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OUTPUT

Robert M. Coen
Bert G. Hickman

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

The research conducted in many Programs and Areas at IIASA -- such as the Energy Program, the Food and Agriculture Program, and the Human Settlements and Services Area -- is often concerned with projections of economic growth. Since 1978, the Task Economic Modeling within System and Decision Sciences Area has contributed to these and related issues through investigations of different methodologies of projecting economic growth. This paper addresses the important question of defining natural growth paths of potential output of an economy, obtained under the assumptions of nearly full employment, with demographic projections of changes in the labour force and projections of average labour productivity being the main driving forces of growth. Many of the results contained in this paper are based on investigations conducted by the authors over a number of years before coming to IIASA; however many of the computational experiments and final conclusions reported here were prepared during their stay with the System and Decision Sciences Area in 1979-80.

THE NATURAL GROWTH PATH OF POTENTIAL OUTPUT

Robert M. Coen and Bert G. Hickman¹

Potential GNP is widely used in empirical macroeconomics as a yardstick for evaluating actual economic performance. In most recent applications, potential GNP in a given year is typically estimated from an aggregate production function, assuming full employment of capital and labor inputs available in that year.

While this approach may be well-suited to analysis of short-term economic fluctuations and stabilization policies, it is not well-suited to assessments of medium- or long-run growth prospects and policies.

Since deviations of actual from potential GNP are likely to affect rates of capital formation, and possibly labor supply growth also, potential GNP as conventionally measured is influenced by past fluctuations in economic activity. For example, the capital stock existing in 1979, which would be used to estimate potential GNP in 1979, is the result of investment expenditures along the actual GNP path; had capital and labor been fully employed each year up to 1979, the 1979 capital stock would almost certainly have differed. Correspondingly, projecting potential GNP into the future requires forecasts of capital formation and labor supply growth, but the usual production-function framework for measuring potential does not provide much guidance for making these forecasts.

In contrast to the empirical literature on potential GNP, the neoclassical theory of macroeconomic growth has been concerned with *time paths* along which both capital and labor are fully employed *each period*. From this theory has emerged the concept of a natural growth rate determined by the growth rates of labor supply and Harrod-neutral technical progress. The neoclassical theory qua theory contains many simplifying assumptions -- constant rates of saving, technical progress, and labor supply growth -- and lays great emphasis on rapid price adjustments and automatic re-investment of savings to achieve equilibrium. Little effort has been made to adapt the general framework of the theory to empirical analyses of growth.

In this paper we present an empirical model of the U.S. economy that joins the production function approach to potential output with the full-employment path concept of neoclassical growth theory. The model is used to calculate what we call the natural growth path of potential output, i.e., potential GNP along a full-employment growth path. As in neoclassical theory, price adjustments play an important role in the model and are endogenously determined along the path; but whereas a fixed saving rate is a key determinant of factor prices in the theory, our supply-side model, which includes no saving function, makes use of two assumptions which we believe to be in accord with U.S. experience -- that the equilibrium real rate of return on capital is constant and that the equilibrium real wage grows at the same rate as average labor productivity. With these two assumptions, we are able to determine jointly the natural growth path of potential GNP and the principal variables associated with it -- the real wage, productivity, labor supply, the natural rate of unemployment, and capital requirements needed to sustain the path.

The supply-side model described and applied in this paper is part of the complete Hickman-Coen (HC) annual macroeconomic growth model of the U.S., which includes equations determining aggregate demand as well as supply. The complete model is expressly designed to study the time path of actual GNP relative to potential GNP, although our concern here is only with the path of potential.

In a previous paper (1980b), we presented provisional estimates of the potential path based on earlier versions of the supply-side equations. Since then, we have completed further testing of alternative specifications of our factor input and production system and our unemployment equation, and we make use of these new results in the present estimates of potential. We also take this opportunity to describe fully the mathematical structure of the supply-side model.

LABOR SUPPLY AT THE NATURAL UNEMPLOYMENT RATE

Population growth is exogenous in the HC model, but average hours and the participation and unemployment rates are not. The model contains labor-force participation equations for 16 age-sex groups and an aggregative equation for hours of work, plus an equation to determine the actual unemployment rate. This set of equations can be used to jointly determine the natural rate of unemployment and the corresponding "full-employment" labor supply as follows.²

Average hours at full employment H^f are a function of the real wage in terms of consumer goods, (W/P) , the natural unemployment rate, U^n , and the proportion of women aged 20 and over in the full-employment labor force, LW^f :

$$(1) \quad H_t^f = \exp[a_0 + a_1 \ln(W/P)_t + a_2 \ln U_t^n + a_3 \ln LW_t^f] \quad .$$

The full-employment labor force in the i -th age-sex group is given by

$$(2) \quad L_{it}^f = N_{it} [b_{0,i} + b_{1,i} (E^f/N)_t + b_{2,i} (LA/N)_t + b_{3,i} (W/P)_t + b_{4,i} H_t^f + b_{5,i} t + b_{6,i} NM_t] \quad ,$$

where the subscript i ranges over the 16 age-sex groups (8 male and 8 female), N_i is the population in the i -th group, E^f is aggregate full-employment employment, N is the noninstitutional population 16 years or older, LA is the number of persons in the armed forces, t is a time index, and NM is the ratio of males aged 16-34 to those 35-64.³ The total labor force and the proportion of women over 20 in the labor force at full employment are given by:

$$(3) \quad L_t^f \equiv \sum_{i=1}^{16} L_{it}^f$$

$$(4) \quad LW_t^f \equiv \sum_{i=11}^{16} L_{it}^f / L_t^f \quad ,$$

and full-employment employment and man-hours are defined by

$$(5) \quad E_t^f \equiv (1 - U_t^n) L_t^f$$

$$(6) \quad M_t^f \equiv (H_t^f) (E_t^f) \quad .$$

The explanation of actual unemployment in the HC model discriminates between the transitory component stressed by search theory and the more enduring structural elements of the natural rate hypothesis. When purged of the transitory component⁴, the unemployment equation relates the natural unemployment rate to three factors:

$$(7) \quad U_t^n = \exp \left[c_0 + c_1 \ln(LA/N)_t + c_2 \ln(ABU)_t + \ln \sum_{i=1}^{16} U_i^{56} LP_{it} \right] \quad .$$

