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Highlights

- We develop a scenario model to support the selection of strategic actions
- The model admits incomplete and action-dependent scenario probability information
- Decision recommendations are based on dominance relations between action portfolios
- Non-dominated portfolios can be used to derive action-specific recommendations
- The model is applied to a real case for building a strategy for a platform ecosystem

Scenario-based portfolio model for building robust and proactive strategies

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Abstract

In order to address major changes in the operational environment, companies can (i) define scenarios that characterize different alternatives for this environment, (ii) assign probabilities to these scenarios, (iii) evaluate the performance of strategic actions across the scenarios, and (iv) choose those actions that are expected to perform best. In this paper, we develop a portfolio model to support the selection of such strategic actions when the information about scenario probabilities is possibly incomplete and may depend on the selected actions. This model helps build a strategy that is *robust* in that it performs relatively well in view of all available probability information, and *proactive* in that it can help steer the future as reflected by the scenarios toward the desired direction. We also report a case study in which the model helped a group of Nordic, globally operating steel and engineering companies build a platform ecosystem strategy that accounts for uncertainties related to markets, politics, and technological development.

Keywords: decision support systems, portfolio selection, scenarios, incomplete probabilities

1. Introduction

To retain their competitive edge, organizations must be able to respond to major changes in their operational environment. By making high-quality strategic decisions, these organizations can mitigate threats and to seize opportunities offered in their changing environment. Traditional strategic planning approaches build on forecasts based on trend extrapolation. Such approaches are, however, inadequate in highly uncertain, intensive and complex environments (Bunn and Salo, 1993; Chermack *et al.*, 2001; Varum and Melo, 2010). Consequently, strategic planning in organizations has increasingly been complemented and even replaced by *scenario planning*, which, instead of focusing on the future that is perceived as the most likely, considers a set of plausible futures, called scenarios (Schoemaker, 1995; Peterson *et al.*, 2003). Specifically, scenarios draw the decision-makers' (DMs') attention to uncertainties and help them build a *robust* strategy that performs relatively well across different operational environments (Wilson, 2000; Lempert *et al.*, 2006; Lindsay, 2015).

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It is often useful to think of strategy not in a holistic sense, but rather as a combination or *portfolio* of (possibly interdependent) courses of action, such as investments in a given technology, project, or business model (Courtney *et al.*, 1997; Luehrman, 1998; Beinhocker, 1999; Raynor and Leroux, 2004; Lin *et al.*, 2005). The portfolio approach enables the development of a large number of alternative strategies with reasonable effort, but also facilitates strategy implementation. Decisions about which action portfolio (i.e., strategy) to select can be supported by methods of Portfolio Decision Analysis (PDA; see Salo *et al.*, 2011 for an overview). In particular, these methods deploy decision-analytic models to capture preferences and uncertainties about the actions' impacts, and use mathematical optimization to identify the most preferred portfolio under resource and other constraints (see, e.g., Jackson *et al.*, 1999; Argyris *et al.*, 2014; Vilkkumaa *et al.*, 2014a; Fasth *et al.*, 2016; Fliedner and Liesiö, 2016).

One approach to scenario-based portfolio selection is to assess the probability of the scenarios, to evaluate the impacts of the actions in each scenario, and, finally, to select the action portfolio with the highest expected utility in light of the available information (e.g., Poland, 1999). This approach, however, entails some challenges. First, it may be difficult to estimate precisely how probable the different scenarios are – for instance, to say that the probability that a given smartphone operating system becomes the industry standard is precisely 40% (Liesiö and Salo, 2012). Second, the actions may affect the scenario probabilities: for example, the probability that a given operating system becomes the industry standard may be increased by large investments made by a major smartphone manufacturer (Hagel III *et al.*, 2008; Toppila *et al.*, 2011). Failing to account for the impacts of such *proactive* actions may lead to poor strategic decisions (Reeves *et al.*, 2012).

In this paper, we develop a scenario model to support the selection of portfolios consisting of strategic actions when (i) information about the scenario probabilities may be incomplete and (ii) the selection of some actions can affect these probabilities. Information about scenario probabilities is modeled by bounding the set of feasible probabilities through constraints that may depend on which actions are selected. Dominance relations are employed to identify those action portfolios that are not outperformed by any other portfolio for any feasible scenario probabilities. To compute these non-dominated portfolios, we develop an efficient computational algorithm that avoids the need to enumerate all feasible portfolios. In this algorithm, (i) the set of feasible portfolios is partitioned with respect to those actions that affect the scenario probabilities, and (ii) a multi-objective zero-one linear programming (MOZOLP) problem is solved within the partitioned sets to identify those non-dominated portfolios that satisfy the resource and other feasibility constraints.

To our knowledge, we present the first decision-analytic portfolio model which accommodates incomplete and action-dependent scenario probability information. In particular, the model provides recommendations for choosing action portfolios that are (i) robust across the range of future scenarios in view of incomplete information about scenario probabilities, and (ii) proactive in that they help steer the course of change by influencing these probabilities. The resulting decision recommendations help prioritize actions by dividing them into three categories: (i) core actions that should be selected (included in all non-dominated portfolios), (ii) exterior actions that should not be selected (not included in any non-dominated portfolios), and (iii) borderline actions (included in some non-dominated portfolios but not all).

We also report a real case study in which this modeling approach was used for building a strategy for a group of Nordic, globally operating steel and engineering companies looking to establish a multi-sided platform ecosystem. The participating companies sought to develop a strategy that would be robust across three alternative scenarios of the future operational environment. Yet, because the ecosystem would be one of the pioneers on the market, its strategy was seen to potentially influence which of the future scenarios would be realized. Our model supported the strategy process by helping to identify those actions that the ecosystem should definitely pursue (core actions) as well as actions in which the ecosystem should make smaller, initial investments for possible later expansion (borderline actions).

The rest of the paper is structured as follows. Section 2 discusses earlier literature on scenario-based strategy building. Section 3 introduces our modeling framework, and Section 4 discusses computational issues. The case study is presented in Section 5, and Section 6 concludes.

2. Earlier approaches to scenario-based strategy development

Scenario planning emerged in the aftermath of World War II as a method for military planning. Later, it was extended to support social forecasting, public policy, and strategic management (Bunn and Salo, 1993; Van der Heijden, 1996; Varum and Melo, 2010). The early scenario planning methodologies can be divided into three schools. First, there is the *intuitive logics* school comprising of qualitative methods for developing scenarios and evaluating strategies against these scenarios (Bunn and Salo, 1993; Bradfield *et al.*, 2005). Second, the *probabilistic modified trends* school generates scenarios by asking experts to provide subjective probability estimates about the occurrence of unprecedented events. Trend-impact analysis (TIA) uses these probabilities and the expected impacts of the events to perturb trends extrapolated from historical data. Cross-impact analysis (CIA) incorporates an additional layer of complexity in that it is also necessary to elicit probabilities for events conditioned on the occurrence or non-occurrence of some other events (Godet, 1987; Bradfield *et al.*, 2005). The third school, *La Prospective* (Godet, 2000), can be viewed as an elaborate, complex and somewhat mechanistic blending of the intuitive logics and the probabilistic modified trend methodologies (Bradfield *et al.*, 2005).

These early methodologies have been criticized for not providing sufficient support for the evaluation of different strategies across the full range of scenarios (Wilson, 2000; Goodwin and Wright, 2001). The comparison of strategies without the help of formal methods is particularly difficult if the DM has multiple objectives (Kahneman and Tversky, 1982; Goodwin and Wright, 2001). In consequence, several approaches have been developed to integrate scenarios within a Multi-Criteria Decision Analysis (MCDA) framework (Stewart, 1997, 2005; Wright and Goodwin, 1999; Belton and Stewart, 2002; Montibeller *et al.*, 2006; Stewart *et al.*, 2013). These approaches help make trade-offs between possibly conflicting objectives and make it possible to compare strategies across all scenarios – for instance, based on the total multi-attribute value (e.g., Goodwin and Wright, 2001; Karvetski and Lambert, 2012), or the *regret* of each strategy in every scenario; here, regret is defined as the difference between the value of the strategy in the given scenario and that of the best-performing strategy in the same scenario (Lempert *et al.*, 2006; Ram *et al.*, 2011).

Many approaches for strategic prioritization use probabilities to describe the relative likelihoods of different scenarios (Kirkwood and Pollock, 1982; De Kluyver and Moskowitz, 1984; Brauers and Weber, 1988; Godet, 2000; Millett, 2003, 2009). Yet, some authors have argued against the use of scenario probabilities, for instance because of the psychological biases associated with subjective probability estimation, including overconfidence (Tversky and Kahneman, 1974; Goodwin and Wright, 2001). The use of probabilities has also been criticized for filtering out important information about vulnerabilities and opportunities, and for forcing stakeholder consensus (Karvetski and Lambert, 2012); moreover, probability estimation has been seen as tantamount to forecasting (Mobasheri *et al.*, 1989). Bunn and Salo (1993), however, point out that if scenario analysis is to support strategic choices, then some judgement about the relative likelihood of scenarios is implicit even in those methods that deliberately attempt to avoid assessing these likelihoods. Unless these judgements are made explicit, both scenario generating teams and executives have been noted to gravitate toward those scenarios that they find most attractive, thereby running the risk of dismissing scenarios that are plausible but unattractive (Millett, 2009). It may therefore be beneficial to make this important aspect explicit by introducing information about scenario probabilities in a flexible way.

In this paper, we assume that the scenarios are exhaustive and mutually exclusive descriptions of possible futures, so that beliefs about their relative likelihoods can be expressed through probabilities. Thus, it is important to highlight that the decision recommendations resulting from this model are to be interpreted subject to the condition that exactly one of the scenarios will in fact be realized in the future. Nevertheless, our model can be used to generate decision recommendations even if the DMs are not willing or able to express any statements about scenario probabilities (Goodwin and Wright, 2001; Montibeller *et al.*, 2006; Ram *et al.*, 2011; Stewart *et al.*, 2013). In this setting, the set of recommended action portfolios consists of Pareto optimal portfolios, i.e., those that are not outperformed by any other portfolio in *each* scenario. Yet, if the resulting decision recommendations are not conclusive enough for making strategic choices, our model provides a justifiable way to accommodate as much information about scenario probabilities as can be elicited with reasonable effort.

3. Model framework for action portfolio selection

3.1. Portfolio selection with complete probability information

Consider a DM who wants to select a portfolio consisting of a subset of m proposed actions. The impacts of these actions are evaluated in n mutually exclusive and collectively exhaustive scenarios. The probabilities of these scenarios are denoted by vector $p = [p_1, \ldots, p_n]$, where p_i is the probability of scenario s_i. By definition, p is in the n-dimensional simplex $\Delta^n = \{p \in [0,1]^n | \sum_{i=1}^n p_i = 1\}$. The real-valued outcome of action j in scenario i, denoted by $x_i^j \in \mathbb{R}$, can represent, for instance, the net present cash flow of the action in scenario i, or the cardinal multi-attribute value of the action, as derived through conventional MAVT analysis (see, e.g., Dyer and Sarin, 1979).

An action portfolio is a subset of the m available actions, represented by a binary row vector $z = [z_1, \ldots, z_m] \in \{0,1\}^m$ where $z_j = 1$ if and only if action j is included in the portfolio. Given scenario probabilities p, the expected utility of portfolio z is defined as

$$\mathrm{EU}(z,p) = \sum_{i=1}^{n} p_i u_i(\sum_{j=1}^{m} z_j x_i^j), \tag{1}$$

where $u_1, \ldots, u_n$ denote the scenario-specific utility functions which are only assumed to be strictly increasing. In particular, we do not assume that the utility functions are either convex or concave, and thereby make no assumptions about the DM's risk attitude. These utility functions can be used to model, for instance, non-constant marginal portfolio value which can be different in each scenario. The elicitation of utility functions is discussed in Section 3.7.

