</sup>

The stochastic model responds to a frequent criticism of traditional optimization models: the inappropriate assumption of a decision-making agent that operates under perfect foresight. Through endogenization of uncertainty, decision making in the model no longer operates under perfect foresight. The model behavior thus approximates the outcome of real-life decision-making situations in which different economic agents with different expectations and risk attitudes show persistent differences in strategies and investment behavior that result in technological diversification.

Model simulations illustrate that compared with traditional deterministic model representations, which assume perfect foresight, endogenization of technological uncertainty yields more diversified technological configurations. Even more important, the model results reveal a pro-innovation bias and no risk aversion in investments into technological change. Diversification thus becomes the optimal response strategy in face of technological uncertainty.

However, model simulations also illustrate that the inclusion of uncertainty leads to technological diversification only along the lines of incremental innovations –in other terms, to technology changes within a “*technolog-*

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<sup>2</sup>A similar type of application using the MARKAL model is reported in Fragnière and Haurie (1995).

<sup>3</sup>This term represents the economic costs (added to the objective function) if a technology turns out to be more expensive than expected.

<sup>4</sup>For details see Golodnikov *et al.* (1995); Messner *et al.* (1996); and Ermoliev and Wets (1988).

*ical neighborhood*” (Foray and Grübler, 1990). Radical technological change does not occur. Diversification into radical technologies is not an “optimal” hedging strategy in this particular model formulation. This means that technologies with currently very high costs and uncertainty ranges do not make it to the market in the stochastic model simulations. For this to occur –as observed in the real world– additional important mechanisms of technological dynamics (and uncertainty reduction) need to be incorporated into the model: learning and R&D.

## 2.2 Learning

In this Section we discuss technological learning, a key driver of technological change and diffusion. We begin by discussing the classical “learning by doing” that usually takes place through (commercial) investments. We then discuss research and development (R&D). We conclude in showing that successful technological learning requires both R&D *and* investments and that both go hand in hand. Investments begin with demonstration efforts (niche market applications), gradually expanding into commercial applications, thus sustaining the technological learning that enables new technologies to ultimately diffuse pervasively.

Although learning is one of the empirically most corroborated phenomenon of technological change, it nevertheless remains uncertain. In other words: R&D and investments are performed in *anticipation* of future returns (learning). This anticipation of learning, but with uncertain outcomes, is the conceptual core of the model presented here.

### 2.2.1 Learning by doing

The performance and productivity of technologies typically increase substantially as organizations and individuals gain experience with them. Long-studied in human psychology, technological learning phenomena were first described for the aircraft industry by Wright (1936), who reported that unit labor costs in air-frame manufacturing declined significantly with accumulated experience measured by cumulative production (output).<sup>5</sup> Tech-

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<sup>5</sup>The aircraft industry however also provides examples that technological learning should not be taken for granted. The other side of “learning by doing” is “forgetting by not doing.” An example of “negative” technological learning is provided by the Lockheed L-1011 Tristar aircraft (Argote and Epple, 1990). Production started in 1972 and

nological learning has since been analyzed empirically for manufacturing and service activities including aircraft, ships, refined petroleum products, petrochemicals, steam and gas turbines, and broiler chickens. Learning processes have also been documented for a wide variety of human activities ranging from success rates of new surgical procedures to productivity in kibbutz farming and nuclear plant operation reliability (Argote and Epple, 1990). In economics, “learning by doing” and “learning by using” have been highlighted since the early 1960s (see e.g., Arrow, 1962; and Rosenberg, 1982). Detailed studies track the many different sources and mechanisms of technological learning (for a succinct discussion of “who learns what?” see Cantley and Sahal, 1980).<sup>6</sup>

Learning phenomena are generally described in form of “learning” or “experience” curves, where typically the unit costs of production decline at a decreasing rate as experience is gained. Because learning depends on the actual accumulation of experience and not just on the passage of time, learning curves are generally measured as a function of cumulative output. Frequently, the resulting exponential decay function is plotted with logarithmically scaled axes so that it becomes a straight line (see *Figure 2*). Because each successive doubling takes longer, such straight line plots should not be misunderstood to mean “linear” progress that can be maintained indefinitely. Over time, cost reductions become smaller and smaller as each doubling requires more production volume. The potential for cost reductions become increasingly exhausted as the technology matures.

Technological learning is a classical example of “increasing returns”, i.e., the more experience is accumulated, the better the performance, the lower the

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reached 41 units in 1974. It subsequently dropped to 6 units in 1977, and then increased again thereafter. The drastic reduction in output led to large scale layoffs and the initially gained experience was lost with the staff turnover. As a result, the planes built in the early 1980s were in real terms (after inflation) more expensive than those built in the early 1970s.

<sup>6</sup>A stylized taxonomy of technological learning mechanisms includes *inter alia*: learning by upscaling (e.g., steam turbines or generators), learning through mass production (e.g., the classical Model T Ford), and learning through both increasing scale and mass production, referred to here as “continuous operation”, i.e., the mass production of standardized commodities in plants of increasing size (e.g., transistors, or base chemicals like ethylene or PVC, where cost reductions through learning have been particularly spectacular, cf. Clair, 1983). This simple taxonomy is confirmed by a statistical analysis of learning rates across many technologies and products (Christiansson, 1995). Learning rates are typically twice as high for “continuous operation” as for either upscaling or mass production alone.

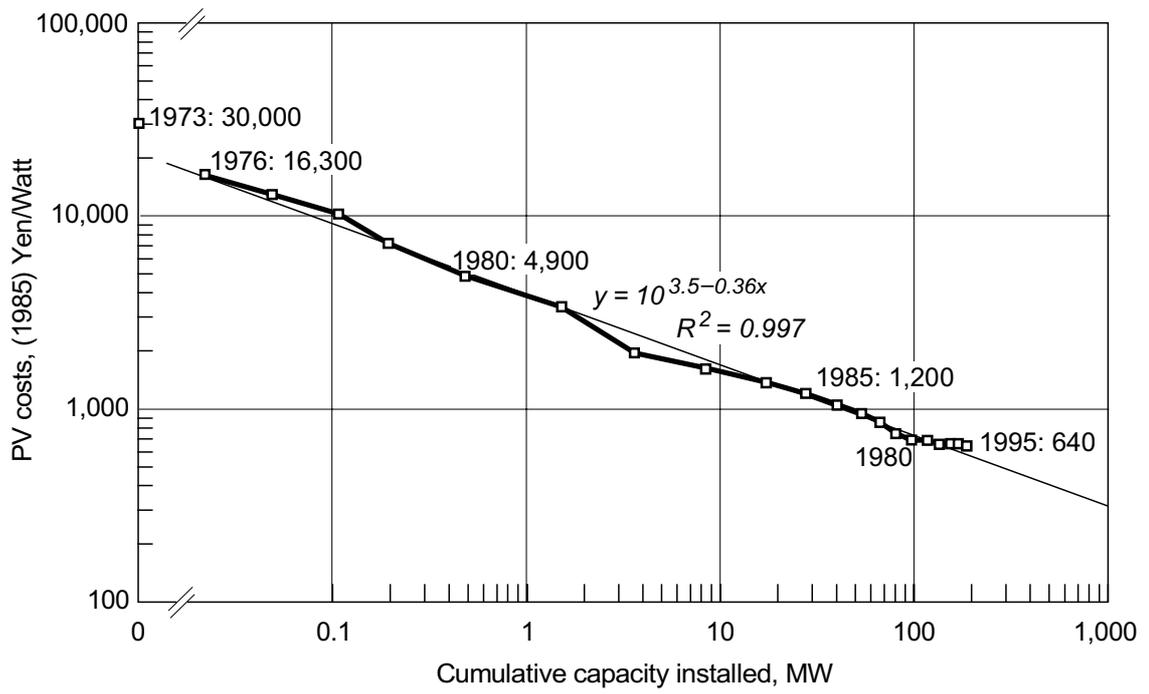


Figure 2: Photovoltaic costs (1985 Yen per kW installed) as a function of cumulative installed capacity (in MW). Japan 1976–1995. Data source: Watanabe (1995) and (1997).

costs of a technology, etc. However, because accumulation of experience takes ever longer (cf. the increasingly “packed” spacing of observations towards the 1990s in *Figure 2*) and is more difficult to achieve, learning itself shows decreasing marginal returns.

*Figure 2* plots the costs of photovoltaic cells per (peak) kW capacity as a function of total cumulative installed capacity for Japan. Starting off from extremely high costs of some 30,000 Yen (in 1985 prices) in the early 1970s, costs fell dramatically: from 16,300 Yen in 1976 to 1,200 Yen in 1985 (i.e., a factor close to 14 in less than 10 years), and then further to 640 Yen in 1995 (another factor 2 within the next 10 years). The resulting learning rate of a 36 percent reduction in costs per each doubling of cumulative installed capacity is at the higher end of the range of learning rates observed in the empirical literature (cf. Argote and Epple, 1990; and Christiansson, 1995). This high learning rate however is less surprising considering the infancy of the technology and the significant progress through R&D<sup>7</sup> that should, in fact, not be separated from “learning by doing” via investments, a point to which we return below.

Despite overwhelming empirical evidence and solid theoretical underpinnings, learning phenomena have been explicitly introduced only into few models of intertemporal choice. The most likely explanation for this paucity of model applications is the difficulties of dealing algorithmically with the resulting non-convexities of the problem solution. A first detailed model formulation was suggested by Nordhaus and Van der Heyden (1983) to assess the potential benefits of enhanced R&D efforts in new energy technologies such as the fast breeder reactor. A first full scale operational optimization model incorporating systematic technological learning was developed by Messner, 1995 (see also Nakićenović, 1996). In a mixed-integer formulation, learning rates for a number of advanced electricity generating technologies were introduced into a linear programming model of the global energy system. These learning rates were assumed to be known *ex ante*. Hence, future technology costs depend solely on the amount of intervening investments that lead to increased experience (installed capacity), that, in turn, stimulates learning and subsequent cost reductions.

The model by Messner (1995) demonstrated the feasibility of including

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<sup>7</sup>Note in particular the substantial cost decreases between 1973 and 1976 prior to any installation of demonstration units.

learning phenomena.<sup>8</sup> The results obtained are especially significant for two reasons.

