

# Co-inform

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**A decision evaluation tool:  
Multi-Criteria Decision Analysis as  
Co-Creation Methodology**

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# 1. Introduction

This report is a part of the work package 5 (**WP5**), which is concerned with building and using policies for intervention strategies and actions in persuading a misinformation-resilient behaviour in citizens, and in supporting evidence-based policy making and journalists' professional practices. The assessment tasks are based on information received from co-creation activities with three key stakeholder groups (citizens, journalists, policy makers), which will primarily take place in Austria, Greece, and Sweden, as part of **WP1**. These assessments are executed using a variety of qualitative and quantitative, formative and summative evaluation techniques, seen from a contextual inquiry approach; outcomes are fed back into the Co-Inform policies (**WP2**), and designs and developments (**WP3, WP4**).

This report describes the integrated decision analytical model developed in T5.4 for evaluating the impact of Co-Inform output on the formation of policies. The methodology includes two steps. The first is a preference elicitation in a workshop setting and the second is an evaluation of the result using elaborated algorithms for multi-stakeholder multi-criteria decisions against under uncertainty. Below, we describe the methodology setup for the workshops performed during the project as well as provide some theoretical background.

The report also includes scientific novel approaches which were developed on the methodology in frames of the project:

- a new methodological approach when eliciting the user criteria
- a new method for handling aggregated distribution
- a new method for checking consistency in user statements

## 2. Background

### 2.1. Elicitation Process

The first phase concerns how to elicit stakeholder preferences. This is done in a workshop format. A typical workshop will last an entire day and will include five main sessions. This consists of the following sessions:

1. The first session starts with the introduction of the overall process, during which the organizers will present the workshop and its objectives as well as the goals of the workshops and the agenda. The policies are explained, and the organizers provide a list of suggested attributes and characteristics under which they can be evaluated, where after the various features are discussed. Participants may also suggest further modifications and extensions.
2. During the second session, the participants have an opportunity to provide a more structured policy feedback regarding modifications, redefinitions and other aspects, followed by a discussion of positive and negative sides of the policy options with respect to these. Other possible policy options, modifications or extensions of the prevailing ones can be introduced.
3. During the third session, the participants discuss more thoroughly the desired features of the policies and possible extensions thereof. All prevailing criteria are discussed. A result of this session is a list of features important for the participants. These features are the criteria for the subsequent MCDA analysis.

4. During the fourth session, the participants rank the (possibly modified) policies *under each criteria* (feature). The participants write down the names of all this on coloured cards, put these on a flipchart and explain their choices.
5. During the fifth session, the criteria are ranked in the same way as the options. Each criterion is first discussed to make sure that participants agree on their definitions. The participants may also change and add further criteria.

## 2.2. Rankings of Policies and Criteria

During sessions 4 and 5, the group of stakeholders are provided with a set of coloured cards with the criteria (policies') names written on them. They are also given a set of blank cards. Then, they are asked to rank the coloured cards from the least important (valuable) to the most important (valuable). The moderator then introduces extra cards without text and explains that they show the relative difference in importance (value) of different criteria (policies). The greater the difference in importance (value) between two criteria (policies), the more blank cards should be positioned in-between these criteria (policies). The semantics for the criteria importance could be as suggested in Table 1. The policy valuation is analogous.

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**Table 1: Blank cards**

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Equal level of cards	Equally important
No blank card	Slightly more important
One blank card	More important (clearly more important)
Two blank cards	Much more important
Three blank cards	Extremely more important

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Table 1. Suggested semantics for the criteria ranking

Following this, a discussion to identify lines of conflicting opinions should follow and be documented. An example of a final result can be seen in Figure 1. The meaning is that criterion 1 is much more important than criterion 3 that, in turn, is equal to criterion 4. Both of these are more important than criterion 2, which is slightly more important than criterion 5.

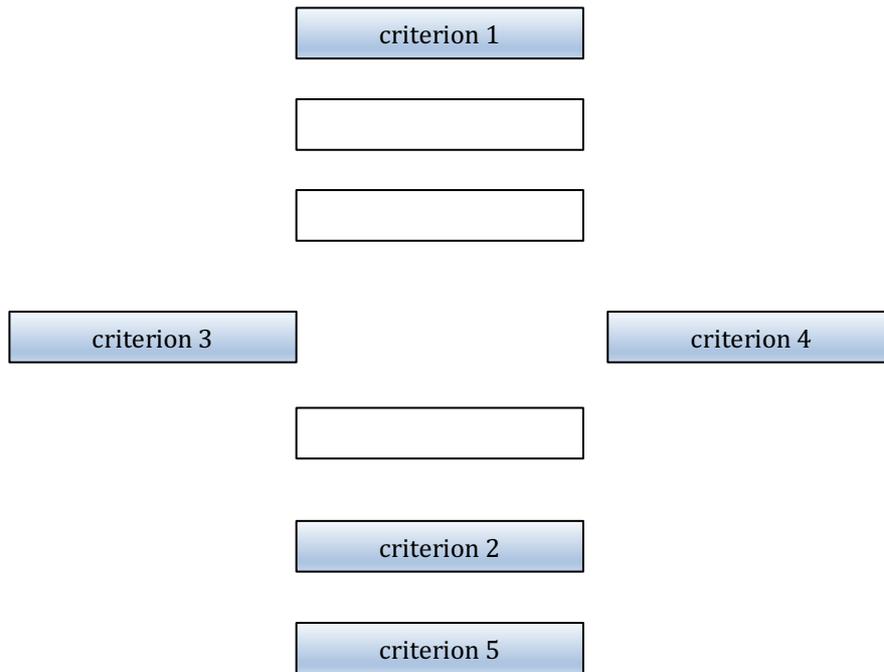


Figure 1. Example of a final criteria ranking

Rankings of the alternatives under each criterion are handled analogously. Figure 2 shows alternatives' ranking under the three first criteria. The meaning is that under criterion 1, alternative 3 is slightly better than alternative 2, that, in turn, is slightly better than alternative 1, and that alternative one is better than alternative 4.