An important determinant of variations in the natural unemployment rate, or the rate which at a given time would be consistent with general equilibrium in the labor and commodity markets (Friedman, [1968]), is shifts in the age-sex composition of the labor force. As an index of the effects of these shifts on the natural rate, we form a weighted average of the unemployment rates by age and sex in a high-employment base year, 1956, the weights LP_{it} being the estimated proportions of the high-employment labor force in each age-sex group.⁵ The rise in this variable during 1953-1977 reflects the growing mismatch between job requirements and job skills as the composition of the labor force increasingly shifted towards teenagers and women. The other determinants of the natural rate include variations in the relative size of the armed forces and in the ratio of average weekly unemployment benefits to average weekly earnings after taxes, ABU.

Equations (1)-(7) can be solved simultaneously for the natural rate of unemployment U^n and the corresponding full-employment supply of man-hours M^f , conditional on given values of the real wage, the population and its age-sex distribution, the armed forces, and the relative level of unemployment benefits.⁶

THE FACTOR DEMAND AND PRODUCTION FUNCTIONS

We turn now to the demand side of the labor market. Demands for labor and capital are interrelated in the HC model, since they are jointly derived from a long-run Cobb-Douglas production function with constant returns to scale:

$$(8) \quad X_t^* = A e^{\gamma t} (K_t^*)^\alpha (M_t^*)^{1-\alpha} \quad , \quad A, \alpha, \gamma > 0 \quad .$$

where X^* is expected output, K^* is desired business fixed capital stock, M^* is desired man-hours, and γ is the rate of technical progress.

Minimizing the production cost function

$$(9) \quad C(X_t^*) = Q_t^* K_t^* + W_t^* M_t^*$$

subject to (1) gives the long-run factor demand functions:

$$(10) \quad M_t^* = [\alpha/(1-\alpha)]^{-\alpha} A^{-1} [(Q^*/W^*)_t]^\alpha X_t^* e^{-\gamma t}$$

$$(11) \quad K_t^* = [\alpha/(1-\alpha)]^{1-\alpha} A^{-1} [(Q^*/W^*)_t]^{-(1-\alpha)} X_t^* e^{-\gamma t} \quad ,$$

where Q^* is the expected implicit rental price of capital and W^* is the expected nominal wage rate.

Adjustment costs inhibit firms from adjusting immediately to variations in the desired inputs of capital and labor. These adjustment costs include external purchase costs and internal installation costs for capital goods and hiring, training and layoff costs for labor. They are represented implicitly by exponential partial adjustment hypotheses:

$$(12) \quad M_t/M_{t-1} = (M_t^*/M_{t-1}^*)^{\lambda_1} \quad , \quad 0 < \lambda_1 \leq 1$$

$$(13) \quad K_t/K_{t-1} = (K_t^*/K_{t-1}^*)^{\lambda_2} \quad , \quad 0 < \lambda_2 \leq 1$$

where λ_1 and λ_2 are the adjustment speeds for labor and capital.

Combining the desired input and adjustment hypotheses yields the short-term or disequilibrium factor demand functions:

$$(14) \quad M_t = \left\{ [\alpha/(1-\alpha)]^{-\alpha} A^{-1} [(Q^*/W^*)_t]^\alpha (X^*)_t e^{-\gamma t} \right\}^{\lambda_1} (M_{t-1})^{1-\lambda_1}$$

$$(15) \quad K_t = \left\{ [\alpha/(1-\alpha)]^{1-\alpha} A^{-1} [(Q^*/W^*)_t]^{-(1-\alpha)} (X^*)_t e^{-\gamma t} \right\}^{\lambda_2} (K_{t-1})^{1-\lambda_2} .$$

Estimation of the short-run demand functions (14) and (15) yields estimated values of the adjustment speeds λ_1 and λ_2 and of the structural parameters of the production function (8) and the long-run factor demand functions (10) and (11). Details of the estimation procedure, including determination of the expected values of output, the wage rate and the rental price of capital, are discussed in the empirical section. It suffices to note here that output is measured as gross private nonresidential product and that the business fixed capital stock is measured net of depreciation at a constant exponential rate.

THE NATURAL GROWTH PATH

Along the natural growth path of potential output both labor and capital are fully employed. Making use first of the labor market condition, potential output may be defined as the level of output that would equate labor supply and demand at the natural rate of unemployment. Since the labor demand function (14) relates man-hours to output and relative factor prices, we may derive an expression for potential output, conditional on the rental-wage ratio, by substituting M_t^f for M_t on the left-hand side and solving for output:

$$(16) \quad X_t^p = A [\alpha/(1-\alpha)]^\alpha [(Q^*/W^*)_t]^{-\alpha} e^{\gamma t} (M_t^f)^{1/\lambda_1} (M_{t-1}^f)^{-(1-\lambda_1)/\lambda_1} .$$

Since full employment would prevail each period along the natural growth path, note that X_t^p depends on the full-employment man-hour supplies in the current and preceding years, irrespective of whether the economy actually operated at full employment in the preceding year.

The expected relative factor price ratio, Q^*/W^* , enters (16) directly, and the real wage, W/P , enters indirectly as a determinant of M^f and also of Q^*/W^* , as will be seen below. To implement (16) historically, actual data on wages and prices could be used, along with equations of the HC model relating Q^*/W^* to observed current and lagged prices, wages and tax parameters; but observed wages and prices are presumably influenced by economic fluctuations around the natural growth path. If the natural path is to be invariant to such fluctuations, it is inappropriate to use actual wages and prices in its construction. Moreover, it is unsatisfactory to treat the real wage and factor price ratio as exogenous variables in ex-ante projections of the natural path.