Portfolios are usually selected subject to limited resources and/or other restrictions. We assume that the set of feasible portfolios $\mathcal{Z}_F \subseteq \{0,1\}^m$ which satisfies these restrictions is defined through q linear feasibility constraints, whose coefficients are contained in matrix $A \in \mathbb{R}^{q\times m}$ and vector $b \in \mathbb{R}^{q\times 1}$ so that

$$\mathcal{Z}_F = \{z \in \{0,1\}^m |\ Az^T \le b\}. \tag{2}$$

This formulation is relatively general as many common constraint types can be modeled as linear inequalities (see Stummer and Heidenberger, 2003; Liesiö *et al.*, 2008; Mavrotas *et al.*, 2008). For instance, a constraint which states that action j can only be selected if action ℓ is selected can be modeled with inequality $z_j \le z_\ell$. Also, a balance constraint which requires that at least m_0 actions of the subset $M_0 \subseteq \{1,\dots,m\}$ are selected can be modeled as $\sum_{j\in M_0} z_j \ge m_0$. If there are synergies (or cannibalization effects) among the actions, the overall value of a set of actions differs from the sum of the individual actions' impacts. These effects can be captured through linear feasibility constraints by introducing dummy actions. For instance, assume that synergy $x_i^\circ > 0$ (or cannibalization effect $x_i^\circ < 0$) occurs in scenario i if at least m_0 actions from the subset M_0 are selected. This synergy effect can be modeled by introducing dummy action $z_\circ$ to the problem with scenario-specific impacts $x_1^\circ,\dots,x_n^\circ$ and linear constraints $\sum_{j\in M_0} z_j - m_0 + 1 \le m \cdot z_\circ \le \sum_{j\in M_0} z_j - m_0 + m$ which ensure that the synergy is realized (i.e., $z_\circ = 1$) if and only if $\sum_{j\in M_0} z_j \ge m_0$. Thus, even though synergies and cannibalization effects are non-additive, they can be modeled by introducing additional dummy binary variables and linear constraints such that the functional form of expected portfolio utility (1) remains the same. Synergies in resource consumption can be modeled in a similar fashion (Liesiö *et al.*, 2008).

A rational DM would seek to maximize the expected utility of the selected portfolio. If scenario probabilities p are known, the feasible portfolio that maximizes this expected utility can be obtained by solving the non-linear zero-one programming problem

$$\max_{z\in\mathcal{Z}_F} \mathrm{EU}(z,p) = \max_{z\in\{0,1\}^m} \left\{ \sum_{i=1}^{n} p_i u_i\Big(\sum_{j=1}^{m} z_j x_i^j\Big) \middle| Az^T \le b \right\}. \tag{3}$$

Throughout the paper, the decision variables of optimization problems are marked beneath the max / min operator. The decision tree corresponding to optimization problem (3) is shown in Figure 1.

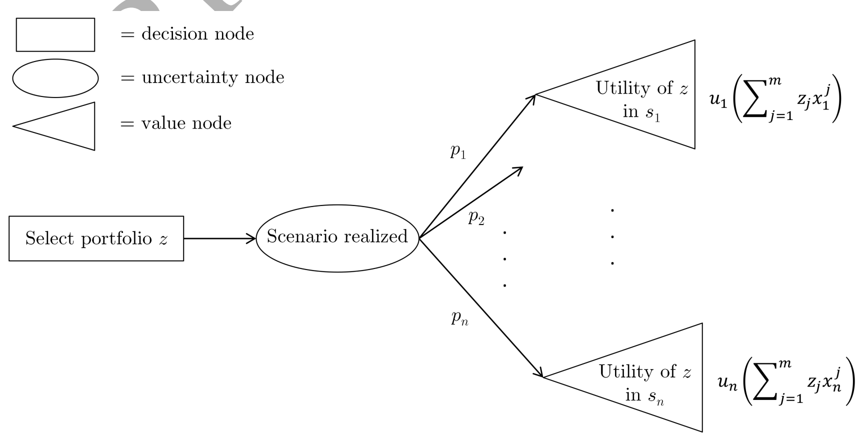


Figure 1: Decision tree for portfolio selection with scenario probabilities which do not depend on the selected actions.

3.2. Action-dependent scenario probabilities

In some contexts, scenario probabilities depend on the selected actions. For instance, if scenarios are characterized by the level of regulation and market demand, a company may steer the course of change toward their desired scenario by making investments in lobbying or marketing (Hagel III *et al.*, 2008; Reeves *et al.*, 2012). We therefore relax the assumption that the probability vector $p = [p_1, \ldots, p_n]$ is an exogenous constant and, instead, consider it to be endogenous so that it may depend on the selected portfolio. Whereas the probabilities of the n scenarios may depend on which actions are selected, we assume that the *contents* of these scenarios are fixed. This is in line with the usual definition of scenarios as descriptions of the external operational environment in which the organization acts (Coates, 2000; Ram *et al.*, 2011).

Technically, we assume that the set of feasible portfolios $\mathcal{Z}_F$ is partitioned into K disjoint sets $\mathcal{Z}^1, \ldots, \mathcal{Z}^K$ ($\cup_{k=1}^K \mathcal{Z}^k = \mathcal{Z}_F$) such that if portfolio $z \in \mathcal{Z}^k$ is selected, then the scenario probability vector is $[P_{k,1}, \ldots, P_{k,n}] \in \Delta^n$. In particular, we assume that for each portfolio of actions that affects scenario probabilities differently, there is a different vector of scenario probabilities. For instance, consider a situation in which the set of feasible portfolios is $\mathcal{Z}_F = \{0,1\}^m$, the selection of action $j = 1$ affects the scenario probabilities in one way, and the selection of both of actions $j = 2$ and $j = 3$ affects them in some other way. Then, $\mathcal{Z}_F$ is partitioned into four sets $\mathcal{Z}^1, \ldots, \mathcal{Z}^4$ which correspond to those portfolios which (i) include at most one of actions $j = 2$ and $j = 3$ but do not include action $j = 1$, (ii) include action $j = 1$ but at most one of actions $j = 2$ and $j = 3$, (iii) contain both actions $j = 2$ and $j = 3$ but not action $j = 1$, and (iv) contain all three actions $j = 1, 2, 3$. That is,

$$\begin{aligned}
\mathcal{Z}^1 &= \big\{z \in \mathcal{Z}_F | z_1 = 0 \wedge (z_2 = 0 \vee z_3 = 0)\big\}, && (4)\\
\mathcal{Z}^2 &= \big\{z \in \mathcal{Z}_F | z_1 = 1 \wedge (z_2 = 0 \vee z_3 = 0)\big\}, && (5)\\
\mathcal{Z}^3 &= \big\{z \in \mathcal{Z}_F | z_1 = 0 \wedge z_2 = z_3 = 1\big\}, && (6)\\
\mathcal{Z}^4 &= \big\{z \in \mathcal{Z}_F | z_1 = z_2 = z_3 = 1\big\}. && (7)
\end{aligned}$$

Given K portfolio sets $\mathcal{Z}^k$, $K \times n$ scenario probabilities need to be estimated. These estimates can be represented by matrix $P \in \Delta_K^n \triangleq \{P \in [0,1]^{K \times n} \mid P_{k,\cdot} \in \Delta^n\}$ with rows $P_{k,\cdot} = [P_{k,1}, \ldots, P_{k,n}]$ such that

$$P = \begin{array}{c} \begin{array}{ccc} \text{Scenario } s_1 & \ldots & \text{Scenario } s_n \end{array} \\ \begin{pmatrix} P_{1,1} & \ldots & P_{1,n} \\ \vdots & \ddots & \vdots \\ P_{K,1} & \ldots & P_{K,n} \end{pmatrix} \end{array} \begin{array}{cc} \text{Portfolios } z \in \mathcal{Z}^1 & \sum_i = 1, \\ \vdots & \vdots \\ \text{Portfolios } z \in \mathcal{Z}^K & \sum_i = 1. \end{array} \tag{8}$$

With action-dependent scenario probabilities, the expected utility maximization problem (3) can be formulated as

$$\max_{z \in \mathcal{Z}_F} \mathrm{EU}(z, P_{\kappa(z),\cdot}), \tag{9}$$

where $\kappa(z)$ denotes the row index of matrix P containing the scenario probabilities resulting from choosing portfolio z, i.e.,

$$\kappa(z) = k \Leftrightarrow z \in \mathcal{Z}^k. \tag{10}$$

The decision tree for this problem is shown in Figure 2.

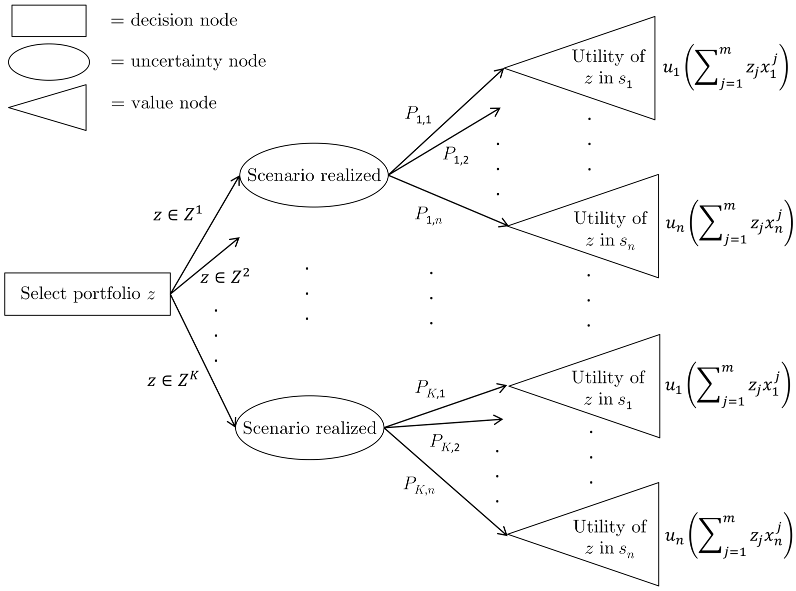


Figure 2: Decision tree for portfolio selection with action-dependent scenario probabilities.

3.3. Incomplete probability information

Due to elicitation costs and time constraints, it may be difficult to obtain estimates for the $K \times n$ probabilities for matrix P in (8). In particular, the precise assessment of scenario probabilities conditioned on the DM's choices may in practice be cognitively too demanding and prone to psychological biases, such as overconfidence (Tversky and Kahneman, 1974; Goodwin and Wright, 2001). Moreover, if the probability estimates are elicited from several experts, it may be challenging to aggregate these estimates into a single probability matrix. Thus, it is instructive to admit incomplete probability information which spans all stated probability estimates, and to examine which decision recommendations are compatible with this information (cf. e.g., White *et al.*, 1982; Hazen, 1986; Walley, 1991; Moskowitz *et al.*, 1993; Liesiö and Salo, 2012).

We model incomplete probability information by set inclusion. That is, instead of a single scenario probability matrix P, we consider a set of feasible probability matrices $\mathcal{P} \subseteq \Delta_K^n$, which satisfy linear constraints that correspond to statements about scenario probabilities. The rows of P are assumed to be independent, whereby these constraints are of the form $\sum_{i=1}^{n} c_i P_{k,i} \leq d$. The set $\mathcal{P}_k$ of feasible k-th row vectors of matrix P is

$$\mathcal{P}_k = \{[P_{k,1}, \ldots, P_{k,n}] \in \Delta^n \mid \sum_{i=1}^{n} c_{i_\ell} P_{k,i} \leq d_\ell \ \forall \ell = 1, \ldots, L_k\}, \tag{11}$$

where L_k is the number of linear constraints on scenario probabilities for action portfolios in set $\mathcal{Z}^k$.

