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AN INTERACTIVE MODEL FOR DETERMINING  
COAL COSTS FOR A CO<sub>2</sub>-GAME

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## PREFACE

There are growing concerns that human activities may lead to global climatic changes. Particular concern is associated with the release of carbon dioxide into the atmosphere, in the future above all from the burning of coal. Questions of the physical effects of different energy policies on climate have been investigated during the last few years under IIASA's Energy Systems Program. More recently, research in the Resources and Environment (REN) Area of IIASA has focused on the relationships of short-term climatic variability and longer term climatic change to human activities, for example, in the agricultural sector. In March of 1980, informal discussions among Jesse Ausubel, and Ingolf Ståhl, John Lathrop and Jennifer Robinson of the Management and Technology (MMT) Area led to the idea that gaming might offer an integrative method for study of the overall problem, from causes, through physical changes, to environmental, economic, and societal effects. At present a collaborative effort is underway between REN and MMT to develop two prototype games, one a board game with primarily an educational purpose, and one an interactive computer game which is seen primarily as a research tool. The overall project is described in a Working Paper entitled "Carbon and Climate Gaming" (WP-80-152). Another Working Paper, "CO<sub>2</sub>: An Introduction and Possible Board Game" (WP-80-153), outlines the CO<sub>2</sub> problem in simple terms and describes a basic framework for a board game. This Working Paper describes the present status of the work on one essential part of the computer game.

## ABSTRACT

A question of great interest in assessing future energy options is whether the burning of carbon, in the future mainly coal, will continue increasing so that the level of CO<sub>2</sub> in the atmosphere rises significantly, perhaps doubling by the middle of the next century. It is widely believed that such an increase in atmospheric CO<sub>2</sub> would lead to an unprecedented warming of the earth's climate and possibly severe consequences for the economy and environment. A project, called Carbon and Climate Gaming, has recently been started at the International Institute for Applied Systems Analysis to examine this issue. An important part of the project deals with the construction of a computer-based game focused on the extraction, trade, and burning of coal during the next half century. The game aims at investigating whether different nations will pursue independent myopic energy policies, leading to a potential "tragedy of the commons," or whether there will be some sort of international cooperation to avoid drastic climatic changes.

For the game it is important to have forecasts of how the costs of extracting coal will develop in various countries over time, dependent on both the actual and the cumulative production quantities. Since these cost functions should appear reasonable to the players of the game it is desirable that the players themselves can, in a short time, construct or revise these functions. This can be done by the computer dialogue system presented here. By answering approximately thirty questions a player determines the parameters of the cost model. Since the player continuously obtains feedback about the implications of his answers and then can revise them, the dialogue can continue until the player obtains a total cost function that appears reasonable.

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Ingolf Ståhl

1. INTRODUCTION

At IIASA, the International Institute for Applied Systems Analysis, there has been considerable research in the field of energy. Among the many topics covered are the future use of coal and the relation between carbon dioxide emission from the combustion of coal and changes in climate.\* One question is, what would be the effects on the climate if the CO<sub>2</sub> contents in the atmosphere were doubled?

Some of the research has indicated that such a doubling of CO<sub>2</sub> might take place a little more than half a century from now, due mainly to the possible rapid increase in the combustion of coal. It could possibly lead to a general increase of global temperature of a couple of degrees, leading, e.g., to a substantial change in conditions for agricultural production in some countries. It should, however, be stressed that there is still a great uncertainty both regarding how much coal will be combusted and what the effects will be of various levels of CO<sub>2</sub> emissions.

A project with a new focus on these two issues has recently been started at IIASA: Carbon and Climate gaming.

The project is a joint effort by Jesse Ausubel, John Lathrop, Jennifer Robinson and the author. The first section of this paper in particular relies heavily on the input of the other members of this team.

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For example, see IIASA 1981 for a general survey of IIASA energy research. For specific information on CO<sub>2</sub> see Williams 1978 and on coal see Grenon 1979.

The project aims at producing two games: One board game with a wide educational purpose and one computer game.

At least in its more developed stages, the computer game will be a research tool intended to raise and to give some very preliminary answers to specific questions about the CO<sub>2</sub> issue. For example, will potentially threatening levels of CO<sub>2</sub> be created or not? What is a likely range of total accumulated CO<sub>2</sub> emissions? If created, what kind of a global market does a CO<sub>2</sub> problem presuppose? Will it be possible for the big coal producing nations to form and enforce some sort of cartel? As the atmospheric CO<sub>2</sub> content increases, will the interest become stronger in control strategies and will strategies of reducing carbon extraction, trade or emissions be preferred? An important question is whether there are institutional scenarios (treaties, cartels and so forth) which will help to avoid the "Tragedy of the Commons" outcome of the CO<sub>2</sub> problem.

Obviously, the answers to these questions will be dependent on the specific character of the game, including the data base used. However, the game will be oriented toward indicating what scenarios are more likely given various information and institutional arrangements. The questions and the tentative answers will be intended mainly to serve as a basis for future discussions both with regard to what kind of research is most urgently needed and which outcome scenarios are acceptable to various interested groups.