We choose instead to endogenize the real wage by imposing the condition that it grow at the same rate as potential labor productivity along the natural growth path:

$$(17) \quad (W/P)_t / (W/P)_{t-1} = (X^P/M^f)_t / (X^P/M^f)_{t-1} .$$

Since the real wage, or rather its reciprocal, is a component of the rental-wage ratio, the latter also becomes endogenous under condition (17). Thus the expected rental-wage ratio is defined as $Q^*/W^* = (P/W)^*(\delta+r)T$, where δ is the depreciation rate, r is the discount rate used for investment decisions, and T is a compound term depending on the parameters of corporate taxation, including the investment tax credit and the tax treatment of depreciation allowances. The real wage in the labor supply equations is in terms of consumer prices, whereas it is the price of capital goods which enters the first term of Q^*/W^* , but we assume (realistically) that the differential trend of capital and consumer goods prices is negligible for present purposes and can be ignored. We assume also that real wage expectations would be realized along the natural growth path, so that

$$(18) \quad (Q^*/W^*)_t = (P/W)_t(\delta+r)T ,$$

since δ and r are fixed constants in our model and only changes in T which are unanticipated and permanent are assumed to affect

the equilibrium capital-labor ratio. For the same reasons, Q^*/W^* falls at the same rate as labor productivity and the real wage rise along the natural path, unless tax policies affecting T are changed.

Rewriting equation (16) in labor productivity form

$$(19) \quad (x^p/M^f)_t = A[(1-\alpha)/\alpha]^{-\alpha} [(Q^*/W^*)_t]^{-\alpha} e^{\gamma t} (M_t^f/M_{t-1}^f)^{(1-\lambda_1)\lambda_1},$$

and using (1) - (7), (17) - (19), and the identity

$$(20) \quad x_t^p \equiv (x_t^p/M_t^f) (M_t^f),$$

we can solve simultaneously for the natural growth paths of labor productivity, output, labor force, employment, unemployment, hours of work, man-hours and the real wage and rental-wage ratios for exogenous values of the demographic and policy variables (population, armed forces, relative unemployment benefits and corporate tax parameters).

It should be stressed that the assumption that the real wage increases secularly at the same rate as man-hour productivity is not only observationally realistic but also consistent with the overall structure of the complete HC model. Nominal prices and wages are endogenous in the complete model, but prices are related to unit labor cost and a constant long-run mark-up, implying constant factor shares in long-run equilibrium and equality in the growth rates of the real wage and labor productivity.

In the standard Solow-Swan neoclassical model the natural rate of growth is determined by the demographically fixed growth rate of the labor force and the rate of Harrod-neutral technical progress. The saving rate determines the supply of capital and the rental-wage ratio adjusts to clear the labor and capital markets and assure full employment of both factors. Our model elaborates the standard growth model on the labor input side by endogenizing the natural rates of unemployment, labor force participation and average hours of work, but its principal departure is in the treatment of capital formation.

Instead of assuming that a constant fraction of output is saved and invested automatically, the HC model contains an explicit investment demand function, equation (15), which can be used to determine the path of full-employment business fixed capital stock, K_t^f , given the natural growth paths of the rental-wage rate and potential output. With equation (15) and the identity relating gross investment and capital stock,

$$(21) \quad I_t^f \equiv K_t^f - (1-\delta)K_{t-1}^f \quad ,$$

added to the earlier equation system, the net and gross business fixed investment expenditures required to sustain the natural growth path are fully determinate. A greater flow of savings could not be absorbed profitably in business fixed capital formation under the given investment conditions, and a smaller flow would be inadequate to attain the required rate of capital deepening to equilibrate the capital-labor and rental-wage ratios.

To expand on the last statement, the optimality condition underlying the HC factor demand system is the cost-minimization requirement that the marginal rate of substitution equal the factor price ratio. Thus in the absence of adjustment lags, the desired capital-labor ratio under the Cobb-Douglas technology is directly proportional to the expected wage-rental ratio:

$$(22) \quad \frac{K^*}{M^*} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{W}{Q} \right)^* = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{W}{P} \right)^* \left(\frac{1}{(\delta+r)T} \right) \quad .$$

As the real wage rises over time with productivity growth, labor becomes increasingly expensive relative to capital, encouraging continuous capital deepening. We have found no evidence in our empirical work on the U.S. economy that the desired real rate of return used in business fixed investment decisions fluctuates with nominal or real interest rates or equity yields (Coen and Hickman, 1980c), and it is held constant at 7 percent after taxes in the model. The real rate of return to capital has remained constant because firms have not absorbed more saving for purposes of business fixed capital formation than was consistent with

maintenance of the required rate of return, and the surplus saving has gone into residential construction, inventory accumulation, or net foreign investment.

In the ex-ante projections presented below, it is assumed that this same condition will continue to prevail during the next 20 years and the necessary investment requirements are determined accordingly. As long as the required business fixed investment share is in line with historical experience, as it is in the projections, it is unlikely that insufficient saving will be available in the next two decades to sustain the natural growth path. We leave to a later date the use of the complete HC model for endogenous projections of total saving and its distribution between domestic and foreign investment and on the domestic side between housing and business fixed capital formation. Meanwhile, our sub-model of production and factor demands and supplies has the considerable virtue and convenience of permitting internally consistent ex-ante projections of the natural growth paths of the principal supply side variables without the weighty additional complications of modelling saving behavior and the distribution of investment resources between domestic and foreign investment over the coming decades.

We differ from the standard growth model also in the inclusion of adjustment lags, which inhibit the complete attainment of the optimal factor combinations along the natural path, as discussed in the following section.

Finally, we note that although we have referred to our solution as a natural growth path, it is not the steady-state concept of conventional growth theory. That concept requires that the real wage increase at the rate of Harrod-neutral technical progress instead of the rate of man-hour productivity. The steady-state concept is unnecessarily restrictive and less meaningful as a benchmark for potential output than the one we have adopted.

THE PRODUCTION FUNCTION AND POTENTIAL OUTPUT

The previous sections have shown how to determine potential output from our structural model of labor supply and demand. In this section we demonstrate how the same path of potential output may be determined and interpreted using the production function of the model.

