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AN INVESTMENT TIMING MODEL FOR SALINITY  
MANAGEMENT VIA NON-CONVECTIVE PONDS

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## PREFACE

The work reported here is part of a REN research program on management of river salinity. A central objective of that program is to assess opportunities for reducing salinity by zero discharge uses of low quality water, such as industrial cooling. The concept of disposing of power plant cooling tower effluent in a salt gradient solar pond (which produces additional energy) emerged as a potentially significant component of the larger regional system being modeled. The sizing and timing of construction for such ponds presented a challenging systems problem by itself, and hence the need for the set of models reported.

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## ABSTRACT

A non-convective pond (NCP) as a solar energy collector can be an effective technological alternative in regions where problems of the disposal of highly saline water persist. In this paper, the use of NCP is studied as an alternative for the economically effective use of saline blowdown from a power plant. The problem considered concerns the determination of a rational scheduling of the construction period for NCP and is analyzed using an iteration procedure involving LP-programming as an iteration step. The results obtained for specific locations in the Colorado river basin are discussed.

## CONTENTS

INTRODUCTION AND SCOPE	1
<u>MATHEMATICAL DESCRIPTION OF THE NCP SYSTEM</u>	3
Assumptions and Notation	5
Economic Relationships	9
Downstream Benefits	11
Notation and Assumptions	11
Objective Function Addition	13
Cooling Water for NCP Energy Production	14
Solution Approach	17
Application in Colorado River Basin	19
Problem 2	27
Problem 3 Results	27
CONCLUSIONS	31
APPENDIX A: SOLUTIONS TO PROBLEM 3	38
REFERENCES	41

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INTRODUCTION AND SCOPE

The concept of using a non-convective pond (NCP) as a solar energy collector is currently attracting the attention of researchers in many regions of the world. The concept has been studied for more than 25 years in Israel, where several ponds have been constructed and some are now producing electricity. In addition to the work in Israel, several research organizations in the US have already produced models of the thermodynamics of such ponds. These include the Solar Research Institute, the Jet Propulsion Laboratory, Utah State University, and MIT.

Very briefly, the concept involves floating a thin layer of fresh water over a few meters depth of highly saline brine. The density of the brine is such that it can approach 100°C without mixing convectively with the lighter although colder surface layer. Since the cooling effect of evaporation is confined to the surface layer, the brine becomes a very effective heat

storage reservoir. The heat can be used directly for such purposes as pre-heating boiler water in fossil fuel plants or it can be converted to electricity by using low temperature turbo-generators.

Two qualities of makeup water are necessary for operation of a NCP. Since some salt is lost from the brine layer by diffusion, a highly saline (approximately 260,000 mg/l) source of brine is required. A much larger source of higher quality water is required to replace evaporation and surface flushing losses. However, the quality of this water can also be quite low since the waste stream from the surface layer can be at approximately 50,000 mg/l total dissolved solids (tds). In addition, if the heat produced is to be converted into electricity, additional cooling water is needed for the conversion process and with appropriate cooling tower technology, this source can also be very low quality water.

We have then a very interesting array of water and salt demands, particularly in a setting where salinity management is an objective. The very factors related to NCP operation which mitigate against economic and environmental objectives in many locations emerge as very positive factors when reduction of river salinity is important. For example, the large evaporation demands of NCP's coupled with the large cooling tower demands for converting the heat to electricity at extremely low efficiency due to the low temperature conversion process (Batty et al., 1982) may tend to make a NCP project appear to be economically and environmentally infeasible if fresh water is used. However, if low quality water is used, an objective may well be

to evaporate as much water as possible in order to keep the salt load out of the main river, thereby creating both economic and environmental net benefits.

The objectives of this report include:

- (1) Develop a generalized model which describes the NCP system in a setting where waste water from an adjacent industrial cooling operation is available. Parameters to be quantified include pond areas, water and salt flows, heat and/or electricity produced, costs incurred, all as functions of time.
- (2) Develop a solution procedure for the NCP model problem including investment timing optimization.
- (3) Apply the model to specific locations in the Colorado River Basin (those sites identified by the US Bureau of Reclamation as "local option" salinity management projects [US Bureau of Reclamation, 1981]).

This report will include no discussion of the NCP thermodynamics but rather will simply accept the estimates of heat produced per unit of pond area at various sites produced by an existing model (SOLPOND) developed by the Solar Energy Research Institute in Golden, Colorado, USA (Henderson and Leboeuf, 1980), and operated for the sites involved by personnel of the USBR Colorado River Water Quality Improvement Program.

#### MATHEMATICAL DESCRIPTION OF THE NCP SYSTEM

If sufficient brine near saturation salinity is available, or if salt is transported to the site for rapid production of brine, the water/salt/pond area quantities for a non-convective solar pond (NCP) can be determined by a relatively simple system of equilibrium equations. The system will include water and

salt mass-balance equations for the NCP itself and for the brine makeup and possibly a fresh water makeup pond. Also required is a salt diffusion function describing the loss of salt from the non-convective layer. The water mass-balance equations must of course include estimates of evaporation as functions of the salinities in each type of pond.

This system can be described rather well by linear functions. If costs of ponds and the energy system revenues are known, an economic analysis can be made by simply comparing annual revenues to annual costs. A mathematical description of such a system is given by Batty, et al., (1982). If, however, a NCP system is to be developed by concentrating brine which is initially much less than saturation, many years may be required before ultimate capacity (the equilibrium state) is achieved. This implies investing capital many years before maximum return is obtained. Further, it may not be efficient to simply produce brine for many years in the ultimate sized makeup pond and then begin total NCP operation at a single future time. Rather, construction of both makeup and NCP cells in several increments may produce maximum net return. Hence, the non-equilibrium problem is basically an investment timing optimization problem which is non-linear both in the objective function (due to discounting costs and revenues over variable time periods) and in one type of constraint.

The notion of using blowdown from a power plant for brine production in a NCP and using other low quality water for freshening of the NCP surface layer and/or brine production requires precisely such a system. A mathematical description



of the system follows. The assumed waterflows between components of the system are shown in Figure 1.

#### Assumptions and Notation

- $w^0$  = annual blowdown from power plant. This highly saline flow must flow to the brine makeup pond.
- $w^1$  = annual amount of low quality water (LQW) available at the site. This water can either be used by the NCP system or returned to the river.
- $sc^0$  = salinity concentration of  $w^0$ .
- $sc^1$  = salinity of  $w^1$ .
- $T_p$  = time elapsed from starting to fill the p-th section of pond 2 to the time when p-th section of pond 3 begins operation.
- $t_p$  = time (year) at which the p-th increments of ponds 2 and 3 will be included into operation.
- $w_p^{12}$  = annual flow from source 1 to pond 2 (the brine production and makeup pond) during period  $T_p$ .
- $w_p^{13}$  = annual flow from source 1 to pond 3 (the NCP) during period  $T_{p+1}$ . This is freshwater makeup for pond 3 increments constructed up to the beginning of period  $T_{p+1}$ .
- $w_p^{23}$  = annual flow from pond 2 to pond 3 during period  $T_{p+1}$  (brine makeup for pond 3 increments constructed up to the beginning of period  $T_{p+1}$ ).
- $w_p^{32}$  = amount of water flushed annually from surface of pond 3 during period  $T_{p+1}$  and received by pond 2.
- $A_p^2$  = increment of area of pond 2 completed at beginning of p-th period (construction is assumed to require one year).

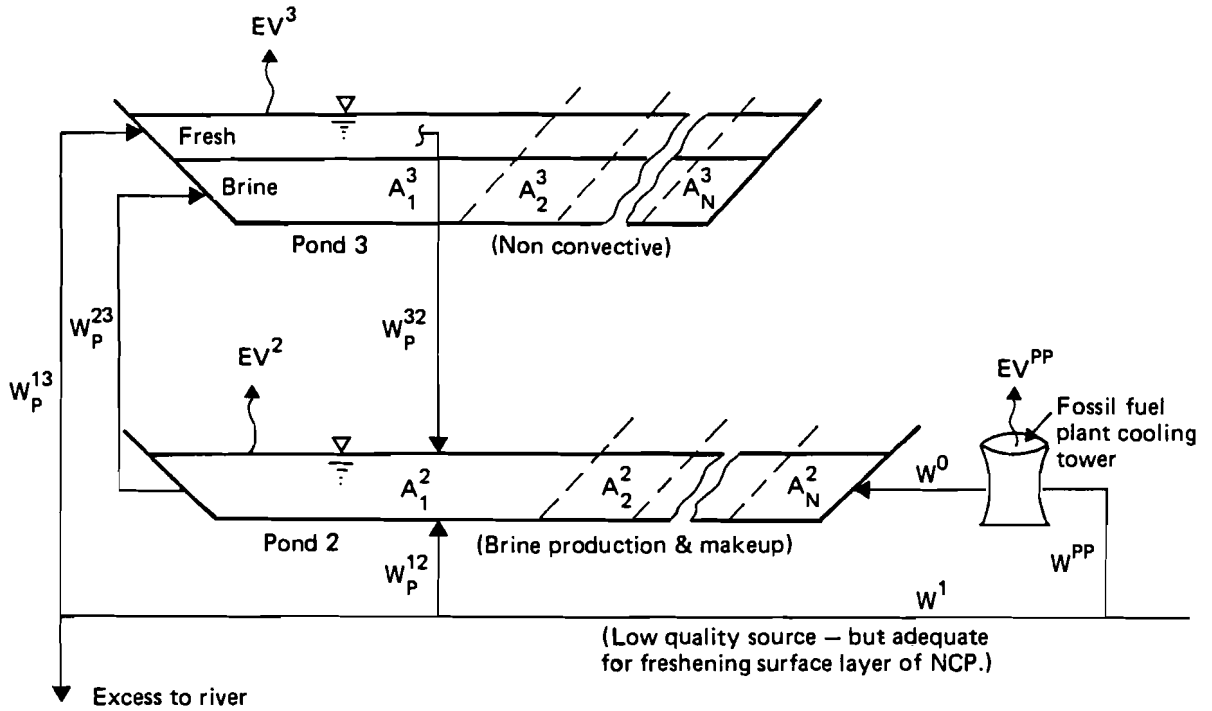


Figure 1. Non-equilibrium system schematic.