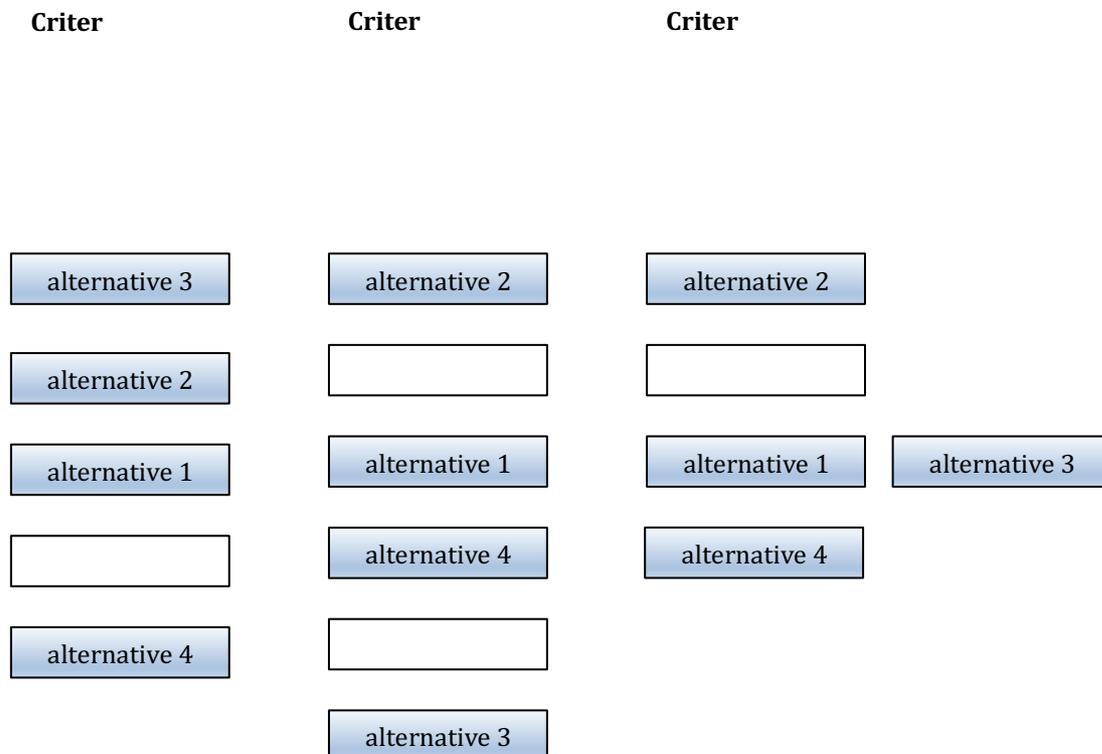


Figure 2. Part of an alternative ranking under some criteria

Thereafter, the entire decision structure can be analysed and tentatively discussed.

## 2.3. Ways forward

### a. Workshop summary

In short, the workshop thus consists of the following: components

1. The policies as well as the criteria structure are presented, i.e., the focus is at the goal and strategy parts. The elicitation is done by stakeholders during workshops, thereafter these are categorized by a moderator in dialogue with the concerned parties.
2. Preference elicitation, where the policies are ranked under each criterion.
3. Weight elicitation, where the criteria are ranked.
4. Documentation of conflicting opinions and rankings.

### b. Theoretical Background

The main theoretical innovation herein is:

- a new methodological approach when eliciting the user criteria
- a new method for handling aggregated distribution
- a new method for checking consistency in user statements

This is presented in detail in [Danielson and Ekenberg, 2019, 2020; Danielson et al, 2019ab].

### c. Model Components

The entire background framework is a decision analytical approach for co-creation in a multi-stakeholder/multi-criteria environment, supported by elaborated decision analytical tools and processes:

- a framework for elicitation of stakeholder preferences
- a decision engine for strategy evaluation
- a machinery for risk analysis
- a set of processes for negotiation
- a set of decision rule mechanisms
- processes for combining these items
- various types of implementations of the above

These components apply to decision components, such as:

- agenda settings and overall processes
- stakeholders
- goals
- strategies/policies/sub-strategies/part-policies, etc
- consequences/effects

- qualifications and sometimes quantifications of the components
- negotiation protocols
- decision rules and processes

A significant subset of this is utilised during WS5.

### 3. Methodology: Multi-Criteria Decision Analysis (MCDA)

Deployment of changed socio-economic conditions must lead to transitions and transformations of entire sectors [Komendantova et al, 2018a]. Such transitions are complex processes, which has political, social, economic and technical dimensions and involve a multitude of stakeholders. Therefore, a holistic, inclusive and comprehensive governance approach to such is essential, since unguided significant socio-technical will lead to many frictions and conflicts. Such changes will thus lead to a socio-technological transition processes, which are combined with and emphasised by shifts in technologies, business models, governance structures, consumption patterns, values and worldviews. Thus, such multi-stakeholder, multi-criteria situations are typical for the planning and decision processes involved herein and a significant issue is of course what methodologies to use.

A multitude of methods for analysing and solving decision problems with multiple criteria have been suggested during the last decades. A common approach is to make preference assessments by specifying a set of attributes that represents the relevant aspects of the possible outcomes of a decision. Value functions are then defined over the alternatives for each attribute and a weight function is defined over the attribute set. One option is to simply define a weight function by fixed numbers on a normalised scale and then define value functions over the alternatives, where these are mapped onto fixed values as well, after which these values are aggregated and the overall score of each alternative is calculated.

One of the problems with the additive model as well as other standard multiple criteria models is that numerically precise information is seldom available, and most decision-makers experience difficulties with entering realistic information when analysing decision problems, and with the elicitation of exact weights, that demands an unreasonably exactness which does not exist. There are other problems, such as that ratio weight procedures are difficult to accurately employ due to response errors. The common lack of reasonably complete information increases this problem significantly. Several attempts have been made to resolve this issue. Methods allowing for less demanding ways of ordering the criteria, such as ordinal rankings or interval approaches for determining criteria weights and values of alternatives, have been suggested, but the evaluation of these models is sometimes quite complicated and difficult for decision makers to accept.

Some main categories of approaches to remedy the precision problem are based on capacities, sets of probability measures, upper and lower probabilities, interval probabilities (and sometimes utilities), evidence and possibility theories, as well as fuzzy measures. The latter category seems to be used only to a limited extent in real-life decision analyses since it usually requires a significant mathematical background on the part of the decision-maker. Another reason is that the computational complexity can be problematic if the fuzzy aggregation mechanisms are not significantly simplified.

