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MATHEMATICAL MODELING OF PHOSPHORUS TRANSFORMATION IN THE LAKE BALATON ECOSYSTEM

A.V. Leonov

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

THE AUTHOR

ALEXANDER LEONOV is on leave from the State Oceanography Institute, Moscow, USSR. At present he is a research scientist at the International Institute for Applied Systems Analysis, Schloss Laxenburg, 2361 Laxenburg, Austria.

PREFACE

One of the principal themes of the Task on Environmental Quality Control and Management in IIASA's Resources and Environment Area is a case study of eutrophication management for Lake Balaton, Hungary. The case study is a collaborative project involving a number of scientists from several Hungarian institutions and IIASA. This paper presents further details of a model for phosphorus transformation processes and phytoplankton growth in the lake (see also WP-80-88). Results are reported for a comparison of the performance of the model with observations recorded for 1977.

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ABSTRACT

An ecological model of the phosphorus system is described. This model includes five phosphorus forms found in water, namely: phytoplankton-P, bacterial-P, dissolved inorganic-P, dissolved organic-P and nonliving particulate-P, and also three phosphorus forms in interstitial water: inorganic, organic and particulate phosphorus fractions. It is assumed that this model will be used as a tool for synthesizing and analyzing the phenomena of eutrophication in Lake Balaton's ecosystem.

The purpose of this study was to obtain the best calibration between existing observation data on Lake Balaton from 1977 and model output. This is considered one of the important steps that must be carried out before application of the model for prediction and management purposes. A hypothesis of three seasonal phytoplankton groups yielded model output that agreed reasonably well with the observation data for total-P, dissolved-P, dissolved organic and inorganic phosphorus, particulate organic-P and phytoplankton chlorophyll "a" in the water of the different basins in Lake Balaton. This provides indirect evidence that the model considered, is a reasonable representation of complex ecological processes in phosphorus transformations and phytoplankton dynamics in the lake. On the basis of simulation results for 1977, the phosphorus material flows and the turnover times of phosphorus fractions in the lake are evaluated. These data provide additional insights for understanding the conditions of phosphorus cycling and the eutrophic state of the basins within Lake Balaton.

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1. INTRODUCTION

Lake Balaton is a large shallow lake in central Europe. It is situated in the western part of Hungary and has a surface area of about 600 km^2. It receives drainage from a catchment area almost ten times larger than the water body itself; i.e. 5,775 km^2 (van Straten et al. 1979). The lake is 75 kms long and 8 kms wide. The average depth is about 3 meters and only a small portion of the lake where it is divided by the peninsula of Tihany, does the depth of the lake reach 11-12 meters. Lake Balaton is a typical example of a water body with different eutrophic conditions in different parts of the lake.

Eutrophication and the phenomena connected with nutrient enrichment of water, such as increased development of plant life, are important limnological problems for various water bodies, both shallow and deep. These problems result entirely from increased productivity and are caused by an imbalanced input of nutrients into the water from the watershed. In shallow lakes and reservoirs, the development of eutrophication is greatly dependent on the sediment-water interactions, because the bottom sediment may be a direct nutrient source and has a significant effect on the nutrient budget of water bodies.

The natural state of eutrophication is dependent on the age of the water body, its geological features, and historical evolution. The development of eutrophication is defined by nutrient levels as a whole. Human activities involving the water body often increase the nutrient inputs, resulting in the acceleration of eutrophication. In every case, the development of eutrophication entails a complex reorganization of the water-ecological system, owing to the disturbance of the balance between the process of nutrient inputs and their biogeochemical cycles within the system.

The existing level of knowledge and available quantitative information concerning water body eutrophication problems are limited and do not answer many questions connected with the acceleration of eutrophication and its undersirable consequences. Generally speaking, the phenomenon is still not completely understood. Two important, classical questions about the eutrophication problem are: what is the cause of excessive fertilization, and what can be done to control it (Lee 1973)?

It is known that of all the biogenic elements which influence aquatic life, phosphorus is the key element and it is considered a major reason for eutrophication in this lake (van Straten et al. 1979). The River Zala* enters the lake at its southwestern part and provides about 50-75 percent of the total water inflow (Csáki et al. 1979; van Straten et al. 1979); it is the primary source of phosphorus in the Lake Balaton ecosystem. This river mainly receives agricultural runoff and some domestic and industrial wastes from a total area of about 2,622 km^2 of the River Zala watershed. Other important tributaries enter the lake in its central part. The water outflow of Lake Balaton, is regulated by a special gate at the northeastern end of the lake.

The River Zala enters Lake Balaton via the shallow Keszthely Bay and the effect of the river is felt not only in the bay itself but also in the neighboring region of Lake Balaton, i.e. the Szigliget Basin. In the last two basins of the lake, Szemes and Siófok, the effect of the Zala River is quite low. As a result, eutrophic conditions decrease from the southwestern to the northeastern end of the lake.

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*The major tributary of Lake Balaton

The role of phosphorus in the eutrophication of various water bodies was discussed in many reports (Bartsch 1972; Lee 1973); Porcella and Bishop 1975). Phosphorus occurs in natural waters in a variety of forms--both dissolved and particulate. Biochemical transformation processes developed in water environments tend to convert phosphorus fractions from one form to another. Dissolved inorganic phosphorus is readily available for phytoplankton growth. In unpolluted waters, it exists at a level which limits plant growth.

The early studies of eutrophication and phosphorus dynamics in water bodies were mostly concerned with qualitative descriptions or, in the best case, with searching for a comparatively simple relationship between phosphorus levels and eutrophication indexes (Stewart and Rohlich 1967; Vollenweider 1968; Hutchinson 1973). In the present study, it is obviously a necessary step to increase the degree of understanding of these relationships by applying systems analysis. At the same time, it has of course been necessary to separate the direct effects on eutrophication--so called man-made impacts--from indirect effects and unrelated factors which cause ecological variations and have important consequences for the ecology of the lake.

During the past few years, water eutrophication as a limnological problem has become the subject of many special studies using mathematical models of different degrees of complexity. The main purpose of these studies is to quantify impacts and assess alternative control strategies in terms of biogenic element balances in various water bodies.

In the Lake Balaton case study mathematical modeling is considered one of the successful methods of studying the eutrophication of Lake Balaton at a comparatively early stage of its development and to obtain a quantitative assessment of this phenomena. Some important characteristics of the water body indicating the state of eutrophication, its trends and the rate of change of the quality of eutrophic waters, may be evaluated by specially constructed models which should be geared to the level of detail of this problem.

This report describes a model of phosphorus compound transformations--an important factor when studying the eutrophication in natural waters--particularly in the case of Lake Balaton. Generally speaking, this model is intended to simulate the dynamics of phosphorus cycling processes and represents an excessive growth of aquatic plant life as a consequence of the changes in quantity of phosphate-phosphorus in the water body.

Comparatively simple phosphorus models used for the study of eutrophication take into account a limited number of phosphorus forms, usually total phosphorus or just two phosphorus fractions, particulate and dissolved phosphorus. Models with a limited number of phosphorus forms have been applied for studying less eutrophic water, when sediments do not exert a significant influence on phosphorus cycling in the water (Lung et al. 1976)

However, a much better understanding of the existing relationship between phosphorus input to a water body, and phosphorus transformation processes within it, may be attained with the help of models constructed on the basis of synthesis of biological and chemical models. These types of models include several phosphorus forms, chemical as well as biological, and integrate the available information concerning their behavior in water bodies. By using this type of model, it is possible to explain the dynamics of phosphorus in the cells of microorganisms and to concentrate on changes of each of the chemical forms of phosphorus present in the water and also the rates of phosphorus interchange between various forms, including biological and chemical. This type of model is very useful for studying complex processes occurring with eutrophication such as phosphorus release from sediment to water, phosphorus loading from a watershed, phosphorus regeneration by microorganisms, and so on. An important stage in understanding the eutrophication of Lake Balaton, is to establish interactions which play a significant role in phosphorus transformations, the magnitudes of rates of individual transformation stages in the phosphorus cycle, and to evaluate the phosphorus material flows in the lake's ecosystem.

2. CONCEPTS BEHIND THE PHOSPHORUS TRANSFORMATION MODEL

Phosphorus is one of the most important elements in natural waters. Its presence often limits the development of various microorganisms and determines the rates of biochemical transformation of organic matter. The concentration of mineral phosphorus compounds must be considered when examining general water problems such as questions of eutrophication, primary production, decomposition of pollutants and the self-purification of water bodies.

The main chemical compounds of the phosphorus system are dissolved organic and inorganic phosphorus, DOP and DIP, respectively. The phosphorus components included in the composition of particulate matter are the biomasses of microorganisms (bacteria, algae, and zooplankton) and the remains of dead organisms (detritus).

Practically all types of microorganisms take part in the transformation of phosphorus compounds. DIP is consumed by algae and bacteria and is the most important biochemical component in organic components which constitute the biomass of living cells. Through the food chain, organic phosphorus of living matter is included in cells of organisms of a higher trophic level, i.e. carnivorous zooplankton. As a result of the life-sustaining functions of the organisms, excretion of dissolved mineral components and organic phosphorus occur together with the formation of particulate organic phosphorus or detritus. DOP may be utilized by heterotrophic bacteria and can also be directly assimilated by phytoplankton. According to well-documented evidence, protozoa and zooplankton play an important role in the transformation of DOP (Watt and Hayes 1963).

The role of bacteria is extremely important in phosphorus transformation. Their uptake of mineral phosphorus may increase the consumption of organic phosphorus under certain conditions (Watt and Hayes 1963; Ajzatullin and Leonov 1977). Bacterial transformation of of phosphorus compounds may be greatly accelerated by the presence of predators (Grill and Richards 1964). In the regeneration of nutrients, the important role played by detritus is established (Rajendran and Venugopalan 1974).

Thus phosphorus transformation occurs as a complex interaction between microorganisms and chemical compounds, of which the rates of change and character are dependent upon environmental factors.

In practice, many schemes of phosphorus compound interactions are used for mathematical modeling of phosphorus transformation in water. Some of the numerous schemes of phosphorous compound interactions used in practice were considered and discussed by Leonov (1978). Ecological models, which are intended for application in studies of phosphorus transformations and phosphorous cycling as a whole, include different types of microorganisms, such as heterotrophic bacteria, protozoa, zooplankton, and phytoplankton.

The concepts and the model considered here are based on the previous studies of phosphorus transformation processes in water environments (Leonov 1978) and also on the field data of measurements of phosphorus compounds obtained from interdisciplinary research in Hungary.*

The general phosphorus model might be usefully constructed as an interactive phosphorus compounds system that involves:

1. Ecological responses--what is the impact of phosphorus transformation on the eutrophication phenomenon?

2. Biochemical mechanisms--how are these responses produced?

3. Prediction power--how will the ecological system behave in the present and in the future?

The model as a whole must consider the phosphorus transformation in the water and in the sediment, because the sediment-water interaction is an important factor in the ecosystem of Lake Balaton (van Straten et al. 1979). The model compartments, or state variables and their interactions, indicated by arrows, are shown in Figure 1.

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*A group of Hungarian specialists presented to IIASA a wide set of field observations made from 1972 to 1978.

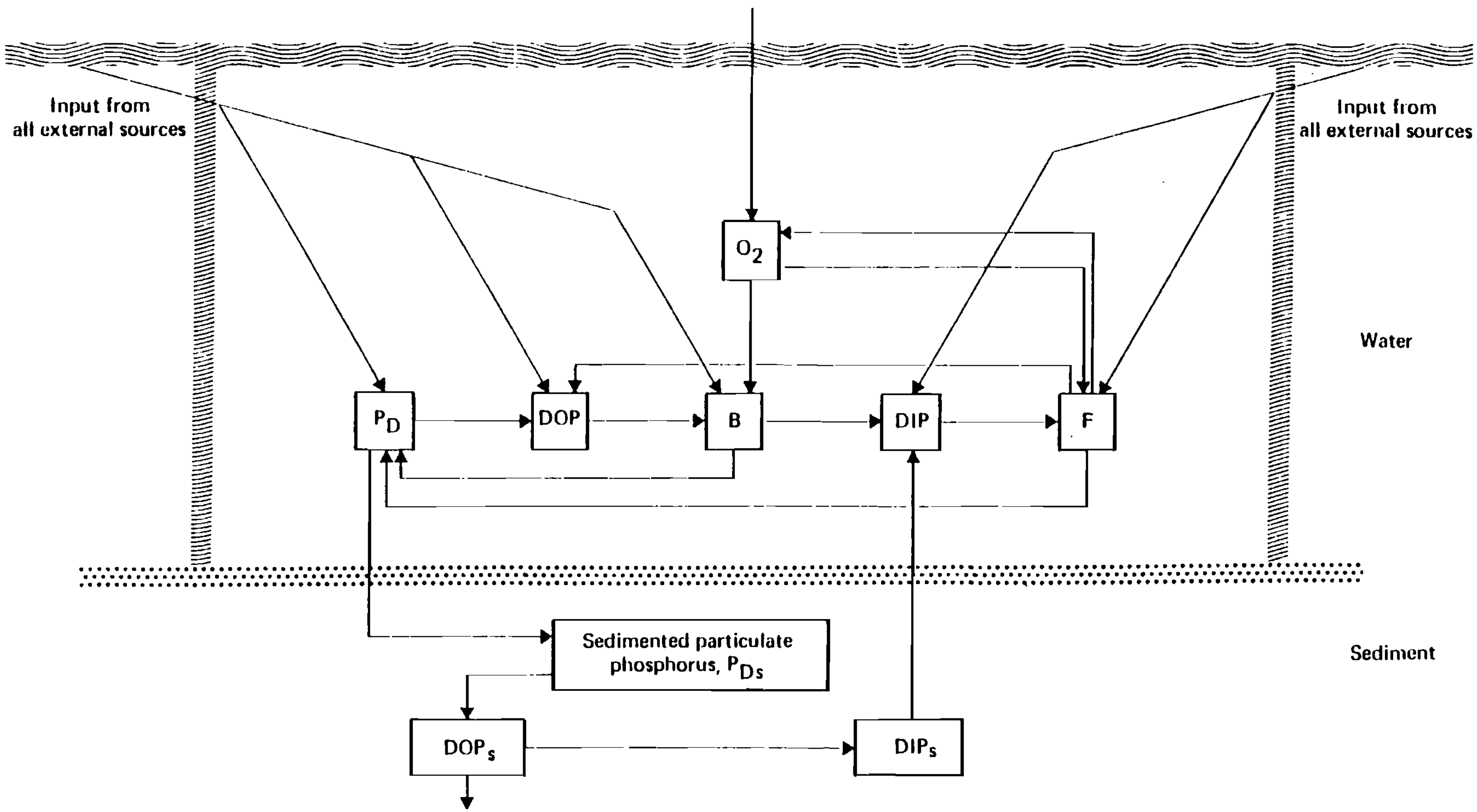


Figure 1. Diagram of compartments of phosphorus transformation and oxygen cycle model.

The main compounds taken into account in the model are:

1. dissolved inorganic phosphorus, DIP;
2. dissolved organic phosphorus, DOP;
3. phosphorus in phytoplankton, F;
4. phosphorus in heterotrophic bacteria, B;
5. non-living particulate phosphorus, P_D.

All of these phosphorus forms are in mg P/ℓ in the model. Dissolved oxygen, O_2 with concentration mg O_2/ℓ, is also introduced in the system modeled as one of the important characteristics of water quality.

The major processes that play a significant role in water ecosystem functioning, particularly in phosphorus transformation are:

1. *phytoplankton production* and *nutrient uptake* which are characterized by a function of temperature, light, and DIP content;
2. *bacterial production* which is temperature-dependent and is an important step of DOP transformation and DIP regeneration;
3. *metabolic excretion of DOP and DIP* by phytoplankton and bacteria, respectively;
4. *nonpredatorial mortality of bacteria and phytoplankton* which is an important mechanism of phosphorus cycling;
5. *decomposition of nonliving particulate phosphorus* in the lake water, is an important stage of phosphorus transformation in the release of chemical energy stored in detritus;
6. *oxygen consumption* due to the respiration of phytoplankton and heterotrophic bacteria, and also *oxygen exchange* through the air-water interface and by photosynthetic production, are important processes which regulate the oxygen content in the water body.

Lake Balaton being shallow, average concentrations of all compounds in the model discussed are taken for each basin.

The first version of the model considered a homogenous phytoplankton population in the water, without subdividing individual groups of species (Leonov 1980). Later on, changes were introduced into the model in order to take into account the possible variation of phytoplankton in time and space. The role of zooplankton is considered unimportant in phosphorus cycling, and in the eutrophication of Lake Balaton (van Straten et al. 1979).

Sediment-water interactions are a very important mechanism in the ecological system of Lake Balaton, in the regulation of nutrient content, and possible enrichment of the water body by DIP. Therefore, it was decided to include in the model individual phosphorus forms in the interstitial water. These are:

1. dissolved inorganic phosphorus in interstitial water, DIP_s;
2. dissolved organic phosphorus in interstitial water, DOP_s;
3. nonliving particulate phosphorus in sediment, P_{Ds}.

Concentrations of all phosphorus fractions in sediment are taken in mg P/ℓ.

The principal mechanisms of benthic phosphorus transformation includes the sedimentation of nonliving particulate phosphorus from water and the consequent phosphorus transformations from particulate to dissolved phosphorus forms, first to DOP_s and then to DIP_s in the interstitial water of the sediment. This approximation is necessary for a simpler representation of phosphorus transformation in the sediments with limited data about processes and phosphorus compound content in sediments. The type of sediments in Lake Balaton are assumed to be similar in different sections of the lake, and to constitute a homogeneous layer. Kinetic reactions control the rates of phosphorus transformation and oxygen cycling in the water and they are functions of exogenous variables, such as temperature. The rate at which all reactions proceed is controlled by local concentrations of the state variables.

The kinetic equations applied in the model are designed to simulate the annual cycle of the main processes which determine the phosphorus transformation, such as phytoplankton production in relation to the DIP supply, bacterial production in relation to the DOP content and degradation of nonliving particulate phosphorus to DOP and the effect of all phosphorus transformation processes on the dissolved oxygen cycle. The model consists of a set of deterministic differential equations describing the dynamics of all model compartments for a one-year period of time, with independent phosphorus inputs for each segment of Lake Balaton.

For increasing the predictive and explanatory power of the phosphorus transformation model discussed here, it was decided to consider the ecosystem as a self-optimizing and self-organizing system following Parker (1972) and Straskraba (1977). In order to do this, it is assumed that in comparison with other processes, nutrition is a basic mechanism which is an adaptive biological function of any organism. This increases the realism of the mathematical model in the description of transformation processes of chemical compounds and trophic interrelationships in relation to environmental changes.

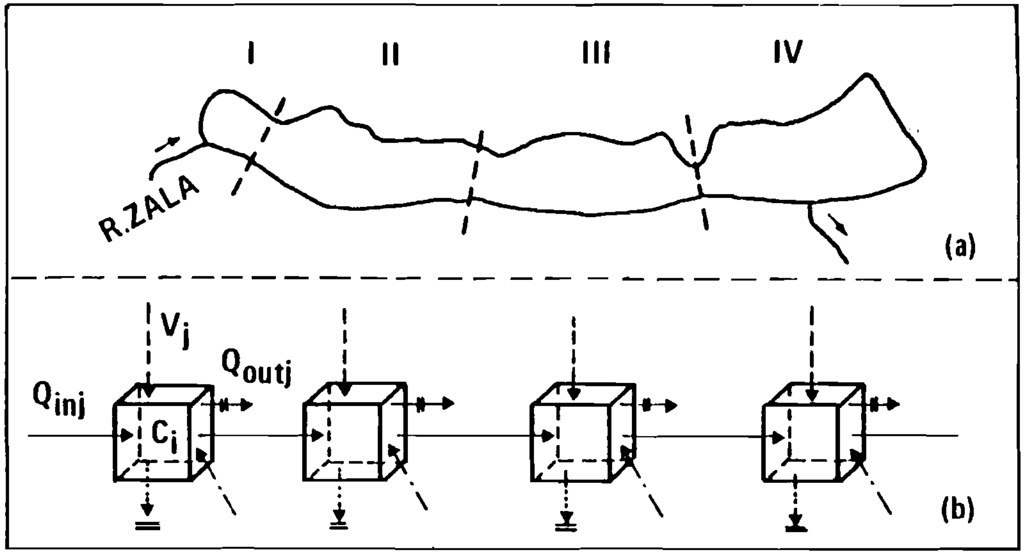
A detailed description of the model's nonlinear equations are the result of a complex kinetic reaction which is presented in the next section of this paper.

3. MATHEMATICAL FORMULATION

The water area of Lake Balaton is subdivided into j number of sectors characterized by specific concentrations of phosphorus compounds, phytoplankton levels and phosphorus loading rates. The subdivision of the lake into four sectors is considered to be quite sufficient for simulating the eutrophication processes in the water ecosystem of Lake Balaton (van Straten et al. 1979). Each sector within the lake has volume V_j and is represented during modeling as a completely mixed system, similar to a stirred tank reactor. The model basins are coupled horizontally by advective transport (Figure 2).

The model itself is constructed on the basis of mass conservation principles for each model compartment which is given by a set of coupled ordinary differential equations. The general form of the model equation is:

$$\frac{dCi}{dt} = R_i + CZ_i + (Q_{in_j}/V_j) \cdot C_i^o + (Q_{pr_j}/V_j) \cdot C_{ir} - (Q_{out_j}/V_j) \cdot C_i \quad , \tag{1}$$



j is number of basins
V_j is volume of each basin
Q_{inj} and Q_{outj} are characteristics of net water advective transport
C_i is concentration of specific constituents
——→ input of C_i by water flow; —‖→ output of C_i by water flow
– – → input of C_i by precipitation;
– · → input of C_i from external sources; ·······→ output of P_D by sedimentation

Figure 2. Subdivision of Lake Balaton by basins (a) and schematic presentation for calculating the concentration distributions

where

C_i and C_i^o are the concentrations of particular compounds in the basin under consideration and former basins, respectively (in mg/ℓ);

Q_{in_j}, Q_{out_j}, and Q_{pr_j} are input, output flow rates and precipitation rates, respectively (all m^3/day);

C_{ir} is DOP and DIP concentrations in rain water (in mg P/ℓ);

R_i is the sum of the reaction rates of biochemical processes taken into account in the model (mg/ℓ-day);

CZ_i is the direct phosphorus loading rate from all external sources (mg P/ℓ-day).

Thus, phosphorus compounds may be increased or decreased in concentration by biochemical reactions, physical advective transport and phosphorus loading. Oxygen may be taken up or produced in conjunction with biochemical transformation, and is transferred by physicochemical reaction processes.

The general form of the model equation shows that the mass balance of all model compartments and the phosphorus system as a whole will be regulated by rates of biochemical reactions and loadings. The effect of hydrodynamical processes on the rates of mass balance changes is considered in the simplest way. It means that among all hydrodynamical processes only the advective transport of substances is taken into account in the given model on a long-term basis.

3.1. Phytoplankton

The mass balance of phytoplankton-phosphorus in the model, is presented by the expression relating the instantaneous rates of nutrient uptake, metabolic excretion and mortality. It also takes into account the effect of water transfer. Thus, the total equation is:

$$\frac{dF}{dt} = \underbrace{(UP_F - L_F - M_F)F}_{\textit{net growth}} + \underbrace{(Q_{in}/V) \cdot F^0}_{\textit{input by water flow}} - \underbrace{(Q_{out}/V) \cdot F}_{\textit{output by water flow}} \qquad (2)$$

where

UP_F, L_F, and M_F are specific rates of uptake, excretion and mortality of phytoplankton (all day^{-1}), whose phytoplankton biomass F, is presented as its phosphorus content (in mg P/ℓ);

F^0 is phytoplankton concentration in the adjacent "upstream" sector of the lake (mg P/ℓ); other parameters are mentioned above.

Most of the eutrophication models developed previously, use the classical Monod kinetic principles for description of nutrient uptake by phytoplankton (Nelson 1971). There are also attempts to describe nutrient phytoplankton uptake as a function of intracellular nutrient level (Nyholm 1978). The present model is constructed on similar principles with a small modification. In considering the question of nutrition for plankton organisms, Sushchenya (1973) found that as food concentration increased, the increased quantity of food consumed was less than expected, giving a progressive decrease in uptake rate. The relation between the quantity of nutrients consumed by plankton per unit of time and food concentration, have a tendency to be asymptotic. A number of available experimental data allow us to apply this regularity to phytoplankton (Finenko and Krupatkina-Anikina 1974; Straskraba 1977) as well as zooplankton organisms (Sushchenya 1973).

The hypothesis used here is that the specific uptake rate of DIP by phytoplankton depends on the phosphorus content in phytoplankton cells and in the water environment. A ratio of intracellular phosphorus content in phytoplankton to available DIP content in the water, regulates the total specific uptake rate of DIP by phytoplankton, when temperature and light conditions are optimal. Thus, the equation for the specific uptake rate of DIP by phytoplankton is formulated as:

$$UP_F = \frac{K_1 \cdot R_{TF} \cdot R_{IF}}{1 + \frac{F}{\beta \cdot DIP}} \qquad (3)$$

where

K_1 is the maximum uptake rate of DIP by phytoplankton (day^{-1});

R_{TF} and R_{IF} are coefficients for correction of the maximum uptake rate for temperature and light conditions, respectively (nondimensional parameters);

β is the coefficient of substrate conversion per unit biomass (nondimensional parameter).

Actually, equation (3) shows that at optimal temperature and light conditions for each level of phytoplankton biomass, there exists an uptake rate defined by phosphorus content in the water environment. A three dimensional graph (Figure 3) shows that curves of uptake rate by phytoplankton at optimal temperature and light, remain of the same shape for each level of phytoplankton biomass, but steepness or configuration of these curves is regulated by ratio $F/\beta \cdot DIP$. A similar three dimensional graph was also referred to by Finenko (1978) for adsorption of inorganic phosphorus by algae, at different phosphate concentrations in the medium and cells, on the basis of experimental data provided by Fuhs et al. (1971). Thus, equation (3) is a modification of the classical Monod approach in which, instead of the Michealis constants, the phytoplankton biomass in phosphorus units is used. Mar (1976) has suggested a similar equation for phytoplankton uptake at the low level of nutrients, in a discussion of Michealis-Menten constants. Bierman and Richardson (1976) used the same equation structure for modeling different phytoplankton species in Saginaw Bay, Lake Huron.