$A_p^3$  = increment of area of pond 3 which begins operation at end of p-th period. Construction is assumed to require 1 year. Note that while all water flows are annual totals up to the p-th period, areas  $A_p^2$  and  $A_p^3$  are only increments added at the beginning and at the end of the p-th period respectively.

$d^2, d^3$  } = minimum depths allowed in ponds 2 and 3.

$ED^2, ED^3$  } = estimated net annual evaporation from ponds 2 and 3 (where net = total evaporation minus precipitation).

N = maximum number of area increments allowed ( $p=1,2,\dots,N$ ).

SAT = salinity of brine desired for NCP.

$SP_p^2, SP_p^3$  } = amounts of salt precipitating annually in pond 2 during period  $T_p$  and in pond 3 during period  $T_{p+1}$ .

$DF^3$  = diffusion coefficient for pond 3 (weight/unit area).

$R(A_p^3), C^2(A_p^2), C^3(A_p^3)$  } = revenue and pond cost functions respectively.  $R(A_p^3)$  is net revenue per year (gross revenue minus operating cost) while the other two functions are costs of the p-th additions to ponds 2 and 3.

r = interest rate.

The physical system can be represented by the following constraints:

- (1) In order to have brine in pond 2 at all times after the initial summer season, the area increments must be large enough to evaporate sufficiently that:

$$\frac{\text{dissolved salt added during p-th period}}{\text{net water added during p-th period}} \geq \text{SAT}$$

or specifically:

$$\frac{(SC^0 \cdot W^0 + SC^1 \cdot W_1^{12} - SP_1) T_1}{(W^0 + W_1^{12} - ED^2 \cdot A_1^2) T_1} \geq SAT \quad (1)$$

$$\frac{(SC^0 \cdot W^0 + SC^1 \cdot W_p^{12} + SC^{32} \cdot W_{p-1}^{32} - SAT \cdot W_{p-1}^{23} + SP_{p-1}^2 - SP_p^2) T_p}{(W^0 + W_p^{12} + W_{p-1}^{32} - W_{p-1}^{23} - ED^2 \cdot \sum_{j=1}^p A_j^2) T_p} \geq SAT \quad (p=2, \dots, N)$$

Note that the time variables ( $T_p$ ) can be eliminated from (1) and (2). (2)

(2) Period  $T_p$  must be long enough to produce sufficient brine in pond 2 during each p-th period to leave minimum depth in pond 2 after transferring  $d^3 \cdot A_p^3$  to pond 3 at the period end. Therefore we have:

$$(W^0 + W_1^{12} - ED^2 A_1^2) T_1 \geq d^2 A_1^2 + d^3 A_1^3 \quad (3)$$

$$(W^0 + W_p^{12} + W_{p-1}^{32} - W_{p-1}^{23} - ED^2 \sum_{j=1}^p A_j^2) T_p \geq d^2 \cdot A_p^2 + d^3 \cdot A_p^3 \quad (p=2, 3, \dots, N) \quad (4)$$

(3) Water balance constraints are needed for pond 3 for all p, but only after the N-th period for pond 2 (since water volume in this pond is continually increasing or decreasing during the first N periods): therefore:

$$W^0 + W_{N+1}^{12} + W_N^{32} - W_N^{23} - ED^2 \cdot \sum_{j=1}^N A_j^2 = 0 \quad (5)$$

$$W_p^{13} + W_p^{23} - W_p^{32} - ED^3 \sum_{j=1}^p A_j^3 = 0 \quad (p=1, 2, \dots, N) \quad (6)$$

(4) Upper limits on water consumptions from low quality sources are required:

$$W_1^{12} \leq W^1 \quad (7)$$

$$W_p^{12} + W_{p-1}^{13} \leq W^1 \quad (p=2, \dots, N+1) \quad (8)$$

(5) Salt precipitation and re-resolution are assumed to be possible in both ponds during the expansion period but only in pond 2 during equilibrium operation (after the completion of all construction). Therefore, salt balance equations can be written in the form:

$$SC^0 \cdot W^0 + SC^1 \cdot W_{N+1}^{12} + SC_{32} \cdot W_N^{32} - SAT \cdot W_N^{23} - SP_{N+1}^2 = 0 \quad (9)$$

$$SC^1 \cdot W_p^{13} + SAT \cdot W_p^{23} - SC_{32} \cdot W_p^{32} + SP_{p-1}^3 - SP_p^3 = 0 \quad (p=1, 2, \dots, N) \quad (10)$$

$$SP_0^3 = SP_N^3 = 0 \quad (11)$$

(6) The final constraint type calculates the brine makeup required to replace salt lost from the non-convective layer by diffusion:

$$SAT \cdot W_p^{23} + SP_{p-1}^3 - DF \sum_{j=1}^p A_j^3 - SP_p^3 = 0 \quad (p=1, \dots, N) \quad (12)$$

### Economic Relationships

Most of the costs related to pond construction can be expressed as functions of area only, the principal one being pond lining to prevent groundwater contamination, and others including land purchase, and site clearing. The volume of the dike is related to depth as well as area, but for normal depths and large pond areas, dike costs are small relative

to other costs and therefore, for fixed depth an assumption that pond construction costs vary linearly with area introduces only insignificant error. Since energy produced from NCP systems also varies linearly with area (for fixed depth) both the costs of mechanical equipment and revenue from energy are also assumed to be linear functions of the pond 3 area. Economies of scale probably exist in regard to mechanical equipment, but for the range sizes expected, this effect was ignored.

These linear cost/area relationships produce a linear objective function for the equilibrium period. However, the expansion period objective function is highly non-linear due to discounting over variable time periods. The objective function (which is defined as present worth of revenue minus costs) can be written as:

$$\sum_{p=1}^N R(A_p^3) \left[ \frac{1}{r} \cdot \frac{1}{(1+r)^{T_p+t_p+1}} \right] - C^2(A_p^2) \frac{1}{(1+r)^{t_{p-1}}} - C^3(A_p^3) \frac{1}{(1+r)^{T_p+t_{p-1}}} \quad (13)$$

where:

$$t_p = \sum_{j=1}^{p-1} T_j + 1 \quad (p=2, \dots, N) \quad (14)$$

the  $R(A_p^3)$  coefficient in function (13) was derived by discounting the future stream of benefits from the p-th addition to its time of initial use, (year t), as a uniform series over a long period  $(1/r)$ , then reducing this to worth at time = 0 by the

appropriate factor  $1/(1+r)^t$ . The cost functions were similarly discounted from the time of beginning construction for each increment.

The problem of maximizing the objective function (13) under constraints (1) through (12) plus (14) will be referred to as Problem 1.

#### Downstream Benefits

The Solution to Problem 1 gives the best investment timing and ultimate capacity configuration for a NCP system from the perspective of a local profit maximizer whose objective ignores possible benefits due to reduction of river salinity below the site. In order to define the problem from the perspective of a salinity management agency, the following additions to Problem 1 are required:

#### Notation and Assumptions

- BEN = annual benefit per unit of salinity reduction at a particular downstream location (point ds).
- $w_0^{ds}, s_0^{ds}, sc_0^{ds}$  = projected future annual average flow of (1) water; (2) salt; and (3) salt concentration at point ds with no salinity management.
- $w_p^{ds}, s_p^{ds}, sc_p^{ds}$  = modeled annual flow of water, salt, and concentration at point ds after completion of the p-th NCP unit.
- $\Delta SC_p^{ds}$  = reduction in salinity concentration at point ds due to p NCP units.

m = the time-lag after which initial downstream benefits begin. This is the time delay in years between diversions from the river and downstream response due to reservoirs, etc.

$W^{PP}$  = power plant water diversion.

The revised water and salt quantities at point ds can be calculated as:

$$W_p^{ds} + W^{PP} + W_p^{12} + W_{p-1}^{13} = W_0^{ds} \quad (p=1, 2, \dots, N+1) \quad (15)$$

$$S_p^{ds} + SC^0 \cdot W^0 + SC^1 \cdot (W_p^{12} + W_{p-1}^{13}) = SC_0^{ds} \cdot W_0^{ds} \quad (p=1, \dots, N+1) \quad (16)$$

The change (reduction) in salinity concentration is:

$$\Delta SC_p^{ds} = SC_0^{ds} - SC_p^{ds} = SC_0^{ds} - S_p^{ds} / W_p^{ds} \quad (p=1, 2, \dots, N+1) \quad (17)$$

Equation (17) is non-linear with respect to W; however, it can be expressed in linear form with good accuracy for small changes in salinity by approximating the derivative of salinity concentration as the finite change due to NCP units as follows:

$$SC \equiv S/W$$

$$d(SC) = \frac{\partial (S/W)}{\partial S} ds + \frac{\partial (S/W)}{\partial W} dW$$

$$= \frac{1}{W} ds - \frac{S}{W^2} dW$$

$$\frac{d(SC)}{SC} = \frac{W}{S} \cdot \frac{ds}{W} - \frac{W}{S} \cdot \frac{S}{W^2} dW$$

$$\frac{d(SC)}{SC} = \frac{ds}{S} - \frac{dW}{W}$$



or

$$\frac{\Delta SC_p^{ds}}{SC_0^{ds}} = \frac{s_0^{ds} - s_p^{ds}}{s_0^{ds}} - \frac{w_0^{ds} - w_p^{ds}}{w_0^{ds}} \quad (p=1, 2, \dots, N+1) \quad (18)$$

Objective Function Addition

The objective function of Problem 1 expresses pond and energy production benefits and costs as present values. It is therefore necessary to discount the annual downstream benefits associated with  $p$  NCP units to time zero. This could be done either by considering the sum of future benefits due to each of  $p$  units (a horizontal decomposition of the total  $\Delta SC^{ds}$  function over time) or by considering a vertical decomposition of the  $\Delta SC^{ds}$  function as shown in Figure 2. The latter method will be used since it allows  $\Delta SC_p^{ds}$  to be used directly as defined by (17).