The computer game focuses on coal, trade and many countries.

Why Coal: The main cause of the problem in the long run as regards the release of carbon dioxide is the burning of coal. Coal is likely to account for two-thirds or more of the emissions in a scenario of serious climatic change. In fact, present estimates of total resources of oil, gas, coal, and other forms of carbon indicate that atmospheric carbon dioxide levels regarded by some experts as critical (for example, a doubling of the present level within the next century) can only be reached by very substantial burning of coal.\* Other carbon resources are simply not available in large enough quantities. Because coal plays this critical role in the CO<sub>2</sub> issue, it is logical to begin game development with emphasis on coal.

Why Trade? About 80% of the coal deposits of the world are in the hands of three big countries: The USSR, the USA, and China. Thus, in discussing possibly dangerous levels of CO<sub>2</sub>, one can conclude in theory that if these three large players do not export any coal and also keep their own coal combustion low, a severe CO<sub>2</sub> emission problem will not arise. However, by far the largest part of future potential coal combustion lies in the world outside of these three players. Much of this coal would come from imports over a long period of time. Hence, the main CO<sub>2</sub> emission threat arises from scenarios, like the one that can be projected from the recent MIT World Coal Study, (1980), where roughly a ten-fold increase

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\*See Table 1, Appendix B (p. 21).

in world coal trade is envisaged. The trade in coal is also of interest in connection with different schemes of international cooperation to reduce or prevent CO<sub>2</sub> emissions. The possibility for the larger countries to limit supplies of coal either on the world market or to specific countries can give "teeth" to attempts at enforcing international trade in coal. This feature is important when discussing whether coal prices will be cartelistic and thus high, discouraging the use of coal, or more formed by competition and thus cost-based, possibly leading to a rapid increase in combustion. The game will attempt to capture the essential aspects of a world coal market as it relates to the CO<sub>2</sub> problem while avoiding the considerable complexities of a detailed market simulation.

Why Many Countries? The computer game will try to represent a world where many countries, acting independently, affect the problem. The first reason for this is that a major portion of energy consumption will be taking place outside of the three big countries in a great many smaller countries. These can act independently and use this independence to their own advantage. Secondly, even if the three big players account for around 80% of total coal resources, the resources of some smaller holders are large from an absolute point of view. Around a dozen countries have probable resources that alone could lead to a level of emissions of the same size as total global emissions during the whole of the last decade.\* Ultimately, one would probably wish to include about twenty countries of different sizes to catch fully the strategic problem. If we limit ourselves to only a handful of actors in all phases of development of the game, we would exclude certain scenarios where international cooperation is impeded by the actions of several relatively small countries.

The playing of this game would take place both at IIASA and outside IIASA, first with scientists, and then with visitors in connection with IIASA workshops on related topics, such as energy policy, environmental protection, etc. Outside of IIASA, the game would be played with interested groups of people from government, industry and academic communities of various countries.

In order to have the computer game played frequently with such persons engaged in energy policy it must be of a convenient duration, for example 3 hours. Allowing for about ten rounds in a game, each round, therefore, calls for only a small number of decisions by each player. The actions of each player at each round of the game include mainly a coal extraction decision, a coal trade decision (supply or demand), a decision on the total amount of energy consumed (implying a certain level of CO<sub>2</sub> emission), and a decision relating to emission control. After market clearing calculations at the end of each round, players are informed about the price of coal, their status as regards coal extraction and coal trade, their own CO<sub>2</sub> emission in the world, as well as their present "welfare" measured in the form of an index. In the early stages of the game, welfare would be largely a function of the size of coal combustion. In the later stages of the game, however, effects of global environmental change would begin significantly to

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\*See Table 2, Appendix B (p. 22).

affect in varying ways the welfare of individual players, depending on the accumulated level of atmospheric CO<sub>2</sub>.

The construction of the computer-based game is to take place in several stages. In the first versions of the game there would only be human players involved. Since the game has to be administratively simple only a limited number of human players can participate. Thus, less than ten countries can be studied in such a manual game. This limitation causes an important problem, since, as noted above, we are ultimately interested in studying a world with many more countries acting independently. A preliminary plan for taking care of this problem is to design a game which can take advantage of the computer's capacity to simulate additional players. This computer-based game might thus have the following form. It would include the three big countries (USSR, USA, China) and four smaller countries. The roles of these seven countries would be played by humans. Besides this, the playing of some ten or more countries would be simulated by the computer. These "robot players" would act partly in the way that the four smaller-country human players acted earlier in this game or in previous games. The important thing is that the action of each small player will, at the moment, seem to him not to be significant to the total outcome.

