THE IIASA HEALTH CARE RESOURCE ALLOCATION SUBMODEL: MODEL CALIBRATION FOR DATA FROM CZECHOSLOVAKIA

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

This paper briefly describes the DRAM (Disaggregated Resource Allocation Model) and its parameterization for seven patient categories, one mode, and two resources in order to analyze Czechoslovakian in-patient hospital care using 1976 data.

Related publications in the Health Care Systems Task are listed at the end of this report.

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ABSTRACT

In many developed countries the problem of allocating resources within the Health Care System (HCS) is perennial. Health Care administrators are continually asking what are the consequences of changing the mix of resources. The disaggregated resource allocation model (DRAM) has been developed to assist Health Care administrators with this problem. The model simulates how the HCS in aggregate allocates limited supplies of resources between competing demands. The principal outputs of the model are the numbers of patients treated in different categories, and the modes and quotas of treatment they receive.

This paper describes how parameters were estimated for DRAM for Czechoslovakian hospital in-patient care. The model was parameterized for seven patient categories (general surgery, general medicine, obstetrics and gynaecology, traumatic and orthopaedic surgery, otorhinolaryngology, paediatrics, and ophthalmology) and two resource types (hospital beds and hospital doctors). The paper ends with a description of how the model could be used to investigate the consequences of changes in the mix of hospital beds and hospital doctors for Czechoslovakian hospital in-patient care.

CONTENTS

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1. INTRODUCTION

In developed countries, the allocation of resources within the Health Care Systems (HCS) is a problem to which governments are giving more and more attention. For example in the U.S., the Federal Government is seeking to control the allocation of health care resources through various medical manpower policies. In Bulgaria, the Government is seeking the right balance for hospital resources, between in-patient care and out-patient care.

DRAM (a disaggregated resource allocation model) is designed to help answer such questions. It is one of the submodels of the HCS Task being developed at the International Institute for Applied Systems Analysis. DRAM was formulated by Gibbs (1978) and further developed by Hughes (1978 a,b,c).

This working paper describes how DRAM was calibrated for hospital in-patient data from Czechoslovakia (CSSR). The paper begins with a brief description of DRAM. This is followed by a section showing how CSSR hospital in-patient care can be formulated in the DRAM format. This leads to a discussion of data to calibrate DRAM. After the details of the calibration process are given, the paper ends by showing how DRAM could be used to investigate the consequences for Czechoslovakian hospital in-patient care of different mixes of resources.

2. HEALTH RESOURCE ALLOCATION MODEL DRAM

Health services cannot be administered in a rigid centralized way. In every country, doctors have clinical control over the treatment of their patients, and it is local medical workers who ultimately determine how to use the resources (e.g. hospital beds, nurses) available to them. The specific question underlying DRAM is: If the decision maker provides a certain mix of resources, how will the HCS allocate them?

There are two assumptions about the behavior of the HCS in the model. First it is assumed that there is never a sufficient supply of resources to meet all the potential (or ideal) demands for them. The model simulates the balance chosen by the many agents in the HCS (doctors, nurses, social workers), between different treatment categories, between alternative combinations (modes) of care within the same treatment category, and between quality of care and numbers treated. The second assumption is that the aggregate behavior of the agents in the HCS can be represented as the maximization of a utility function whose parameters can be inferred from results of previous choices. Thus when the model is parameterized, it can be used to estimate the consequences of different allocations of resources.

The model in mathematical terms is as follows:

x_{jk} = numbers of individuals in the j^{th} patient category who receive resources in the k-th mode of care (per head of population per year)

X_{jk} = the ideal number of individuals in the j^{th} patient category who should receive resources in the k-th mode of care (per head of population per year)

$y_{jk\ell}$ = supply of resource type ℓ received by each individual in the j^{th} patient category in the k^{th} mode of care.

$Y_{jk\ell}$ = the ideal levels of supply of resource ℓ for each individual in the j^{th} patient category in the k^{th} mode of care*

R_{ℓ} = the availability of resource type ℓ (per head of population per year)

C_{ℓ} = marginal cost of resource ℓ when all demands are satisfied.

The utility function which the various agents in the HCS seek to maximize is taken to be

$$Z(x,y) = \sum_j \sum_k g_{jk}(x_{jk}) + \sum_j \sum_k \sum_\ell x_{jk}\, h_{jk\ell}(y_{jk\ell}) \qquad (1)$$

subject to $\quad \sum_j \sum_k x_{jk}\, y_{jk\ell} = R_\ell \qquad \forall \ell$

where
$$g_{jk}(x) = \frac{\sum_\ell C_\ell X_{jk} Y_{jk\ell}}{\alpha_j} \left\{ 1 - \left(\frac{x}{X_{jk}} \right)^{-\alpha_j} \right\}$$

$$h_{jk\ell}(y) = \frac{C_\ell Y_{jk\ell}}{\beta_{jk\ell}} \left\{ 1 - \left(\frac{y}{Y_{jk\ell}} \right)^{-\beta_{jk\ell}} \right\}$$

α_j (>0) is a parameter measuring the relative importance of treating the ideal number of individuals X_{jk}. $\beta_{jk\ell}$ (>0) is a parameter measuring the relative importance of achieving the ideal level $Y_{jk\ell}$. The utility function Z, depicts the many agents who control the allocation of health care resources as seeking to attain ideal levels of service (X) and supply (Y), but where the urge to increase the actual levels of service (x) and supply (y) decreases with increasing values of x and y, according to the parameters α and β. The costs of different resources are introduced so that the marginal increases in Z, when ideal levels are achieved (x = X, y = Y), equal the marginal

*In the sequel, x,y are used to denote $\{x_{jk}\}$,$\{y_{jk\ell}\}$ respectively, with a like notation for similarly subscripted variables.

resource costs. Beyond these levels, extra resources are only useful as assets and not for treating patients.

Hughes (1978c) has shown that the solution of the optimization problem at equation 1 is as follows

$$y_{jk\ell} = Y_{jk\ell}(\lambda_\ell)^{\frac{-1}{\beta_{jk\ell}+1}} \tag{2}$$

$$x_{jk} = X_{jk}(\mu_{jk})^{\frac{-1}{\alpha_j+1}} \tag{3}$$

where μ_{jk} is a weighted sum

$$\mu_{jk} = \frac{\sum\limits_{\ell} C_\ell Y_{jk\ell} \nu_{jk\ell}}{\sum\limits_{\ell} C_\ell Y_{jk\ell}}$$

of the terms

$$\nu_{jk} = \left((\beta_{jk\ell} + 1)\lambda_\ell^{\frac{\beta_{jk\ell}}{\beta_{jk\ell}+1}} - 1 \right) / \beta_{jk\ell} .$$

and where λ_ℓ are the solutions of the following set of equations

$$0 = -R_\ell + \sum_j \sum_k X_{jk} Y_{jk\ell} (\lambda_\ell)^{\frac{-1}{\beta_{jk\ell}+1}} (\mu_{jk})^{\frac{-1}{\alpha_j+1}} \quad \text{for all } \ell$$

The algorithm for determining the solutions (equations 2 and 3) has been developed by Hughes and Wierzbicki (1980). This algorithm has been programmed, and requires no specialized software. Experience has shown that the computer program is easily transferred from computer to computer.

3. AN APPLICATION OF DRAM TO CZECHOSLOVAKIAN HOSPTIAL IN-PATIENT DATA

This section describes the DRAM variables chosen for Czechoslovakian hospital in-patient care. In all that follows, we will assume there is only one mode of care, i.e. in-patient care. The section begins with a brief description of how hospital in-patient care is organized in Czechoslovakia.

3.1. Hospital In-patient Care in CSSR

Czechoslovakia is a federation of two states - the Czech Socialist Republic (CSR) and the Slovak Socialist Republic (SSR). Health care is administered from the Health Ministry of each state. These ministries control all aspects of health care which are defined in the Act "On the Health Care of the People" issued on 17th March 1966 - No. 20 of the collection. CSR is divided for administrative purposes (including health care) into 8 regions, one of which is Prague, the capital of CSSR. SSR is also divided into 4 regions, one of which is Bratislava, the capital of SSR.

Health care is considered to have two aspects, therapeutic and preventive. These two aspects are realized in two forms of care - ambulatory care (policlinics) and hospital care. In CSSR there are three types of hospitals. Type I hospitals serve areas with populations of about 50,000, Type II hospitals serve areas with populations of about 200,000; and Type III hospitals serve areas with populations of about 1,000,000. Type III hospitals are considered teaching hospitals. The range of available specialities increases from Type I to Type II hospitals, and from Type II to Type III hospitals. The relevant details are given in Makovický et al. (1978).

3.2. Treatment Categories

In defining the treatment categories it is necessary to take into account certain conditions imposed by the calibration method (see Appendix A). It is assumed that the same utility function Z(x,y) (equation 1) holds for each of the 12 administrative regions of Czechoslovakia. Given that there is sufficient variation in the resource levels, then the shape of the utility function can be inferred. This means that for the treatment categories chosen, each area should be self sufficient, i.e. if general medicine is chosen, almost all general medicine patients should be treated in the area. Thus, treatment categories which are regarded as "national specialties" are excluded.

Another requirement for chosen treatment categories arises from the fact that in the DRAM formulation, the resource levels are treated as continuous variables. This means that the basic unit of each resource (e.g. a hospital bed year) should be small compared to the total amount of resources allocated to a treatment category in each of the regions under consideration. Hence treatment categories should not be too small. For example, this would exclude a treatment category consisting of only occupational medicine.

Having taken into account the above, the following treatment categories were chosen:

-- General surgery
-- General medicine
-- Obstetrics and gynaecology
-- Traumatic and orthopaedic surgery
-- Otorhinolaryngology
-- Paediatrics
-- Ophthalmology

The above treatment groups also constitute seven of the largest acute specialties in the UK. Data on admission rates (per head of resident population) for each treatment category for each of the twelve Czechoslovakian regions, (including Prague and Bratislava) for 1976 were obtained from CSSR zdravotnictí (1977: 189), the Czechoslovakian year book on Health Statistics. Data from all types of hospitals were used.

3.3. Hospital In-patient Resources

The question arises: Which are the most important health care resources for hospital in-patient care? The most important would appear to be beds, hospital doctors, nurses, and operating theaters. Feldstein (1967) was perhaps the first to demonstrate the elasticity of admission rates to bed supply using data from the UK. Recently Rousseau and Gibbs (1980) have done the same for data from Canada. The bed supply has therefore been included in our model.

The hospital doctor supply is also an important determinant for hospital admission rates. Many authors have noted this. For example, van der Gaag, et al (1975), have demonstrated, using Dutch data, that referrals to hospitals are positively correlated to hospital doctor (specialist) supply. Hospital doctors have therefore been included in the model.

The level of nurse supply has not been included in our model, largely because Feldstein (1967) could not demonstrate any relation between admission rates and level of nursing. This analysis may be out of date now, and perhaps needs repeating. In Czechoslovakia, for instance, health care planners are concerned about a shortage of nurses, especially in large urban areas.

Little appears to be known about the relationship between the supply of operating theaters and admission rates. Possibly this is because data on the usage of operating theaters by treatment group are not readily available. Unfortunately no adequate Czechoslovakian data were available, and so this resource was excluded from the model. Perhaps more detailed study of this resource would be fruitful.

Thus, we decided to calibrate DRAM for the two resources: beds and doctors. This choice was supported by a recent analysis by Rudge (1978) who found that for general surgery in the Trent Regional Health Authority (UK), the most important supply variables for predicting hospital admissions were hospital beds and hospital doctors.

Having decided which resources are to be used it is necessary now to consider how these resources are to be measured. The unit for hospital beds was taken to be available beds per 1000 population in the particular region. This means that the supply variable ($y_{jk\ell}$) is available bed-days per patient. This has the advantage over the more usual measure of occupied bed-days per patient (i.e. length of stay) of eliminating the separate estimation of occupancy rates.

With regard to hospital doctors, there are several possible measures. The aim is to find the measures which best explain the variations in admission rates and supply levels per patient. Examples of possible measures are:

(a) The number of hospital doctors (including anesthetists pathologists, surgeons) involved with a particular treatment category

(b) The number of hospital doctors of all grades belonging to the specialities which treat a particular treatment category (For example, if the treatment is "general medicine", then this measure would be the number of doctors within the general medicine specialty.)

(c) The number of senior hospital doctors (second degree specialist in CSSR, and consultants in UK) belonging to the specialities which treat a particular treatment category

(d) The number of anesthetists involved with a particular treatment category

These measures are not exclusive, since, for instance, measures (c) and (d) could be used simultaneously. However, some of these measures may be difficult to calculate, as it would be difficult to allocate the time of a pathologist or an anesthetist to the various treatment categories. For the purpose of this study measure (b) has been adopted. (The units used are doctor-days per 1000 population - 1 doctor year = 225 doctor-days). Subsequently, whenever we refer to hospital doctors, it will be to this definition. In support of this choice, Rudge (1978) reported that in some instances measure (b) explains general surgical admission rates (in the Trent Regional Health Authority, UK) better than measure (c).