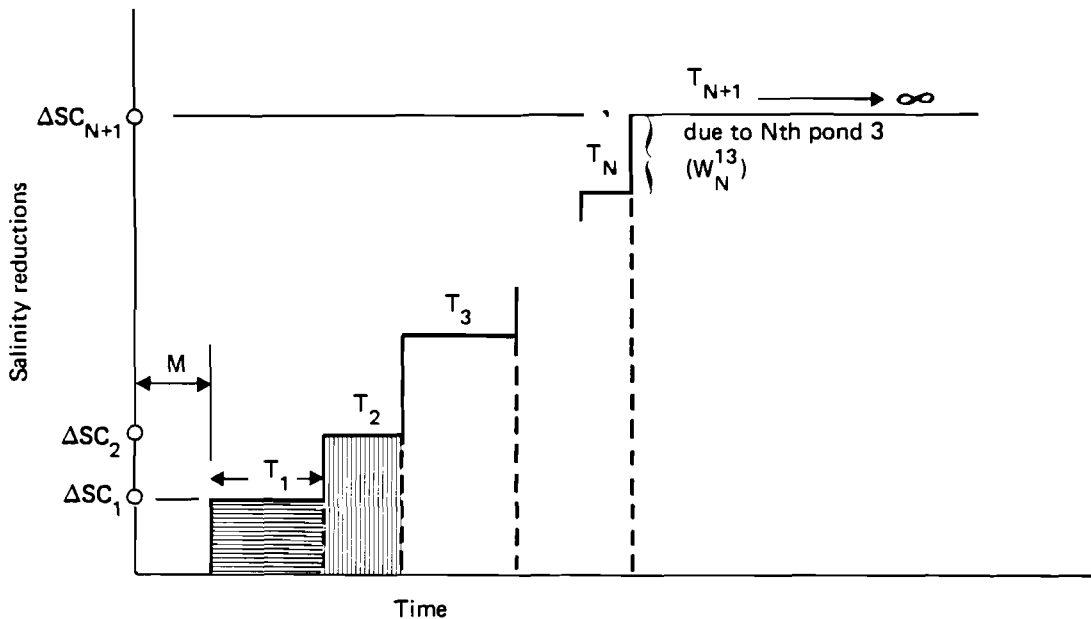


Figure 2. Change in downstream salinity over time.

The objective function coefficients can be quantified by using a uniform series present worth factor:

$$\frac{(1+r)^T p - 1}{r(1+r)^T p}$$

to discount each rectangle in Figure 2 to its value at its time of beginning and then discounting to time zero by the appropriate  $1/(1+r)^n$  factor. This produces the following function:

$$BEN \sum_{p=1}^N \left[ \frac{1}{(1+r)^{t_p}} \cdot \left\{ \frac{(1+r)^T p - 1}{r(1+r)^T p} \right\} \cdot \Delta SC_p^{ds} \right] + \frac{BEN}{r} \left[ \frac{1}{(1+r)^{t_{N+1}}} \cdot \Delta SC_{N+1}^{ds} \right] \quad (19)$$

where:

$$t_p = \sum_{j=1}^{p-1} T_j + m \quad (20)$$

and:

$$t_{N+1} = \sum_{j=1}^N T_j + m \quad (21)$$

If we therefore add equations (15), (16) and (18) to the constraint set of Problem 1 and terms given by (19) to the original objective function (13), the maximization problem obtained will then include downstream benefits due to salinity reduction. This will be referred to as Problem 2.

#### Cooling Water for NCP Energy Production

So far, we have NCP models which can be used either to maximize profit for a local NCP entrepreneur (Problem 1) or to maximize net regional benefits including those due to preventing downstream salinity damages (Problem 2). These models are adequate if the energy produced by the NCP is used in the form

of heat. Direct use of heat may well be desirable, for example, for preheating the process water in an adjacent fossil fuel power plant. However, if the NCP heat is to be converted to electricity, cooling water will be required for the heat exchange process. The water demand for this cooling is in fact almost an order of magnitude higher per MW of electricity than for a fossil fuel plant, because of the low efficiency (about 60%) of the thermal to electrical conversion process (Batty et al., 1982). This compares to about 38% efficiency for coal/electricity conversion. The large resulting water demand would be a very negative aspect of NCP economics and environmental impact if the water source is to be high quality water. However, from a salinity management perspective, a demand for large quantities of (saline) cooling water is precisely what we seek.

A revised NCP schematic including the cooling water component addition is shown in Figure 3. The changes to previous models required to quantify the new parameters follow. The revised model will be used to formulate Problem 3. The notation is defined by Figure 3. The water balance at the NCP cooling tower can be expressed as:

$$W_p^{1c} = W_p^{c2} + EV_p^c \quad (p=1,2,\dots,N) \quad (22)$$

Since energy production is assumed to be a linear function (k) of the area of the NCP (on an annual average basis, with fixed use factor) water demand can be quantified as:

$$EV_p^c = K \cdot \sum_{j=1}^P A_j^3 \quad (23)$$

Also, a salt balance through the cooling tower is required as follows:

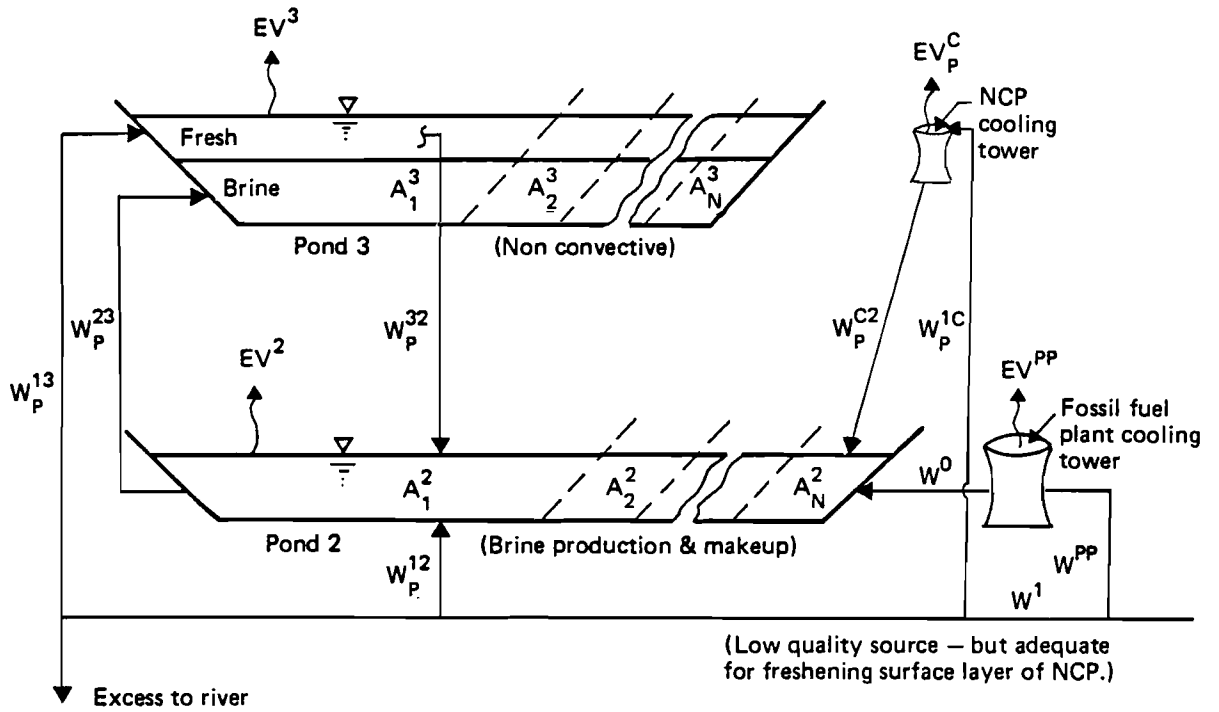


Figure 3. NCP schematic including thermal/electric conversion water demand.

$$w_p^{c2} = \frac{SC^1}{SC^0} \cdot w_p^{1c} \quad (p=1,2,\dots,N) \quad (24)$$

The following additional terms should also be added to (the left side of) previous equations in order to account for water and salt balance changes:

<u>Equation</u>	<u>New Term</u>
(2)	$\left\{ \begin{array}{l} SC^0 \cdot w_{p-1}^{c2} \quad (\text{add to numerator}) \\ w_{p-1}^{c2} \quad (\text{add to denominator}) \end{array} \right.$
(4)	$w_{p-1}^{c2}$
(5)	$w_N^{c2}$
(6)	$-w_p^{3c}$
(8)	$w_{p-1}^{1c}$
(9)	$SC^0 \cdot w_N^{c2}$
(10)	$-w_p^{3c} \cdot SC^{32}$
(15)	$w_{p-1}^{1c}$
(16)	$SC^1 \cdot w_{p-1}^{1c}$

No modification of the objective function is required. The revised problem including all the relationships used in Problem 2 plus (22), (23), (24), and changes to (2) through (6) given above will be referred to as Problem 3.

#### Solution Approach

Problems 1, 2, and 3 are the non-linear mathematical programming type and in principle, non-linear computational algorithms can be applied for obtaining their solutions. But as

is frequently the case, the optimal solution to a mathematically formulated problem is not the best for a corresponding real system. Any mathematical model is always an approximation of the reality and can not in principle encompass all the aspects of the real system under analysis, therefore the time and effort spent on obtaining mathematically precise optimal solutions is often not justified. In this study, rather than trying to obtain an optimal solution, we used a simplified heuristic procedure providing for a local improvement of an initial solution.