Consider, for instance, the previous example where the selection of action 1 affects the scenario probabilities in one way, and the selection of both of actions 2 and 3 affects them in some other way. In this case, the set of feasible portfolios $\mathcal{Z}_F$ was partitioned into four sets $\mathcal{Z}^1, \ldots, \mathcal{Z}^4$ defined by (4)-(7). Consider a setting with three scenarios. Stating that scenario s_2 is more probable than scenario s_3 regardless of what actions are selected can be modeled as

$$P_{k,2} \geq P_{k,3} \ \forall k = 1, \ldots, 4. \tag{12}$$

A statement that the selection of action 1 makes the realization of scenario s_1 more probable than the realization of one of scenarios s_2 and s_3 can be modeled as

$$P_{k,1} \geq P_{k,2} + P_{k,3} \quad \text{for } k = 2, 4. \tag{13}$$

Similarly, consider a statement that choosing actions 2 and 3 jointly (i) increases the probability of scenario s_1 so that it is at least 50% and (ii) decreases the probability of scenario s_3 so that it is at most 10%. Then, we have constraints

$$\left.\begin{array}{l} P_{k,1} \geq 0.5 \\ P_{k,3} \leq 0.1 \end{array}\right\} \text{ for } k = 3, 4. \tag{14}$$

Using constraints (12)-(14), the set of feasible probability matrices for the four portfolio sets $\mathcal{Z}^k,\ k = 1, \ldots, 4$ and three scenarios becomes

$$\begin{aligned} \mathcal{P} = \{P \in \Delta_4^3 \quad | \quad & P_{k,2} \geq P_{k,3}\ \forall k = 1, \ldots, 4, \\ & P_{k,1} \geq P_{k,2} + P_{k,3} \quad \text{for } k = 2, 4, \\ & P_{k,1} \geq 0.5 \text{ for } k = 3, 4, \\ & P_{k,3} \leq 0.1 \text{ for } k = 3, 4\}, \end{aligned} \tag{15}$$

so that

$$\begin{aligned} \mathcal{P}_1 &= \{[P_{1,1}, \ldots, P_{1,4}] \in \Delta^4 \mid P_{1,2} \geq P_{1,3}\}, \\ \mathcal{P}_2 &= \{[P_{2,1}, \ldots, P_{2,4}] \in \Delta^4 \mid P_{2,2} \geq P_{2,3},\ P_{2,1} \geq P_{2,2} + P_{2,3}\}, \\ \mathcal{P}_3 &= \{[P_{3,1}, \ldots, P_{3,4}] \in \Delta^4 \mid P_{3,2} \geq P_{3,3},\ P_{3,1} \geq 0.5,\ P_{3,3} \leq 0.1\}, \\ \mathcal{P}_4 &= \{[P_{4,1}, \ldots, P_{4,4}] \in \Delta^4 \mid P_{4,2} \geq P_{4,3},\ P_{4,1} \geq P_{4,2} + P_{4,3},\ P_{4,1} \geq 0.5,\ P_{4,3} \leq 0.1\}. \end{aligned}$$

3.4. Dominance structures

If information about scenario probabilities was complete, the DM would select the feasible portfolio $z \in \mathcal{Z}_F$ with the highest expected utility $\text{EU}(z, P_{\kappa(z),\cdot})$. However, different selections of the scenario probability matrix P from the feasible set $\mathcal{P}$ associate different expected utilities with each portfolio z. To determine which portfolios outperform others, we define dominance as follows.

Definition 1. *Portfolio z dominates z' with regard to the set of feasible probability matrices $\mathcal{P}$ denoted $z \succ_{\mathcal{P}} z'$ if and only if*

$$\text{EU}(z, P_{\kappa(z),\cdot}) \geq \text{EU}(z', P_{\kappa(z'),\cdot}) \qquad \textit{for all } P \in \mathcal{P} \tag{16}$$

$$\text{EU}(z, P_{\kappa(z),\cdot}) > \text{EU}(z', P_{\kappa(z'),\cdot}) \qquad \textit{for some } P \in \mathcal{P}, \tag{17}$$

where $\kappa(\cdot)$ is given by (10).

Thus, portfolio z dominates portfolio z' if (i) the expected utility of z is at least as high as that of z' for all feasible scenario probabilities, and (ii) the expected utility of z is strictly higher than that of z' for at least some feasible scenario probabilities. Even though different scenario probabilities may be used for the computation of the expected utilities for portfolios z and z', the relation $\succ_{\mathcal{P}}$ is transitive, which is a desirable property for any partial ordering used for normative decision support. All proofs are in Appendix A.

Lemma 1. *The dominance relation $\succ_P$ is transitive.*

Dominance between two portfolios can be readily checked by minimizing and maximizing the utility difference between them subject to the requirement that scenario probabilities P belong to $\mathcal{P}$. These are linear problems whose optimal solutions are attained at some extreme point matrix of $\mathcal{P}$. The set $\text{ext}(\mathcal{P})$ of such extreme point matrices is

$$\text{ext}(\mathcal{P}) = \{P \in \Delta_K^n \mid P_{k,\cdot} \in \text{ext}(\mathcal{P}_k)\ \forall k = 1, \dots, K\}, \tag{18}$$

where $\mathcal{P}_k$ is the set of feasible k-th row vectors of P as defined in (11), and $\text{ext}(\mathcal{P}_k)$ is the set of extreme points of $\mathcal{P}_k$. That is, the set $\text{ext}(\mathcal{P})$ consists of matrices whose rows are different combinations of the extreme points of the sets $\mathcal{P}_k$.

To check dominance relations among many portfolios using the same probability information, it is typically faster to determine the set of extreme points $\text{ext}(\mathcal{P})$ first and to then compare the expected utility differences between the portfolios in these points. This is because the number of linear programming problems needed to identify $\text{ext}(\mathcal{P})$ grows linearly as a function of the number of extreme points (e.g., Dyer and Proll, 1982), whereas the number of linear programming problems needed to establish dominance relations through (16)-(17) grows polynomially as a function of the number of portfolios. Moreover, in practical problems, there are typically fewer than a dozen extreme points, while there may be hundreds of portfolios. Efficient algorithms to compute the set $\text{ext}(\mathcal{P})$ are presented by, e.g., Matheiss and Rubin (1980), Dyer and Proll (1982), and Avis and Fukuda (1992).

Theorem 1. *Let $z, z' \in \mathcal{Z}_F$, and let the set of feasible probability matrices be $\mathcal{P} \subseteq \Delta_K^n$. Furthermore, denote*

$$D(P) = \left(\sum_{i=1}^{n} P_{\kappa(z),i} u_i\Big(\sum_{j=1}^{m} z_j x_i^j\Big) - \sum_{i=1}^{n} P_{\kappa(z'),i} u_i\Big(\sum_{j=1}^{m} z'_j x_i^j\Big) \right).$$

Then, $z \succ_{\mathcal{P}} z'$ if and only if

$$\min_{P \in \text{ext}(\mathcal{P})} D(P) \geq 0 \text{ and} \tag{19}$$

$$\max_{P \in \text{ext}(\mathcal{P})} D(P) > 0, \tag{20}$$

where $\text{ext}(\mathcal{P})$ is the set of extreme points of $\mathcal{P}$.

Figure 3 illustrates dominance relations among three portfolios $z^1, z^3 \in \mathcal{Z}^1$ and $z^2 \in \mathcal{Z}^2$ for two scenarios s_1 and s_2 such that the set of feasible probability matrices is

$$\mathcal{P} = \{P \in \Delta_2^2 \mid P_{1,1} \leq 0.6,\ P_{2,1} \geq 0.5\}.$$

Then, $\text{ext}(\mathcal{P}_1) = \{[0, 1], [0.6, 0.4]\}$ and $\text{ext}(\mathcal{P}_2) = \{[0.5, 0.5], [1, 0]\}$, and thus

$$\text{ext}(\mathcal{P}) = \left\{ \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0.6 & 0.4 \\ 1 & 0 \end{bmatrix} \right\}. \tag{21}$$

Portfolio z^2 is dominated by portfolio z^1, because its expected utility (ranging from 0.1 to 0.5) is less than or equal to that of portfolio z^1 (ranging from 0.5 to 0.75) for all extreme point matrices in (21). Also, portfolio z^1 dominates portfolio z^3, because its expected utility is 0.15 units greater than that of portfolio z^3 for each feasible extreme point $[P_{1,1}, P_{1,2}] \in \text{ext}(\mathcal{P}_1) = \{[0, 1], [0.6, 0.4]\}$.

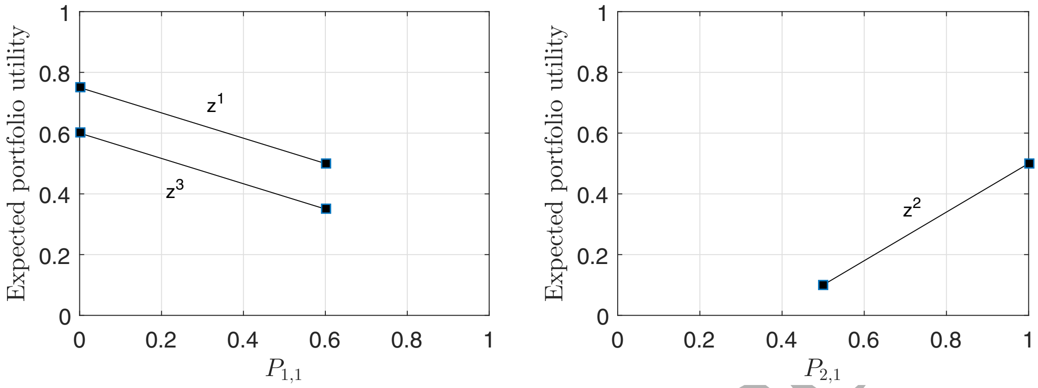


Figure 3: Expected utilities of portfolios $z^1, z^3 \in \mathcal{Z}^1$ and $z^2 \in \mathcal{Z}^2$.

Because a rational decision maker would not choose a dominated portfolio, it is reasonable to focus on feasible portfolios which are not dominated by any other feasible portfolio.

Definition 2. *The set of non-dominated portfolios with regard to the set of feasible probability matrices $\mathcal{P}$ is*

$$\mathcal{Z}_N(\mathcal{P}) = \{z \in \mathcal{Z}_F \mid \nexists z' \in \mathcal{Z}_F \text{ such that } z' \succ_{\mathcal{P}} z\}. \tag{22}$$

A non-dominated portfolio is both (i) robust in that it is not outperformed by any other feasible portfolio and (ii) proactive in that it accounts for the effect that the actions may have on scenario probabilities. In the example of Figure 3, there is only one non-dominated portfolio so that $\mathcal{Z}_N(\mathcal{P}) = \{z^1\}$.

3.5. Additional information

During the decision support process, additional statements about scenario probabilities may be elicited. Such statements correspond to additional linear constraints on scenario probabilities, which reduce the set of feasible probability matrices to $\tilde{\mathcal{P}} \subseteq \mathcal{P}$. Unless $\tilde{\mathcal{P}}$ is a subset of the 'border' of $\mathcal{P}$, then $\mathcal{Z}_N(\tilde{\mathcal{P}})$ is a subset of $\mathcal{Z}_N(\mathcal{P})$, meaning that the introduction of additional probability information may reduce the set of non-dominated portfolios but cannot generate new non-dominated portfolios. However, if $\tilde{\mathcal{P}}$ is a subset of the border of $\mathcal{P}$, then $\mathcal{Z}_N(\tilde{\mathcal{P}})$ may contain two portfolios whose expected utilities coincide on this border, while one has strictly lower expected utility everywhere else in $\mathcal{P}$ and, thus, does not belong to $\mathcal{Z}_N(\mathcal{P})$.

This situation is illustrated in Figure 3, where portfolio z^1 dominates z^2. Assume that the additional information reduces the set of feasible probabilities for portfolios $z^1, z^3 \in \mathcal{Z}^1$ to the single point $\tilde{\mathcal{P}}_1 = [P_{1,1}, P_{1,2}] = [0.6, 0.4]$, and that for portfolio $z^2 \in \mathcal{Z}^2$ to the single point $\tilde{\mathcal{P}}_2 = [P_{2,1}, P_{2,2}] = [1, 0]$. Here, the expected utilities of portfolios z^1 and z^2 are equal (=0.5) so that z^1 no longer dominates z^2. To rule out this possibility, we assume that $\tilde{\mathcal{P}}$ includes some points from the relative interior of $\mathcal{P}$.

Theorem 2. *Let $\tilde{\mathcal{P}} \subseteq \mathcal{P}$ such that $\text{int}(\mathcal{P}) \cap \tilde{\mathcal{P}} \neq \emptyset$. Then, $\mathcal{Z}_N(\tilde{\mathcal{P}}) \subseteq \mathcal{Z}_N(\mathcal{P})$.*

Because $\mathcal{P}$ and $\tilde{\mathcal{P}} \subseteq \mathcal{P}$ are closed, convex, and bounded polytopes, $\mathrm{int}(\mathcal{P}) \cap \tilde{\mathcal{P}} = \emptyset$ if the extreme points of $\tilde{\mathcal{P}}$ lie on the same face of $\mathcal{P}$. Algorithms for enumerating the faces of a convex polytope are presented by Fukuda and Rosta (1994). Having established the set $\mathrm{ext}(\tilde{\mathcal{P}})$ of extreme points of $\tilde{\mathcal{P}}$ with a suitable algorithm (e.g., Avis and Fukuda, 1992), it remains to check whether all points in $\mathrm{ext}(\tilde{\mathcal{P}})$ satisfy the equation for the hyperplane corresponding to some face of $\mathcal{P}$.