For lack of a better assumption, it is assumed in the first version of the model that the optimal temperature for phytoplankton activity is 24-26^0, and that phytoplankton activity at a low temperature, close to 0^0C, decreases by five times. This assumption is in reasonable agreement with available information about phytoplankton activity in Lake Balaton during winter (Herodek and Oláh 1973). The rate reduction factor, R_{TF}, is calculated in the model in accordance with dependence shown in Figure 4a using the expression (Leonov 1980):

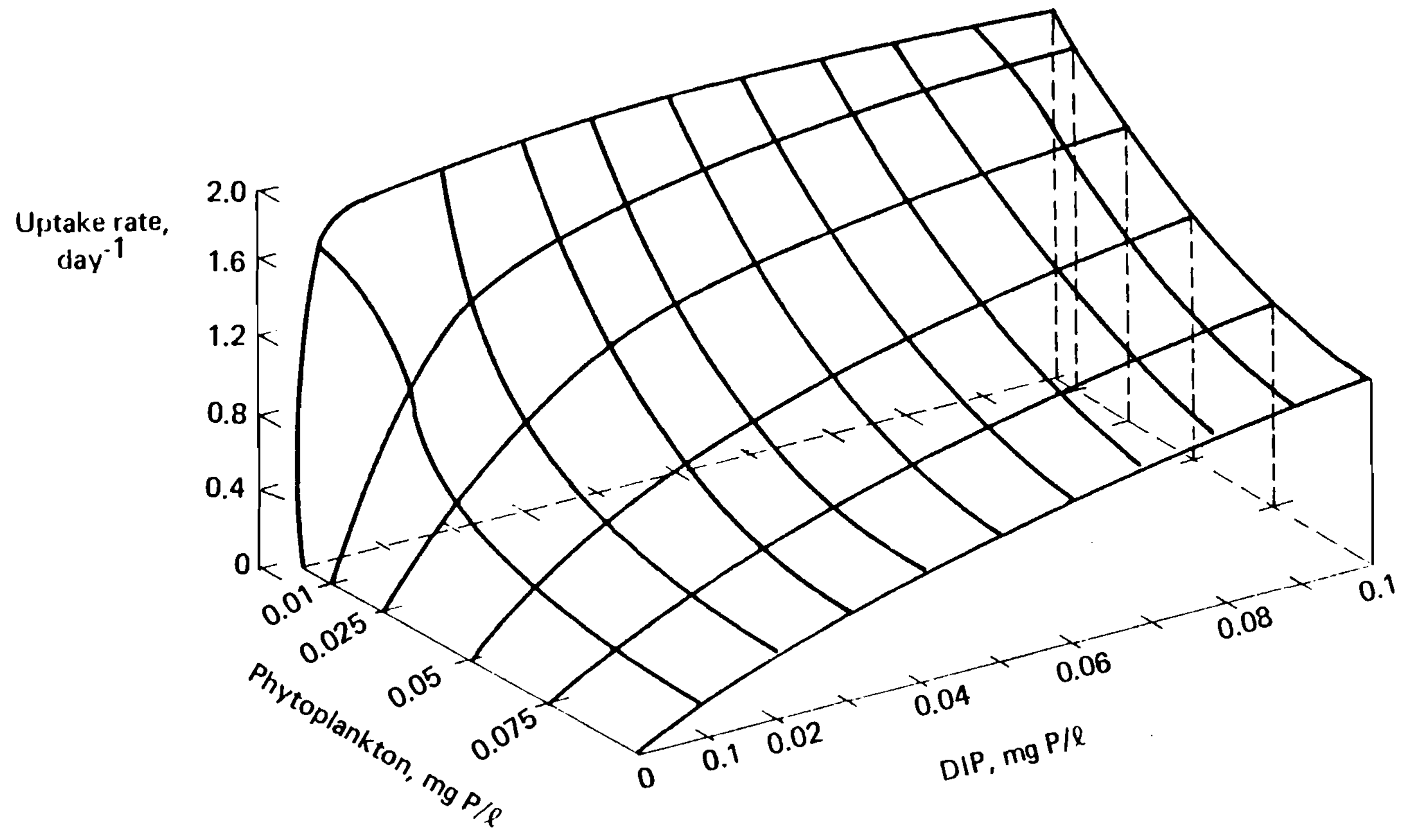


Figure 3. Functional relations of uptake rate of DIP by phytoplankton with nutrient content and biomass of phytoplankton, constructed on equation (3): $K_1 = 2.0\ day^{-1}$; $R_{TF} = R_{IF} = 1$; $\beta = 1$.

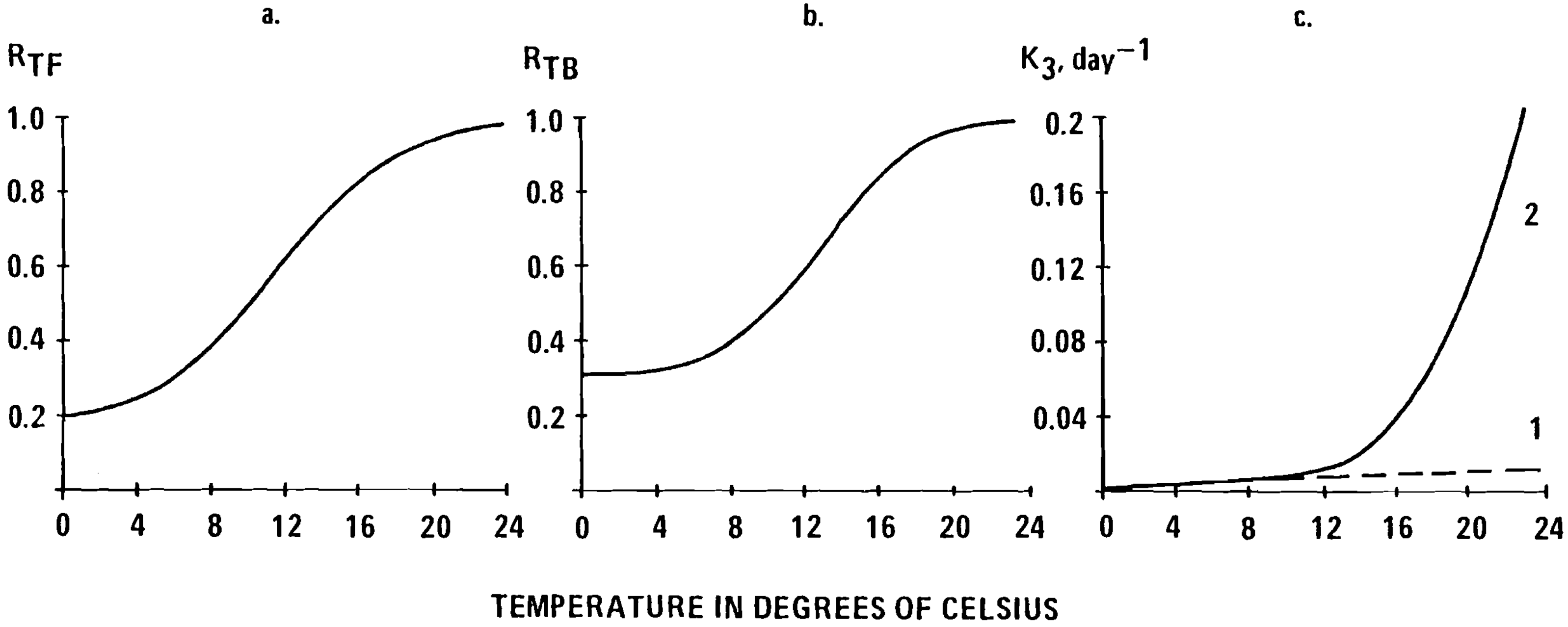


Figure 4. Rate characteristics as a function of temperature

a. uptake rate reduction factor for phytoplankton,
b. uptake rate reduction factor for bacteria
c. rate constant of decomposition of nonliving particulate phosphorus to DOP: lines 1 and 2 are constructed on equation (24) and (24a), respectively.

$$R_{TF} = 0.2 + \frac{0.022(e^{0.31T}-1)}{1 + 0.028e^{0.31T}} \quad (4)$$

where

T is water temperature in ^{0}C.

Nonoptimal light conditions reduce the specific uptake rate, UP_F. The effect of light on phytoplankton growth has been studied by various authors, namely by Steele (1962), Vollenweider (1965), and Jassby and Platt (1976). The results of these studies have been presented in comparatively simple mathematical equations used in various mathematical models of water ecological systems (Di Toro et al. 1971). The same principles were used in the given model. The functional form for the effect of light, R_{IF}, is given by a similar expression to that applied by Di Toro et al. (1971):

$$R_{IF} = \frac{e}{K_e \cdot h} [\exp(-r_x) - \exp(-r_1)] \quad (5)$$

where

$$r_x = \frac{I}{I_{opt}} [\exp(-K_e \cdot h)] \quad (6)$$

$$r_1 = \frac{I}{I_{opt}} \quad (7)$$

where

h is the depth that is considered in the model as constant, and equal to 0.5 m;

I is daily average light intensity in (cal/cm^2-day);

I_{opt} is the optimal light intensity that is assumed to be equal 350 cal/cm^2-day;

K_e is the extinction coefficient.

Although it is known that magnitudes of extinction coefficient in Basins II-IV in a large degree depends on the suspended solid concentration (van Straten 1980a), this fact is not taken into account in a given model. The extinction coefficient is calculated as a function of phytoplankton concentration using the following expression:

$$K_e = K_a + K_b \cdot \text{Chl "a"} \qquad (8)$$

K_e has units m^{-1}; K_a and K_b are constants; chlorophyll "a" has units μg/ℓ and its value is recalculated from phytoplankton phosphorus using a simple ratio:

$$\mu g \text{ Chl "a"}/\ell = \gamma \cdot mg\ P/\ell \qquad (9)$$

where

γ is a constant stoichiometric coefficient.

In the given model, the daily course of light intensity, I, following Golterman (1975) is given by:

$$I = I_{max} \cdot h\ [1 + \cos \frac{2\pi (t_{now} - t_{peak})}{f}] \qquad (10)$$

where

I_{max} is maximum light intensity;
t_{now} is current time of day in hours;
t_{peak} is time of maximum light intensity (12 o'clock);
f is photoperiod in hours.

Values of I_{max} are calculated from:

$$I_{max} = 2 \cdot I_{av}/f \qquad (11)$$

where

I_{av} is mean daily light intensity.

The formulation of the phytoplankton excretion rate L_F, is the result of a previous study (Leonov 1978). The excretion rate is considered to be a fraction of the specific uptake rate of nutrients by phytoplankton:

$$L_F = r_F \cdot UP_F \qquad (12)$$

where

r_F is the coefficient representing the fraction of excretion over uptake.

It is assumed that this fraction is not constant, and it depends on the nutrient content in the water, through the specific rate of nutrient uptake described by equation (3). Thus, the excretion rate expression is given by:

$$r_F = \frac{(a_1/a_2)UP_F}{(1/a_2) + UP_F} + 1 - (a_1/a_2) \tag{13}$$

where

a_1 and a_2 are coefficients with the dimensions of the days.

Equation (13) shows that the specific excretion rate is defined by the term $(1-a_1/a_2)$ at limited nutrient content when the specific uptake rate is low. When the DIP content is not limited, the fraction of excretion depends on values of specific uptake rate, and this dependence is asymptotic. Therefore, the first term of equation (13) is presented by the Monod expression.

For describing phytoplankton mortality M_F, the hypothesis used is that the biomass of phytoplankton and specific uptake rate of nutrients by phytoplankton, are factors which regulate the rate of mortality of phytoplankton, when the role of predators is neglected. Therefore, the expression for mortality rate of phytoplankton is given by:

$$M_F = \frac{v_1 \cdot F}{UP_F} \tag{14}$$

where

v_1 is constant $[(\text{mg } P/\ell)^{-1} \cdot (\text{day})^{-2}]$.

This equation shows that when food is abundant, i.e. the specific uptake rate is high and the level of biomass is low, then survival of phytoplankton is high and the biomass of phytoplankton has a tendency to increase. When food is limited, the specific uptake rate is low, then survival is low and biomass decreases. Thus, increased mortality must occur when nutrient content is limited. The constant v_1 regulates this interaction. A three-dimensional graph illustrates this dependence (Figure 5).

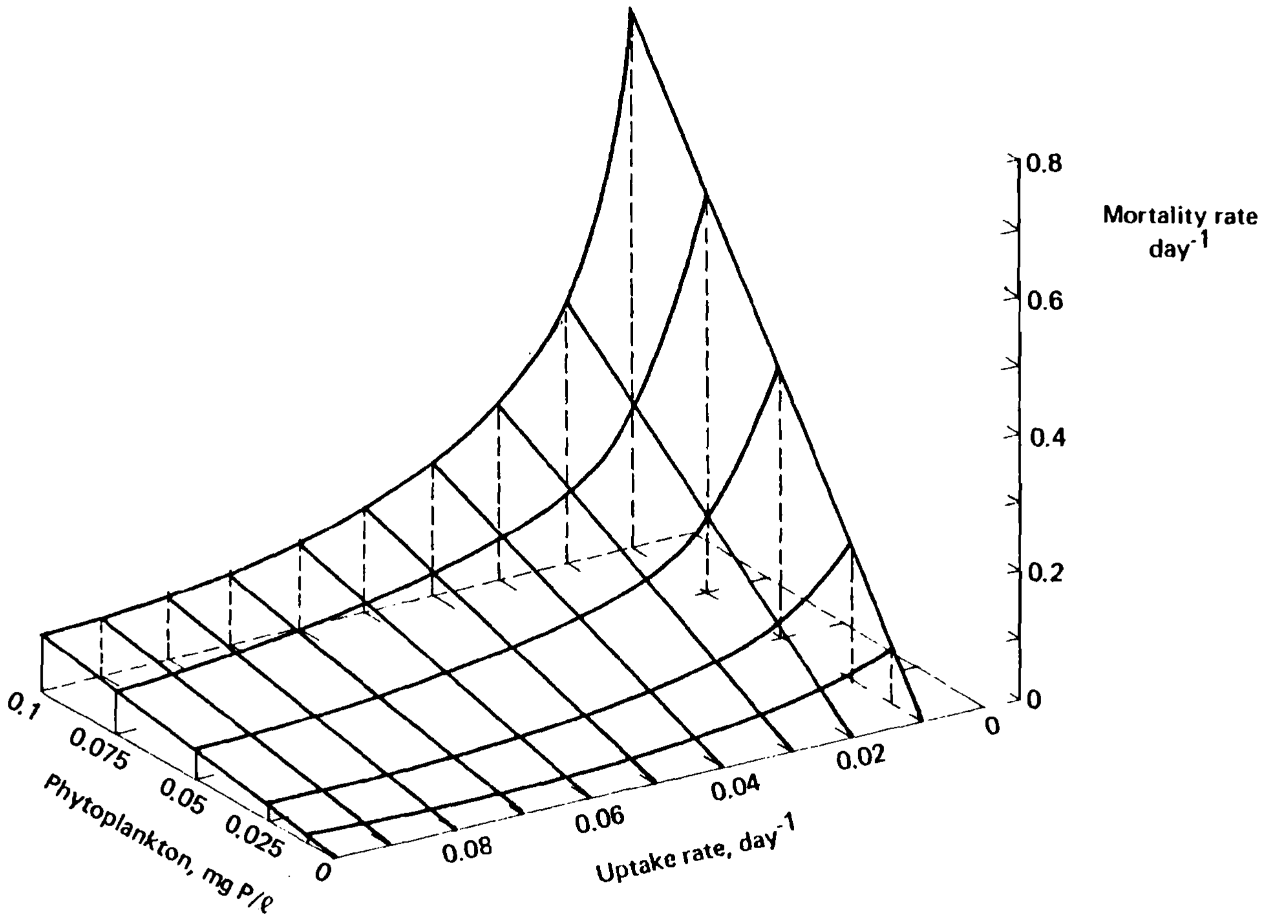


Figure 5. Response surface of phytoplankton mortality rate versus phytoplankton biomass and uptake rate, constructed on equation (4): $v_1 = 0.08$.

Two three-dimensional graphs shown in Figure 6 illustrate a complex dependence between the rate of phytoplankton production and phytoplankton biomass and nutrient level (Figure 6a) and temperature and light (Figure 6b). These plots are constructed using equations (3-14) at the following values of constraints: $K_1 = 2$; $a_1 = 0.057$, $a_2 = 0.075$; $v_1 = 0.08$; $K_e = 1.7$; $\beta = 1$. Plots are very similar to those presented by Straskraba (1978), which explain the hypothesis of multiple resource kinetics of phytoplankton photosynthesis.

3.2. Bacteria

The equation for bacterial kinetics is analogous to that used for phytoplankton:

$$\frac{dB}{dt} = \underbrace{(UP_B - L_B - M_B) \cdot B}_{\textit{net growth}} + \underbrace{(Q_{in}/V) \cdot B^0}_{\textit{input by water flow}} - \underbrace{(Q_{out}/V) \cdot B}_{\textit{output by water flow}} \qquad (15)$$

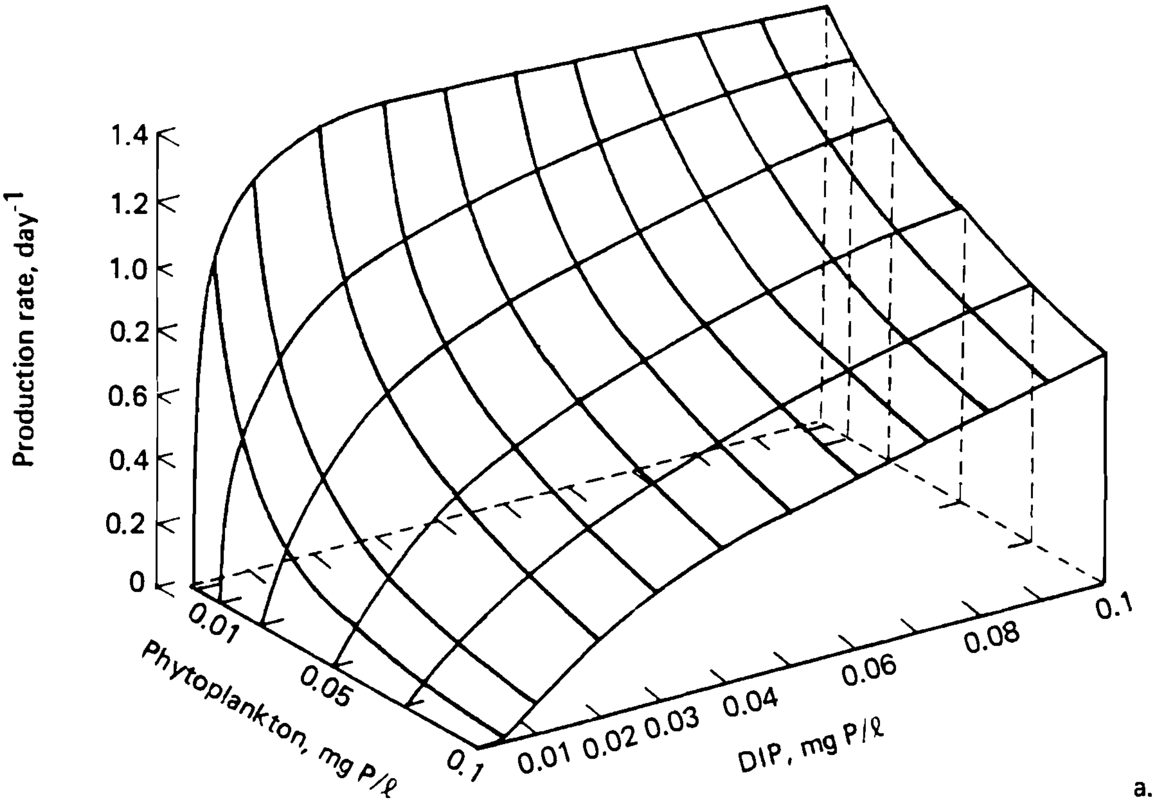
where

UP_B, L_B and M_B are specific uptake, excretion and mortality rates of heterotrophic bacteria, whose bacterial biomass, B, is expressed as its phosphorus content in mg P/ℓ.

B^0 is bacterial concentration in the upstream basin of the lake (mg P/ℓ).

The same assumptions as for phytoplankton were used for the formulations of these terms for heterotrophic bacteria. Therefore, the specific rates of uptake, excretion and mortality are given by equations similar in structure to those for phytoplankton. These equations (16-19) are presented in Table 1.

It is assumed that the maximum activity of heterotrophic bacteria occurs at 22-25°C (Leonov 1979). Just the same values



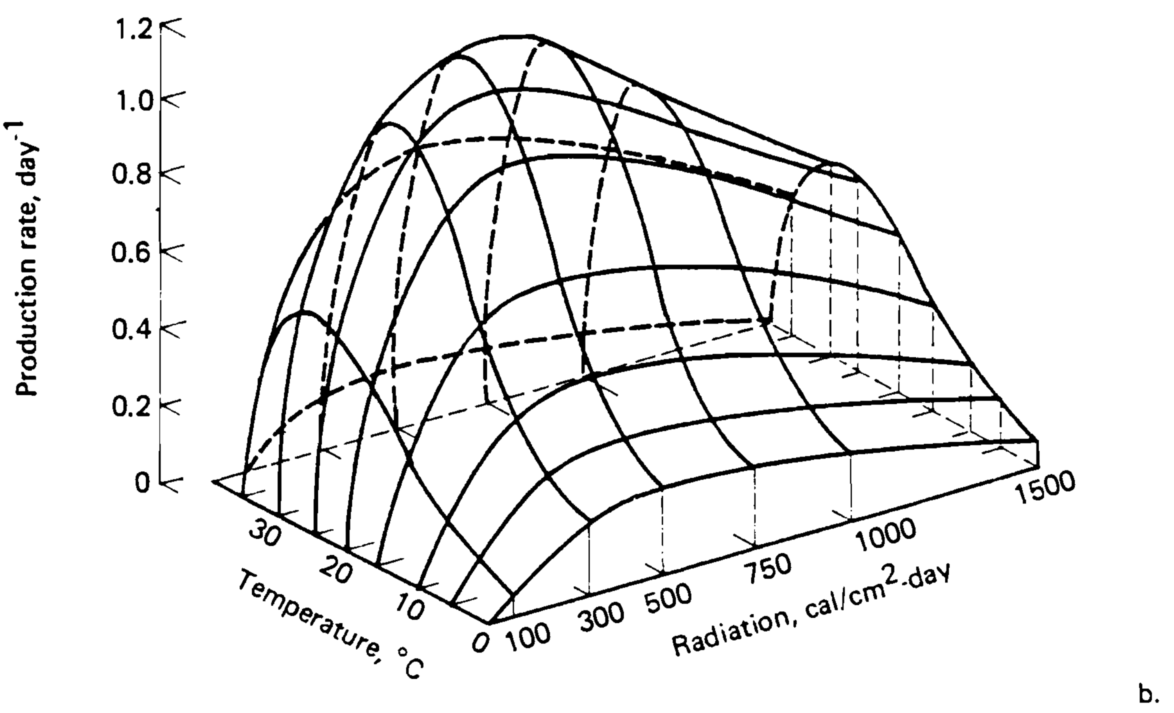


Figure 6. Complex dependence of rate of phytoplankton production on environmental factors such as nutrient level (a) and temperature and light (b).

Table 1. Equations for description of bacterial metabolism.

Bacterial functions	Equations	Numbers
Specific uptake rate	$UP_B = \dfrac{K_2 \cdot R_{TB}}{1 + \dfrac{B}{DOP}}$	(16)
Specific excretion rate	$L_B = r_B \cdot UP_B$	(17)
Fraction for excretion from uptake	$r_B = \dfrac{(a_3/a_4)\,UP_B}{(1/a_4) + UP_B} + (1 - a_3/a_4)$	(18)
Specific mortality rate	$M_B = v_2 + \dfrac{v_3 \cdot B}{UP_B}$	(19)

Note:

R_{TB} = temperature uptake rate reduction factor

K_2 = maximum uptake rate of DOP (day^{-1})

a_3 and a_4 = coefficients with the dimensions of days

v_2 = rate of natural mortality of bacteria (day^{-1})

v_3 = coefficient which regulates mortality as a function of biomass level and nutrient content $[(mg\ P/\ell)^{-1} \cdot (day^{-2})]$

of temperature is observed in Lake Balaton during summer when bacteria is most active (Oláh 1969) and the intensive self-purifying ability of Lake Balaton water is found (Oláh 1969). The following equation is used in the model to describe the temperature influence on bacteria growth according to the dependence shown in Figure 4b:

$$R_{TB} = 0.3 + \frac{3.68 \cdot 10^{-3}(e^{0.403T} - 1)}{1 + 5.25 \cdot 10^{-3}e^{0.403T}} \qquad (20)$$

where

T is temperature in ^{0}C.

3.3. Other Phosphorus Compounds

The major phosphorus compounds in the water environment included in the model, are nonliving dissolved organic phosphorus (DOP) and dissolved inorganic phosphorus (DIP) (Figure 1). The mechanisms of phosphorus transformations consider the stage of bacterial consumption of DOP an important step for DIP formation, because bacteria excrete DIP into the water. The phytoplankton take up DIP, and this stage is one of the sources of DOP which is excreted by phytoplankton during metabolism. The second source of DOP is the stage of nonliving particulate phosphorus transformation which is assumed to be a first-order reaction with a temperature-dependent rate coefficient. Nonliving particulate-P, P_D, includes detrital organic phosphorus and other suspended material of different origin. Detrital organic phosphorus consists of particulate compounds produced by phytoplankton and bacteria. The sink of nonliving particulate phosphorus included in the formulation is sedimentation. Thus, the equations used for description of DIP, DOP, and P_D transformations are:

$$\frac{dDIP}{dt} = \underbrace{L_B \cdot B}_{\text{bacterial excretion}} - \underbrace{UP_F \cdot F}_{\text{phyto-plankton uptake}} + \underbrace{CZ_2}_{\text{external input}} + \underbrace{s_2 \cdot DIP_s}_{\text{sediment release}}$$

$$+ \underbrace{(Q_{pr}/V) \cdot DIP_r}_{\text{input with rainfall}} + \underbrace{(Q_{in}/V) \cdot DIP^0}_{\text{input by water flow}} - \underbrace{(Q_{out}/V) \cdot DIP}_{\text{output by water flow}} \qquad (21)$$

$$\frac{dDOP}{dt} = \underbrace{K_3 \cdot P_D}_{\text{detritus decay}} + \underbrace{L_F \cdot F}_{\text{phyto-plankton excretion}} - \underbrace{UP_B \cdot B}_{\text{bacterial uptake}} + \underbrace{CZ_1}_{\text{external input}}$$

$$+ \underbrace{(Q_{pr}/V) \cdot DOP_r}_{\text{input with rainfall}} + \underbrace{(Q_{in}/V) \cdot DOP^0}_{\text{input by water flow}} - \underbrace{(Q_{out}/V) \cdot DOP}_{\text{output by water flow}} \qquad (22)$$

$$\frac{dP_D}{dt} = \underbrace{M_F \cdot F}_{\text{phytoplankton mortality}} + \underbrace{M_B \cdot B}_{\text{bacterial mortality}} - \underbrace{K_3 \cdot P_D}_{\text{decay}} - \underbrace{s_1 \cdot P_D}_{\text{net sedimentation}}$$

$$+ \underbrace{CZ_3}_{\text{external input}} + \underbrace{(Q_{in}/V) \cdot P_D^{\,0}}_{\text{input by water flow}} - \underbrace{(Q_{out}/V) \cdot P_D}_{\text{output by water flow}} \qquad (23)$$

where

K_3 and s_1 are rate coefficients for nonliving particulate phosphorus decomposition and sedimentation (both day^{-1});

CZ_1, CZ_2 and CZ_3 are input rates of DOP, DIP and P_D into the lake, from all external sources (all mg P/ℓ-day);

s_2 is first-order rate of DIP input from sediment (day^{-1});

DIP_r and DOP_r are concentrations of phosphorus forms in rain water (mg P/ℓ).