Data on the availabilities of beds and hospital doctors for each treatment category and each region were taken from CSSR zdravotnictí (1977: 219,217 respectively). The totals for the seven treatment categories for each region are given in Table 1. The table displays a sufficiently wide range of resource availabilities to calibrate DRAM. Furthermore, the values of the resource availabilities for the two resource types are uncorrelated (test statistic not significant at the 25% level).

Table 1. Resource Availabilities for the Seven Treatment Categories - Czechoslovakia 1976.

Region	Available bed-days per 1000 population (1 bed-year=365 bed-days)	Hospital doctor-days per 1000 population (1 Doc-year=225 doc-days)
Praha	2271	91
Stredocesky	2634	111
Jihoscesky	2312	96
Zapadocesky	2355	96
Severocesky	2435	112
Vychodocesky	2525	86
Jihomoravsky	1970	79
Severomoravsky	2352	98
Bratislava	960	61
Zapadoslovensky	1715	89
Stredoslovensky	1799	107
Vychodoslovensky	1870	107

4. PARAMETER ESTIMATION FOR DRAM - CZECHOSLOVAKIAN HOSPITAL IN-PATIENT DATA

The problem of calibrating DRAM for Czechoslovakian hospital in-patient data is now considered. Estimates are required for three groups of parameters:

(1) The ideal levels X,Y at which patients would be admitted and would receive resources, if there were no constraints on resource availability

(2) The power parameters α,β which reflect the relative importance of achieving the ideal levels X and Y (For instance, if an α is relatively high then it is relatively more important to achieve the corresponding X.)

(3) The relative costs C of the different resources - in this case hospital beds and hospital doctors

In what follows the parameter $\{X,Y,\alpha,\beta\}$ will be estimated from actual allocations of resources. If estimates of the ideal levels (X,Y) derived from morbidity surveys and surveys of clinical opinion were available, then these could have been used. The power parameters (α,β) are not as readily interpreted as the ideal levels (X,Y), and therefore surveys of informed opinion may not provide useful estimates. The cost parameters will be determined exogenously.

In estimating the parameter set $\{X,Y,\alpha , \beta\}$ the approach of Hughes (1978c) will be followed. This assumes the utility function Z, is applicable both to the whole of Czechoslovakia and to each of the individual regions in Czechoslovakia. A necessary condition of this assumption is that for a chosen category all patients within this category are treated in the region in which they live - hence, the data requirements mentioned in section 3.2. In practice a small percentage of patients will be treated in regions other than the one they live in. Net patient flows of 2-3% probably do not introduce to much inaccuracy. Net flows greater than this should be adjusted for, for example, by making appropriate adjustments to the regional populations. It was considered that such adjustments were unnecessary for the

Czechoslovakian data for the treatment groups chosen.

The parameter estimation procedure is explained in detail in Appendix A. Briefly, the procedure is as follows. Given that the utility function Z is applicable to each region, each region provides an independent data point to estimate $\{X,Y,\alpha,\beta\}$. To estimate these parameters, each region (i.e. each data point) should be allocated to one of two groups. The number of data points in each group should be approximately equal. Initial estimates of (X,Y) are provided, (α,β) are then estimated using the first data set. These estimates of (α,β) give us a new (X,Y) which are estimated from the second data set. With these new (X,Y), further (α,β) are estimated using the first data set... and so on until successive estimates of (X,Y,α,β) only change by a small amount.

4.1. Preliminary Analysis and Further Definitions

Before carrying out the estimation procedure given above, it is useful to examine the plots of admission rates and bed supply per patient for each treatment category against total hospital bed supply for all seven categories, and similarly for hospital doctors. These plots for the Czechoslovakia data are given in Figures 1-4. (The Bratislava data have been excluded from the graphs and all subsequent calculations.)

In the one-resource version of DRAM, the model assumptions imply that for each patient category, the admission rates and supply levels per patient should monotonically increase as total resource supply increases. This result should be born in mind when Figures 1 to 4 are examined. When the actual plots of admission rates and supply levels per patient against total bed availability fail to follow this pattern, a one-resource (in this case beds) version of DRAM will not fit well to the actual results. The same implication holds for the plots against total doctor availability. However, failure of the actual data to follow this pattern does not imply that a two-resource DRAM will not fit the actual data as there are likely to be interactions between the two resources not indicated in the figures.

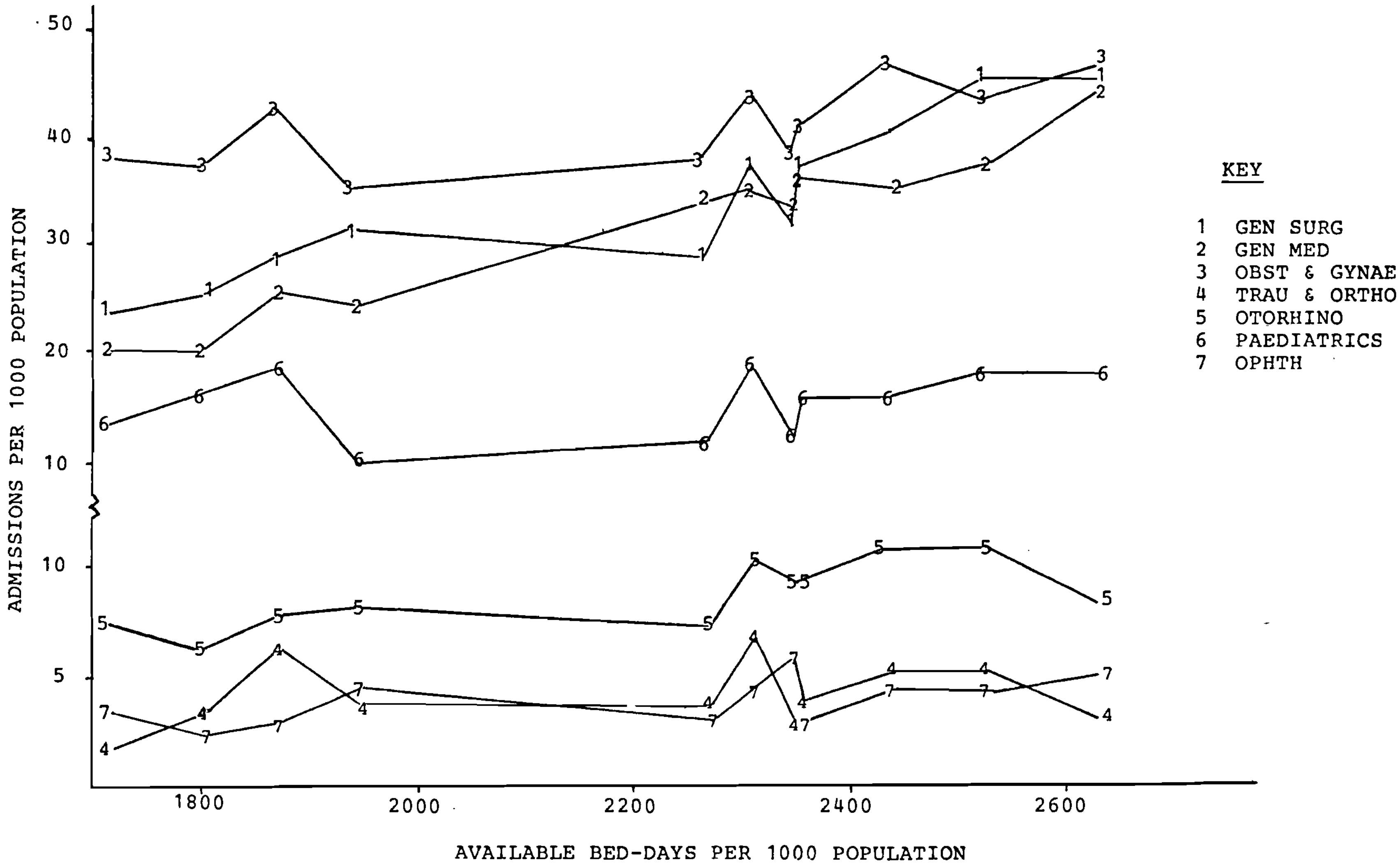

Figure 1. Admission rates - 1976.

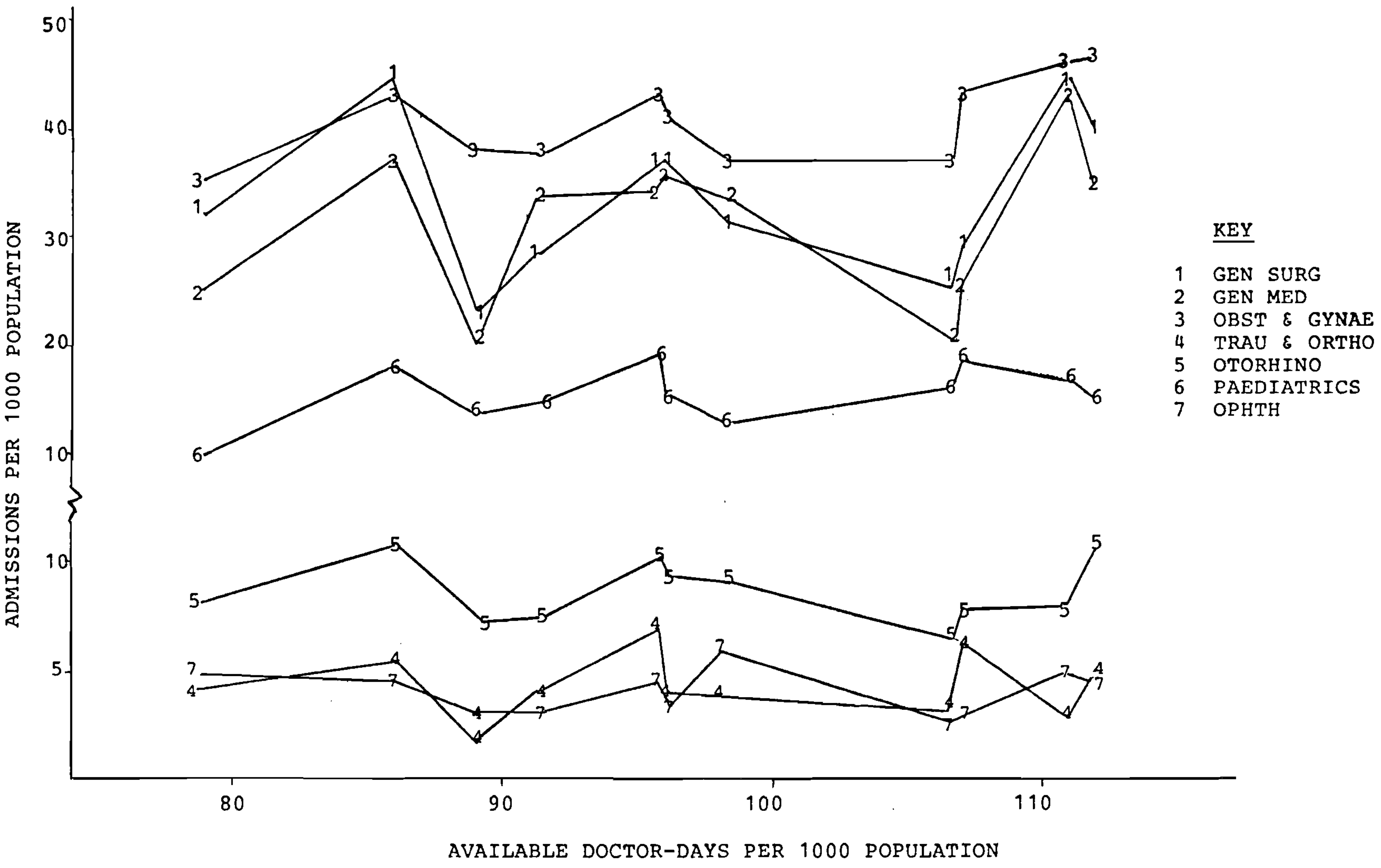

Figure 2. Admission rates - 1976.