First, the results indicate that providing for technological learning can lead to radical technological change. Learning enables the diffusion of technologies that are very different in their technological and economic characteristics of those predominantly used today. (The radical technologies that diffuse in the model simulations are, incidentally, also less carbon-intensive.) The resulting technology dynamics in the model yield diffusion patterns that are remarkably consistent with the theoretical and empirical findings of the diffusion literature (cf. Grübler, 1991, 1992): slow, but early, growth in niche markets where initial experience is gained, subsequent widespread diffusion that however, ultimately saturates when the technology eventually matures. This is in stark contrast to the typical “flip-flop” behavior of optimization models in which technological change (cost reductions) is introduced exogenously. There, the initial necessary gradual slow growth in niche markets and the resulting required upfront investments are entirely missed out simply because the learning that leads to the cost reductions postulated come at no cost.

Secondly, the model simulations with an optimization framework of Messner (1995) demonstrate that upfront investments into new technologies stimulate future costs decreases and can be economically optimal, even if at the time of investment a new technology is more expensive and has lower performance than existing ones. The results also contradict the policy advice (e.g., Wigley *et al.*, 1996<sup>9</sup>) that environmental policies such as emissions regulations should be delayed in anticipation of future technology improvements. The viewpoint of technological learning suggests that earlier action is better. Such early action may not necessarily imply the adoption of strict environmental targets but rather might consist of enhanced R&D and niche application efforts that stimulate technological learning.

There remain however two shortcomings in the modeling approaches discussed thus far. First, even if the empirical literature and statistical studies (e.g., Christiansson, 1995) give some hints about possible rates and mechanisms of learning in the past we remain uncertain about the rates at which a particular technology may improve in the future. Thus, instead of treating learning rates as (deterministically) known *ex ante* one needs to con-

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<sup>8</sup>On the methodology see Messner (1995) and (1997).

<sup>9</sup>For counterarguments see also Grubb, 1996.

sider uncertainty explicitly. Second, viewing technological change as result of R&D *and* investments, it is insufficient to consider only investments, even if investments constitute the dominant share in total expenditures into new technologies. Both domains are considered explicitly in the model presented here.

### 2.2.2 Learning through (Applied) R&D

The importance of RD&D (research, development, and demonstration) as source of technological change is evident and needs no further discussion here. The demonstration component of RD&D, which takes up the highest share in total RD&D costs, is well captured in the learning curve formulation presented above. However, one needs also to consider R&D (research and development) costs explicitly. In other words: include applied research<sup>10</sup> efforts in our considerations here.

As a representative conceptual and empirical model we follow the formulation of Watanabe (1995), who draws on the experience with MITI’s “sunshine” technology program. The data are particularly suited for illustrating our main argument because they include both public and private R&D expenditures and are also exceptionally comprehensive. (As a rule it is very difficult to get a complete overview of technology specific R&D expenditures by private industry.) The Watanabe model has also the added benefit of empirical parametrization obtained through statistical/econometric analysis of long time-series data. We use the example of photovoltaic cells (PVs) as illustration.

In essence, the model of applied R&D (see *Figure 3*) describes a positive feedback loop (a “virtually spin cycle” in the terminology of Watanabe, 1995): public R&D (together with other incentives) stimulates industry R&D, and both increase the “technology knowledge stock”<sup>11</sup> of a particu-

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<sup>10</sup>We recognize the importance of basic R&D as laying the groundwork, typically in form of new scientific knowledge, for applied RD&D and subsequent technological change (cf. Rosenberg, 1990). However, considering the frequently long lead times between the generation of new basic scientific knowledge and first commercial applications as well as the generic nature of scientific knowledge, i.e., it is relevant for more than just a few particular technologies; basic research is not treated separately in our discussion and model.

<sup>11</sup>This is the sectoral or technology specific equivalent of the knowledge stock introduced in the production function models of the so-called “new growth theory” (e.g., Romer, 1986, and 1990), that can also exhibit increasing returns. Evidently there are likely additional interindustry and cross-national R&D spillover effects (cf. Mansfield, 1985), including

lar technology, which leads to performance and cost improvements. These (amongst other incentives) in turn stimulate demand, increasing size of niche markets, learning (and hence further cost reductions), widening markets (production increases), that all feed back as a further stimulus for industrial R&D.

A flow diagram as well as the associated empirically derived model parameter estimates of the Watanabe model is shown in *Figure 3*.

For our purposes here, the model is particularly suited to demonstrate the close interlinkages between research and development *and* demonstration as well as the importance of the interplay between public and private R&D. One of the interesting findings of Watanabe (1995) is also the identification of the time lag between actual R&D expenditures and their returns in form of improved technology performance i.e., lowered costs in this case. This time lag is estimated by Watanabe (1985) to be less than three years illustrating a rather effective application of improved technical knowledge gained through systematic R&D improving design, production methods, etc. Combined, they result in rapidly falling technology costs.

Retaining this time lag of three years, we replot the learning curve from *Figure 2* above, but this time using RD&D expenditures (including R&D and investment costs) as independent variable (*Figure 4*).

Over the period 1973 to 1995 a total of 206 billion Yen (in constant 1985 money)<sup>12</sup> were spent on photovoltaics in Japan. 78 percent (162 billion Yen) of that amount were expenditures in actual investments in PV capacity, and 22 percent (44 billion Yen) on R&D proper. These statistics confirm the dominance of investments in niche markets and early deployment in total RD&D expenditures (and support our model simplification of adding R&D costs to investments rather than the other way around). Even more important is that R&D and investments cannot be treated separately as sources of technological learning. A linear model of the form that R&D precedes actual investments (demonstration in niche markets, even early commercial

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those from purchases of equipment (cf. OECD, 1996) that increase this technology knowledge stock too (working however, in both directions of give *and* take). These spill-over effects are not treated in the simplified aggregate model presented here.

<sup>12</sup>This equals approximately 2.5 billion US\$ in 1995 prices and exchange rates.

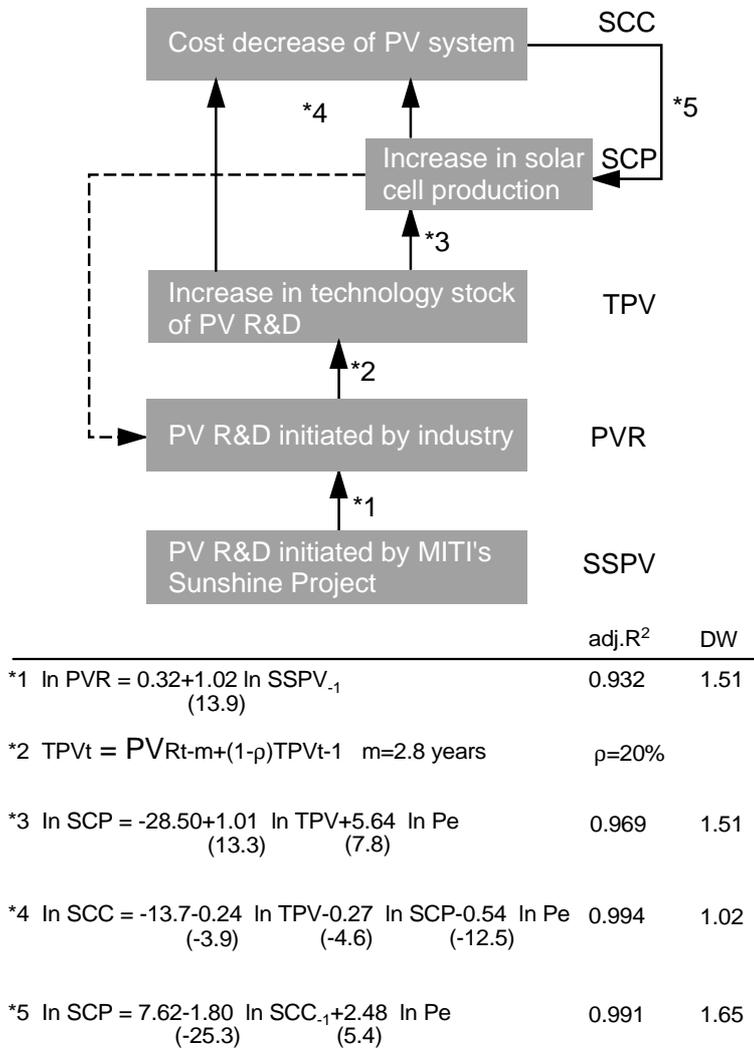


Figure 3: Positive feedback model of RD&D of Japanese photovoltaic development: Major relationships, feedbacks and model parameters estimated from empirical data over the period 1976 to 1990. Source: Watanabe, 1995.

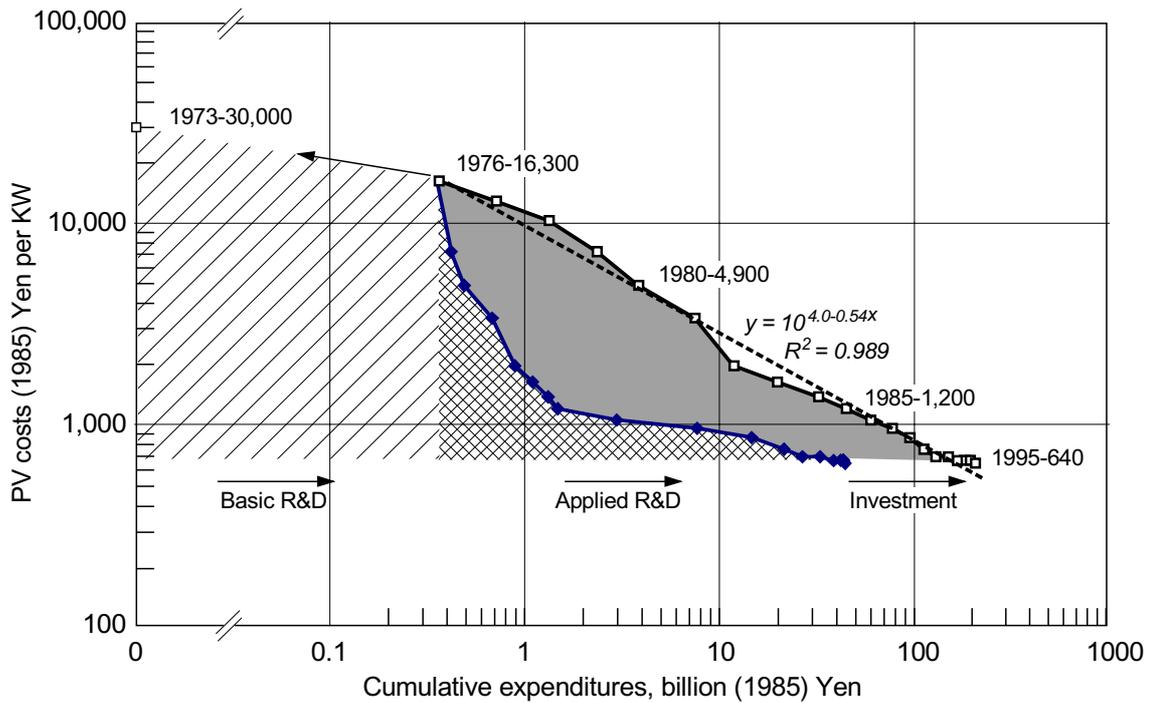


Figure 4: Photovoltaic costs (1985 Yen per kW installed) as a function of cumulative RD&D expenditures (billion (1985) Yen). Japan 1976–1995. Note that both applied R&D expenditures (lagged three years prior to investments) as well as demonstration (i.e., investment) costs are shown. The declining costs of PVs correlate well with total aggregate RD&D expenditures along a classical learning curve pattern with an over 50 percent reduction in costs for each doubling of cumulative expenditure (a proxy for the technology knowledge stock). Data source: Watanabe, 1995, 1997.