For the evaluations herein, we will therefore utilise a method and software for integrated multi-attribute evaluation under risk, subject to incomplete or imperfect information. The software originates from our earlier work on evaluating decision situations using imprecise utilities, probabilities and weights, as well as qualitative estimates between these components derived from convex sets of weight, utility and probability measures. To avoid some aggregation problems when handling set membership functions and similar, we introduce higher-order distributions for better discrimination between the possible outcomes. For the decision structure, we use the common tree formalism but refrain from using precise numbers. To alleviate the problem of overlapping results, we suggest a new evaluation method based on a resulting belief mass over the output intervals, but without trying to introduce further complicating aspects into the decision situation. During the process, we consider the entire range of values as the alternatives presented across all criteria as well how plausible it is that an alternative outranked the remaining ones, and thus provided a robustness measure. Because of the complexity in these calculations, we use the state-of-the-art multi-criteria software DecideIT or Decision Wizard for the analysis, which allows for imprecision of the kinds that exist here [Danielson and Ekenberg, 2007]. Versions of DecideIT have been successfully used in a variety of decision situations [Ekenberg et al, 2018], such as larch-scale energy planning [Komendantova et al, 2018b], allocation planning [Larsson et al, 2018], demining [Ekenberg et al, 2018], financial risks [Danielson and Ekenberg, 2018], gold mining [Mihai et al, 2015] and many others.

Figure 3 shows of the multi-criteria multi-stakeholder tool Decision Wizard, developed for group decisions regarding infrastructure policy making [Larsson et al, 2018] in Swedish municipalities, using the CAR method of [Danielson and Ekenberg, 2016c].

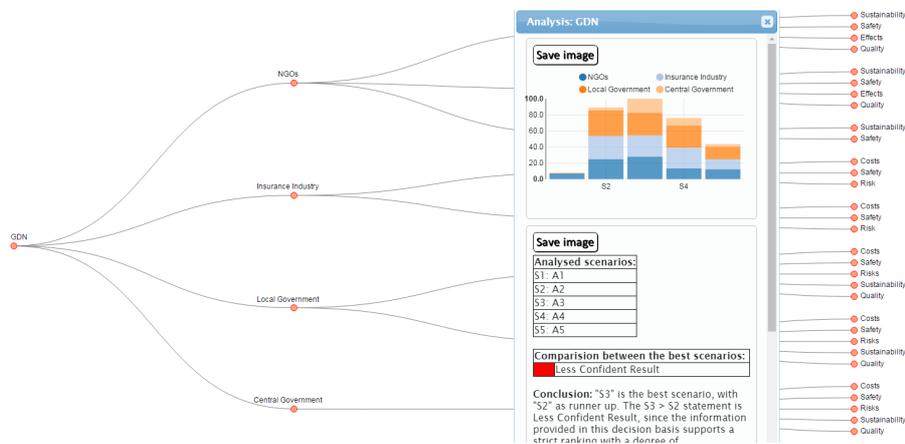


Figure 3. The Group Decision tool Decision Wizard – a simplification of DecideIT

### 3.1. Decision Modelling

Typically, a multi-criteria decision situation is modelled like a tree, such in the figure below, where the  $w$ :s are criteria weights and the  $v$ :s are values of alternatives under the different criteria.

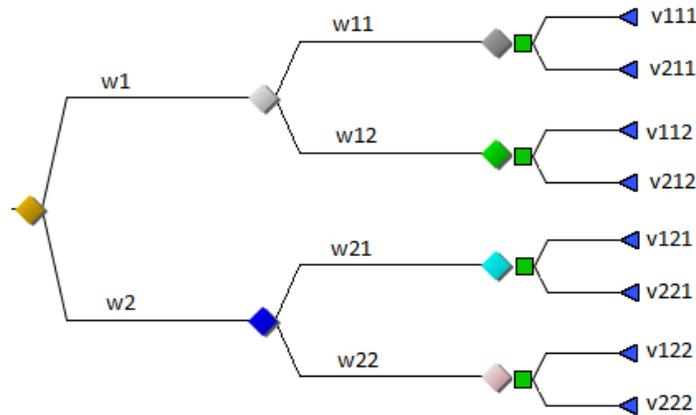


Figure 4. A multi-criteria decision tree

The normalisation constraint means that the weights are restricted by the equation  $\sum w_j = 1$ , where  $w_j$  denotes the weight of a criterion  $G_j$  and the weight of sub-criterion  $G_{jk}$  is denoted by  $w_{jk}$ . Denote the value of alternative  $a_i$  under sub-criterion  $G_{jk}$  by  $v_{ijk}$ .

A common value function for evaluating alternatives in the analyses is a weighted average of the components involved. For instance, consider an alternative  $A_i$  under two criteria, with the respective weights  $w_1$  and  $w_2$ . The overall value of this alternative can be calculated by a weighted average:

$$E(A_i) = \sum_{j=1}^2 w_j \sum_{k=1}^2 w_{jk} v_{ijk}$$

This can straightforwardly be generalized to multi-criteria decision trees of arbitrary depth and solved as corresponding multi-linear equations.

One of the problems with most models for criteria ranking is that numerically precise information is seldom available. We have solved this in part by introducing surrogate weights as before. This, however, is only a part of the solution since the elicitation can still be uncertain and the surrogate weights might not be a fully adequate representation of the preferences involved, which of course, is a risk with all kinds of aggregations. To allow for analyses of how robust the problem is to changes of the input data, we will also introduce intervals around the surrogate weights as well as around the values of the options. Thus, in this elicitation problem, the possibly incomplete information is handled by allowing the use of intervals (cf., e.g., [Danielson et al, 2013]), where ranges of possible values are represented by intervals (in combination with pure orderings without the use of surrogate weights at all, if the latter turns out to be inadequate).

There are thus several approaches to elicitation in MCDA problems, and one partitioning of the methods into categories is how they handle imprecision in weights and values, such as fixed numbers, comparative statements, representing orderings or intervals.

Computationally, methods using fixed numbers are very easy to solve, while systems of relational or interval constraints normally require more elaborated optimization techniques. On the other hand, if the model only accepts fixed numbers, we impose constraints that might severely affect the decision quality. If we allow for imprecision in terms of intervals and relations, we usually get a more realistic representation of the problem. These can, for instance, be represented by interval statements, such as  $w_i \in [y_i - a_i, y_i + b_i]$ , where  $0 < a_i \leq 1$  and  $0 < b_i \leq 1$ , or comparative statements, such as  $w_i \geq w_j$ .