This procedure is based on the following properties of all the problems considered here. In each problem, the variables can be divided into two classes:

- (1) An N-vector of time periods  $T=(T_1, \dots, T_N)$ ;
- (2) A vector of all other variables (incremental areas, water flows, etc.).

A common property of all 3 problems is that for fixed vector  $T$ , all the problems are linear. Based on this property, we used the following iteration procedure: At each  $k$ -th iteration with vector  $T^k$  fixed we solve the corresponding LP problem (maximizing the objective function with respect to all the variables except  $T$ ), then we appropriately change vector  $T^k$  to obtain a new vector  $T^{k+1}$ , solve the new LP problem (with  $T^{k+1}$  as the value of vector  $T$  in the objective function and in the constraints) and so on. The procedure stops when the current value of the objective function is satisfactory, starts to decrease, or when the current LP problem has no solution. Clearly, the effectiveness of the procedure depends on the method for choosing the direction of change for vector  $T^k$ , and also the length of the step along

this direction. In our study, this direction was chosen by considering the gradient of the objective function with respect to  $T$  at a current point, and also slacks in the constraints of the type (3)-(4) containing variables  $T_p$ . As can be seen from the formulations of our problems, a solution producing slacks in these constraints can be improved by decreasing the values of the corresponding variables  $T^P$ .

The block diagram of the procedure is illustrated in Figure 4. It was implemented using a UNIX operating system on a VAX 11/780 computer. The software package which has been developed is a very user-friendly interactive system, which provides great flexibility in the analysis and produces adequate solutions with reasonable computational effort.

#### Application in Colorado River Basin

A US Bureau of Reclamation special report on Saline Water Use and Disposal Opportunities in the Colorado Basin (USBR, 1981) identifies seven locations within the upper basin (and others in the lower basin) where proposed coal-fired electric generating plants are each located less than 100 miles from a significant source of low quality water. Costs of collecting and transporting that fraction of the LQW which can reasonably be diverted are shown in Table 1. Also included are the estimated power plant water demand and waste quantities assuming that only low quality water is used for cooling tower makeup and that a high salinity technology cooling tower is used which concentrates the blowdown to 120,000 mg/l tds. Other assumptions are that both investment costs and future benefit streams are discounted at 7 $\frac{3}{8}$ % (this will be varied later) minimum brine depth

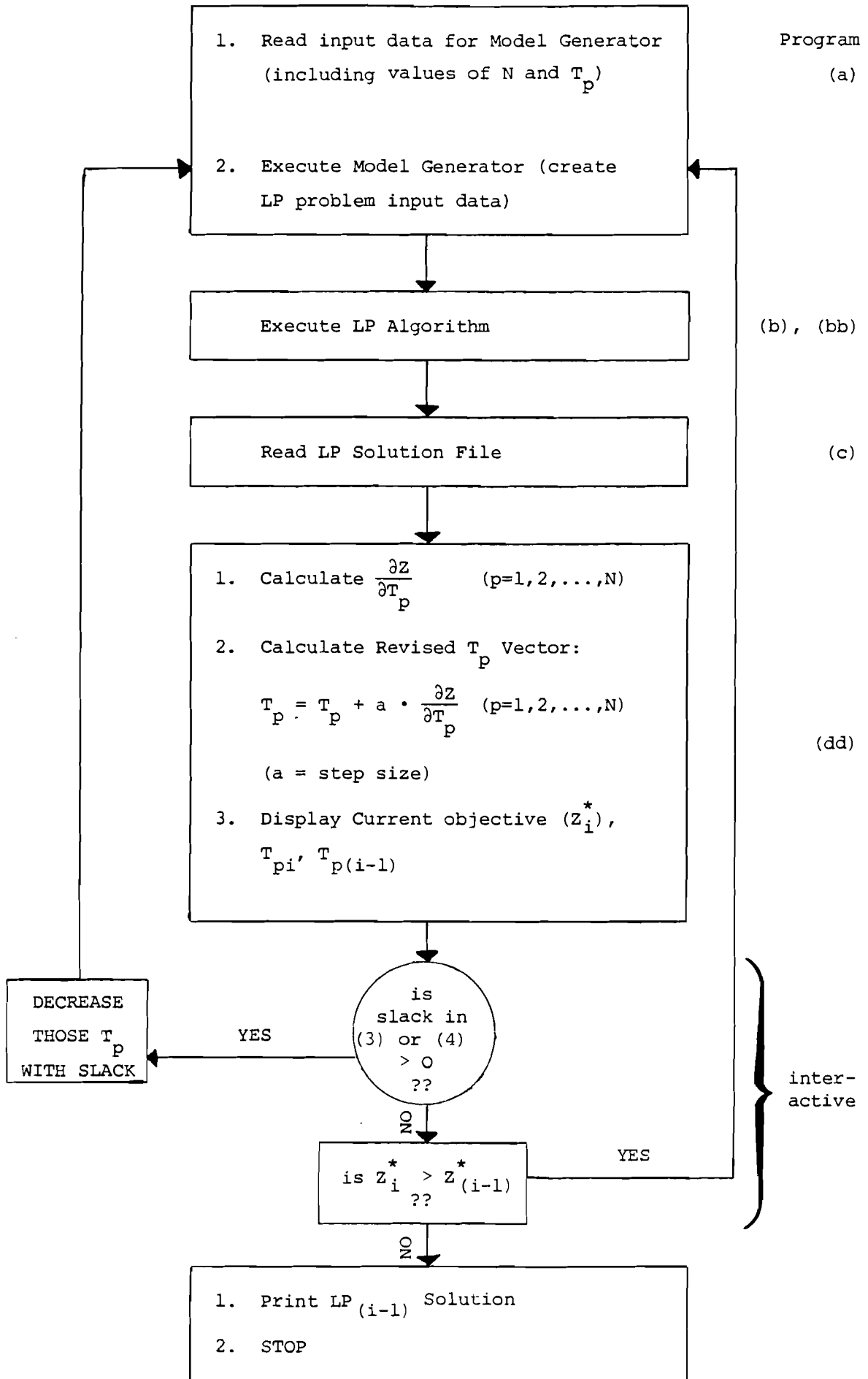


Figure 4. Solution flow diagram.



Table 1. Upper Colorado NCP information (water and salt dimensions are  $10^3 m^3$  and  $mg/l$  resp.).

Site	Total Available LQW	Salt Conc.	Power Plant			LQW Remaining (W)	NCP Elec. ( $\frac{KWH}{M^2}$ )
			Capacity (MW)	Makeup (WPP) 1	Blowdown (W <sup>0</sup> ) 1		
Big Sandy	17,900	5000	350	5,725	302	12,170	20
Uintah Basin	16,780	4500	800	13,030	661	3,750	27
Price River	10,490	4000	500	8,100	383	2,380	27
San Rafael	8,390	3600	400	6,470	296	1,925	27
Grand Valley	33,560	3300	800	12,960	497	20,600	27
Lower Gun.	21,220	2900	800	12,830	494	8,400	27
McElmo Cr.	25,670	2700	1600	25,670	990	0	30

Source: Israelsen et al., (1980).

in pond 3 is 2.0 meters; salt diffusion rate is  $.023 \text{ ton/m}^2/\text{yr}$ , pond costs are  $\$4.8/\text{m}^2$  and  $\$10/\text{m}^2$  for ponds 2 and 3 respectively (the latter includes electric generating equipment); revenue from sale of electricity is  $\$1.5/\text{m}^2$  (area of pond 3). The reference flow in the river downstream from the project locations (at Lee's Ferry) is 11.1 million acre feed/yr and the pre-management salinity at this point is 581 mg/l. The downstream benefits for reducing salinity are  $\$.27(10^6)$  per mg/l. Finally, the NCP cooling tower water demand is estimated at  $0.5 \text{ m}^3$  per  $\text{m}^2$  of pond 3 area. Justification of these coefficient values is discussed in Hughes, Orlovsky and Narayanan (1982), and will not be presented here. Two of the coefficients, revenue/ $\text{m}^2$  and NCP cooling tower water demand are varied from the values given above for two sites (Big Sandy and McElmo Creek) in proportion to the unit energy production given in Table 1.

Evaporation estimates at these sites are given in Table 2. These quantities are based upon lake evaporation estimates from Hughes, et al. (1974) as adjusted for both salinity and heat advection.

Table 2. Evaporation depth (in inches).

Site	Elevation (ft)	Annual Lake Evap.	Total Pond 2 Evap.	Pond 2	Pond 3
				Evap. Minus Precip.	Evap. Minus Precip.
Big Sandy	6500	29.6	23.6	5.6	7.2
Uintah B.	4800	37.5	30.0	17.0	22.0
Price River	5400	36.5	29.2	17.2	22.0
San Rafael	5500	36.7	29.4	17.4	22.2
Grand Valley	5000	38.5	30.8	16.8	21.5
Lower Gun.	5000	38.5	30.8	16.8	21.5
McElmo Cr.	5000	67.6	54.1	46.1	57.0

Problem 1 Results

This problem includes no water demand for heat/electricity conversion and no calculation of downstream benefits. The "optimal solutions" in this mode are interesting in that the sequence of pond increments added over time neither increase nor decrease monotonically but rather assume an alternating increasing/decreasing pattern.

For example, consider the Lower Gunnison solution shown in Table 3. In general, the "Low Quality" source is much too fresh to produce brine in a reasonable time (in a reasonably sized pond) therefore, only the very salty blowdown ( $W^0$ ) from the power plant cooling tower (assumed to be a high salinity type tower) is used to produce brine directly. The LQW ( $W^1$ ) is used only for fresh layer makeup in the NCP and eventually for brine after concentration to 50,000 mg/l in the NCP (Figure 5).

Table 3. Solution to Problem 1.