applications<sup>13</sup>) is not supported by the data.<sup>14</sup>

Functionally, total RD&D costs and technology costs again exhibit a classical learning curve relationship as shown in *Figure 4*. (The fact that the resulting learning curve parameter is with 54 percent per doubling higher than that given in *Figure 2* above [36 percent per doubling] is self-evident: with falling costs simply more capacity can be installed per unit expenditure.)

This simplifies our basic model considerably as both R&D and investments taken together can be modeled by a single learning curve,<sup>15</sup> whose actual value is however subject to uncertainty. This constitutes the essence of our simple model of endogenized technological change integrating uncertainty, R&D, and technological learning.

### 3 The Model

Our optimization model of technology choice is conceptually simple. (For a mathematical description and parameter values see the Mathematical Appendix.)

We suppose one primary resource, whose extraction costs increase over time as a function of resource depletion, while being sufficiently large for not resulting in absolute resource scarcity over the entire simulation horizon (set rather extremely at 200 years). The economy demands one homogeneous good, the demand for which increases over time. Three technologies are in principle available to perform the transformation from primary resource to the good demanded: “Existing,” “Incremental,” and “Revolutionary.”

The “Existing” technology is assumed to be an entirely mature one, i.e. its characteristics (costs and resource conversion efficiency) do not change over time. The “Incremental” technology represents its incremental improvement counterpart with a slight efficiency advantage, but with currently higher (by a factor 2) costs. The “Incremental” technology has potential for technological learning; the mean learning rate assumed is set at 10 percent (for

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<sup>13</sup>There can be quite an overlap between these two types of investments. Consider for example the case of PVs: their use in remote locations constitutes both an important demonstration effort, but in many cases may constitute already a commercial investment as well. This is an additional reason of not separating artificially R&D, from demonstration and subsequent early commercial investments.

<sup>14</sup>For further evidence see also Mori *et al.*, 1992; and Baba *et al.*, 1995.

<sup>15</sup>Technically this is done simply by increasing the intercept of the learning curve referring to investments alone through a fixed R&D component percentage.

each doubling of cumulative production capacity<sup>16</sup> installed) which is characteristic for incremental technological change.

As mentioned above, this learning rate reflects the “expectations” of the returns on RD&D efforts invested into the “Incremental” technology. The learning rate is of course uncertain and we represent this through an uncertainty range around the mean value adopted based on a lognormal distribution function.<sup>17</sup>

As its name suggests, our assumed “Revolutionary” technology is radically different. It hardly requires any resource inputs, and thus offers a substantial efficiency premium. But that premium comes at a very high cost: Initial costs are assumed to be a factor 40 higher than those of the “Existing” technology in our base case simulations. However, high costs also imply high potentials for technological learning, and we assume a mean learning rate of 30 percent (per doubling of capacity) which is consistent with empirical examples of radical technological change. Again, the exact learning rate is uncertain, represented by a lognormal distribution function around the mean value (cf. the Mathematical Appendix). And uncertainty of course is also larger than that of incremental technological change: we assume that the dispersion around the mean learning rate is three times the uncertainty range of the less risky “Incremental” technology.

Current costs are assumed to be known perfectly which reflects the common-sense notion that actual investment eliminates any uncertainty on *current* costs of new technologies (provided of course a supplier is found). As mentioned above, costs are assumed to include both actual investment costs and R&D costs.<sup>18</sup> Resource quantities/prices are also assumed to be perfectly known as well as the future evolution of demand (this restrictive assumption is relaxed in simulations reported below which also treat also future demand as uncertain).

Formally, our model works as follows. The learning rates of the “Incremental” and “Revolutionary” technology are treated as random values. This means that future technology (investment) costs are a random function of the

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<sup>16</sup>As simplification (in order for not having to compute the costs of under-utilized installed capacity) we assume 100 percent capacity utilization for all three technologies.

<sup>17</sup>We also test below the sensitivity of our model results to using alternative functions.

<sup>18</sup>For the “Incremental” technology the R&D component is assumed to be comparatively small, whereas for the “Revolutionary” technology R&D costs are much higher. Typical empirical examples of R&D intensive technologies indicate that R&D costs can account for up to 30 percent of total RD&D costs.

intervening cumulative investments. The probabilistic characteristics of these random values can be derived from the uncertainty distribution functions of the corresponding learning rates. In our model we are using a simultaneous approximation of these random future cost values by  $N$  sample functions of the learning rate, where  $N$  is the sample size (see the Mathematical Appendix for further details). For this simultaneous sampling of parameter values, the non-convex and non-smooth optimization problem is solved by applying a combination of a simple global search procedure, a modified Nelder-Mead algorithm, and a BFGS quasi-Newton minimization. The solution path of the optimal technology strategy for our problem with  $N$  approaching infinity converges to an optimal solution of the original stochastic problem.

For each sample  $N$  we integrate the expected costs into the objective function that consists of three parts. Part 1) corresponds to the expected value for a deterministic formulation. Part 2) in the objective function represents the risk (costs) of having overestimated the technological learning rate, i.e. realized investment costs are higher than expected. The additive term is assumed to be quadratic, i.e. the costs added to the objective function grow quadratically with the deviation of costs from sample  $N$  to the mean expected value. Part 3) is its benefit counterpart, i.e. when costs turn out to be lower than the expected (mean) value due to learning rates that are higher than expected. This part added to the objective function is assumed as a linear term.

In our approach, “risk” and “benefits” are non-symmetric and cannot be expressed simply in terms of mean and variance of corresponding economic gain and losses. This reflects our interpretation of reality characterized by asymmetry of the costs associated with under- or overestimating future costs and hence one’s future competitive position. Underestimating costs is penalized more heavily in competitive markets than overestimation. (Though in our stylized model we only have a single decision agent that however, does *not* operate with perfect foresight.)<sup>19</sup> Cost underestimation risks the very survival on the market, whereas cost overestimation yields “merely” lower profits than expected. In other words, our model (perhaps conservatively) values survival higher than profitability.

The model is solved for a sufficiently large sample  $N$ , where the size of

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<sup>19</sup>We are currently working on an extension of the model with multiple agents that can have different valuations of risks and benefits associated with making a wrong “bet” on future learning rates.

$N$  has been determined through successive experiments. Several successive model runs with the same sample size (but because of the stochastic draw of course consisting of different subsamples of learning rates)  $N$  are compared. If no major changes in the solution structure and the objective function can be observed then  $N$  is considered sufficiently large. The model then determines an overall intertemporal cost minimum given all the individual realized objective functions (plus and minus the quadratic and linear risk and benefit costs terms respectively) of sample  $N$  using a discount rate of 5 percent.<sup>20</sup> The resulting model solution represents the optimal technological investment diversification strategy *vis à vis* uncertain returns from RD&D needed to promote technological learning.

## 4 Base Case Simulations

In this Section we report quantitative simulations performed with the model using a discount rate of 5 percent and the other model parameters such as initial costs and learning rates set at their base case value given in the Section above and in the Mathematical Appendix. In turn, these base case assumptions are varied further in the Sensitivity runs reported in the next Section below.

We start with the results in terms of the share of various technologies in new capacity additions over time as shown *Figure 5*.

Simulation runs 0 and 1 represent the more conventional view of technology as either static (run 0) or determined exogenously (run 1). The significance of the run with static technology is what does *not* result: neither the “Incremental” nor the “Revolutionary” technologies ever make it. Over the entire simulation horizon all additional capacity growth is supplied by the “Existing” technology (dotted line at 100 percent share level in *Figure 5*). Run 1 portrays a typical pattern of models that employ exogenous technological change: at some future time (2020 in our case) a new technology massively enters the picture due to an exogenously prespecified cost reduction.<sup>21</sup> Similar patterns occur in models deploying an exogenous “backstop” technology which enters the market only due to resource depletion effects or additional exogenous constraints, e.g. environmental limits. “Running out

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<sup>20</sup>This as it turns out very critical parameter is varied below in the sensitivity analysis.

<sup>21</sup>We assume simply that the costs of the “Incremental” technology fall like “manna from heaven” to the level of the “Existing” technology by 2020.