3.6. Implications for decision support

It is reasonable to recommend only portfolios in the set $\mathcal{Z}_N(\mathcal{P})$, because any portfolio outside this set of non-dominated portfolios is outperformed by at least one non-dominated portfolio. Furthermore, by Theorem 2, no portfolio outside $\mathcal{Z}_N(\mathcal{P})$ can become non-dominated as a result of introducing additional information about scenario probabilities, unless this information $\tilde{\mathcal{P}}$ contains no interior points of $\mathcal{P}$. It is therefore advisable to start with loose statements about scenario probabilities so that the feasible region does not become empty, and to tighten these statements only if the initial recommendations are not conclusive enough (Moskowitz *et al.*, 1989; Salo and Hämäläinen, 2010),

Deciding which one of the non-dominated portfolios to select can be cognitively demanding, especially if the number of non-dominated portfolios $|\mathcal{Z}_N(\mathcal{P})|$ is high. Yet, the set of non-dominated portfolios can be examined to derive recommendations about whether a given action should be included in the portfolio or not. Such action-specific recommendations are based on the concept of *core index*, defined as follows (cf. Liesiö *et al.*, 2007).

Definition 3. *For a given set $\mathcal{P}$ of feasible probability matrices we define*

$$
\begin{aligned}
&\textit{Core index of action } j: && CI_j(\mathcal{P}) = |\{z \in \mathcal{Z}_N(\mathcal{P}) | z_j = 1\}| / |\mathcal{Z}_N(\mathcal{P})| \\
&\textit{Core actions}: && \mathcal{X}_C(\mathcal{P}) = \{j \in \{1, \dots, m\} \mid CI_j(\mathcal{P}) = 1\} \\
&\textit{Borderline actions}: && \mathcal{X}_B(\mathcal{P}) = \{j \in \{1, \dots, m\} \mid 0 < CI_j(\mathcal{P}) < 1\} \\
&\textit{Exterior actions}: && \mathcal{X}_E(\mathcal{P}) = \{j \in \{1, \dots, m\} \mid CI_j(\mathcal{P}) = 0\}.
\end{aligned}
$$

All core actions should be selected, because they belong to all non-dominated portfolios even if additional information about scenario probabilities was given. Similarly, all exterior actions can be rejected, because they do not belong to any non-dominated portfolios even in light of additional information. This result is formalized in Corollary 1.

Corollary 1. *Let $\tilde{\mathcal{P}} \subseteq \mathcal{P}$ such that $\mathrm{int}(\mathcal{P}) \cap \tilde{\mathcal{P}} \neq \emptyset$. Then, $\mathcal{X}_C(\mathcal{P}) \subseteq \mathcal{X}_C(\tilde{\mathcal{P}})$ and $\mathcal{X}_E(\mathcal{P}) \subseteq \mathcal{X}_E(\tilde{\mathcal{P}})$.*

Action-specific recommendations facilitate decision-making by helping to identify core actions that should definitely be pursued and exterior actions that should not, after which further discussion can be focused on a smaller set of borderline actions. Nevertheless, when deciding which combination of borderline actions to ultimately select, it is important to ensure that the resulting portfolio is feasible and non-dominated, i.e., belongs to set $\mathcal{Z}_N(\mathcal{P})$. Recommendations for selecting one out of $|\mathcal{Z}_N(\mathcal{P})|$ non-dominated portfolios can be based on decision rules that have been developed to identify preferred alternatives when the model parameters (such as scenario probabilities or attribute weights) are set-valued (see, e.g., Sarabando and Dias, 2009). Examples of robust decision rules are *maximin* and *minimax regret* (Kouvelis and Yu, 1997; Salo and Hämäläinen, 2001). The maximin portfolio z^{mm} yields the highest worst-case expected utility, whereas the minimax regret portfolio

z^{mmr} results in the lowest maximal regret in expected utility when compared to the best-case performance of any other portfolio. More formally,

$$\text{Maximin portfolio:} \quad z^{mm} \in \operatorname*{argmax}_{z \in \mathcal{Z}_N(\mathcal{P})} \min_{P \in \mathcal{P}} \mathrm{EU}(z, P_{\kappa(z),\cdot}),$$

$$\text{Minimax regret portfolio:} \quad z^{mmr} \in \operatorname*{argmin}_{z \in \mathcal{Z}_N(\mathcal{P})} \max_{\substack{z' \in \mathcal{Z}_N(\mathcal{P}) \\ P \in \mathcal{P}}} \Big[\mathrm{EU}(z', P_{\kappa(z'),\cdot}) - \mathrm{EU}(z, P_{\kappa(z),\cdot})\Big].$$

Other commonly used decision rules include *maximax* and *Hurwicz rule* (Hurwicz, 1951; Salo and Hämäläinen, 2001). The maximax portfolio yields the highest best-case expected utility and can be computed by replacing the min operator by a max operator in the above definition of the maximin portfolio z^{mm}. The Hurwicz portfolio maximizes the weighted average of the worst-case and best-case expected utility and can be computed by replacing $\min_{P\in\mathcal{P}} \mathrm{EU}(z, P_{\kappa(z),\cdot})$ by $w \cdot \min_{P\in\mathcal{P}} \mathrm{EU}(z, P_{\kappa(z),\cdot}) + (1-w) \cdot \max_{P\in\mathcal{P}} \mathrm{EU}(z, P_{\kappa(z),\cdot})$ in the definition of z^{mm}, where $w \in [0,1]$ is the weighting coefficient. Yet, recommendations based on maximax amd Hurwicz decision rules are not robust in that their worst-case performance can be relatively poor (unless the weighting coefficient w is close to 1, in which case Hurwicz rule is equivalent to the maximin rule).

3.7. Elicitation of utility functions

The choice of a suitable approach for eliciting the utility functions u_i depends on the application under consideration. The most straightforward case is when (i) outcomes are measured on a single attribute and (ii) the utilities of outcomes are not contingent on the scenario (i.e., $u_i(\cdot) = u(\cdot)$ for all $i \in \{1, \dots, n\}$). Then, the utility function $u(\cdot)$ can be assessed through standard approaches that utilize hypothetical lotteries between outcomes, such as certainty- and probability-equivalent techniques (Clemen, 1996). As an alternative approach, Wakker and Deneffe (1996) propose the gamble-tradeoff method which does not require the specification of numerical values for lottery probabilities. Arguably, this method is less sensitive to misconceptions about probabilities and behavioral violations of the assumptions of expected utility theory.

In applications where outcomes are measured with respect to multiple attributes, a common approach is to convert all outcomes onto a single monetary scale (cf. 'pricing-out' approach; see, e.g., Clemen and Smith, 2009). Then, outcomes x_i^j in the scenario model are unidimensional, whereby the above methods can be used to assess the utility function over their value scale. As an alternative to the pricing-out approach, a multiattribute portfolio value function can be built to aggregate the multiattribute outcomes of a combination of actions to an overall portfolio value (Golabi *et al.*, 1981; Liesiö, 2014). For instance, the additive-linear portfolio value function (Golabi *et al.*, 1981) uses a standard additive value function to evaluate the overall value of each action, and portfolio value is then obtained as the sum of the overall values of those actions that are included in the portfolio. In this case, outcome x_i^j in our scenario model would correspond to the overall value of action j in scenario i. The above methods can be used to assess a utility function over this portfolio value scale to capture the DM's risk preferences (cf. utility over value approach by Matheson and Abbas, 2005). However, this requires that the DM is able to compare lotteries (or gamble-tradeoffs) between multiattribute consequences of action portfolios.

In some applications it may be appropriate to relax the assumption of the same utility function across scenarios. Indeed, some studies that use linear-additive multiattribute value functions to compute the decision alternatives' scenario-specific values report that the attribute-specific value functions and attribute weights vary across scenarios (see, e.g., Montibeller *et al.*, 2006). In such cases, the above methods can be used to elicit multiattribute portfolio value functions for each scenario separately. Moreover, these approaches can be used to specify the utility function (possibly over value) for a single scenario, say $u_1(\cdot)$. The question is then how to assess utility functions $u_2(\cdot), \ldots, u_n(\cdot)$ such that the utilities from these functions are commensurable with those obtained from u_1. This can be achieved by considering outcome levels ℓ_1^0 and ℓ_1 in scenario s_1, and levels ℓ_2^0 and ℓ_2 in scenario s_2, where ℓ_1^0, ℓ_2^0 correspond to worst outcomes in scenarios s_1 and s_2, respectively. Without loss of generality, the scenario-specific utility functions can be scaled such that $u_1(l_1^0) = u_2(l_2^0) = 0$. The DM is asked to consider two alternative portfolios: The first one yields outcomes ℓ_1^0 and ℓ_2 in scenarios s_1 and s_2, respectively, while the second one yields outcomes ℓ_1 and ℓ_2^0. Then, given a fixed level of ℓ_1, the DM is asked to consider a situation where the scenarios are equally likely and to adjust the level of ℓ_2 until the two portfolios are equally preferred. Setting the expected utilities of the resulting portfolios equal yields $u_1(\ell_1) = u_2(\ell_2)$. Repeating this line of questioning for multiple different levels of ℓ_1 makes it possible to assess the utility function u_2 with desired accuracy. A similar procedure can be used to elicit each of the remaining utility functions $u_3(\cdot), \ldots, u_n(\cdot)$.

It is important to highlight that the above approach for eliciting scenario-specific utility functions assumes that the DM can comprehend the concept of scenarios being equally likely. This assumption seems reasonable in practical applications, and much of decision-theoretic work on expected utility builds on it (De Groot, 1970). However, there also exists a substantial body of literature on State-Dependent Utility Theory (SDUT) that avoids this assumption by developing axiomatisations of expected utility, in which both unique state-specific utilities and state probabilities are derived from preferences between decision alternatives (Schervish *et al.*, 1990; Karni and Schmeidler, 2016). Based on this theory, both scenario probabilities and scenario-specific utility functions could be derived by asking questions about the DM's preferences between hypothetical portfolios. However, the benefits from following this route seem to be outweighed by the heavy workload of the resulting elicitation procedure, especially because our model does not require exact numerical values for scenario probabilities.

4. Computation of non-dominated portfolios

In principle, the set of non-dominated portfolios $\mathcal{Z}_N(\mathcal{P})$ could be computed by first enumerating all feasible portfolios and then by checking the dominance relations by using Theorem 1. However, computation can be more efficient if the partition of portfolios $z \in \mathcal{Z}_F$ into sets $\mathcal{Z}^1, \ldots, \mathcal{Z}^K$ is utilized. In particular, let $\mathcal{Z}_N^k(\mathcal{P})$ denote the set of portfolios that are non-dominated among $\mathcal{Z}^k$, i.e.,

$$\mathcal{Z}_N^k(\mathcal{P}) = \{z \in \mathcal{Z}^k \mid \nexists z' \in \mathcal{Z}^k \text{ s.t. } z' \succ_{\mathcal{P}} z\}.$$

For each non-dominated portfolio $z \in \mathcal{Z}_N(\mathcal{P})$ there exists a set $\mathcal{Z}^k$ among which z is non-dominated, i.e., $z \in \mathcal{Z}_N^k(\mathcal{P})$. This result is formally stated by the following lemma.

Lemma 2. *Let $\mathcal{P} \subseteq \Delta_K^n$. Then, $\mathcal{Z}_N(\mathcal{P}) \subseteq \mathcal{Z}_N^1(\mathcal{P}) \cup \ldots \cup \mathcal{Z}_N^K(\mathcal{P})$.*

An implication of this lemma is that if sets $\mathcal{Z}_N^1(\mathcal{P}), \ldots, \mathcal{Z}_N^K(\mathcal{P})$ are known, then $\mathcal{Z}_N(\mathcal{P})$ can be readily determined by checking dominance relations (Theorem 1) between all pairs (z, z') of portfolios that are included in different sets $z \in \mathcal{Z}_N^k(\mathcal{P}),\ z' \in \mathcal{Z}_N^\ell(\mathcal{P}),\ k \neq \ell$. This is because the transitivity of the dominance relation guarantees that any dominated portfolio is dominated by at least one non-dominated portfolio.