Decomposition of nonliving particulate phosphorus to DOP is a temperature-dependent process (Zison et al. 1978) and therefore, in the given model, the rate constant of this process, K_3, is regulated by temperature. In initial simulation runs (Leonov 1980) a linear relationship was used for temperature correction of the first-order rate coefficient, K_3:

$$K_{3(T)} = K_{3(0^0)} \cdot T \qquad (24)$$

where

T is water temperature in ^{0}C;

$K_{3(T)}$ and $K_{3(0^0)}$ are the reaction rates at T and 0^0C.

For Keszthely Bay basin, a good approximation was arrived at with $K_{3(0^0)} = 0.0005 \text{ day}^{-1}$ (Leonov 1980). However, the value of $K_{3(0^0)}$ for other basins of Lake Balaton (where input of particulate phosphorus from the watershed is much lower) did not give satisfactory results, and it was evident that decomposition of P_D must be more intensive (at least 5-10 times) when the temperature is more than 15°C.

In the next step of model examination, a linear equation (24) in the model was changed to an exponential equation (24a):

$$K_{3(T)} = \frac{1.2 \cdot 10^{-4} \, (e^{0.351T} - 1)}{1 + 3 \cdot 10^{-4} e^{0.351T}} \qquad (24a)$$

Figure 4c shows that values of K_3 calculated using equation (24) and (24a), are practically similar for temperatures of 0-12°C. When the temperature is higher than 12°C, the values of K_3 calculated by equation (24a) are exponentially increased. The parameters of equation (24a) were chosen after several simulation runs and a comparison of modeling results with available observation data.

3.4. Oxygen

Dissolved oxygen content in the water is defined by simultaneous processes of respiration of phytoplankton and bacteria, atmospheric reaeration and photosynthetic production.

Thus, the equation for the dynamics of dissolved oxygen is:

$$\frac{dO_2}{dt} = \underbrace{g_1 \cdot UP_F \cdot F}_{\textit{photosynthesis}} - \underbrace{K_4 (O_2 - O_2^{sat})}_{\textit{reaeration}} - \underbrace{h_1 \cdot L_B \cdot B}_{\textit{bacterial respiration}}$$

$$- \underbrace{h_2 \cdot L_F \cdot F}_{\textit{phytoplankton respiration}} + \underbrace{(Q_{in}/V) \cdot O_2^0}_{\textit{input by water flow}} - \underbrace{(Q_{out}/V) \cdot O_2}_{\textit{output by water flow}} \qquad (25)$$

where

0_2 and 0_2^0 are the concentrations of oxygen in the basin under study and the former (upstream) basin of the lake, respectively (mg $0_2/\ell$);

K_4 is the first-order rate constant of atmospheric reaeration (day^{-1});

h_1 and h_2 are stoichiometric coefficients;

0_2sat is oxygen concentration at saturation level (mg $0_2/\ell$);

g_1 is a regulation parameter.

The first-order rate constant of atmospheric reaeration, is a temperature-dependent parameter. Exponential temperature dependence is used in this model for temperature correction of this constant:

$$K_{4(T)} = K_{4(20^0)} \cdot 1.05^{(T-20)} \tag{26}$$

where

T is temperature in degrees centigrade;

$K_{4(20^0)}$ is the rate at 20^0C;

1.05 is temperature coefficient.

Photosynthetic production of oxygen, is considered here to be connected with assimilation of nutrients by phytoplankton i.e., to a specific uptake rate, UP_F. Coefficient g_1 is a regulator of photosynthesis and it is equal to:

$$g_1 = \begin{cases} g_2 & \text{for daylight hours} \\ 0 & \text{for nighttime hours} \end{cases} \tag{27}$$

where

g_2 is a stoichiometric coefficient i.e., ratio of oxygen to phytoplankton phosphorus.

The concentration of oxygen in the water at saturation level, is given by the polynomial temperature function (L. Wang and Wang Mu-Hao 1976):

$$O_2^{sat} = 14.61996 - 0.4042 \cdot T + 0.00842 \cdot T^2 - 0.00009 \cdot T^3 \qquad (28)$$

where

T is temperature in degrees centigrade.

3.5. Phosphorus Compounds in Sediment

The mechanisms of phosphorus transformations in sediment assumed by the model include the input of nonliving particulate phosphorus by sedimentation, and the consequent transformation of phosphorus from the particulate form to dissolved forms (first to DOP and then to DIP). Typical processes are biochemical oxidation and decomposition of sedimented particulate phosphorus material, oxidation and reduction of associated inorganic compounds, chemical and physical adsorption, precipitation and dissolution of phosphorus to and from mineral forms (Williams and Mayer 1972). However, the theoretical background and kinetics of most of these reactions have still been incompletely studied. It seems justifiable to consider phosphorus transformation in sediment in a simplified form.

In the model, first-order kinetic terms have been adopted to describe the sedimentation of particulate phosphorus from water into the sediment-water interface, the transformation of sedimented particulate phosphorus first to DOP and then to DIP and the release of DIP from the sediment into the water. The mechanisms of phosphorus transformation which contributed to the model, are presented in the following equations:

$$\frac{d\, P_{Ds}}{dt} = s_1 \cdot P_D - s_3 \cdot P_{DS} \qquad (29)$$

$$\frac{d\, DOP_s}{dt} = s_3 \cdot P_{DS} - s_4 \cdot DOP_S - CZ_4 \qquad (30)$$

$$\frac{d\, DIP_s}{dt} = s_4 \cdot DOP_S - s_2 \cdot DIP_S \qquad (31)$$

where

s_1 and s_2 are mentioned above (see equations (23) and (21);

s_3 and s_4 are first-order temperature-dependent rate constants of transformations of P_{Ds} to DOP_s and DOP_s to DIP_s, respectively (day^{-1});

CZ_4 is the rate of DOP conservation in the sediment (mg P/ℓ - day).

The exponential temperature dependence shown by equation (26) appears to be simplest and justifiable in the application for temperature correction of s_3 and s_4.

4. DATA BASE

A detailed analysis of all the available data required for simulating phosphorus compounds and phytoplankton dynamics in relation to the eutrophication problem was made by van Straten et al. (1979) for different basins of the lake. The present report contains only a brief description of the data collected in 1977.

All the existing data at IIASA on Lake Balaton may be subdivided into three groups:

1. Physical, meteorological and hydrological data.
2. Tributary stream influence and watershed nutrient loading data.
3. Open water phosphorus, nitrogen and phytoplankton data.

These data have been computerized at IIASA's computer center. Data from the first and second groups were used in simulation runs as direct input characteristics that regulate the rates of change of all processes considered by the model.

The first group of data include information on the mean daily measurements of water temperature, solar radiation, photoperiod and also water balance characteristics. The variations of daily average water temperature, solar radiation and photoperiod for 1977, are shown is Figure 7. The water balance data for each basin in Figures 8-11, together with the functional relationship represented by the model is used in computation. Water balance data include the weekly measurements of discharge flow rates of the River Zala, monthly average input-output flow rates, and precipitation rates for all basins of Lake Balaton. The volumes of each basin are presented in Table 2.

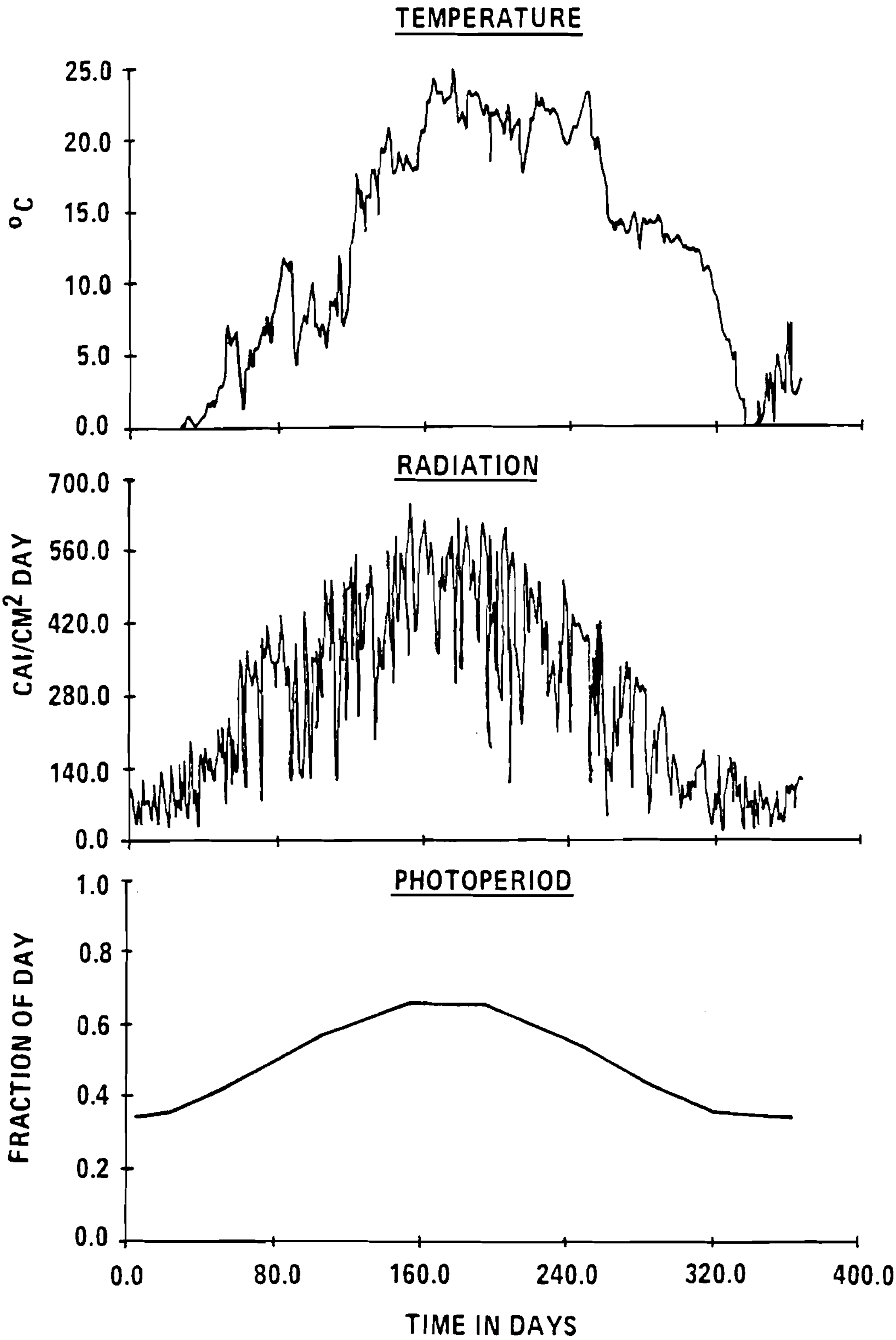


Figure 7. Variation of daily average values of water temperature, solar radiation, and photoperiod for Lake Balaton in 1977.

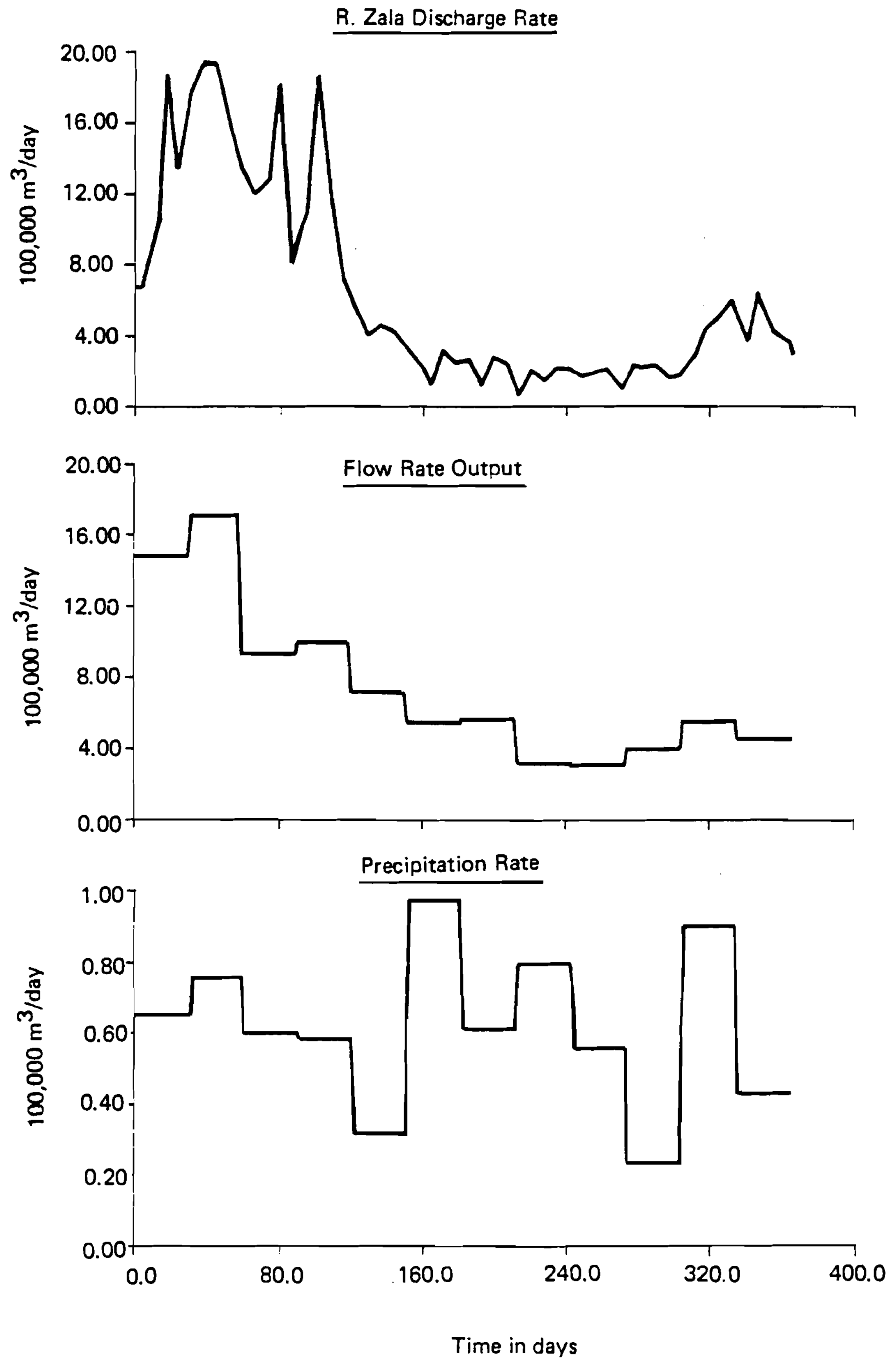


Figure 8. Water balance data for 1977, Lake Balaton, Basin I.

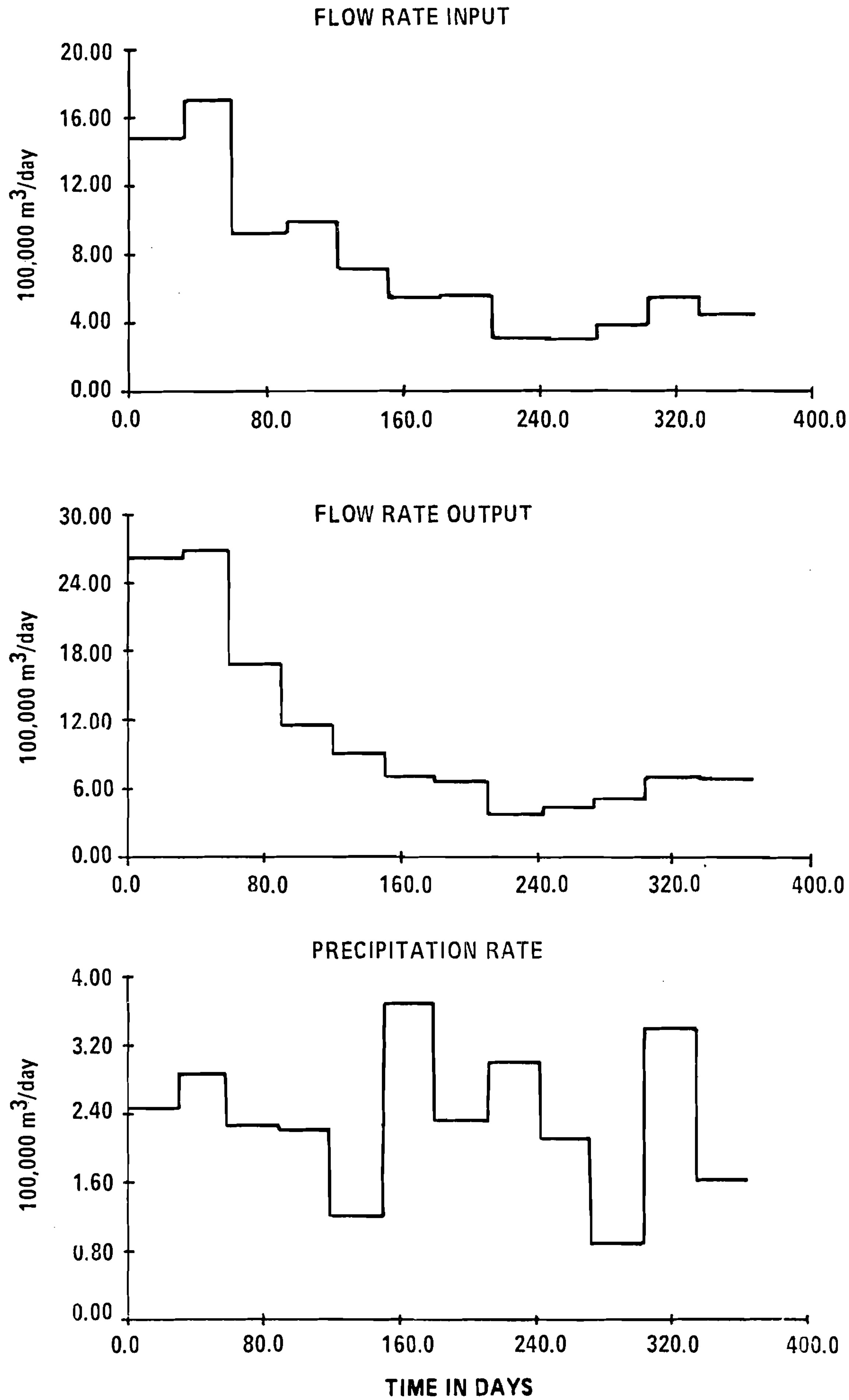
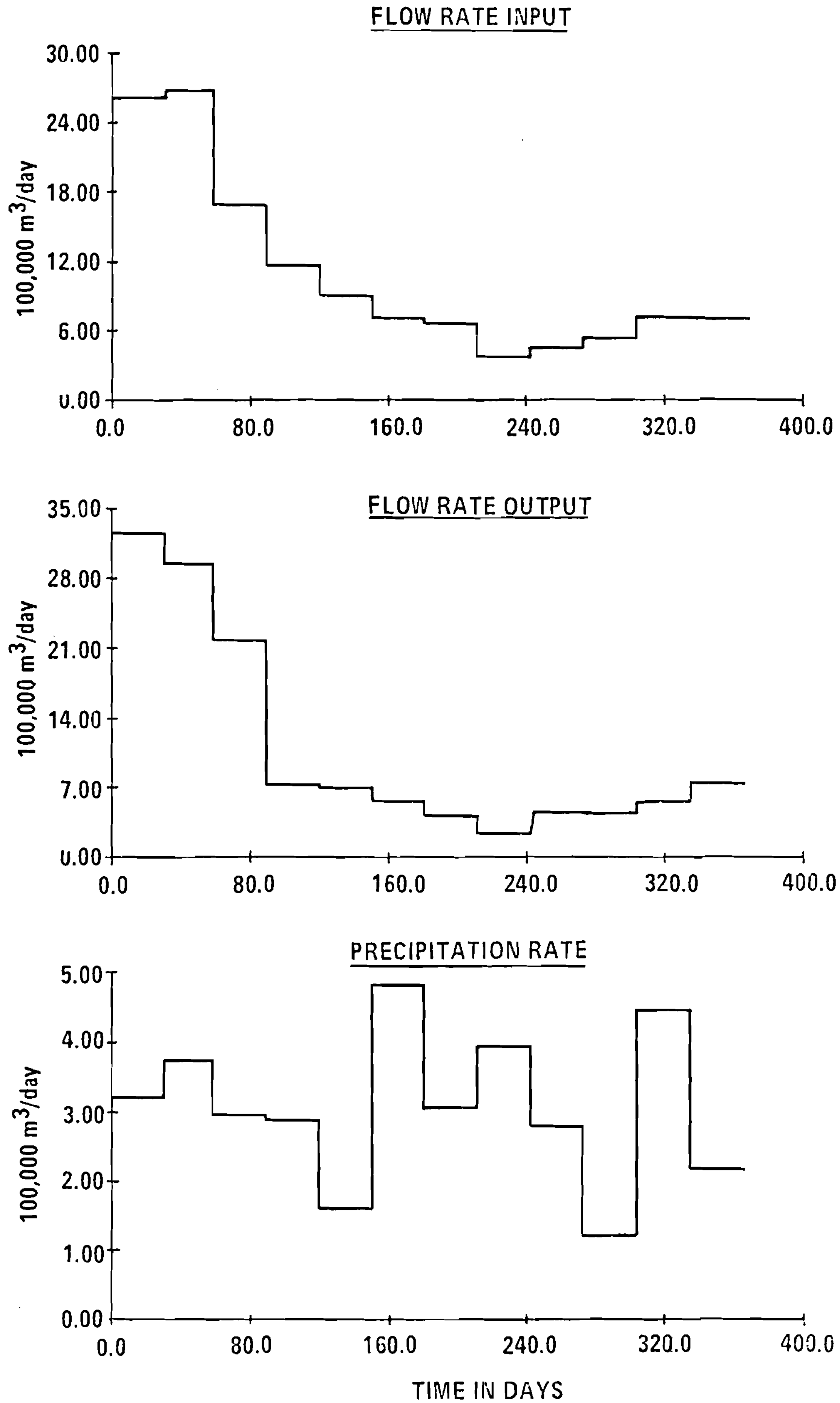


Figure 9. Water balance data for Lake Balaton, 1977, Basin II.


FLOW RATE INPUT
30.00
24.00
18.00
12.00
6.00
0.00
100,000 m3/day
0.0
80.0
160.0
240.0
320.0
400.0
FLOW RATE OUTPUT
35.00
28.00
21.00
14.00
7.00
0.00
100,000 m3/day
0.0
80.0
160.0
240.0
320.0
400.0
PRECIPITATION RATE
5.00
4.00
3.00
2.00
1.00
0.00
100,000 m3/day
0.0
80.0
160.0
240.0
320.0
400.0
TIME IN DAYS

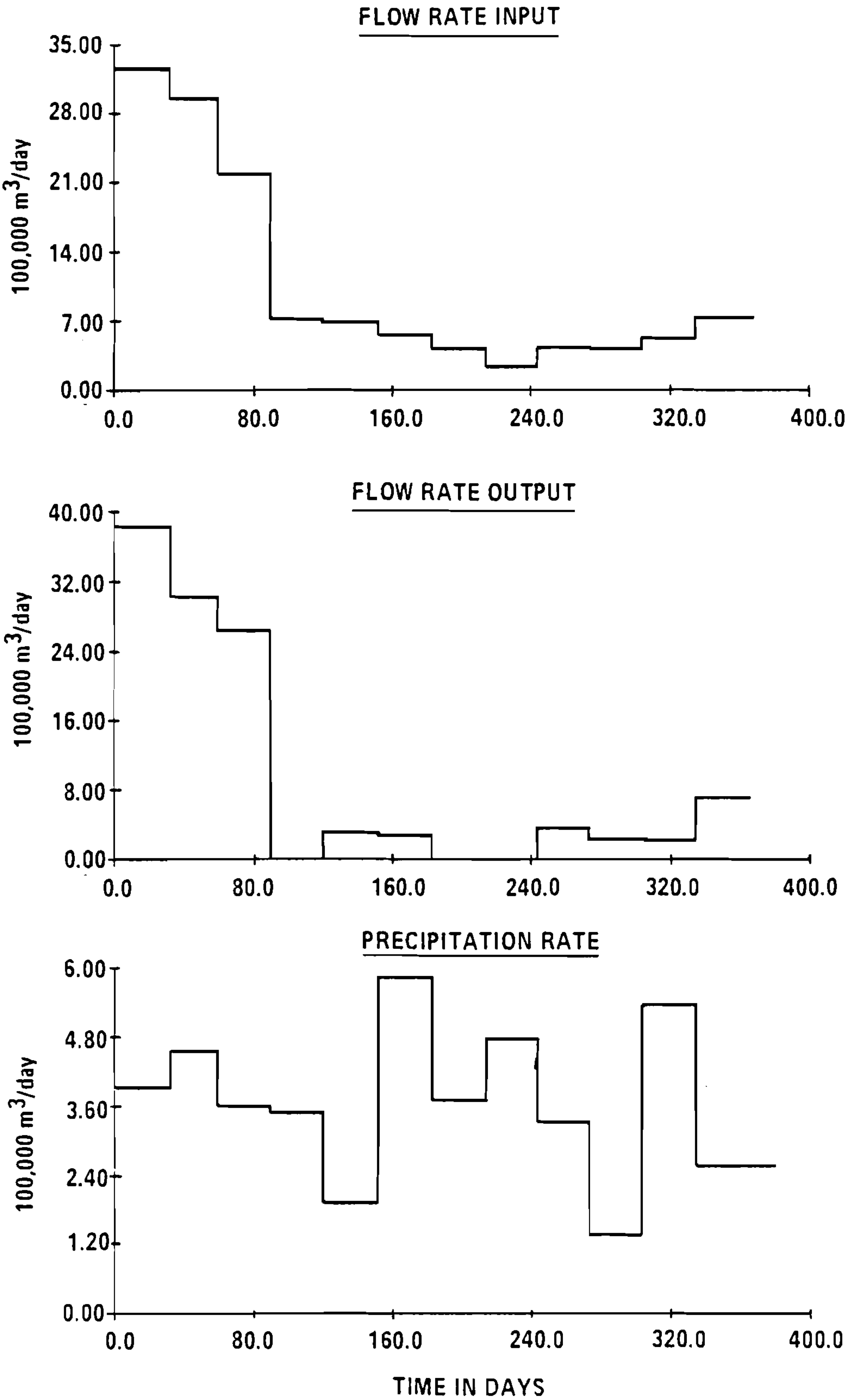


Figure 11. Water balance data for 1977, Lake Balaton, Basin IV.