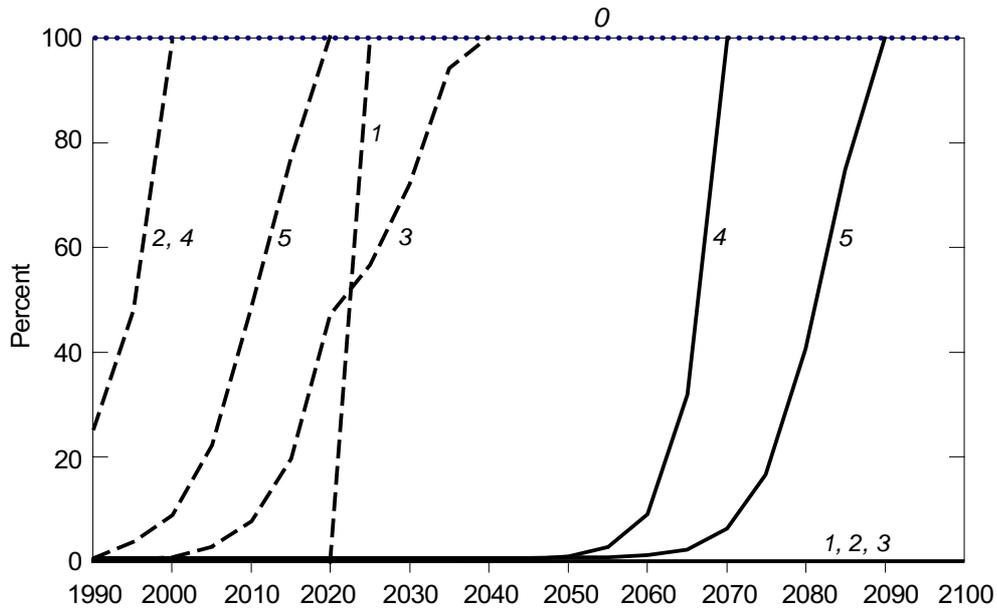


Figure 5: Share (in percent) of three technologies in new capacity additions, “Existing” (dotted lines), “Incremental” (dashed lines), and “Revolutionary” (solid lines). Note that for clarity of exposition only growing shares are reported (and the symmetrical declining shares of technologies being substituted are omitted). The simulation runs shown include:

0: static technologies

1: exogenous improvements in “Incremental” technology only

2: learning of “Incremental” technology only

3: uncertain learning of “Incremental” technology only

4: learning of “Incremental” *and* “Revolutionary” technology

5: uncertain learning of “Incremental” *and* “Revolutionary” technology (retained as standard base case BC30 in the discussion below).

For a discussion see text.

of resources” and environmental limits are not considered in the base case simulations.

More interesting are the results from simulation runs 2 and 3. There we allow technological learning for the “Incremental” technology (but not for the “Revolutionary” one). In run 2, the learning rate (10 percent cost reductions for each doubling of installed capacity) is assumed to be perfectly known. Combined with the model’s perfect foresight, this results in a very rapid market introduction. Allowing for uncertainty in the potential returns of technological learning (run 3) yields a more cautious diversification strategy with delayed and gradual experimentation, starting with a low level of installed capacity which is gradually stepped up.

Simulation runs 4 and 5, repeat the runs 2 and 3, but this time also allowing learning for the “Revolutionary” technology. Run 4 assumes perfect knowledge of future returns to RD&D (learning), whereas run 5 assumes that learning rates are uncertain. This run 5 with uncertain mean learning rates of 10 and 30 percent for the “Incremental” and “Revolutionary” technology respectively is retained subsequently as base case for the sensitivity analysis (and is denoted as BC30).

Because of high initial costs and much greater uncertainty the market entry of the “Revolutionary” technology is delayed into the future (compare runs 4 and 5 in *Figure 5*). However, it is important to emphasize that from very early on investments even in the “Revolutionary” technology do indeed occur (and such investments are optimal in the sense of the model formulation). These small initial investments, which are critical for continued technological learning, appear rather invisible on the linear scale of *Figure 5*. It is important to emphasize again that the “Revolutionary” technology makes it to the market *only* if learning occurs.

Overall, the most significant results of the model simulations is the demonstration of an entirely endogenous mechanism that drives technological change: expected returns from RD&D which are uncertain but potentially large, make (gradual) technological experimentation and learning the optimal strategy. The decision agent in our model acts entirely rationally by investing upfront into RD&D in expectations of returns in form of performance improvements, cost reductions, etc. His/her rationality is however “bounded” (Simon, 1982) by the inevitable uncertainty about the benefits and costs of such investments.

*Figure 6* shows the results for one of our base case simulations (run 5 [BC30] from *Figure 5* above) as *total* market share (in total installed capac-

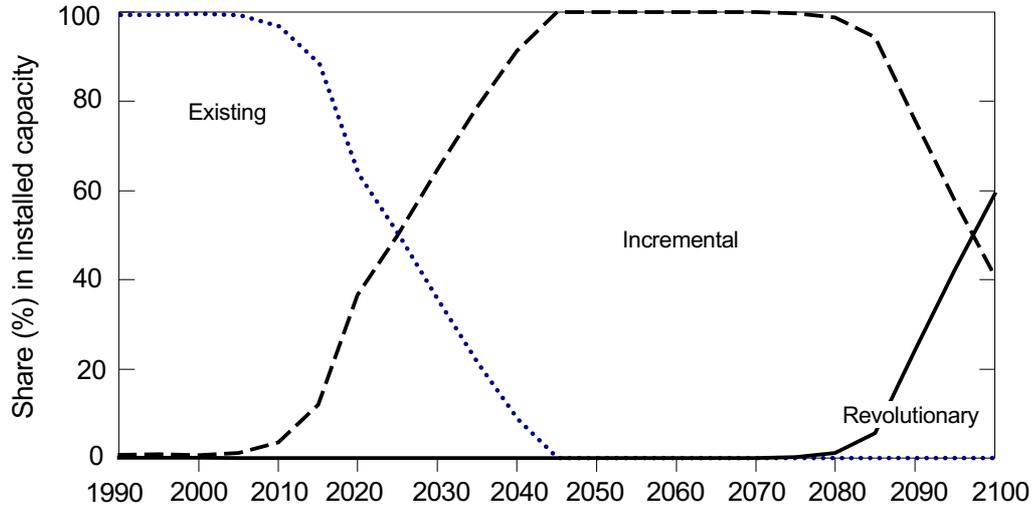


Figure 6: Market shares (in percent) of the three technologies in total installed capacity under uncertain technological learning with base case parameter specifications (simulation run 5 of *Figure 5* above). Note in particular the smooth S-shaped diffusion patterns.

ity) of each of the three technologies. The result is a pattern of technological evolution characterized by a “sequence of replacements” (Montroll, 1978) of older by newer technologies. This technological structural change is consistent with the diffusion patterns observed historically (cf. Nakićenović, 1997) and formulated by diffusion theory (cf. Rogers, 1983). Technologies enter into small niche markets slowly, but with declining costs (through learning) diffuse more rapidly and widely until markets are saturated and technological improvement possibilities (learning potentials) become exhausted. The result, graphically, is the familiar S-shaped curve pattern.

## 5 Sensitivity

In the simulations reported thus far we have used uncertainty distributions only around the base case parameter values of the learning rate while assuming all other salient model parameters and input variable as perfectly known. We now report several sensitivity analyses that relax successively these simplifying assumptions. We have explored the model’s sensitivity in the following domains:

1. variations in the base case parameters of initial starting costs and mean learning rates for the stochastic sampling;
2. variations in the (shape of the) distribution functions of technological uncertainty;
3. different discount rates;
4. treating energy demand as uncertain; and finally,
5. including the possibility of an environmental constraint in form of an emission tax, whose likelihood of occurrence and extent are uncertain.

### 5.1 Variation in start-up costs and learning rates

A model that incorporates increasing returns is obviously highly sensitive to the parameter values adopted in the simulations. Two parameters are of particular importance in our model: 1) the initial start-up costs assumed from which technological learning (cost reductions) begin, and 2) the learning rate (percent cost reduction per doubling of cumulative installed capacity). Even small variations of parameter values in particular ranges can lead to radically different model outcomes. The resulting non-linear behavior is illustrated in *Figure 7* which compares the “Revolutionary” technology with the (static) “Existing” one.

The sensitivity analysis illustrates that it is the learning rate parameter that is the most influential over whether a currently expensive technology makes it to the market (see for instance the flatness of the curve with a 50 percent learning rate even when varying the initial costs between 10,000 to 100,000 \$/kW). Conversely, if the learning rate is low, even initially low start-up costs do not help much. This (together with simple plausibility) confirms our approach in treating the learning rate as stochastic while assuming that initial costs are well-known. Nevertheless, for some parameter combinations comparatively small variations in initial costs can make a large difference: varying the initial costs from 10,000 to 30,000 for a mean learning rate of 30 percent delays the break-even point of the “Revolutionary” technology by more than five decades in this particular example.

### 5.2 Distribution of uncertainty

As an additional sensitivity analysis we have also varied the *type* of uncertainty distribution function around the mean learning rate. Even maintaining the same variance, using alternative distribution functions to the lognormal

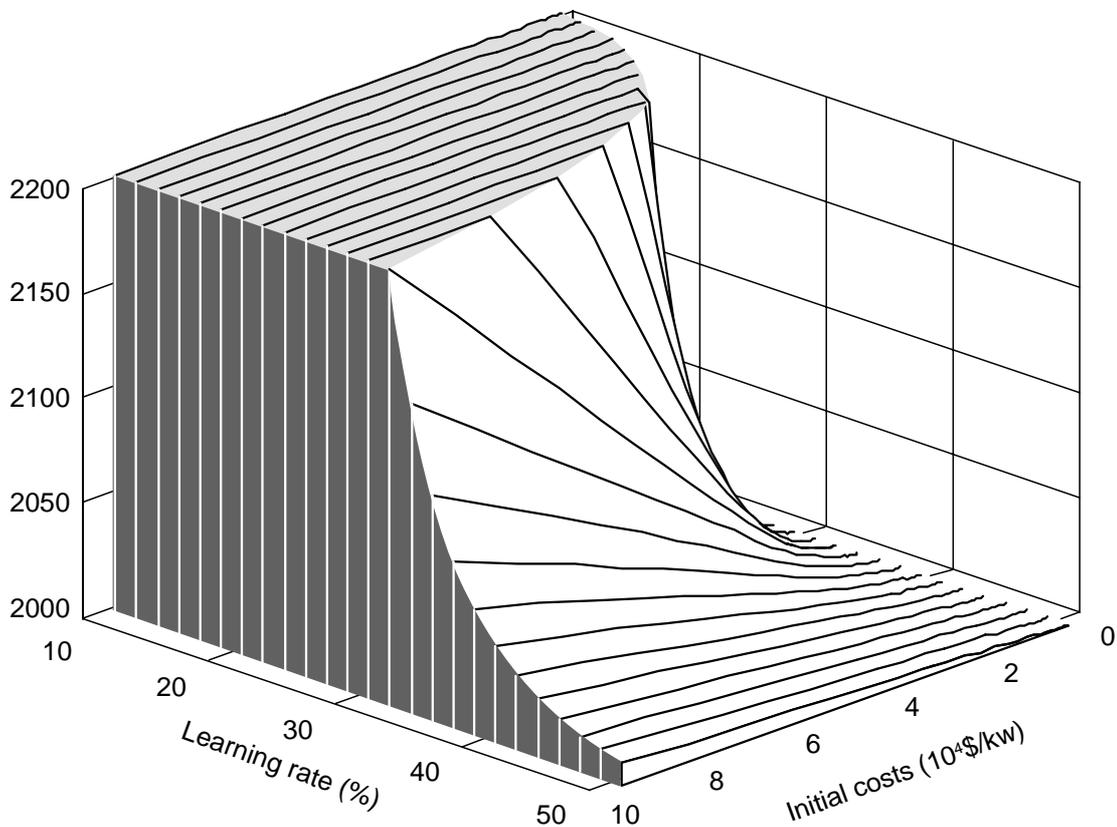


Figure 7: Time (year) by when the “Revolutionary” technology reaches economic break-even with the “Existing” one as a function of initial start-up costs (in 10,000 US\$ per kW) and learning rates (percent cost reduction per doubling of cumulative installed capacity). The vertical axis represents time. Low values indicate the technology becomes rapidly competitive (right-hand side, lower corner), high values (left- and right-hand side upper corners) indicate it never would become competitive. Note in particular the non-linear domain of the parameter space, where even small variations in parameter values result in quite different model outcomes. Our model achieves optimal investment solutions by sampling stochastically drawn samples in this parameter space around the mean value of the learning rate adopted.