		$T_p$									$\Sigma = 60$
		3	4	3	7	7	9	9	9	9	
Variable	$P$	1	2	3	4	5	6	7	8	9	Totals
	$W_p^{12}$		0	0	0	0	0	0	0	0	60
$W_p^{23}$		22	131	0	177	239	333	429	529	529	
$W_p^{32}$		130	346	422	1037	1400	1957	2515	3104	3104	
$A_p^2$		617	243	477	0	1223	681	1043	1047	1802	7136
$W_p^{13}$		245	587	865	1960	2646	3698	4753	5865	5865	
$A_p^3$		250	426	129	1193	700	1072	1076	1134	0	5983

Objective value = \$478,000.

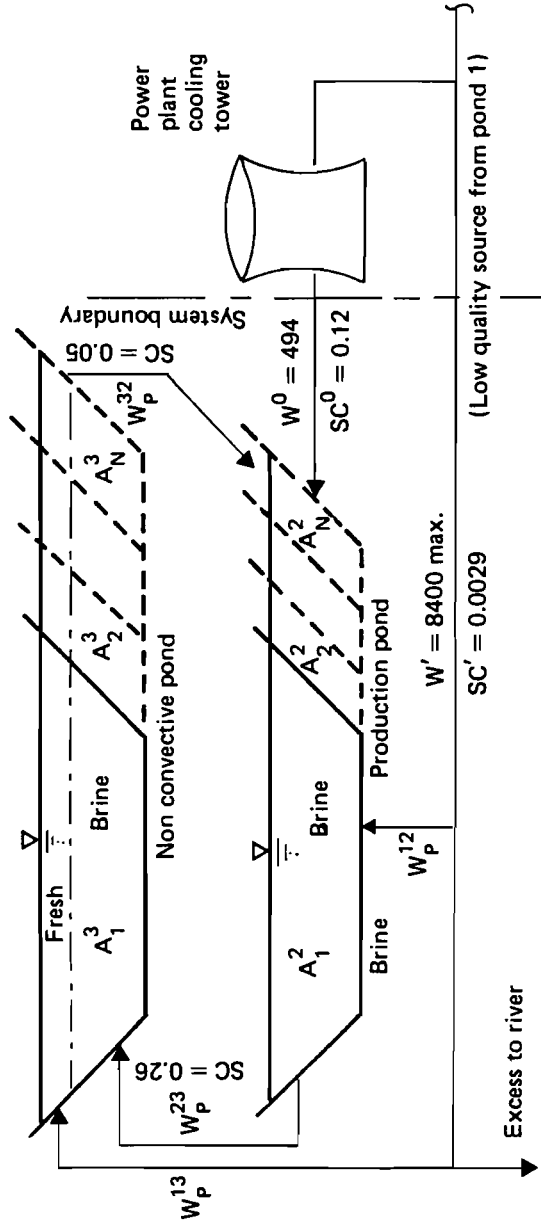


Figure 5. Example application of model 1. (Dimensions are  $10^3 M^3$  for water, and metric tons/ $M^3$  for salt concentration).

Since pond 2 receives the same amount of  $W^0$  each year, but an increasing amount of  $W_p^{32}$  (during  $P$  for which an increase in  $A^3$  occurred during  $p-1$ ) the  $A_p^2$  increments are sized accordingly. Also, since  $A^3$  increments produce net revenue but  $A^2$  increments incur only costs, the optimal strategy is to delay construction of  $A^2$  additions but expedite  $A^3$  additions. This fact coupled with the lag 1 relationship between  $A^2$  and  $A^3$  (the first  $p$  increments of  $A^3$  produce return flow to  $p+1$  increments of  $A^2$ ) produce a system where rational strategy is to iteratively increase and decrease both the  $A^2$  and  $A^3$  additions as shown in Figure 6. The total pond sizes in operation however, grow as shown in Figure 7.

The selection of  $N=9$  is arbitrary since the 8400 units of  $W^1$  are not used completely even after 60 years (only 5865 are used). If  $N$  were increased,  $W_N^{13}$  would continue to increase until  $W^1$  was exhausted. This is so because both revenue and costs are discounted at equal rates and therefore, if there is a net profit for a particular  $N$ , a net profit will occur at any  $N$  and  $T_p$  vector. However, the present worth of any additional benefit as total time exceeds 40 years approaches zero. Therefore, the more interesting questions are related to pond areas, water diversions and net returns after more reasonable planning periods such as 20-30 years. The 60-year duration example is presented only to demonstrate the pattern  $T_p$  assumes over long periods.

The water quantities and pond area portions of the solution displayed in Table 3 are not sensitive to changes in costs, revenue, or interest rate so long as a net positive revenue is maintained. This is so because the basic strategy of making

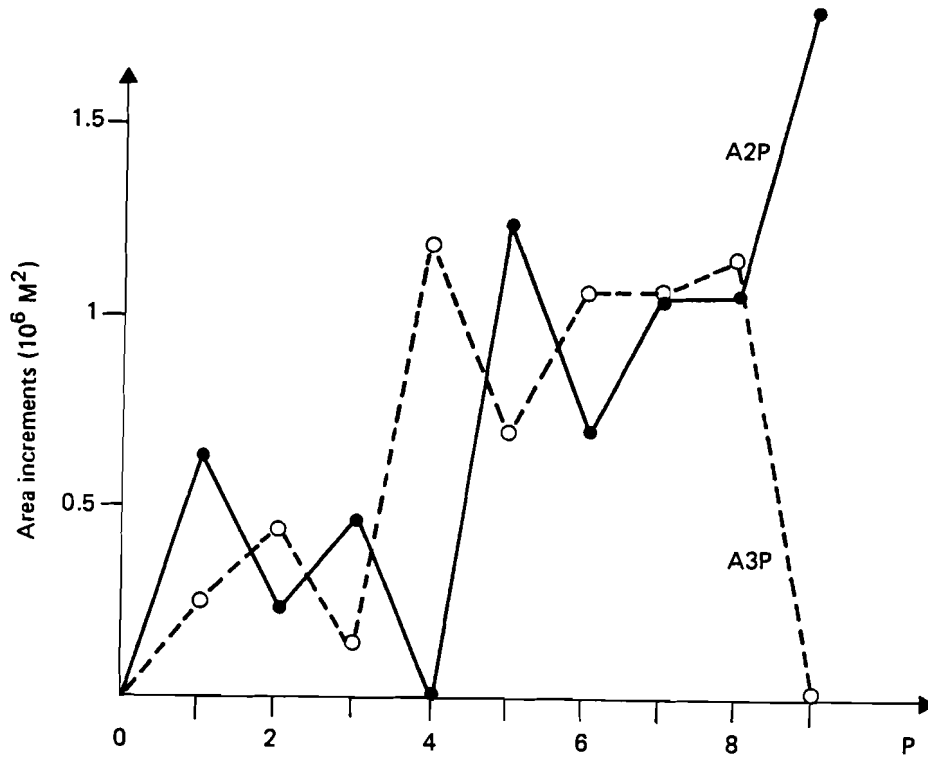


Figure 6. Area expansion per interval.

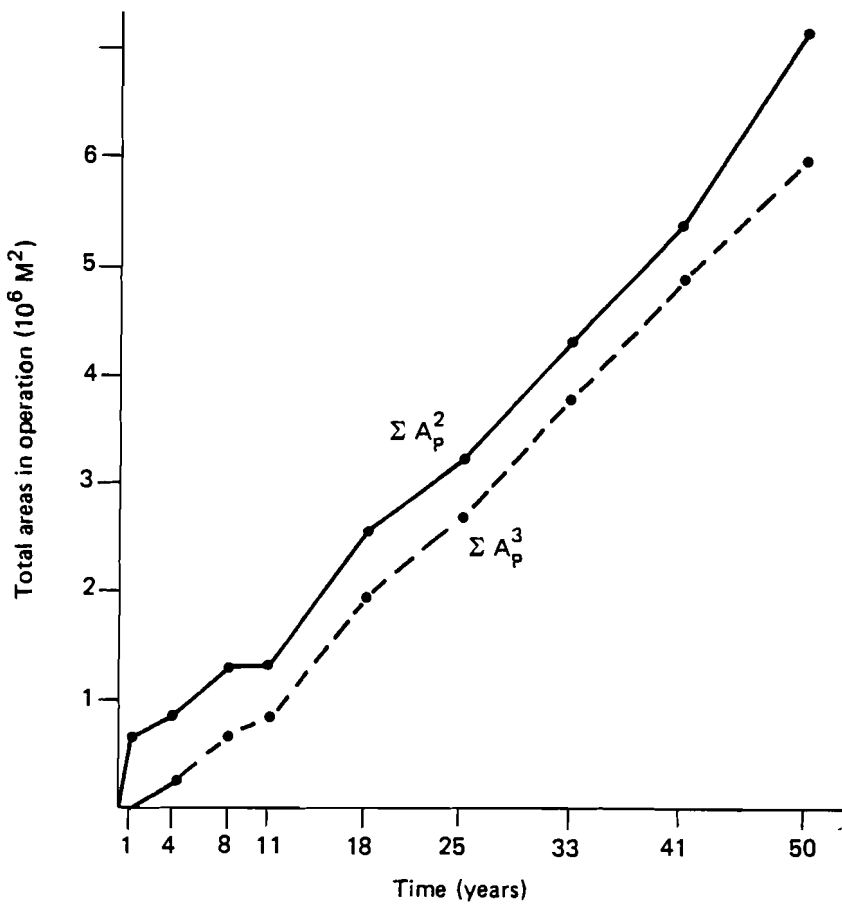


Figure 7. Total pond areas in use.

brine only from  $W^0$  and using  $W^1$  for pond 3 makeup is stable over a wide range of  $W^1$  salinity concentrations. The value of the objective function however, is very sensitive to changes in money-related coefficients.

For Problem 1, it is necessary for the NCP to make a net profit in order to produce a solution of interest (a non-zero  $A^3$  solution). In the case of Problem 2 and 3, however, the downstream benefit may produce a non-zero solution even with an on-site NCP net loss.

