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SOME COMMENTS ON THE ESTIMATION OF DEMAND RELATIONS FOR POLAND

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FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of the IIASA Food and Agriculture Program since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. For greater realism the models in this scheme are being kept descriptive rather than normative. In the end it is proposed to link models to twenty countries, which together account for nearly 80 per cent of important agricultural attributes such as area, production, population, exports, imports and so on.

In the course of the work on the development of models of centrally planned economies, the difficulties of estimating parameters of consumer behavior in such economies where consumer markets cannot be assumed to be in equilibrium become apparent. Since the understanding of consumer behavior is critically important in formulating plans and desigining policies that facilitate the realization of plans, we have explored alternative approaches to this problem. This paper proposes one possible approach to estimating consumer demand relations.

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1. INTRODUCTION

The purpose of the present paper is to outline some major methodological problems connected with the estimation of demand relations in contemporary Poland. After presenting some crucial specific features of the market mechanism in that country, the author points to the fact that, because of the lack of immediate price or supply adjustments, situations may occur in which the level of supply of a commodity falls below the level of demand. Such situations introduce bias if the demand relations are estimated by conventional methods. To avoid such bias special methods must be designed, and the author presents some of these methods. Next, a linear expenditure system is considered, and the final section of the paper is devoted to the problem of determining whether it is possible, under conditions of insufficient supply, to define a measure of demand shifts induced by such market disequilibrium.

2. MARKET MECHANISM IN POLAND

When considering the market for consumer goods in Poland one must conclude that there are really two different markets, although these are to some extent interrelated. First one must mention the state (or cooperative) market operated by socialized enterprises. On this market consumer goods are sold at prices determined by central state authorities or by appropriate local or sectoral organisms. The second market is a private one. It is operated by private individuals, and prices there are determined by the law of supply and demand, although the prices of some commodities are subject to state control. The private market deals primarily with such commodities as are produced by private farms and which are not sold to state trade organizations, or which are made by craftsmen. The share of the private market is small, although for some goods (fruits and vegetables, curios, second-hand transactions) its role is really significant. The level of prices on the private market follows to some extent the movement of prices charged by state and cooperative organizations, especially when it comes to a general inflationary component. However, since it is the state and cooperative market which is predominant, we shall concentrate on its analysis, or more specifically, on its mechanism.

The first important observation to be made is that owing to the determination of prices by the state, no immediate dependence of price level on demand and supply is observed. If corresponding shifts of price occur, they are usually made with much delay, such delay often being on the order of several years.[1]

Second, one must note that the supply of commodities produced by the socialized sector does not immediately depend on price level, the main supply-determining factor being economic plans. Such plans reveal state preferences, and although enterprises have in most cases the possibility of modifying the plan, the range of their possible maneuvers is quite limited.

Third, one must note that consumers are in principle free to make their decisions about what they will buy with their money, this freedom being limited by the existing structure and level of supply of various consumer goods. Consequently, the consumers buy such commodities which will best satisfy their preferences. One might add that various studies of demand pattern in Poland have revealed considerable inertia in their purchasing habits, which causes tradition to be one of the important demand-determining factors.

[1] It is interesting to build price or supply equations to look for an adequate lag system and thus to make inferences about the speed with which planners react to changing economic conditions.

3. THE SITUATION OF INSUFFICIENT SUPPLY

The fact that there is no instantaneous feedback influence of demand on price and on supply makes it possible to consider demand equations in a market mechanism model as statistically independent of the other two typical equations, namely, those representing the variations of price and of supply. A further implication of this fact is that the demand equation can be independently estimated, usually by ordinary least squares method. Obviously this greatly simplifies the computations.

On the other hand, one should not overlook one important pitfall. The lack of immediate adjustment of price (or of supply) to demand level makes it possible to encounter such situations (within months, quarters, and years) when the actual level of supply of a given commodity is less than the level of demand for it. If, within the sample, there is only one such period, such an atypical observation can simply be dropped out. If their number is larger, however, the problem becomes an important one. Since time series data are usually short, throwing away a number of observations means a significant decrease of sample size together with incurring all the known perils and consequences of basing one's inference on a small amount of data.

Retaining the observations which pertain to periods of inadequate supply means that the statistical data for such periods do not reveal demand but rather supply. Therefore, time series referring to the values of the dependent variable of the demand equation consist of a mixture of data referring either to demand or to supply. The statistical consequences of such a situation will be more fully considered in the next section.

Let us denote by N_t the phenomenon of inadequate supply of a given commodity in time t, i.e., the fact that in time t the level of supply is lower than the level of demand for this commodity. Let us also denote by J_N the set of all such sample periods for which the phenomenon N_t has occurred.