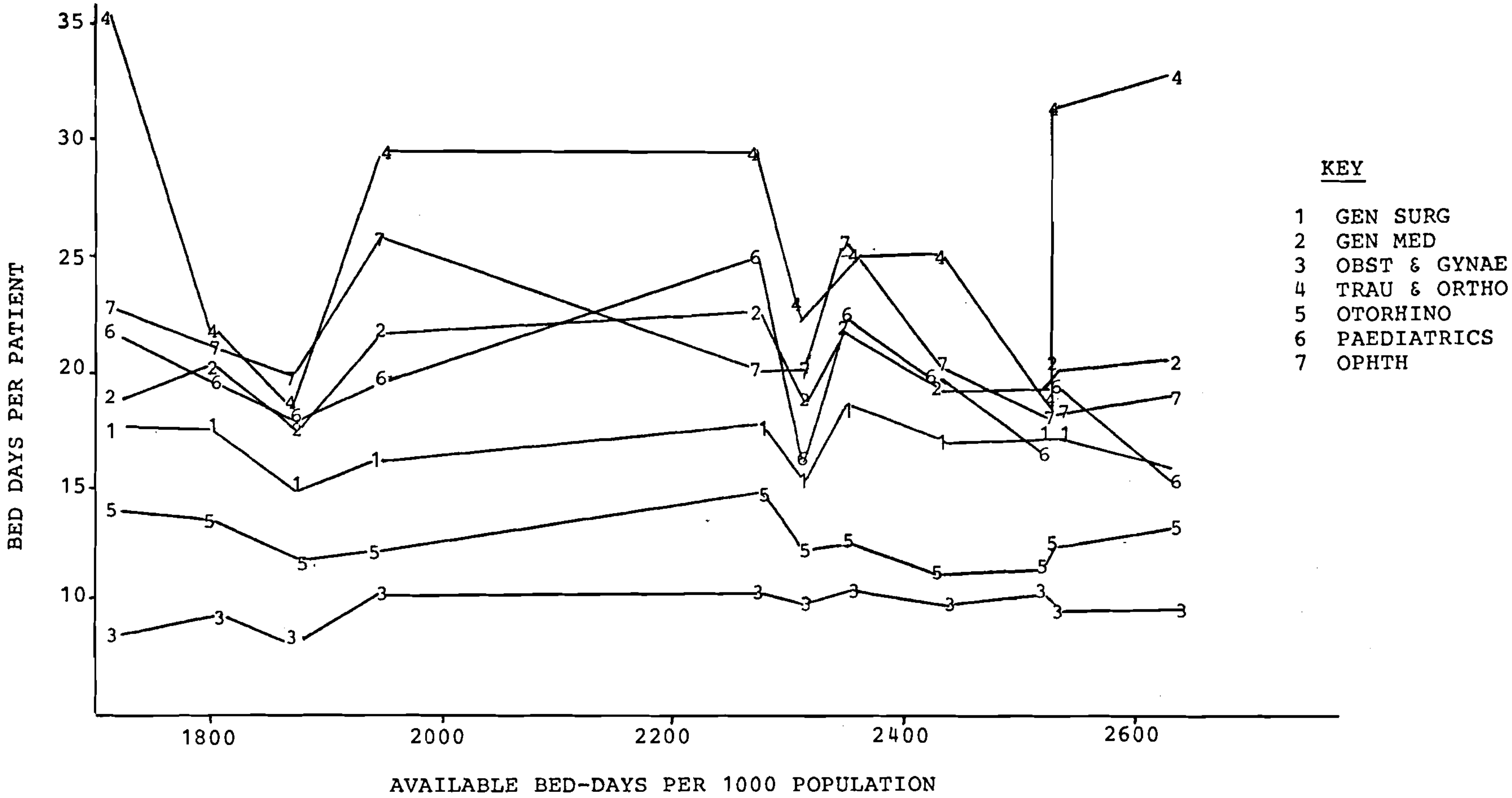

Figure 3. Supply levels (beds) - 1976.

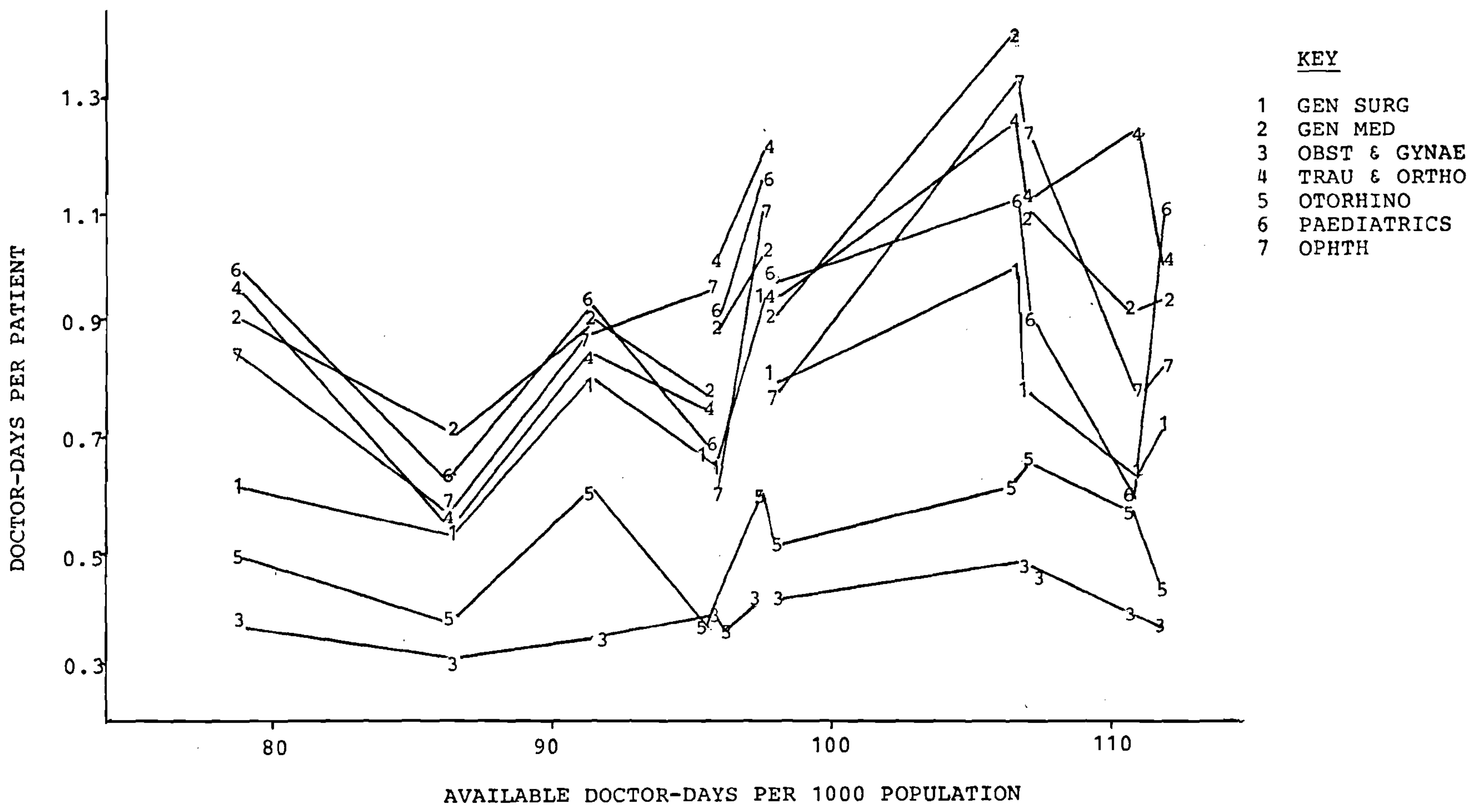

Figure 4. Supply levels (doctors) - 1976.

Figure 1 gives the plots of admission rates against total bed supply. It indicates that the admission rates for general medicine increase as total bed supply increases. Obstetrics and gynaecology show a similar tendency. Further, the figure suggests that the admission rate for general medicine is more elastic to total bed supply than the admission rate for obstetrics and gynaecology. This should imply that the α for general medicine is less than the α for obstetrics and gynaecology. Thus, some of the admission rates follow the pattern mentioned earlier, and for these a bed supply model should reproduce the actual results.

Figure 2 shows the admission rates plotted against total doctor supply. This graph shows, in comparison to Figure 1, that less of the variation is related to the resource supply. For example, the variation in general surgical admission seems to be unrelated to overall doctor supply. However, Figure 1 indicates general surgical admissions are related to overall bed supply. Thus for general surgical admission rates, the total bed supply is a better explanatory variable than the total doctor supply. In general, the total doctor supply does not seem to correlate with admission rates as well as the total bed supply.

Figure 3 gives the plots of bed-days per patient against total bed supply. It indicates that the resource supply per patient is less sensitive to overall bed supply than are admission rates. Feldstein (1967) also made this observation. Figure 4 shows that the supply of doctor days per patient is fairly insensitive to total doctor supply.

Ideally, it would be interesting to investigate further these variations in admission rates and supply levels per patient. For instance, different measures of bed and doctor supply could be tried. Other resource types may also relate to the variations in admission rates. Alternatively, disaggregating the treatment categories might lead to a model which fits the data better. However, such analyses would require access to technical experts and unpublished material, and are therefore impractical for an Institute not situated in Czechoslovakia.

The parameter estimation process was carried out in three stages. Models were calibrated for bed supply and doctor supply separately. Then a two resource (beds and doctors) model was calibrated. This process is described in the next section. Before doing this, it is necessary to extend further the notation of section 2. The model parameters will be estimated from 11 data points. The actual data for data point i $(i = 1,2,N)$ will be represented thus - $x_j(i)$, $y_{j\ell}(i)$ with the mode subscript k removed as there is only one mode. Thus the amount of resource type ℓ used at data point i is

$$\sum_j x_j(i) y_{j\ell}(i) = R_\ell(i)$$

Further, let $\hat{x}_j(i)$ and $\hat{y}_{j\ell}$ be the predicted levels using DRAM given a particular parameter set (X,Y,α,β) and resource availabilities $R_\ell(i)$ at data point i. The following measures of goodness-of-fit can then be defined

$$SS\hat{x}_j = \sum_i \left(\frac{x_j(i) - \hat{x}_j(i)}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - \hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

where w_j is weighted average of $x_j(i)$
and $v_{j\ell}$ is a weighted average of $y_{j\ell}(i)$. As an indication of the goodness-of-fit of DRAM, it is useful to make the following comparisons

$$SS\hat{x}_j \quad \text{with} \quad SS\bar{x}_j = \sum_i \left(\frac{x_j(i) - w_j}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} \quad \text{with} \quad SS\bar{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - v_{j\ell}}{v_{j\ell}} \right)^2$$

4.2. Parameter Estimation for DRAM with One Resource - Hospital Beds

DRAM was parameterized firstly for one resource - hospital beds. The parameters for this model were estimated using the techniques described in Appendix A. They are given in Table 2.

With regard to admission rates, $SS\hat{x}_j$ are much smaller than $SS\bar{x}_j$ for general surgery and general medicine, implying that DRAM reproduces the actual results better than taking the average of the corresponding $x_j(i)$. For traumatic and orthopaedic surgery and paediatrics, $SS\hat{x}_j$ approximately equals $SS\bar{x}_j$ indicating that the model does not reproduce tha actual results any better than taking the mean of the actual $x_j(i)$. For the remaining three categories, $SS\hat{x}_j$ is about half of $SS\bar{x}_j$.

Low α's indicate that the admission rate is elastic to the supply of hospital beds, and vice versa. For example, for general medicine $\alpha_j = 0.001$, and for obstetrics and gynaecology $\alpha_j = 4.1$, implying that general medicine admission rates are more elastic to hospital bed supply than obstetrics and gynaecology admission rates. This observation was also made earlier from Figure 1.

With regard to the supply of hospital beds per patient the results indicate that for all categories the supply of bed-days per patient is inelastic to total bed supply (i.e. β_{j1} is large). This view is supported by Figure 3. Put another way, this means that the total bed supply is not an explanatory variable with regard to bed supply per patient. When β_{j1} is large (as in this case), Y_{j1} is an estimate of the average bed supply per patient for category j.

In summary, the bed supply model reproduces the actual behavior better for admission rates than for the supply levels per patient. The model predictions, compared to actual results for admission rates and supply levels are given in Figure 5 and 6 for selected patient categories.

Table 2. One-resource (hospital beds) DRAM parameter estimates of Czechoslovakian hospital in-patient care.

