# A VINTAGE CAPACITY MODEL OF PRODUCTION

Jeffrey A. Parker

October 1980
WP-80-143



INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

# A VINTAGE CAPACITY MODEL OF PRODUCTION

Jeffrey A. Parker

## I. INTRODUCTION

In recent years, many econometric studies of production processes have been undertaken at both the industry and aggregate levels. Such studies represent an attempt to use aggregated, observable data on prices and flows of industry inputs and outputs to estimate important characteristics of production technology such as elasticities of substitution between inputs to production. Many studies have been based on an assumption of continuous minimum-cost equilibrium and make no distinction between short and long run effects. Such an assumption can lead to a fundamental misspecification of the model and biased estimates of substitution elasticities.

As an alternative to this static equilibrium model, a vintage model is developed here which embodies a polar opposite assumption about production technology -- that once production units have been installed, their technologies are fixed. This assumption corresponds more closely to the nature of production in some sectors of the economy. The structure of the vintage capacity model is such that it can be estimated using data and econometric techniques almost identical to those used for the static equilibrium model. This suggests the feasibility of using both models to generate alternative estimates of important parameters of production technology.

In Section II of this paper, the structure of the static equilibrium model is briefly reviewed and its common features with the vintage capacity model are noted. In Section III, the vintage capacity model is presented, highlighting the differences between the two models. The conclusion discusses possible implementations of the model and ways in which the model could be extended.

## II. THE STATIC EQUILIBRIUM MODEL

Econometric models of production processes usually begin by assuming the existence of a twice differentiable, concave production function of the form

$$X = f(Z_1, Z_2, \ldots, Z_n; A_1, A_2, \ldots, A_m) \quad , \tag{II.1}$$

where $Z_i$ represents a quantity of the ith input to production, $A_1$ through $A_m$ are exogenous variables which influence the technology of production,[1] and X is the maximum amount of the single output which can be produced using the given amounts of inputs in the given states of the exogenous influences. This function is often assumed to be homogeneous of degree one in the set of inputs. For simplicity we will deal only with such "constant returns to scale" production functions.

Because detailed data on the inputs and outputs of individual plants cannot usually be obtained, the above function is assumed to represent the aggregate production function of an industry or group of related industries, aggregated spatially over a state, region, or entire nation and temporally over a quarter or year. This allows it to be applied using aggregate data on inputs and outputs, which are often published by industry or government sources.

If we postulate that production in the industry is carried out in such a way as to minimize the cost of producing an exogenously fixed quantity of output subject to exogenously given input prices, then there exists a twice-differentiable cost function dual to the production function which has the following form:

$$\text{(II.2)} \qquad \frac{C}{X} = g(P_1, P_2, \ldots, P_n; A_1, A_2, \ldots, A_m) \quad .$$

The function g gives the minimum average cost per unit of output which can be attained consistent with the production function given the prevailing input prices and exogenous conditions. It is written as an average cost function to embody the hypothesis of constant returns to scale. The function g must also be homogeneous of degree one in the set of prices to assure the absence of "money illusion" under conditions of perfectlybalanced inflation or deflation.

Using Shephard's (1953) Lemma, the cost-minimizing quantities of the factor inputs are given by

$$Z_i^* = \frac{\partial C}{\partial P_i}(P_1, P_2, \ldots, P_n; A_1, A_2, \ldots, A_m) \quad ,$$

or in terms of input-output ratios

$$\text{(II.3)} \qquad \left(\frac{Z_i}{X}\right)^* = g_i(P_1, P_2, \ldots, P_n; A_1, A_2, \ldots, A_m) \quad .$$

Equations (II.3) constitute an interdependent system of cost-minimizing demand functions. The interdependence arises from the fact that

$$\text{(II.4)} \qquad \begin{aligned} \frac{\partial (Z_i/X)}{\partial P_j} &= g_{iy}(P_1, P_2, \ldots, P_n; A_1, A_2, \ldots, A_m) \\ &= \frac{\partial^2 (C/X)}{\partial P_i \partial P_j} = \frac{\partial^2 (C/X)}{\partial P_j \partial P_i} = \frac{\partial (Z_j/X)}{\partial P_i} \quad . \end{aligned}$$

Thus the partial derivative of the demand for the ith factor of production with respect to the jth price is equal to that of the jth factor with respect to the ith price.