## 2. REASONS FOR THE INTERACTIVE SYSTEM

As mentioned, one of the fundamental aspects of the game is to focus on the price of coal, which to a large extent will be dependent on the coal extraction cost functions of the various countries. For instance, the question of whether a coal cartel can develop or not will be dependent on whether small countries can fairly rapidly expand their output. Whether this in turn is feasible will depend on how steeply the coal extraction costs of these countries rise with increasing extraction. Hence it appears reasonable to devote a significant share of our game construction efforts to the estimation of the coal extraction cost functions.

It should be stressed that these estimates are of a special nature for several reasons.

The forecasts are very long term. The game will concern at least fifty years of time since it is after the year 2000 that the total CO<sub>2</sub>-contents in the atmosphere could possibly reach such levels that there could be a significant impact on human conditions.

We need, as mentioned, estimates for several (roughly a score of) individual countries.

The forecasts have to be functions, i.e., dependent not only on time, i.e., year of extraction, but also on the quantity extracted. These forecasted functions are made for the specific purpose of the game outlined above. If they also, as a byproduct, are of interest independent of the game it would be welcome, but not specially strived for.



In the game one can in particular envisage the following uses of these coal cost extraction functions.

a) For the computation in each round of total results due to the decision on a specific coal extraction quantity. This is obviously the most important use of the function. The computer would, on the basis of coal extraction, coal trade and coal burning compute the change in welfare level (some sort of adjusted GNP) for the country.

b) For information to each player in each round prior to making his decision on coal extraction. Each player could then from the computer obtain a table indicating what his total extraction cost would be at various levels of extraction.

c) For information to each player at the start of the game. As discussed further below, the extraction costs depend on how well the coal seams are located. This in turn depends on how much coal has been extracted up to the time of the decision. Hence the most suitable form of representing future coal costs appear to be to project how extraction costs develop, given an initial mining quantity (for 1980) and a fixed annual percentage change (possibly 0) of the extraction rate.

Since the game deals with a kind of scenario generation it is obvious that the demands for precision and accuracy cannot be very strong. Since we deal with very long term forecasts all figures will really only be at best "guestimates". One should in this connection remember that also in reality many long term energy decisions are based on some kind of "guestimates".

In fact we believe that one of the most important things is that the players consider these forecasts to be reasonable. As mentioned above, we plan to have the game played with various energy experts. Many of them will probably have their own ideas of what constitutes a reasonable forecast, in particular if the expert plays the role of his own country. If these players are then not at ease with the cost forecasts generated by the model, the players might very well be less serious than otherwise. The playing of the game might then lose much of its value as a research instrument. Because of this we regard it essential that the participants in the game have a possibility to make their own changes of the cost model prior to the actual playing of the game.

These requirements obviously make it impossible directly to use cost forecasts produced by others. First of all, the requirement to give the participants a possibility to change the functions makes it important to have an interactive model. Furthermore, our demand for long term forecasts generally involves a longer view than most other coal studies. This does not, however, mean that these other coal studies are not important to us. Rather these would constitute one of the sources of information for the model. In this connection we want to mention in particular the available data base at IIASA on coal extraction costs, gathered by the WELMM group in connection with IIASA's Energy Systems Program.

Another source of data for the model would be various experts. Above we mentioned the possibility of letting the players in the game alter the cost functions. We should also aim at involving other experts in constructing cost functions independently of the playing of the game. Since we deal with possibly twenty countries, it is not reasonable to have only one single or a couple of scientists develop the data. Rather one would try to involve a great many experts from various countries. Many such persons are likely to pass through IIASA over the time span of a year.

Both of these two activities, the modification of the model by the players as well as the actual construction of "best possible guestimates" by experts from various countries, point at the need for a computer dialogue system. As regards the possibility of letting the players change the model prior to the playing of the game, only a computer dialogue can make it possible to get the new input right into the game. This is necessary since the kind of people we want to involve in a game may only be available for an evening of game playing. As regards the collection of "guestimates" from the experts, we obviously also have a limited time, and a computer dialogue will generally be the fastest method, particularly, if one wants to give some feed-back to the expert about the long term implications of certain assumptions, e.g. regarding growth rates.

Furthermore, a computer dialogue has in this case the advantage that you only get the answers that you want. In particular, for our game, we want quantitative estimates, not qualitative opinions. A problem we have found, when using "man-expert dialogue" for the collection of data, is the following: The experts want to give only qualitative opinions, sometimes of a methodological character, in some cases not even related to the specific question. If a human would be like the computer, forcing a certain kind of answer, he would be considered very rude and, therefore, probably not be as successful as the computer in doing this task.

Finally, the computer dialogue method has the advantage of being usable in teleconferencing. Thus we could, from IIASA in Austria, obtain guestimates from experts in remotely located countries. This is of importance since we are interested in modelling many different countries.

### 3. GENERAL STRUCTURE OF THE MODEL

The first consideration of importance is the size of the model. This is in turn mainly determined by the length of time that one can expect to involve the players or the experts in the dialogue. In order that the expert shall be expected not only to have the time to answer but also to hold his interest and give the best possible answers, the number of questions asked by the computer has to be fairly limited. From our experience with a similar dialogue system (Stahl 1980) we believe that around thirty questions (excluding very simple yes/no questions) to be a practical maximum.