Let further d_t denote demand in time t and $\bar{d}_t$ the level of sales observed in time $t \in J_N$. Without any loss of generality we can write

$$\bar{d}_t = d_t - z_t \tag{1}$$

where z_t can be called the level of insufficiency of supply. Variable z_t is non-negative and assumes the value zero when the level of supply is sufficient to meet demand and $z_t > 0$ for all $t \in J_N$.

For nonlinear (Cobb-Douglas type) demand models the level of insufficiency of supply is better expressed in relative terms. Thus, instead of (1), it is more convenient to use the formula (with $0 < z' \leq 1$):

$$\bar{d}_t = z_t'd_t \tag{2}$$

Of course, both z_t and z_t' are unobservable statistically. On the other hand, it is possible to single out from past experience not only all such time periods which belong to J_N but also to form a general idea of how z_t changed in the past (for instance if it had a constant value or if it exhibited a time trend).

4. THE CONSEQUENCES OF INSUFFICIENT SUPPLY

We shall try now to establish the consequences of insufficient supply for the estimation of demand relation. For this purpose we shall assume the demand relation to be linear.

$$D_t = \sum_{i=1}^{k} \alpha_i X_{it} + \xi_t \tag{3}$$

and the estimation method that of ordinary least squares. The main and most general result of analysis of the consequences of insufficient supply can be presented by means of the following theorem:

Theorem I. If the relation (3) is estimated by ordinary least squares and the set J_N is not empty then the estimate $\bar{a}_i$ of parameter α_i is equal to

$$\bar{a}_i = a_i - b_{z|x_i} \tag{4}$$

where a_i is the estimate of α_i which would be obtained if J_N was an empty set, and by $b_{z|x}$ we denote the ordinary least squares estimate of parameter β_i in the following relation

$$z_t = \sum_{i=1}^{k} \beta_i X_{it} + \eta_t \tag{5}$$

To prove (4) it suffices to observe that $\bar{a}_i$ is obtained as the ratio of two determinants W_{α_1} and W, namely

$$W = \begin{vmatrix} \Sigma x_{1t}^2 & \Sigma x_{1t}x_{2t} & \dots & \Sigma x_{1t}x_{kt} \\ \Sigma x_{2t}x_{1t} & \Sigma x_{2t}^2 & \dots & \Sigma x_{2t}x_{kt} \\ \vdots & \vdots & & \vdots \\ \Sigma x_{kt}x_{1t} & \Sigma x_{kt}x_{2t} & \dots & \Sigma x_{kt}^2 \end{vmatrix} \tag{6}$$

and

$$W_{\alpha_1} = \begin{vmatrix} \Sigma \bar{d}_t x_{1t} & \Sigma x_{1t}x_{2t} & \dots & \Sigma x_{1t}x_{kt} \\ \Sigma \bar{d}_t x_{2t} & \Sigma x_{2t}^2 & \dots & \Sigma x_{2t}x_{kt} \\ \vdots & \vdots & & \vdots \\ \Sigma \bar{d}_t x_{kt} & \Sigma x_{kt}x_{2t} & \dots & \Sigma x_{kt}^2 \end{vmatrix} \tag{7}$$

Since the way the explanatory variables of demand relation are allotted their subscript is conventional, we put i = 1, which simplifies the notation. Thus, any explanatory variable of (3) can be thought of as appearing in the first place on the right-hand side of the equation.

However, because of (1), W_{α_1} can be rewritten as the difference of two determinants.

$$W_{\alpha_1} = \begin{vmatrix} \Sigma\, d_t x_{1t} & \Sigma\, x_{1t}x_{2t} & \cdots & \Sigma\, x_{1t}x_{kt} \\ \Sigma\, d_t x_{2t} & \Sigma\, x_{2t}^2 & \cdots & \Sigma\, x_{2t}x_{kt} \\ \vdots & \vdots & & \vdots \\ \Sigma\, d_t x_{kt} & \Sigma\, x_{kt}x_{2t} & \cdots & \Sigma\, x_{kt}^2 \end{vmatrix} + \qquad (8)$$

$$- \begin{vmatrix} \Sigma\, z_t x_{1t} & \Sigma\, x_{1t}x_{2t} & \cdots & \Sigma\, x_{1t}x_{kt} \\ \Sigma\, z_t x_{2t} & \Sigma\, x_{2t}^2 & \cdots & \Sigma\, x_{2t}x_{kt} \\ \vdots & \vdots & & \vdots \\ \Sigma\, z_t x_{kt} & \Sigma\, x_{kt}x_{2t} & \cdots & \Sigma\, x_{kt}^2 \end{vmatrix}$$

or $W_{\alpha_1} = W_{\alpha_1}^{(1)} + W_{\alpha_1}^{(2)}$. Now, it is easily seen that the ratio $W_{\alpha_1}^{(1)}/W$ gives a_i, while $W_{\alpha_1}^{(2)}/W$ gives the estimate of β_i. This completes the proof.