The goal of production modeling is to discover the nature and properties of the function g (or equivalently those of the production function f). To make this problem empirically tractible, a functional form must be posited for g (or for f) which, although it restricts it to belong to a specified family of functions, contains unknown free parameters which allow the precise shape of the function to be inferred econometrically from empirical data.

Because the choice of functional form is quite important, we must consider the bases on which this choice should be made. Two properties commonly sought in functional forms are flexibility and facility of econometric parameter estimation. A functional form is considered flexible if it allows important measures of interactions and responses in production such as elasticities of substitution to be freely estimated over a wide range of feasible values. The econometric estimation of cost parameters is most easily accomplished for functional forms for which the cost function and its derivatives are (or can easily be transformed to be) linear in their parameters.

A number of functional forms offering flexibility and linearity have been proposed and employed in empirical production studies. The most popular has been the translog function introduced by Christenson, Jorgenson, and Lau (1973). Others include the generalized Leontief of Diewert (1971), the generalized square root quadratic of Diewert (1974), and most recently the generalized Box-Cox function of Berndt and Khaled (1979), which includes as special cases each of the three aforementioned forms. Each of these functional forms provides a second-order approximation to any arbitrary cost function -- that is, for some suitable transformation of the arguments, each corresponds to a second-order polynomial approximation. Each form is also linear in its parameters, leading to ease of econometric estimation. Because of the interdependencies implied by (II.4), the equations must be estimated as a system subject to linear cross-equation constraints on the coefficients.

Having chosen a suitable functional form, the static equilibrium model is completed by the addition of one critical assumption: that producers are always able to act in such a way as to keep aggregate input-output ratios continuously at their long run cost-minimizing levels. If this condition holds, then the parameters determining the shape of the cost function can be estimated by applying linear regression techniques to equations (II.2) and (II.3).[2] For example, if the generalized Leontief function is used, the factor demand equations can be written

$$\text{(II.5)} \qquad \left(\frac{Z_i}{X}\right)_t = \sum_{j=1}^{n} \beta_{ij}\left(\frac{P_j}{P_i}\right)_t^{\frac{1}{2}} + \sum_{k=1}^{m} d_{ik}A_{k,t} \quad .$$

Let us now consider the conditions under which this equilibrium assumption is valid. Production in the industrial sector of technologically advanced economies is characterized by a large concentration of durable capital plant and equipment. Because these production units are long-lived, the same units will be used to produce output over many periods. If factor proportions are to shift instantaneously to their new long run equilibrium levels following a change in relative factor prices, then these durable production units must either be replaced by new units embodying the changed technology or the technology of existing units must be changed. Because the former adjustment mechanism directly contradicts the observed durability of production units, adjustment to equilibrium must occur through changes in the technology of existing units.

Furthermore, if equilibrium is to be immediately restored, not only must the technologies of existing units be changeable, but they must by costlessly changeable over precisely the same set of alternative technologies as newly-produced units. In other words, if the production units themselves are not replaced each period, then the technology which they use to produce output must be freely replaceable in each period. Because of this assumption of ex post as well as ex ante malleability of production technology, these models are often called "putty-putty" models.

A brief example may help elucidate the nature of technology which is imposed by the putty-putty hypothesis. Consider a production process which uses three inputs: labor, capital, and fuel. Suppose that after many durable production units (embodied in capital plant and equipment) have been put in place, the wage rate falls relative to the costs of other factors. The optimal technology of production will now use more labor relative to fuel, and new plants which are built will be designed to substitute more of the now cheaper human resources for the relatively more costly fuel. The putty-putty hypothesis implies that precisely the same degree of substitution of labor for fuel can be made in already existing production units as in units being planned and constructed.

While this may sometimes be the case, many industrial production processes are designed to run with virtually fixed proportions of inputs. In such cases, a specific number of persons are required to operate the process and the machinery demands a fixed amount of electricity, gas, or other fuel in order to function. By adding an extra human laborer to operate it, one cannot, for example, reduce the wattage of a machine and thus the amount of fuel it requires to operate.