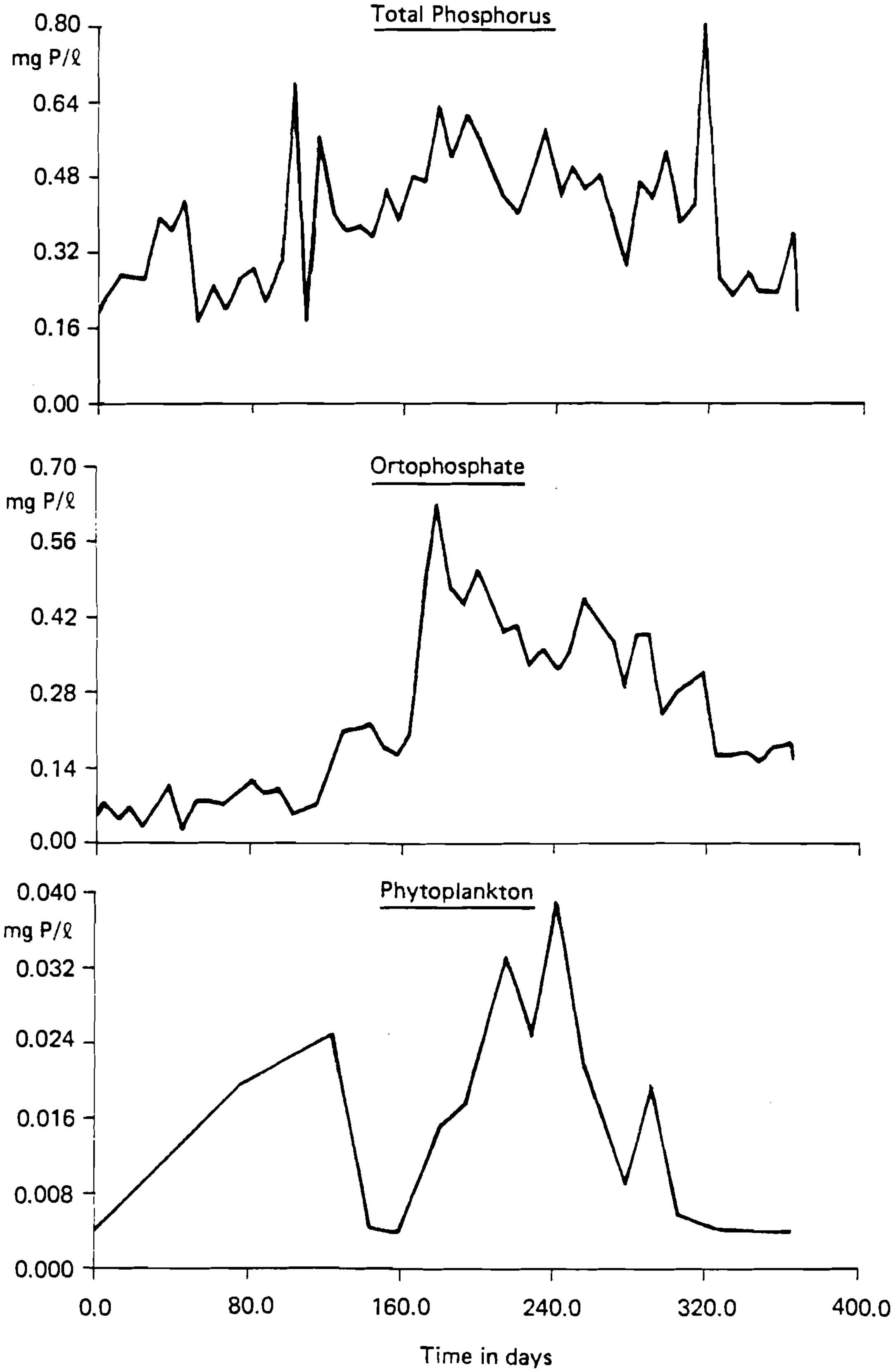


Figure 12. Fluctuations of phosphorus concentration in River Zala discharge in 1977.

Table 2. Lake Balaton basin volumes.

Basins	I	II	III	IV
Volume, m^3	82.10^6	413.10^6	600.10^6	802.10^6

The second group of data includes information on external phosphorus loads which is one of the most important quantitative factors in model application. In the case of Lake Balaton, these loads originate from domestic sewage, industrial wastewater, runoff from agriculture and forest land, and direct precipitation on the lake surface (Jolánkai 1979). Weekly measurements of phosphorus concentrations for 1977, namely total phosphorus, orthophosphate-phosphorus, and phytoplankton-phosphorus* at the mouth of the main tributary, are shown in Figure 12. Concentrations of DIP and DOP in the rainfall were assumed to be equal to 0.1 and 0.06 mg P/ℓ, respectively.** Together with the water balance data presented above, these phosphorus concentrations allowed inclusion of the direct effect of the River Zala discharges and precipitation on the water quality of Lake Balaton for 1977.

Quantitative information on the contributions of other nutrient sources mentioned above is incomplete. These are only tentative estimates of present total and available phosphorus loads in Lake Balaton (van Straten et al. 1979). They allow one to take into consideration the longitudinal distributions of the non-point source pollution and sewage input into the basins and also indicate that part of the total phosphorus load may be considered as being available for phytoplankton growth.

Table 3 includes estimates of phosphorus loads for 1977 (in mg P/ℓ), that are used for the simulation of phosphorus transformations.

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*Values of phytoplankton-phosphorus were recalculated from results of a few measurements of phytoplankton chlorophyll "a" concentration (μg/ℓ) using a constant stoichiometric coefficient $\gamma=2128$ in equation (11).

**Calculated by S. Herodek (1979).

Table 3. Annual phosphorus loads (mg P/ℓ) of different Lake Balaton basins, 1977.

P-Load \ Basins	I	II	III	IV
Particulate-P	.6 †	.12	.037	.019
Orthophosphate-P	.4 †	-	-	-
Direct & indirect sewage PO_4-P	.09	.044	.031	.037
Total P-load	1.09	.164	.068	.055

† River Zala load.

The third group of data includes the measurements of phosphorus compound concentrations and phytoplankton in different points of Lake Balaton. Dissolved inorganic phosphorus or orthophosphate-phosphorus (PO_4), total dissolved phosphorus (TDP), particulate inorganic phosphorus (PIP)*, and total phosphorus (TP), are directly measured compounds of the phosphorus system. Concentrations of these phosphorus forms measured in various points of Lake Balaton in 1977 were averaged for each basin considered in modeling (van Straten et al. 1979) and these calculated values are presented in Table 4. Concentrations of the remaining phosphorus compounds were calculated from those directly measured:

1. Dissolved organic phosphorus, $DOP=TDP-PO_4$;
2. Particulate phosphorus, PP=TP-TDP;
3. Particulate organic phosphorus, POP=PP-PIP.

These values of phosphorus compounds, in mg P/ℓ calculated for different basins of Lake Balaton, are presented in Table 5.

Available measurements of chlorophyll "a" in µg/ℓ averaged for various basins of the water body, are presented in Table 6.

Phytoplankton and phosphorus compound concentrations from the third group of data were used for comparison with modeling results of phytoplankton and phosphorus dynamics in the different basins of Lake Balaton.

*This phosphorus fraction is not taken into account in the model considered here.

Table 4. Directly measured values of phosphorus compounds in mg P/ℓ for different basins of Lake Balaton, 1977.

Days since 1st Jan.	Total phosphorus TP				Total dissolved phosphorus, TDP				Orthophosphate-phosphorus, PO_4				Particulate inorganic phosphorus, PIP			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
74	.069	.059	.039	.031	.022	.013	.013	.013	.002	.004	.003	.004	.005	.005	.005	.005
87	.078	.052	.038	-	.012	.010	.010	-	.004	.006	.005	-	.015	.009	.007	-
88	-	-	.129	.082	-	-	.009	.008	-	-	.006	.002	-	-	.049	.028
108	.080	.070	.060	-	.020	.012	.010	-	.005	.005	.005	-	.016	.010	.013	-
109	-	-	.040	.037	-	-	.009	.007	-	-	.004	.003	-	-	.007	.003
143	.094	.065	.037	.030	.025	.024	.020	.019	.003	.005	.005	.003	.007	.010	.006	.007
164	.074	.060	.032	.024	.042	.022	.020	.014	.006	.005	.004	.003	.008	.007	.005	.002
193	.093	.078	.049	.033	.027	.025	.016	.012	.004	.003	.004	.004	.019	.013	.007	.006
220	.081	.052	.035	.029	.022	.021	.015	.011	.003	.006	.003	.004	.008	.007	.005	.002
298	.080	.071	.047	.029	.029	.026	.019	.019	.006	.003	.005	.004	.007	.006	.006	.005
326	.081	.064	.044	.026	.028	.016	.014	.011	.009	.005	.004	.004	.008	.008	.006	.005

Table 5. Values of phosphorus compound concentrations in mg/Pℓ calculated as differences of the main phosphorus forms for different Lake Balaton basins, 1977.

days since 1st Jan.	Dissolved organic phosphorus, DOP				Total particulate phosphorus, TPP				Particulate organic phosphorus, POP			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
74	.020	.010	.010	.009	.047	.046	.026	.018	.042	.041	.021	.013
87	.008	.005	.005	-	.066	.041	.028	-	.051	.032	.022	-
88	-	-	.004	.006	-	-	.120	.073	-	-	.071	.045
108	.015	.007	.005	-	.060	.058	.049	-	.044	.048	.037	-
109	-	-	.005	.004	-	-	.031	.030	-	-	.025	.027
143	.022	.019	.015	.016	.069	.041	.018	.011	.062	.032	.012	.004
164	.036	.018	.016	.010	.032	.038	.012	.011	.024	.031	.007	.008
193	.023	.022	.012	.009	.066	.053	.033	.021	.047	.041	.026	.015
220	.019	.015	.012	.007	.059	.032	.020	.018	.051	.025	.015	.016
298	.023	.023	.014	.015	.051	.045	.028	.010	.044	.039	.022	.005
326	.019	.012	.010	.007	.053	.047	.030	.015	.045	.039	.025	.010

Table 6. Measurements of chlorophyll "a" in μg/ℓ for different Lake Balaton basins, 1977.

days since January 1st	Basins			
	I	II	III	IV
74	48.1	40.6	25.1	-
75	-	-	7.5	5.6
87	51.5	56.2	32.3	-
88	-	-	26.2	13.5
108	-	40.0	24.0	-
109	38.7	46.3	18.8	11.4
149	12.3	12.7	8.2	8.6
157	14.6	14.4	8.5	4.4
164	14.4	15.5	7.3	4.5
172	15.3	13.4	7.3	4.6
186	24.8	11.2	4.0	2.5
193	30.9	17.4	7.9	4.3
200	38.2	19.4	11.6	3.9
228	54.4	29.2	10.0	5.9
249	76.5	43.4	14.2	3.1
263	23.8	22.3	8.3	2.5
270	28.0	29.8	9.9	5.3
284	32.5	24.0	9.3	5.7
298	19.8	17.1	9.2	4.9
326	12.1	12.4	10.0	4.4

5. SIMULATION RESULTS

The model parameters and coefficients described above, together with phosphorus loading rates, water temperature, solar radiation and water balance measurements are necessary input data for the given model. The specification of the model parameters, and comparison of simulation results with field observation data, constitutes the model calibration process. Some preliminary results in application of the given model for analysis of phosphorus compound and phytoplankton dynamics in 1977 for the most polluted area of Lake Balaton--Keszthely Bay, were presented and discussed in a special report (Leonov 1980). In the present content, practically the same values for the rate constants are used for the simulation of phosphorus transformation and phytoplankton dynamics in different basins of the lake.

A comparison of the given model calculations and 1977 data for phosphorus compounds and chlorophyll "a", are shown in Figures 13-16 for basins I through IV, respectively. Observed values of phosphorus and chlorophyll "a" concentrations are shown in these figures by points, while curves are the results of simulations. Particulate organic phosphorus (shown by simulation curves) includes the contents of phosphorus in living matter and detritus.

The Runge-Kutta-4 method was used to solve the differential equations with a constant time step equal to 0.1 day. The oxygen cycle in the water body, and phosphorus transformation in sediment, were not considered at this step of the study and therefore the corresponding parameters are not discussed below. Some of the input data that include the concentrations of phosphorus compounds in Lake Balaton basins on the first day of January, 1977, and values of the kinetic parameters used for the set of model runs in Figures 13-16, are listed in Table 7. The concentration of phosphorus compounds in Table 7 were received by interpolation of averaged values of measurements for each basin (van Straten et al. 1979). The order of magnitudes of kinetic parameters presented in Table 7 was estimated previously (Leonov 1980). As Table 7 shows, the similar values of kinetic parameters excluding the maximum uptake rate for phytoplankton (K_1)

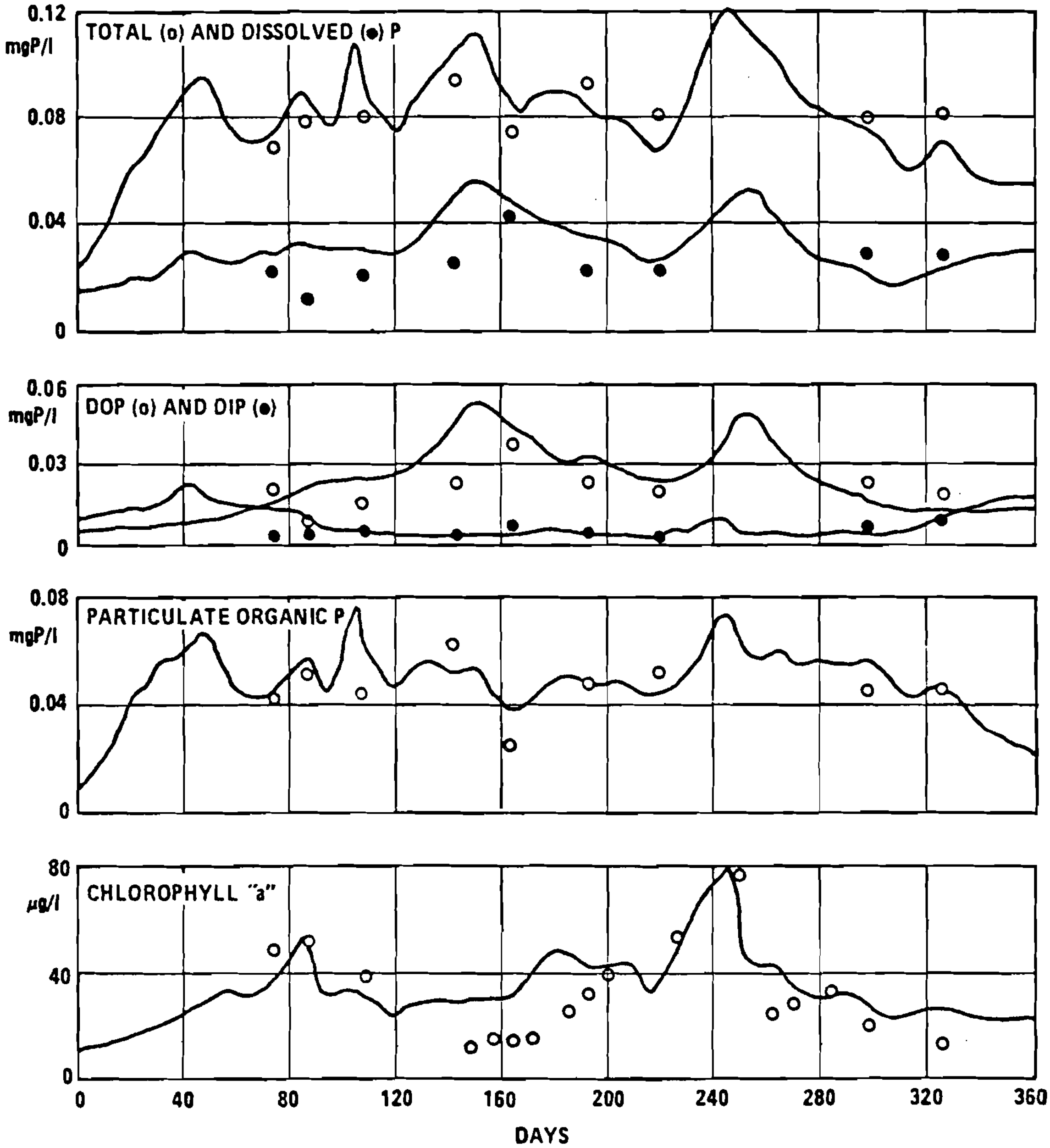


Figure 13. Comparison between model output (curves) and field data (points). Lake Balaton, Basin I, 1977.

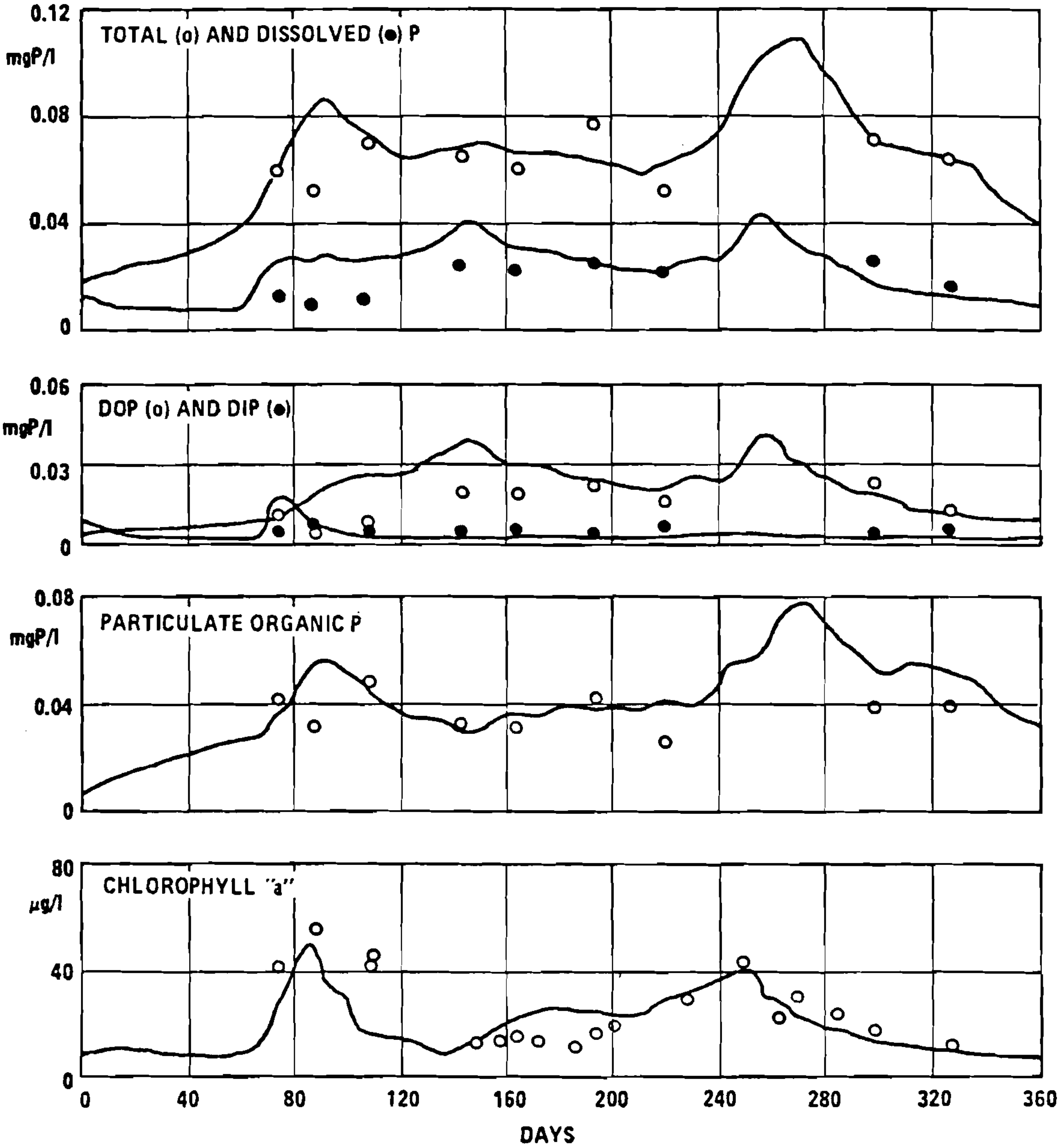


Figure 14. Comparison between model output (curves) and field data (points).
Lake Balaton, Basin II, 1977.

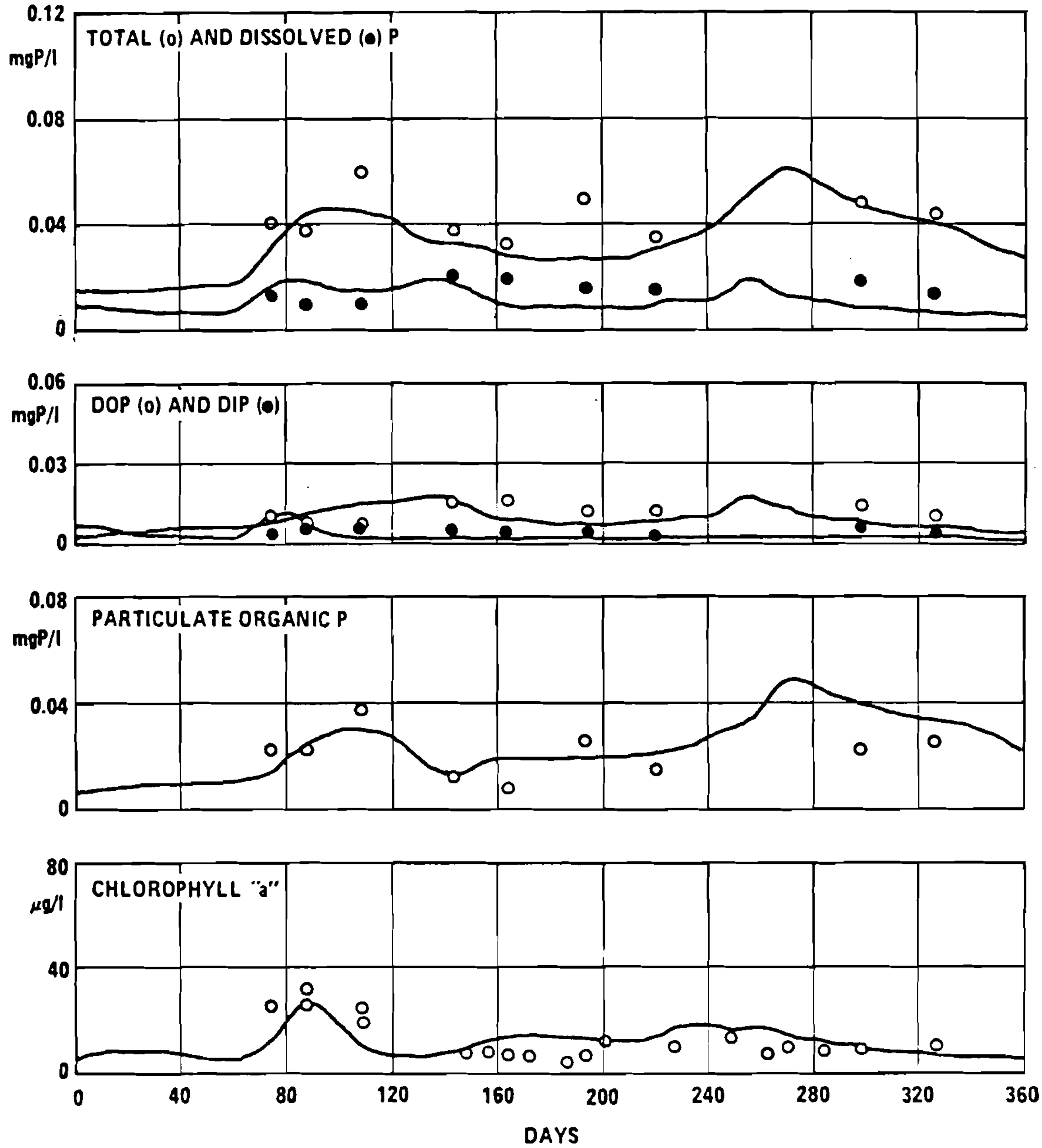


Figure 15. Comparison between model output (curves) and field data (points).
Lake Balaton, Basin III, 1977.