Treatment Category	Admission rates X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Supply levels: bed-days per patient Y_{j1}	β_{j1}	$SS\hat{Y}_{j1}$	$SS\bar{Y}_{j1}$
General Surgery	60	.001	.101	.501	17	100	.052	.052
General Medicine	55	.001	.055	.635	20	100	.061	.059
Obstetrics and Gynaecology	45	4.1	.068	.100	10	100	.049	.047
Traumatic and Orthopaedic Surgery	7	.75	1.275	1.228	25	100	.565	.612
Otorhinolaryngology	13	.66	.161	.333	12	100	.095	.092
Paediatrics	18	3.1	.388	.409	20	100	.222	.220
Ophthalmology	8	.001	.219	.476	22	100	.160	.160

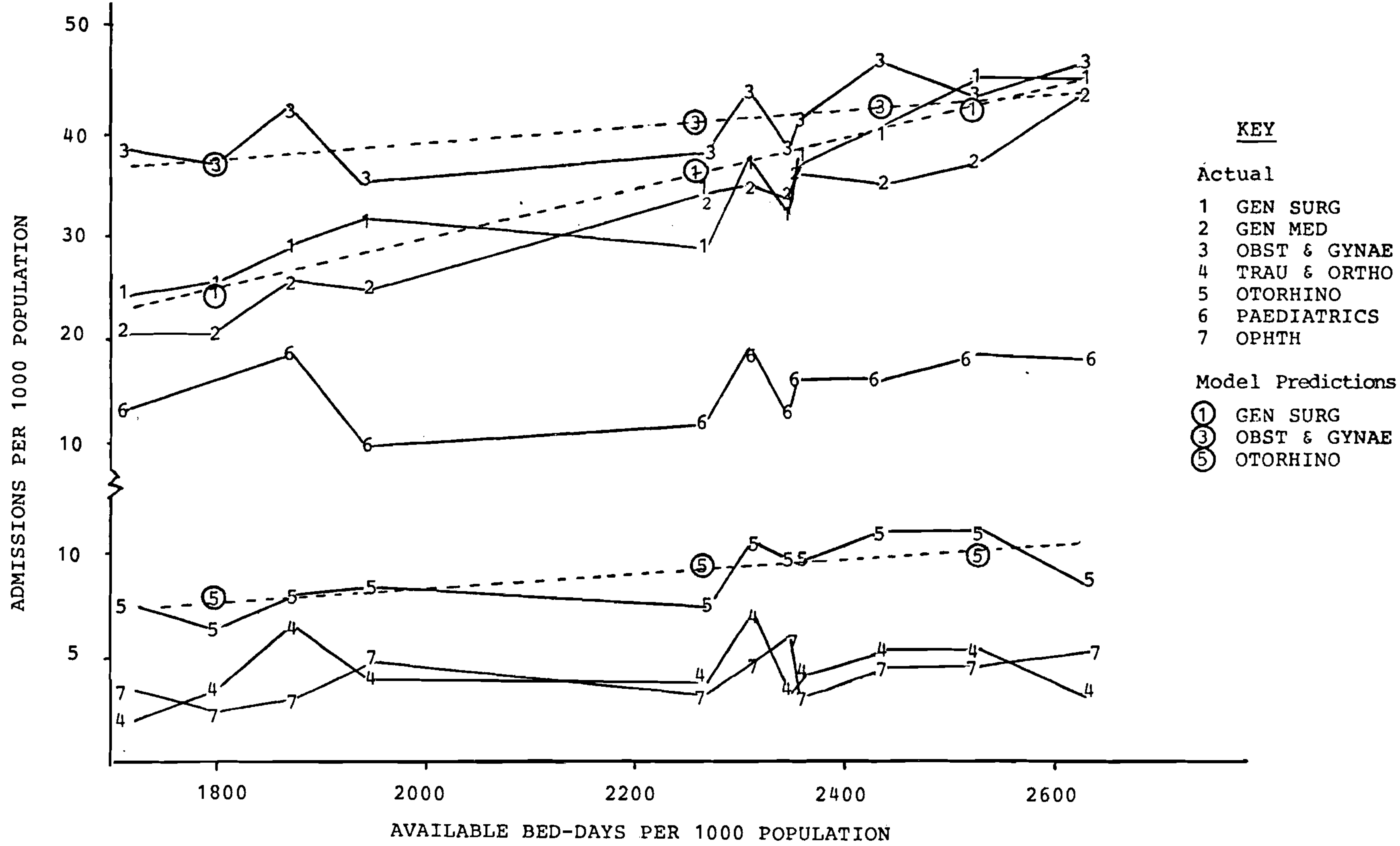

Figure 5. Admission rates - 1976.

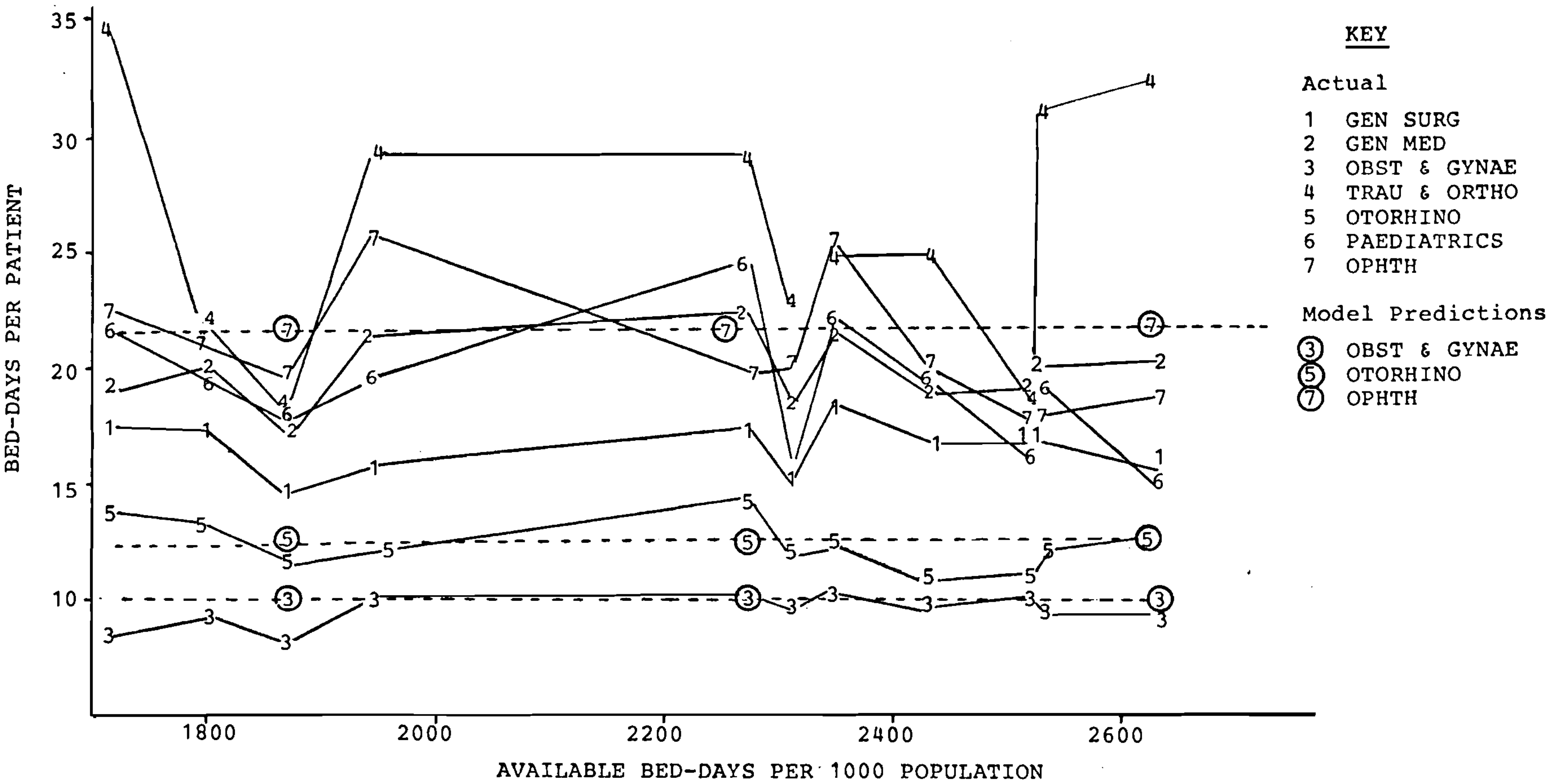

Figure 6. Supply levels (beds) - 1976.

4.3. Parameter Estimation for DRAM with One Resource - Hospital Doctors

The parameter estimates for DRAM when the resource is hospital doctors are given in Table 3. $SS\hat{x}_j$ and $SS\bar{x}_j$ are approximately equal, except for obstetrics and gynaecology, and paediatrics. Similarly, $SS\hat{y}_{j\ell}$ and $SS\bar{y}_{j\ell}$ are approximately equal except for obstetrics and gynaecology, and traumatic and orthopaedic surgery. Even in these four cases, $SS\bar{x}_j$ and $SS\bar{y}_{j1}$ are only reduced by 20%, indicating that the total doctor supply is only explaining a small part of the actual variations. Thus the hospital doctor model does not reproduce the actual admission rates as well as the hospital bed model. Further the doctor supply per patient does not, in general, seem to be related to total doctor supply. Similar remarks were made in section 4.1. The model predictions compared to actual results are given in Figures 7 and 8 for selected patient categories.

Table 3. One-resource (hospital doctors) DRAM parameter estimates of Czechoslovakian hospital in-patient care.

Treatment category	Admission Rates				Supply levels: doctor-days per patient			
	X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$
General Surgery	45	1.2	.488	.501	0.81	8.2	.384	.401
General Medicine	41	1.0	.600	.635	1.43	.80	.360	.421
Obstetrics and Gynaecology	52	1.3	.068	.100	.63	.51	.137	.176
Traumatic and Orthopaedic Surgery	8	.05	1.276	1.228	1.94	.001	.398	.520
Otorhinolaryngology	12	1.0	.330	.333	1.03	.001	.412	.382
Paediatrics	22	.71	.331	.409	1.37	1.0	.500	.448
Ophthalmology	4	10	.483	.476	1.82	.001	.794	.733

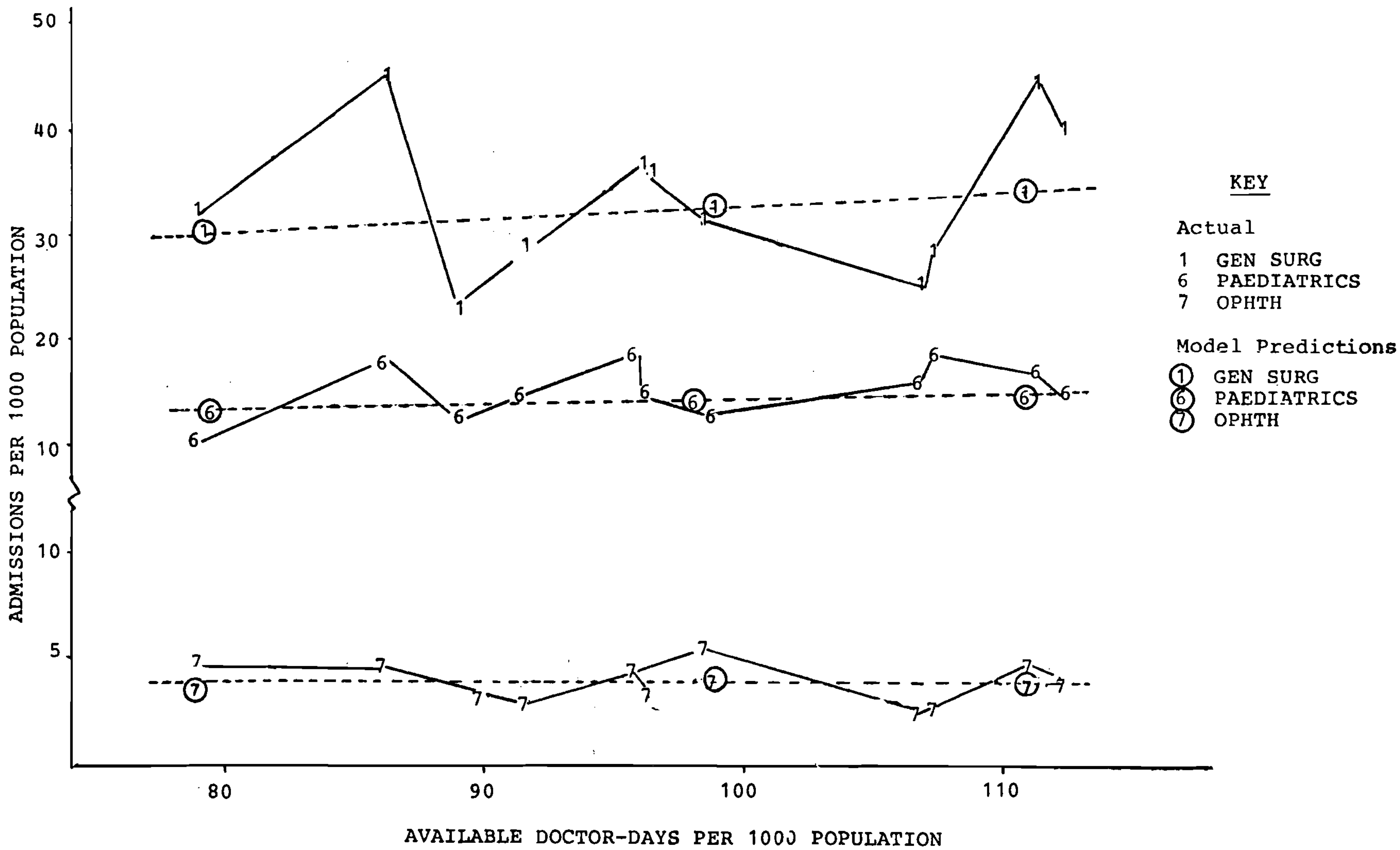

Figure 7. Admission rates - 1976.