An even more unreasonable situation results when one considers the role of capital as one of the inputs to production. As a result of the change in relative factor prices, labor should now be substituted for capital as well as for fuel. New production units will be designed to use more labor, with a reduction in capital input achieved by reducing the size and cost of machinery, plant, and equipment. The putty-putty hypothesis, as before, implies that the same substitution will be made with already existing production units. However, the size and cost of the machinery, plant, and equipment constituting a production unit is fixed and paid for at the time the unit is installed. For already existing units, this capital quantity is fixed in history and cannot usually be reduced.[3] Thus the putty-putty hypothesis is not consistent with the concept of fixed capital which is specific to the individual firm or industry.

Because these implications of the putty-putty hypothesis are unrealistic for many industries, it is likely that models of these industries based on it will seriously overestimate the speed with which factor proportions respond to changes in relative prices. Because such models assume that the full long run adjustment is reflected in the observed one-year response, it is likely that the estimated parameters of the models imply elasticities of substitution which seriously understate the true long run parameters. Policies based on the conclusions of such models may have far stronger long run impacts than anticipated or desired.

The vintage capacity model provides an alternative model built on an assumption which is the polar opposite of the putty-putty hypothesis. This alternative structure may prove more effective in modeling industries where production is characterized by durable production units with inflexible technologies.

## III. THE VINTAGE CAPACITY MODEL

The vintage capacity model retains much of the structure and approach of the static equilibrium model. The most important difference is in the assumption made about the nature of technology and substitution among factors for production units which have already been installed. In place of the putty-putty hypothesis of the static equilibrium model, it substitutes the assumption that once production units have been installed, their technologies are fixed and no ex post substitution among factors of production is possible. Because of the ex post rigidity it implies, this assumption is often called the "putty-clay" hypothesis.

With putty-clay production units, it is no longer possible for producers to alter input factor proportions freely to keep input-output ratios at the long run minimum cost levels when prices change. Because the technology of existing units is fixed, only newly-added capacity can be changed to embody the new optimal technology. It is assumed that the set of new production units added each year -- that year's vintage -- is built with the feasible technology which minimizes costs at the factor prices and conditions prevailing in that year.

In any year, the total stock of capacity is composed of many vintages, each having a different production technology. The newest vintage, added in the current year, uses inputs in the proportions which minimize costs at current prices. The previous year's vintage embodies the technology which minimizes costs at the previous year's prices, and so on back to the oldest vintage still remaining in the stock of capacity, which has a technology based on prices which may be forty years old or more.

In the vintage capacity model, one must be careful to distinguish between "fixed factors of production" such as capital and "variable factors of production." Fixed factors in the model are capital-type inputs which are bought and paid for at the time a production unit is built and whose "employment" cannot be varied in response to changes in the rate of capacity utilization. Variable factors are inputs such as labor, fuels, and raw materials, which are used on a year-by-year basis and whose levels of employment can fluctuate depending on the quantity of output the vintage is required to produce. The following discussion focuses on the demand for variable factors of production. Fixed factors are treated in a post-script to this section.

The input-output ratios observed for the overall stock of capacity can be written as a weighted average of the fixed ratios of all of the vintages comprising it, with weights given by the shares of total output produced by each vintage. Using the notation of the model,

$$\text{(III.1)} \qquad \left(\frac{Z_i}{X}\right)_t = \sum_{s=0}^{L} \frac{X_{t,t-s}}{X_t} \left(\frac{Z_i}{X}\right)^*_{t-s} ,$$

where the left-hand variable is the ratio of the quantity of input i consumed by the entire stock of capacity in year t to total output. $X_{t,t-s}$ denotes the output of vintage t-s in year t and $X_t$ is total production. $\left(\frac{Z_i}{X}\right)_{t-s}$ is the fixed input-output ratio for input i embodied in vintage t-s. This ratio is the cost-minimizing value based on prices and conditions which prevailed in year t-s and follows the form of (II.3).