These considerations limit the complexity of the cost model. We must look for a structure of the model that, within a given level of complexity (mainly given by the number of parameters), incorporates the most fundamental aspects of the cost relations. Therefore, suggestions for increased complexity with regard to one aspect should be matched by lower complexity regarding some other aspect.

A further point is that we find it suitable, at least initially, to attempt to use the same model for all countries. The use of several different cost models for the computer game would imply a more complex game model and with a given project time, decrease the time to be spent on game playing.

Since the model has to be fairly simple, it must concentrate on those features which are the most important ones for the problem studied, i.e. the CO<sub>2</sub> question. This implies, e.g., that it is more important that the model is representative for underground mining than for surface mining. The reason for this is that a serious CO<sub>2</sub> problem would most likely first occur after the burning of a cumulative amount of around one teraton of coal. The coal available from surface mines would probably constitute only a small portion of such a quantity.

Furthermore, we have not made any distinction between "horizontal" and "vertical" location of mining seams. We have found it suitable to define all coal costs of the model as the costs of extracting the coal from the mine and bringing it to one specific location in each country. Since we are particularly interested in coal brought out on the world market and since a country's locally extracted coal sometimes will have to compete with imported coal, we will generally refer all costs of each country to one specific large port, generally located in an industrial area.\* The important thing is that this approach leads to a far less complicated model than one which represents the mining of coal and the transport of coal within the country separately. Such a model would require some kind of optimizing routine. When expanding output, one would have to calculate, if the best strategy is to go deeper into the ground, i.e., get higher mining costs, or go further away and get higher transport costs.

Since trade and hence also ocean shipping (including the loading of ships) will be covered in a separate trade model, the coal costs of this model will only cover extraction and transport within the country. This transport can be by train, by barges, or by pipe-line, e.g., in the form of slurry. In the latter case the coal costs would cover not only costs of adding water but also the de-watering process.

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\*For some of the large players, e.g., the USSR and the US it might in later stages be necessary to divide the country into two sections; e.g., for the US one section delivering to an east coast port, one to a west coast port.

We shall furthermore for the sake of simplicity not make any distinction between different kinds of coal. All tons shall in principle be given in tce, (tons of coal equivalent). For bituminous coal one could very well use original metric tons, since it would involve only a very small difference, but for subbituminous coal and lignite etc, one would have to do corrections.

As regards the general structure of the model it appears, however, necessary to make a distinction between old and new mines. The main reason for making this distinction is to obtain a clear picture of the possible financial problems that can arise when coal production in a country rises rapidly and hence a large part of the production must come from new mines.

New mines are characterized by large initial investments. Many of these investments, such as land purchases, development costs, investments in new railroads etc., have a very long life. Since each period of the game concerns five years or possibly even a decade, it appears suitable to use the following simplified treatment of the investments:

All investments are divided into two categories.

- I. Those that are made at the opening of the new mine and have a fairly long life span, (e.g., at least 10 or 20 years).
- II. All other investments.

It appears that category I would in most cases represent the bulk of the investments in terms of present value.

All category I costs are assigned to the period of the start of the new mine as investment costs. No depreciation is calculated, even if the costs also concern investments that would later be replaced. The error involved in this approximation is in this context small.\*

All category II investments, involving both replacements and more continuously made investments, e.g., reconstructions, for increasing efficiency, are lumped together with other non-wage costs.

It should be stressed that we hence deal with all investments as expenditures, rather than as traditional costs. Besides simplicity, this has the advantage of facilitating the above mentioned focus on the financial problems.

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\*If one, for instance, assumes that some equipment is purchased only once and has an eternal life when it really is replaced every 20 years, at a constant price, then there is an error in the present value of 13 percent, if one calculates with a discount rate of 10 percent. For equipment with a 10 year life the error increases to 37 percent. The total error is, however, less, since life is not eternal and more importantly the machines with, e.g., 10-20 years life time constitutes only a smaller part of total investments.

Finally the coal cost model can be seen as consisting of two major parts:

- A. A model for calculating the coal cost of a certain country, for a certain production policy. The only input is then the production policy, but all the parameters are given. This is the cost function model.
- B. A model for extracting the parameters of this model from the expert or the player. This is the model of the computer dialogue system.

Since model B cannot be understood without knowledge of model A, model A will be presented first, in the next two sections (4 and 5), allowing us to return to model B in section 6.

#### 4. COSTS IN OLD MINES

We first have to distribute total production of a certain year between production in old mines and production in new mines.

Let  $q_t$  be the decision supplied by the players on production in period  $t$ . Furthermore, since an increase in production above capacity requires investments in new capacity, we define production capacity:

$$\bar{q}_t = \max(\bar{q}_{t-1}, q_t) .$$

Production to take place in old mines is then:  $q_t^o$  ,  
 where  $q_t^o = q_t$  , if  $q_t \leq \bar{q}_{t-1}$

and  $q_t^o = \bar{q}_{t-1}$  , if  $q_t > \bar{q}_{t-1}$  .