Systems of such equations can be solved, and aggregations of decision components in these formats can be optimized, by using the methods from [Danielson and Ekenberg, 2007]. The disadvantage here is that many decision-makers sometimes perceive these methods difficult to understand and accept, because of complex computations and loss of user transparency.<sup>1</sup>

In this case, the performance of the different alternative options is estimated. Together with the surrogate weights, they thus provide the decision base for the multi-criteria analysis. Using the weighted aggregation principle, we will combine the multiple criteria and stakeholder preferences with the valuation of the different options under the criteria surrogate weights. This will be further described below.

The results of the process are (i) a detailed analysis of each option's performance compared with the others, and (ii) a sensitivity analysis to test the robustness of the result.

During the process, we consider the entire range of values as the alternatives presented across all criteria as well how plausible it is that an alternative will outrank the remaining ones, and thus provide a robustness measure.

### 3.2. Criteria ranking

Barron and Barrett [1996a] argue that the elicitation of exact weights demands an unreasonably exactness which does not exist. There are other problems, such as that ratio weight procedures are difficult to accurately employ due to response errors [Barron and Barrett [1996b]]. The common lack of reasonably complete information increases this problem significantly. Several attempts have been made to resolve this issue. Methods allowing for less demanding ways of ordering the criteria, such as ordinal rankings or interval approaches for determining criteria weights and values of alternatives, have been suggested, but the evaluation of these models is sometimes quite complicated and difficult for decision makers to accept.

The utilisation of ordinal or imprecise importance information to determine criteria weights is a way of handling this and some authors have suggested surrogate weights as representative numbers assumed to represent the most likely interpretation of the preferences expressed by a decision-maker or a group of decision-makers. The idea is to enable decision-makers to utilise the information they are able to supply and then generate representative weights from some underlying distribution and investigate how well they perform. One such type is derived from ordinal importance information [Barron, 1992; Jia et al, 1998; Stilwell et al, 1981;

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<sup>1</sup> This should be kept in mind here as always when working with aggregation methods of whatever kind and this should affect how the elicitation mechanisms and policies that are used.

Katsikopoulos et al, 1981; Butler et al, 1997; Ahn and Park, 2008], where decision-makers supplies ordinal information on importance and the information is the subsequently converted into surrogate weights corresponding to and consistent with the extracted ordinal information. Often used such are rank sum (RS) weights, rank reciprocal (RR) weights, and centroid (ROC) weights [Danielson and Ekenberg, 2014]. ROC is a function based on the average of the corners in the polytope defined by the simplex  $S_w = w_1 > w_2 > \dots > w_N$ ,  $\sum w_i = 1$ , and  $0 \leq w_i$ , where  $w_i$  are variables representing the criteria weights. The weights then become the centroid (mass point) components of  $S_w$ . The ROC weights are then, for the ranking number  $i$  among  $N$  items to rank, given by Eq. 1.

$$w_i^{ROC} = \frac{\sum_{j=i}^N \frac{1}{j}}{N} \quad (1)$$

For instance, [Barron and Barret, 1992] introduced a process utilising systematic simulations for validating the selection of criteria weights, when generating surrogate weights as well as “true” reference weights. It also investigated how well the result of using surrogate numbers matches the result of using the “true” numbers. This is however heavily dependent on the distribution used for generating the weight vectors.

Still the problem there is to elicit stakeholder information. Different elicitation formalisms have been proposed by which a decision-maker can express preferences. Such formalisms are sometimes based on scoring points, as in point allocation (PA) or direct rating (DR) methods. In PA, the decision-maker is given a point sum, e.g. 100, which they distribute among the criteria. Sometimes, it is pictured as putty with the total mass of 100 being divided and put on the criteria. The more mass, the larger weight on a criterion, the more important it is. When the first  $N-1$  criteria have received their weights, the last criterion’s weight is automatically determined as the remaining mass. Thus, in PA, there is  $N-1$  degrees of freedom (DoF) for  $N$  criteria. DR, on the other hand, puts no limit on the total number of points to be allocated. The decision-maker allocates as many points as desired to each criterion. The points are subsequently normalized by dividing by the sum of points allocated. When the first  $N-1$  criteria have received their weights, the last criterion’s weight still has to be assigned by the decision-maker. Thus, in DR, there are  $N$  degrees of freedom for  $N$  criteria. Regardless of elicitation method, the assumption is that all elicitation is made relative to a weight distribution held by the decision-maker.

We have earlier investigated various aspects of this in a couple of articles and compared state-of-the-art weight methods, both ordinal (ranking only) and cardinal (with the possibilities to express strength) [Danielson et al, 2014; Danielson and Ekenberg, 2016ab] in order to devise methods requiring as little cognitive load as possible. We also used these together with ranked values (utilities) and suggested a multi-stakeholder decision method that has been applied in, e.g., [Komendantova et al, 2018]. This method fulfils several desired robustness properties and is comparatively stable under reasonable assumptions.

### 3.3. Elicitation methods

The crucial issue in all these methods is how to assign surrogate weights while losing as little information as possible and ensuring correctness when assigning the weights. Providing ordinal rankings of criteria seems to avoid some of the difficulties associated with the elicitation of exact numbers. It puts fewer demands on decision-makers and is thus, in a sense, effort-saving. Furthermore, there are techniques for handling ordinal rankings with various success. A limitation of this is naturally that decision-makers usually have more knowledge of the decision situation than a pure criteria ordering, often in the sense that they have an idea regarding importance relation information containing strengths. In such cases, the surrogate weights may be an unnecessarily weak representation, why we have also investigated whether the methods can be extended to accommodate information regarding relational strengths as well, while still preserving the property of being less demanding and more practically useful than other types of methods.

One well-known class of methods is the SMART family, where [Edwards, 1971,1977] proposed a method to assess criteria weights. The criteria are ranked and then 10 points are assigned to the weight of the least important criterion ( $w_N$ ). Then, the remaining weights ( $w_{N-1}$  through  $w_1$ ) are given points according to the decision-maker's preferences. The overall value  $E(a_j)$  of alternative  $a_j$  is then a weighted average of the values  $v_{ij}$  associated with  $a_j$  (Eq. 2):

$$E(a_j) = \frac{\sum_{i=1}^N w_i v_{ij}}{\sum_{i=1}^N w_i} \quad (2)$$

The most utilised processes for converting ordinal input to cardinal use automated procedures and yield exact numeric weights. For instance, [Edwards and Barron, 1994, 1996a] proposed the SMARTER method for eliciting ordinal information on importance before converting it to numbers, thus relaxing information input demands on the decision-maker. An initial analysis is carried out where the weights are ordered such as  $w_1 > w_2 > \dots > w_N$  and then subsequently transformed to numerical weights using ROC weights and then SMARTER continues in the same manner as the ordinary SMART method.