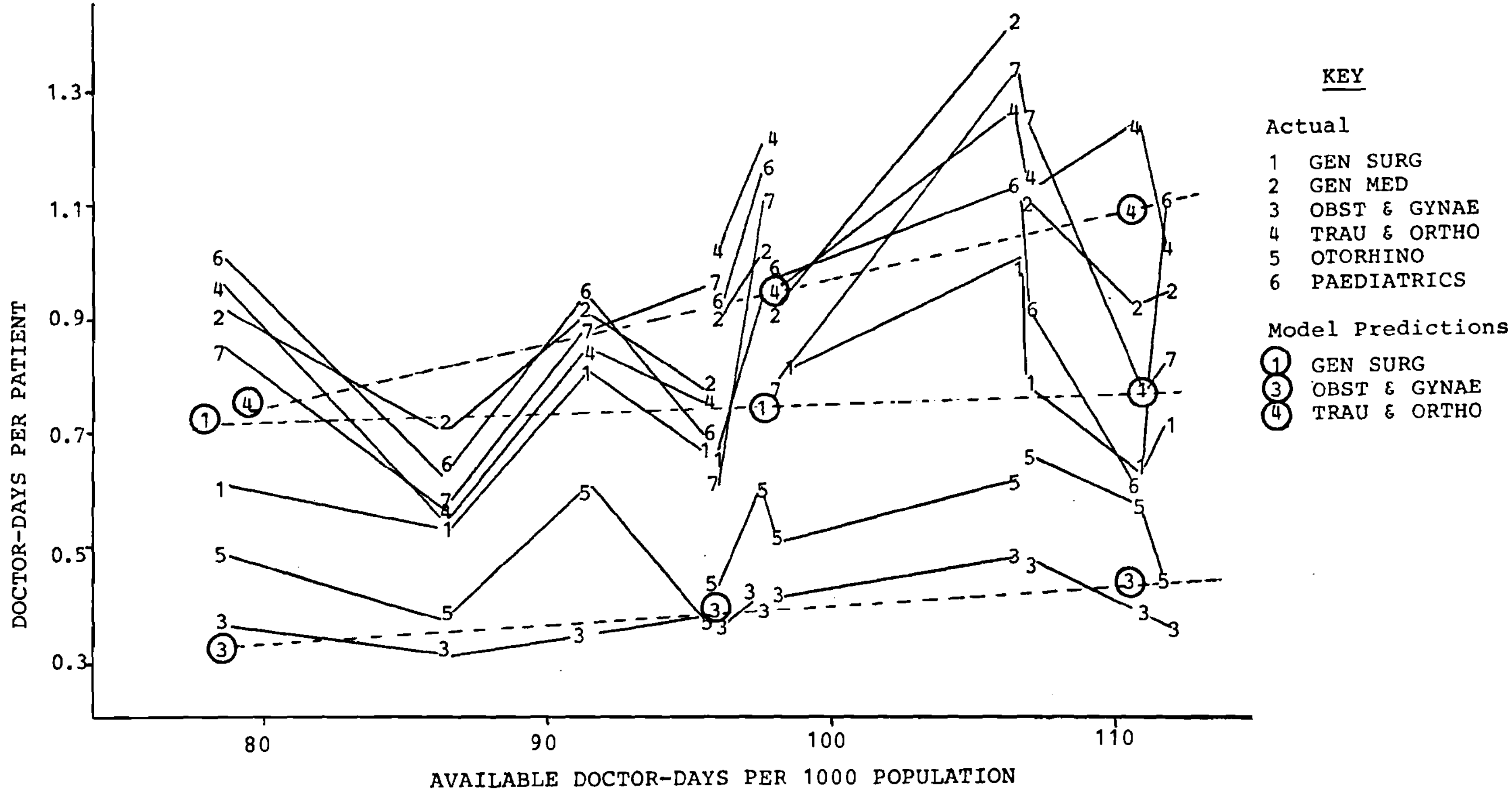

Figure 8. Supply levels (doctors) - 1976.

4.4. Parameter Estimation for DRAM with Two Resources - Hospital Beds and Hospital Doctors

To calculate the parameters for DRAM for two resources, hospital beds and hospital doctors, it is necessary to estimate the relative marginal costs of these resources (C_ℓ) when all needs for health care are met. First of all, it has been assumed that the ratios of the marginals costs for these two resources are the same for current resource levels as for the resource levels needed to satisfy all demands for health care. The marginal costs of hospital beds and hospital doctors were estimated using the following linear regression equation:

Total hospital costs = Constant + (Cost of bed-year) x (No. of bed-years) + (Cost doctor-year) x (No. of hospital doctor years).

Data on the total hospital costs, the number of available beds and the number of available hospital doctor for each of the 12 Czechoslovakian regions for 1976 was taken from the Czechoslovakian Year Book on 1976 Health Statistics (1977). The least squares estimators of the above costs gave the following cost ratio:

a doctor-day = 5 bed-days

(one bed-year = 365 bed-days, one doctor-year = 225 doctor-days)

Using the above ratio, the parameters for the two-resource version of DRAM were estimated. The estimates are given in Table 4. In section 4.2 and 4.3, it was suggested that total bed supply was a much better explanatory variable for admission rates than total doctor supply. Comparison of Tables 2 and 4 shows that the estimates of X_j, α_j and $SS\hat{x}_j$ are very similar. This suggests again that for the admission rates the total bed supply is the more important variable.

All the β_{j1} are large, thus the two-resource model is unable to reproduce the variations in bed-days per patient better than the one-resource model (see Table 2). However, the goodness-of-fit of the two-resource model is much better for

Table 4. Two-resource (hospital beds and hospital doctors) DRAM parameter estimates of Czechoslovakian hospital in-patient care.

Treatment category	Admission rates X_j	α_j	$SS\hat{x}_j$	$SS\bar{x}_j$	Supply levels: bed-days per patient Y_{j1}	β_{j1}	$SS\hat{y}_{j1}$	$SS\bar{y}_{j1}$	Supply levels: doctor-days per patient Y_{j2}	β_{j2}	$SS\hat{y}_{j2}$	$SS\bar{y}_{j2}$
General Surgery	59	.001	.095	.501	17	100	.052	.052	.97	.066	.144	.401
General Medicine	53	.001	.073	.635	20	100	.060	.059	1.32	.040	.078	.421
Obstetrics and Gynaecology	46	3.7	.066	.100	10	100	.048	.047	.49	.36	.039	.176
Traumatic and Orthopaedic Surgery	7	.55	1.318	1.228	25	100	.565	.612	1.29	.001	.261	.520
Otorhinolaryngology	15	.19	.188	.333	12	100	.095	.092	.68	.001	.217	.382
Paediatrics	18	1.9	.393	.409	20	100	.223	.220	1.24	.18	.332	.448
Ophthalmology	7	.001	.219	.476	22	100	.161	.160	1.32	.001	.201	.733

doctor-days per patient than the appropriate one resource model (see Table 3). This is because for any pair of patient categories, the ratio of doctor-days per patient for each of the resource levels, is approximately constant. The model seeks to reproduce this behavior, by making all the β_{j2} approximately equal, hence

$$y_{j2} = Y_{j2}\lambda^{-1/(\beta_2+1)} \text{ from equation 2 when all } \beta_{j2} = \beta_2$$

Thus $\dfrac{y_{p2}}{y_{q2}} = \dfrac{Y_{p2}}{Y_{q2}}$ for any pair of patient categories p and q at a particular resource level, and hence all resource levels;

i.e. $\dfrac{y_{p2}}{y_{q2}}$ is constant for all resource levels.

The doctor-supply model was not able to reproduce this supply per patient behavior because it was simultaneously trying to reproduce the admission rates behavior.

Thus the two-resource model is able to reproduce the variation in admission rates as well as the bed-supply model. With regard to resource supply per patient, the two-resource model is an improvement over the doctor-supply model, for doctor supply per patient, and gives the same results as the bed-supply model for bed supply per patient. Table 5 gives the ratios $SS\hat{x}/SS\bar{x}$ and $SS\hat{y}/SS\bar{y}$ from Table 4, i.e. an indication of how much variation the model has reproduced. The model has reproduced most variation for general surgery and general medicine, and least for traumatic and orthopaedic surgery, and paediatrics. To improve these results, it would be necessary to carry out more detailed analysis. For instance, it would be interesting

(a) To see whether there were any benefits to be gained from disaggregating the patient categories
(b) To consider whether there are resource measures of hospital beds and hospital doctors better than the ones chosen
(c) To consider whether other resources should be introduced

into the model, e.g. nurses, operating theatres, anesthetists diagnostic services

(d) To check that cross regional flows of patients are not introducing a bias

All these more detailed analyses will require access to unpublished statistics and necessitate discussions with health care planners in Czechoslovakia.

Table 5. Two-resource DRAM ratios for $SS\hat{x}_j/SS\bar{x}_j$ and $SS\hat{y}_{j\ell}/SS\bar{y}_{j\ell}$.

Treatment category	$SS\hat{x}_j/SS\bar{x}_j$	$SS\hat{y}_{j1}/SS\bar{y}_{j1}$	$SS\hat{y}_{j2}/SS\bar{y}_{j2}$
General Surgery	.19	1.00	.36
General Medicine	.11	1.02	.19
Obstetrics and Gynaecology	.66	1.02	.22
Traumatic and Orthopaedic Surgery	1.07	.92	.50
Otorhinolaryngology	.56	1.03	.57
Paediatrics	.96	1.01	.74
Ophthalmology	.46	1.01	.27

5. ILLUSTRATIVE EXAMPLES OF THE USE OF THE TWO-RESOURCE DRAM FOR CZECHOSLOVAKIAN IN-PATIENT HOSPITAL CARE

In the previous section, the estimation of the parameters for a model of Czechoslovakian in-patient hospital care was described. In this section, the use of this model will be discussed.

The two-resource DRAM has been parameterized using 1976 data. If it is to be used as a predictive tool for future health care planning, the question arises whether any of the parameter set $\{X,Y,\alpha,\beta\}$ vary with time. For $\{X,Y\}$, this question could be answered by carrying out longitudinal analyses on such variables as admission rates, lengths of in-patient stay, etc.

For instance, in the UK, the length of in-patient stay for many treatment groups has been declining. Such a change could be incorporated into DRAM by reducing the appropriate Y. $\{\alpha,\beta\}$ represent the relative importance of achieving the ideal levels $\{X,Y\}$. Longitudinal studies on such variables are likely to be more difficult to carry out. However, longitudinal studies are beyond the scope of the current paper and for the purposes of illustration, it will be assumed that $\{X,Y,\alpha,\beta\}$ do not change with time.

In 1976, for the seven patient categories used in the model, the resource allocations for the whole of Czechoslovakia were 2119 bed-days per 1000 population and 96 hospital doctor-days per 1000 population. Health care planners in Czechoslovakia can use the model to investigate the consequences of changing this mix of resources. First however, we must demonstrate that the model reproduces quite closely the actual allocation of resources in 1976. Table 6 shows that the actual allocations and the model predictions are quite close.

Suppose the resource mix is changed to 1800 bed-days per 1000 population and 110 doctor-days per 1000 population. Table 6 gives the model predictions for this resource mix. In making this change of resource mix, the model indicates that in general fewer patients will be treated, but the levels of hospital doctor care per patient will rise. Further the model indicates the differential rates of decrease for the admission. For instance, it is estimated that admission rates for general surgery and general medicine will decline by 20% (following from the relatively low α_j's). However, the admission rates for obstetrics and gynaecology will only decline by 5% (following from the relatively high α_j). With regard to the levels of hospital doctor supply per patient, these are expected to rise by about 35% for all patient categories.

In using DRAM to make predictions of future admission rates and supply levels per patient, it is important to consider the accuracy of these predictions. Appendix B considers this problem, suggesting a model for the variance of these estimates.

Table 6. Allocations of health care resources in Czechoslovakia.