Production to take place in new mines is  $q_t^n$  ,

where  $q_t^n = 0$  , if  $q_t \leq \bar{q}_{t-1}$

and  $q_t^n = q_t - q_t^o$  , if  $q_t > \bar{q}_{t-1}$  .

We then assume a production function for the old production to be divisible into two components:

- 1) Labor costs
- 2) Other current expenses

The labor costs per ton produced are thus:

$$h_t W e^{wt} ,$$

where  $h_t$  is the number of man hours required per ton in period  $t$ .

$W$  is the average hourly wage rate initially (e.g. 1980) and  $w$  is the annual percentage increase in the wage rate. In case of piece rate, one might think that the development of  $w$  should

depend on productivity changes. Since we cover fairly long periods, it is however more likely that  $w$  follows the general wage trend of the country, and that  $w$  is fairly independent of productivity increases in just mining.

In this connection it should be stated that the user is free to calculate in constant prices or in real prices. The important thing is that he is consistent. Since constant prices are usually expressed in terms of consumer prices, constant price calculation might still involve changes in wages and prices of investment goods.

It remains to define  $h_t$ . In order to do this, we must first define the average age of the mines in the country in period  $t$ :

$$a_t = \frac{q_t^o}{q_t} (a_{t-1} + \Delta t) ,$$

where  $\Delta t$  is the length of each period and  $a_{t-1}$  is the average age in the prior period. Furthermore, we define the change in average age from  $t-1$  to  $t$ :

$$\Delta a_t = a_t - a_{t-1} .$$

We also define the average labor productivity, i.e., at the end of period  $t$ :

$$h_t = \frac{q_t^o}{q_t} h_t^o + \frac{q_t^n}{q_t} h_t^n ,$$

where  $h_t^o$  and  $h_t^n$  refer to productivity in old and new mines.

We can then define the average productivity of the old mines in period  $t$ :

$$h_t^o = h_{t-1}^o e^{-k\Delta t} e^{b\Delta a_t}$$

Hence, we have two different types of changes:

Labor requirement decreases annually by  $100k$  percent due to learning effects and smaller continuously made investments.  $k$  is a parameter, (like  $b$ ,  $g$ ,  $c$  etc., below) which is determined indirectly by the computer dialogue presented in section 6.

Labor requirement increases on the other hand due to the obsolescence of the mine and the fact that within a specific

mine one has to go down to more deeply located coal seams as the mine gets older. This is reflected in the factor

$$e^{b\Delta a_t},$$

implying that labor requirement increases with the increase of the average age of the mines by 100b percent of each year of age increase.

For mines in which production is fairly constant year after year this implies that the labor requirement increases exponentially with cumulative production, e.g., as one goes down deeper and deeper in an underground mine.

The productivity in the new mines,  $h_t^n$ , will be defined in section 5.

As regards current non-wage costs, we have decided on keeping the model simpler. It appears that in most mines these costs are less important and we must, as mentioned, keep total complexity limited. The expression for the non-wage costs per ton are:

$$M_0 e^{gt} e^{-ct}$$

where  $M_0$  is the cost of other resources currently used in the initial period (1980).  $100 g$  is the annual percentage increase in the cost of other resources due to price increases. We assume that non-labor costs decrease annually by  $100 c$  percent due to learning and smaller investments, but for the sake of simplicity, we do not allow these costs to change with the age of the mine.

### 5. COSTS IN NEW MINES

As regards the cost of production in new mines, we distinguish between three components: Investment costs, wage costs and other current costs, which are added to obtain total costs in new mines.

#### 1) Investment costs:

Before looking further at these, it should be remembered that the production decision might automatically involve an investment decision on new capacity  $q_t^n$  since

$$q_t = \bar{q}_{t-1} + q_t^n, \text{ if } q_t > \bar{q}_{t-1}$$

Let us first define the investment costs for the case of constant prices for investment goods. Then the investment costs depend on how well located the seams (initially to be mined) of this new mine are. The measure of how well located a coal seam is will be in terms of how large a percent of total resources have been used at the time of the start of the new mine, i.e., total production up to now, called  $s_t$ , set in relation to total resources,  $R$ .

It should in this connection be stressed that the definition of resources is allowed to be a very subjective one. The important thing is that one gets a reasonable development of the cost curve as one goes into less well located seams. Hence the important thing is that as the fraction  $s_t/R$  gets larger, total costs increase steeply, e.g., exponentially. In order to obtain such an increase we use a multiplier:

$$e^{ms_t/R}$$

For a similar function see Grenon (1979, p. 92).

$s_t$  is simply calculated as:

$$s_t = \sum_{j=1}^{t-1} q_j + s_0$$

where  $s_0$  is total production up to now (e.g., 1980).