The most well-known ratio scoring methods is the Analytic Hierarchy Process (AHP). The basic idea in AHP [Saaty, 1977,1980] is to evaluate a set of alternatives under a criteria tree by pairwise comparisons. The process requires the same pairwise comparisons regardless of scale type. For each criterion, the decision-maker should first find the ordering of the alternatives from best to worst. Next, he or she should find the strength of the ordering by considering pairwise ratios (pairwise relations) between the alternatives using the integers 1, 3, 5, 7, and 9 to express their relative strengths, indicating that one alternative is equally good as another (strength = 1) or three, five, seven, or nine times as good. It is also allowed to use the even integers 2, 4, 6, and 8 as intermediate values, but using only odd integers is more common.

As an, as we have claimed in [Danielson et al, 2016c], a better alternative to these, we have suggested the CAR method and shown that this is more robust and efficient than methods from the SMART family and AHP. We will also combine this method with a SWING model variety from [Danielson and Ekenberg, 2019]. We will develop this further below.

### 3.4. Expanded CAR Method

Simos proposed a simple procedure, using a set of cards, trying to indirectly determine numerical values for criteria weights [Simos, 1990ab]. It is a relatively simple method for easily expressing criteria hierarchies while introducing some cardinality if needed. It has been widely applied and has been well-received by real decision-makers, but it has some serious problems why we use a significantly changed variety and save just the card model with white and coloured cards as described above. After the card ranking, the surrogate numbers can be computed. A constant value difference, 'u', between two consecutive cards is assumed here. A blank card between two consecutive coloured cards signifies a difference of  $2 \cdot u$ , and two white cards represent a difference of  $3 \cdot u$ , etc.

However, one problem with the Simos method is that it is not robust when the preferences are changed and that it has some other contra-intuitive features, such as that it only picks one of the weight vectors satisfying the model, while there can, of course, be an infinite number of them. Furthermore, because the weights are determined differently depending on the number of cards in the subsets of equally ranked cards, the differences between the weights also change in an uncontrolled way when the cards are reordered. This is why [Figuera and Roy, 2002] suggested a revised version, where there is a more robust proportionality when these blank cards are used. It is accomplished by requesting the decision-makers to state how many times more important the most important criterion or criteria group is—compared to the least important. This addition seemingly solves some problems but introduces the complication that the decision-maker has to reliably and correctly estimate a proportional factor 'z' between the largest and the smallest criteria weights.

We, therefore, use a variant of the Simos method for elicitation purposes and kept the card ranking part while changing the evaluation significantly compared to the Simos method and its revisions. At that point, the participants already know the criteria well from the previous sections of the workshops. The key challenge in our workshops is to elicit a collective ranking. Most methods for ranking and weighting deal with individuals, we have to do it as a group effort. This is the main reason to opt for the card-ranking through a silent negotiation, not the calculation behind it.

Each criterion is written on a coloured card and arranged horizontally on a table. Then each of the participants successively rank the cards from the least important to the most important by moving the cards to a vertical arrangement, where the highest-ranked criterion is put on top and so forth. If two criteria are considered to be of equal importance, they are put on the same level. This process goes on for four rounds, where the number of moves for each round are 8, 5, 3 and 2. Furthermore, the first and third round are concluded by an open discussion before the following round. The ranking procedure last 120 minutes or until a final ranking is achieved that the participants find acceptable.

It is true that the decreasing number can be disputed and is a weak point of the method since it induces / forces the participants to act strategically in relation to the information they got during the process. So, when this method is used, the potential conflicts must come to the open and be dealt with. In some cases, by working with a set of final ranking in the evaluations, where it turns out whether the differences are of importance or not. After the first ordinal ranking is finalized, the participants are asked to introduce preference strengths in the ranking by introducing the blank cards during three additional rounds (with three, two and one move). The number of white cards (i.e. The strength of the rankings between criteria) is also interpreted verbally:

While being more cognitively demanding than ordinal weights, these are still less demanding than, for example, AHP weight ratios or point scores like. In an analogous manner as for ordinal rankings, the decision-maker statements can by using these be converted into weights.

The final rankings of the workshops and its rationales are then presented to the other participants during an introductory presentation round.

### 3.5. Preference strengths

In analogy with the ordinal weight functions above, counterparts using the concept of preference strength can straightforwardly be derived.

1. Assign an ordinal number to each importance scale position, starting with the most important position as number 1.
2. Let the total number of importance scale positions be  $Q$ . Each criterion  $i$  has the position  $p(i) \in \{1, \dots, Q\}$  on this importance scale, such that for every two adjacent criteria  $c_i$  and  $c_{i+1}$ , whenever  $c_i >_{s_i} c_{i+1}$ ,  $s_i = |p(i+1) - p(i)|$ . The position  $p(i)$  then denotes the importance as stated by the decision-maker. Thus,  $Q$  is equal to  $\sum s_i + 1$ , where  $i = 1, \dots, N-1$  for  $N$  criteria.
3. Then the cardinal counterparts to the ordinal ranking methods above can be found by using the results from [Danielson and Ekenberg, 2016b], where the ordinal SR weights were given by Eq. 3

$$w_i^{SR} = \frac{\frac{1}{i} + \frac{N+1-i}{N}}{\sum_{j=1}^N w_j^{SR}} \quad (3)$$

4. and using steps 1–3 above, the corresponding preference strength SR weights (CSR, Eq. 4) are obtained as

$$w_i^{CSR} = \frac{\frac{1}{p(i)} + \frac{Q+1-p(i)}{Q}}{\sum_{j=1}^N \left( \frac{1}{p(j)} + \frac{Q+1-p(j)}{Q} \right)} \quad (4)$$

Using the idea of importance steps, ordinal weight methods in general are easily generalised to their respective counterparts. In the same manner, values (or utilities) can be ranked, either ordinally (ranking only) or cardinally (additionally expressing strength).

Already in [Danielson and Ekenberg, 2016c], we combined cardinal weights with cardinal values into the CAR method and assessed the method by simulations as well as a large number of user cases. The CAR method was found to outperform SMART and AHP on terms of performance and the ease of use (the cognitive load), but some of the users still wanted a method with even less cognitive load so we tried to satisfy this while still preserving reasonable requirements of correctness. The CAR method follows the three-step procedure below.