In the derivation of the desired input functions (10) and (11), the production function (8) is viewed as a planning relation between equilibrium input and output levels. The corresponding disequilibrium production function for actual current output is

$$(23) \quad X_t = Ae^{\gamma t} (k_t K_{t-1})^\alpha (m_t M_t)^{1-\alpha} ,$$

where k_t and m_t are indexes of the intensity of utilization respectively of the measured inputs K_{t-1} and M_t . The intensity of use of capital stock can be increased, for example, by operating equipment at a faster rate, increasing the number of machine-hours per day or week, or diminishing downtime by postponing maintenance. Similarly, a man-hour may represent a greater amount of effective labor input as workers are induced to work at a faster pace and with fewer or shorter breaks, although the scope for variations in intensity of labor input is smaller than for capital.

Variations in the intensity of factor utilization occur in the process of adjusting man-hours and capital stock to changes in the desired or equilibrium quantities. Desired inputs may change because of variations in expected output or the rental-wage ratio. Adjustment is not instantaneous, however, because of adjustment costs. Meanwhile, the existing inputs of K and M must be used at intensities that differ from the equilibrium rates of factor utilization. Since the principal source of variation in utilization rates is the discrepancies between the actual and desired inputs, we assume that the intensity of use of each factor is proportional to the degree of disequilibrium in each:⁷

$$(24) \quad k_t = K_t^*/K_{t-1} .$$

$$(25) \quad m_t = M_t^*/M_t \quad .$$

Moreover, since the observed changes in measured inputs are proportional to the desired changes by hypotheses (12) and (13), the intensity indexes can be defined and measured in terms of observable variables:

$$(26) \quad k_t = (K_t/K_{t-1})^{1/\lambda_2}$$

$$(27) \quad m_t = (M_t/M_{t-1})^{(1-\lambda_1)/\lambda_1} \quad .$$

Potential output can now be determined from the production function (23) by substituting full-employment man-hours and capital stock and measuring the effective capital and labor inputs at their natural intensities k^n and m^n :

$$(28) \quad x_t^p = Ae^{\gamma t} (k_t^n K_{t-1}^f)^\alpha (m_t^n M_t^f)^{1-\alpha} \quad .$$

It might be thought that the natural utilization intensities would be unity, but this is the case only under stationary conditions. Since adjustment costs lead firms to adjust measured inputs to the desired levels with some lag, we would expect to observe firms using factor inputs which are below desired levels even if the economy is experiencing steady growth. Thus the natural intensities associated with *growth* equilibrium will exceed unity by amounts which depend on the natural growth rates of output and the wage-rental ratio and on the adjustment speeds λ_1 and λ_2 , which govern the rapidity with which firms adjust their inputs. The natural utilization intensities are determined endogenously in the natural path solution as:

$$(29) \quad k_t^n = (K_t^f/K_{t-1}^f)^{1/\lambda_2}$$

and

$$(30) \quad m_t^n = (M_t^f/M_{t-1}^f)^{(1-\lambda_1)/\lambda_1} \quad .$$

Cyclical fluctuations in output relative to the natural growth path will be accompanied, of course, by corresponding fluctuations of the actual utilization intensities relative to their natural levels.

The equivalence of equations (16) and (28) for the determination of potential output is easily demonstrated. The expected factor price ratio enters (16) because it determines the desired capital-labor ratio. Substituting the expansion path equation (22) into (16) we obtain:

$$(31) \quad X_t^P = Ae^{\gamma t} (K^*/M^*)_t^\alpha (M_t^f)^{1/\lambda_1} (M_{t-1}^f)^{-(1-\lambda_1)/\lambda_1} .$$

According to our factor intensity hypothesis, along the potential path the desired factor inputs are expressible as:

$$(32) \quad K_t^* = k_t^n K_{t-1}^f$$

and

$$(33) \quad M_t^* = m_t^n M_t^f .$$

Substituting (32) and (33) into (31) and making use of (30), we arrive at expression (28).

Although numerically equivalent to (16), the production function expression (28) for potential output is more than a redundant curiosity. It serves as a reminder that capital as well as labor requirements must be satisfied along the natural growth path, as already stressed, even though capital stock does not appear explicitly in equation (16). More important, equation (28) embodies a useful interpretation of potential output growth in terms of the growth rates of capital and labor, and their intensities and the rate of technical progress, as will be seen below.

ENERGY AND POTENTIAL OUTPUT⁸

Should energy be included as a separate input in the aggregate production function as a means of analyzing the effects of higher energy costs on potential GNP growth? If this is done, the output variable must be changed to include the real value of intermediate energy inputs in addition to the real value added by capital and labor which comprises real GNP. Analysis of substitution among energy, capital and labor then must distinguish between substitution in the production of gross output, and the resulting effects on real value added or GNP. The effect of an increase in the relative price of energy must be obtained in two steps, first deriving the effect on the production of gross output, and second, correcting the change in gross output by the change in intermediate energy input to derive the change in potential GNP.

The foregoing is perhaps a natural approach in an input-output framework, in which the prices and quantities of gross outputs are carried along with those of intermediate inputs, nominal value added is the difference between the nominal values of gross output and intermediate inputs, and real value added is obtained by deflating the values of gross output and intermediate inputs by their respective prices and differencing the resulting real quantities.

Another approach to the measurement of real value added, which does not rest on the double-deflation method and is conceptually superior, has been suggested by Arrow (1974). As he argues, the most natural meaning of real value added arises from the estimation of production functions and the attribution of a special role to capital and labor as primary factors. Real value added should be measurable in terms of capital and labor, and the construction of an aggregate for them can be justified only if they are separable in production from energy and other intermediate inputs. Thus the production function for output is assumed to have the nested form:

$$(34) \quad Y = Y[X(K,M), E]$$

where Y is gross output, K , M and E are respectively the inputs of capital, labor and energy, and X is real value added, a function of K and M alone.