We next define  $I_0$  as the investment costs when 0 percent of resources have been used. The investment costs at time  $t$ ,  $I_t$ , (assuming no price increase for investments) when 100  $s_t/R$  percent of resources have been used are then  $I_0 e^{ms_t/R}$ . Since the investment costs  $I$  at a time when 100  $s_0/R$  percent of resources have been used are  $I_0 e^{ms_0/R}$ , we have that  $I_0 = I e^{-ms_0/R}$ . Hence it can be written as  $I e^{m(s_t-s_0)/R}$ .

Since we furthermore want to allow investment costs per ton to change annually by 100  $i$  percent, we write total investment costs, resulting from a production of  $q_t^n$  as

$$q_t^n I e^{it} e^{m(s_t-s_0)/R}$$

## 2) Wage costs

We here first have to depict the productivity at the time of the start of the new mine:

$$h_t^n$$



This productivity is dependent first of all on a technical progress component, allowing for, e.g., a 100 k percent annual decrease in man-power requirement, for simplicity the same as in old mines. Furthermore, we allow for a decrease in productivity, i.e., an increase in man-power requirement due to less easily accessible mines. We here assume that man-power per ton increases exponentially with the percentage of resources used, in a way similar to how investment costs rise.

Hence we write:

$h_t^n = h_0^n e^{m'(s_t - s_0)/R_e - kt}$ , where  $m'$  is a constant similar to  $m$  above. The total wage cost in new mines is

$$q_t^n h_t^n w_t e^{wt}$$

3) Other current costs

For the sake of simplicity, we here assume the same development as regards the old mines.

6. INTERACTIVE MODEL FOR ESTIMATING THE PARAMETERS

With the mathematical form of the model given, it is the task of the experts/players to determine the parameters. This is done in a man-computer dialogue. The best way to report on this is to present the printout of a dialogue session with comments. This is presented in an appendix.

It should be mentioned that the user has received some prior information, roughly equivalent to that given in this article, regarding e.g., the interpretation of the words "ton", "resources", "prices", etc.

In the left margin of the computer printout we have set out figures at the questions (given by the computer) and symbols at the answers (given by the player). These figures and symbols are not part of the computer dialogue, but written out as an "interface" between the computer printout and comments.

It should be mentioned that in the example the starting year  $t_0$  is 1980.

In (1) (question 1), the age  $A_0$  is determined. In (2)  $W$ , the wage rate at time  $t_0$  is read. In (3) and (4) a year  $T$  and a wage  $W_1$  are given. Then  $w$  is computed as  $\ln(W_1/W)/(T-t_0)$ .

It should here be stressed that  $T$  can be either a historical year (e.g. 1975), allowing historical data to be used for  $W$  or a

year in the future in which case one would use some other forecast for  $W_1$ . (If one happens mainly to think directly in terms of percentage changes one can set  $T$  to  $t_0+1$ , e.g., 1981.)

In order to show the partial forecast that comes from these parameters,  $We^{wt}$  is presented for  $t = t_0, t_0+5$  etc. (For reasons of space we present in the appendix only the first two years and the last year of the forecast.)

After this forecast, as well as after every other forecast, the computer gives the player a chance to revise the parameters responsible for the forecast. Hence, if he answers NO, the computer will in this case bring him back to (2).

Next at (6) - (8) and at (10) we input  $h_0^0, T, h_1$  and  $h_2$ .

$k = -\ln(h_2/h_0^0)/10$  and  $b = k + \ln(h_1/h_0^0)/(T-t_0)$  are then computed.

The computer at (9) and (11) makes the forecasts of  $h_0^0 e^{(b-k)t}$  and  $h_0^0 e^{-kt}$  for  $t = t_0, t_0+5$  etc.

Next at (12) - (14) and at (16) we input  $M_0, T, M_1, M_2$  and we obtain  $c = -\ln(M_2/M_0)/10$ ;  $g = c + \ln(M_1/M_0)/(T - t_0)$ . At (15) and (17) we obtain forecasts of  $M_0 e^{(g-c)t}$  and  $M_0 e^{-ct}$  for  $t = t_0, t_0+5$  etc.

If we have proceeded this far with all partial forecasts acceptable, we make a forecast of how the total costs per ton in old mines will develop over time. If this forecast is not acceptable one is brought back to (1).

Otherwise one continues to the questions regarding new mines. Here at (19) one inputs  $s_0$  and at (20)  $R$ . The computer checks that this is the desired value  $s_0/R$ .

Next at (22) one supplies  $I$  and at (23) and (24)  $T$  and  $I_1$ . The investment price change  $i$  is next computed as  $\ln(I_1/I)/(T-t_0)$ . Then a forecast  $Ie^{it}$  is presented for  $t = t_0, t_0+5$  etc.

At (26) one states  $I_2$ , the investment at a level  $\bar{s} = \min(\max(2s_0/R, 0.2), 0.1)$ . On the basis of this we compute  $m$  as  $\ln(I_2/I)/(\bar{s} - s_0/R)$ .