As can be seen from (8), the insufficient supply situation bias vanishes when z_t is uncorrelated with X_i. Let us suppose, on the other hand, that this correlation does exist. The sign of $b_{z|x_1}$ depends on the character of the X_i variable and on the time behavior of the level of insufficiency of supply.[2] Other ways leading to the annihilation of the effects of insufficient supply will be presented in the next section.

5. ESTIMATION OF INSUFFICIENT SUPPLY BIAS

It is possible to derive a number of theorems stating the conditions under which the demand relation (3) can be modified so as to make the bias disappear. The most general one can be stated as follows.

Theorem II. Let the demand relation to be estimated be of the form (3) and let the situation N occur for at least one sample period, so that the set J_N is not empty. If the variable z_t exhibits such sample variation that it can be described by the function

$$h(t) = \sum_{j=1}^{L} c_j h_j(t) \qquad (9)$$

where $h_1(t), h_2(t), \ldots, h_L(t)$ are known functions[3] and $c_1, c_2, \ldots, c_L$ are constant coefficients not all equal to zero, then the bias $b_{z|x_1}$ defined by (4) vanishes if the set of explanatory variables of relation (3) is increased by adding to it as new,

[2] For Polish conditions, for example, the correlation of Z with both income and price variables is likely to be positive. Since for the income variable α_1 is usually positive, this means that insufficiency of supply results in increasing the estimated parameter value. But, since the coefficient standing for the own price of the commodity is usually negative, the result will be that of decreasing the absolute value of the estimate, etc.

[3] For instance, $h_j(t)$ may be defined as t^j or as $h_j(t) = \ln t$, etc.

additional explanatory variables $X_{k+1,t} = h_1(t), \ldots X_{k+L,t} = h_{L(t)}$.

The proof of this theorem is very similar to that of Theorem 1. It suffices to note the fact that for the augmented[4] demand relation the determinant W_{α_1} has the form

$$W_{\alpha_1} = \sum_{j=0}^{L} W_{\alpha_1}^{(j)} \tag{10}$$

where

$$W_{\alpha_1}^{(o)} = \begin{vmatrix} \Sigma\, d_t x_{1t} & \Sigma\, x_{1t}x_{2t} & \ldots & \Sigma\, x_{1t}x_{kt} & \Sigma\, x_{1t}h_1(t) & \ldots & \Sigma\, x_{1t}h_L(t) \\ \Sigma\, d_t x_{2t} & \Sigma\, x_{2t}^2 & \ldots & \Sigma\, x_{2t}x_{kt} & \Sigma\, x_{2t}h_1(t) & \ldots & \Sigma\, x_{2t}h_L(t) \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \Sigma\, d_t x_{kt} & \Sigma\, x_{kt}x_{2t} & \ldots & \Sigma\, x_{kt}^2 & \Sigma\, x_{kt}h_1(t) & \ldots & \Sigma\, x_{kt}h_L(t) \\ \Sigma\, d_t h_1(t) & \Sigma\, h_1(t)x_{2t} & \ldots & \Sigma\, h_1(t)x_{kt} & \Sigma\, h_1^2(t) & \ldots & \Sigma\, h_1(t)h_L(t) \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \Sigma\, d_t h_L(t) & \Sigma\, h_L(t)x_{2t} & \ldots & \Sigma\, h_L(t)x_{kt} & \Sigma\, h_L(t)h_1(t) & \ldots & \Sigma\, h_L^2(t) \end{vmatrix} \tag{11}$$

while the determinants $W_{\alpha_1}^{(j)}$ are similar to $W_{\alpha_1}^{(o)}$ but for the fact that their first column is equal to the following vector

$$V_j = \begin{vmatrix} c_j\, \Sigma\, h_j(t)\, x_{1t} \\ c_j\, \Sigma\, h_j(t)\, x_{2t} \\ \cdot \\ \cdot \\ \cdot \\ c_j\, \Sigma\, h_j(t)\, x_{kt} \\ c_j\, \Sigma\, h_j(t)\, h_1(t) \\ \cdot \\ \cdot \\ \cdot \\ c_j\, \Sigma\, h_j(t)\, h_L(t) \end{vmatrix} \tag{12}$$

As is easily seen for $j > 0$, the determinants $W_{\alpha_1}^{(j)}$ all have two proportional columns and hence are equal to zero. The estimate of α_i reduces then to the ratio of determinant (11) to the determinant of the system of normal equations which correspond to the augmented demand relation, i.e.