If we define the rate of capacity utilization of vintage t-s in year t as

$$\text{(III.2)} \qquad \mu_{t,t-s} \equiv \frac{X_{t,t-s}}{Q^a_{t-s}} ,$$

then we can further decompose (III.1) as

$$\text{(III.3)} \quad \left(\frac{Z_i}{X}\right)_t = \sum_{s=0}^{L} \frac{Q^a_{t-s}}{Q_t} \; \frac{\mu_{t,t-s}}{\mu_t} \left(\frac{Z_i}{X}\right)^*_{t-s} \; .$$

where $Q^a_{t-s}$ is the total capacity of new production units added in period t-s (the size of vintage t-s), $Q_t$ is total capacity in year t, and $\mu_t$ is the overall rate of capacity utilization. It is assumed in the development of the vintage capacity model that only aggregate variables can be observed, and not those applying specifically to individual vintages. This is consistent with the type of data needed to apply the static equilibrium model.

If a functional form is chosen for the cost function for which the derivatives in equation (II.3) are linear in their parameters, then (III.3) will be similarly linear. For example, consider the generalized Leontief cost function,

$$\text{(III.4)} \quad \left(\frac{C}{X}\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} P_i^{\frac{1}{2}} P_j^{\frac{1}{2}} + \sum_{i=1}^{n} \sum_{k=1}^{m} \alpha_{ik} P_i A_k \quad ,$$

with $\beta_{ij} = \beta_{ji}$ imposed for normalization. Substituting the derivatives of (III.4) into (III.3) yields

$$\text{(III.5)} \quad \left(\frac{Z_i}{X}\right)_t = \sum_{j=1}^{n} \beta_{ij} \left[ \sum_{s=0}^{L} \frac{Q^a_{t-s}}{Q_t} \frac{\mu_{t,t-s}}{\mu_t} \left(\frac{P_j}{P_i}\right)^{\frac{1}{2}}_{t-s} \right] + \sum_{k=1}^{m} \alpha_{ik} \left[ \sum_{s=0}^{L} \frac{Q^a_{t-s}}{Q_t} \frac{\mu_{t,t-s}}{\mu_t} A_{k,t-s} \right] .$$

The constraints $\beta_{ij} = \beta_{ji}$ appear as linear cross-equation restrictions on the coefficients of this system of equations. Although (III.5) is linear in its parameters, two problems prevent it from being easily estimated. The first is that we have constrained the model to use only aggregate variables. Data on the utilization rates of individual vintages (which appear in equation (III.5)) cannot usually be observed. Secondly, the summations across vintages involve prices from L years before the first year of the sample. Because L may be forty or more, a sample beginning in 1950 would require price data back to the first decade of this century. Consistent time series data are rarely available over such a long span of time.

Fortunately, a single simplifying assumption makes the solution of both problems possible. Because vintage utilization rates are not observed, we substitute a simple model allowing them to be replaced by a function of known data. The function used is of an exponential form, which permits a convenient differencing transformation to eliminate the lengthy lags on the right-hand side.

The relative rates of utilization of capacity of different vintages result from of decisions made by producers about how intensively the available vintages should be used in production. To minimize current production costs, subject to producing the required volume of output, producers should use most intensively those vintages which can produce most cheaply at currently prevailing prices. Thus the utilization rate of each vintage depends mainly on two factors: 1) the rate of utilization of the overall stock of capacity, and 2) the current cost of producing with that vintage relative to other vintages in the stock. Increased overall capacity utilization can be expected to increase the rate of utilization of each vintage. A reduction in the cost of producing with a vintage relative to others will increase its relative rate of utilization.

Usually, the vintages having lowest costs are the newest ones. Because they embody the most recent advances and discoveries in production technology, more recent vintages are usually more efficient than older ones. In addition, if relative input prices tend to move in a consistent direction over time, then more recent vintages will reflect price conditions closer to current prices than older vintages.[4] If the sample period over which the parameters of the model are being estimated is characterized by rapid technical progress and non-cyclic movements in relative factor prices, then we can consider relative utilization rates to depend chiefly on age -- newer vintages being used more heavily, older ones less heavily.