**First**, the values of the alternatives under each criterion are elicited in a way similar to the weights described above:

- 1A.** For each criterion in turn, rank the alternatives from the worst to the best outcome.

**1B.** Enter the strength of the ordering. The strength indicates how strong the separation is between two ordered alternatives. Similar to weights, the strength is expressed in the notation with ' $>_i$ ' symbols.

**Second**, the weights are elicited with a swing-like procedure in accordance with the discussion above.

**2A.** For each criterion in turn, rank the importance of the criteria from the least to the most important.

**2B.** Enter the strength of the ordering. The strength indicates how strong the separation is between two ordered criteria. The strength is expressed in the notation with ' $>_i$ ' symbols.

**Third**, the usual weighted overall value is calculated by multiplying the centroids of the weight simplex with the centroid of the alternative value simplex.

As described in [Danielson and Ekenberg, 2016b], the same description can be used to introduce the three candidate methods called C+O, O+C, and O+O depending on whether a cardinal or ordinal procedure is used for the representation of weights and values respectively. In the original CAR method all the steps 1A, 1B, 2A, 2B, and 3 was performed in that order. The steps in the three candidate methods that we suggest are performed as follows: In O+C, step 1B is omitted, resulting in the sequence 1A, 2A, 2B, and 3 in order. In C+O, step 2B is omitted instead, resulting in the sequence 1A, 1B, 2A, and 3 in order. Finally, in O+O, both steps 1B and 2B are omitted, resulting in the sequence 1A, 2A, and 3 in order.

### 3.6. P-Swing

The P-SWING method (detailed described in [Danielson and Ekenberg, 2019]) – is designed to overcome some of the typical problems associated with elicitation processes. The method consists of an amended swing-type technique at its core. However, whereas a traditional SWING session only contains from-worst-to-best swings, the suggested method adheres to the core ideas while allowing for intermediate comparisons as well. This will aid the convergence of the weights for the criteria. Furthermore, there is no use of zero alternatives or similar synthetic constructs, and instead many more available real data points are utilised. Based on this, we provide an integrated framework for elicitation, modelling and evaluation of multi-criteria decision problems.

The P-SWING method emanates from two observations on desirable properties (in addition to a swing-like procedure) for an elicitation technique to possess:

- The focus during the elicitation should only be on the existing real-life alternatives without any abstract additions.
- When constructing the ordering of the criteria weights, the procedure should not be limited to extreme points (the endpoints of the value scales), but should rather allow the use of all values actually asserted.

Based on these desiderata, we have designed an elicitation technique that extends the SWING methodology by introducing partial assignments and interval constraints. This extension is applicable to all SWING-related methods and has been coined P-SWING (Partial SWING), which is formalised in the following section and then exemplified by extending an existing MCDM method.

To recap, cardinal ranking methods (represented by the CAR method) were superior to other classes of methods, but the elicitation component could be improved. We therefore propose the P-SWING method, consisting of an amended swing-type technique at its core. The basis is that while a traditional SWING session embraces only from-worst-to-best swings, P-SWING employs intermediate comparisons as well. This will rapidly aid the convergence of the weights for the criteria. Furthermore, there is no use for zero alternatives or similar synthetic constructs, and instead many more real data points are utilised.

### 3.7. Evaluations under Strong Uncertainty

Now we turn our attention to the general evaluation of the entire decision problem. We will use the evaluation method from [Danielson et al, 2020]. In the type of multi-criteria decision problems, we consider, we hold that strong uncertainty exists if the decision is also made under risk, with uncertain consequences for at least one criterion, in combination with imprecise or incomplete information with respect to probabilities, weights, and consequences or alternative values. Decision evaluation under strong uncertainty and computational means for evaluating these models should both be capable of embracing the uncertainty in the evaluation rules and methods and provide evaluation results reflecting the effects of uncertainty for the subsequent discrimination between alternatives.

We will call our representation of a combined decision problem a multi-frame. Such a frame collects all information necessary for the model in one structure. One part of this is the concept of a graph.

**Definition:** A graph is a structure  $\langle V, E \rangle$  where  $V$  is a set of nodes and  $E$  is a set of node pairs. A tree is a connected graph without cycles. A rooted tree is a tree with a dedicated node as a root. The root is at level 0. The adjacent nodes, except for the nodes at level  $i-1$ , to a node at level  $i$  is at level  $i+1$ . A node at level  $i$  is a leaf if it has no adjacent nodes at level  $i+1$ . A node at level  $i+1$  that is adjacent to a node at level  $i$  is a child of the latter. A (sub-)tree is symmetrical if all nodes at level  $i$  have the same number of adjacent nodes at level  $i+1$ . The depth of the tree is  $\max (n \mid \text{there exists a node at level } n)$ .

**Definition:** A criteria-consequence tree  $T = \langle CUAUNU\{r\}, E \rangle$  is a tree where  
 $r$  is the root,  
 $A$  is the set of nodes at level 1,  
 $C$  is the set of leaves, and  
 $N$  is the set of intermediary nodes in the tree except those in  $A$ .

In a multi-frame, represented as a multi-tree, user statements can either be range constraints or comparative statements (see below); they are translated into inequalities and collected together in a value constraint set. For probability and weight statements, the same is done into a node constraint set. We denote the values of the consequences  $c_i$  and  $c_j$  by  $v_i$  and  $v_j$  respectively. User statements have the following forms for real numbers  $a_1, a_2, b_1, b_2, d_1$  and  $d_2$ :

- *Range constraints:*  $v_i$  is between  $a_1$  and  $a_2$ , denoted  $v_i \in [a_1, a_2]$  and translated into  $v_i > a_1$  and  $v_i < a_2$ .
- *Comparisons:*  $v_i$  is larger than  $v_j$  by an amount ranging from  $d_1$  to  $d_2$ , denoted  $v_i - v_j \in [d_1, d_2]$  and translated into  $v_i - v_j > d_1$  and  $v_i - v_j < d_2$ .

A constraint set is said to be consistent if it can be assigned at least one real number to each variable so that all inequalities are simultaneously satisfied. Consequently, we get potential sets of functions with an infinite number of instantiations.