#### Problem 2

Problem 2 represents the addition of downstream salinity benefits but still maintains energy production in the form of heat. In a salinity management setting problem, 3 is a more interesting model since it has a significant additional diversion of water for cooling of the heat/electricity conversion process. Therefore, in the interest of limiting the number of detailed applications, no Problem 2 examples will be presented here.

#### Problem 3 Results

The model including both downstream benefits and NCP cooling tower water demand was applied at each of the seven upper basin locations. The assumed configuration at each site is as shown in Figure 2, which pre-supposes that a LQW diversion and delivery system has been constructed and that a high salinity technology cooling tower has been provided at the adjacent power plant. Costs of the delivery system and added costs due to using LQW in the fossil fuel plant are not con-

sidered in the NCP model. The objective here is only to answer the question--given a power plant with known highly concentrated blowdown and known limit on LQW, what sizes (if any) and timing of NCP ponds should be constructed? A regional model which does consider all costs related to the power plant, NCP, LQW delivery system, and alternatives other than the one assumed here are reported in a separate publication (Hughes, Orlovsky and Narayanan, 1982). NCP model applications reported here were used as input data for that more global model.

The complete solutions for each site are included in Appendix A and will only be summarized here. The pattern of investment timing is similar for each location at which the pond concept is feasible. The climate at the Big Sandy site is such that low levels of both solar radiation and evaporation make this location unsuitable for development of a NCP. Five of the other six sites have similar characteristics which appear to make the NCP concept feasible at the cost and revenue levels given previously while the other site (McElmo) is particularly well suited for the NCP concept.

The optimal timing of pond expansion (the  $T_p$  vector) follows a very similar pattern for all sites. That is, to begin with rather short periods and increase them over time. For example, the following timing vectors (years before constructing the next pond increment) occur:

2, 3, 4, 5, 6, 7, 8  
2, 4, 4, 5, 6, 7, 9  
2, 3, 4, 5, 6, 7, 9



This pattern is different from that of the Problem 1 solution which for the typical example (Lower Gunnison) was: 3, 4, 3, 7, 7, 9, 9, ...

The size of ponds added at each of these times is also somewhat different than for Problem 1, but still follows an alternately increasing/decreasing sequence. For example, the incremental and accumulative sequences for Lower Gunnison are shown in Figures 8 and 9, respectively.

The previous discussion assumed that the power plant cooling tower makeup was entirely from lower quality water available at the site. Using such water adds significant costs to cooling tower investment and/or operation; therefore, another mode of operation which may be of interest is using high quality makeup water, but still concentrating the blowdown to a high salinity for disposal in a NCP. Use of high quality power plant makeup produces less blowdown and therefore less brine for the NCP but slightly more LQW for the "fresh" layer makeup and more importantly, lower power plant costs.

Solutions at each site for this mode of operation are given in Appendix A. The revised makeup and blowdown quantities are given in Table 4.

Table 4. Water demand and supply for HQW use by power plant ( $10^3 m^2$ ).

Site	Total LQW Available	Makeup ( $W^{PP}$ )	Blowdown ( $W^0$ )	LQW Available To NCP
Big Sandy	17,900	5,520	120	17,900
Uintah	16,780	12,510	274	16,780
Price	10,490	7,880	172	10,490
San Rafael	8,390	6,300	137	8,390
Grand Valley	33,560	12,610	274	33,560
Lower Gun.	21,220	12,610	274	21,220
McElmo Cr.	25,670	25,230	550	25,670

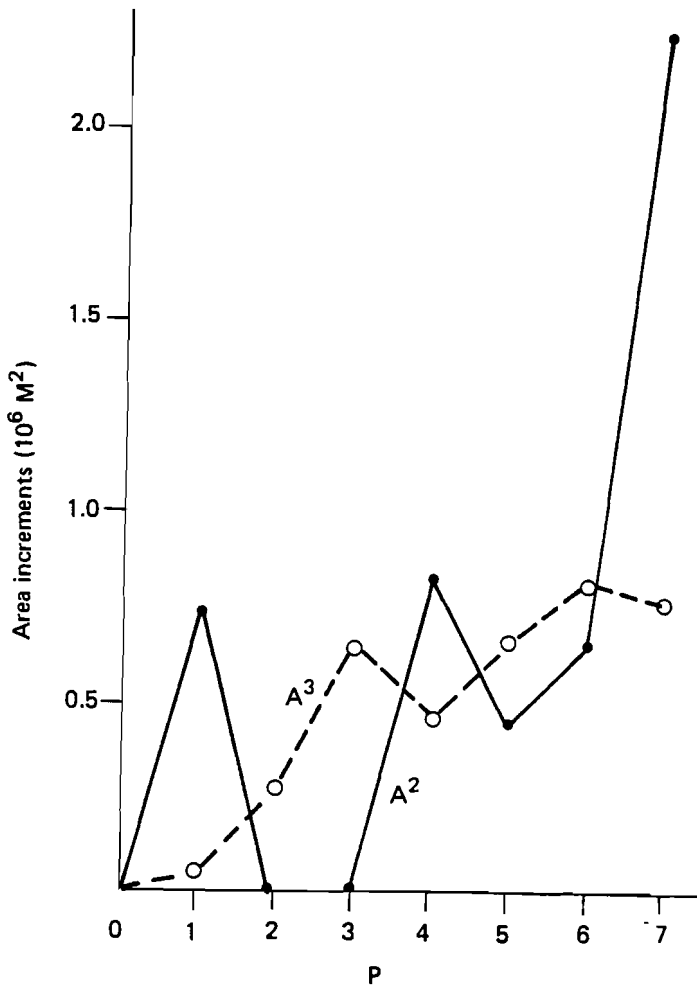


Figure 8. Lower Gunnison area increments if power plant uses only LQW.

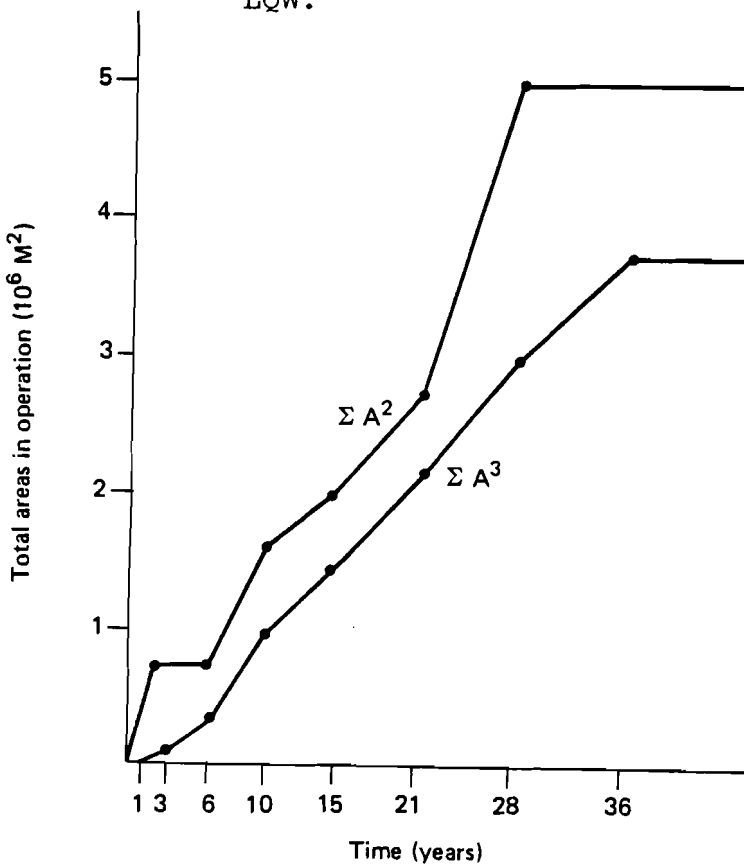


Figure 9. Lower Gunnison total areas if power plant uses only LQW.

## CONCLUSIONS

Conclusions which can be drawn from the solutions produced by the NCP models include the following:

- (1) The amounts of water used and size of ponds required are not sensitive to estimates of cost, revenue or interest rate except in a discrete way. That is, since the total sizes of pond 2 and 3 converge toward a reasonably constant ratio for most Colorado Basin sites (see Figure 10) and since revenue is a linear function of area ( $A^3$ ), for any given relative magnitude of unit pond costs and energy revenue, the system tends to either make a net profit or a net loss as water and salinity quantities vary over a wide range. If the system makes a net profit, the model solution is to use all of the power plant blowdown to produce brine and to use the other LQW as fresh layer makeup. If, however, a net loss occurs, the solution is to provide only pond 2 (as a conventional evaporation pond) to dispose of brine while  $A^3 \rightarrow 0$ .
- (2) The net profit (loss) produced by a NCP system is on the other hand, very sensitive to cost, revenue, and interest rate levels. The impact of various interest rates is shown in Figure 11.
- (3) Generalizations about the size of NCP pond areas as a function of power plant size are displayed in Figure 12. Those data points substantially below the upper line (Price and San Rafael) represent sites where the supply of LQW is limiting when LQW is used by the power plant. If LQW supply is not limiting, areas approximately on the