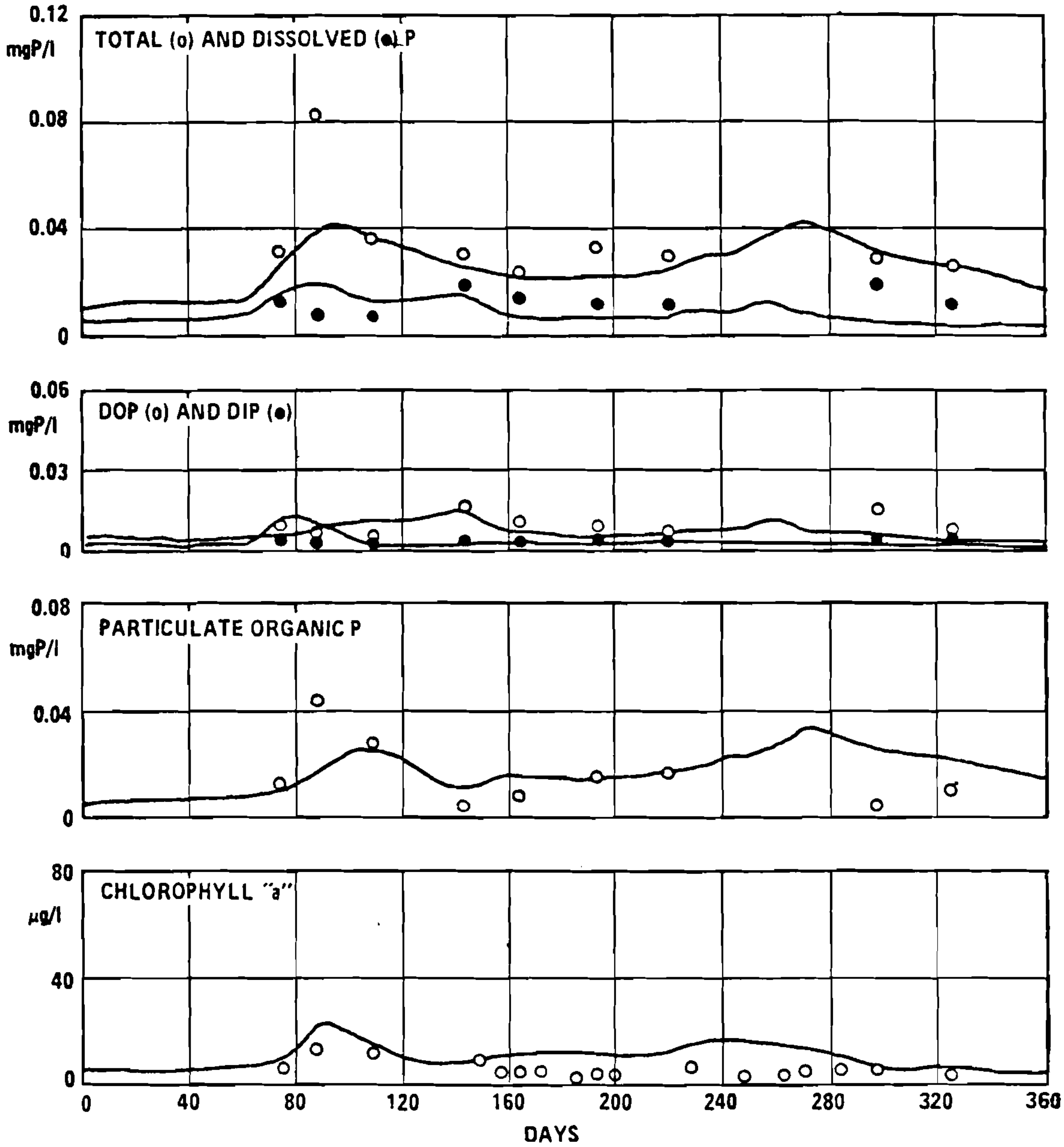


Figure 16. Comparison between model output (curves) and field data (points). Lake Balaton, Basin IV, 1977.

Table 7. Values of input data used for phosphorus transformation modeling in Lake Balaton, 1977.

Parameters	Units	Symbols	Basins			
			I	II	III	IV
State variables						
Dissolved inorganic phosphorus	mgP/ℓ	DIP	.010	.008	.006	.004
Dissolved organic phosphorus	"	DOP	.005	.004	.003	.002
Phytoplankton phosphorus	"	F	.005	.004	.003	.002
Bacterial phosphorus	"	B	.001	.0008	.0004	.0003
Nonliving particulate phosphorus	"	P_D	.002	.002	.002	.002
Chlorophyll "a"	μg/ℓ	Chl	10.6	8.5	6.4	4.3
Rate constants and other parameters						
Maximum uptake rate for phytoplankton	day^{-1} at 20°C	K_1	2.8	2.8	1.9	1.4
Excretion efficiency of phytoplankton	day	a_1	.057	.057	.057	.057
" " " "	"	a_2	.075	.075	.075	.075
Phytoplankton mortality as function of biomass and nutrient levels	$(mgP/\ell)^{-1}$ day^{-2}	v_1	.2	.2	.2	.2
Coefficient of substrate conversion by phytoplankton	dimensionless	β	.6	.6	.6	.6
Maximum uptake rate for bacteria	day^{-1} at 20°C	K_2	.3	.3	.3	.3
Excretion efficiency of bacteria	day	a_3	.3	.3	.3	.3
" " "	"	a_4	.45	.45	.45	.45
Natural mortality of bacteria	day^{-1}	v_2	.053	.053	.053	.053
Bacterial mortality as a function of biomass and nutrient levels	$(mgP/\ell)^{-1}$ day^{-2}	v_3	1.0	1.0	1.0	1.0
Particulate phosphorus decomposition rate	day^{-1} at 20°C	K_3	.1	.1	.1	.1
Extinction coefficient	m^{-1}	K_a	1.8	1.8	1.5	1.5
" "	—	K_b	.0088	.0088	.0088	.0088

and extinction coefficient (K_a) were used for simulation of phosphorus dynamics in Lake Balaton basins. The analysis of existing data on Lake Balaton studies is the basis of the suggestion that values of K_1 and K_a may be taken as changeable parameters for diffrent Lake Balaton basins. Available information on Lake Balaton shows that the composition of phytoplankton species and phytoplankton activity are quite different in various parts of Lake Balaton (Herodek 1977). Generally, Basin I is characterized by the biggest number of phytoplankton species, while the lowest species number were observed in Basin IV (Tamas 1969, 1972, 1975). According to this, the phytoplankton activity is about three (Herodek and Tamas 1975) to eight (Herodek and Tamas 1974) times higher at Keszthely Bay than at Tihany. It was also observed the spatial heterogeneity of some of the physical, chemical, and biological indexes of water quality along the Lake Balaton axis or from Basin I to Basin IV (Bella 1953). Taking into account the fact that the concentrations of DIP in various parts of Lake Balaton are about 0.004-0.006 mg P/ℓ and phytoplankton levels averages 3-4 times higher in Basin I than Basin IV (van Straten et al. 1979), it is possible to assume that there is a different equilibrium between the phytoplankton and DIP in each of these basins. In one's turn this may seriously effect the conditions and rates of phosphorus cycling in various parts of Lake Balaton. According to model structure, parameters K_1 and K_a may be considered as taking into account the spatial heterogeneity of phytoplankton and physical-chemical properties of water in the Lake Balaton ecosystem.

Another set of input data for model runs takes into account the effect of the River Zala on the Lake Balaton ecosystem and includes the concentrations of main phosphorus fractions, total phosphorus, orthophosphate, and phytoplankton-phosphorus in the River Zala discharge water (Figure 12). On the basis of these direct measurements of phosphorus concentrations the contents of nonliving particulate phosphorus in the discharge water were calculated as differences between total phosphorus, orthophosphate, and phytoplankton-phosphorus.

According to available data the River Zala has an expressed influence on the heterotrophs number in Keszthely Bay (Oláh 1974). Becuase there is no data about bacterial biomass in units of phosphorus, the concentration of bacterial phosphorus in the River Zala discharge water was assumed to be constant during the year and equal to 4.10^{-4} mg P/ℓ. The concentration of DOP in this dischardge water, because the information is absent, was assumed to be negligibly low. The next set of input data includes the rates of external phosphorus loads. The application of annual average rates of external P-load in preliminary model calculations showed that the given model as well as other models (van Straten 1980b) can not describe the many details of phytoplankton dynamics in 1977. Therefore, a question was raised which seasonal pattern of external P-load should be in model simulation runs to represent the observed dynamics of phosphorus compounds and phytoplankton in different basins of Lake Balaton. After several simulation runs the seasonal pattern of external P-load that give reasonable agreement with observation in all basins (Figure 13-16) was evaluated. The rates of external P-load used for simulation runs discussed now are presented in Table 8. As follows from Table 8 the total annual P-load values for each basin are close to estimates of phosphorus loading for 1977 (Table 3).

Sedimentation is one of the main processes which regulates phosphorus cycling in the lake. In the given model, it is assumed that sedimentation is restricted to nonliving particulate phosphorus. One of the purposes of the simulation runs was to estimate the possible values of sedimentation rates for various basins of Lake Balaton and to calculate the net flux of nonliving particulate phosphorus from the water to the sediment. In an earlier report (Leonov 1980), the sedimentation rate was treated as a time-variable parameter. A significant fluctuation of the particulate phosphorus concentration in comparatively short periods of time testifies to this (Tables 4 and 5).

In the given approach, the sedimentation rate is defined according to the ecological model considered here, but not according to the nature of the sedimentation process. The present

Table 8. Monthly average values of external P-load rate (mg P/ℓ day) used for modeling the phosphorus transformation in Lake Balaton basins, 1977 (to Figures 13-16).

Basin	Keszthely		Sligliget		Szemes		Siófok	
Months / P-input	CZ_2	CZ_3	CZ_2	CZ_3	CZ_2	CZ_3	CZ_2	CZ_3
I	0	0	0	$2\cdot10^{-4}$	0	0	0	0
II	$1\cdot10^{-4}$	0	0	$6\cdot10^{-4}$	0	0	0	0
III	$5\cdot10^{-4}$	0	$2\cdot10^{-3}$	$6\cdot10^{-4}$	$9\cdot10^{-4}$	$5\cdot10^{-5}$	$9\cdot10^{-4}$	$5\cdot10^{-5}$
IV	0	0	$3\cdot10^{-4}$	$1\cdot10^{-4}$	0	$2\cdot10^{-4}$	0	$2\cdot10^{-4}$
V	0	0	0	$1\cdot10^{-4}$	0	0	0	0
VI	0	0	0	$1\cdot10^{-4}$	0	0	0	0
VII	0	0	0	0	0	0	0	0
VIII	$2\cdot10^{-3}$	0	$6\cdot10^{-4}$	0	$2\cdot10^{-4}$	$1\cdot10^{-4}$	$2\cdot10^{-4}$	$1\cdot10^{-4}$
IX	$4\cdot10^{-4}$	0	$1\cdot10^{-3}$	$1.5\cdot10^{-3}$	0	$1\cdot10^{-3}$	0	$4.5\cdot10^{-3}$
X	0	0	0	$1\cdot10^{-4}$	0	$5\cdot10^{-5}$	0	0
XI	0	0	0	$6\cdot10^{-4}$	0	0	0	0
XII	0	0	0	$2\cdot10^{-4}$	0	0	0	0
Total P-load mg P/ℓ year	0.09		0.219		0.075		0.054	

approach is motivated primarily by the lack of viable alternatives about the sedimentation of phosphorus compounds, although there is an example of an attempt to simulate the sedimentation process in Lake Balaton on the basis of data on the dynamics of suspended solids and wind spread (Somlyódy 1980). In this paper for purpose of simplification, it was suggested that net sedimentation, as a whole, may be characterized by monthly average rate values, although rates of sedimentation in lakes frequently exhibit variations in a small time range due to changes in climate regime, water dynamics, wind or local modifications by bathymetric features (Sly 1978). In reality, these processes will interfere with sedimentation and resuspend the deposits from the bottom sediments to the water.

As a matter of fact, parameter s_1 in equation (23), which describes the sedimentation rate, reflects at the least, the net effect of two simultaneous processes, namely sedimentation and resuspension. In order to describe the sedimentation in rigorous terms, it is necessary to consider the hydrodynamics of the lake and the effect of wind on the resuspension of sediment. Therefore, in the given model, parameter s_1 is extremely uncertain. It is difficult to be sure whether or not a correlation with physical phenomena, such as wind, can be obtained, and it is not clear how this relationship can be quantified.

The yearly course of average monthly net sedimentation rates used in the simulation runs considered now, is shown in Figure 17. The pattern of net sedimentation rates in Lake Balaton basins were chosen after several simulation runs. Similar values of sedimentation rates were used for Basins III and IV and the order of sedimentation rate values for Basin III is in agreement with rates evaluated by Somlyódy (1980).

The magnitude of sedimentation rates for Basin I excluding the period of May are 4 to 5 times higher than for Basins II and III, respectively. As results of phosphorus observations in Lake Balaton show, the concentration of particulate phosphorus in Basin I during May, 1977, has increased. At the same time input of particulate phosphorus by the River Zala discharge water has

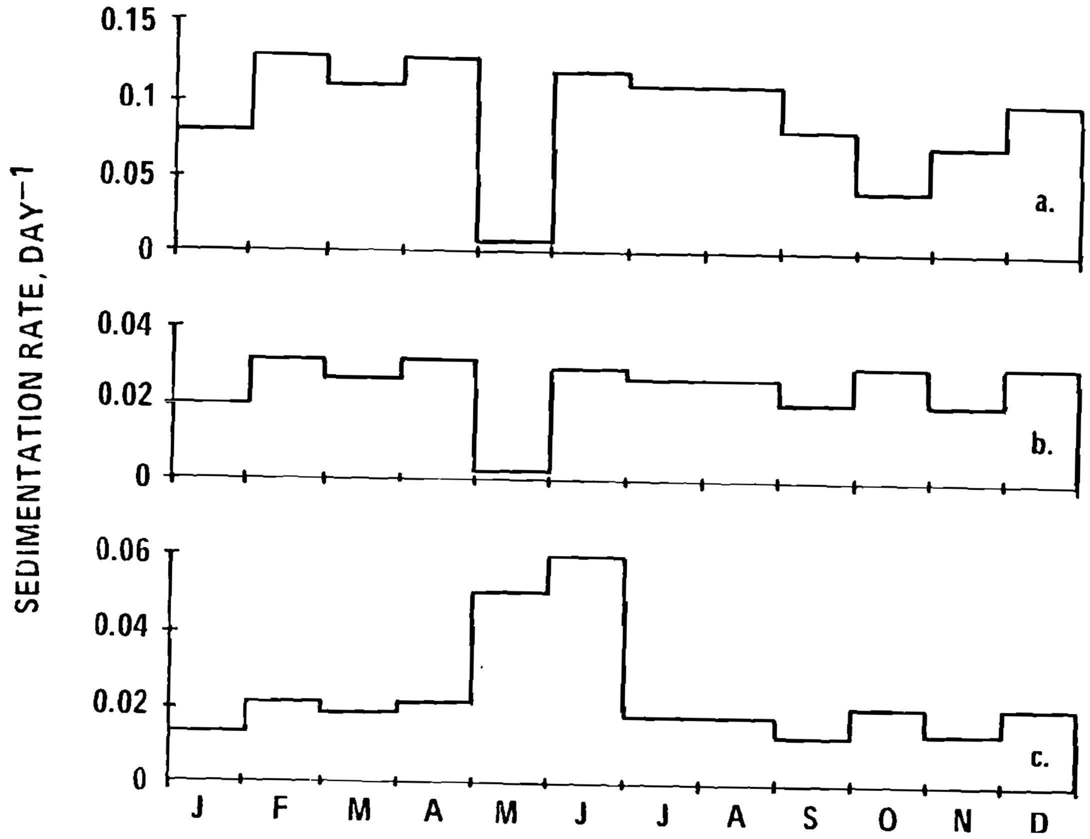


Figure 17. Yearly course of average monthly sedimentation rates, used for modelling phosphorus transformations in Lake Balaton basins, 1977.

a. for Basin I
b. for Basin II
c. for Basins III and IV.

decreased about 5 times in comparison with those in April (Table 12). The biomass of living matter in May, 1977, did not increase so significantly to reach the particulate phosphorus concentration of 0.06 mg P/ℓ. Therefore the analysis of the processes in the model shows that only the decreasing net sedimentation rate in May can increase the concentration of particulate phosphorus in this period in Basin I, and there is no other possibility in the model to describe the increasing particulate phosphorus in May. After several calculations it was found that the low net sedimentation rate, 0.005 day^{-1}, in comparison with values for other months can describe the increasing particulate phosphorus in May, 1977, in Basin I (Figure 13).

The similar time pattern of net sedimentation rates, as for Basin I, was used for simulation of phosphorus dynamics in Basin II (Figure 17). In Figure 14 the low sedimentation rate in May appeared to be satisfactory, to describe the dynamics of particulate phosphorus in Basin II.

For Basins III-IV the rates of net sedimentation in principle were lowest and they fluctuated around 0.02 day^{-1}. In the first simulation runs the constant value of the sedimentation rate was used for Basins III-IV. However, in attempt to describe the dynamics of all phosphorus fractions (including the particulate phosphorus in Basin III-IV) as close as possible to observations, it was found that rates of net sedimentation in May-June must be 2-3 times higher than 0.02 day^{-1} because the concentration of particulate phosphorus in these months were lowest in Basins III-IV. In accordance with the model structure this fact is difficult to explain because the sedimentation process is considered the simplest way and that only one parameter (s_1) is indepent on other factors. However, some alternative explanations applicable to different time patterns of net sedimentation rates in Basins I-IV may be suggested. First, there is a different effect of sediment in Basins I-IV on the phosphorus content in water taking into account the available information about the spatial heterogeneity of Lake Balaton sediment (Ponyi et al. 1972 and Literâthy et al. 1981) although on the basis of one field survey by Dobolyi (1980), he concluded that there is a slight change in sediment phosphorus concentration in various parts of Lake Balaton. The second possible explanation is the difference of wind action on the exchange processes in sediment-water interface (IIASA's data on wind shows differences in various parts of Lake Balaton). For example, Table 9 shows statistically analyzed data on wind based on averaged speeds and frequencies of various parts of Lake Balaton.

Table 9. Wind speed frequencies (%) and average wind speeds in different parts of Lake Balaton, May, 1977.

Basins	Wind speeds, m/sec				average wind speed m/sec
	0 - 2	2 - 4	4 - 6	> 6	
Keszthely	63.5	25.1	10.3	1.0	1.9
Szemes	42.1	29.4	22.0	6.5	2.9
Siófok	51.8	27.6	14.6	6.0	2.5

In an attempt to find a correlation between wind data and sedimentation rates used for phosphorus transformation modeling in 1977, one serie of wind data for Siófok available at IIASA for this year was statistically analyzed. Table 10 consists of the results of this analysis. The range of raw wind data was based on speed and direction. The frequency distributions of winds, with respect to direction and velocity, were calculated for each month of 1977. Plots of empirical data in Figure 18 show some dependencies between monthly wind frequencies of a certain speed, and monthly sedimentation rates used for modeling phosphorus transformation in Basin I. Figure 18 shows that some frequencies of wind speed give more or less good correlations with sedimentation rates on a monthly basis. Obviously, these dependencies may be used in the future for a preliminary estimation of sedimentation rates based on wind data.

In the results of simulation runs presented in Figures 13-16, the hypothesis of one phytoplankton population was used. As a whole, this hypothesis allows us to describe quite satisfactorily, the phytoplankton growth in relation to the nutrient content in different basins of Lake Balaton. In Basin I, simulation results of phytoplankton chlorophyll "a" values during the summer months, are two times higher than measured chlorophyll "a" concentrations

Table 10. Wind frequencies based on speed and direction for each month of 1977 (measurement at Siófok).

Wind speed m/sec	Wind direction	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1-2	N	11.6	7.2	7.4	8.9	8.0	10.2	7.7	8.8	6.8	7.6	5.1	8.4
	NE	11.2	6.6	7.4	3.5	8.0	2.3	4.1	4.4	4.5	4.6	2.3	5.2
	E	15.6	4.0	12.8	7.5	13.9	5.5	4.1	8.3	8.6	13.4	9.6	12.7
	SE	2.7	3.0	2.8	2.6	4.4	3.7	4.1	4.9	2.7	5.6	1.8	1.4
	S	4.5	9.7	6.0	9.3	10.8	12.0	11.3	11.2	8.6	11.8	15.6	22.6
	SW	4.0	8.2	3.7	2.6	3.6	3.2	7.7	3.4	5.0	11.8	7.8	4.2
	W	4.0	5.0	2.8	4.4	2.7	6.5	3.6	2.4	4.1	3.6	2.8	1.4
	NW	1.8	2.5	0.9	0.9	0.4	0.9	2.7	1.5	1.8	3.1	0.9	0.9
2-6	N	8.0	8.1	8.3	12.7	14.2	12.9	5.4	9.7	11.8	5.1	5.5	5.6
	NE	7.5	0.5	1.4	0.8	2.2	1.4	0.4	2.4	3.2	0.5	1.9	1.4
	E	5.8	0.5	1.8	0.4	1.3	0.5	-	0.5	2.8	1.0	0.9	3.3
	SE	0.4	-	0.5	0.4	1.3	-	-	1.0	0.5	1.0	0.9	-
	S	3.6	7.1	9.7	3.5	8.5	10.7	4.1	5.9	2.3	8.2	10.1	3.3
	SW	7.5	9.1	6.0	5.3	5.8	3.2	5.0	5.3	10.0	10.7	10.1	13.6
	W	4.0	8.0	6.4	5.3	4.4	2.8	8.6	4.4	5.0	4.0	5.1	3.7
	NW	0.5	2.5	3.2	6.1	4.5	2.8	8.6	3.9	4.5	0.5	3.7	1.8
0-6	N	19.6	15.3	15.7	21.6	22.2	23.1	13.1	18.5	18.6	12.7	10.6	14.0
	NE	18.7	7.1	8.8	4.3	10.2	3.7	4.5	6.8	7.7	5.1	4.2	6.6
	E	21.4	4.5	14.6	7.9	15.2	6.0	4.1	8.8	11.4	14.4	10.5	16.0
	SE	3.1	3.0	3.3	3.0	5.7	3.7	4.1	5.9	3.2	6.6	2.7	1.4
	S	8.1	16.8	15.7	12.8	19.3	22.7	15.4	17.1	10.9	20.0	25.7	25.9
	SW	11.5	17.3	9.7	7.9	9.4	6.4	12.7	8.7	15.0	22.5	17.9	17.8
	W	8.0	13.0	9.2	9.7	7.1	9.3	12.2	6.8	9.1	7.6	7.9	5.1
	NW	2.3	5.0	4.1	7.0	4.9	3.7	11.3	5.4	6.3	3.6	4.6	2.7
6-10	N	4.0	3.0	5.9	12.7	2.2	9.7	9.9	4.9	8.1	1.0	4.2	2.8
	NE	-	-	0.5	-	-	0.5	-	-	-	-	0.5	-
	E	-	-	-	-	-	-	-	-	0.5	-	-	-
	SE	-	-	-	-	-	-	-	-	-	-	-	-
	S	1.3	5.5	0.5	-	0.4	0.5	-	1.0	-	4.0	0.9	-
	SW	0.5	1.5	1.4	0.4	0.4	1.9	1.3	2.9	0.5	-	2.7	0.5
	W	-	0.5	1.4	3.0	0.4	0.5	2.3	3.4	2.3	0.5	1.4	0.5
	NW	1.0	4.0	3.2	3.6	2.2	4.6	4.1	3.9	3.2	-	1.9	1.9
>10	N	0.5	-	3.2	2.6	0.4	3.2	2.3	-	3.2	1.0	4.2	1.9
	NE	-	-	-	-	-	-	-	-	-	-	-	-
	E	-	-	-	-	-	-	-	-	-	-	-	-
	SE	-	-	-	-	-	-	-	-	-	-	-	-
	S	-	2.0	-	-	-	-	0.9	0.5	-	1.0	-	-
	SW	-	1.0	-	-	-	-	0.9	0.5	-	-	-	0.5
	W	-	-	-	-	-	-	-	1.5	-	-	-	1.9
	NW	-	0.5	2.8	3.5	-	0.5	0.9	3.4	-	-	-	0.5
Total Wind	N	24.1	18.3	24.8	36.9	24.8	36.0	25.3	23.4	29.9	14.7	19.0	18.7
	NE	18.7	7.1	9.3	4.3	10.2	4.2	4.5	6.8	7.7	5.1	4.7	6.6
	E	21.4	4.5	14.6	7.9	15.2	6.0	4.1	8.8	11.9	14.4	10.5	16.0
	SE	3.1	3.0	3.3	3.0	5.7	3.7	4.1	5.9	3.2	6.6	2.7	1.4
	S	9.4	24.3	16.2	12.8	19.7	23.2	16.3	18.6	10.9	25.0	26.6	25.9
	SW	12.0	19.8	11.1	8.3	9.8	8.3	14.9	12.1	15.5	22.5	20.6	18.8
	W	8.0	13.5	10.6	12.7	7.5	9.8	14.5	11.7	11.4	8.1	9.3	7.5
	NW	3.3	9.5	10.1	14.1	7.1	8.8	16.3	12.7	9.5	3.6	6.5	5.1

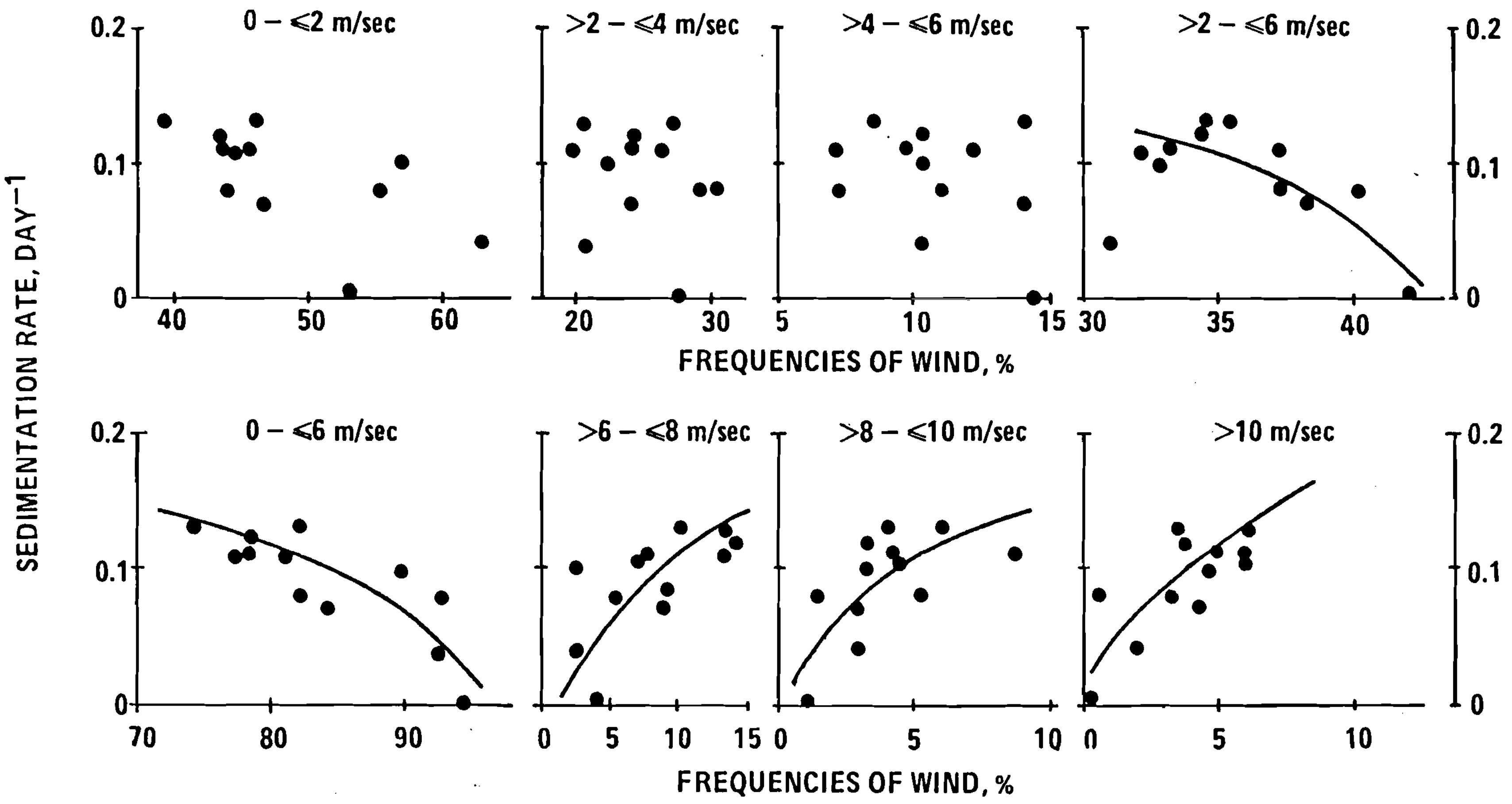


Figure 18. Dependences between monthly average frequencies of wind, ranged on speeds, and monthly average sedimentation rates used for phosphorus transformation modeling, Keszthely Bay, 1977.

