A method for calculation of program package elements for singular clusters

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1. Problem statement

Let us consider a linear dynamic controlled system, described by an ordinary differential equation

$$x(t) = A(t)x(t) + B(t)u(t) + f(t), \quad t \leq 0 \leq T$$ (1)

Open-loop control (program) $u_c(t)$ is a measurable function on $[0, T] 	imes P$, $f(t)$ is a convex compact set. The initial state of the system $x(0) \in X_0$, $X_0 \in R^n$, where $X_0$ is a finite known set. The terminal condition $x(T) \in M_R \subset R^m$, where $M$ is a closed and convex set, should hold.

A linear signal $y_c = Qx(t)$, where $Q(t)$ is left piecewise continuous matrix function, $Q(t) \in R^{m \times n}, t \in [0, T]$, is observed by the controlling side.

The problem of positional guidance is formulated as follows - based on the given arbitrary $\gamma > 0$ choose a closed-loop control strategy with memory, whatever the system’s initial state $x(0)$, the system’s motion $x(t)$, corresponding to the chosen closed-loop strategy and starting at the time $t_0$ from the state $x(t_0)$ reaches the state $x(t_0)$ belonging to the $\gamma$-neighbourhood of the target set $M$ at the time $t$.

2. Program packages method

Homogeneous system, corresponding to (1)

$$x(t) = A(t)x(t)$$

For each $x_0 \in X_0$ its solution is given by the Cauchy formula:

$$x(t) = F(t, t_0)x_0, t \in [0, T], \text{ is the fundamental matrix.}$$

Homogeneous signal, corresponding to an admissible initial state $x_0 \in X_0$:

$$y_c(t) = Q(t)x(t), \quad t \in [0, T].$$

Let $G = \{x(t) | x(t) \in X_0\}$ be the set of all admissible signals and $\{x(t)\}$ be the set of all admissible initial states $x_0 \in X_0$, corresponding to the homogeneous signal $y_c(t) \in G$ till time point $t \in [0, T]$.

$$X_0(y_c(\cdot)) = \{x_0 \in X_0 | x(\cdot) \text{ corresponding to } y_c(\cdot) \text{ is found such that} \text{ at time point } t \in [0, T]\}.$$  

Program package $(x_0^u, y^u_c)$ is an open-loop controls family $(x_0^u(t), y^u_c(t))$ satisfying non-anticipatory condition: for any homogeneous signal $y_c$, any time $t \in [0, T]$ and any admissible initial states $x_0 \in X_0$ the equality $u^u_c(t) = u^u(t)$ holds for almost all $t \in [0, T]$.

3. Algorithm for singular clusters

An algorithm for the package guidance problem solution is proposed in paper [4], however, in some cases (so-called singular clusters of the initial states set) it may not be applied, because the minimum condition degenerates. In this work an additional algorithm for tackling such cases is presented.

At a step $i = 0, 1, \ldots$ of the algorithm for arbitrary cluster $X_0(x_0(t)), x_0(t) = 1, \ldots, R(t)$, $k = 1, \ldots, K$ a non-empty set $w_0$ of indices is estimated $j, 1 \leq j \leq m$ such that $j \in w_0$ corresponds non-empty set $y_j \subset \{x_1, \ldots, x_n\}$ such that

$$\sum_{x_0 \in (X_0 \cap Y_j)} D_{2j}^u(0) \geq \varepsilon (t \in Y_j).$$ (2)

The problem of estimation of the components $(\omega_{\omega_{\omega_{\omega}}}(t))(j)$ of the guiding program package elements on the sets $s_j, j \in w_0$ reduces to the following system of equations:

$$\begin{cases}
\sum_{x_0 \in (X_0 \cap Y_j)} f_{1j}(x_0) = e_{1j}, \\
\sum_{x_0 \in (X_0 \cap Y_j)} f_{2j}(x_0) = e_{2j},
\end{cases}$$  

(3)

where

$$e_{1j} = \|x(t) - x_0\|, \quad e_{2j} = \|x(t) - x_0\|$$

Let us introduce functions

$$\mu_{1j} = \langle x_0, \mu_{1j} \rangle, \quad \mu_{2j} = \langle x_0, \mu_{2j} \rangle, \quad \mu_{1j}, \mu_{2j} \in R^m$$

where

$$\begin{cases}
\mu_{1j} = (\mu_{1j}(1), \ldots, \mu_{1j}(m)), \\
\mu_{2j} = (\mu_{2j}(1), \ldots, \mu_{2j}(m)).
\end{cases}$$