(i.e. functions with long tails like Weibull or Gamma distributions) can make a difference in model outcomes. Even with the same variance, the model is sensitive to the existence of even extremely low-probability “outliers” parameter values. For instance, consider the technological “expectations” of future learning rates: even in adopting the same mean and variance as in the base case simulations but including the very small possibility of an extreme high learning rate yields a different model outcome. The model moves in direction of earlier and higher upfront investments into RD&D. Typically, the penetration curves (run 5 in *Figure 5*) is shifted to the left by one to two decades. Thus, if there is even a slight chance of ultimately doing better than in the base case simulation the model results in an accelerated innovation pattern. Obviously, the relationship goes in both ways: even a slight chance of much lower than expected learning (or even no learning at all) can mean delayed introduction of new technologies (or even their entire disappearance from the market).

Such model runs with long-tailed uncertainty distributions reflect also reality, especially for radically new technologies. Empirical distributions of future technological “expectations” frequently show slightly higher frequencies towards the extreme tails reflecting notorious technological “optimism” and “pessimism” (cf. *Figure 1* above). Taken together, the existence of such widely different expectations about future cost improvement (learning) potentials may enhance technological innovation, rather than hinder it, because innovation is usually carried out by agents who are optimistic about a particular technology. In this aspect our optimization model can in fact portray quite similar behaviors as simulation models developed within the framework of evolutionary economic theories (cf. the insightful model of Silverberg *et al.*, 1988).

### 5.3 Discount rates

The influence of the discount rate on any intertemporal optimization problem is evident and needs no further exposition. We have performed sensitivity analyses for discount rates of 3 and 7 percent in addition to the base case value of 5 percent reported above. Overall, technological change patterns in the model varied as a function of the discount rate. *Ceteris paribus*, higher discount rates result in postponed technological investment, experimentation and learning. This result was to be expected considering the decisive influence of the discount rate on the objective function. The most inter-

esting insight from this sensitivity analysis was obtained when combining high discount rates (7 percent) with uncertainty of technological learning. In this simulation experiment, the “Revolutionary” technology, which invariably appeared as a robust technological diversification strategy in all earlier simulations (albeit with different timing and market penetration profiles) did *not* make it to the market. In other words, high intertemporal discounting combined with high technological uncertainty favors “no change” technology strategies.

## 5.4 Uncertainty in demand

We also have explored the sensitivity of the model to uncertainties in demand. Because growth in demand is the result of complex interacting demographic, economic, and lifestyle forces we can expect its future evolution to be highly uncertain. Perhaps demand is even more uncertain than technological parameters. Hence the interest to explore its implications on technology RD&D strategies.

For the demand uncertainty analysis we adopt a somewhat different procedure. Instead of sampling within one singular uncertainty distribution around the mean expected value of a 13-fold increase between 1990 and the year 2100, we divide the uncertainty distribution into four subsamples (see *Figure 8*) and perform the stochastic sampling on basis of these subsamples. We do not assign relative probabilities to these four samples as our interest lies in examining different technology strategies that emerge from four distinct expectational domains of future demand. As previously, probabilities are assigned to draws *within* each of the four subsamples.

As a result we obtain four distinct solutions (technology trajectories) corresponding to alternative technology strategies in face of demand uncertainty. These are reported in *Figure 9* for the “Revolutionary” technology. For comparison we also show simulations with uncertain technology learning rates (around mean values of 30 and 40 percent respectively) and a simulation run with an uncertain emission tax (cf. the discussion in the next Section below).

*Figure 9* illustrates the wide variation in future diffusion pathways of the “Revolutionary” technology as a function of differences in rates of technological learning, demand, and environmental limits. Obviously, if the potential for technological learning is higher, then new technologies penetrate the market earlier (cf. the difference between the base case scenarios BC30 and BC40 in *Figure 9*). This is also the case if one is uncertain whether environmental

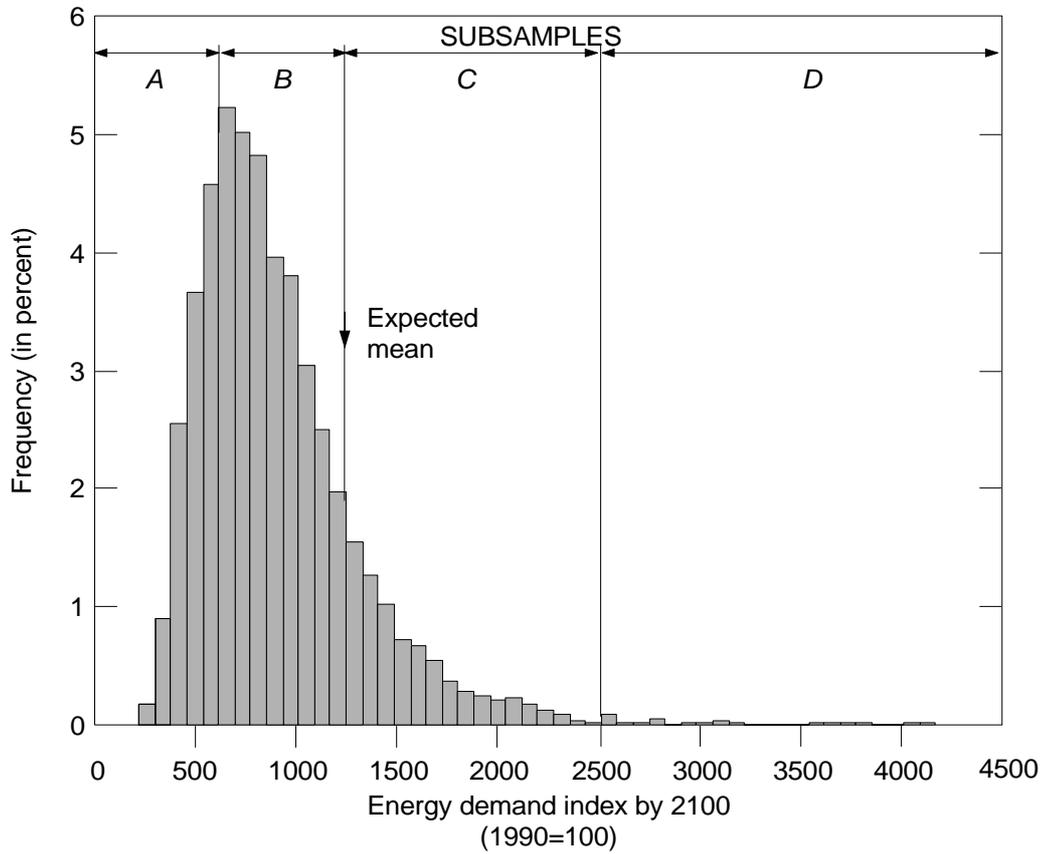


Figure 8: Uncertainty distribution of demand (index 1990 = 100) by 2100 around the mean value of 1250 which is used for analysis of model sensitivity to uncertainty in demand. Also shown are the four subsamples A, B, C, and D used for stochastic sampling. Samples B and C represent “normal” uncertainties below (sample B) and above (sample C) the expected mean value; samples A and D represent low probability possibilities that the demand could be vastly lower (A) or higher (D) than expected.

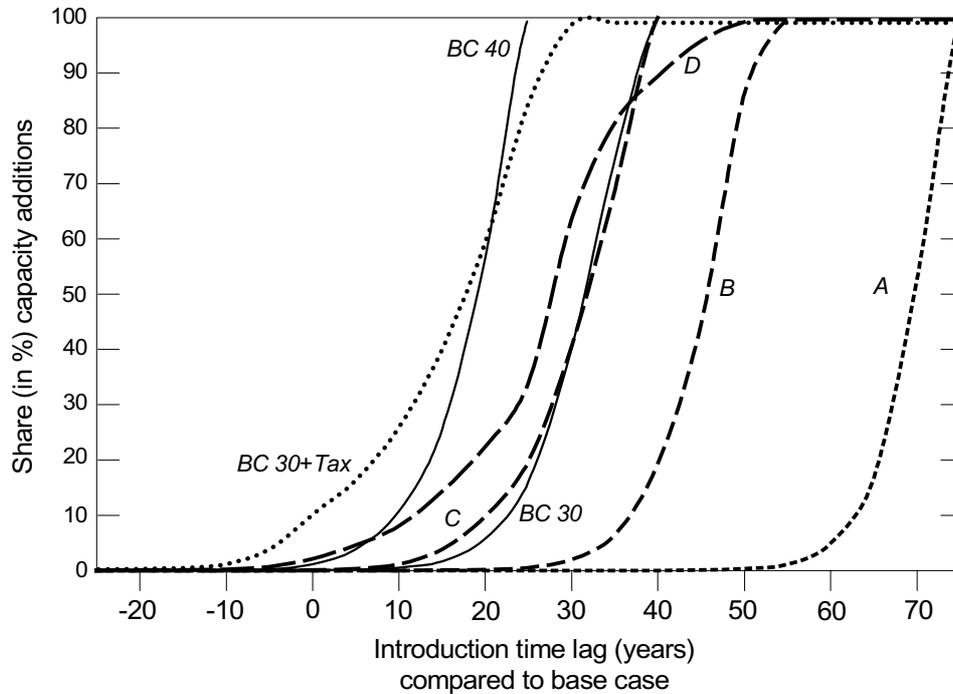


Figure 9: Shares in new capacity additions (in percent) of a “Revolutionary” technology in three different scenario classes:  
 solid lines: base case and sensitivity run (stochastic uncertainty with mean learning rate of 30 and 40 percent respectively) denoted as BC30 and BC40;  
 dashed lines: base case with additional uncertainty of demand for four domains of demand uncertainty denoted as A, B, C, and D (A and B represent different degrees of realized demand being lower than expected, C and D indicate domains where demand could turn out higher than expected);  
 dotted line: base case (BC30) with an uncertain environmental constraint (emission tax), denoted as BC30+Tax. The time axis shows the (positive or negative) diffusion lag (in years) compared to the base case (BC30) simulation.