	Treatment category	Actual allocation for 1976 R_1=2119 bed-days per 1000 pop. R_2=95.6 doctor-days per 1000 pop.	Model prediction R_1=2119 bed-days per 1000 pop. R_2=95.6 doctor-days per 1000 pop.	Model prediction R_1=1800 bed-days per 1000 pop. R_2=110 doctor-days per 1000 pop.
ADMISSION RATES PER 1000 POP.	Gen. Surg.	33.1	32.1	25.7
	Gen.Med.	30.1	29.0	23.4
	Obst. & Gynae	40.1	40.1	38.2
	T & O Surgery	4.2	4.7	4.0
	Otorhino.	8.6	8.7	7.2
	Paed.	15.2	15.0	13.9
	Opth.	4.1	4.0	3.2
BED-DAYS PER PATIENT	Gen. Surg.	16.6	16.6	16.6
	Gen. Med.	19.7	20.3	20.3
	Obst. & Gynae	9.5	9.8	9.7
	T & O Surgery	24.4	25.3	25.2
	Otorhino.	12.4	12.4	12.3
	Paed.	19.5	19.5	19.4
	Opth.	21.8	21.7	21.7
DOCTOR-DAYS PER PATIENT	Gen. Surg.	0.73	0.74	0.99
	Gen. Med.	0.96	1.00	1.35
	Obst. & Gynae	0.40	0.40	0.50
	T & O Surgery	0.98	0.96	1.32
	Otorhino.	0.53	0.51	0.70
	Paed.	0.94	0.97	1.27
	Opth.	0.90	0.98	1.35

This model gives minimum variance predictions at the average of the supply levels used to parameterize the model, i.e. in this case, the 1976 resource allocations for Czechoslovakia. Table 7 gives the minimum standard deviations.

An alternative way of presenting the results from the model would be to estimate the admission rates and supply levels for a whole range of total resource levels, for example for all combinations of 1800, 2100, 2400, 2700 bed-days per 1000 population and 80, 95, 110 doctor-days per 1000 population. Having done this, one would take each patient category and show how admission rates and supply levels vary with total resource levels. Figure 9 gives a possible way of illustrating the results for general medicine. In this graphical representation the health care planner can see how admission rates and supply levels vary with total resource levels. For example, for general medicine, if the resource mix is changed to 2400 bed-days per 1000 population and 98 doctor-days per 1000 population, then the predictions of the admission rate and the supply of doctor-days per patient is 34 admissions per 1000 population and 0.90 doctor-days per patient. The estimate for bed supply per patient is 20 bed-days per patient.

6. CONCLUSIONS

This paper has extended the DRAM methodology of Gibbs and Hughes. The total methodology now has four parts:

(1) The development of the model and its solution
(2) The choice of suitable patient categories and resource measures
(3) The estimation of model parameters
(4) The use of the model and consideration of prediction errors

The above methodology has been applied to hospital in-patient care in Czechoslovakia. DRAM has been parameterized for seven patient categories, one mode, and two resources. The two

Table 7. Estimates of minimum standard deviations for predictions of admission rates and supply levels per patient using data from Table 4.

Treatment category	Admission rates per 1000 pop. (x_j)	Bed-days per patient (y_{j1})	Doctor-days per patient (y_{j2})
General Surgery	3.8	1.4	.10
General Medicine	3.0	1.8	.10
Obstetrics and Gyanecology	3.8	.8	.03
Traumatic and Orthopaedic Surgery	1.8	6.8	.18
Otorhinolaryngology	1.4	1.4	.09
Paediatrics	3.5	3.4	.20
Opthalmology	0.7	3.2	.15

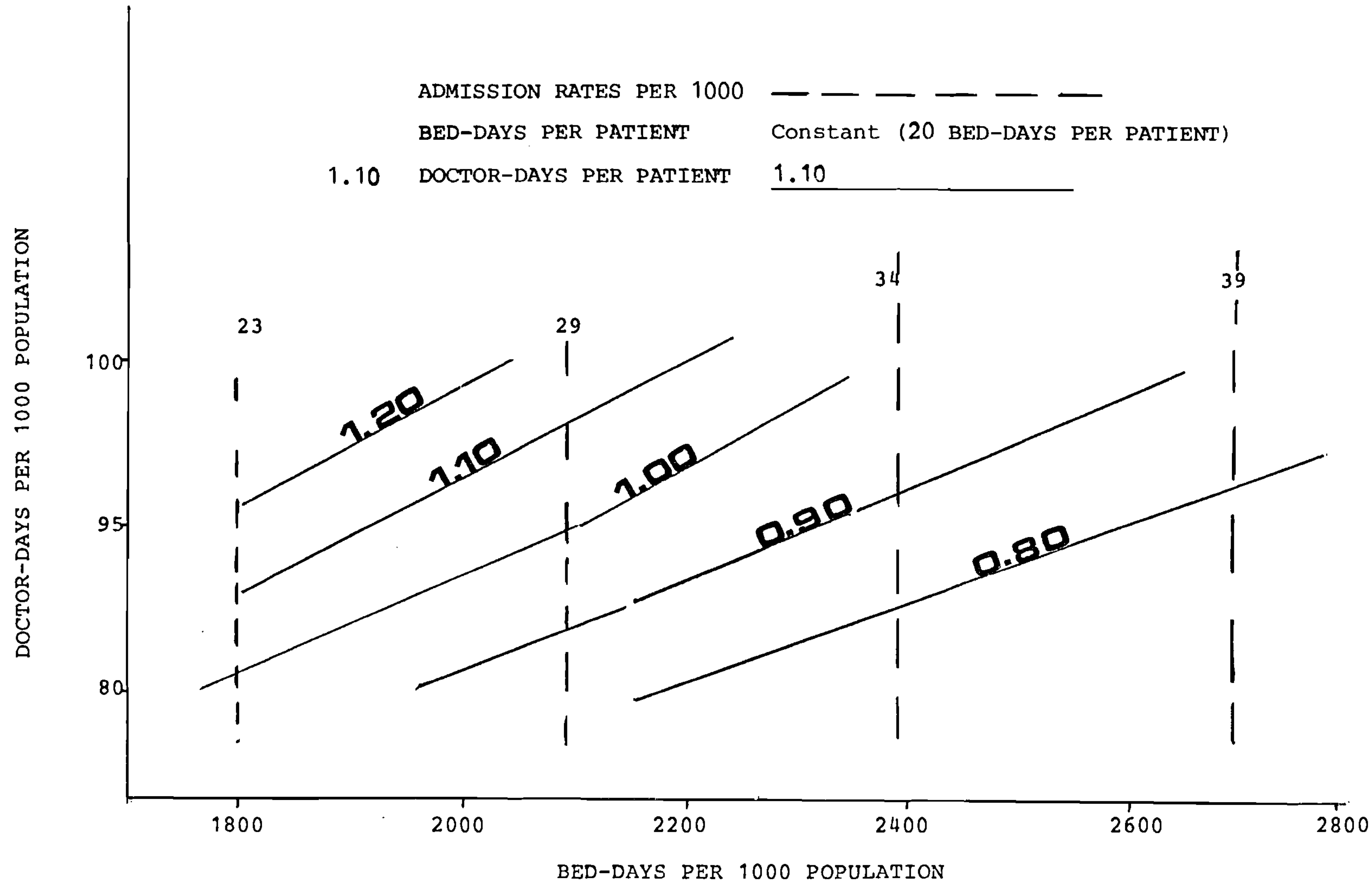

Figure 9. Two-resource (hospital beds and hospital doctors) DRAM model predictions for general medicine in Czechoslovakia.

resources were hospital beds and hospital doctors. The results indicated that the supply of hospital beds was a more important factor in reproducing the behavior of admission rates than the supply of hospital doctors. Both the models including hospital beds as a resource type did not seem to be able to reproduce the variation in bed-days per patient. In addition, the model with hospital doctors as resource type did not seem to be able to reproduce the variation in doctor-days per patient. However, when this resource type was taken in conjunction with the supply of hospital beds, DRAM was able to reproduce much of the variation in doctor-days per patient. Having parameterized DRAM, illustrations were given describing how the two-resource model could be used to help health care planners.

Throughout the parameterization procedure, only data from readily available sources were used. The resulting DRAM is able to reproduce the observed variation better for some treatment categories than for others. In order to produce an improved model, access would be necessary to health care planners and to sources of unpublished data, to carry out more detailed analyses. For instance, it would be interesting to consider whether

- Any improvements could be gained by disaggregating the patient categories
- There are better resource measures of hospital beds and hospital doctors than the ones chosen
- Other resource types should be introduced into the model, e.g. operating theaters, nurses, anesthetists, diagnostic services
- Cross regional flows of patients are not introducing a bias

The above are suggestions for improving the model of hospital in-patient care described in this paper. The authors would like to end the paper with some suggestions for further work in a wider context. The model described in this paper was parameterized using 1976 data. Similar data are available for the years 1977, 1978, and 1979. It would therefore be interesting to see how the DRAM parameters change with time. As mentioned earlier, this information is important when predictions for future years are being made.

Hospital in-patient care is largely concerned with the therapeutic aspect of health care. However, the importance of preventive medicine is increasing in Czechoslovakia. It would therefore be useful to parameterize DRAM for out-patient (policlinic)care, in order to assist health care planners allocate resources for this type of care. The important resource types are thought to be policlinic doctors and technical support staff.

APPENDIX A: PARAMETER ESTIMATION FOR DRAM

1. Introduction

To estimate the DRAM parameters (X,Y,α,β) for Czechoslovakian hospital in-patient care, the approach of Hughes (1978c) was followed. The approach is described in largely qualitative terms. The technical details can be found in Hughes (1978c).

It is assumed the utility function Z (equation 1), is applicable both to the whole of Czechoslovakia and also to each of the individual regions in Czechoslovakia. Thus each region provides an independent data point to estimate (X,Y,α,β). The available data points are split into two approximately equal groups. Initial estimates of (X,Y) are provided, and the (α,β) are estimated using the first data set (details given below). Given these estimates of (α,β), new (X,Y) are then estimated from the second data set (details also given below). Given these new (X,Y) further (α,β) are then estimated using the first data set and so on until successive estimates of (X,Y,α,β) only change by a small amount.

Before discussing these two estimation procedures, it is necessary to introduce additional notation. Following the notation introduced in section 4.1, the N data points are defined as

$x_{jk}(i)$, $y_{jk\ell}(i)$, $R_\ell(i)$ $\qquad$ $i = 1...N$. λ_ℓ is the Lagrange multiplier associated with each resource constraint

$$\sum_j \sum_k x_{jk} y_{jk\ell} = R_\ell$$

2. Estimates of (α,β) Given (X,Y)

To start the estimation process, λ_ℓ must be provided externally for each resource type. The same λ_ℓ is used for all data points. More will be given later about the choice of λ_ℓ.

Hughes (1978c) has indicated that in a certain sense unbiased estimates of (α,β) can be determined by solving iteratively the following set of equations given (X,Y):

$$\ln\left(x_{jk}(i)\right) = a^x_{jk} + \left(\frac{1}{\alpha_j+1}\right) \sum_\ell A_{jk\ell} \ln\left(R_\ell(i)\right) + \varepsilon^x_{jk}(i) \; \forall \; j,k,i$$

$$\ln\left(y_{jk\ell}(i)\right) = a^y_{jk\ell} + \frac{1}{(\beta_{jk\ell}+1)} \sum_m B_{\ell m} \ln\left(R_m(i)\right) + \varepsilon^y_{jk\ell}(i) \; \forall \; j,k,\ell,i.$$

where a^x_{jk} , a^y_{jk} are unknown constants

$A_{jk\ell}$, $B_{\ell m}$ are known functions of α and β, given X,Y and λ

and ε^x_{jk}, $\varepsilon^y_{jk\ell}$ are random uncorrelated error terms with zero means. Within the above iteration process, there is a mechanism to maintain the nonnegativity conditions on (α,β). If at the end of an iteration an α or β is estimated to be negative, then the parameter is set to 0.001 if the prediction error for the parameter is small, otherwise it is reset to some arbitrary level (normally 1 or 5). The estimation of (α,β) is depicted in Figure A1.