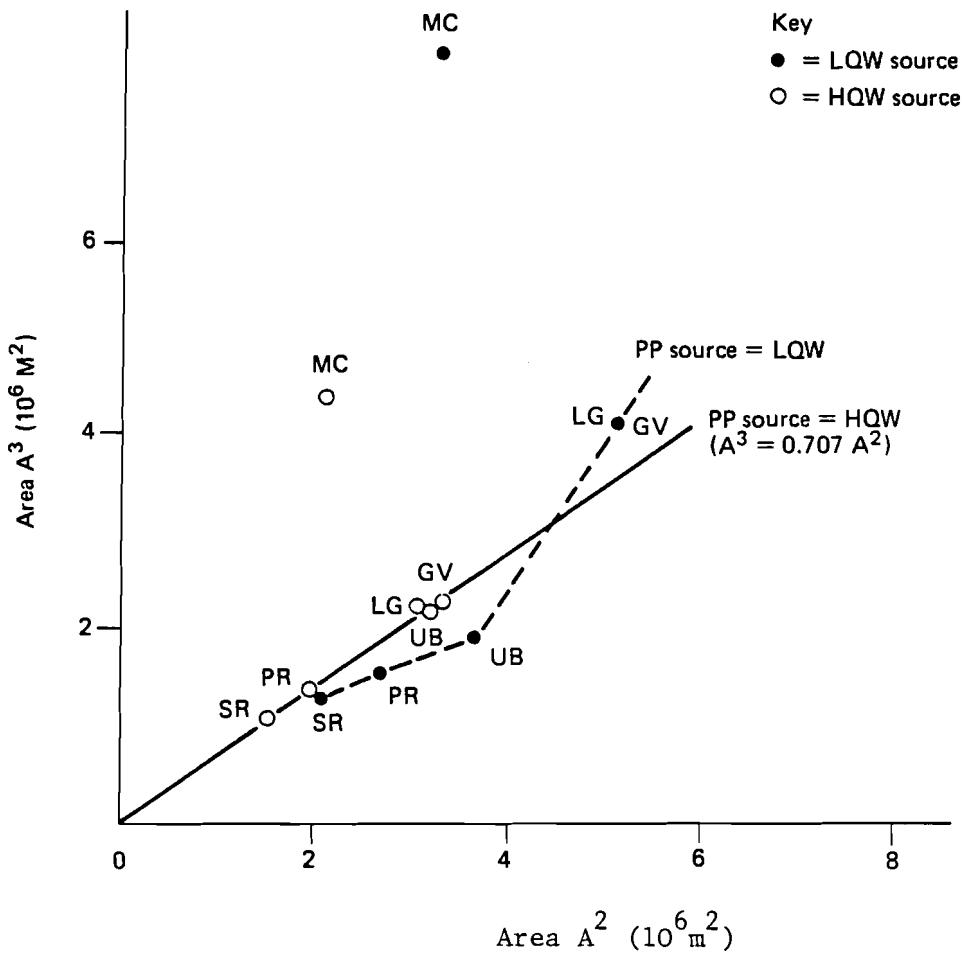


Figure 10. Comparison of pond 2 and 3 areas (after 30 years).

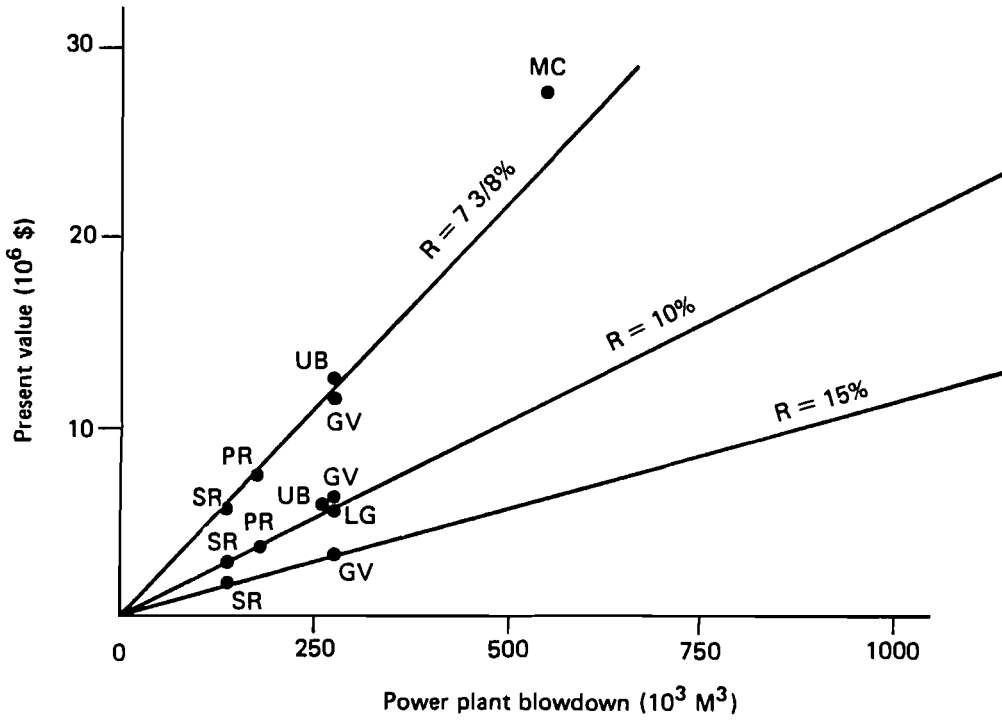


Figure 11. Net return sensitivity to discount rate.

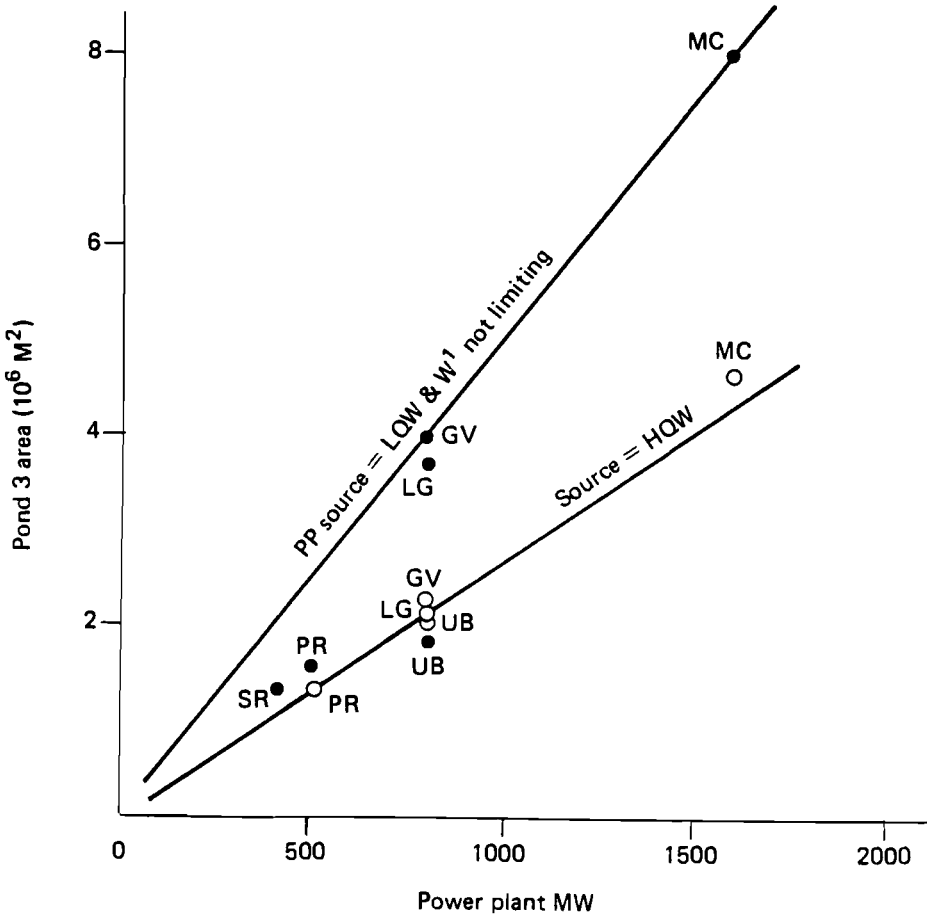


Figure 12. NCP area - power plant capacity relation.

upper line can be expected after 30 years of operation. Since the HQW power plant makeup produces much smaller amounts of brine, the LQW supply is not limiting (or only barely limiting) at all sites and the lower line represents expected areas of NCP. A similar correlation between NCP and power plant energy capacity is displayed in Figure 13.

- (4) Since the size of NCP is highly correlated with brine availability and therefore with quantity of power plant blowdown (assuming a fixed salinity concentration of blowdown) the net return from a NCP system can be roughly predicted as a function of blowdown. Such a relationship is displayed in Figure 14.

A final caveat for interpreting the foregoing information: in order to judge whether a NCP should or should not be constructed, it is not sufficient to analyze only the net return of the NCP sub-system (as was done in this paper.) Rather, it is necessary to expand the system boundaries to include other sub-systems such as energy and agricultural production which are impacted by the NCP water demand (see Hughes and Narayanan for such an analysis.) The NCP system may for instance either increase or decrease the net cost of the related power plant. If LQW is used for cooling, this obviously increases the power plant producers' costs. On the other hand, if the NCP eliminates the need for a conventional zero discharge pond, this eliminates a major power production cost. Particularly, if HQW is used by the power plant, a NCP which accepts its blowdown will produce a net decrease in power plant costs. The point is that a NCP may by itself show a net loss but still improve economic