constraints may appear in the future and how severe such constraints might be. Again RD&D (learning) and the consequent gradual increased diffusion of new technologies are optimal response strategies in face of uncertainty. The downside risk for RD&D is obviously the case when demand growth is much lower than expected. Although not as extreme as in the earlier sensitivity analysis with a much higher discount rate (in which radical technological change did not occur at all), the penetration of radical technologies is postponed. In *Figure 9*, the extreme low demand sample (run A) results in a five decade delay in the introduction of the “Revolutionary” technology, when compared to the base case (BC30) simulation.

The most interesting case in the simulations is that of the possibility that future demand is much higher than expected. The probability of extreme high demands (cf. subsample D in *Figure 8* above) is extremely low in our example. Nonetheless, facing this uncertainty, in addition to the uncertainty on the rate of technological learning, does not lead to any moratorium in RD&D and experimentation in radical technologies. Because of the possibility of technological learning, experimentation is the optimal response strategy in face of demand uncertainty on the upper end. It represents an adaptive strategy that allows satisfying even high demand with low cost technologies, despite accelerated resource depletion.

What are the implications of above model simulations for near- to medium-term investment strategies? *Figure 10* shows the actual time profiles of new installed capacity for the “Revolutionary” technology from the simulations reported in *Figure 9* above. Invariably, with higher potential for technological learning, higher future demand, or an uncertain environmental limit (the simulated carbon tax) investment profiles are shifted earlier. The time shift is shown in *Figure 10*, where the time profiles are renormalized to the base case simulation BC30 at  $t=0$ . Thus, if indeed such possibilities are within the realm of current policy concerns (and we feel that they are very much so), then the implications on technology strategy are the same as in our simple model: learn earlier, in order to prepare for later surprise.

## 5.5 An uncertain emission tax

Finally, let us address environmental issues as possible drivers of technological change. The existence, timing, and extent of possible future environmental constraints, e.g. in form of emission limits or taxes, are highly uncertain.

Consequently we study the model’s behavior when subjected to an un-

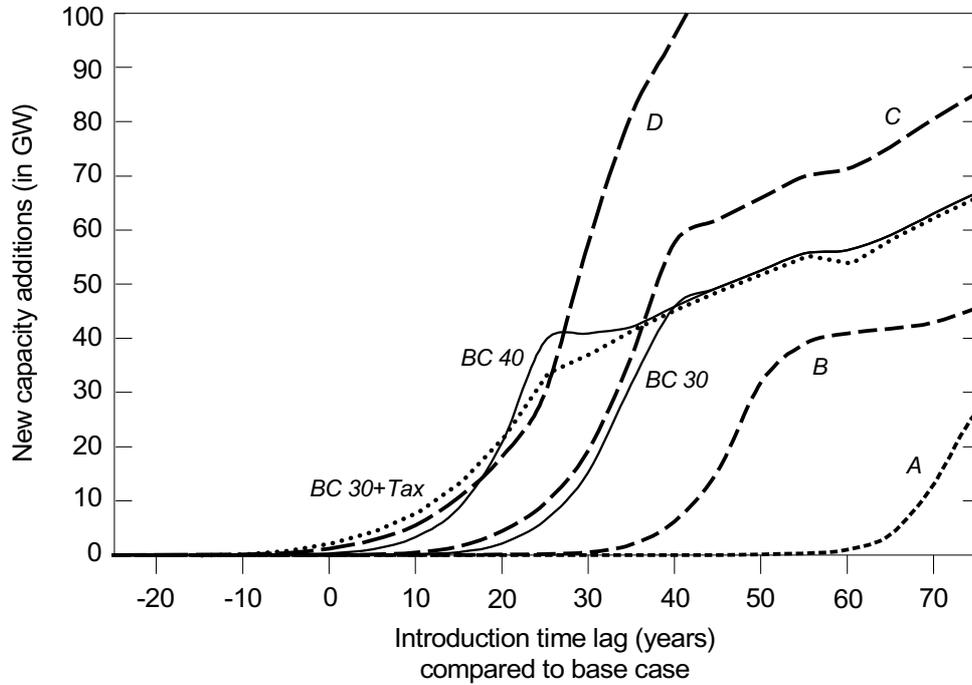


Figure 10: Results from *Figure 9* but shown as new capacity additions (GW) of a “Revolutionary” technology in three different scenario classes (cf. *Figure 9* above for a more detailed definition). The time axis shows the (positive or negative) diffusion lag (in years) compared to the base case (BC30) simulation.

Solid lines: base case (uncertain learning rates with mean of 30 and 40 percent respectively) denoted BC30 and BC40;

Dashed lines: base case with additional uncertainty of demand (A and B represent simulations in which realized demand could turn out lower than expected, in C and D demand could turn out much higher);

Dotted line: base case (BC30) with uncertain emissions tax (BC30+Tax).

certain environmental constraint. The existence, magnitude, and the timing of the constraint are treated as uncertain. This is done as follows: First we assume a cumulative probability distribution of the occurrence of the emission tax over the entire time horizon. Starting (at quasi zero) in the year 1990, the probability that an emission tax (for instance on emissions of carbon dioxide) is implemented or not increases over time (i.e. uncertainty about occurrence and timing is reduced). For instance, in the illustrative distribution function used for this sensitivity analysis we assume that there is a one third chance that some tax would be implemented sometime in the future (and hence a two thirds chance that it would not). In conformity with the problem at hand (even though it complicates our computations) we also consider the timing of introduction and the absolute amount of the emission tax to be highly uncertain. Concerning timing, we assume a probability of introduction increasing to 50 percent by 2050, and to 99 percent by the year 2100 (*if* the tax is established *at all*). Concerning the absolute level of the emission tax, we also assume a distribution with a very small probability of an unpleasant surprise (i.e. a relatively high tax level). Formally this is done by drawing a Weibull distribution around the mean expected value of the tax, set at US\$50 per ton (carbon)<sup>22</sup> with a 99 percent probability that it would not exceed US\$125 per ton (C).

The result of the introduction of this additional uncertainty on the technology diffusion of the “Revolutionary” technology is reported in *Figure 9* above. The existence of a possible environmental constraint alters the patterns of technological change substantially. Again, RD&D and investments that enable subsequent technological learning are shifted earlier in time to prepare for the possibility of facing a costly future environmental constraint. In that respect, the possibility of an environmental constraint yields similar patterns as the possibility that future demand could be much higher than expected, or that learning rates might be higher than anticipated. In all three cases, that represent the most important unknowns for the energy sector, earlier RD&D and investments are the optimal response strategy. Short-term investments into RD&D in new technologies are higher in order to stimulate learning, even if these new technologies ultimately penetrate on a massive scale only many decades in the future. In essence our model results indicate

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<sup>22</sup>For illustrative purposes we assume conversion efficiencies and carbon emissions per unit output (electricity) representative for conventional and advanced coal systems (“Existing” and “Incremental” technologies of our model) and of solar PVs as example of a “Revolutionary” technology.

as optimal strategy *vis à vis* future contingencies: “get prepared as early as possible for potential surprises that might strike later.”

We conclude this overview of our model runs by pointing out a final potential “surprise” emerging from our model runs. Allowing for technological learning in an endogenous model of technological change could result in pronounced discontinuities of future emission levels. These in fact might drop substantially, not through an exogenous “shock” such as taxation or emission limits, but through the endogenous dynamics of technological change (cf. *Figure 11*). Such a view is obviously in stark contrast to the typical “business as usual” emission trajectories, which embrace either a static or incrementalist technological change perspective. Our model results strongly suggest that this divergence in future emission pathways might be not only an issue of uncertainty of the future *per se* (e.g. of resource availability), but also how technological change is represented in models: exogenous, or endogenous.<sup>23</sup>

## 6 Conclusion

We have developed a model of endogenous technological change, which is driven by expectations of uncertain returns from investments into research, development, demonstration, and commercialization of new technologies. Such technologies are initially unattractive, but they offer (uncertain) potential for future improvements. Lower costs, once realized, allow widespread adoption, i.e. technology diffusion. As in the real world, investments result in (uncertain) technological learning and are a main driver of technological change.

Our model represents technological change as resulting from the (rational) strategies of economic agents that know that technological change does not come as a free good. Rather, improved technologies require dedicated efforts and expenditures, and agents act accordingly. Albeit they remain uncertain about the ultimate outcome of their strategies (i.e. there is a difference between “technological expectations” and the ultimately realized technological learning). In this sense, technological change arises out of the “bounded” economic rationality of pursuing technological R&D *and* invest-

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<sup>23</sup>Of course we can also imagine endogenous technological change trajectories leading to higher emissions rather than lower ones –for example, energy-intensive hypersonic or space travel.