Functionally, we shall represent such behavior by

$$\text{(III.6)} \qquad \mu_{t,t-s} = \mu_t (1 - \gamma)^s \frac{1}{k_t} \quad ,$$

where $\gamma$ is an unknown but fixed parameter, and $k_t$ is a scale factor which depends on $\gamma$ and the age distribution of the stock of capacity. $k_t$ is chosen so that the aggregation identity

$$\text{(III.7)} \qquad \mu_t = \sum_{s=0}^{L} \mu_{t,t-s} \frac{Q^a_{t-s}}{Q_t}$$

holds. Substituting (III.6) into (III.7) and solving yields[5]

$$\text{(III.8)} \qquad k_t = \sum_{s=0}^{L} (1 - \gamma)^s \frac{Q^a_{t-s}}{Q_t} \quad .$$

Notice that equation (III.6) implies that utilization rates of various vintages are an exponential function of their ages. In particular, the ratio of the utilization rates of two consecutive periods are in the constant ratio

$$\text{(III.9)} \qquad \frac{\mu_{t,t-s-1}}{\mu_{t,t-s}} = 1 - \gamma \quad .$$

Substituting the utilization rate hypothesis of (III.6) into the expression for overall input-output ratios (III.3) yields

$$\text{(III.10)} \qquad \frac{k_t(\gamma) Z_{i,t}}{\mu_t} = \sum_{s=0}^{L} Q^a_{t-s}(1-\gamma)^s \left(\frac{Z_i}{X}\right)^*_{t-s} \quad .$$

with $k_t$ written as a function of $\gamma$ to emphasize its dependency on that parameter. Equation (III.10) contains only observable variables and for the generalized Leontief function it is linear in all unknown parameters except $\gamma$.

The presence of prices from the early pre-sample period can be eliminated by noting that

$$\text{(III.11)} \qquad (1-\gamma) \frac{k_{t-1}(\gamma) Z_{i,t-1}}{\mu_{t-1}} = \sum_{s=1}^{L+1} Q^a_{t-s} (1-\gamma)^s \left(\frac{Z_i}{X}\right)^*_{t-s}$$

Substituting (III.11) into (III.10) yields

$$\text{(III.12)} \qquad \frac{k_t(\gamma) Z_{i,t}}{\mu_t} = (1-\gamma) \frac{k_{t-1}(\gamma) Z_{i,t-1}}{\mu_{t-1}} + Q^a_t \left(\frac{Z_i}{X}\right)^*_t - Q^a_{t-L-1} (1-\gamma)^{L+1} \left(\frac{Z_i}{X}\right)^*_{t-L-1} \quad .$$

Equation (III.12) has a straight-forward interpretation. The left-hand variable is the overall input of factor i in year t adjusted for the rate of capacity utilization. On the right-hand side, overall input is decomposed into three additive components. The first represents input required by production units carried over from the preceding year. Demand from these units is discounted by the quantity $(1-\gamma)$ to reflect the increased age and correspondingly reduced utilization rates of the units. This term is likely to be the largest in quantitative measure. It embodies the continuity of capacity from one period to the next which is essential to the dynamic structure of the model.

The second term captures the demand for inputs of the new vintage being added to the stock in the current period. The technology of this vintage is based on current prices and conditions. Although this term may be of smaller magnitude than the first, it is of equally critical importance. It is through this term that changes over time in relative factor prices affect the overall employments of factors.

The final term removes the demand by units which are currently being retired from the stock. The retirements term is most likely small in magnitude and of relatively minor importance. Not only are the old vintages being retired likely to be small in capacity terms relative to more recent ones, but their importance is discounted heavily by their very low rates of utilization. For example, if $\gamma=.1$ and L is forty, the retiring capacity is being utilized only 1.3% as intensively as the newly-added vintage. Applying this utilization rate to a vintage which may be only ten to twenty percent as large as the new vintage results in a term of negligible magnitude in comparison with the other effects.