To our knowledge, algorithms for solving the set $\mathcal{Z}_N^k(\mathcal{P})$ directly do not exist. Hence, we use an approach in which the set $\mathcal{Z}_N^k(\Delta_K^n)$ of non-dominated portfolios among $\mathcal{Z}^k$ given no probability information is solved first, after which the set $\mathcal{Z}_N^k(\mathcal{P})$ is obtained through dominance checks between all portfolios in $\mathcal{Z}_N^k(\Delta_K^n)$ (Theorem 1). Here, we utilize the fact that set $\mathcal{Z}_N^k(\mathcal{P})$ is a subset of $\mathcal{Z}_N^k(\Delta_K^n)$ (cf. Theorem 2). To solve the set $\mathcal{Z}_N^k(\Delta_K^n)$, we note that the difference between the expected utilities of portfolios $z, z' \in \mathcal{Z}^k$ at any extreme point of Δ_K^n is $u_i(\sum_{j=1}^m z_j x_i^j) - u_i(\sum_{j=1}^m z'_j x_i^j)$ for some $i = 1, \dots, n$ (cf. Theorem 1). If u_i is strictly increasing, then $u_i(\sum_{j=1}^m z_j x_i^j)) \geq (>) u_i(\sum_{j=1}^m z'_j x_i^j)$ if and only if $\sum_{j=1}^m z_j x_i^j \geq (>) \sum_{j=1}^m z'_j x_i^j$. Thus, the set $\mathcal{Z}_N^k(\Delta_K^n)$ can be obtained by solving the Pareto optimal solutions to the n-objective zero-one linear programming (MOZOLP) problem

$$\underset{z \in \mathcal{Z}^k}{\text{v–max}}[\sum_{j=1}^m z_j x_1^j, \sum_{j=1}^m z_j x_2^j, \dots, \sum_{j=1}^m z_j x_n^j], \tag{23}$$

for which there exist several solution algorithms (Villareal and Karwan, 1981; Kiziltan and Yucaoğlu, 1983; Liesiö *et al.*, 2008; Gutjahr *et al.*, 2010). This result is formally stated by the following lemma.

Lemma 3. *Let the set of feasible probability matrices be $\mathcal{P} = \Delta_K^n$, and let $z, z' \in \mathcal{Z}^k$. Then, $z \succ_{\mathcal{P}} z'$ if and only if*

$$\begin{bmatrix} \sum_{j=1}^m z_j x_1^j \\ \vdots \\ \sum_{j=1}^m z_j x_n^j \end{bmatrix} \gneq \begin{bmatrix} \sum_{j=1}^m z'_j x_1^j \\ \vdots \\ \sum_{j=1}^m z'_j x_n^j \end{bmatrix},$$

where $\gneq$ denotes that the inequality is strict on at least one element.

Based on the above results and the pairwise dominance check of Theorem 1, the algorithm to obtain the set of non-dominated portfolios $\mathcal{Z}_N(\mathcal{P})$ can be formulated as follows:

1. For each $k \in \{1, \dots, K\}$, obtain $\mathcal{Z}_N^k(\Delta_K^n)$ by solving the MOZOLP problem (23).
2. For each $k \in \{1, \dots, K\}$, obtain $\mathcal{Z}_N^k(\mathcal{P})$ by pairwise dominance checks within $\mathcal{Z}_N^k(\Delta_K^n)$:
$$\mathcal{Z}_N^k(\mathcal{P}) \leftarrow \{z \in \mathcal{Z}_N^k(\Delta_K^n) \mid \nexists z' \in \mathcal{Z}_N^k(\Delta_K^n) \ s.t. \ z' \succ_{\mathcal{P}} z\}.$$
3. For each $k \in \{1, \dots, K\}$, obtain $\mathcal{Z}_N(\mathcal{P})$ by pairwise dominance checks between $\mathcal{Z}_N^k(\mathcal{P})$ and $\mathcal{Z}_N^\ell(\mathcal{P})$ for each $\ell \neq k$:
   (a) $\mathcal{Z}_N^k(\mathcal{P}) \leftarrow \{z \in \mathcal{Z}_N^k(\mathcal{P}) \mid \nexists z' \in \bigcup_{\substack{\ell=1,\dots,K \\ \ell \neq k}} \mathcal{Z}_N^\ell(\mathcal{P}) \ s.t. \ z' \succ_{\mathcal{P}} z\}$,
   (b) Set $\mathcal{Z}_N(\mathcal{P}) \leftarrow \mathcal{Z}_N^1(\mathcal{P}) \cup \dots \cup \mathcal{Z}_N^K(\mathcal{P})$.

4.1. Computational tests

The effort of computing the set $\mathcal{Z}_N(\mathcal{P})$ increases as a function of the number of actions m, the number of scenarios n, the number of extreme points of the set of feasible scenario probability matrices $\mathcal{P}$, and the number K of sets into which the action-dependent scenario probabilities partition the set of feasible portfolios $\mathcal{Z}_F$. Table 1 illustrates the average time required for computing the set $\mathcal{Z}_N^k(\mathcal{P})$ and the size of this set as a function of the number of scenarios n and the number of actions m. For each combination of n and m, 100 problem instances were solved using RPM-Decisions software[1] in Step 1 and Matlab in Steps 2 and 3 of the algorithm on a standard laptop (2.60 GHz, 8 GB memory). In each instance, there was one budget constraint corresponding to one third of the combined cost of all proposed actions. The constraints on scenario probabilities corresponded to a complete ranking and a lower bound $1/(3n)$ on the least probable scenario (i.e., $P_{k,1} \geq P_{k,2} \geq \ldots \geq P_{k,n} \geq 1/(3n)$) so that the number of extreme points of the set of feasible $P_{k,\cdot}$ was n. In each problem instance, the actions' impacts and costs were generated from a uniform distribution. In each scenario, a logarithmic utility function $u_i(\cdot) = \ln(\cdot)$ was used to map portfolio impact to portfolio utility.

Table 1a shows that, for instance, it takes on average $987.9s \approx 16\,\text{min}$ to compute the set $\mathcal{Z}_N^k(\mathcal{P})$, when there are $m = 50$ actions and $n = 5$ scenarios. If the set of feasible portfolios $\mathcal{Z}_F$ is partitioned into $K = 8$ sets, the combined computation time of $\mathcal{Z}_N^k(\mathcal{P})$ for all $k = 1, \ldots, 8$ (i.e., Steps 1 and 2 in the algorithm) is $8 \times 987.9s \approx 2\text{h}$.

		n		
		3	4	5
m	30	0.4	1.2	3.8
	40	1.5	9.5	70.0
	50	5.7	108.1	987.9

(a) Average computation time for set $\mathcal{Z}_N^k(\mathcal{P})$ in seconds.

		n		
		3	4	5
m	30	7	10	11
	40	11	15	21
	50	17	26	36

(b) Average number $|\mathcal{Z}_N^k(\mathcal{P})|$ of non-dominated portfolios.

Table 1: Average computation time and size of $\mathcal{Z}_N^k(\mathcal{P})$ as functions of the number of actions m and the number of scenarios n. Number of problem instances for each combination of n and m is 100.

The computation time for carrying out the pairwise dominance checks in Step 3 of the algorithm is negligible compared to the combined computation time of Steps 1 and 2. Figure 4 illustrates the computational effort required by these dominance checks as a function of the number of portfolios in each $\mathcal{Z}_N^k(\mathcal{P})$ for different values of K. With 50 actions and five scenarios, there are approximately 36 non-dominated portfolios in each set $\mathcal{Z}_N^k(\mathcal{P})$, $k \in \{1, \ldots, K\}$ (see Table 1b). Given $K = 8$, the combined computation time for carrying out the pairwise dominance checks for each of the 36 portfolios in each portfolio set is 0.11ms.

5. Application to ecosystem strategy building

5.1. Case description

In the fall of 2015 a group of Nordic, globally operating steel and engineering companies were developing a multi-sided, economic ecosystem around a technology platform called SmartSteel. With the help of digital marking on raw materials and cloud storage, the SmartSteel platform would enable a real-time documentation and tracking of all activities in the manufacturing process of steel into a final construction. In addition to generating reliable audit trails and reducing documentation costs and errors, data collected through this platform would create new technology- and service-related business opportunities for the ecosystem.

Platform ecosystems are a relatively new phenomenon (Evans and Gawer, 2016; Van Alstyne *et al.*, 2016), and multisided data exchange has not been used in engineering (Hermann *et al.*, 2016). Hence, the participating companies felt that the strategy for developing the ecosystem should be robust across alternative scenarios of the future operational environment. Moreover, because SmartSteel ecosystem was one of the pioneers on the

[1] http://rpm.aalto.fi/rpm-software.html

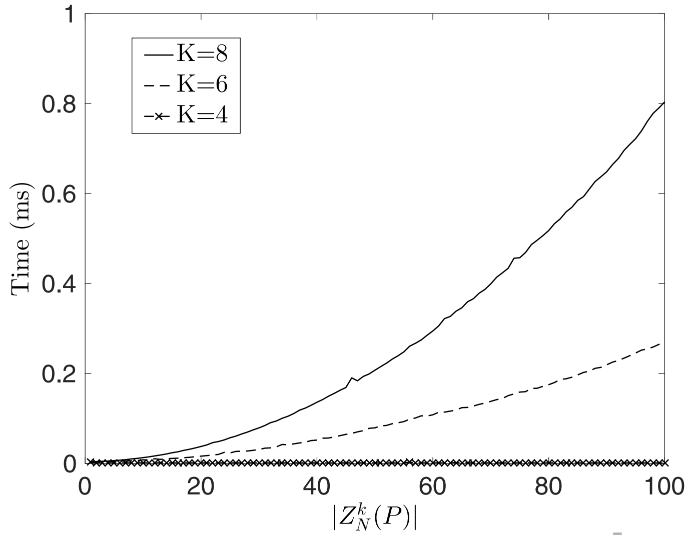


Figure 4: Computation times in milliseconds for pairwise comparisons between all portfolios in sets $\mathcal{Z}_N^k(\mathcal{P})$ and $\mathcal{Z}_N^\ell(\mathcal{P})$ for each $k = 1, \ldots, K$, $\ell \neq k$ for different values of K and $|\mathcal{Z}_N^k(\mathcal{P})| = |\mathcal{Z}_N^\ell(\mathcal{P})|$.

market, the strategy it would adopt was seen to have a potential impact on which of the alternative scenarios would be realized.

The strategy process was carried out with a team of R&D leaders from the participating companies in four workshops between which data was collected using web-based questionnaires. At the first stage of this process, the participants developed three alternative scenarios for the operational environment of global platform ecosystems in year 2030. Then, the participants defined a set of actions that would need to be taken for the SmartSteel ecosystem to succeed in each scenario. Finally, the participants provided subjective assessments about (i) the performance of these actions in each scenario, (ii) the scenario probabilities, and (iii) the actions' impacts on these probabilities.

5.2. Definition of scenarios and actions

To define scenarios, we utilized morphological analysis (Godet, 2000; Ritchey, 2006). The process of morphological analysis begins by identifying (i) key uncertainties which affect the operational environment and (ii) the possible outcomes of these uncertainties. Then, the consistency of each pair of outcomes on each pair of key uncertainties is assessed. Based on these assessments, a small number (e.g., three to five) of internally consistent and sufficiently dissimilar outcome combinations are selected to serve as bases for scenario descriptions (Schoemaker, 1995; Peterson *et al.*, 2003; Raynor and Leroux, 2004).

In our case, the workshop participants identified five key uncertainties for global platform ecosystems (technological development, globalisation, internet, political environment, and consumer values), each with three potential outcomes (see Figure 5). Based on pairwise consistency assessments, EIDOS Option Development[2] tool was used to visualize the dissimilarity and internal consistency of all $3^5 = 243$ combinations of outcomes (Figure 6). This visualization supported the creation of three consistent and sufficiently dissimilar scenarios for further analysis: 'Internet havens', 'Fast transition', and 'Stuck in tar'. Brief descriptions of these scenarios are

[2]https://www.parmenides-eidos.com/eidos9/us/

given in Figure 7.

Key uncertainties				
Technological development	Globalisation	Internet	Political environment	Values
Slow	Fast	Reliable	Stable	Ecology
Fast	Asymmetric	Unreliable	Volatile	Consumption
Stuck	Regional	Super fast	Conflicts	Speed

Figure 5: Key uncertainties and their possible outcomes.