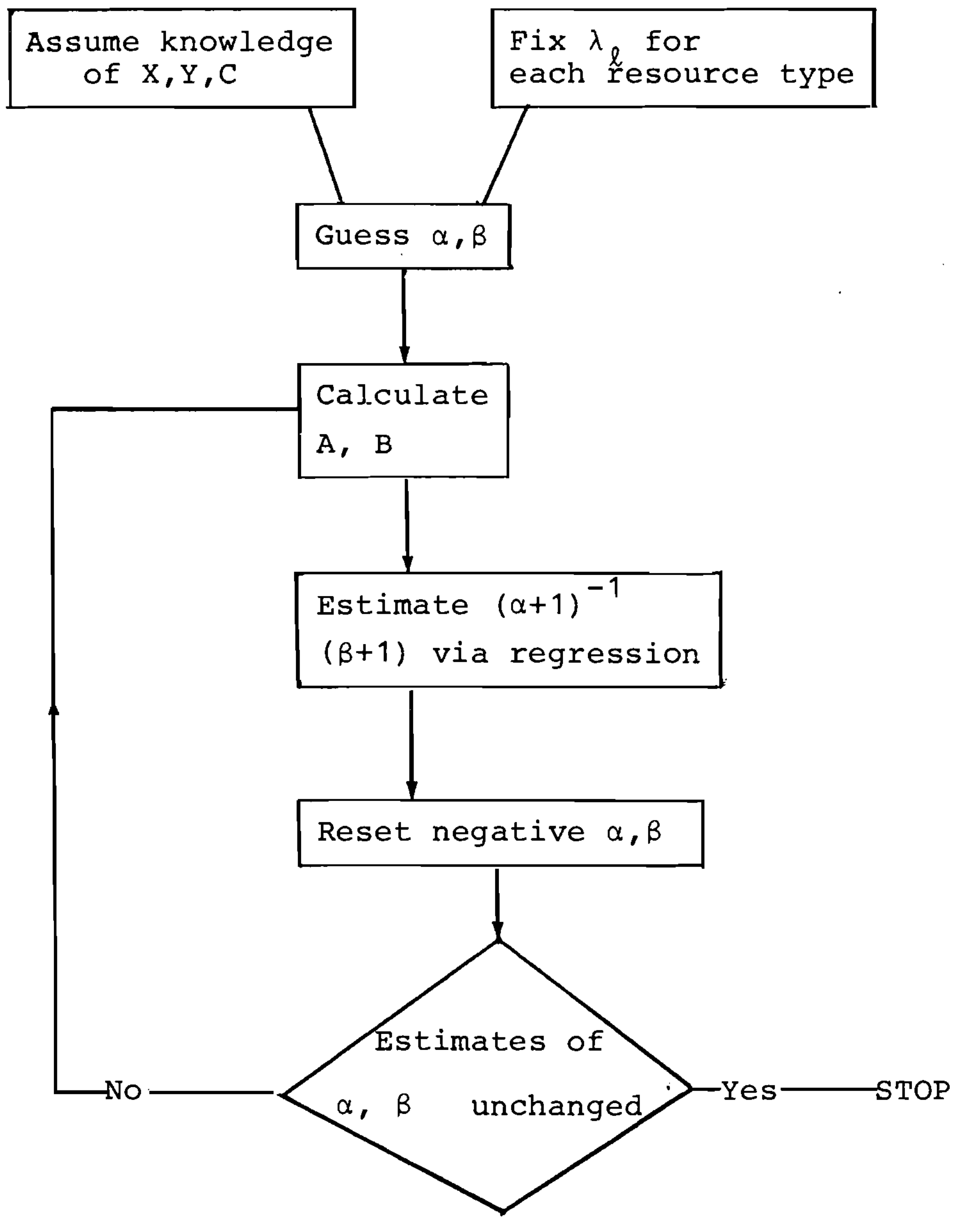

Figure A1. Estimation of $\{\alpha,\beta\}$.

3. Estimates of (X,Y) Given (α,β)

Hughes (1978c) shows that

$$X_{jk} = x_{jk}(\mu_{jk})^{\frac{1}{\alpha_j+1}} \quad \text{from equation 3}$$

$$Y_{jk} = y_{jk\ell}(\lambda_\ell)^{\frac{1}{\beta_{jk\ell}+1}} \quad \text{from equation 2}$$

where μ_{jk} is a function of α,β,Y and λ. Thus given (α,β), (X,Y) can be estimated iteratively if λ_ℓ is known. Hughes shows that if we can specify θ_ℓ, the ratio of type ℓ resources at ideal levels to current usage, i.e.

$$\sum_j \sum_k X_{jk}Y_{jk\ell} = \Theta_\ell \sum_j \sum_k x_{jk}y_{jk\ell} \qquad \forall\ell$$

then λ_ℓ can be determined.

The above is the procedure for the first data point. For the second (and succeeding) data points the value of the ideal resource needs (i.e. $\sum_j \sum_k X_{jk}Y_{jk\ell}$) specified for the first data point is used similarly to determine λ_ℓ for the second (and succeeding) data points. Thus the specification of Θ_ℓ at the first data point is used to fix λ_ℓ for each of N data points. Each data point provides an estimate of (X,Y). A weighted average of the N estimates of (X,Y) is then produced. The estimation of (X,Y) is depicted in Figure A2.

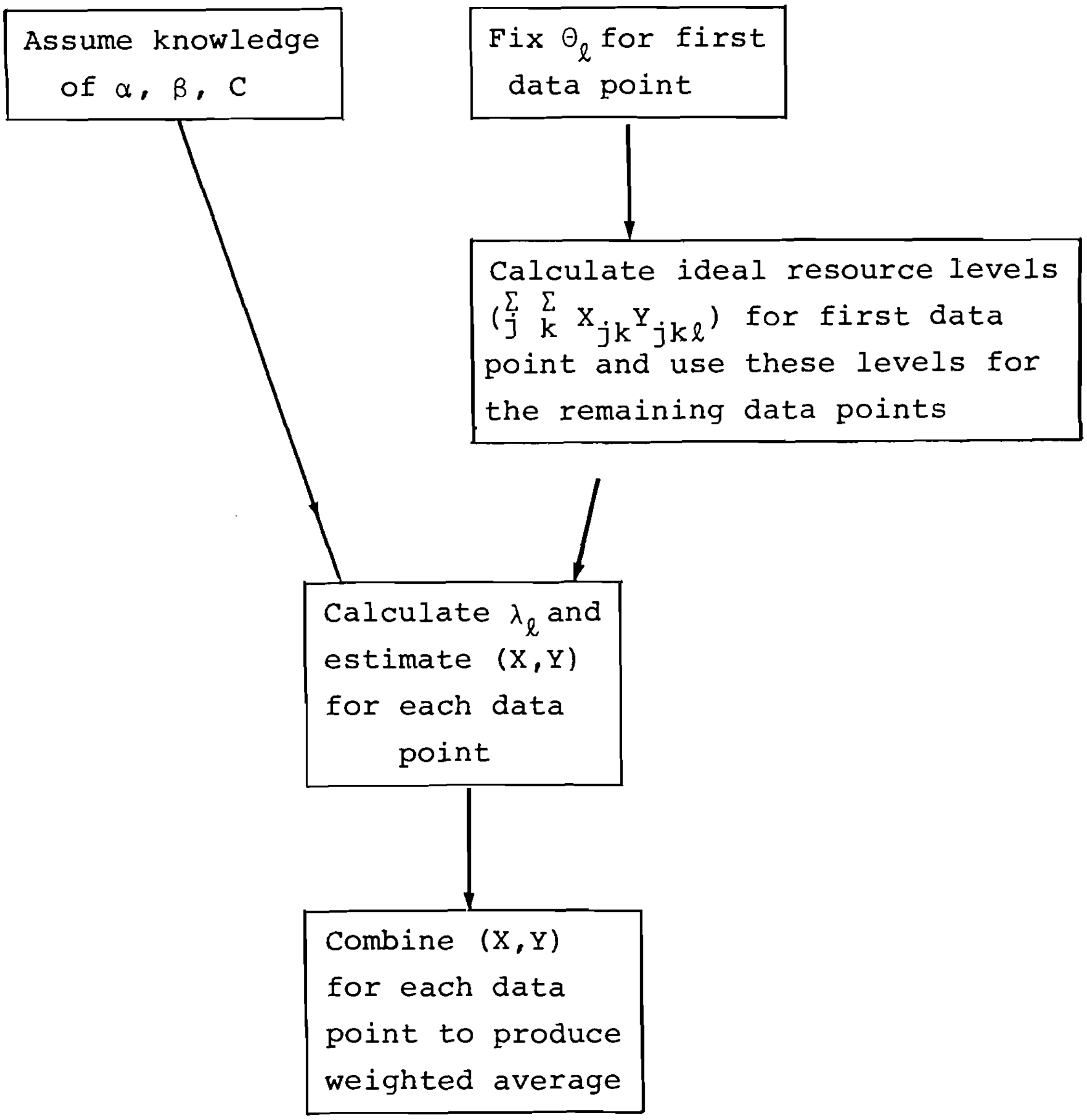

Figure A2. Estimation of ideal levels.

4. The Linkage Between the Estimation Procedures

The two estimation procedures are linked in the following manner.

(1) The estimates of (X,Y) are used as input for the other procedure. This is similar for (α,β).

(2) Both estimation procedures require the input of values for λ_ℓ. These should be consistent in the following sense. Consider parameter estimation when there is one resource type and ten data points (five data points for each procedure). In (X,Y) estimation, setting Θ_1 means that λ_1 is fixed for the five data points, e.g.

	R_1	λ_1
Data point 1	600	1.5
" 2	540	1.8
" 3	520	2.0
" 4	510	2.2
" 5	480	2.6

If data points 6-10 have an average resource level of 535, then λ for the (α,β) estimation should satisfy $1.8<\lambda<2.0$.

Arising from the second of the two linkage mechanisms, is the fact that Θ_ℓ must be provided externally. Θ_ℓ is the ratio of type ℓ resources at ideal levels to current usage at a particular data point. Health care planners should be able to provide an approximate estimate of this ratio. The complete parameter estimation process is given in Figure A3.

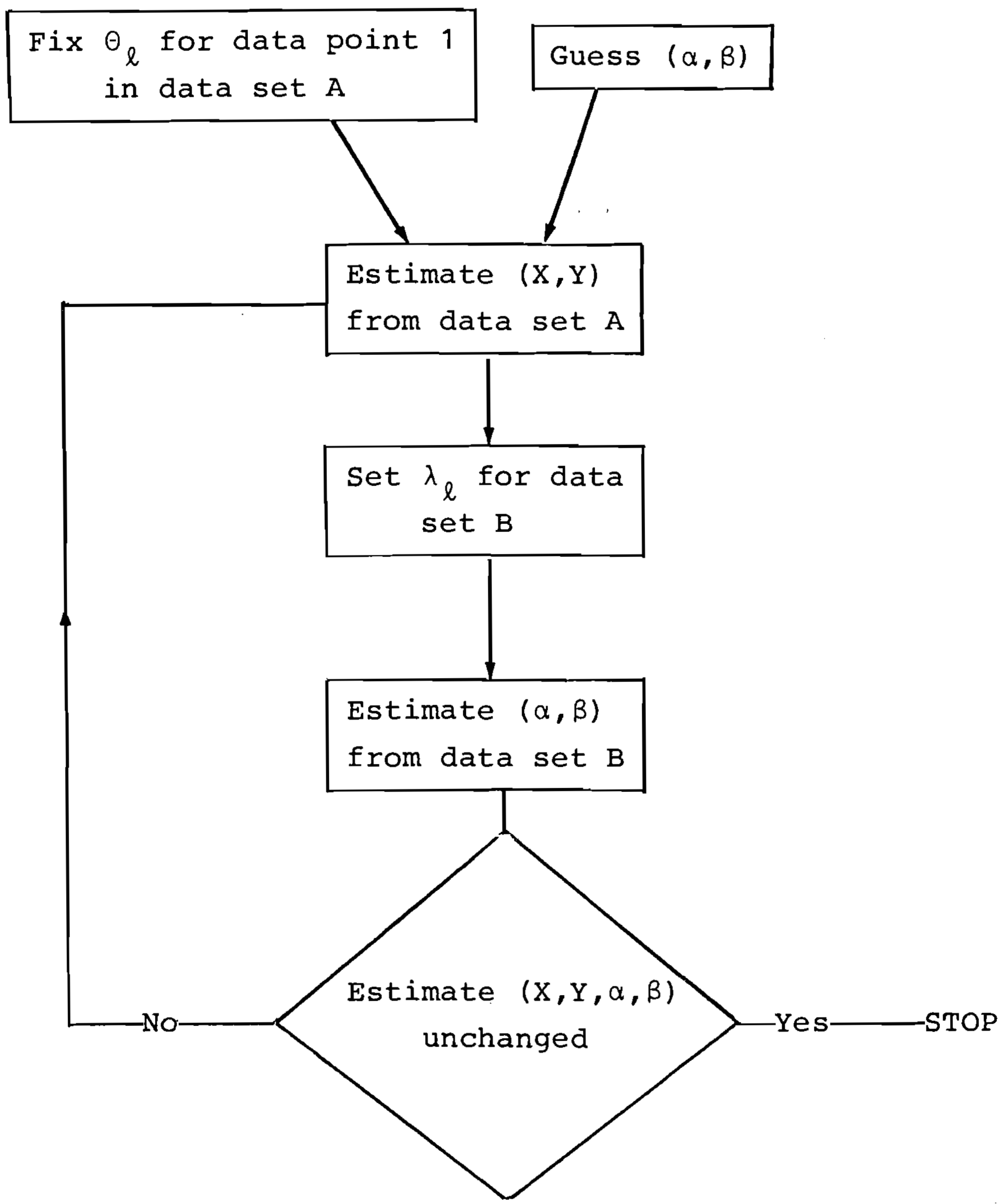

Figure A3. The parameter estimation process.

5. Measure of Goodness-of-Fit

In addition to the parameter estimation mentioned above, it is also useful to have some way of deciding whether successive sets of (X,Y,α,β) are "better". In addition, it is useful to consider whether different values of Θ_ℓ give rise to "better" parameter sets. Lastly, it is interesting to see if certain parameters from the set (X,Y,α,β) are fixed exogeneously, whether the estimation procedure produces "improved" parameter sets.