Next a forecast is given for  $Ie^{-ms_0/R} e^{ms}$  for  $s = 0.05, 0.15$  etc. In order to check that this really is the desirable value of  $m$  we reverse the presentation, talking of remaining resources, instead of used ones.

Next we proceed to productivity development in new mines.

At (28) we input  $h_0^n$  and at (29)  $h_3$ , i.e., the manpower requirement, if 100  $\bar{s}$  percent of resources have been used.  $m'$  is then computed as  $\ln(h_3/h_0^n)/(\bar{s}-s_0/R)$ . A forecast is then given of how productivity changes with remaining resources by giving  $h_0^n e^{-ms_0} e^{m's}$  for  $s=0.05$ ., 0.15 etc.

This completes the read in of the parameters of the new mine. Next the computer gives a full forecast of the development of cost per ton of a new mine. Since this cost in a certain year is dependent not only on the year, but also on earlier production, the computer asks for a full production plan with

$q_t = q_0 e^{zt}$ , where 100  $z$ , asked for in (32), is the annual increase in production.

The computer then, for each year, calculates  $s_t$ , i.e., cumulative production up to now. On the basis of this, it calculates productivity as well as investment costs per ton in a new mine. Finally, by adding up investment costs, wage costs and current non-wage costs, total costs per ton in a new mine are calculated and presented for  $t = t_0, t_0+5$  etc.

One is then allowed to repeat this forecast with different production figures, by going back to (31). If one is not satisfied with some parameter, one can go back to (19).

Otherwise, one has come to the last phase at (35) when one can test run the total model by once again specifying a production plan,  $q_0 e^{zt}$ , in (35) and (36). One also has to specify present production capacity,  $\bar{q}_{-1}$ , in order to divide total production  $q$  into  $q^n$  and  $q^o$ .

One then obtains a forecast with two cost figures, one per ton and one of total costs. One then gets a chance to try a new production policy, by going back to (35). Finally, if one is not satisfied one can go back to (1) and start the whole process over. Otherwise the dialogue is finished and the final parameter values are saved.

It should finally be mentioned that we have up to now only made a few tests of this model and it is quite likely that we will revise it as we gain more experience. We welcome any comments.

APPENDIX A: EXAMPLE OF A COMPUTER DIALOGUE SESSION

- (1) estimate average age of mines in 1980  
A<sub>0</sub> 20
- (2) estimate wage per hour in coal mining in 1980  
W 8
- (3) for estimation of wage per hour in coal mining any other  
year than 1980  
state year  
T 1975
- (4) estimate wage rate year 1975  
W<sub>1</sub> 6.5
- (5) you have made the following forecast for  
wage-rate  
1980: 8.00  
1985: 9.85  
2030: 63.81  
is this acceptable? if yes push return button  
if no print no
- (6) estimate for typical old mine of average age man  
hour requirement per ton in 1980  
h<sub>0</sub><sup>o</sup> 0.9  
estimate for same typical mine man hour requirement  
per ton in some other year
- (7) state year  
T 1975
- (8) state man hours per ton  
h<sub>1</sub> 0.85

- (9) you have made the following forecast for man hour requirement :
- |       |      |
|-------|------|
| 1980: | 0.90 |
| 1985: | 0.95 |
| 2030: | 1.59 |
- is this acceptable? if yes push return button  
if no print no
- (10) estimate what hypothetical man hour requirement in this mine would be in 1990 provided one then has not gone to significantly less well located seams
- $h_2$  0.75
- (11) for the hypothetical case of no change in seam location you have made the following forecast for man hour requirement :
- |       |      |
|-------|------|
| 1980: | 0.90 |
| 1985: | 0.82 |
| 2030: | 0.36 |
- is this acceptable? if yes push return button  
if no print no
- (12) estimate total non-wage costs per ton in 1980
- $M_0$  4
- (13) estimation of total non-wage cost per ton given year
- T 1975
- (14) state costs
- $M_1$  3.6
- (15) you have made the following forecast for total non-wage costs :
- |       |       |
|-------|-------|
| 1980: | 4.00  |
| 1985: | 4.44  |
| 2030: | 11.47 |
- is this acceptable? if yes push return button  
if no print no
- (16) give hypothetical estimate of total non-wage costs per ton in 1990 provided all prices remain constant
- $M_2$  4.2
- (17) for hypothetical case of no price increases you have made the following forecast for total non-wage costs :
- |       |      |
|-------|------|
| 1980: | 4.00 |
| 2030: | 5.11 |
- is this acceptable? if yes push return button  
if no print no