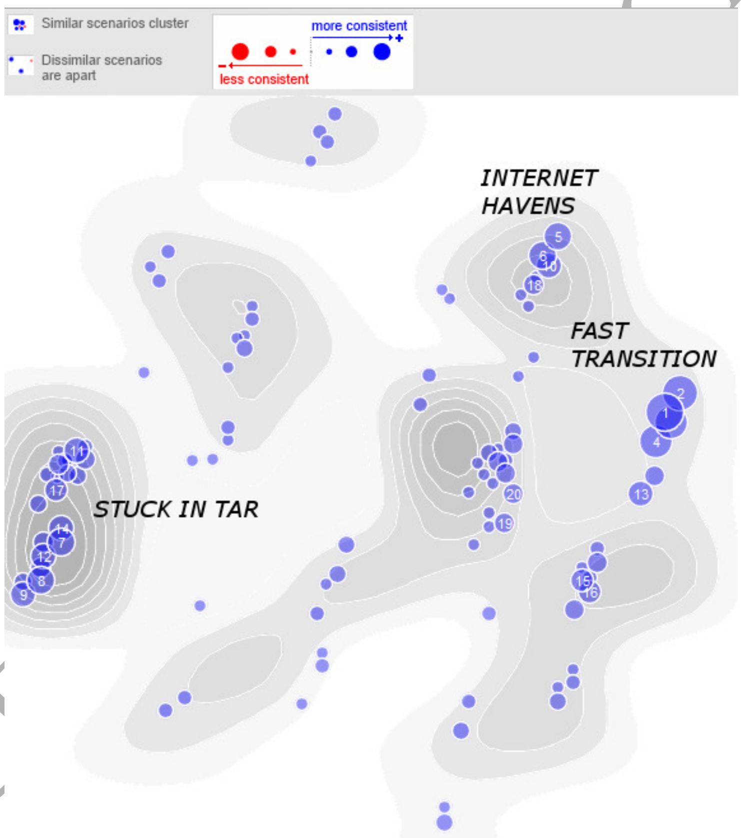


Figure 6: Visualization of the similarity and internal consistency of different combinations of outcomes of the five key uncertainties.

Once the scenarios had been defined, the participants were asked to develop courses of action that would need to be taken now to enable success for the SmartSteel ecosystem in each future scenario. Moreover, the participants were asked to think of actions that could be taken to increase or decrease the likelihood of different scenarios being realized. For each scenario, 4-15 actions were identified. By combining similar actions, a list of altogether $m = 23$ actions was generated. This list is shown in Figure 8, where those seven actions that were seen by the workshop participants to have a potentially significant impact on scenario probabilities are marked

Scenario 1: Internet havens 2030
World politics are very turbulent when different power structures are fighting for dominance. Open conflicts and information warfare have generated major disruptions on the Internet, whereby platform ecosystems are characterized by closed, sector-specific networks. These ecosystems can generate scaling advantages by transferring sector-specific solutions to other sectors (e.g., from defense to medical, aviation, and food industries) but due to troubled global Internet, global consumer markets do not benefit from this development.
Scenario 2: Fast transition by 2030
Documentation solutions based on EU-level regulation become the global standard, leading to a unique competitive advantage of European manufacturers compared to cheap Chinese steel. Competition between platforms is fierce, whereby they are enhancing in efficiency, transparency, and adaptability. These developments lead to a situation in which each member of the ecosystem is specializing in their core area. Even if the transaction costs of transferring from one ecosystem to another are high, increased competition leads to failures of some of the platforms.
Scenario 3: Stuck in tar 2030
Global political conflicts are violent and technology development investments are focusing on military solutions. Technology is used for the disruption of global Internet and building national secured systems. All this is possible because consumers are willing to give up some of their consumption. Robustness and security are emphasized, making it crucial for small players to form coalitions. Yet, open collaboration models in coalitions are hard to maintain due to unreliable Internet and the need to ensure the image of high security.

Figure 7: Descriptions of the three scenarios.

with asterisks.

5.3. Feasibility constraints

Because the actions were short thematic descriptions rather than detailed execution plans, we assumed that each of these actions, if implemented, would consume roughly the same amount of resources. Therefore, a single feasibility constraint was imposed to limit the number of selected actions in the portfolio. More specifically, the set of feasible portfolios was

$$\mathcal{Z}_F = \{z \in \{0,1\}^{23} | \sum_{i=1}^{23} z_i \leq b\}, \tag{24}$$

where $b \in \{0, \dots, 23\}$ is the limit on the number of actions that could be selected.

5.4. Assessment of actions' impacts and scenario probabilities

Assessments about the actions' impacts and scenario probabilities were gathered using a web-based questionnaire. First, each respondent was asked to assess the impact x_i^j of each action j in each scenario $s_i,\ i \in \{1, 2, 3\}$ on a scale 0-100. The average scenario-specific assessments are shown in Figure 8. Due to limited time, we did not elicit scenario-specific utility functions, but decided to use a single linear utility function $u_i(x) = x$ instead.

After having assessed the actions' impacts, the respondents were asked to rank the scenarios in order of their probability of occurence. Because the respondents' rank orderings were different, it was decided that a lower bound of 10% would be set on the probability of each scenario. When asked about the actions' impacts on scenario probabilities, the respondents agreed on the four statements shown in Figure 9. Although there are in total $2^4 = 16$ combinations of conditions on selected actions that impose different statements, some of these combinations imply the same set of feasible probabilities. For instance, when conditions 3 and 4 hold (i.e., both projects z_6 and z_7 are selected), then the set of feasible probabilities is the same regardless of whether neither,

Action	s_1: Internet havens	s_2: Fast transition	s_3: Stuck in tar
z_1: Traceability requirements*	80	60	59
z_2: Sustainability requirements*	74	51	33
z_3: Information security*	73	74	81
z_4: Sector-focused development*	79	29	74
z_5: EU steel legislation*	68	73	75
z_6: Open interfaces*	83	70	45
z_7: Partnering*	77	66	85
z_8: Platform investment	69	75	79
z_9: New sources of income	68	66	71
z_{10}: Central corporation / foundation	49	64	83
z_{11}: Service business	59	70	67
z_{12}: Certification	43	81	67
z_{13}: Speed and agility	49	85	52
z_{14}: Institutional actors	63	40	72
z_{15}: Cost savings	44	59	56
z_{16}: Technology business	59	40	57
z_{17}: Early investments	54	61	37
z_{18}: Passive location tag	38	61	45
z_{19}: Active tag	49	36	31
z_{20}: Energy storage tag	45	21	33
z_{21}: Sensor layer for IoT	36	35	15
z_{22}: No novel technologies	11	28	26
z_{23}: Antenna capability	33	13	16

Figure 8: Average assessments about the actions' impacts x_i^j in scenarios s_i, $i \in \{1,2,3\}$ on a scale 0-100. Actions z_j that have an impact on scenario probabilities are marked with asterisks.

one, or both of conditions 1 and 2 hold. As a result, there are only seven unique probability sets, which are shown in Table 2.

Portfolio sets:	**Probability sets:**
$\mathcal{Z}^1 = \{z \in \mathcal{Z}_F \mid z_1, \ldots, z_7 = 0\}$	$\mathcal{P}_1 = \{P_{1,\cdot} \in \Delta^3 \mid P_{1,i} \geq 0.1\ \forall i\}$
$\mathcal{Z}^2 = \{z \in \mathcal{Z}_F \mid \sum_{j=1}^3 z_j \geq 1 \wedge (z_6 = z_7 = 0)\}$	$\mathcal{P}_2 = \{P_{2,\cdot} \in \Delta^3 \mid P_{2,i} \geq P_{2,3} \geq 0.1,\ i = 1,2\}$
$\mathcal{Z}^3 = \{z \in \mathcal{Z}_F \mid z_4 + z_5 \geq 1 \wedge z_j = 0\ \forall j \in \{1,2,3,6,7\}\}$	$\mathcal{P}_3 = \{P_{3,\cdot} \in \Delta^3 \mid P_{3,2} \geq P_{3,3} \geq 0.1,\ P_{3,1} \geq 0.1\}$
$\mathcal{Z}^4 = \{z \in \mathcal{Z}_F \mid z_6 = 1 \wedge z_j = 0\ \forall j \in \{1,2,3,7\}\}$	$\mathcal{P}_4 = \{P_{4,\cdot} \in \Delta^3 \mid P_{4,2} \geq P_{4,i} \geq 0.1,\ i = 1,3\}$
$\mathcal{Z}^5 = \{z \in \mathcal{Z}_F \mid z_6 = 0 \wedge z_7 = 1\}$	$\mathcal{P}_5 = \{P_{5,\cdot} \in \Delta^3 \mid P_{5,2} \geq 2P_{5,3},\ P_{5,1} \geq P_{5,3} \geq 0.1\}$
$\mathcal{Z}^6 = \{z \in \mathcal{Z}_F \mid \sum_{j=1}^3 z_j \geq 1 \wedge z_6 = 1 \wedge z_7 = 0)\}$	$\mathcal{P}_6 = \{P_{6,\cdot} \in \Delta^3 \mid P_{6,2} \geq P_{6,1} \geq P_{6,3} \geq 0.1\}$
$\mathcal{Z}^7 = \{z \in \mathcal{Z}_F \mid z_6 = z_7 = 1\}$	$\mathcal{P}_7 = \{P_{7,\cdot} \in \Delta^3 \mid P_{7,2} \geq 2P_{7,3},\ P_{7,2} \geq P_{7,1} \geq P_{7,3} \geq 0.1\}$

Table 2: Portfolio and probability sets. For instance, if portfolio z does not contain action z_6 but does contain action z_7, then z belongs to portfolio set $\mathcal{Z}^5$, for which the set of feasible probabilities is $\mathcal{P}_5$

.

#	Condition		Statement
1	z_1: Traceability requirements z_2: Sustainability requirements z_3: Information security	Select at least one	IH is at least as likely as ST FT is at least as likely as ST
2	z_4: Sector-focused development z_5: EU steel legislation	Select at least one	FT is at least as likely as ST
3	z_6: Open interfaces		FT is at least as likely as ST FT is at least as likely as IH
4	z_7: Partnering		FT is at least twice as likely as ST IH is at least as likely as ST

Figure 9: Actions' impacts on scenario probabilities. IH = Internet havens, FT = Fast transition, ST = Stuck in tar.

5.5. Computation

The set of non-dominated portfolios was computed on a standard laptop (2.60 GHz, 8 GB memory) for each value of $b \in \{0, \ldots, 23\}$ (limit on the number of actions that can be selected). The computation of sets $\mathcal{Z}_N^1(\mathcal{P}_1), \ldots, \mathcal{Z}_N^7(\mathcal{P}_7)$ in Steps 1 and 2 of the algorithm was done by using RPM-Decisions software, which implements the dynamic programming algorithm by Liesiö *et al.* (2008). Pairwise dominance checks to obtain set $\mathcal{Z}_N(\mathcal{P})$ in Step 3 were done by Matlab 2016b. The combined computation time for all 24 sets of non-dominated portfolios corresponding to different values of $b \in \{0, \ldots, 23\}$ was less than two minutes.

5.6. Results

Figure 10 shows the number of non-dominated portfolios $|\mathcal{Z}_N(\mathcal{P})|$ as a function of the limit b on the number of selected actions. The number of non-dominated portfolios is highest (i.e., 36) when the portfolio contains at most either six or seven actions. Yet, this number is considerably lower than the number of feasible portfolios in either case: $\sum_{j=0}^{6} \binom{23}{j} = 145,499$ and $\sum_{j=0}^{7} \binom{23}{j} = 390,656$, respectively.