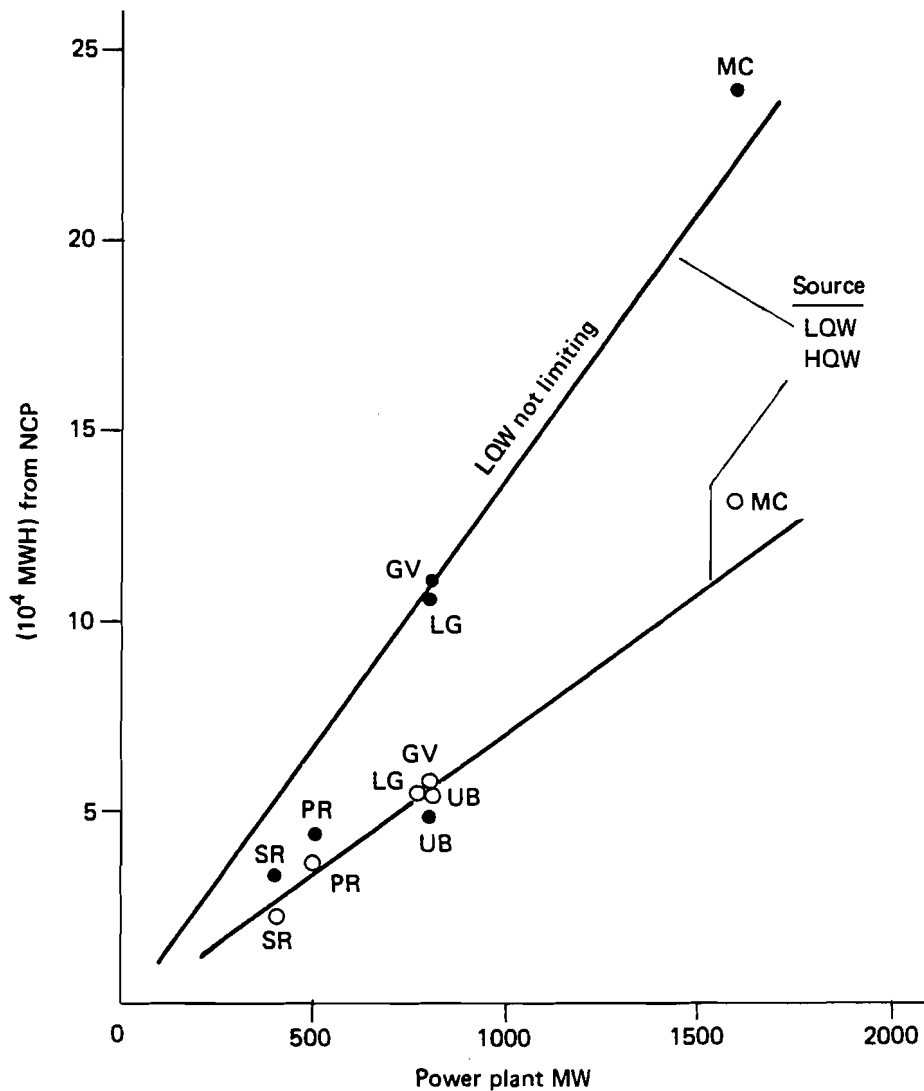


Figure 13. Comparison of NCP and power plant energy production.

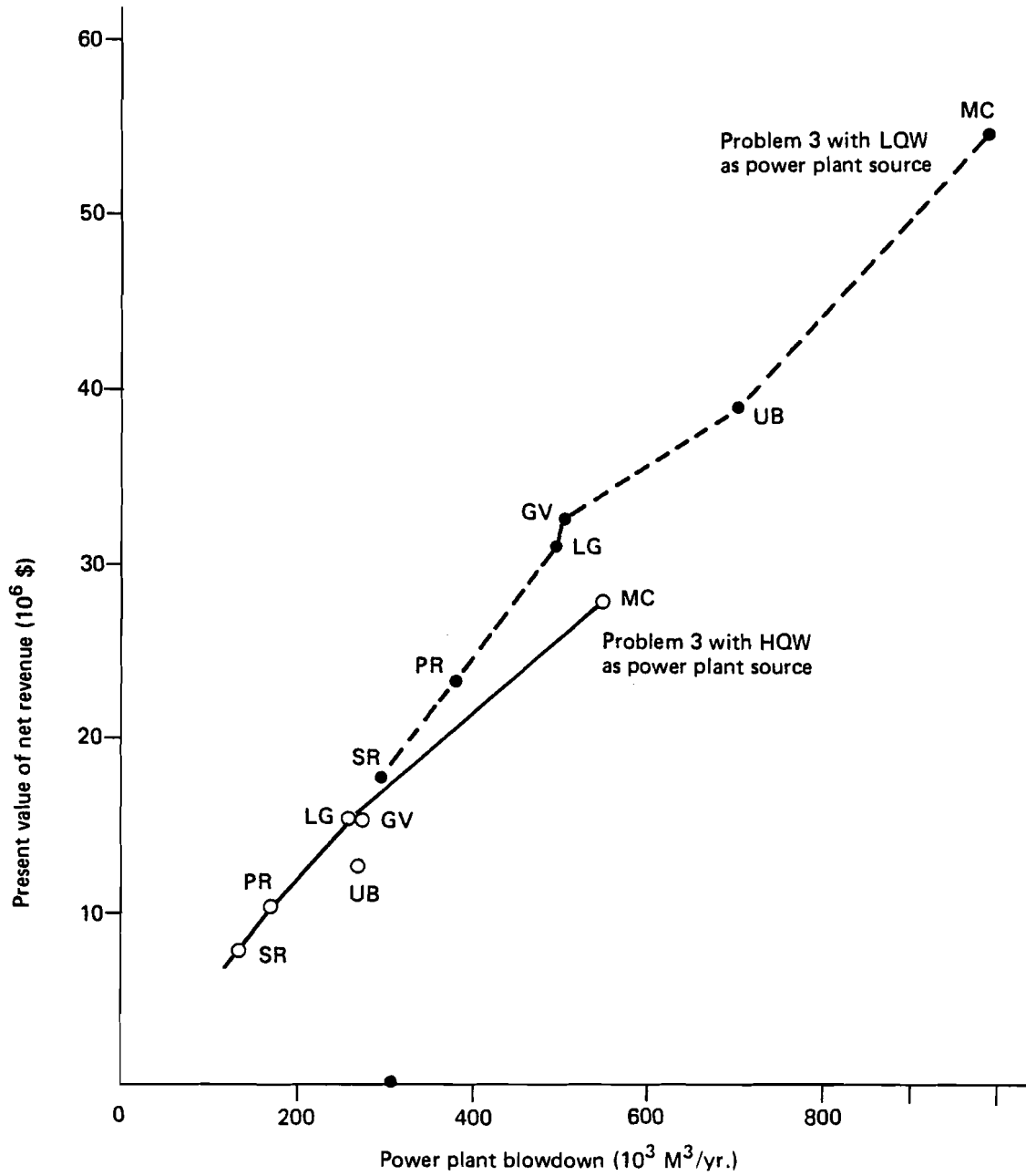


Figure 14. New profit of NCP systems.



efficiency of the larger system. Conversely, the NCP may by itself show a net profit but still cause a net loss in the larger system.

APPENDIX A: SOLUTIONS TO PROBLEM 3

Table A1. Summary of Problem 3 Solutions with LQW used for cooling.

Site	Item	P								$W_7^{13}$	$W_7^{1C}$	$\Delta SC_{7-7}$ ( $10^{-7}$ )	(10 <sup>3</sup> \$) Objec. Function
		1	2	3	4	5	6	7	Total				
Big Sandy	$A_P^2$	521	0	287	271	369	461	569	2497				
	$A_P^3$	0	408	257	350	437	540	749	2741	2806	1430	42	20,917
	$T_P$	2	4	4	5	6	7	9					
Uintah	$A_P^2$	897	0	599	569	2214	0	0	4279				
	$A_P^3$	140	510	551	744	503	0	0	2447	2478	1271	77.8	39,103
	$T_P$	2	3	4	5	6							
Price	$A_P^2$	520	0	354	337	458	994	0					
	$A_P^3$	80	295	320	434	318	91	0	1540	1577	803	37.5	23,105
	$T_P$	2	3	4	5	6	7						
San Rafael	$A_P^2$	401	0	258	244	327	816	0	2049				
	$A_P^3$	63	227	246	329	344	67	0	1276	1267	658	34	17,720
	$T_P$	2	3	4	5	6	7						
Grand Valley	$A_P^2$	761	0	0	864	482	681	812	3602				
	$A_P^3$	58	298	659	489	642	824	1120	4144	4087	2130	58	32,216
	$T_P$	2	3	4	5	6	7	9					
Lower Gun	$A_P^2$	704	0	608	489	642	767	2565	5777				
	$A_P^3$	180	537	505	662	792	940	898	4517	4418	2314	61	30,661
	$T_P$	2	3	4	5	6	7	8					
McElmo Creek	$A_P^2$	572	0	0	571	342	429	1519	3433				
	$A_P^3$	234	614	1283	1062	1330	1557	1906	7982	13,476	4397	87	54,755
	$T_P$	2	3	4	5	6	7	9					

Table A2. Summary of Problem 3 solutions with HQW source for power plant.  
(Areas are  $10^3M^2$ , flows are  $10^3M^3$ , and  $T_p$  in years).

Site	Item	P								Use of LQW		$(10^{-7})$ $\Delta SC_7$	$(10^3 \$)$ Objec. Function
		1	2	3	4	5	6	7	Total	$w_7^{13}$	$w_7^{1c}$		
Big Sandy	$A_p^2$								1434				
	$A_p^3$	0	162	102	139	174	214	297	1089	1026	523	16	7,132
	$T_p$	2	4	4	5	6	7	9					
Uintah	$A_p^2$	419	0	0	525	283	407	1482	3094				
	$A_p^3$	31	162	367	274	394	476	466	2170	2198	1387	38.7	12,343
	$T_p$	2	3	5	5	6	7	8					
Price	$A_p^2$								1954				
	$A_p^3$	19	101	232	172	249	303	298	1376	1409	718	22.8	10,296
	$T_p$												
San Rafael	$A_p^2$								1497				
	$A_p^3$	19	101	232	172	250	240	221	1054	1046	543		7,972
	$T_p$	2	3	4	5	6	7	8					
Grand Valley	$A_p^2$								3061				
	$A_p^3$	32	163	361	268	379	452	500	2156	2127	1108	30.7	15,624
	$T_p$	2	3	4	5	6	7	8					
Lower Gunnison	$A_p^2$								2935				
	$A_p^3$	32	163	359	267	375	445	425	2067	2022	1059	29.2	15,444
	$T_p$	2	3	4	5	6	7	8					
McElmo Creek	$A_p^2$	318	0	0	317	190	238	843	1907				
	$A_p^3$	130	340	712	590	739	862	1059	4434	7486	2221	43	27,852
	$T_p$	2	3	4	5	6	7	8					

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