[4] In what will follow the augmented demand relation will denote relation (3) with additional explanatory variables.

$$W = \begin{vmatrix} \Sigma x_{1t}^2 & \Sigma x_{1t}x_{2t} & \dots & \Sigma x_{1t}x_{kt} & \Sigma x_{1t}h_1(t) & \dots & \Sigma x_{1t}h_L(t) \\ \Sigma x_{2t}x_{1t} & \Sigma x_{2t}^2 & \dots & \Sigma x_{2t}x_{kt} & \Sigma x_{2t}h_1(t) & \dots & \Sigma x_{2t}h_L(t) \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \Sigma x_{kt}x_{1t} & \Sigma x_{kt}x_{2t} & \dots & \Sigma x_{kt}^2 & \Sigma x_{kt}h_1(t) & \dots & \Sigma x_{kt}h_L(t) \\ \Sigma h_1(t)x_{1t} & \Sigma h_1(t)x_{2t} & \dots & \Sigma h_1(t)x_{kt} & \Sigma h_1^2(t) & \dots & \Sigma h_1(t)h_L(t) \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \Sigma h_L(t)x_{1t} & \Sigma h_L(t)x_{2t} & \dots & \Sigma h_L(t)x_{kt} & \Sigma h_L(t)h_1(t) & \dots & \Sigma h_L^2(t) \end{vmatrix} \qquad (13)$$

However, $W_{\alpha_i}^{(o)}/W$ is the "correct" ordinary least squares estimate of α_i, and this completes the proof of the theorem.

Two important conclusions follow from Theorem II. These can be stated as follows:

Conclusion I. If $J_N = J_N^{(1)} \cup J_N^{(2)}$ where $J_N^{(1)}$ is a set of sample periods such that for all $t \in J_N^{(1)}$ the level of insufficiency of supply is equal to a constant level $c1 \neq 0$ and $J_N^{(2)}$ is a set of sample periods such that for all $t \in J_N^{(2)}$ there is $z_t = c2 \neq 0$ and $c1 \neq c2$, then the bias due to insufficient supply can be eliminated by augmenting the demand relation (3) by two dummy variables, V_1^t and V_2^t, say, such that $V_1^t = 1$ for all $t \in J_N^{(1)}$ and $V_1^t = 0$ for all $t \notin J_N^{(1)}$ and $V_2^t = 1$ for all $t \in J_N^{(2)}$ and $V_2^t = 0$ for all $t \notin J_N^{(2)}$.

Conclusion I corresponds to the situation when (9) represents a step function. If h(t) has $j \neq 1$ steps then the conclusion presented above can be generalized stating that the augmented demand relation must contain j different and appropriately defined dummy variables.

Conclusion II. If within the sample period variable z_t exhibits a time trend which can be described by function

$$f(t) = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \dots + \gamma_s t^s \qquad (14)$$

then to eliminate the bias due to a situation of insufficient supply, s additional explanatory variables must be introduced into the demand relation, namely $t, t^2, \dots, t^s$.

From the point of view of practical applications it is important to get reliable information about the variation of z_t within the sample period. Since the level of insufficiency of supply is not directly observable, one must make use of indirect approaches. Either one uses past experience of market conditions and makes inferences about the growing or lessening gap between demand and supply or, alternatively, one may first estimate the relation (3) as it stands and then compute the residuals. Large negative residuals will suggest the periods when situation N_t may have occurred.

To conclude the argument let us finally add that if instead of linear relation (3) there is a nonlinear one of the Cobb-Douglas type, then to augment it the dummy variables and the time variables enter it in an exponential way, i.e., as $\exp(\alpha V)$ or $\exp(\alpha t^i)$.

6. A LINEAR EXPENDITURE SYSTEM

The present section will be devoted to the presentation of a simple linear expenditure system which was devised by this author and which was used in some empirical studies aimed at analyzing the pattern of consumers' expenditure in Poland.

The following assumptions underlie the model:

(a) Expenditure in various years is measured in real terms.

(b) The expenditure system is linear and additive.

(c) The expenditure model refers to total expenditure, to expenditure on groups of commodities and finally to subgroups of commodities.

(d) The model has a hierarchical structure; i.e., the higher level expenditure is assumed predetermined when it comes to modeling the expenditure for the lower hierarchical level.