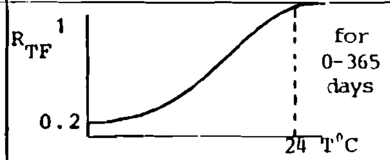
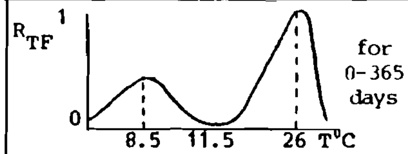
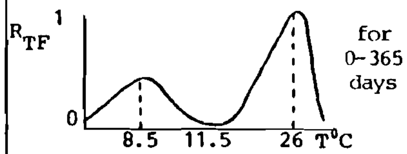
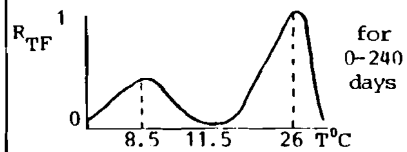
(Figure 13). The difference in conditions for phytoplankton in Basins I and II, is that the River Zala provides a continuous DIP input during the entire year in Basin I, while DIP contents in other basins during most of the year are limiting for active phytoplankton growth. For Basins III and IV, a low nutrient level is especially marked during the summer months.

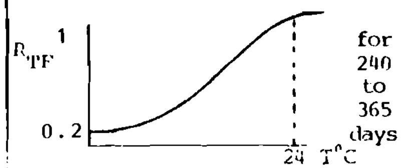
The model presented in this report, allows us to check the different hypotheses concerned with the peculiarities of phytoplankton growth in Lake Balaton, in order to arrive at a better model approximation of phytoplankton dynamics for Basin I as the most important part of the entire lake. It is a well-known fact, that diatoms are the dominating species in the cooler months, while blue-green algae are abundant during the summer months (van Straten et al. 1979). The principal difference of these phytoplankton populations is that blue-greens have a relatively low level of chlorophyll "a" (Vörös 1979). A mixed phytoplankton population may be generated during the autumn months (Herodek 1979). Thus, in the simulation runs, an attempt was made to describe phytoplankton dynamics in 1977, using the hypotheses of two and three seasonal phytoplankton groups. The main assumption for these phytoplankton populations, may be related to the different effects of temperature on phytoplankton groups and the various nutrient requirements. In the given model, the effect of temperature on phytoplankton development, is described by parameter R_{TF} in equation (4), while the nutrient requirements of phytoplankton may be connected with values of parameter v_1 in the term that describes the specific rate of phytoplankton mortality in equation (14).

Table 11 includes the information about three model experiments which are examined in the hypotheses mentioned above. Results of these model experiments are presented in Figure 19. In these calculations, the same input data discussed before were used.

In the hypothesis of one phytoplankton population (Figures 13-16), the exponential temperature function and constant for the whole year value of $v_1 = 0.2$, were used (Table 11, Figure 19a).

Table 11. Model Experiments for examining the hypotheses connected with phytoplankton development in Lake Balaton (for Figure 19).

<table>
<tr><th>NN</th><th>Hypotheses concerning the number of phytoplankton species</th><th>Temperature Function (R_{TF})</th><th>Nutrient Requirements (v_1)</th></tr>
<tr><td>from Figure 13</td><td>one population
(for Figure 19a)</td><td>
</td><td>v_1 = 0.2 for 0-365 days</td></tr>
<tr><td>1.</td><td>two populations
(for Figure 19b)</td><td>
</td><td>v_1 = 0.2 for 0-365 days</td></tr>
<tr><td>2.</td><td>two populations
(for Figure 19c)</td><td>
</td><td>v_1 = 0.2 for 0-120 days
v_1 = 0.5 for 120-240 days
v_1 = 0.2 for 240-365 days</td></tr>
<tr><td>3.</td><td>three populations
(for Figure 19d)</td><td>


</td><td>v_1 = 0.2 for 0-120 days
v_1 = 0.5 for 120-240 days
v_1 = 0.2 for 240-365 days</td></tr>
</table>

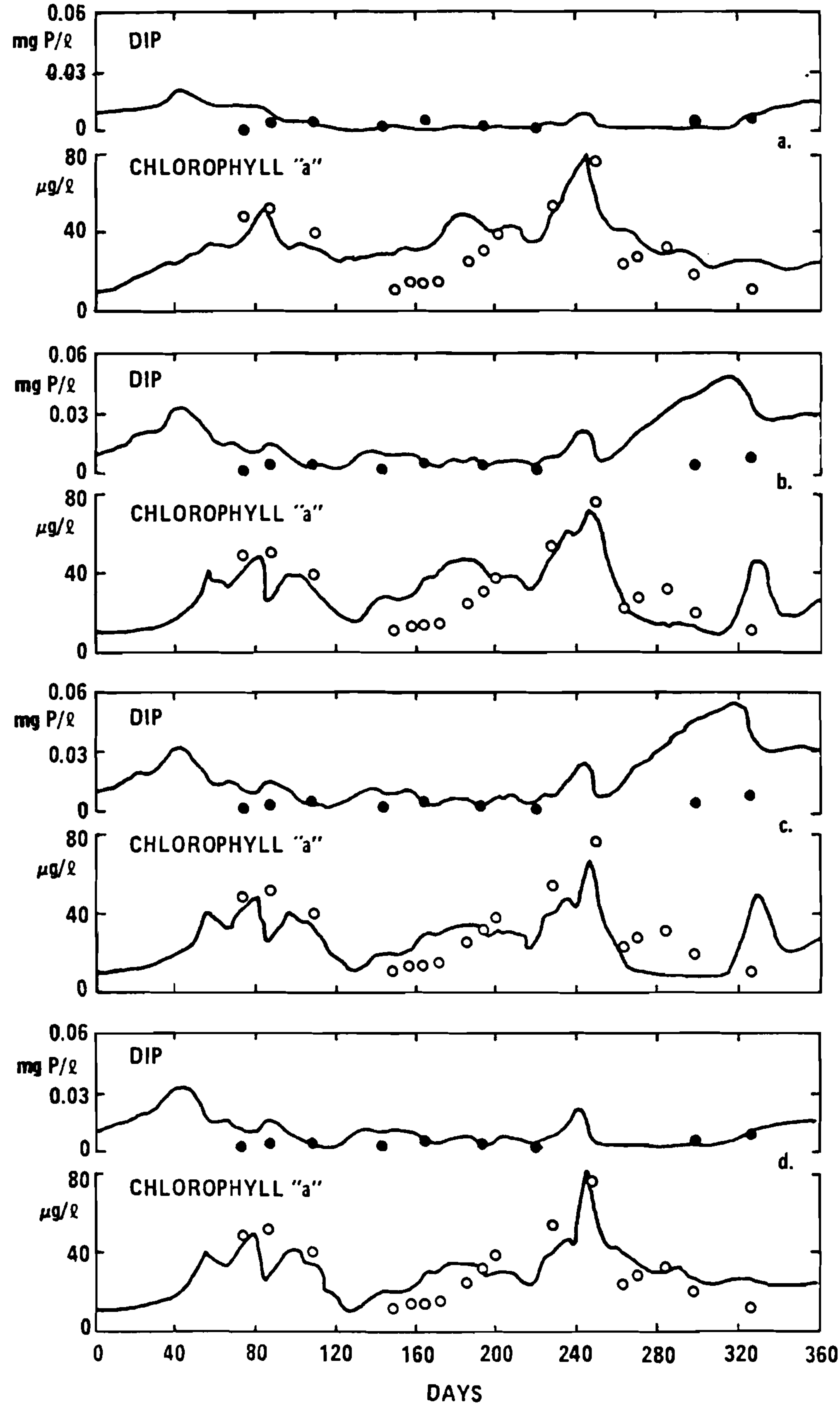


Figure 19. Results of modeling experiments for examination of hypotheses about phytoplankton development in Lake Balaton, Keszthely Bay, 1977. (To Table 10) a. from Fig. 13; b. run 1, c. run 2, d. run 3.

In the next calculation, a "two temperature-peaks function" suggested by van Straten (1979), was used for a description of different temperature effects on two phytoplankton groups. This function is:

$$R_{TF} = w_s \cdot \max\left[0, \frac{T_{cr} - T}{T_{cr} - T_{opt}} \exp\left(1 - \frac{T_{cr} - T}{T_{cr} - T_{opt}}\right)\right] \quad (32)$$

where

w_s is the height of the function peak in range 0-1;
T_{cr} and T_{opt} are critical and optimal temperatures for certain phytoplankton groups ; and
T is the current temperature of water.

This equation is used for each phytoplankton group. The shape of the curve that may be described by equation (32) is shown in Table 11. During modeling , it was reproduced by the following parameter values: $w_s = 0.45$; $T_{cr} = 11.5^0C$ and $T_{opt} = 8.5^0C$ for first phytoplankton group and $w_s = 1.0$; $T_{cr} = 29^0C$, $T_{opt} = 26^0C$ for the second phytoplankton group.

In the first model experiment, the bi-modal temperature function was used instead of the exponential function. The results of this experiment are shown in Figure 19b. The differences in concentration of DIP and chlorophyll "a" during (see Figures 19a and 19b) 0-240 days, are not very great. A comparison of these figures shows that at the beginning of the year (0-50 days) the concentration of DIP is 1.5-2 times higher (Figure 19b), because phytoplankton develop slowing during this period, and later (50-240 days) the DIP concentrations are practically the same.

It must be noted that the bi-modal temperature function made it possible for the phytoplankton to respond sensitively to temperature fluctuation from one day to another in the winter-spring period. Phytoplankton oscillations in the 40-110 day period are the results of changing temperature conditions (Figure 7).

The bi-modal temperature function describes the decreasing R_{TF} in the temperature range 10-13^{O}C which corresponds to the end of spring and it is assumed that chlorophyll "a" concentrations will decrease in that period in comparison to the results obtained by the use of the exponential temperature function. Actually, the phytoplankton level is decreased to 14 mg Chl"a"/ℓ in the beginning of May, however, it remains on this level for a short time interval--120-130 days (Figure 19b). Modeling results show that increasing chlorophyll "a" concentrations after the 130th day, coincides with temperature increases up to 15°C and higher.

For the autumn months (240-330 days), the differences in simulation results and data presented in Figure 19a and 19b, are more significant than for the earlier months. Activity of phytoplankton is sharply decreased because the temperature drops to 11-14°C in the autumn months. As a result of this, during the period, the level of DIP is unrealistically high. It is higher than measured DIP concentrations, by 4-5 times. When temperature reaches 8-9°C (end of November to beginning of December), the model shows that the phytoplankton generate a third peak of chlorophyll "a".

After a comparison of the results presented in Figure 19a and Figure 19b, it is possible to come to some conclusions concerning the application of the bi-modal temperature function:

1. It makes phytoplankton sensitive to temperature fluctuations in winter, spring and autumn months;

2. It does not change the high level of phytoplankton in summer months;

3. It forms an unrealistically high concentration of DIP in autumn months;

4. It gives a third peak of chlorophyll "a" at the end of November to the beginning of December.

In the second model experiment, it was suggested that nutrient requirements of summer phytoplankton must be higher than phytoplankton from other seasonal groups. Therefore, coefficient v_1 for the summer period, was taken to be 2.5 times higher than that for other months (Table 11).

The results of model experiment 2 are presented in Figure 19c. A comparison of Figures 19b and 19c shows that summer phytoplankton levels come closer to measured chlorophyll "a" concentrations in the case of high nutrient requirements of the summer phytoplankton groups (Figure 19c). This hypothesis practically does not change the concentration fluctuation of DIP during the summer period. It is also clear that bi-modal temperature functions can not give a good approximation for the autumn months. This suggests in accordance with all assumptions that mixed phytoplankton population in autumn must have other temperature dependencies.

In the third model experiment, the exponential temperature function was used for autumn months (Table 10). The results of this model experiment are presented in Figure 19d. They show a fairly good agreement with observed data for the year analyzed. Thus, the hypothesis of the three phytoplankton groups in the first basin of Lake Balaton that have differences in temperature dependences and nutrient requirements, gives quite an accurate model representation of phosphorus compounds and chlorophyll data measured in 1977.

This hypothesis was used for modeling phosphorus transformation in connection with phytoplankton dynamics in other basins of Lake Balaton. The results of simulation are presented in Figures 20-23 for Basin I through IV, respectively. They show that a three phytoplankton group hyphothesis gives a good reproduction of observed phosphorus compounds and phytoplankton dynamics for 1977 in all the basins considered.

6. ANALYSIS OF PHOSPHORUS TRANSFORMATION PATHWAYS

Since the model permits computation of the dynamics of phosphorus compounds in 1977 (Figures 20-23), considerable insight into the ecosytem behavior may be gained by studying the phosphorus transformation flows in various basins of the lake. From an ecological point of view, the following processes are very important for a quantitative assessment of phosphorus transformations in the lake's ecosystem: phosphorus-loading, particulate phosphorus sedimentation and oxidative transformation to DOP, and transformation of phosphorus by phytoplankton and bacteria. A quantitative assessment of these processes is the basis for understanding the internal phosphorus cycling in the lake and the eutrophication of this water body.

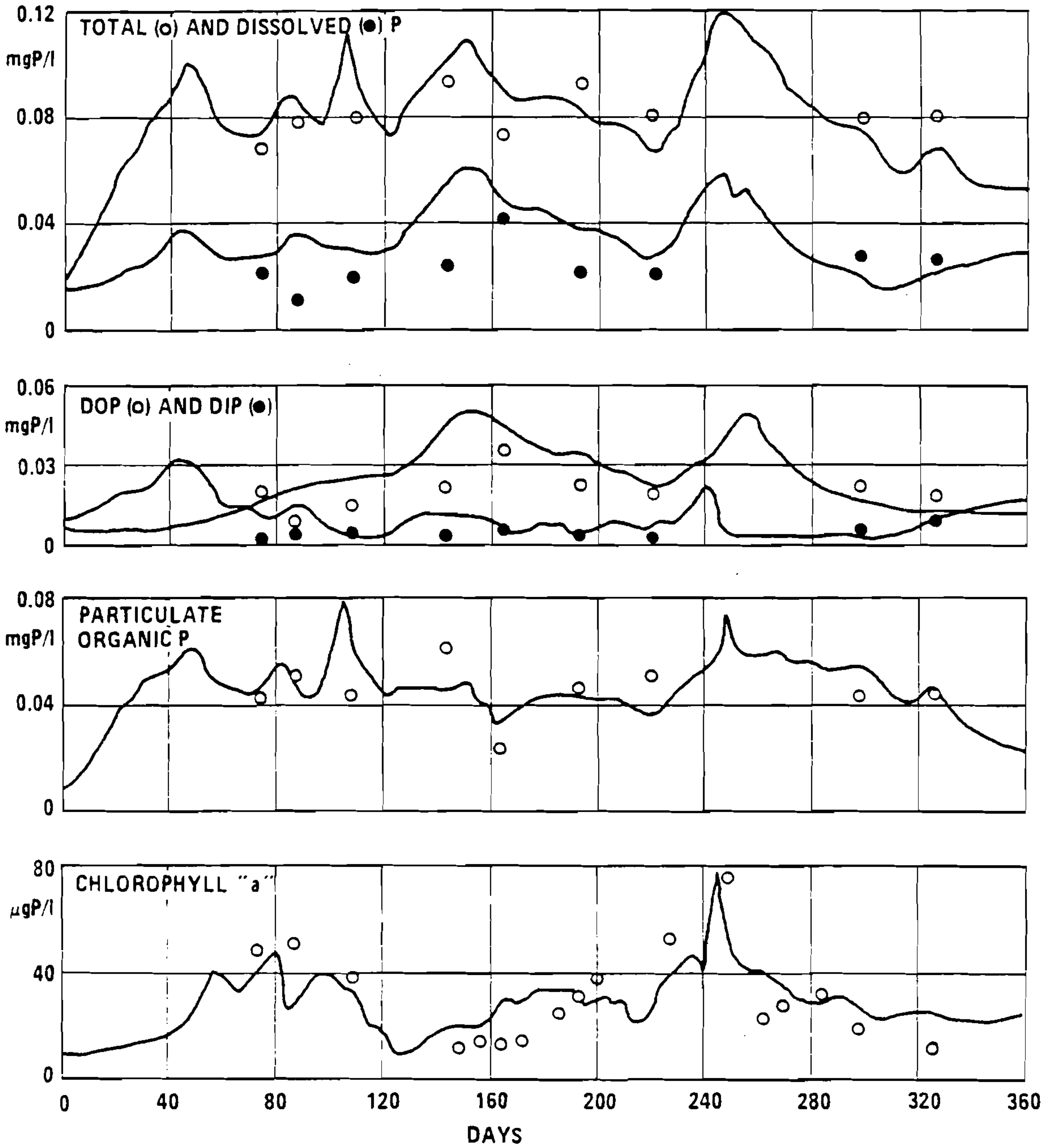


Figure 20. Comparison between model output (curves) and field data (points). Lake Balaton, 1977, Basin I – Hypothesis of three phytoplankton groups.

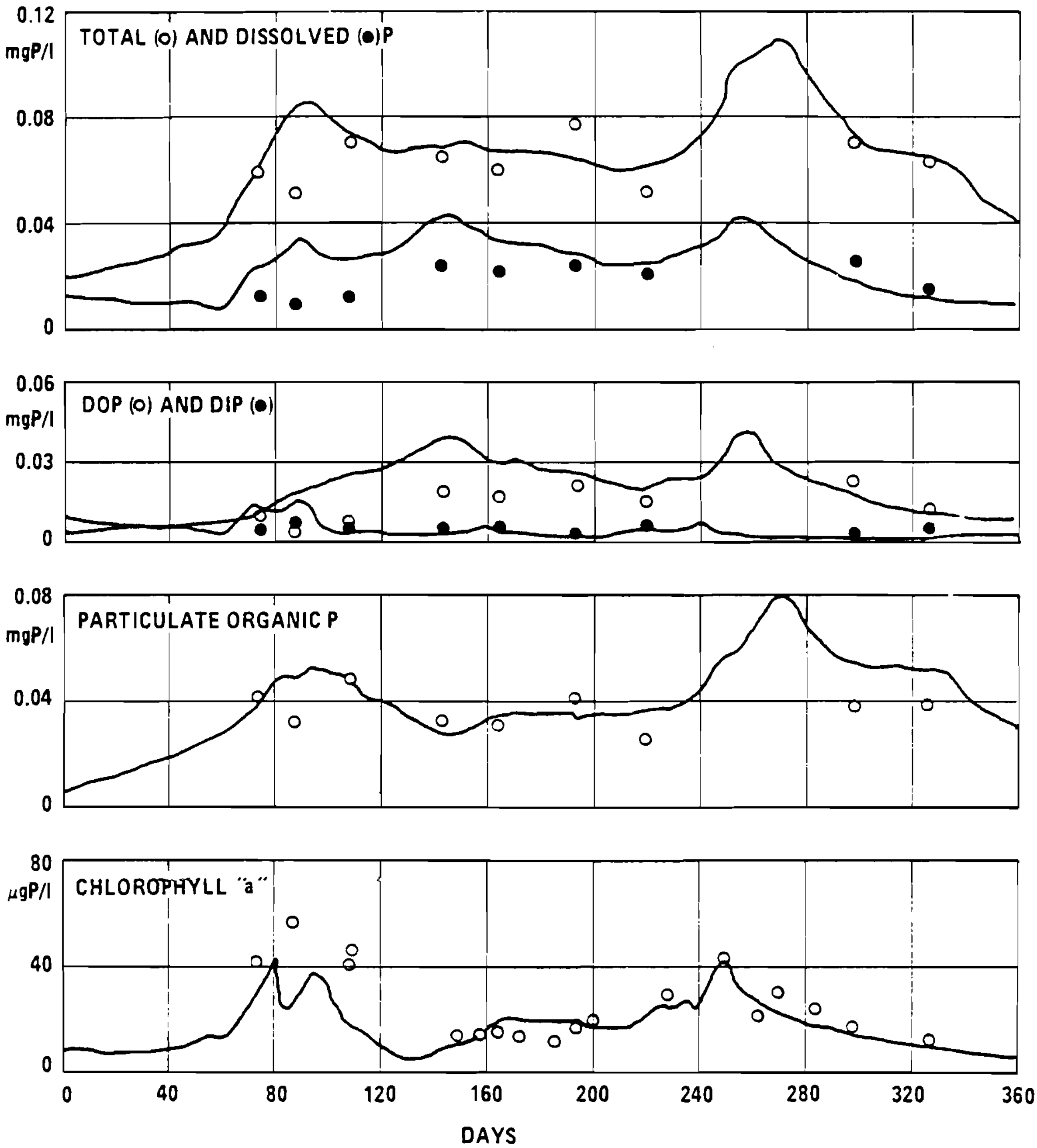


Figure 21. Comparison between model output (curves) and field data (points). Lake Balaton, 1977, Basin II – Hypothesis of three phytoplankton groups.

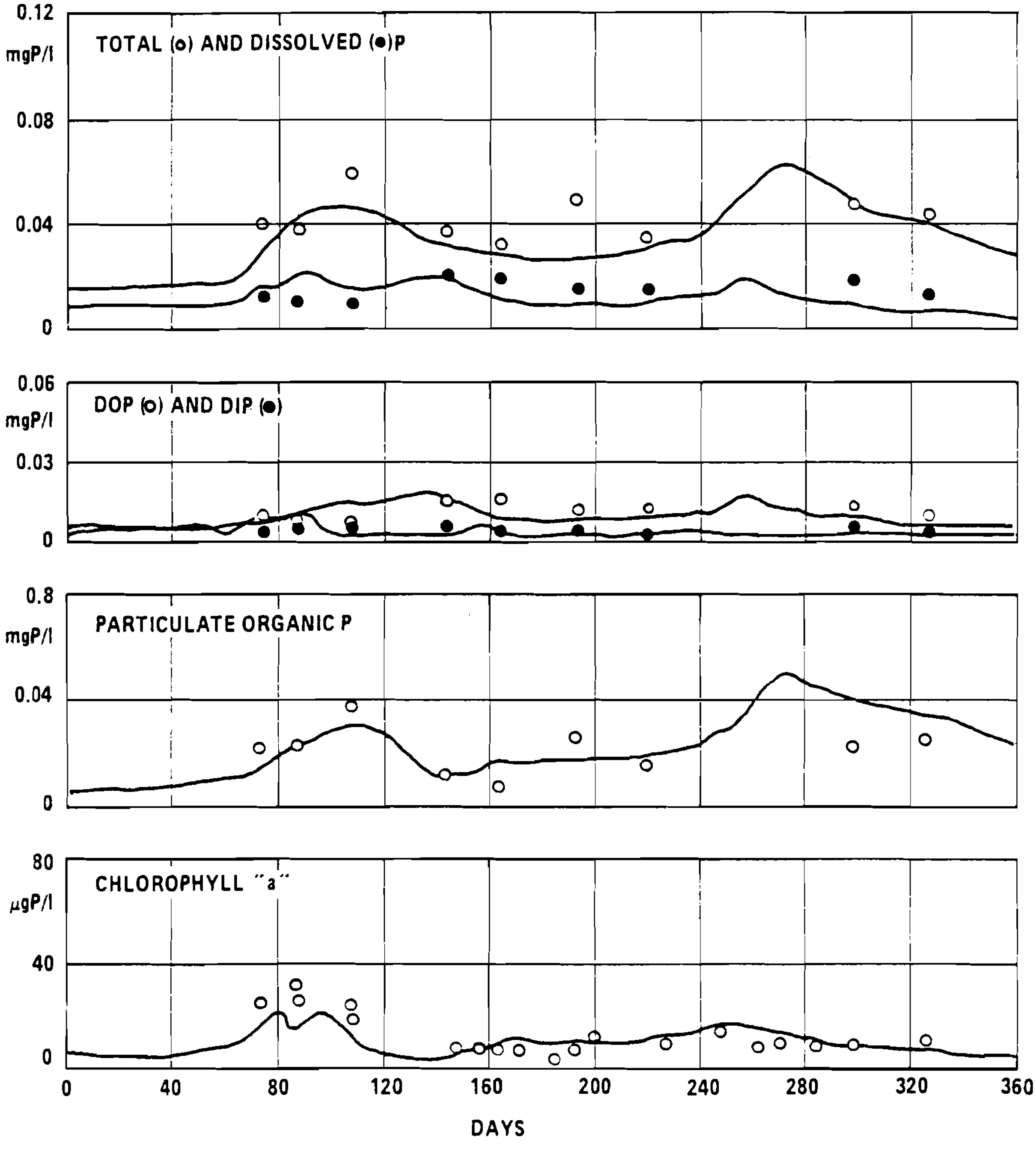


Figure 22. Comparison between model output (curves) and field data (points). Lake Balaton, 1977, Basin III – Hypothesis of three phytoplankton groups.