An immediate consequence of (34) is that real value added is invariant to changes in E as long as K and M remain fixed. Thus changes in the price and quantity of energy will affect production of real value added only when they induce changes in K or M . Indeed, E does not even appear in the production function for real value added, which is

$$(35) \quad X = X(K, M) \quad ,$$

in general form and equation (28) in the specialized form of the HC model. But this means that *potential* GNP (aggregate real value added) will not be directly affected by changes in energy supply conditions in the short run, before capital formation and the full-employment labor supply have responded to the change in energy costs.⁹ *Actual* GNP may change in the short run, however, as changes in energy prices impact on real income and effective demand, thereby altering current production, and hence the current inputs of capital and labor services, relative to their rates under full-employment conditions of labor supply and normal intensities of use of capital and labor.

The growth paths of labor and capital inputs, and hence of potential output, will be affected over time as the economy adjusts to a higher real price of energy. To the extent that a permanent increase of the real OPEC price requires a once-for-all reduction in the level of the real wage, it induces a small once-for-all change in the full-employment labor force in our model with its endogenous labor force participation equations. Similarly, the reduction in the real wage induces a once-for-all decrease in the desired capital-labor ratio with resultant depressant effects on capital stock and potential output growth during the adjustment period as we will see below.¹⁰

PARAMETER ESTIMATES

In this section we present estimates of the factor demand and production function parameters underlying the subsequent calculations of the natural growth path. Joint estimates of the parameters of equations (14) and (15), obtained by the maximum likelihood method and corrected for first-order autocorrelation in the labor and investment disturbances are shown in Table 1. The factor demand functions were fitted for output (X) as measured by gross private nonresidential product (GNP less housing rent and income originating in government). Measured labor input (M) is given by private manhours in systems (2) and (4) and by private manhours adjusted for labor quality in systems (1) and (3). The labor quality index is similar to that of Perry (1977), who weights each year's employment in each age-sex group by the product of average hours and earnings for the group in a base year, except that we need only adjust for the differences in average hourly earnings among the groups, since our basic manhour measure of labor input already accounts for differences in hours worked. The stock of business fixed capital (K) is measured net of depreciation at a constant exponential rate. The wage rate (W) is defined as average hourly earnings before tax in the private sector. The implicit rental price of capital (Q) is the product of the three terms indicated in equation (18) with the price of capital goods measured by the implicit deflator for nonresidential fixed investment and P and W set at their expected levels.

The estimates in Table 1 are the outcome of extensive testing of alternative measures of price and output expectations and the rental price of capital and of alternative breaks in the trend of technical progress since World War II, as documented in Coen and Hickman (1980c). The discount rate in the rental price of capital is a constant after-tax rate of return of 7 percent per annum, since this measure out-performed 12 alternative formulations of Q involving variable nominal and real interest rates and equity yields. The variables entering the factor demand functions are not actual but expected output and factor prices. After considerable testing of autoregressions of varying lags and on both levels and changes, a second-order autoregression was selected to determine

TABLE 1. JOINT ESTIMATES OF LABOR AND INVESTMENT DEMAND FUNCTIONS, 1950-1978

SYSTEM	Estimated Parameters and t-Statistics								SEEM SEFK
	α	λ_1	λ_2	γ	γ_1	γ_2	ρ_1	ρ_2	
(1)	.3106 (5.06)	.7063 (12.93)	.1661 (5.40)	.0166 (6.79)	-.0069 (3.37)		.3034 (2.37)	.4784 (2.38)	.0078 .0065
(2)	.2554 (3.91)	.7329 (13.90)	.1638 (5.40)	.0185 (7.06)	-.0089 (3.93)		.3808 (3.41)	.4785 (2.43)	.0076 .0066
(3)	.2761 (4.33)	.6963 (13.14)	.1615 (5.20)	.0176 (7.16)	-.0044 (1.68)	-.0069 (1.49)	.2964 (2.28)	.4788 (2.33)	.0075 .0066
(4)	.2234 (3.30)	.7254 (14.20)	.1608 (5.28)	.0195 (7.34)	-.0066 (2.36)	-.0063 (1.35)	.3776 (3.33)	.4841 (2.42)	.0073 .0066

Note: ρ_1 and ρ_2 are the auto-correlation coefficients and SEEM and SEFK the standard errors of estimate respectively for the labor and investment functions. The estimated trend rate of technical progress beginning in 1950 is given by γ , whereas γ_1 and γ_2 are estimated additive reductions in the trend rate of progress beginning respectively in 1969 and 1974.

Source: R.M. Coen and B.G. Hickman (1980c), Tables 5 and 6.

the expected investment price deflator entering expected Q , whereas the expected nominal wage changes at a constant rate adjusted for lagged changes in the consumer price deflator. A first-order autoregression for output fits well over the sample period and was used to determine expected output in our initial runs, but the output predictions differed so little from the actuals that we assumed equality of actual and expected output in the final estimates.

Systems (1) and (2) differ only in the fact that manhours are adjusted for quality changes as proxied by wage differentials in (1) but not in (2). The fit of the labor function deteriorates slightly when manhours are adjusted for quality differences, but that of the investment function improves a little. The estimate of α is larger and more significant and the autocorrelation in the labor disturbance is lower for the version with quality adjustment, and we regard these as favorable features of system (1) and adopt it as our preferred specification.

The estimated rate of technical progress decelerates beginning in 1969 in systems (1) and (2), dropping from 1.66 percent to 0.97 in the first case and from 1.85 to 0.96 in the second. Previous tests showed that an assumed deceleration beginning in 1967 instead of 1969 resulted in similar patterns but poorer fits. In systems (3) and (4) we test for an additional trend deceleration beginning in 1974, again with and without adjustment for changes in labor quality. The fits improve slightly but the coefficients for the 1974 trend break have low t-ratios. The estimated annual rate of technical progress for system (3) drops from 1.76 percent in 1950-68 to 1.32 in 1968-73 and 0.63 in 1973-78, and the corresponding figures for system (4) are 1.95, 1.29 and 0.66. Apart from the weak statistical support for the hypothetical trend break after 1973, such a radical deceleration of technical progress seems implausible to us, and for these reasons we prefer the estimated systems (1) and (2) over (3) and (4). Thus our preferred specification remains system (1), but we report below on the principal implications of the more pessimistic alternatives given by (2) and (4). Finally, we note that the hypothesis of a once-for-all downshift in the level of technology

during 1974-75 was decisively rejected in all tests or alternative patterns of technical progress reported in Coen and Hickman (1980c).