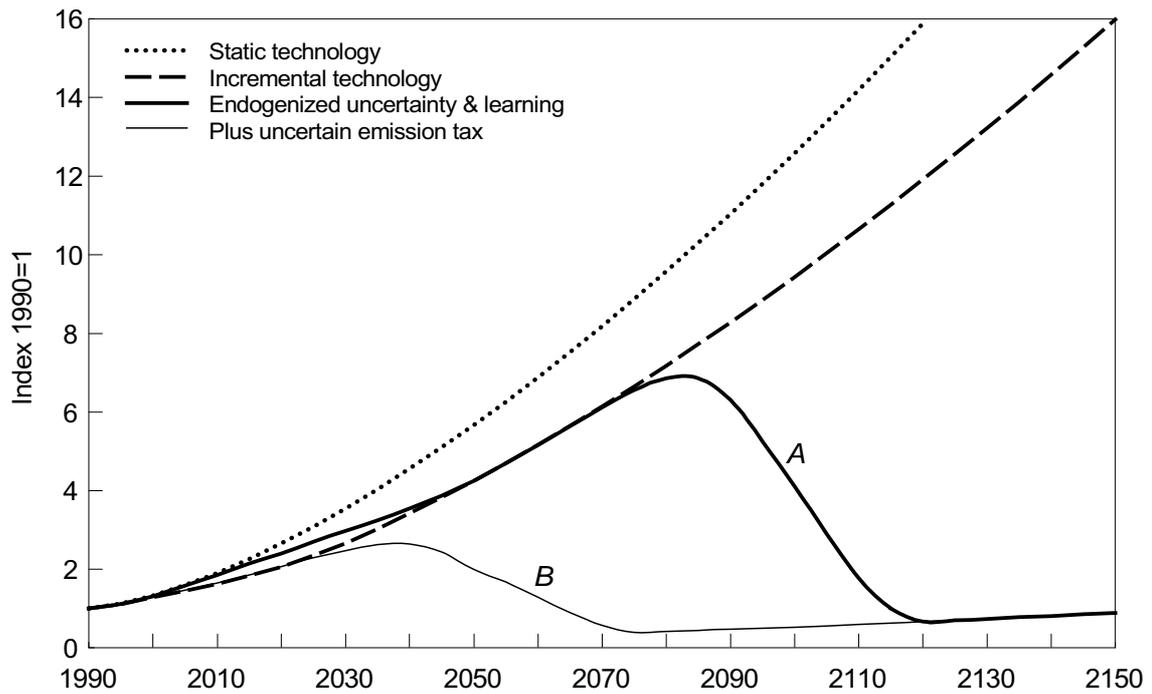


Figure 11: Carbon Emissions of a one-region, three technology model (including a currently expensive, but potentially promising zero-carbon option). Emissions (index, 1990=1) resulting from alternative representations of technological change (all other model specifications are identical): Static technology (dotted line), incremental technological change (dashed line), full endogenized uncertainty and learning (solid line A), combined with an uncertain emission tax (solid line B).

ments in anticipation of future returns in form of performance improvements, cost reductions, etc. that are also main drivers of technology diffusion. As in the real world, uncertainty in outcomes of (returns to) of R&D and investment efforts is a key feature governing technological evolution.

At the analytical level, we were able to demonstrate a model formulation involving simultaneously increasing returns and uncertainty within an intertemporal optimization framework. The most important result is the demonstration of an entirely endogenous mechanism of technological change. In other words, we could show that the non-convex, non-smooth, stochastic optimization problem resulting from stochasticity (uncertainty) *and* increasing returns (learning) combined, is solvable. Moreover, the patterns of technological change and diffusion exhibited by the model are consistent with those observed historically and in the diffusion literature. As such, our model responds to the frequent (and justified) critique of diffusion studies and models as being phenomenological, lacking a clear endogenous causality mechanism.

In model runs with plausible patterns of uncertainty and technological learning, technologies that are economically unattractive today (e.g. a factor 40 higher costs) nonetheless diffuse into the market within 4–5 decades. Such diffusion is economically optimal, but requires upfront investments into R&D, demonstration (niche markets), and gradually into expanding commercial investments, all of which lead to pervasive diffusion. These upfront investments initiate a process of technological improvements and cost reductions (learning) that is further sustained during subsequent pervasive diffusion.

Our model results also show, that investments into technological learning (RD&D) constitutes an optimal contingency strategy *vis à vis* uncertainty in future demand and the possible emergence of environmental regulations.

After demonstrating the feasibility of a model of endogenous technological change, much remains to be done on both the conceptual and modeling levels. Clearly, the highly stylized structure of the model must be expanded to, at least rudimentarily, resemble the complexity of existing technological systems. This constitutes a prerequisite also to study in more detail the critical issue of technological *interdependence*, i.e. technological change in one domain (e.g. hydrogen cars) is insufficient if not accompanied by corresponding changes in other technologies (e.g. hydrogen production, transport, and distribution infrastructures). The complex issues of spillovers and learning externalities –such as advances in general scientific knowledge or the possibility of “free riding” on someone else’s learning efforts– also have not been

addressed by our simple, one actor model. In a next step a multi-region, multi-actor model will be required to study the models behavior in the presence of technological interdependence and positive or negative spillovers.

Despite its simplifications and limitations our model of endogenized technological change offers several important insights that might be relevant to policy. All are intuitive but have become corroborated through model simulations. Foremost is the conclusion never to dismiss the market potential for a new technology based on its current state (e.g. costs). Through appropriate strategies (i.e. learning) the costs and performance of technologies can change drastically, as amply illustrated by technology history and also our model simulations. Uncertainty is an important driver than can retard change, or, rather lead to a more cautious and gradual learning strategy and resulting investment profile. High uncertainty, combined with low expectations of learning potential can even stall radical change altogether, as does using a high discount rate of intertemporal choice.

However, there are also cases in which higher uncertainty can lead to accelerated technological learning as a robust and low cost hedging, or contingency strategy *vis à vis* extreme outcomes. Our results have indicated, for example that surprises in form of much higher demand than expected or (uncertain) environmental limits lead to strategies of early learning rather than delay. It is in this domain that our model results may contribute also to the current policy debate. We know that our knowledge on the future evolution of demand, be it for energy, raw materials, food, or environmental amenities is extremely uncertain. We also remain ignorant even about most basic drivers, such as how many people will inhabit Planet Earth some 100 years from now. No model can help to resolve these fundamental uncertainties of the future. But a view of endogenized technological change can yield some insights into possible strategies of how we can prepare for such contingencies. From the perspective of the results obtained with our model the answer is: *invest*. Invest in R&D, demonstration (niche markets), in gradually expanding commercial markets, preparing pervasive diffusion. Such investments strictly entail acting sooner than later. Absent investment, the technological change needed to face future contingencies will not occur.

*“Unlike resources found in nature, technology is a manmade resource whose abundance can be continuously increased, and whose importance in determining the world’s future is also increasing”* (Starr and Rudman, 1973).

## Acknowledgments

We gratefully acknowledge the contributions of our IIASA colleagues Yuri Ermoliev and Sabine Messner. This model evolved and the optimization problem became solvable through their continuous advice and numerous helpful suggestions. Its shortcomings remain ours. We have also benefited from comments and suggestions by David Victor and Nebojša Nakićenović on earlier drafts of this paper. Its shortcomings remain ours too.

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## Mathematical Appendix

The model contains three technologies  $i$ ,  $i = 1, 2, 3$ , that share a common single resource, whose extraction costs increase over time. Total annual production of all three technologies must satisfy an exogenously given demand

$$D(t) \leq \sum_i X_i(t), \quad i = 1, 2, 3,$$

where  $D(t)$  is demand at time  $t$ ,  $X_i$  is the annual production of technology  $i$  and the time horizon extends from 1990 to 2200. Capacity constraints assure that annual production for each technology does not exceed installed capacity (100% capacity utilization is assumed).

$$X_i(t) \leq C_i(t),$$

$C_i$  is the total installed capacity of technology  $i$ . Capacities are built up by annual new installations

$$C_i(t) = \int_{t-\tau_i}^t Y_i(\tau) d\tau,$$

where  $Y_i$  is the annual new installation of technology  $i$  and  $\tau_i$  is the plant life of technology  $i$ .  $\tau_i$  is assumed (with 30 years) equal for all technologies.

The input-output relation for technologies is given by an efficiency rate (of 30, 40, and 90 percent conversion efficiency respectively). All three technologies share the same input resource. Annual extraction is sum of resources consumed by each technology

$$R(t) = \sum_i \frac{1}{\eta_i} X_i(t),$$

where  $\eta_i$  is the conversion efficiency of technology  $i$ . Extraction costs are a function of cumulative extraction  $y$

$$\begin{aligned} c^R(t) &= f(\bar{c}^R(t)), \\ \bar{c}^R &= \int_{-\infty}^t R(\tau) d\tau \equiv \int_{1990}^t R(\tau) d\tau + \bar{c}(1990). \end{aligned}$$

#### *Technological learning*

In the deterministic case future (investment) costs for technologies are a function of cumulative installed capacity

$$c_i^I(t) = A_i \bar{C}_i(t)^{-b_i},$$

where  $\bar{C}_i(t)$  is the cumulative installed capacity of technology  $i$  by time  $t$ .

$$\bar{C}_i(t) = \int_{-\infty}^t C_i(\tau) d\tau \equiv \int_{1990}^t C_i(\tau) d\tau + \bar{C}_i(1990),$$

$b_i$  is the progress ratio ( $1 - 2^{-b_i}$ , is the learning rate, expressed in percent cost reduction per doubling of cumulative capacity) and  $A_i$  is the initial investment cost at 1GW.

In the stochastic case the progress ratio  $b$  is assumed to be a random value  $\beta$  with a given distribution function<sup>24</sup>. As a result, future investment costs are a random function of  $\beta$ .

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<sup>24</sup>A distribution is completely specified when its distribution function is given. For more details see Devroye, 1986.

$$c_i^I(t) = A\bar{C}_i(t)^{-\beta}.$$

*Objective function.*

The objective function is the sum of three components

$$\begin{aligned} \mathbf{E} \left\{ \int_{1990}^{2200} c_i^I(\tau; \omega) Y_i(\tau) d\tau \right\} + \int_{1990}^{2200} \delta(\tau) \left( c^R(\tau) R(\tau) + c_i^{OM} X_i(\tau) \right) d\tau + \\ + \rho \int_{1990}^{2200} \delta(\tau) \mathbf{E} \max \left\{ 0, \left[ \mathbf{E} c_i^I(\tau; \omega) - c_i^I(\tau; \omega) \right]^2 Y_i(\tau) \right\} d\tau - \\ - \int_{1990}^{2200} \delta(\tau) \mathbf{E} \min \left\{ 0, \left[ \mathbf{E} c_i^I(\tau; \omega) - c_i^I(\tau; \omega) \right] Y_i(\tau) \right\} d\tau \rightarrow \mathbf{min} \end{aligned}$$

where  $\delta(t)$  is the discount rate; and  $t, c_i^{OM}$  are the specific O+M cost of technology  $i$  (assumed to be constant over time).  $\mathbf{E}$  stands for expectation, with  $\omega$  being an element from a probability space. Part two represents the risk associated with overestimating learning rates (future investment costs of a technology are higher than expected) with its associated risk factor  $\rho$ . Part three represents the benefits in case of underestimating the learning rate (future investment costs of a technology are lower than expected). As shown above, we apply a quadratic formulation for the risk term and a linear one for the benefit term as one of many possible formulations.<sup>25</sup> Note that in the deterministic formulation parts two and three of the objective function given above do not appear.