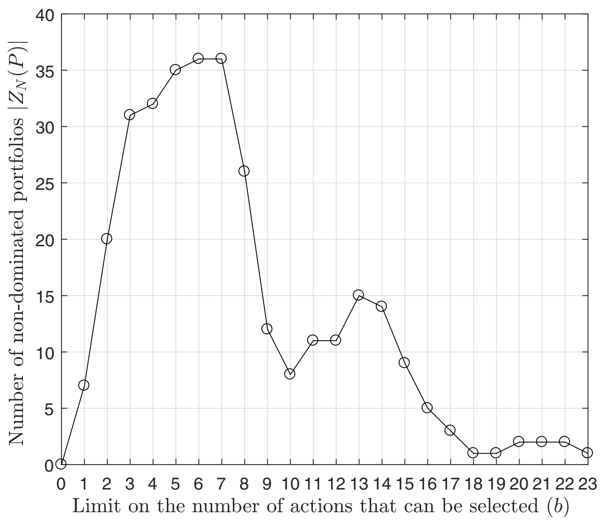
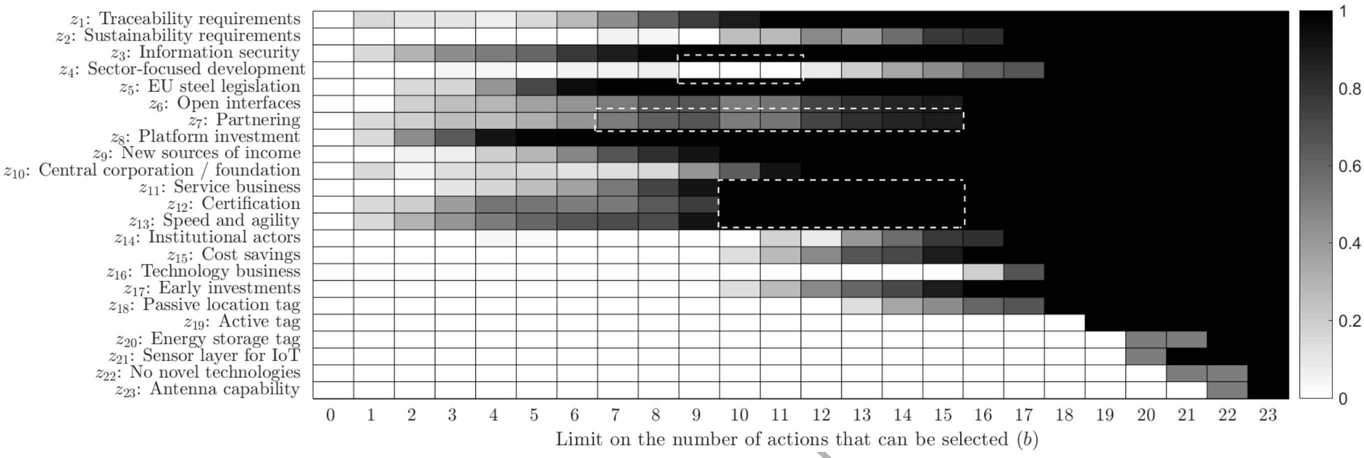
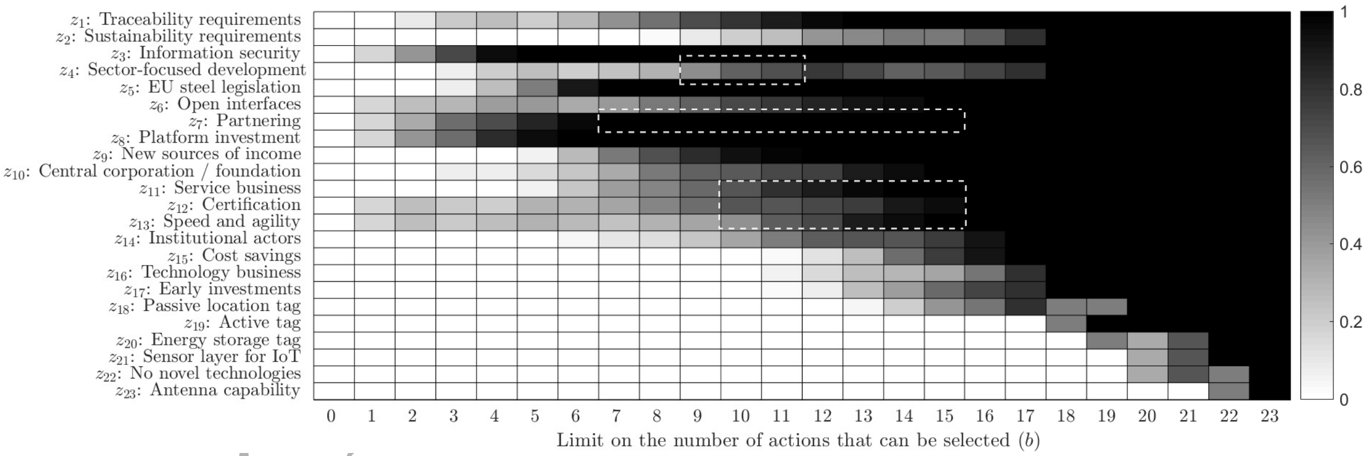


Figure 10: The number of non-dominated portfolios given different limits b on the number of selected actions.

The compositions of the non-dominated portfolios for different values of b are illustrated in Figure 11a by the actions' core indices (cf. Definition 3). The darker the shade of the cell, the higher the core index. All non-dominated portfolios containing fewer than ten actions are combinations of eleven actions: z_1, z_3, and z_5 through z_{13}. Moreover, actions z_5 and z_8 ('EU steel legislation' and 'Platform investment') are included in all non-dominated portfolios that may contain at least seven actions, as is z_3 ('Information security') in all non-dominated portfolios containing at least nine actions.



(a) Actions' core indices, when action-dependent probability information is taken into account.



(b) Actions' core indices, when action-dependent probability information is neglected.

Figure 11: Actions' core indices given different limits b on the number of selected actions.

For comparison, Figure 11b shows the actions' core indices when the actions' impacts on scenario probabilities are neglected. The differences between the non-dominated portfolios resulting from action-dependent probability information and action-independent probability information reflect the possibility of making proactive choices which help steer the future toward the desired scenarios. For instance, without the action-dependent information, z_4 ('Sector-focused development') is a borderline action for all values of b between 9 and 11 (topmost dashed rectangle in Figure 11b). However, if the action-dependent information is taken into account, then z_4 should not be selected within this range of b (topmost dashed rectangle in Figure 11a). This is because z_4 would be an optimal choice in scenario 'Stuck in tar', the probability of which is decreased by selecting actions such as z_3 and z_5 ('Information security' and 'EU steel legislation'). For the same reason, z_7 ('Partnering') is a core action in portfolios containing 7 to 15 actions when action-dependent probability information is neglected, but only a borderline action when action-dependent probability information is taken into account (middle dashed rectangles).

On the other hand, actions z_{11}, z_{12}, and z_{13} ('Service business', 'Certification', and 'Speed and agility') are included in all non-dominated portfolios containing 10 to 15 actions when action-dependent probability information is taken into account, but only in some non-dominated portfolios when such information is neglected (bottom dashed rectangles in Figures 11a and 11b). This is because these three actions are optimal choices in the 'Fast transition' scenario, the probability of which is increased by selecting actions such as z_3 and z_5.

In order to prioritize actions, it was decided that those portfolios which contained $b = 8$ actions (ca. one third of the 23 proposed actions) would be studied in more detail. The core indices corresponding to these 26 non-dominated portfolios are shown in Figure 12. Maximin and minimax regret decision rules were applied to obtain a portfolio-level recommendation, which turned out to be the same for both rules (i.e., $z^{mm} = z^{mmr}$). Actions included in this recommended portfolio are marked with (R). Looking at the actions' scenario-specific performances in Figure 8, it can be seen that the recommended portfolio is a balanced combination of (i) actions that perform relatively well across all scenarios ('Platform investment', 'EU steel legislation', 'Information security', and 'New sources of income') and (ii) actions that perform very well in one scenario ('Open interfaces' and 'Traceability requirements' in 'Internet havens' scenario; 'Speed and agility' in 'Fast transition' scenario; and 'Partnering' in 'Stuck in tar' scenario). Moreover, this portfolio belongs to set $\mathcal{Z}^7$, in which the realization of 'Fast transition' scenario is most likely (Table 2).

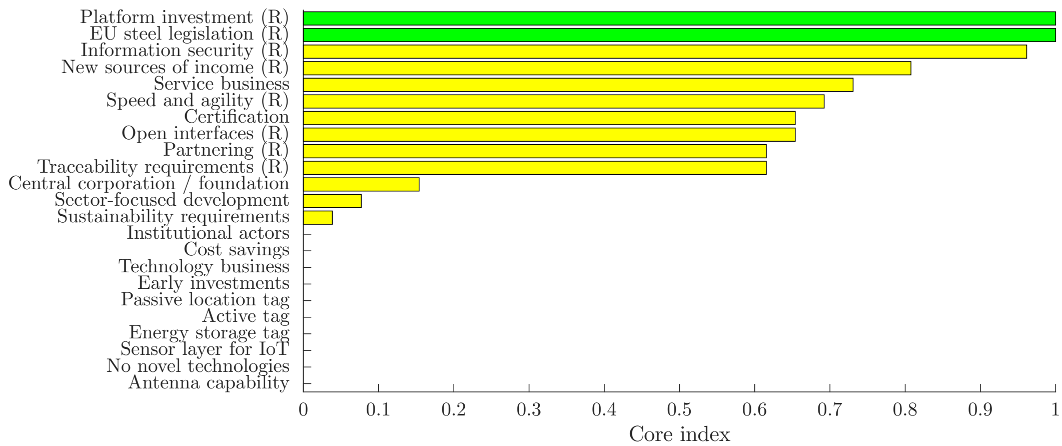


Figure 12: Core indices corresponding to the 26 non-dominated portfolios, when the limit on the number of selected actions is $b = 8$. Actions included in the portfolio recommended by both maximin and minimax regret decision rules are marked with (R).

Finally, to study the sensitivity of our results to the choice of a linear (risk neutral) utility function $u_i(x) = x$, we recomputed the results for a logarithmic (risk averse) utility function $u_i(x) = \ln x$. The actions' core indices for the logarithmic utility function are shown in Figure 13. Comparing Figures 11a and 13 shows that the choice of utility function makes hardly any difference. In fact, the sets of non-dominated portfolios $\mathcal{Z}_N(\mathcal{P})$ are exactly the same for linear and logarithmic utility function whenever at least $b = 10$ actions can be selected.

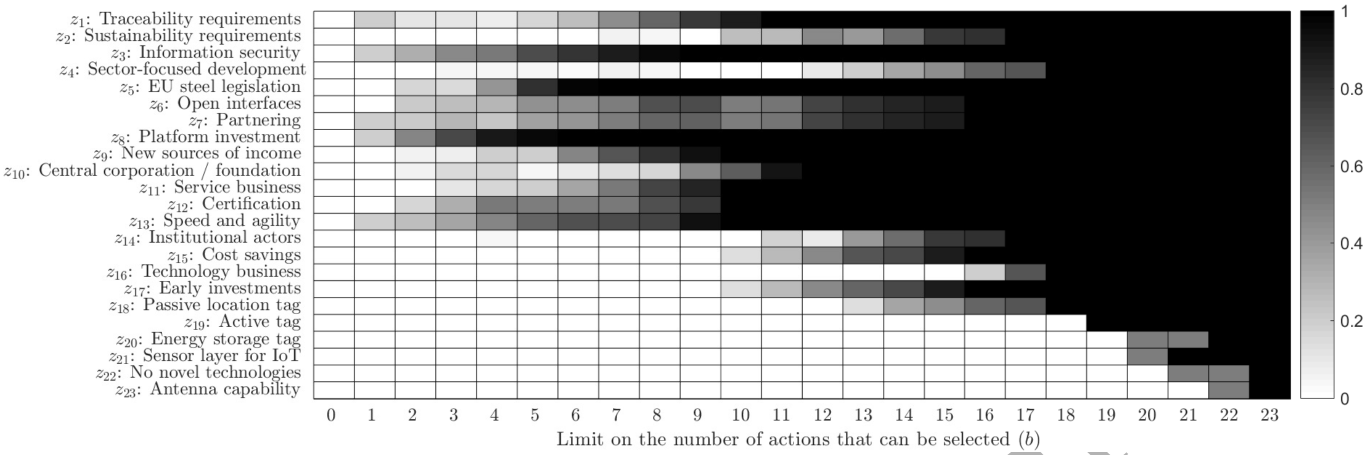


Figure 13: Actions' core indices with logarithmic utility function $u_i(x) = \ln x$.

Experiences from the above process suggest several benefits in using the proposed model to support strategy development. The model was transparent and could be readily explained to managers without a strong mathematical background. Due to the use of incomplete probability information, the managers could provide probability statements they were comfortable with, which is likely to have increased their confidence in the resulting decision recommendations. Moreover, these recommendations could be intuitively justified by comparing the scenario-specific impacts of those actions that were included in most recommended portfolios with those that were included in none. Because the actions were short thematic descriptions rather than detailed execution plans, the generation of several decision recommendations instead of a single 'optimal' action portfolio was appreciated. In fact, given the uncertainties about the future operational environment, the managers were reluctant to make large, irreversible investments in actions whose benefits were highly contingent on how these uncertainties would unfold. Based on the actions' core indices, the managers were able to identify (i) core actions that should be pursued immediately and (ii) borderline actions in which small initial investments should be made to create opportunities for later expansion or abandonment.