PROVISIONAL ESTIMATES OF THE NATURAL GROWTH PATH, 1955-2000

At the time of writing we have not settled on a best estimate of the historical growth path or on a plausible range for its future evolution. In the following discussion we therefore explore some of the uncertainties and examine the sensitivity of the results to key assumptions, without attempting a final resolution of the issues.

Let us begin by examining the principal characteristics of the growth path computed from the preferred set of factor demand parameters, system (1) of Table 1. The parameters of the model of full-employment labor supply (equations 1-7) are taken from Coen and Hickman (1980a). The simultaneous model consisting of equations 1-7, 15, and 17-21 is solved dynamically for 1953-2000, using as initial conditions the actual 1952 values of manhours and capital stock. The transitory effects of using actual instead of full-employment values as initial conditions are quickly absorbed and the natural growth path is firmly established by 1955. Actual values of the exogenous variables are used through 1977 or 1978 and later observations are extrapolations or projections. The population projections are from the Bureau of the Census (1977). The exogenous components of the wage-rental ratio and the armed forces and unemployment insurance variables are held constant after 1977.

A further adjustment is necessary to account for the exogenous upshift in the rental-wage ratio in 1973-75 resulting from the quadrupling of OPEC oil prices. This external shock raised the ratio of the investment price deflator to the nominal wage rate by 6.5 percent between 1973 and 1975, after which time the endogenous downtrend resulting from productivity growth was resumed, and we have introduced a corresponding once-for-all upshift in the path of the rental-wage ratio (equation 18) in those years.

As noted above, private non-residential product (XNR) is the output concept in our production system. Estimated potential

output was converted to a GNP basis by adding the actual values of real income originating in government and real residential rent during the sample period and by holding the ratio of GNP to XNR constant at the 1978 level in the ex-ante projections.

Some results of this first simulation are reported in Table 2. The estimated natural rate of unemployment, U17, averaged 4.7 percent in 1955-1968, 5.6 in 1969-1973 and 7.0 in 1974-1979. The dominant factor explaining the increase is the downtrend of the proportion of the working age population in the armed forces, LA/N, although shifts in the composition of the labor force also raised U17 moderately and steadily throughout the period. The level of U17 is projected to decline gradually during the next two decades, reflecting the coming-shifts of the population structure toward a generally older workforce with a higher proportion of experienced workers. This is an arbitrary path for U17, of course, since both LA/N and ABU, the relative unemployment benefits variable, are held constant at their 1977 levels in the projections for 1978-2000. If the relative size of the armed forces were assumed instead to increase, for example, U17 would decrease more than projected in Table 2.

The natural growth rate of output decreased from 3.6 percent in 1955-68 to 2.6 in 1968-73 and 2.8 in 1973-79. The rate of increase of manhour productivity diminished even more sharply in 1968-73 and 1973-79, but the impact on output growth was mitigated by accelerating manhour growth.

Our projection indicates that potential productivity growth will accelerate markedly in 1979-85 and will increase the natural growth rate despite a deceleration in manhour growth. The natural growth rate is projected to resume its decline after 1985, largely because manhour growth will slow.

Simple growth models assume a constant labor force participation rate and constant hours of work, so that the growth rates of output per manhour and of output per capita are the same. Neither participation rates nor hours are constant in the U.S. economy, however, and it is apparent from Table 2 that manhours increased less rapidly than population before 1968 and more

TABLE 2. NATURAL UNEMPLOYMENT AND GROWTH RATES AND INVESTMENT RATIOS, SELECTED PERIODS, 1955-2000
(IN PERCENT)

Period	Natural ^{a)} Unemploy- ment Rate	Natural Growth Rates ^{b)}					Ratios of Investment to GNP ^{a)}	
		Potential Output (GNP)	Capital Stock	Manhour Manhours	Manhour Produc- tivity	Output ^{c)} per Capita	Net	Gross
1955-1968	4.7	3.6	4.0	0.7	2.9	2.2	2.2	9.3
1968-1973	5.6	2.6	3.6	1.2	1.3	1.6	2.1	9.7
1973-1979	7.0	2.8	2.8	1.8	1.1	2.1	1.7	9.5
1979-1985	6.8	3.1	3.1	1.4	1.7	2.2	1.8	9.5
1985-1993	6.4	2.5	2.8	0.8	1.7	1.6	1.7	9.5
1993-2000	6.2	2.4	2.5	1.0	1.4	1.8	1.5	9.5

a) Averages for period.

b) Exponential rates between endpoints of periods.

c) Potential GNP per person - Population projections for 1977-2000 are from Bureau of the Census (1977).

rapidly than population thereafter, so that output per capita rose more uniformly over the period and is projected to do so until 1985.¹¹

Net investment requirements along the natural growth path diminished in 1973-79 as growth decelerated and the optimal capital-labor ratio fell in reaction to the exogenous 1973-75 upshift in the rental-wage ratio. A similar reduction is found in the gross investment shares. The net share is projected to decrease somewhat toward the end of the century, but the gross share remains constant throughout the forecast period.

The contributions of capital deepening -- increases in the ratio of capital to manhour input -- and other factors to the natural rates of manhour productivity growth are shown in Table 3. The breakdown is based on the production-function formulation of potential output (equation (28)), normalized to labor productivity form and with the labor quality index added:

$$(36) \quad (X^P/M^f)_t = Ae^{\gamma t} (\ell_t^n)^{1-\alpha} (m_t^n)^{1-\alpha} (k_t^n)^\alpha (K_{t-1}^f/M_t^f)^\alpha ,$$

where ℓ_t^n is the index of labor quality along the natural growth path. Thus changes over time in potential labor productivity can be decomposed as weighted geometric averages of the rates of change respectively of technical progress, labor quality, the intensities of utilization of manhours and capital stock, and the equilibrium capital-labor ratio.