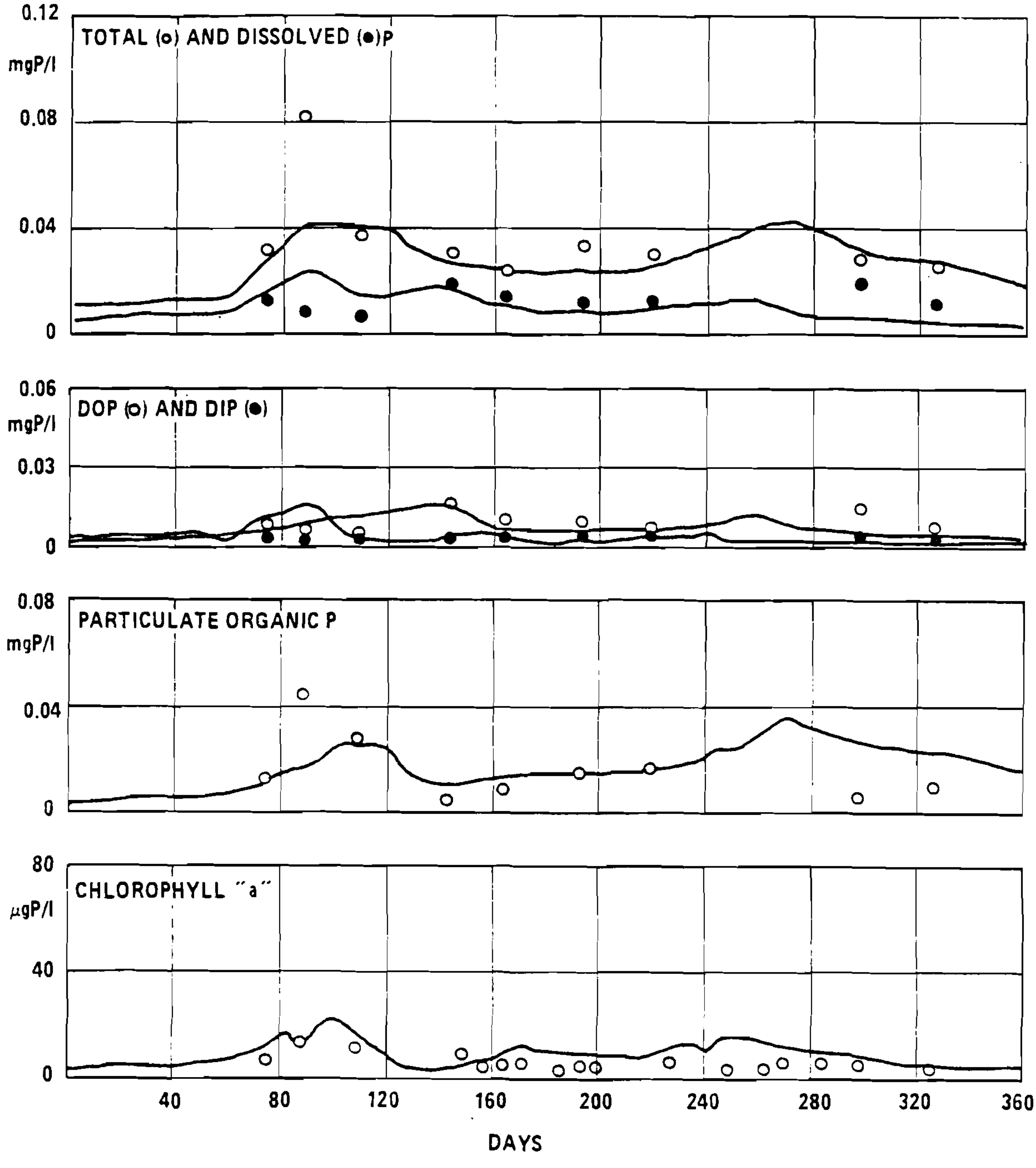


Figure 23. Comparison between model output (curves) and field data (points). Lake Balaton, 1977, Basin IV – Hypothesis of three phytoplankton groups.

This section of the report includes an analysis of the main phosphorus transformation flows within Lake Balaton that are evaluated by the model.

6.1. Phosphorus Loading

Water transfer, precipitation and additional phosphorus input from external sources are the main processes considered by the model in the total phosphorus loading of different Lake Balaton basins. It is interesting to estimate the role of these processes from a quantitative perspective for each basin.

Table 12 shows the monthly amounts of phosphorus forms entering the lake with water discharged by the River Zala. This information is received at analysis of input data demonstrating the effect of the River Zala on the Lake Balaton ecosystem. As shown in Table 12 during January-April 1977, most of the phosphorus i.e. 55-78 percent enters the lake in particulate form. The role of dissolved inorganic phosphorus in the River Zala discharge water, becomes significant from May although the actual contribution of DIP is slightly changed in all months. The role of other phosphorus forms in water discharged from the River Zala may be considered of lesser importance from a qualitative perspective.

The annual input of particulate-P and DIP by River Zala water flow comprised of the total 56.5 percent and 39.2 percent respectively, while the annual input of phytoplankton-P and bacterial-phosphorus, are estimated to be equal to 4.2 percent and 0.1 percent, respectively. Total annual P-input in 1977 from River Zala to Lake Balaton is equal to 0.99 mg P/ℓ.

Table 13 contains information which allows an estimate to be made of the role of water transfer in P-loading in all the basins. The role of the River Zala's water flow is very important in P-loading of Basin I. However, in other basins, the water flow in the lake transfers significantly smaller amounts of P: just 0.05 mg P/ℓ in Basin II, and 0.013 mg P/ℓ in Basin IV.

In the spring and autumn 34-64 percent of phosphorus is transferred by water flow from one basin to another in particulate form. During the summer months, the role of DOP in phosphorus transfer becomes important and water flow transfers from 31 percent

Table 12. Dynamics of phosphorus inputs (in mg P/ℓ and percent of total P-load) from River Zala to Keszthely Bay in 1977.

Months	nonliving particulate phosphorus		DIP		Phytoplankton-phosphorus		Bacterial-phosphorus		Total input of P
	mg P/ℓ	percent	mg P/ℓ	percent	mg P/ℓ	percent	mg P/ℓ	percent	mg P/ℓ
I	0.099	77.8	0.025	19.7	0.003	2.4	0.00018	0.1	0.12718
II	0.158	74.4	0.046	21.7	0.008	3.8	0.00026	0.1	0.21226
III	0.064	54.6	0.044	37.5	0.009	7.8	0.00019	0.1	0.11719
IV	0.138	76.2	0.033	18.2	0.010	5.5	0.00017	0.1	0.18117
V	0.028	45.1	0.032	51.6	0.002	3.2	0.00006	0.1	0.06206
VI	0.012	27.2	0.031	70.4	0.001	2.3	0.00004	0.1	0.04404
VII	0.004	8.9	0.039	86.6	0.002	4.4	0.00003	0.1	0.04503
VIII	0.006	19.3	0.023	74.2	0.002	6.4	0.00003	0.1	0.03103
IX	0.003	9.4	0.027	84.3	0.002	6.2	0.00003	0.1	0.03203
X	0.006	18.7	0.025	78.1	0.001	3.1	0.00003	0.1	0.03203
XI	0.027	44.2	0.033	54.1	0.001	1.6	0.00005	0.1	0.06105
XII	0.014	31.1	0.030	66.6	0.001	2.2	0.00007	0.1	0.04507
Total input of P, mg P/ℓ	0.559	56.5	0.388	39.2	0.042	4.2	0.00114	0.1	0.99014

Table 13. An estimation of the role of water transfer in P-loading in Lake Balaton basins, 1977.

Basin of Lake Balaton	Season	nonliving particulate phosphorus		DIP		DOP		Phyto-P		Bacterial-P		Total P
		mg P/ℓ	percentage	mg P/ℓ	percentage	mg P/ℓ	percentage	mg P/ℓ	percentage	mg P/ℓ	percentage	mg P/ℓ
I	Spring	.2303	64.1	.1079	30.0	.0	.0	.0210	5.8	.00042	0.1	0.35962
Keszthely	Summer	.0224	18.8	.0927	77.6	.0	.0	.0042	3.5	.00014	0.1	0.11944
Bay	Autumn	.0356	28.6	.0856	68.6	.0	.0	.0034	2.7	.00011	0.1	0.12471
	Annual	.5590	56.5	.3880	39.2	.0	.0	.0420	4.2	.00114	0.1	0.99014
II	Spring	.0069	41.5	.0019	11.4	.0049	29.4	.0027	16.2	.00025	1.5	0.01665
	Summer	.0015	16.5	.0009	9.9	.0037	40.9	.0016	17.6	.00137	15.1	0.00907
Sligliget	Autumn	.0026	36.2	.0005	6.9	.0020	27.8	.0014	19.4	.00070	9.7	0.00720
	Annual	.0193	37.9	.0087	17.1	.0124	24.4	.0080	15.7	.00251	4.9	0.05091
III	Spring	.0056	43.0	.0014	10.7	.0040	30.7	.0018	13.8	.00023	1.8	0.01303
	Summer	.0012	20.7	.0003	5.2	.0024	41.5	.0008	13.8	.00109	18.8	0.00579
Szemes	Autumn	.0035	52.6	.0001	1.5	.0017	25.6	.0007	10.5	.00065	9.8	0.00665
	Annual	.0145	42.4	.0033	9.6	.0097	28.4	.0045	13.2	.00218	6.4	0.03418
IV	Spring	.0016	34.0	.0008	16.9	.0014	29.7	.0008	16.9	.00012	2.5	0.00472
	Summer	.0003	22.9	.0001	7.6	.0004	30.5	.0002	15.3	.00031	23.7	0.00131
Siofok	Autumn	.0014	56.8	.0001	4.0	.0004	16.2	.0003	12.1	.00027	10.9	0.00247
	Annual	.0048	36.9	.0022	16.9	.0032	24.6	.0020	15.4	.00082	6.2	0.01302

to 41 percent of P in the form of DOP. Phytoplankton-P is evidently transferred by water flow to all the basins during the year. Therefore, significantly less P is transferred by water flow in the form of DIP and bacterial-phosphorus.

Figure 24 shows the monthly dynamics in 1977 of DOP- and DIP-inputs due to precipitation in the different basins. Annual inputs of DOP by precipitation are equal to 0.0163, 0.0123, 0.0109 and 0.01 mg P/ℓ for Basin I through IV, respectively. Annual inputs of DIP are 0.0271, 0.0204, 0.0182 and 0.0166 mg P/ℓ for Basin I through IV, respectively.

The role of individual processes such as water transfer, precipitation and external sources in total P-loading of basins, is estimated in Table 14. As shown, the role of water transfer in total P-loading is decreased from Basin I to Basin IV, while the role of precipitation is increased. Input of phosphorus from external sources is the main process in P-loading for Basins II, III and IV.

Table 14. Role of individual processes in P-loading Lake Balaton basins, 1977.

Basin	Total annual P-load mg P/ℓ	Processes in P-loading					
		water transfer		Precipitation		External Sources	
		mg P/ℓ	per-cent	mg P/ℓ	per-cent	mg P/ℓ	per-cent
I	1.12408	0.99014*	88.1	0.04344	3.9	0.0905	8.0
II	0.30260	0.05091	16.8	0.03267	10.8	0.21902	72.4
III	0.13828	0.03418	24.7	0.02906	21.0	0.07504	54.3
IV	0.09366	0.01302	13.9	0.02664	28.4	0.054	57.7

* River Zala loading

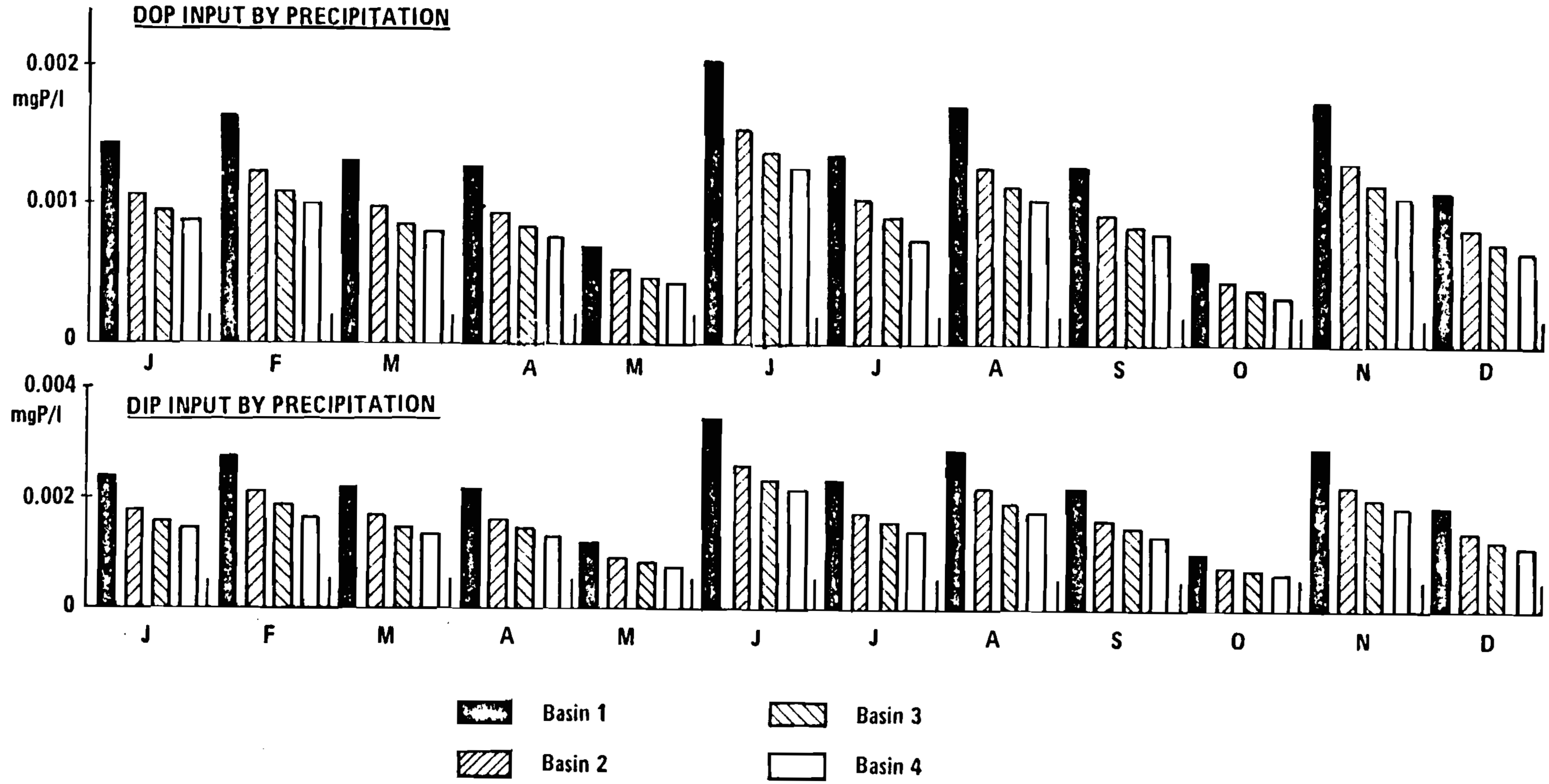


Figure 24. Monthly inputs of DOP and DIP by precipitation for different Lake Balaton basins, 1977.

6.2 Composition and Transformation of Particulate Phosphorus

The particulate organic phosphorus in the lake has three main components, namely phytoplankton, bacteria and detritus. An analysis of simulation results shows that the total amount of particulate organic phosphorus, in different basins of Lake Balaton in 1977, varied by a factor of 3-5. The model discussed here allows one to get direct estimates of the fractions of living and nonliving particulate organic matter.

Calculations show that living particulate phosphorus comprise, on an average, 20-40 percent of the total, and that this increases up to 60-70 percent in periods of active development of phytoplankton and bacteria. The proportion of phytoplankton-phosphorus to bacterial-phosphorus is not constant during the year: phytoplankton phosphorus dominates in the winter and spring, and during this period it contributes 85-95 percent of the total particulate (living) phosphorus. The role of bacteria becomes essential in the summer and autumn when bacteria make up 40-65 percent of living phosphorus matter.

6.2.1 *Nonliving Particulate Phosphorus and its Decomposition*

When making a quantitative study of phosphorus transformation, living as well as dead particulate matter have to be taken into account. This is because a major portion of particulate organic phosphorus is composed of dead organic matter or detritus, which in fact makes up 60-80 percent of the total particulate phosphorus for most of the year. Just during the summer months, when organisms are most active in their development, the role of detritus in providing the particulate phosphorus pool decreases as much as 30-40 percent. Table 15 consists of quantities of dead particulate phosphorus contributed by various processes. All the processes may be subdivided into external and internal.

The very great quantity of nonliving particulate phosphorus in Basin I results from the River Zala discharge. The effect of this source is even more prominent in winter and spring. For example, in Basin I, 65.3 percent of the total pool of nonliving particulate phosphorus is provided by input from the River Zala discharge water (Table 15) in spring. According to assumptions made, the input of nonliving particulate-P from the watershed area and generally from external sources, combine

Table 15. Quantities of nonliving particulate organic phosphorus (mg P/ℓ) provided by various processes in Lake Balaton basins in 1977. (simulation)

Basin	Season	Water Transfer		Phytoplankton mortality		Bacterial mortality		Input from external sources		Total P-input in particulate phosphorus	Sources			
											external		internal	
		mg P/ℓ	per-cent	mg P/ℓ	per-cent	mg P/ℓ	per-cent	mgP/ℓ	per-cent	mg P/ℓ	mg P/ℓ	per-cent	mg P/ℓ	per-cent
I	Spring	.2303	65.3	.1134	32.2	.0087	2.5	.0	.0	.3524	.2303	65.3	.1221	34.7
	Summer	.0224	7.1	.1598	50.5	.1340	42.4	.0	.0	.3162	.0224	7.1	.2938	92.9
	Autumn	.0356	13.8	.1319	51.3	.0898	34.9	.0	.0	.2573	.0356	13.8	.2217	86.2
	Annual	.5590	44.8	.4509	36.1	.2383	19.1	.0	.0	1.2482	.5590	44.8	.6892	55.2
II	Spring	.0069	7.1	.0616	63.4	.0096	9.9	.0190	19.6	.0971	.0190	19.6	.0781	80.4
	Summer	.0015	7.2	.0760	36.3	.1290	61.6	.0030	1.4	.2095	.0030	1.4	.2065	98.6
	Autumn	.0026	1.2	.0599	27.6	.0881	40.7	.0660	30.5	.2166	.0660	30.5	.1506	69.5
	Annual	.0193	3.3	.2109	36.3	.2327	40.1	.1180	20.3	.5809	.1180	20.3	.4629	79.7
III	Spring	.0056	11.2	.0295	58.8	.0076	15.1	.0075	14.9	.0502	.0075	14.9	.0427	85.1
	Summer	.0012	1.2	.0342	34.9	.0596	60.8	.0030	3.1	.0980	.0030	3.1	.0950	96.9
	Autumn	.0035	3.1	.0282	24.9	.0503	44.3	.0315	27.7	.1135	.0315	27.7	.0820	72.3
	Annual	.0145	5.2	.0997	35.9	.1214	43.8	.0420	15.1	.2776	.0420	15.1	.2356	84.9
IV	Spring	.0016	4.1	.0273	69.1	.0061	15.4	.0045	11.4	.0395	.0045	11.4	.0350	88.6
	Summer	.0003	0.4	.0275	35.3	.0472	60.5	.0030	3.8	.0780	.0030	3.8	.0750	96.2
	Autumn	.0014	1.9	.0223	30.1	.0368	49.8	.0135	18.2	.0740	.0135	18.2	.0605	81.8
	Annual	.0048	2.4	.0818	40.8	.0929	46.3	.0210	10.5	.2005	.0210	10.5	.1795	89.5

to contribute 10-20 percent of the total particulate phosphorus annually in Basins II-IV. The role of this source is very small during the summer months--only 1.4-3.8 percent of phosphorus input is provided by it, while the most significant quota of phosphorus is provided by internal sources for most of the year. Phytoplankton mortality and bacterial mortality are taken into account in the model as the main internal sources of particulate matter. Phytoplankton mortality provides 36-40 percent of particulate phosphorus annually in all the basins considered by the model. The role of this process is very significant in the spring (Table 15) in comparison with other sources. Bacterial mortality contributes most particulate phosphorus in the summer-autumn period, namely 35-62 percent, to different basins of Lake Balaton.

Dead organic phosphorus is an important source of DOP, therefore it may be considered one of the most important energy sources for bacteria. Quantitative estimates of amounts of detritus which decompose to DOP are essential for a better understanding of the role of suspended organic matter in the transformation cycles of phosphorus. Such estimates were arrived at by calculations made during the analysis of simulation results of phosphorus transformations in the different basins of the lake. The data derived from this analysis are summarized month by month in Table 16.

The decomposition of nonliving particulate phosphorus is a temperature-dependent process, therefore, its role in phorphorus transformation in the January-April period when temperature is low, is extremely small. The effect of this process may be considered important in the May-September period--about 0-05-0.06 mg P/ℓ in each month during this period had been transformed from detritus to DOP, in Basins I and II. Contributions of this process in Basins III and IV are 0.01-0.04 mg P/ℓ, for the same period. Total annual contributions of nonliving particulate phosphorus decomposition to DOP are practically similar in Basins I and II (Table 16), while in Basins III and IV they are two times lower.

6.2.2 *Sedimentation of Nonliving Particulate Phosphorus*

Among all the processes involved sedimentation is one of the major ones in the regulation of phosphorus content in the water.

Table 16. Monthly contributions of detritus decomposition to DOP in mg P/ℓ in Lake Balaton basins, 1977 (simulation).

Months	Basins			
	I	II	III	IV
I	.0000	.0000	.0000	.0000
II	.0004	.0002	.0001	.0001
III	.0023	.0021	.0007	.0004
IV	.0023	.0020	.0012	.0009
V	.0497	.0330	.0127	.0107
VI	.0561	.0545	.0207	.0161
VII	.0624	.0601	.0266	.0214
VIII	.0548	.0572	.0294	.0234
IX	.0613	.0676	.0366	.0253
X	.0138	.0203	.0138	.0091
XI	.0043	.0063	.0043	.0029
XII	.0001	.0001	.0001	.0001
Annual flow of P_D to DOP	.3075	.3034	.1462	.1104

Quantitative estimates of phosphorus which has settled into the sediment are important. They are a key factor in the interpretation of ecological processes and in the analysis of phosphorus cycling between water and sediment when studying eutrophication problems.

In the model results considered, estimates of phosphorus losses in sediment were made for each basin on a monthly basis. These estimates are given in Table 17. The largest amount of particulate phosphorus had settled in Basin I, while the total annual phosphorus losses in Basins II and III and IV are 4, 8, and 11 times lower than in Basin I. As a further comparison, it is interesting to note that the percentages of annual phosphorus loss by sedimentation show approximately the same values: 75%, 78%, 79% and 83% from a total phosphorus load for Basin I through IV, respectively.

The composition of total phosphorus which settled into sediment is changeable during the year. In winter, the suspended material entering Lake Balaton via the river dominates. In spring, summer and autumn, detritus, generated by biochemical processes in the lake, is the predominant type of particulate phosphorus that settles into the sediment.

6.3. Bacterial Phosphorus Transformation

The mechanisms of phosphorus cycling in the water environment to a large degree depends on bacterial activity. At this step in the study, a quantitative assessment of bacterial significance in phosphorus transformation processes in the lake is desirable. In the given model, the role of bacteria in phosphorus cycling is taken into account in processes of DOP uptake, DIP excretion and detritus formation. It is interesting to note the bacterial efficiency in these processes and also to make an estimate of the seasonal changes of bacterial net production rates in different basins of Lake Balaton. These assessments are presented in Table 18. A total analysis of simulation results, show that bacteria take up a significant part of phosphorus as DOP in the summer and start of autumn, when water temperature is close to the optimum for bacterial activity. The model results about bacterial activity coincide with those received

Table 17. Monthly losses to sediments of particulate phosphorus (mg P/ℓ) for Lake Balaton basins, 1977 (modeling).

Months	Basins			
	I	II	III	IV
I	0.0528	0.0039	0.0013	0.0010
II	0.1665	0.0156	0.0030	0.0018
III	0.1033	0.0224	0.0049	0.0029
IV	0.1608	0.0349	0.0133	0.0095
V	0.0011	0.0008	0.0166	0.0141
VI	0.0542	0.0121	0.0089	0.0070
VII	0.0457	0.0108	0.0032	0.0026
VIII	0.0480	0.0120	0.0041	0.0033
IX	0.0590	0.0200	0.0073	0.0049
X	0.0404	0.0439	0.0200	0.0132
XI	0.0609	0.0263	0.0113	0.0075
XII	0.0494	0.0320	0.0145	0.0097
Annual P-losses	0.8422	0.2347	0.1085	0.0775

Table 18. Assessments of bacterial phosphorus transformations (mg P/ℓ) in Lake Balaton basins, 1977.

Basin	Season	DOP uptake	DIP excretion	Detrital formation
I	Spring	.0270	.0106	.0087
	Summer	.2402	.0938	.1340
	Autumn	.1344	.0520	.0898
	Annual	.4084	.1584	.2383
II	Spring	.0288	.0113	.0096
	Summer	.2180	.0845	.1290
	Autumn	.1270	.0484	.0881
	Annual	.3804	.1466	.2327
III	Spring	.0215	.0087	.0076
	Summer	.0973	.0367	.0596
	Autumn	.0709	.0266	.0503
	Annual	.1941	.0732	.1214
IV	Spring	.0178	.0070	.0061
	Summer	.0772	.0290	.0472
	Autumn	.0504	.0187	.0368
	Annual	.1487	.0558	.0929

by Olâh (1969a;1974). He found a maximum number of heteotrophs at the end of summer and beginning of autumn for different parts of Lake Balaton. Accordingly, the simulation results in this period, the bacteria has the highest biomass about 0.012-0.015 mg P/ℓ and they take up 0.12-0.24 mg P/ℓ-month in Basins I-II. The bacterial activity in phosphorus transformation is 2-3 times lower in Basins III-IV; in summer and at the beginning of autumn their DOP uptake is 0.05-0.1 mg P/ℓ-month while their biomasses are about 0.005-0.007 mg P/ℓ in these basins. Corresponding annual bacterial uptake of DOP is 0.38-0.41 mg P/ℓ in Basins III-IV. Bacterial DOP uptake is estimated to be equal to 85%, 95%, 97%, and 99% of total DOP inputs from all the sources considered in Basins I, II, III, and IV, respectively.

The release of dissolved phosphorus as inorganic orthophosphate, due to the metabolism of bacteria, is equal to 37-40 percent of the total bacterial uptake of phosphorus in an observed range of DOP concentrations in Lake Balaton basins. In a quantitative assessment, this process is very important for maintaining DIP levels in the water body. In Basin I, the annual bacterial excretion of DIP is estimated to be equal to 24 percent of the total DIP input from all sources. During June-September, the bacterial DIP excretion in this basin is even comparable to DIP input from the River Zala. In other basins, the role of bacterial excretion, contributing to the overall balance of DIP, is higher, and it is estimated to be equal to 52-57 percent.

Mortality of bacteria is a main factor in regulating bacterial biomass levels in the absence of predators. It is estimated to be equal to about 32-35 percent of the total phosphorus uptake by bacteria in the spring. Thus, the quota is increased to 56-73 percent in the summer-autumn period in all the basins.