**Definition:** Given a criteria-consequence tree  $T$ , let  $N$  be a constraint set in the variables  $\{n_{...i...j...}\}$ . Substitute the intermediary node labels  $x_{...i...j...}$  with  $n_{...i...j...}$ .  $N$  is a node constraint set for  $T$  if, for all sets  $\{n_{...i1}, \dots, n_{...im}\}$  of all sub-nodes of nodes  $n_{...i}$  that are not leaves, the statements  $n_{...ij} \in [0,1]$  and  $\sum_j n_{...ij} = 1, j \in [1, \dots, m]$  are in  $N$ .

A probability node constraint set relative to a criteria-consequence tree then characterizes a set of discrete probability distributions. Weight and value constraint sets are analogously defined. Weight and probability node constraint sets also contain the usual normalization constraints ( $\sum_j x_{ij} = 1$ ) requiring the probabilities and weights to total one.

**Definition:** A multi-frame is a structure  $\langle T, N \rangle$ , where  $T$  is a criteria-consequence tree and  $N$  is a set of all constraint sets relative to  $T$ .

The probability, value and weight constraint sets thus consist of linear inequalities. A minimal requirement is that it is consistent—i.e. there must exist some vector of variable assignments that simultaneously satisfies each inequality in the system.

**Definition:** Given a consistent constraint set  $X$  in the variables  $\{x_i\}$ ,  $x_{\max}(x_i) =_{\text{def}} \sup(a \mid \{x_i > a\} \cup X \text{ is consistent})$ . Similarly,  $x_{\min}(x_i) =_{\text{def}} \inf(a \mid \{x_i < a\} \cup X \text{ is consistent})$ . Furthermore, given a function  $f$ ,  $x_{\text{argmax}}(f(x))$  is a solution vector that is a solution to  $x_{\max}(f(x))$ , and  $x_{\text{argmin}}(f(x))$  is a solution vector that is a solution to  $x_{\min}(f(x))$ .

The set of orthogonal projections of the solution set is the orthogonal hull, consisting of all consistent variable assignments for each variable in a constraint set.

**Definition:** Given a consistent constraint set  $X$  in  $\{x_i\}_{i \in [1, \dots, n]}$ , the set of pairs  $\langle x_{\min}(x_i), x_{\max}(x_i) \rangle$  is the orthogonal hull of the set.

The orthogonal hull is the upper and lower probabilities (weights, values) if  $X$  consists of probabilities (weights, values). The hull intervals are calculated by first finding a consistent point. Thereafter, the minimum and maximum of each variable are found by solving linear programming problems. Because of convexity, the intervals between the extremal points are feasible—i.e. the entire orthogonal hull has been determined.

### 3.8. Introducing Second-Order Beliefs

We will first extend the representation to obtain a more granulated representation of a decision problem. Often when we specify an interval, we probably do not believe in all values in the intervals equally: we may, for example, believe less in the values closer to the borders of the intervals. Additional values are nevertheless added to cover everything that we perceive as possible in uncertain situations. These additions give rise to belief distributions indicating the

different strengths with which we believe in the different values. Distributions over classes of weight, probability and value measures have been developed into various models, such as second-order probability theory.

In the extended model, we introduce a focal point to each of the intervals used as parameters for belief distributions for probabilities, values and criteria weights. We can then operate on these distributions using additive and multiplicative combination rules for random variables. The detailed theory of belief distributions in this sense is described in [Danielson et al, 2019, 2020].

To make the method more concrete, we introduce the unit cube as all tuples  $(x_1, \dots, x_n)$  in  $[0,1]^n$ . A second-order distribution over a unit cube  $B$  is a positive distribution  $F$  defined on  $B$  such that

$$\int_B F(x) dV_B(x) = 1,$$

where  $V_B$  is the  $n$ -dimensional Lebesgue measure on  $B$ .

We will use second-order joint probability distributions as measures of beliefs. Different distributions are utilized for weights, probabilities and values because of the normalization constraints for probabilities and weights. Natural candidates are then the Dirichlet distribution for weights and probabilities and two- or three-point distributions for values. In brief, the Dirichlet distribution is a parameterized family of continuous multivariate probability distributions. It has a probability density function given by a function of those parameters, such that  $\alpha_1, \dots, \alpha_k > 0$  depends on a beta function and the product of the parameters  $x_i$ .

More precisely, the probability density function of the Dirichlet distribution is defined as

$$f_{dir}(p, \alpha) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} p_1^{\alpha_1-1} p_2^{\alpha_2-1} \dots p_k^{\alpha_k-1}$$

on a set  $\{p = (p_1, \dots, p_k) \mid p_1, \dots, p_k \geq 0, \sum p_i = 1\}$  where  $(\alpha_1, \dots, \alpha_k)$  is a parameter vector in which each  $\alpha_i > 0$  and  $\Gamma(\alpha_i)$  is the Gamma function.

The Dirichlet distribution is a multivariate generalization of the beta distribution and the marginal distributions of Dirichlet are thus beta distributions. The beta distribution is a family of continuous probability distributions defined on  $[0, 1]$  and parameterized by two parameters,  $\alpha$  and  $\beta$ , defining the shape of the distribution.

For instance, if the distribution is uniform, the resulting marginal distribution (over an axis) is a polynomial of degree  $n-2$ , where  $n$  is the dimension of a cube  $B$ . Let all  $\alpha_i = 1$ , then the Dirichlet distribution is uniform with the marginal distribution

$$f(x_i) = \int_{B_i^-} dV_{B_i^-}(x) = (n-1)(1-x_i)^{n-2}$$

However, for our purposes, we need a bounded Dirichlet distribution operating on a user-specified  $[a_i, b_i]$  range instead of the general interval  $[0,1]$ . Bounded beta distributions are then derived—the so-called four-parameter beta distributions, also defined only on the user-specified range. We then define a probability or weight belief distribution as a three-point bounded Dirichlet distribution  $f_3(a_i, c_i, b_i)$  where  $c_i$  is the most likely probability or weight and  $a_i$  and  $b_i$  are the boundaries of the belief with  $a_i < c_i < b_i$ .