6. Discussion and conclusions

To succeed in an unpredictable environment, a company must adopt a robust strategy that can perform well in a variety of possible future environments (Lindsay, 2015; Ilmola and Rovenskaya, 2016). Yet, adopting a purely reactive stance may lead to suboptimal decisions. For instance, in young, high-growth industries with low entry barriers, high innovation rates, and unpredictable demand, a company may be able to radically shift the course of industry development through some innovative move. On the other hand, a mature industry that is either fragmented, or stagnant and ripe for disruption, is likely to be similarly malleable (Reeves *et al.*, 2012). In such cases a company can be better off by executing a proactive strategy through which it seeks to shape the operational environment toward the desired direction (Hagel III *et al.*, 2008).

In this paper, we have developed a scenario-based portfolio model to support the building of a strategy that is (i) robust in that it performs relatively well across the possible future scenarios and (ii) proactive in that it helps steer the course of change toward the desired scenario. In particular, our model generates recommendations about which portfolio of strategic actions should be selected when information about scenario probabilities may be incomplete and depend on selected actions. This model can account for factors that make traditional methods of, e.g., cost-benefit analysis difficult to apply, namely (i) different kinds of action interdependencies, including balance constraints and logical interdependencies, and (ii) actions that yield utility only indirectly through increasing the probability of the most desirable scenarios. To facilitate decision-making, the model helps identify core actions that should definitely be pursued and exterior actions that should not, after which further discussion can be concentrated on a smaller set of borderline actions.

Importantly, recommendations about individual actions as well as entire action portfolios can be generated even in the absence of information about scenario probabilities. In this case, the set of non-dominated portfolios consists of Pareto optimal portfolios, i.e., those that are not outperformed by any other portfolio in each scenario (Goodwin and Wright, 2001; Montibeller *et al.*, 2006; Stewart *et al.*, 2013). Moreover, this set contains decision recommendations suggested by scenario models that do not use scenario probabilities, such as the portfolio that yields the smallest worst-case regret in scenario-specific portfolio utility (Lempert *et al.*, 2006; Ram *et al.*, 2011). Yet, if information about scenario probabilities is elicited during the strategy process, the updated set of non-dominated portfolios can be computed on the fly by carrying out pairwise dominance checks among the non-dominated portfolios in the original set. This makes it possible to provide interactive decision support in workshops, for instance.

There are several avenues for future work. First, the model could be extended to explicitly account for multi-period portfolio selection processes in which the DM has the opportunity to revisit the initial selection decision later (cf. Huchzermeier and Loch, 2001; Kettunen *et al.*, 2010; Vilkkumaa *et al.*, 2015). Second, uncertainty about the actions' impacts in different scenarios could be accommodated. Earlier work on this subject suggests that, to curtail the complexity of the model, such action-specific uncertainties should be characterized as intervals (Liesiö *et al.*, 2008; Vilkkumaa *et al.*, 2014b), instead of dividing the scenarios further into sub-scenarios, each of which would correspond to specific realizations of these action-specific uncertainties. Finally, the model could be integrated with a game-theoretic framework to support the selection of actions when the actions' impacts and scenario probabilities can be affected by the actions of others. Such an integrated framework could support corporate strategy development, in which key uncertainties often relate to the actions of competitors.

Acknowledgements

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Appendix A. Proofs

Proof of Lemma 1: Assume $z \succ_{\mathcal{P}} z$ and $z' \succ_{\mathcal{P}} z''$. Then for any $P \in \mathcal{P}$: $\mathrm{EU}(z, P_{\kappa(z),\cdot}) \geq \mathrm{EU}(z', P_{\kappa(z'),\cdot}) \geq \mathrm{EU}(z'', P_{\kappa(z''),\cdot})$. Also, there exists $P \in \mathcal{P}$ such that $\mathrm{EU}(z, P_{\kappa(z),\cdot}) > \mathrm{EU}(z', P_{\kappa(z'),\cdot}) \geq \mathrm{EU}(z'', P_{\kappa(z''),\cdot})$. These inequalities together imply that $z \succ_{\mathcal{P}} z''\square$.

Proof of Theorem 1: We show that the two conditions of Definition 1 for portfolios z and z' hold if and only if the minimization and maximization conditions of Theorem 1 hold.

$$\begin{aligned}
\mathrm{EU}(z, P_{\kappa(z),\cdot}) \geq \mathrm{EU}(z', P_{\kappa(z'),\cdot}) \quad \forall P \in \mathcal{P} \quad &\Leftrightarrow \\
D(P) \geq 0 \quad \forall P \in \mathcal{P} \quad &\Leftrightarrow \\
\min_{P\in\mathcal{P}} D(P) \geq 0 \quad &\Leftrightarrow \\
\min_{P\in\mathrm{ext}(\mathcal{P})} D(P) \geq 0, &
\end{aligned}$$

where $D(P) = \sum_{i=1}^n P_{\kappa(z),i} \cdot u_i(\sum_{j=1}^m z_j x_i^j) - \sum_{i=1}^n P_{\kappa(z'),i} \cdot u_i(\sum_{j=1}^m z'_j x_i^j)$, and the last equivalence follows from that fact that the sets of the row vectors $P_{\kappa(z),\cdot}$ of $P \in \mathcal{P}$ are convex polytopes, and $\sum_{i=1}^n P_{\kappa(z),i} \cdot u_i(\sum_{j=1}^m z_j x_i^j)$ is linear in $P_{\kappa(z),\cdot}$. These convexity and linearity properties also imply the second condition, i.e.,

$$\begin{aligned}
\exists P \in \mathcal{P} \text{ s.t.} \quad \mathrm{EU}(z, P_{\kappa(z),\cdot}) > \mathrm{EU}(z', P_{\kappa(z'),\cdot}) \quad &\Leftrightarrow \\
\exists P \in \mathcal{P} \text{ s.t.} \quad D(P) > 0 \quad &\Leftrightarrow \\
\max_{P\in\mathcal{P}} D(P) > 0 \quad &\Leftrightarrow \\
\max_{P\in\mathrm{ext}(\mathcal{P})} D(P) > 0.\square &
\end{aligned}$$

Proof of Theorem 2: Assume contrary to the claim that $\exists z' \in \mathcal{Z}_N(\tilde{\mathcal{P}}),\ z' \notin \mathcal{Z}_N(\mathcal{P})$. Then, $\exists z \in \mathcal{Z}_F$ such that $z \succ_{\mathcal{P}} z'$, which is equivalent to

$$D(P) \geq 0 \quad \forall P \in \mathcal{P} \quad \wedge \quad \exists P \in \mathcal{P} \text{ s.t. } D(P) > 0. \tag{A.1}$$

Because $\tilde{\mathcal{P}} \subseteq \mathcal{P}$, it holds that $D(P) = \sum_{i=1}^n P_{\kappa(z),i} \cdot u_i(\sum_{j=1}^m z_j x_i^j) - \sum_{i=1}^n P_{\kappa(z'),i} \cdot u_i(\sum_{j=1}^m z'_j x_i^j) \geq 0\ \forall P \in \tilde{\mathcal{P}}$.

By assumption, there exists $\hat{P} \in \mathrm{int}(\mathcal{P}) \cap \tilde{\mathcal{P}}$, and by (A.1) there exists $P^* \in \mathcal{P}$ s.t. $D(P) = \sum_{i=1}^n P_{\kappa(z),i} \cdot u_i(\sum_{j=1}^m z_j x_i^j) - \sum_{i=1}^n P_{\kappa(z'),i} \cdot u_i(\sum_{j=1}^m z'_j x_i^j) > 0$. Let $P^\circ = \hat{P} + \varepsilon(\hat{P} - P^*)$. Because $\hat{P} \in \mathrm{int}(\mathcal{P})$, $\exists \varepsilon$ such that $P^\circ \in \mathcal{P}$. By rearranging the terms we have

$$\hat{P} \;=\; \frac{1}{1+\varepsilon} P^\circ + \frac{\varepsilon}{1+\varepsilon} P^* \equiv \alpha P^\circ + \beta P^*.$$

Note that $\alpha, \beta > 0$. But then,

$$D(\hat{P}) = D(\alpha P^\circ + \beta P^*) = \alpha D(P^\circ) + \beta D(P^*) > 0,$$

where the second equality follows from the linearity of $D(P)$ in P, and the last inequality from $D(P^\circ) \geq 0$ by (A.1), $D(P^*) > 0$, and $\alpha, \beta > 0$. This last inequality, together with the fact that $D(P) \geq 0\ \forall P \in \tilde{\mathcal{P}}$, implies that $z \succ_{\tilde{\mathcal{P}}} z'$. Thus, $z' \notin \mathcal{Z}_N(\tilde{\mathcal{P}})$, which is a contradiction$\square$.

Proof of Corollary 1: The result follows directly from the result $\mathcal{Z}_N(\tilde{\mathcal{P}}) \subseteq \mathcal{Z}_N(\mathcal{P})$ of Theorem 2$\square$.

Proof of Lemma 2: Assume $z \in \mathcal{Z}_N(\mathcal{P}) \subseteq \mathcal{Z}_F$. By Definition 2, $\nexists z' \in \mathcal{Z}_F$ such that $z' \succ_{\mathcal{P}} z$. Because $\mathcal{Z}^1 \cup \ldots \cup \mathcal{Z}^K = \mathcal{Z}_F$, there exists k such that $z \in \mathcal{Z}^k$. Now, $\nexists z' \in \mathcal{Z}^k \subseteq \mathcal{Z}_F$ such that $z' \succ_{\mathcal{P}} z$, whereby

$z \in \mathcal{Z}_N^k(\mathcal{P})\square$.

Proof of Lemma 3: Because $z, z' \in \mathcal{Z}^k$, we have $P_{\kappa(z),\cdot} = P_{\kappa(z'),\cdot} = P_{k,\cdot}$. Thus by Theorem 1, $z \succ_{\mathcal{P}} z'$ if and only if $\min_{P\in\text{ext}(\mathcal{P})} D(P) \geq 0$ and $\max_{P\in\text{ext}(\mathcal{P})} D(P) > 0$, where

$$D(P) = \left(\sum_{i=1}^{n} P_{k,i} u_i(\sum_{j=1}^{m} z_j x_i^j) - \sum_{i=1}^{n} P_{k,i} u_i(\sum_{j=1}^{m} z'_j x_i^j)\right)$$

Furthermore, since $\mathcal{P} = \Delta_K^n$, for any extreme point matrix $P \in \text{ext}(\Delta_K^n)$ there exists one scenario i^* such that $P_{k,i^*} = 1$ while $P_{k,i} = 0$ for all $i \neq i^*$. Hence, we may write

$$\min_{P\in\text{ext}(\Delta_K^n)} D(P) = \min_{i\in\{1,\ldots,n\}} \left(u_i(\sum_{j=1}^{m} z_j x_i^j) - u_i(\sum_{j=1}^{m} z'_j x_i^j)\right)$$
$$\max_{P\in\text{ext}(\Delta_K^n)} D(P) = \max_{i\in\{1,\ldots,n\}} \left(u_i(\sum_{j=1}^{m} z_j x_i^j) - u_i(\sum_{j=1}^{m} z'_j x_i^j)\right).$$

Therefore, $\min_{P\in\text{ext}(\Delta_K^n)} D(P) \geq 0$ holds if and only if

$$u_i(\sum_{j=1}^{m} z_j x_i^j) \geq u_i(\sum_{j=1}^{m} z'_j x_i^j) \ \forall\ i \in \{1,\ldots,n\} \Leftrightarrow \sum_{j=1}^{m} z_j x_i^j \geq \sum_{j=1}^{m} z'_j x_i^j \ \forall\ i \in \{1,\ldots,n\},$$

since each u_i is a strictly increasing function. Similarly, $\max_{P\in\text{ext}(\Delta_K^n)} D(P) > 0$ holds if and only if there exists $i \in \{1,\ldots,n\}$ such that $\sum_{j=1}^{m} z_j x_i^j > \sum_{j=1}^{m} z'_j x_i^j$, which proves the Lemma$\square$.

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