The dynamics of calculated bacterial net production rates for all the basins is shown in Figure 25. It was found that these values vary seasonally in accordance with temperature and DOP concentration changes. During January - April, all the processes in bacterial metabolisms are balanced and the bacterial net production rate is very close to zero. Bacterial uptake of DOP begins to be noticeable as of May. Approximately at the end of May, bacterial net production reaches the maximum

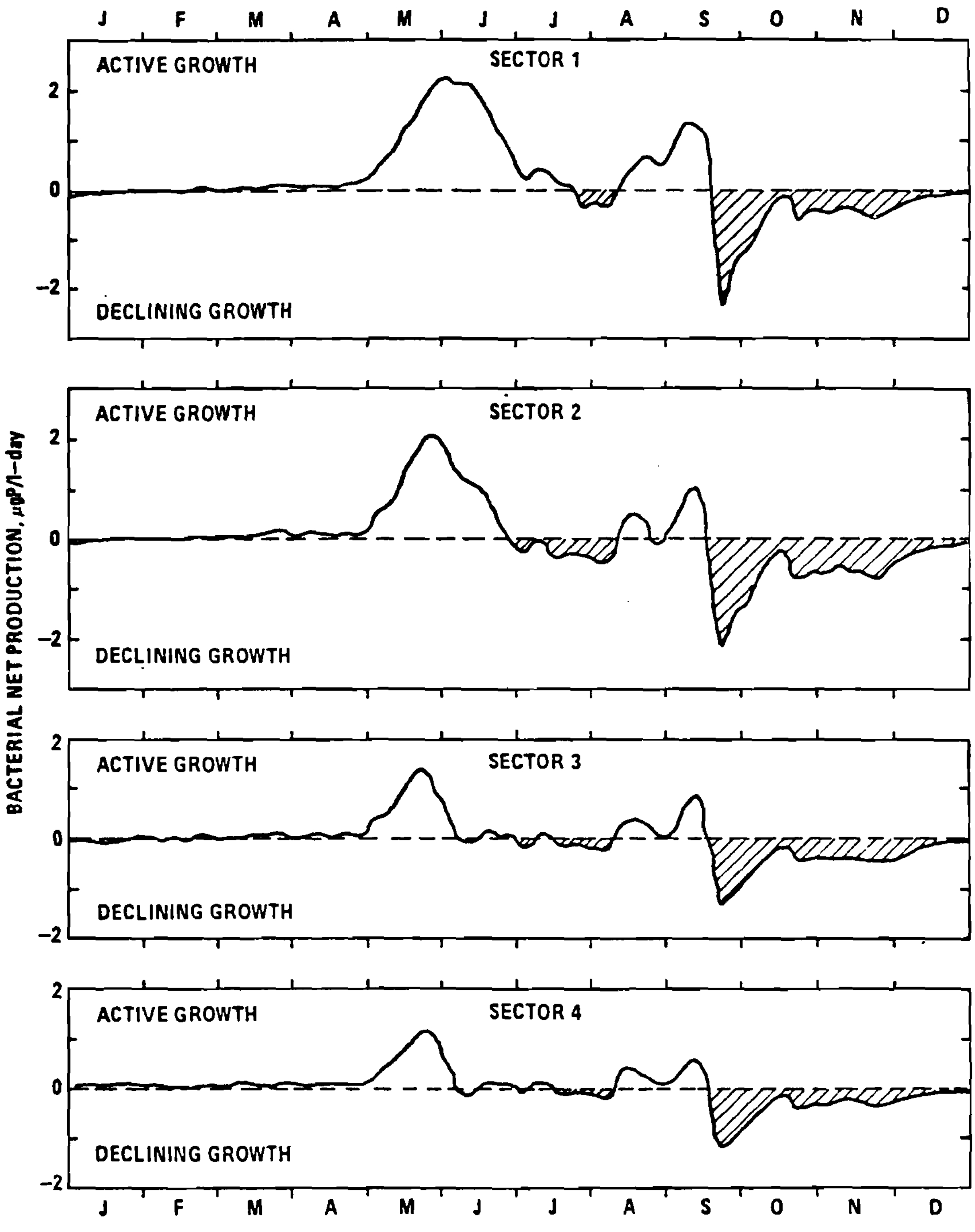


Figure 25. Dynamics of bacterial net production in different basins of Lake Balaton, 1977.

level attainable. This is due to increasing water temperature. These values in the peak period, are equal to 1-2 mg P/ℓ-day in various basins.

In the July-September period, the effect of bacteria on phosphorus transformation is very noticeable: the monthly uptake of DOP is 0.07-0.08 mg P/ℓ (Basin I-II) and 0.03-0.04 mg P/ℓ (Basin III-IV); they excrete 0.01-0.03 mg P/ℓ as DIP and they form 0.02-0.05 mg P/ℓ as detrital phosphorus in all basins for the period of the year mentioned. An analysis of bacterial-phosphorus transformation shows that during the summer, just 10-20 percent of total phosphorus taken up is used by bacteria for production of their own cell material. The role of excretion is especially significant for the maintenance of phytoplankton production in summer and autumn, when DIP input from the watershed area is reduced to 15-25 percent of the total. Thus although the model results as a whole confirm a conclusion of Oláh (1969a; 1974) about slight spatial differences of bacterial numbers, it also show quantitatively the bacterial functional differences in phosphorus transfer motion in different Lake Balaton basins.

6.4. Phytoplankton Phosphorus Transformation

One of the key factors of water body eutrophication is the dynamics of phytoplankton with relation to processes of phosphorus transformation, in time and space. Phytoplantkon growth in water is controlled by the combined influence of temperature, light and nutrition. Only through a quantitative estimate of all the processes defining phytoplankton metabolism and control of its growth is it possible to assess and predict the lake system's responses to a wide range of state variable changes in concentration. Some quantitative assessments of phytoplankton activity were presented and discussed in previous sections of this report. Table 19 consists of quantitative data on the main phytoplankton activity in phosphorus transformation processes. These data are the results of special calculations based on the simulation results presented in Figures 20-23.

About 25 percent of the phosphorus taken up is excreted by phytoplankton as DOP. A significant part of DOP is produced by phytoplankton through the detritus chains. The quota of detritus formed by phytoplankton is about 67-85 percent of the total uptake of phosphorus in phytoplankton metabolism. This quota is smallest in summer, when the nutrient level limits phytoplankton growth to a great extent. Features in the behavior of phytoplankton in the lake's ecosystem for the given set of environmental conditions of 1977, may be studied and understood in detail with the help of graphs showing the dynamics of phytoplankton net production rates (Figure 26), and values of phytoplankton activities in phosphorus tranformation (Table 19).

The intensive growth of phytoplankton in Basin I at the end of February, was due to favorable physical conditions and nutrient levels. Only a low concentration of DIP can explain the absence of an intensive net production of phytoplankton at this period in the other basins. Favorable conditions for phytoplankton growth are observed in all basins from mid-March till the beginning of April.

Spring peaks of phytoplankton net production rates are about 2 μg Chl "a"/ℓ-day for Basins I-II and 1 μg Chl "a"/ℓ-day for Basins III-IV. Amounts of DIP taken up by phytoplankton in spring bloom, are estimated to be 0.13, 0.08, 0.038, and 0.036 mg P/ℓ for Basins I, II, III, and IV, respectively.

After the spring peak of phytoplankton, growth is limited by DIP content in April and at the beginning of May in all the basins. For this period, the phytoplankton DIP uptake is smaller than its mortality and metabolic excretion losses, the phytoplankton biomass decreases slightly and phytoplankton net production is negative.

To approximately mid-May, phytoplankton growth increases due to DIP input by bacterial regeneration. In summer, uptake of DIP by phytoplankton is most essential in comparison with DIP uptake in other seasons. It is estimated to be 0.237, 0.111, 0.049, and 0.04 mg P/ℓ for four basins from Keszthely to Siófok, respectively, although the DIP content in these basins are almost similar, about 0.004-0.006 mg P/ℓ. This is a result of equilibrium between all biochemical processes defining nutrient cycling in the water. The data presented in previous sections of

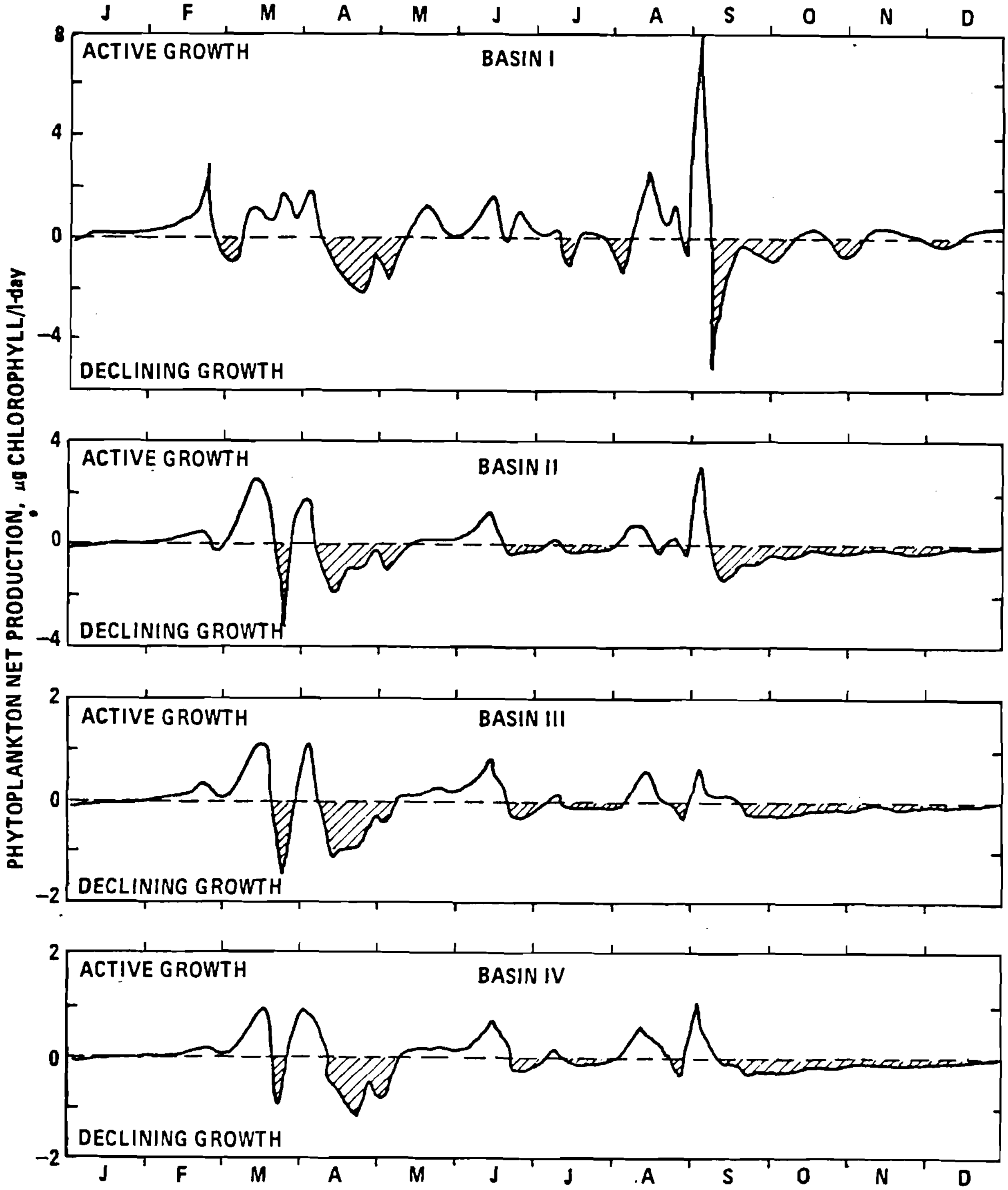


Figure 26. Dynamics of phytoplankton net production in different basins of Lake Balaton, 1977.

Table 19. The influence of phytoplankton in phosphorus transformation processes in Lake Balaton basins, 1977 (mg P/ℓ).

Basin	Season	DIP uptake	DOP excretion	Detrital formation
I	Spring	.1344	.0339	.1134
	Summer	.2370	.0608	.1598
	Autumn	.1735	.0442	.1319
	Annual	.6223	.1583	.4509
II	Spring	.0829	.0209	.0616
	Summer	.1107	.0280	.0760
	Autumn	.0708	.0177	.0599
	Annual	.2822	.0710	.2109
III	Spring	.0376	.0094	.0295
	Summer	.0490	.0123	.0342
	Autumn	.0337	.0083	.0282
	Annual	.1307	.0326	.0997
IV	Spring	.0358	.0089	.0273
	Summer	.0403	.0100	.0275
	Autumn	.0261	.0064	.0223
	Annual	.1089	.0271	.0818

this report make it possible to estimate the efficiency of individual processes in phosphorus cycling, in connection with phytoplankton dynamics and nutrient levels for different basins. These data take into account the combined effect of external phosphorus sources and internal processes of phosphorus transformation.

An analysis of simulation results show that although the bacterial excretion of DIP in May-June is effective, the nutrient level is not enough for maintaining active phytoplankton growth under favorable physical conditions, in all basins, and in July, the active growth phase changes to a negative growth phase. In mid-August, there is a significant increase in phytoplankton growth as a result of the complex effect of bacterial DIP regeneration, DIP inputs from watershed areas that may include all possible sources mentioned previously, and favorable physical conditions. This relatively short phase of active phytoplankton growth is continued till the beginning of September, when phytoplankton growth becomes limited by the DIP content. In the next few months, phytoplankton growth is regulated mainly by temperature and light conditions.

In a quantitative sense, the effect of phytoplankton in phosphorus cycling is very significant, especially in Basin I. Suffice it to say that the annual uptake of DIP by phytoplankton estimated to be 0.622 mg P/ℓ in Keszthely Bay, is about 63 percent of the total phosphorus input from the watershed area, in Basin I.

7. INTERNAL PHOSPHORUS CYCLING

A useful way of assessing the lake's ecosystem behaviour, is to estimate the amounts of phosphorus stored in various compounds and individual fluxes of phosphorus between the different compartments, chemical as well as biological. It is assumed to be important, because qualitatively, the reactions comprising the phosphorus cycling and sources of phosphorus in the lake's ecosystem are well known in principle, however, the quantitative information vital to the understanding of the phenomena of eutrophication in Lake Balaton is lacking. Special calculations based on simulation results have been made, to compare the annual amounts of phosphorus transfer by the various transformation pathways considered in the given model. Results of these calculations are summarized in Figure 27.

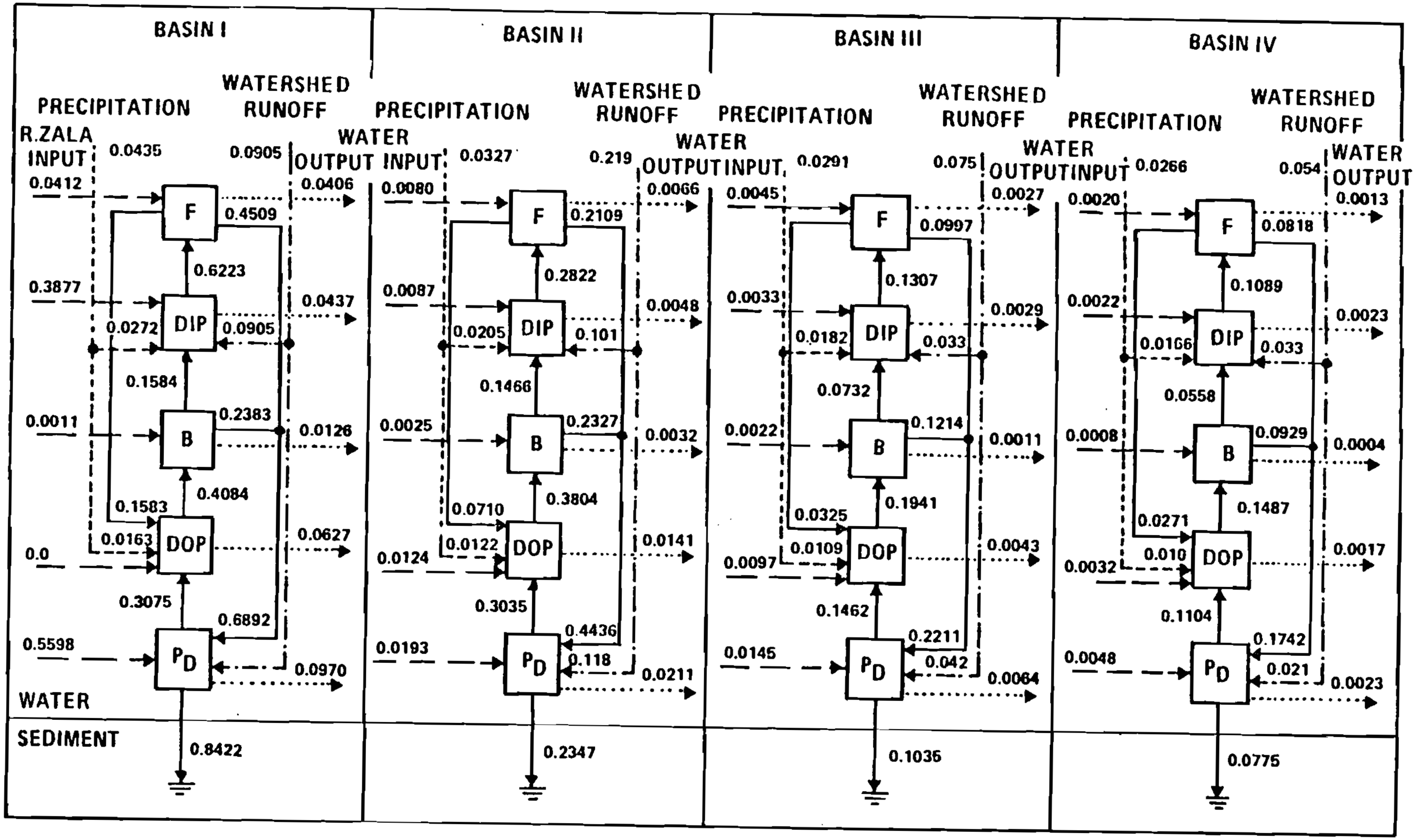


Figure 27. Quantitative representation of annual phosphorus material flows (mgP/l) in Lake Balaton basins for the environmental conditions of 1977.

- – – – → P-input by water flow
- ··········→ P-output by water flow
- –·–·–→ P-input by watershed runoff
- – – – – –→ P-input by precipitation
- ———→ P-cycling by internal transformation processes.

With the help of these data, we can estimate the phosphorus annual budget of Lake Balaton for the environmental conditions of 1977. Taking into account the hypotheses about the phytoplankton development, external phosphorus loading and sedimentation, and also the complex effects of the main abiotic factors such as temperature, radiation, water balance, loading of the River Zala, phosphorus transformations, and the fact that the model output satisfactorily tallies with observed data, it is possible to say that the estimated phosphorus budget for Lake Balaton in 1977, although being tentative, is based on an actual set of measurements. The results of analysis show that in 1977, only 2-13 percent of the total phosphorus load in the basins was retained in the water and recycled by biochemical transformation processes; 75-83 percent of the total phosphorus settled, and 9-23 percent was transferred by water flow from one basin to another.

The phosphorus in the water is recycled repeatedly. The movement of phosphorus compounds in water may be considered as biogeochemical cycles, determined by the activity of microorganisms and physical-chemical reactions. In this way, a cycle may be described in terms of compound pools that include some quotas of matter in soluble form, organically bound in living and non-living particulate matter. An estimate can be made of the rate of phosphorus recyling and sizes of different phosphorus pools; this can be used for defining the features of water ecosystem behaviour. In this case, very important information may be obtained with the help of flow rates between different pools, or actual contributions of various processes to the total balance of matter in the ecosystem studied. The results of special calculations of annual phosphorus movements presented in Figure 27, allow us to obtain quantitative information concerning the contributions of the key processes in phosphorus cycling, namely the decomposition of organic phosphorus and the formation of mineral phosphorus in the total balance of phosphorus in the lake. These processes provide favorable conditions for the growth of microorganisms, owing to the continued recycling of inorganic compounds and decomposition of organic substances.

A better understanding of the system functions in phosphorus recycling may be obtained through an analysis of turnover times of this element and its individual fractions (Pomeroy 1974). The model's output allows us to calculate the values of turnover times for phosphorus compounds, using the quantities of phosphorus in the model compartments, i.e. phosphorus pools, and the rates of flux between them. Thus, the mean annual turnover times of all the phosphorus compartments were obtained for each basin, by dividing the phosphorus pool sizes averaged for one year, by the rate of phosphorus flux through the pools, for the same period of time. Flux rate, in this case, was defined as the average of the input and output rates of the pools. The results of calculating the turnover times of phosphorus fractions and total phosphorus, obtained with the aid of the model presented in this report, are summarized in Table 20. Although the mean annual turnover times of phosphorus fractions are not greatly different, a variance may be observed. Turnover time as a whole for phosphorus fractions, range from 1 day to 15 days. The phosphorus pool turnover in the bacteria's biomass is approximately 1 day in all the basins. The phytoplankton phosphorus biomass turnover is somewhat slower--about 5-8.5 days. Thus, the given model is consistent with one of the major ecological postulates that the turnover time increases, the higher the position of the germs in the trophic structure (Odum 1967).

Dissolved inorganic phosphorus has a turnover time of 5-11 days. This assessment of turnover time for DIP obtained for the Lake Balaton ecosystem by the present model, is in accordance with the measured turnover time of 5-10 days for phosphates in other lakes as well (Golterman 1975). The orthophosphate-phosphorus turnover time is 8 days in the lake as a whole. This has been corroborated by Olah et al. (1977). The turnover time for DOP is practically the same for the various basins considered by the model and is equal to 5-6 days. It is found that differences between the mean annual turnover times of nonliving particulate phosphorus in all basins within Lake Balaton appear to be greater than turnover times of other phosphorus fractions (Table 20). These differences in turnover times of

Table 20. Comparison of mean annual turnover times (in days) for phosphorus fractions in Lake Balaton basins (analysis of simulation results for environmental conditions of 1977).

Phosphorus fraction	Keszthely Bay	Szigliget Basin	Szemes Basin	Siófok Basin	Range
DIP	5.0	6.5	11.4	10.1	5 - 11.4
DOP	6.0	5.6	6.2	4.8	4.8 - 6.2
Nonliving particulate-P	0.6	7.4	15.4	10.7	0.6-15.4
Phytoplankton-P	5.0	6.0	8.5	7.6	5.0 - 8.5
Bacterial-P	0.9	0.8	0.9	1.2	0.8 - 1.2
Total P	12	35	58	55	12 - 58

nonliving particulate-P in various basins, show the significant role played by suspended matter in phosphorus transformations in the lake. In Keszthely Bay, this phosphorus fraction has a turnover time of 0.6 day, while in other basins, turnover time of nonliving particulate-P is almost ten times higher, and it is estimated to be equal to 7.4-15.4 days. It is possible to explain the obtained differences in turnover times of nonliving particulate phosphorus in Lake Balaton basins by the fact that all processes connected with this fraction are much faster in Keszthely Bay than in other basins. It must be noted that the value of turnover time for nonliving particulate-P obtained for Basins II-IV is in general agreement with the average turnover time for labile detritus--1-2 weeks (Watson and Loucks 1979). The total phosphorus turnover time in Lake Balaton is approximately 0.5-2 months on an average. An assessment of turnover times for total P suggest that phosphorus is retained and recycled to a different extent in the various basins. In Keszthely Bay, the turnover time of total phosphorus is shortest--about 0.5 month, in the Szigliget basin it is equal to about 1 month, and in Basin III-IV the turnover time of total phosphorus is estimated to be equal to about 2 months.

8. CONCLUSIONS AND RECOMMENDATIONS

The application of mathematical models in limnological research opens up the possibility of studying and predicting ecosystem functions and changes in eutrophication, as a result of the combined effect of abiotic, biotic, and particularly anthropogenic influences. Many important quantitative characteristics describing the water ecosystem behavior that cannot be obtained directly from concentration values of chemical and biological compounds present in the water, may be easily estimated by ecological models specially constructed for studying limnological questions.

This report describes the mathematical model of biogeochemical phosphorus cycling in water. It was applied for the analysis of phosphorus transformation processes, in conjunction with phytoplankton dynamics and eutrophication phenomena as a whole in Lake Balaton's ecosystem. The main purpose of this study was to use this model for simulation of the seasonal dynamics of compounds and phytoplankton, based on a real set of measurements of water temperature, radiation, phosphorus loads and water balance. The specific objectives of the given study are to increase the understanding of the dynamic behaviour of Lake Balaton as an ecological system. A data set for 1977 was used at this stage of the study. It appears reasonable to claim that the model output for the four basins of Lake Balaton is quite satisfactory for all phosphorus fractions considered in the model and for phytoplankton, when the hypothesis for three seasonal groups was used.

In order to analyze the simulation results from the point of view of phosphorus control, special calculations of phosphorus flows were made for all phosphorus-dependent activities and sources. In this case, the model gives a considerable degree of insight into the behavior of the Lake Balaton ecosystem and, in particular, phosphorus transformation processes, which form the basis for estimating the role of external nutrient load and internal processes of phosphorus cycling in different basins of the lake. Estimated phosphorus mass flows quantitatively explain the importance of individual processes in total phosphorus cycling, on the basis of observed data, while calculated rates of bacterial and phytoplankton net production indicate a direct response of microorganisms on the content of nutrients in the water. These estimations may be used for a quantitative explanation of changeable conditions in a trophic state and nutrient limitations in four basins of the lake. They may also be considered quite important for the formulation of management alternatives in the future.

A detailed analysis of phosphorus compounds and phytoplankton dynamics indicate that the spring peak of phytoplankton biomass is primarily controlled not only by increasing light, temperature and nutrients accumulated in water during winter, but also by additional external inputs of nutrients. It is estimated that the mid-summer minimum of phytoplankton is due completely to the scarcity of DIP. The fall peak of phytoplankton is a complex interaction of nutrients regeneration by bacteria and external nutrient inputs. These processes are more essential in Basins I-II.

On the basis of model results, pool sizes, flux rates and turnover times were calculated for all phosphorus fractions considered in the model. Results indicate that the range of turnover times of phosphorus compounds is about 1 day to 15 days. Turnover times of total phosphorus is shortest in Basin I and it is equal to about 2 weeks, while in other Lake Balaton basins total phosphorus turnover time is 1-2 months.

The primary emphasis in this report is on the scientific analysis and validity of the phosphorus model intended for Lake Balaton ecosystem studies. It is recommended that this model be used to analyze phosphorus compounds and phytoplankton dynamics for other years, for instance, 1976 and 1978, and to apply the values of rate constants, so as to give a satisfactory description of ecosystem behavior in 1977. This work provides the basis, not only for model validation, but also for understanding the response of Lake Balaton's ecosystem to the different environmental conditions and external loads.

Application of the given model to analyze oxygen dynamics in the water and phosphorus dynamics in sediment requires the data for verification of model constants on the basis of comparison of model output with a real set of measurements.

It is now obvious that for application of this model for prediction purposes, the description of the sedimentation process must be improved. One possibility is to construct a submodel of sedimentation,where the sedimentation rate will be estimated on the basis of wind data. It must be noted also that at the current stage of the Lake Balaton ecosystem study, the hydrodynamics of Lake Balaton were given in the ecological model in the simplest way possible. Results of the first model calculations make it possible to suggest that a more complete description of eutrophication phenomena in Lake Balaton must include better estimations of hydrodynamic effects upon biological and chemical processes in the water, and especially the influences of wind-induced circulation on inter-basin exchange processes and wind-induced resuspension of sediments.

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