Furthermore, it is only in the negligible third term that early pre-sample prices enter equation (III.12). Thus it is operationally convenient to allow the effect of the third term to be subsumed in the general stochastic error term which is attached in the econometric specification. For the generalized Leontief function, the final econometric specification of the set of variable factor demand equations is

$$\frac{k_t(\gamma)Z_{i,t}}{\mu_t} - (1-\gamma)\frac{k_{t-1}(\gamma)Z_{i,t-1}}{\mu_{t-1}} = \sum_{j=1}^{n}\beta_{ij}Q_t^a\left(\frac{P_j}{P_i}\right)_t^{\frac{1}{2}} + \sum_{k=1}^{m}\alpha_{ik}A_{k,t} + \epsilon_{it} \quad \text{(III.13)}$$

As mentioned above, the demand for capital must be treated differently than the demand for variable factors of production. The capital input of a plant or production unit is inherant in the physical manifestation of the unit. If the rate of output desired from a machine declines, some of the operators associated with its functioning may be fired and the input of fuel to the machine may be reduced. However, the capital input cannot be changed since it is embodied in the physical existence of the machine. In addition, the flow of expenditures on capital input to a production unit is concentrated at the beginning of its life when it is built or purchased. Since in this simplified form of the model we are ignoring gestation lags, we shall suppose that all of the investment expenditures on each production unit are made in the year in which it is added to the stock of capacity. The real value of the investment associated with a vintage is the capital-output ratio embodied in the vintage multiplied by its full capacity size

$$\text{(III.14)} \quad I_t = Q_t^a \left(\frac{Z_K}{X}\right)_t^* ,$$

where $I_t$ is real investment expenditures and $\left(\frac{Z_K}{X}\right)_t^*$ is the minimum-cost input-output ratio for capital at period t prices. Substituting the generalized Leontief function yields the following linear investment function with stochastic disturbance term added:

$$\text{(III.15)} \quad \frac{I_t}{Q_t^a} = \sum_{j=1}^{n} \beta_{Kj} \left(\frac{P_j}{P_i}\right)_t^{\frac{1}{2}} + \sum_{k=1}^{m} d_{Kk} A_{k,t} \quad .$$

The system of equations (III.13) and (III.15) is linear in all its parameters except $\gamma$. For a given specification of the joint distribution of the stochastic $\epsilon_{i,t}$ terms, a pseudo-linear estimation procedure can be carried out. Given any value of $\gamma$, the time series $k_t(\gamma)$ can be constructed and the left-hand variables formed for each input. Linear regression tehniques can then be appled to this system, imposing the cross-equation coefficient restrictions $\beta_{ij} = \beta_{ji}$. This procedure can then be repeated for different values of $\gamma$. The final estimate of $\gamma$ is that value $\hat{\gamma}$ for which the least squares criterion function reaches its minimum (or the concentrated likelihood function reaches its maximum). The estimates of the and parameters are those from the linear system based on $\gamma = \hat{\gamma}$.

Equations (III.13) and (III.15) are strikingly similar to the estimating equations of the corresponding static equilibrium model, (II.5). The same estimation methods can be applied to both, although the estimation of the vintage capacity model is slightly more complicated due to the presence of the non-linear parameter $\gamma$.

With two exceptions, the same data is used to estimate both models. The static equilibrium model requires a measure of the size of the capital stock of the industry. The construction of such data from observed time series on real capital investment can be a difficult task, since one must estimate an initial level of the stock and a rate of depreciation for it. The vintage capacity model does not require capital stock data. As was seen above, the demand for capital is formulated in terms of real investment expenditures -- a directly observable data series. The stock variable of the vintage capacity model is not measured as capital but rather in terms of capacity. For many important industries in the economy, detailed data on capacity, including stocks, additions, and retirements, are readily available in published sources. Thus the data requirements of the vintage capacity model may actually be less stringent than those imposed by the static equilibrium model.

## IV. CONCLUSION

The vintage capacity model provides to production modelers an alternative system to the static equilibrium model, based on a different hypothesis about the structure of production technology. The models share a common set of data requirements and can be estimated using similar econometric techniques. Because of these similarities, two potential applications of the model may be suggested. Firstly, for many industries the assumptions of the vintage capacity model more closely approximate reality than those of the static equilibrium model. In the form discussed in Section III, the vintage capacity model could be most fruitfully applied to industries characterized by large, fixed, durable, and inflexible production units. It is less appropriate for industries where production is carried out using small, decentralized, flexible facilities which are often remodeled or replaced.