(e) Furthermore, expenditure is assumed to depend on relative prices defined as ratios of two price indexes.

For the highest level, i.e. for the total expenditure formation, the following equation has been used.

$$W_t = \alpha_1 Y_t + \alpha_2 M_t + \alpha_o + \eta_t \qquad (15)$$

where W denotes total expenditure, Y is total income and M represents a variable expressing either the general state of the national economy or the consumers' expectations.[5] The second stage of the model mechanism refers to the formation of expenditure of the various commodity groups, the number of groups being G. Here, the typical equation explaining the expenditure on the i-th group of commodities is defined as

$$W_{it} = \beta_{i1} W_t + \sum_{\substack{j=1 \\ j \neq i}}^{G} \gamma_{ij} P_{jt} + \beta_{io} + \xi_{it} \qquad (16)$$

In (16) P_j represents the price variable. As has already been mentioned earlier, the price variable is defined as the ratio of two price indices. In its numerator appears the price index for the j-th commodity group, and in the denominator the price index for the i-th group $(i \neq j)$. In practice some price variables may a priori be eliminated when it is felt that no price influence for two given groups exists. Finally there are also equations explaining the formation of expenditure in various subgroups, the number of subgroups in the i-th group being equal to S_i. At this stage of modeling the group expenditures are assumed to be predetermined, and the model tries to explain the distribution of W_i into subgroup expenditures W_{ih}. The typical equation now has the form

$$W_{iht} = \beta_{iht} W_{it} + \sum_{\substack{l=1 \\ l \neq h}}^{S_i} \gamma_{ihl} P_{hlt} + \beta_{iho} + \xi_{iht} \qquad (17)$$

Here P_{hlt} is the price variable for the subgroup level and is defined as the ratio of the price index for the l-th subgroup within the i-th group to the price index for the h-th subgroup within the i-th group of commodities.

In order to achieve additivity of the expenditure system some constraints are introduced, namely that for each i = 1, 2,...,G there be

[5] In the latter case variable M has the subscript t+1 instead of t.

$$\sum_{h=1}^{s_i} W_{iht} = W_{it} \tag{18}$$

and that summing up of group expenditures gives the total level of expenditure, i.e.

$$\sum_{i=1}^{G} W_{it} = W_t \tag{19}$$

Using the relation (18) one arrives at the following constraints imposed on the parameters referring to subgroup equations

$$\sum_{h=1}^{S_i} \beta_{ihl} = 1 \tag{20}$$

and

$$\sum_{\substack{l=1 \\ l \neq h}}^{S_i} \gamma_{ihl}\bar{P}_{hl} + \beta_{iho} = 0 \qquad \begin{array}{l} i = 1, 2, \ldots, G \\ h,l = 1, 2, \ldots, S_i \end{array} \tag{21}$$

Similarly, for the level of group expenditures, the relation (19) leads to the following constraints

$$\sum_{i=1}^{G} \beta_{i1} = 1 \tag{22}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{G} \gamma_{ij}\bar{P}_j + \beta_{io} = 0 \qquad i = 1, 2, \ldots, G \tag{23}$$

The constraints (20)-(23) must be used in the process of estimation of the model. In (21) variable $\bar{P}_{hl}$ and $\bar{P}_j$ in (23) denote the arithmetic means of the corresponding variables observed in the sample period.

7. SUBSTITUTABILITY OF EXPENDITURES WHEN SUPPLY IS INSUFFICIENT

To conclude, we would like to point out still another aspect of demand analysis under conditions of inadequate supply. The insufficient level of supply of some commodity may--and usually will--lead to increased purchases of another commodity which is not in short supply and which can be used as a substitute for the missing one. A measure of such induced substitution is therefore needed. Several possible solutions can be thought of. The simplest one is

$$\lambda_1 = \frac{\Delta W_{iht}}{\Delta W_{jlt}} \tag{24}$$

which expresses the ratio of observed changes of expenditures on two compared subgroups.[6] The weakness of this measure, however, is that the changes of expenditures may also have been induced by factors other than the insufficiency of supply. A better measure is provided by

$$\lambda_2 = \rho_{hl} \cdot \frac{\sigma_h}{\sigma_l} \tag{25}$$

where ρ_{hl} denotes the coefficient of correlation of the random components appearing in the equations for expenditures on the h-th and l-th subgroups

[6] Let us note that here the subgroups need not belong to the same expenditure group.

respectively, and σ_l, σ_h denote standard deviations of the random components. If $\lambda_2 < 0$ then one can say that the expenditures on the two compared subgroups are substitutable.

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