The following measure of goodness-of-fit has been used to compare parameter sets:

$$SS = \sum_j \sum_i \left(\frac{x_j(i) - \hat{x}_j(i)}{w_j} \right)^2 + \sum_j \sum_\ell \sum_i \left(\frac{y_{j\ell}(i) - \hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

where

(1) $x_j(i)$, $y_{j\ell}(i)$ $(i=1\ldots N)$ are the actual data points and

$\hat{x}_j(i)$, $\hat{y}_{j\ell}(i)$ $(i=1\ldots N)$ are the predicted levels from DRAM given a particular parameter set and resource availabilities at data point i are

$R_\ell(i) = \sum_j x_j(i) y_{j\ell}(i)$;

(2) w_j and $v_{j\ell}$ are scaling factors, set as follows -

w_j is an average (possibly weighted) of $x_j(i)$, $i=1\ldots N$

$v_{j\ell}$ is an average (possibly weighted) of $y_{j\ell}(i)$, $i=1\ldots N$;

(3) the modal subscript has been omitted.

In practice it is useful to split this measure into the following sections:

$$SS\hat{x}_j = \sum_i \left(\frac{x_j(i) - \hat{x}_j(i)}{w_j} \right)^2$$

$$SS\hat{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - \hat{y}_{j\ell}(i)}{v_{j\ell}} \right)^2$$

Thus

$$SS = \sum_j SS\hat{x}_j + \sum_j \sum_\ell SS\hat{y}_{j\ell}$$

6. Computational Procedure

Experience has shown that the parameter estimation procedure given in Figure A3 converges about half the time within 6 to 9 iterations. Convergence is assumed when the change in parameter estimates is about 4%.

If there is no sign of convergence after seven iterations, the process should be stopped. Frequently, in such cases parameter estimates are oscillating. Often this arises when the actual admission rates (or resource supply levels per patient) exhibit great variation independent of total resource availability.

Whether the estimation procedure converges or not the function SS should be calculated and

$$SS\hat{x}_j \text{ compared with } SS\bar{x}_j = \sum_i \left(\frac{x_j(i) - w_j}{w_j} \right)^2 \qquad \forall_j$$

$$SS\hat{y}_{j\ell} \text{ compared with } SS\bar{y}_{j\ell} = \sum_i \left(\frac{y_{j\ell}(i) - v_{j\ell}}{v_{j\ell}} \right)^2 \qquad \forall_{j\ell}$$

If $SS\hat{X}_p > SS\bar{X}_p$ then $\hat{w}_p$ is a better predictor of the actual results than $x_p(i)$. In a one-resource model, this normally arises when $x_p(i)$ is independent of total resource supply. In such circumstances a better model fit (i.e. smaller SS) is normally achieved if X_p is fixed at w_p and α_p is set to a large number in the parameter estimation process.

A similar approach should be adopted if

$$SS\hat{y}_{j\ell} > SS\bar{y}_{j\ell} \quad \text{for a particular } j\ell.$$

As a result of the above comparison there are four options.

(1) Parameter estimation procedure converged and no (X,Y,α,β) fixed. The (X,Y,α,β) should be regarded as the best estimates the method can produce.

(2) Parameter estimation procedure converged and some (X,Y,α,β) fixed. The parameter estimation procedure given in Figure A3 should be run again. Convergence should occur again and after calculating SS, no further (X,Y,α,β) should be fixed. The second set of (X,Y,α,β)

should be regarded as the best estimate the method can produce.

(3) Parameter estimation procedure did not converge and no (X,Y,α,β) fixed. This seems an unlikely event. In such cases perhaps the data points should be reallocated to the two groups, and the parameter estimation process started again.

(4) Parameter estimation procedure did not converge and some (X,Y,α,β) fixed. The parameter estimation procedure given in Figure A3 should be run again and SS calculated. Further $SS\hat{x}_j$ and $SS\hat{y}_{j\ell}$ comparisons should be carried out and more (X,Y,α,β) fixed if necessary, and so on.

Normally a maximum of two runs of the procedure given in Figure A3, should produce usuable (X,Y,α,β).

7. Fixing the Value of Θ_ℓ

Using the Czechoslovakian data, DRAM parameters were estimated for two one-resource models. The resources were hospital beds and hospital doctors. In the first instance, Θ_1 was fixed so that the actual resource levels were 52-77% of ideal levels. In the second instance, Θ_1 was fixed so that actual resources were 40-58% of the ideal levels. In each case using the estimated parameter set, DRAM produced estimates of admission rates $(\hat{x}_j)$ and supply levels per patient $(\hat{y}_{j\ell})$ which were approximately linearly related to total resource supply for the range of resource availabilities from which the parameters were estimated. To see whether these choices of Θ_1 lead to error, consider Figure A4. This figure shows how two admission rates x_1, x_2 (or supply levels per patient) relate to total available resources under the DRAM hypothesis. For resource levels in range A, x_1 and x_2 are approximately linear, whereas in range B this is not so. A linear model fitted to x_1 in range B will exhibit bias as shown.

Thus to check whether Θ_1 leads to error in the two above cases, it is necessary to examine the error terms $[x_j(i)-\hat{x}_j(i), y_{j\ell}(i)-\hat{y}_{j\ell}(i)]$ to see if the sign is consistantly $-ve_j$ then +ve and finally -ve, as the total available resources increases.

This was done, and there was no evidence that the sign of the errors related to the total available resources in this way. Thus the choices of θ_1 were considered reasonable.

Admission Rate
(Supply levels
per patient)

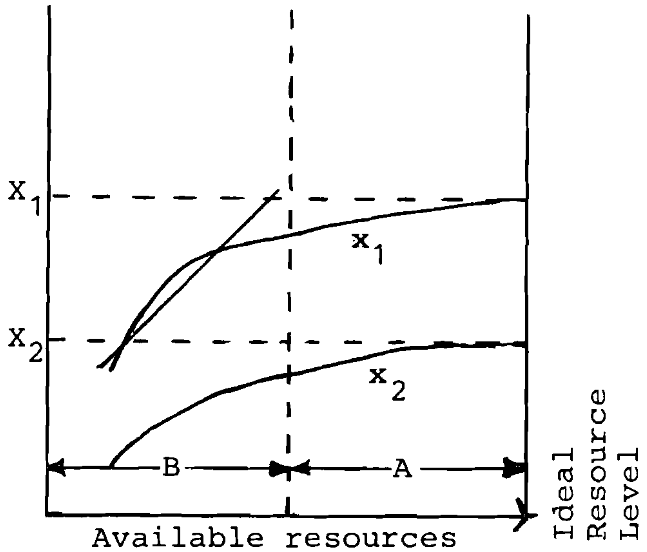

Figure A4. Possible variations in admission rates under the DRAM hypothesis.

The aim of the work is to produce DRAM for Czechoslovakian in-patient care. The resource levels per head of total population for the whole of Czechoslovakia will lie approximately in the middle of the range of resource levels per head of population for the individual regions. Provided that predictive runs of the Czechoslovakian DRAM do not involve total resources very different from those used in the estimation process, then the above analysis has indicated that the θ_1 chosen are satisfactory. For DRAM runs outside this range of resources, more precise estimates of θ_1 are probably needed.

Computational experience in varying θ_ℓ indicates that as θ_ℓ increases so do the estimates of (X,Y). Uncertainty about θ_ℓ implies, therefore, uncertainty about the estimates of (X,Y). Hence, interpreting the estimates of (X,Y) as predictions of "ideal levels" of care must be done with some caution.

APPENDIX B: PREDICTION ERRORS FOR x_{jk} and $y_{jk\ell}$ ESTIMATED BY DRAM

Suppose the DRAM parameters (X,Y,α,β) have been estimated from N data points, $x_{jk}(i)$, $y_{jk\ell}(i)$ $i=1...N$. If DRAM with this parameter set is now used to estimate x_{jk} and $y_{jk\ell}$ for given levels of resource R_ℓ, what confidence can be placed in these estimates? Can we estimate the variance of the difference between the prediction and an observed value? DRAM is a non-linear model and to produce an analytically exact solution to these problems would be very difficult. Instead, a simplified approach has been adopted. The following model will be assumed for x_{jk} (and similarly for $y_{jk\ell}$)

$$x_{jk} = F(X,Y,\alpha,\beta,R) + e_{jk}$$

where (a) $F(X,Y,\alpha,\beta,R)$ is the value of x_{jk} when DRAM is run with parameter set (X,Y,α,β) and resource levels R. Thus in the notation introduced in section 4.1

$$\hat{x}_{jk}(i) = F(X,Y,\alpha,\beta,R(i)) \text{ for data point i.}$$

(b) e_{jk} are independent random variables with mean = 0, and variance σ^2_{jk}.

Using this model, the variance of x_{jk} has contributions from σ^2_{jk} and from the inaccuracies in estimating the parameter set (X,Y,α,β). It will be assumed that these contributions are independent, and hence additive. Each of these two contributions will be considered in turn.

(1) σ^2_{jk} can be estimated from $\sum_i \left(\hat{x}_{jk}(i) - x_{jk}(i)\right)^2$, but what divisor should be used? Suppose there are J patient categories, K treatment modes, and L types of resources. DRAM predicts JK x_{jk}'s and JKL $y_{jk\ell}$'s - in total JK(L+1) predictions. DRAM requires J(1+K+2KL) parameters, i.e. J(1+K+2KL) degrees of freedom can be considered lost. A further NL degrees of freedom are lost because there are L resource constraints at each data point. Thus the number of degrees of freedom considered lost per prediction is

$$\frac{J(1+K+2KL)+NL}{JK(L+1)}.$$ When J=7, K=1, L=2, N=10, this ratio

is approx 3. Thus 3 degrees of freedom can be considered lost from $\sum_i \left(\hat{x}_{jk}(i) - x_{jk}(i)\right)^2$ and hence σ^2_{jk} could be estimated by $\sum_i \frac{\left(\hat{x}_{jk}(i) - x_{jk}(i)\right)^2}{N-3}$.

(2) In order to take into account inaccuracies in estimating (X,Y,α,β) it would be useful to know the variance and covariance matrix of the estimates of this parameter set. The parameter estimation process described in Appendix A, does not give this. In any case because DRAM is non-linear, it would be analytically difficult to use this matrix to estimate the errors. A more simplified approach has therefore been adopted. To a first order approximation $F(X,Y,\alpha,\beta,R)$ can be thought of as a linear function in R_ℓ. Figures 5-8 suggests this is reasonable. Thus for two resources,

$$x_{jk}(i) = a_{jk}+b_{jk}\left(R_1(i)-\bar{R}_1\right)+c_{jk}\left(R_2(i)-\bar{R}_2\right)+e_{jk}$$

where $\bar{R}_1 = \frac{1}{N} \sum_i R_1(i)$

$R_2 = \frac{1}{N} \sum_i R_2(i)$ and a,b, and c are coefficients to be estimated.

Using least square regression techniques to estimate a_{jk}, b_{jk} and c_{jk} it can be shown [see Draper and Smith (1966)] for resource levels R_1 and R_2, that

$$\text{Var}\ (x_{jk}) = \sigma^2_{jk} \left[1 + \frac{1}{N} + \frac{(R_1 - \bar{R}_1)^2}{A} \sum_i (R_2(i) - \bar{R}_2)^2 \right.$$

$$\left. + \frac{(R_2 - \bar{R}_2)^2}{A} \sum_i (R_1(i) - \bar{R}_1)^2 - \frac{2(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)}{A} \sum_i (R_1(i) - \bar{R}_1)(R_2(i) - R_2) \right]$$

where

$$A = \sum_i (R_1(i) - \bar{R}_1)^2 \sum_i (R_2(i) - \bar{R}_2)^2 - \left[\sum_i (R_1(i) - \bar{R}_1)(R_2(i) - \bar{R}_2) \right]^2 .$$

For the 11 data points used to parameterize the model for Czechoslovakian hospital in-patient care,

$$\text{Var}\ (x_{jk}) = \sigma^2_{jk} \left[1 + \frac{1}{11} + \frac{(R_1 - R_1)^2}{935135} + \frac{(R_2 - R_2)^2}{1153} - \frac{2(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)}{169,459} \right]$$

The minimum variance at $R_1 = \bar{R}_1$ and $R_2 = \bar{R}_2$

is $1.091\ \sigma^2_{jk}$.

At $R_1 = 2600$ $R_2 = 80$

$$\text{Var}\ (x_{jk}) = \sigma^2_{jk}(1 + 0.091 + 0.169 + 0.251 + 0.080)$$

$$= 1.591\ \sigma^2_{jk}$$

In conclusion the above analysis has indicated a possible way to estimate the variance of the DRAM estimates of x_{jk} (and $y_{jk\ell}$). Further analysis may prove useful.

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