Secondly, the model could be useful to provide alternative estimates of production parameters in cases where the relative merits of the assumptions of the two models are not clear. For most industries, the technologies of existing production units are neither as freely flexible as the static equilibrium model assumes, nor totally fixed as the vintage capacity model assumes. The two models may be used together to provide alternative estimates under polar boundary conditions. In Parker (1980), the vintage capacity model is applied to the U. S. electric utility industry.

Numerous extensions can be made to the basic structure of the vintage capacity model. In many industries, the construction of production units requires more than one year's time. Lengthy and variable gestation lags can be easily attached to the model and may be important if the technologies of production units are fixed at the time their installation process begins. The model can easily be generalized to include the possibilities of increasing returns to scale and multiple outputs.

Two possible extensions of the vintage capacity model bear further careful study. One is the generalization of the putty-putty and putty-clay hypotheses to a model which allows some ex post factor substitution without requiring equally free flexibility both ex ante and ex post. Fuss and McFadden (1978) have developed such a model, but its estimation requires detailed data on the inputs and outputs of individual plants.

The second extension is to develop a less restrictive and more rigorous model of vintage capacity utilization. Although it is likely to complicate the non-linearity of the model, such a system would permit the analysis of important questions relating to economic obsolescence. If the model can be extended in these two ways without requiring additional data or greatly complicating the econometric estimation, it will provide an even more useful tool for the econometric analysis of production.

FOOTNOTES

[1] Such variables might include indices of the state of technical progress, weather and other natural conditions, and variables such as pollution controls relating to the legal and regulatory environment of the industry.

[2] Although most studies estimate only the linear demand functions (II,3), the cost function (II.2) is also a linear function in its parameters and contains only variables which are observable. For maximum efficiency in estimation, the cost function should be included in the system of equations.

[3] One could imagine reducing the capital-labor ratios of existing production units by removing and selling labor-saving components of the machine or plant, if that is physically possible. However, while a firm may do this in isolation, it is doubtful that the industry or economy as a whole could reduce its capital-labor ratio in this way. If the reduction in wages relative to capital costs is general throughout the industry or economy (as it would be if aggregate factor prices changed), then no other firm or industry will want to purchase these labor-saving components, since they too will be attempting to reduce their capital-labor ratios.

[4] However, if relative input prices move in a cyclical manner, this effect will be in the opposite direction. For example, if labor is cheap relative to fuel in 1950, expensive in 1960, and cheap again in 1970, then in 1970 labor-intensive 1950 plants might be cheaper to use than fuel-intensive plants of the 1960 vintage.

[5] Notice that the computation of k requires knowledge of the size of vintages back L years into the pre-sample period. If this data is unavailable, then some approximation or extrapolation technique must be used, such as equally-sized vintages or a constant rate of growth of vintage size.

REFERENCES


Berndt, E.R., and M.S. Khaled. December 1979. Parametric productivity measurement and choice among flexible functional forms. Journal of Political Economy 87:1220-45.

Christensen, L.R., D.W. Jorgenson, and L.J. Lau. February 1973. Transcendental logarithmic production frontiers. Review of Economics and Statistics 55:28-45.

Diewert, W.E. May/June 1971. An application of the Shephard duality theorem: A generalized leontief production function. Journal of Political Economy 79:481-507.

Diewert, W.E. February 1974. Functional forms for revenue and factor requirements functions. International Economic Review 15:119-30.

Fuss, M., and D. McFadden. 1978. Flexibility versus efficiency in ex ante plant design. An Economic Approach to Production Theory, edited by M. Fuss and D. McFadden. Amsterdam: North-Holland.

Parker, J.A. 1980. A Vintage Capacity Model of the Electric Utility Industry. Unpublished Ph.D. dissertation, Stanford University, Stanford, Calif.

Shephard, R. 1953. Cost and Production Functions. Princeton: Princeton University Press.