Deepening was second only to technical progress as a source of potential productivity growth during the postwar years and is projected to remain so in the future. The principal sources of the sharp reduction of productivity growth during 1968-73 were reductions in the rates of technical progress, capital deepening, and the natural rate of capacity utilization, with a secondary contribution from decreases in labor quality. The further deterioration of productivity growth after 1973 reflects the decrease in the natural rate of capital deepening resulting from the OPEC shock to the rental-wage ratio and the accompanying deceleration of the natural growth rate. Conversely, potential productivity growth is projected to increase

TABLE 3. CONTRIBUTING SOURCES AND ANNUAL RATES OF
CHANGE OF POTENTIAL MANHOUR PRODUCTIVITY,
HISTORICAL 1955-1979 AND PROJECTED 1980-2000
(IN PERCENT)

Period	Product- ivity	Technical Progress	Labor Quality	Labor Utiliz.	Capital Utiliz.	Capital Deepening
1955-1968	2.87	1.66	0.01	0.01	0.18	1.01
1968-1973	1.33	0.97	-0.20	0.03	-0.26	0.78
1973-1979	1.09	0.97	0.01	0.01	-0.26	0.36
1979-1985	1.72	0.97	0.28	-0.04	0.01	0.51
1985-1993	1.67	0.97	0.21	-0.01	-0.11	0.62
1993-2000	1.40	0.97	-0.04	0.02	-0.02	0.47

in future years, assuming that the real price of oil does not again increase substantially.¹²

This projected reversal of productivity growth is a consequence of the transitory nature of the adjustment to the OPEC shock. The effect of the once-for-all increase in the rental-wage ratio was to induce a once-for-all reduction in the desired capital-labor ratio and a corresponding adjustment to a lower growth path. Without the oil shock, the growth rate of potential productivity would have been 1.6 instead of 1.1 percent in 1973-79. The potential growth path would have been 2.9 percent higher by 1979, but the subsequent natural growth rates of productivity and potential GNP would have been unchanged from those in Tables 2 and 3. Thus our model of the natural growth path explains the reduced growth rate of potential productivity during 1973-79 as the reflection of a temporary deceleration during the transition to a permanently lower path rather than as a permanent reduction in the natural growth rate itself, such as might result from some unexplained break in the rate of technical progress after 1973.

ALTERNATIVE ESTIMATES OF POTENTIAL GROWTH RATES, 1955-2000

In this section we compare the estimated growth rates resulting from four alternative estimates of the parameters of the factor demand system. The model of labor supply and the assumptions on exogenous variables are the same in all four simulations. The results are presented in Table 4. The first column, repeated from Table 2, refers to the estimated growth rates using our preferred factor demand specification, system 1. The following columns correspond to parametric systems 2-4 in Table 1.

Our preferred specification indicates that the potential growth rate declined from 3.6 percent in 1955-68 to 2.6 in 1968-73 and 2.8 in 1973-79. It is projected to rise moderately in 1979-85 and then to decline sharply once more in the last 15 years of this century.

Neglecting the adjustment for labor quality but continuing to assume no reduction in the rate of technical progress since 1969, as in system (2), results in a similar reading of the 1973-79

TABLE 4. ALTERNATIVE ESTIMATES OF POTENTIAL GROWTH RATES,
HISTORICAL AND PROJECTED, 1955-2000
(Annual Percentage Change^{a)})

PERIOD	ALTERNATIVE ESTIMATES ^{b)}			
	(1)	(2)	(3)	(4)
1955-1968	3.6	3.6	3.6	3.6
1968-1973	2.6	2.7	2.9	2.9
1973-1979	2.8	2.8	2.4	2.5
1979-1985	3.1	2.7	2.6	2.3
1985-1993	2.5	2.1	2.0	1.7
1993-2000	2.4	2.3	1.9	1.9

a) Exponential rates between endpoints of periods

b) See text for explanation of alternatives

period but a definitely bleaker forecast for the coming years. The bleaker outlook reflects the absence of the favorable productivity effects of the projected improvement in labor quality incorporated in the first system and shown in column 3 of Table 3.

Retaining the labor quality adjustment but assuming an additional permanent deceleration of technical progress after 1974, as in system (3), alters the historical picture considerably. The deceleration of potential GNP is smaller than before in 1968-1973 but greater in 1973-1979. The ex-ante growth rates for 1979-2000 are about half a percentage point lower than in the preferred system in reflection of the reduced rate of technical progress after 1973.

Finally, dropping the labor quality adjustment but retaining the pessimistic assumption on technical progress implies yet a further reduction in future growth rates is shown in column 4.

INFLUENCE OF THE NATURAL UNEMPLOYMENT RATE

The strong uptrend of the estimated natural unemployment rate during 1955-1979 is the outcome of simulation with a particular set of parameter estimates for equation (7), in which the armed forces variable LA/N receives heavy weight. Since military recruits are drawn primarily from the population of young males with its extraordinarily and chronically high unemployment rate, we expect LA/N to be an important variable in the equation for U17, but perhaps we have overestimated its strength in the present application. It is therefore interesting to compare the present results with an alternative simulation employing a different version of equation (7) but with all other equations and parameters as before. This is done in Table 5. The columns headed (a) are repeated from Table 2, whereas those labeled (b) are the results of the alternative simulation.

The new estimates of U17 are considerably lower and rise considerably less during 1955-1979, with the rise basically reflecting the slow uptrend due to changes in the demographic mix of the labor force and only minimally influenced by the armed forces variable. Just as in the first simulation and for the same

TABLE 5. ALTERNATIVE ESTIMATES OF NATURAL GROWTH RATES
FOR DIFFERING NATURAL UNEMPLOYMENT PATHS, 1955-2000

	Natural		Natural Growth Rates							
	Unemploy- ment Rate		Potential GNP		Manhours		Manhour Produc- tivity		Output per capita	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
1955-1968	4.7	4.2	3.6	3.6	0.7	0.7	2.9	2.9	2.2	2.1
1968-1973	5.6	4.6	2.6	3.1	1.2	1.8	1.4	1.4	1.6	2.2
1973-1979	7.0	4.9	2.8	3.0	1.8	1.9	1.1	1.1	2.1	2.2
1979-1985	6.8	4.8	3.1	3.1	1.4	1.4	1.7	1.7	2.2	2.2
1985-1993	6.4	4.5	2.5	2.5	0.8	0.8	1.7	1.7	1.6	1.6
1993-2000	6.2	4.4	2.4	2.4	1.0	1.0	1.4	1.4	1.8	1.8

Notes: Simulation (a) is repeated from Table 2.
Simulation (b) uses alternative parameter estimates for equation (7) but the parameters of the other equations in the system are the same as before.
See footnotes to Table 2.

reasons, the natural rate is projected to decline gradually during the remainder of the century.