- (18) you have made the following forecast of the development of total costs per ton of typical old mine existing 1980  
1980: 12.00  
1985: 14.87  
2030: 124.48  
is this acceptable? if yes push return button  
if no print no
- (19) how much coal has been produced over the years up to now in millions of tons  
s<sub>0</sub> 12000
- (20) how large are total coal resources in millions of tons  
R 100000
- (21) 12.00 percent of your resources have been used  
is this acceptable? if yes push return button  
if no print no
- (22) estimate investment costs in 1980 per ton for new mine  
I 80
- (23) estimate investment costs in some other year for typical new mine provided it concerns equally well located coal seams  
state year  
T 1975
- (24) state investment costs  
I<sub>1</sub> 70
- (25) regarding investment with location as in 1980 you have made the following forecast for investment costs :  
1980: 80.00  
1985: 91.43  
2030: 304.10  
is this acceptable? if yes push return button  
if no print no
- (26) estimate hypothetical investment cost 1980 if you already used 24.00 percent of resources, i.e., 24000 mill tons  
I<sub>2</sub> 130
- (27) regarding investment costs you have made the following forecast for changes due to changes in percentage of remaining coal resources  
remaining percentage cost  
95.00 percent 60.26  
85.00 percent 90.32  
35.00 percent 682.90  
is this acceptable? if yes push return button  
if no print no
- (28) give man hour requirement per ton in 1980 when 12.00 percent of coal used  
h<sub>0</sub> 0.8

(29) estimate man hour requirement in new mine if one today had used 24.00 percent of coal resources i.e., 24000 mill ton  
h<sub>3</sub> 1.3

(30) regarding man power requirement per ton  
you have made the following forecast for changes due to changes in percentage of remaining coal resources

| remaining percentage | cost |
|----------------------|------|
| 95.00 percent        | 0.60 |
| 85.00 percent        | 0.90 |
| 35.00 percent        | 6.82 |

is this acceptable? if yes push return button  
if no print no

(31) to test your assumptions regarding costs of new mines  
give initial annual production in millions of tons  
q<sub>0</sub> 700

(32) give annual increase in production in percentage  
z 1.5

(33) you have now made the following forecast for the development of total cost per ton in typical new mine

|      |         |
|------|---------|
| 1980 | 90.40   |
| 1985 | 118.81  |
| 2030 | 2710.68 |

do you want to try with different production figures?

(34) do you want to put in new parameters for case of new mines

(35) to test total model  
give initial annual production in million of tons  
q<sub>0</sub> 700

(36) give annual increase in production in percentage  
z 1.5

(37) give total production capacity at start of 1980  
q<sub>-1</sub> 800

| year | cost per ton | total production (million tons) | total costs (in millions) |
|------|--------------|---------------------------------|---------------------------|
| 1980 | 11.20        | 700                             | 7840                      |
| 1985 | 13.83        | 754                             | 10432                     |
| 2030 | 120.11       | 1481                            | 177989                    |

do you want to try with different production figures?

(39) is this acceptable? if yes push return button  
if no print no

APPENDIX B: STATISTICS ON COAL RESOURCES



Table 1. Carbon wealth in Gt<sup>a</sup>.

|                           | Reserves | Resources        |
|---------------------------|----------|------------------|
| Coal                      | 430      | 7000             |
| Oil                       | 70       | 200 <sup>b</sup> |
| Gas                       | 30       | 100 <sup>c</sup> |
| Tropical moist<br>forests |          | 250              |

<sup>a</sup>(1 Gt = 1 billion metric tons = 10<sup>9</sup> metric tons)

<sup>b</sup>Possibly a maximum of 500 Gt from unconventional sources, such as shale oil.

<sup>c</sup>A possible maximum of Gt from unconventional gases.

SOURCE: Ausubel (1980).

Table 2. Approximate World Distribution of Coal Resources  
(in gigatons carbon)

| Huge holdings |      | Large holdings |     | Small holdings |    |
|---------------|------|----------------|-----|----------------|----|
| USSR          | 3300 | Australia      | 180 | Czechoslovakia | 12 |
| U.S.          | 1700 | FRG            | 170 | Yugoslavia     | 7  |
| China         | 1000 | UK             | 110 | Brazil         | 7  |
|               |      | Poland         | 80  | GDR            | 7  |
|               |      | Canada         | 80  | Japan          | 6  |
|               |      | Botswana       | 70  | Colombia       | 6  |
|               |      | India          | 40  | Zimbabwe       | 5  |
|               |      | South Africa   | 40  | Mexico         | 4  |
|               |      |                |     | Swaziland      | 3  |
|               |      |                |     | Chile          | 3  |
|               |      |                |     | Indonesia      | 2  |
|               |      |                |     | Hungary        | 2  |
|               |      |                |     | Turkey         | 2  |
|               |      |                |     | Netherlands    | 2  |
|               |      |                |     | France         | 2  |
|               |      |                |     | Spain          | 2  |
|               |      |                |     | North Korea    | 1  |
|               |      |                |     | Romania        | 1  |
|               |      |                |     | Bangladesh     | 1  |
|               |      |                |     | Venezuela      | 1  |
|               |      |                |     | Peru           | 1  |

SOURCE: Based on data from World Energy Conference (1978).  
Very rough estimate of carbon wealth in Gt has been  
obtained by multiplying coal resources in  $10^9$  tons  
coal equivalent by carbon fraction of 2/3.

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