The input of the algorithm is

$$\mu_{1j} = (1, \ldots, \mu_{1j}(m)), \quad \mu_{2j} = (1, \ldots, \mu_{2j}(m)) \in \mathbb{R}^m$$

After the step $h = 1, 2, \ldots$ of the algorithm the following is known

$$\mu_{1j}(1), \ldots, \mu_{1j}(m), \mu_{2j}(1), \ldots, \mu_{2j}(m)$$

Let us describe the step $h = 1, 2, \ldots$ of the algorithm.

1. The following approximation is calculated

$$\begin{cases}
\mu_{1j}(1), \ldots, \mu_{1j}(m), \\
\mu_{2j}(1), \ldots, \mu_{2j}(m)
\end{cases}$$  

(4)

4. The set of indices $\omega_{\omega_{\omega_{\omega}}}(t)$ is estimated and for each $j \in \omega_{\omega_{\omega_{\omega}}}(t)$ the equality $u_{\omega_{\omega_{\omega_{\omega}}}}(t) = u_{\omega_{\omega_{\omega_{\omega}}}}(t)$ holds for almost all $t \in [0, T]$.

5. On the sets $(\omega_{\omega_{\omega_{\omega}}}(t))(j)$ of $j \in \omega_{\omega_{\omega_{\omega}}}(t)$ the components $(\omega_{\omega_{\omega_{\omega}}}(t))(j)$ of the guiding program package elements are

$$\omega_{\omega_{\omega_{\omega}}}(t) = \arg\max \sum_{x_0 \in (X_0 \cap Y_j)} D_{2j}^u(x_0).$$

6. If $m(t) = m(t) \bigcup_{j \in \omega_{\omega_{\omega_{\omega}}}} (t) \neq 0$, then $(\omega_{\omega_{\omega_{\omega}}}(t))(j)$ is known on the whole set $(\omega_{\omega_{\omega_{\omega}}}(t))(j)$, and the problem is solved. Otherwise the algorithm proceeds to the next step.

7. The following approximation is calculated

$$\begin{cases}
\sum_{j \in \omega_{\omega_{\omega_{\omega}}}(t)} u_{\omega_{\omega_{\omega_{\omega}}}}(t) = u_{\omega_{\omega_{\omega_{\omega}}}}(t), \\
\sum_{j \in \omega_{\omega_{\omega_{\omega}}}(t)} D_{2j}^u(x_0) = D_{2j}^u(x_0)
\end{cases}$$  

(5)

and the algorithm proceeds to the step $h = 1 + 1$.

4. Example

Let us consider a linear control system

$$x_{\omega_{\omega_{\omega}}}(t) = x_{\omega_{\omega_{\omega}}}(t)$$

on the time segment $[0, 1]$.

$$\begin{cases}
m(t) = \begin{cases}
1, & j \in \omega_{\omega_{\omega}}(t), \\
0, & j \notin \omega_{\omega_{\omega}}(t)
\end{cases}, \\
\omega_{\omega_{\omega}}(t) = \omega_{\omega_{\omega}}(t)
\end{cases}$$  

(6)

where

$$\begin{cases}
x_{\omega_{\omega_{\omega}}}(t) = 1, & j \in \omega_{\omega_{\omega}}(t), \\
x_{\omega_{\omega_{\omega}}}(t) = 0, & j \notin \omega_{\omega_{\omega}}(t)
\end{cases}$$

(7)

Since the solvability criterion holds, the package guidance problem is solvable. Using the algorithm from [4] the guiding program package elements for $s_1$ and $s_2$ can be calculated. However, for the cluster $X_0 = \{x_0, x_1\}$ on the time interval $[0, 1]$ there is an ambiguity in the minimum condition and the standard algorithm is not applicable. The presented method allows to reduce the problem to the system of two integral equations:

$$\begin{cases}
x_{\omega_{\omega}}(t) = 1, & j \in \omega_{\omega}(t), \\
x_{\omega_{\omega}}(t) = 0, & j \notin \omega_{\omega}(t)
\end{cases}$$

(8)

Its solution gives $x_{\omega_{\omega}}(t) = 1, t \in [0, 1]$.  

References