The striking feature of the comparison is that potential manhour productivity is virtually unaffected by the differential behavior of U17. Productivity growth is determined primarily by technical progress, the complex of factors governing the rate of capital deepening, and the improvements in labor quality associated with compositional changes in the labor force. None of these determinants is importantly affected by variations in the natural rate of unemployment even in our simultaneous framework.

What *is* noticeably affected is the growth of labor supply at the natural unemployment rate. Although obscured by the averages shown in Table 4, the natural rate in the first simulation rose from 4.3 percent in 1968 to 6.4 in 1973 during the demobilization from the Vietnam War and induced a correspondingly substantial reduction in the growth of potential manhours, GNP, and GNP per capita over that interval.

A slight reduction in the growth rates of manhours and GNP is apparent also in the comparison for 1973-79, although the impact is much more moderate since U17 rose only from 6.4 to 6.9 over that interval. Finally, since the projected downtrends in U17 during 1980-2000 are at the same rate in the two simulations, the natural growth rates are the same along both paths.

The important lessons from the comparison are these. First, productivity growth is unaffected by a change in the natural rate of unemployment. Second, by affecting labor supply, a change in U17 can induce a temporary change in the growth rates of GNP and GNP per capita. Third, if after a change U17 remains at the new level, the natural rate of growth will return to its normal trend rate, but along a path which is permanently higher or lower because of the acceleration or deceleration of growth during the transition period.

PLANNED POLICY SIMULATIONS

Since our natural growth model is a structural system including a number of policy instruments, it can be used to investigate alternative policies to affect the growth path. Given the necessary uncertainties attending any attempt to quantify future trends in labor supply and productivity, such policy simulations are perhaps the most important use to be made of the model. We conclude with a summary of our plans in this regard.

The model includes two classes of policy instruments. The first class may be used to affect investment demand and productivity growth, whereas the second impinges on labor supply.

Investment demand may be augmented or moderated by altering the tax component of the rental price of capital through changes in the investment credit, the depreciation rules for tax accounting, or the corporate tax rate. We reported briefly in Coen and Hickman (1980b) on the simulated effects during 1980-2000 of a permanent 20 percent reduction in the rental price of capital beginning in 1980, but we plan more extensive investigations of this important topic in the near future.

The second class of policy variables includes the scale of unemployment benefits, which affects the natural unemployment rate and potential labor supply. In a sense the size of the armed forces may also be regarded as a policy instrument to affect the natural unemployment rate, although it has not been directed to this purpose in past years. If the exogenous ratio of armed forces to population were assumed to follow a different path than in, say, the preferred simulation for the 1980's and 1990's reported in Table 2, one could regard the results as measuring the impact either of a realistic military scenario or some equivalent employment policy impinging directly on the young male population.

Finally, the model may be used to study the effects of alternative assumptions about exogenous variables other than policy instruments. In this connection, the most important item on our research agenda is to test the responses to alternative assumptions on the future time path of the real price of oil, as we already have done retrospectively for the oil shock of 1974.

NOTES

1. The authors are Professors of Economics at Northwestern University and Stanford University respectively. This research was supported by the National Science Foundation under Grant DAR77-27746. Jeffrey A. Parker provided valuable assistance with the data and computations.
2. For simplicity we neglect the complications in the actual model arising from the distinction between private and public employment, hours, and wage rates, and also the distinction between the high-employment and the natural unemployment rates. For details, see Coen and Hickman (1980a).
3. NM is included in conformity with the Easterlin (1978) hypothesis that changes in this ratio affect the participation rates of younger women positively and those of older women negatively. It does not appear in the participation equations for males.
4. See Coen and Hickman (1980a) for the theoretical derivation of the actual unemployment equation and its transitory variables. Since writing (1980a), we have modified the unemployment equation by deleting the lagged wage surprise and employment variables.
5. Our high-employment labor force estimates are obtained in an intermediate step as the solutions of equations (1) and (2) in the text, subject to the constraint that unemployment rates by age and sex remain constant over time at their 1956 base-year levels. For details, see Coen and Hickman (1980a).
6. We have presented such conditional estimates and projections of full-employment labor supply and unemployment, 1951-2000, in Coen and Hickman (1980a).

7. Even if adjustment were instantaneous, the inputs could be over or under utilized if expected and actual outputs differ (Coen and Hickman (1976), Ch. 1). Our empirical results indicate that systematic deviations of expected from actual output are of secondary importance and can be neglected.
8. This section draws on material from Hickman (1979).
9. It is sometimes argued that the 1973-74 OPEC shock rendered part of the capital stock obsolete. Such extraordinary obsolescence would permanently reduce K and hence V at the time of occurrence. This idea was tested during the work on estimation of the factor demand and production system described in the next section, but elimination of two percent of K in 1974 had negligible effects on the parameters of the system estimated through 1979.
10. Thus our specification of potential GNP as a function of K and M alone, whereas gross output depends also on E , is consistent with the view that K and E are net complements when responding to energy price changes even though they are technical substitutes in the production of gross output. Unlike Berndt and Wood (1979), however, our specification stresses that interdependence of the prices of energy and capital goods is the basic source of K - E complementarity on the macroeconomic level.
11. See Table 5 and accompanying discussion in Coen and Hickman (1980a) for details on growth rates of the noninstitutional population, labor force, private employment, private hours, and private manhours during the sample and projection periods. An additional source of variation between manhour and population growth rates is changes in the ratio of noninstitutional to total populations.
12. The effects of the increases in real OPEC prices in the rental wage ratio during 1979 are not included in these simulations, but they are much smaller than in 1973-75.

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