In the model runs defined above, the objective function is substituted by a discrete time formulation. The resulting stochastic optimization problem is solved on the basis of simultaneous approximation of the random function by  $N$  sample functions with sufficiently large  $N$  (see Messner *et al.*, 1996).

### *Carbon tax*

The uncertain emergence of a carbon emission tax is modeled in the following way. We assume that the eventual establishment of the tax is uncertain with a given occurrence probability of 0.33 (i.e. there is a chance of one out of

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<sup>25</sup>F. the sensitivity model run 12 below in which we test an alternative formulation where the benefits part entering the objective function is assumed as a quadratic term and the risk part as a linear term.

three that a tax would be established at all). The introduction time (in case the tax would be established) is also unknown with an expected cumulative distribution function that goes from 0 in 1990 to 50% in 2050 reaching 99% by 2100. In the model runs a Weibull distribution around a mean tax value of 50 \$/tC and probability of 99 percent of the tax being lower than 125 \$/tC was assumed. The carbon tax is added to the objective function

$$p^{tax} \left\{ \int_{1990}^{2200} p^{\tau} \left\{ \begin{array}{l} \mathbf{E} \left( c^c(\omega) \frac{\mu_i}{\eta_i} X_i(\tau) \right) + \\ \rho \mathbf{E} \max \left\{ 0, [\mathbf{E} c^c(\omega) - c^c(\omega)]^2 \frac{\mu_i}{\eta_i} X_i(\tau) \right\} - \\ \mathbf{E} \min \left\{ 0, [\mathbf{E} c^c(\omega) - c^c(\omega)] \frac{\mu_i}{\eta_i} X_i(\tau) \right\} \end{array} \right\} d\tau \right\},$$

where  $p^{tax}$  is the probability that the tax will be established at all;  $p^t$  is the probability that, if established, the tax will be introduced before time  $t$ ;  $c^c$  is the uncertain carbon tax value and  $\mu_i$  are the emissions of technology  $i$ , in tons elemental carbon ( $C$ ) per unit input.

The following summary gives an overview of the assumptions and model parameter values used for the model simulations and sensitivity analyses.

#### Technologies $i$ :

- $i = 1$  – “existing” technology,
- $i = 2$  – “incremental” technology,
- $i = 3$  – “revolutionary” technology.

#### Demand $D(t)$ :

*Base Case*

$$D(t) = 100 + (t - 1990)^{1.5}.$$

*Linear demand growth*

$$D(t) = 100 + 14.4914(t - 1990).$$

*Unknown demand*

$$D(t) = 100 + (t - 1990)^{\xi},$$

where  $\xi$  is a normally distributed random value with mean 1.4 and variance  $\sigma^2, \sigma = 0.1$ .

**Life time  $\tau_i$ :**

$$\tau_i = 30, i = 1, 2, 3.$$

**Efficiencies  $\eta_i$ :**

$$\eta_1 = 0.3, \eta_2 = 0.4, \eta_3 = 0.9.$$

**Resource cost  $C^R(t)$ :**

$$c^R(t) = \min \left[ 1000, 16.97 \exp \left( 16 \bar{c}^R(t) / 10^6 \right) \right]$$

based on a quantity-cost supply curve for coal resources suggested by Rogner, 1996. For our model simulations we limit the maximum extraction costs to 1000 \$/t.

**Learning rates for investment cost  $b_i$  and  $\beta_i$  :**

*Deterministic case*

$$b_1 = 0, \text{ no learning,}$$

$$b_2 = 0.152003,$$

10% cost reduction per doubling of installed capacity,

$$b_3 = 0.514573,$$

30% cost reduction per doubling of installed capacity.

*Stochastic case*

$$\mathbf{E}\beta_1 = 0, \mathbf{Var}\beta_1 = 0, \text{ no learning,}$$

$$\mathbf{E}\beta_2 = 0.152003, \mathbf{Var}\beta_2 = (0.1 \mathbf{E} \beta_2)^2,$$

$$\mathbf{E}\beta_3 = 0.514573, \mathbf{Var}\beta_3 = (0.1 \mathbf{E} \beta_3)^2.$$

**Initial investment costs  $A_i$ :**

$$A_i = 1000\$/kW,$$

$$A_2 = 2000\$/kW,$$

$$A_3 = 40000\$/kW.$$

**Operation and maintenance (O+M) costs  $c_i^{OM}$ :**

$$c_1^{OM} = 30\$/kW, c_2^{OM} = C_3^{OM} = 50\$/kW .$$

**Carbon emission coefficients  $\mu_i$ :**

$$\mu_1 = \mu_2 = 0.8, \mu_3 = 0.1 .$$

**Distribution function for carbon tax introduction date** under condition that carbon tax will be established sometime in the future we assume a lognormal distribution function with mean 2050 and  $\delta = 0.1$ .

**Distribution function for carbon tax value** We assume a Weibull distribution with mean of 50\$/tC and standard deviation of 25. This means that with probability 99470 \$/tC.

**Optimization techniques and implementation** Due to the rather complicated nature of the problem and the experimental status of suggested approach, MATLAB was chosen as a main platform for model implementation and testing. Also a number of additional mathematical tools, like Mathematica and Maple, were used in order to analyze the problem and test alternative approaches and part of the optimization code. For the global optimization search procedure a generic method similar to the approach suggested by Saltjanis (1989) and Törn and Zilinskas (1989) was used. A partially re-designed code from Dolezal *et al.* (1991) was adapted for the MATLAB environment. Results from the global search step were then used as starting points for a local optimization procedure based on a Broyden-Davidon-Fletcher-Powell algorithm (see Broyden, 1967, and Johnson, 1976). This procedure switches to a Nelder-Mead type of algorithm (see Himmelblau, 1972) in cases when the approximation of derivatives become numerically too imprecise as a result of the stochastic approximation techniques involved.

**Sensitivity Analysis** The following table summarizes altogether 22 simulation runs and sensitivity cases performed. Main parameter values assumed as well as model results are summarized. As a simple common metric we report model results in measuring the changing diffusion patterns for two technologies: the time when the “incremental” technology reaches 50 percent market share in annual new installations, and the

time the “revolutionary” technology reaches 0.5 and 50 percent market share in new installations respectively. Note that due to the highly non-linear nature of the model, dates for reaching these market shares are in some simulations only approximative

N	Case	Year when technology will reach new installations share of		
		50% - "Incremental"	0.5 % - "Revolutionary"	50 % - "Revolutionary"
<i>Learning rates</i>				
1	No learning ( $b_i=0$ )	2040 (jumps from 0 to 100%)	Never	Never
2	Exogenous for incremental (linear declining from the value 2000 to 1000 by 2050, and constant after that)	2025 (jumps from 0 to 100%)	Never	Never
3	Deterministic learning, but just for "Incremental" technology ( $b_2=10\%$ , $b_3=0$ )	1995	Never	Never
4	Stochastic learning but just for "Revolutionary" technology ( $Eb_2=10\%$ , $b_3=0$ )	2020	Never	Never
5	Deterministic learning base case ( $b_2=10\%$ , $b_3=30\%$ )	1995	2020	2070 (jumps from 32% to 100%)
6	Deterministic learning for both technologies, but with 40% for "Revolutionary" ( $b_2=10\%$ , $b_3=40\%$ )	2015	2045	2070
7	Stochastic learning base case ( $Eb_2=10\%$ , $Eb_3=30\%$ )	2010	2060	2080
8	Stochastic with 5% vs. 10% in base case for $\sigma$ of "Revolution" technology ( $Eb_2=10\%$ , $Eb_3=30\%$ )	2010	2050	2070
9	Stochastic with 20% vs. 10% in base case for $\sigma$ of "Revolution" technology ( $Eb_2=10\%$ , $Eb_3=30\%$ )	2010	2075	2105
10	Stochastic learning with gamma distribution vs. normal in base case ( $Eb_2=10\%$ , $Eb_3=30\%$ )	2005	2055	2070

N	Case	Year when technology will reach new installations share of		
		50% - "Incremental"	0.5 % - "Revolutionary"	50 % - "Revolutionary"
11	Stochastic learning with Weibull distribution vs. normal in base case (Eb2=10%, Eb3=30%)	2005	2075	2105
12	Stochastic learning with "inverted" objective function - quadratic for benefits and linear for losses	2000	2050	2070
<i>Discount rate</i>				
13	Deterministic base case with 3% discount	1990	2010	2035 (jumps from 42% to 100%)
14	Stochastic base case with 3% discount	1990	2050	2105
15	Deterministic base case with 7% discount	2015 (jumps from 34% to 100%)	2065	2085
16	Stochastic base case with 7% discount	2020 (jumps from 32% to 100%)	Due to the fact that the objective function is very flat near global optimum in this case, the number is very sensitive to stochastic approximation use and vary from one run to another	
<i>Carbon tax</i>				
17	Stochastic base case with stochastic tax (see Mathematical Appendix for details)	between 2000 and 2005	2035	2070
<i>Stochastic demand</i>				
18	Stochastic base case, but with demand as a random function $D(t)=100+(t-1990)^\gamma$ . $\gamma$ is normally distributed with $E\gamma=1.4$ and $\sigma=0.1$	2010	2055	2085
19	As in case 17, but with additional constraint $D(2100)<625$	2020	2100	2120 (jumps from 50% to 100%)
20	As in case 17, but with additional constraint $625<D(2100)<1250$	2015	2075	2095
21	As in case 17, but with additional constraint $1250<D(2100)<2500$	2005	50	2055
22	As in case 17, but with additional constraint $D(2100)>2500$	2000	2045	2080