For values, the generalization to a trapezoid from a triangle is analogous. We will utilize either a two-point distribution (uniform, trapezoidal) or a three-point distribution (triangular). When

there is a large uncertainty regarding the underlying belief distribution in values and we have no reason to make any more specific assumptions, a two-point distribution modelling upper and lower bounds (the uniform or trapezoid distributions) is preferred. On the other hand, when the modal outcome can be estimated, the beliefs are more congenially represented by three-point distributions. The Beta and Erlang belief distributions are widely used in many models and generally give results similar to triangular distributions; we use the latter as a representative for the class of three-point distributions. This is thus a description when there are only limited sample data, particularly in cases where the variable relationships are known as well as the minimum, maximum and modal values. The PERT distribution is a classic example and the mean value of these three-point value belief distributions  $f_3(a_i, c_i, b_i)$  is  $\mu(\lambda) = (a_i + b_i + \lambda c_i) / (\lambda + 2)$ , with special cases  $\lambda = 1$  for a triangular distribution and  $\lambda = 0$  for a two-point uniform or trapezoid distribution.<sup>2</sup> Because triangular distributions are less centre-weighted than other three-point distributions, the risk of underestimation is less, which is why there are no particular reasons to use any other distribution for practical purposes.

### 3.9. The Evaluation Model

We will use a generalization of the ordinary expected value for the evaluation—i.e. the resulting distribution over the generalized expected utility is

$$E(A_i) = \sum_{i_1=1}^{n_0} w_{ii_1} \sum_{i_2=1}^{n_{i_1}} w_{ii_1 i_2} \cdots \sum_{i_{m-1}=1}^{n_{i_{m-2}}} p_{ii_1 i_2 \cdots i_{m-2} i_{m-1}} \sum_{i_m=1}^{n_{i_{m-1}}} p_{ii_1 i_2 \cdots i_{m-2} i_{m-1} i_m} v_{ii_1 i_2 \cdots i_{m-2} i_{m-1} i_m}$$

given the distributions over the random variables  $p$  and  $v$ . There are only two operations of relevance here, multiplication and addition.

Let  $G$  be a distribution over the two cubes  $A$  and  $B$ . Assume that  $G$  has a positive support on the feasible distributions at level  $i$  in a general decision tree, as well as on the feasible probability distributions of the children of a node  $x_{ij}$  and assume that  $f(x)$  and  $g(y)$  are the marginal distributions of  $G(z)$  on  $A$  and  $B$ , respectively. Then the cumulative multiplied distribution of the two belief distributions is

$$H(z) = \iint_{\Gamma_x} f(x)g(y)dx dy = \int_0^1 \int_0^{\frac{z}{x}} f(x)g(y)dx dy = \int_z^1 f(x)G\left(\frac{z}{x}\right) dx$$

where  $G$  is a primitive function to  $g$ ,  $\Gamma_z = \{(x,y) \mid x \cdot y \leq z\}$ , and  $0 \leq z \leq 1$ .

Let  $h(z)$  be the corresponding density function. Then

$$h(z) = \frac{d}{dz} \int_z^1 f(x)G(z/x)dx = \int_z^1 f \frac{f(x)g(z/x)}{x} dx.$$

The addition of the products is the standard convolution of two densities restricted to the cubes. The distribution  $h$  on a sum  $z = x + y$  associated with the belief distributions  $f(x)$  and  $g(y)$  is therefore given by

<sup>2</sup> Beta-PERT usually has  $\lambda = 4$  and Erlang-PERT has  $\lambda = 3$ . However, higher values of  $\lambda$  tend to underestimate the uncertainties involved.

$$h(z) = \frac{d}{dz} \int_0^z f(x)g(z-x)dx.$$

Then we can obtain the combined distribution over the generalized expected utility.

As in most of risk and decision theory, we assume that a large number of events will occur and a large number of decisions will be made. This way, the expected value becomes a reasonable decision rule and, at the same time, the belief distributions over the expected values tend to normal distributions or similar. Note that even when assuming that the expectations are estimated a large number of times and consequently can be approximated by a normal distribution, there are three particular observations to be made:

The resulting distributions will be normal only when the original distributions are symmetrical, which of course is not usually the case for beta and triangular distributions. The result then will instead be skew-normal.

Even if the original distributions are symmetrical, the non-linear multiplication operator breaks the symmetry. The result then will again be skew-normal.

To obtain a resulting normal (or skew-normal) distribution, both the original distributions and their aggregations must allow for long tails. This is not generally the case in our models since our estimates have lower and upper limits.

We therefore use a truncated skew-normal distribution, generalizing the normal distribution by allowing for non-zero skewness and truncated tails. This is accomplished by introducing a shape parameter  $\alpha$ , where the standard normal distribution has  $\alpha = 0$ , and where  $\alpha = 1$  yields the distribution of the maximum of two independent standard normal variates. We can then conveniently represent truncated (skew-)normal distributions as probability distributions of (skew-)normally distributed random variables that are bounded. The skewness of the distribution increases along with the absolute value of  $\alpha$ , and when  $|\alpha| \rightarrow \infty$ , we obtain folded normal or half-normal distributions. Distributions are right-skewed when  $\alpha > 0$  and left-skewed when  $\alpha < 0$ . Assume that a distribution  $X$  has a normal distribution within the interval  $(a, b)$ . Then  $X, a < X < b$ , has a truncated normal distribution and its probability density function is given by a four-parameter expression that tends to normality as the intervals are widened.

The analyses are supported by the tool decideIT. Figure 5 shows an example of one of the result windows from the tool.

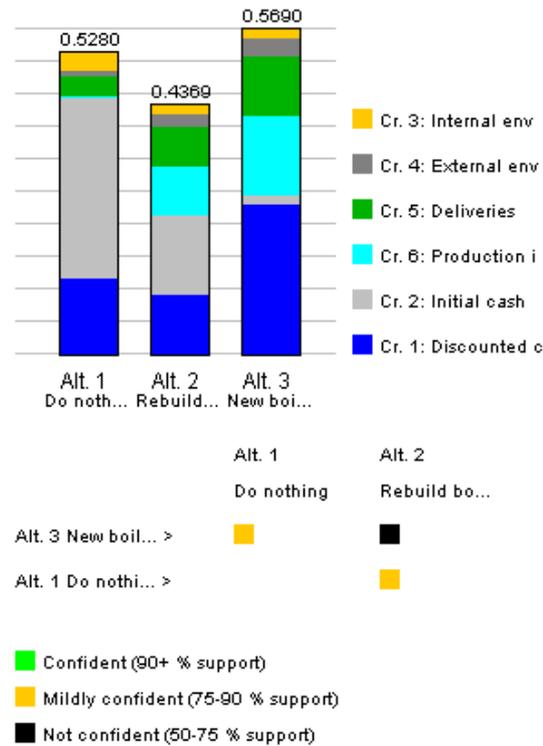


Figure 5. Main decision evaluation result

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