Working Paper

LAKE EUTROPHICATION MANAGEMENT
OPTIMIZATION MODELING - APPROACHES
WITH APPLICATION TO LAKE BALATON

Trevor C. Hughes

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International Institute for Applied Systems Analysis
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PREFACE

One of the principal projects of the Task on Environmental Quality Control and Management in IIASA's Resources and Environmental Area is a case study of eutrophication management for Lake Balaton, Hungary. The case study is a collaborative project involving a number of scientists from several Hungarian institutions and IIASA (for details see WP-80-187).

Most of the Balaton models to date have focused upon simulating the physical and biochemical processes which determine the nutrient loading from the watershed and the resulting lake water quality. This study uses the loading/lake response information from previous work to identify least cost management alternatives for improving lake quality. Two approaches to economic optimization models are developed in generalized form and then applied to the Balaton problem.

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LAKE EUTROPHICATION MANAGEMENT OPTIMIZATION MODELING - APPROACHES WITH APPLICATION TO LAKE BALATON

INTRODUCTION

The problem of management of lake water quality is inherently multiobjective in nature, with at least two types of objectives--one related to maximizing water quality and another for minimizing cost. If, however, the water quality objective can be quantified in terms of management of a single parameter or even a single water quality index which combines the (commensurate) levels of more than one parameter, the problem can be formulated mathematically with only a single objective function by expressing the other objective as a constraint which will be parameterized (varied over the range of interest). In such a situation, there are several ways of formulating the problem, each with particular implications in regard to algorithms which may be appropriate, computational effort required, and depending on whether or not a two stage simulation/optimization combination of models is required.

Some definitions appropriate to the unavoidable jargon in this paper will be useful. The distinction between optimization and simulation models is the conventional one--an "optimization model" maximizes (minimizes) a constrained formal objective function while maintaining quantitative functional relationships among system parameters. The term "simulation model" refers to a set of functional relationships which, for each run, quantify the state of the system (that state which is uniquely associated with the specified value of each "decision variable").

The phrase "input coefficients" refers to constants which are used to quantify the functional relationships in optimization models. The phrase "decision variable" refers to those parameters which are in some way controllable by management action.

This paper will include: (1) a brief discussion of possible model structures, their advantages and disadvantages; (2) an example of a generalized mixed integer programming (MIP) optimization model which is self-contained in the sense that it calculates internally both the management induced changes in nutrient loading and the lake water quality response; (3) an example of an all integer (AIP) optimization model which is much simpler than the MIP model but which requires a simulation model for its input coefficients; (4) an example of such a management simulation model which calculates nutrient loading changes and resulting phytoplankton levels; (5) results of application of both the simulation and the AIP optimization models to the Lake Balaton management problem.

The nutrient which is considered limiting to production in Lake Balaton and which therefore is shown explicitly in the notation of these models, is phosphorous. The models are also applicable conceptually for management of other parameters such as nitrogen, however, in practice this is usually not realistic.

The kinds of management actions considered for reduction of nutrient loadings include advanced (tertiary) treatment of municipal sewage and construction of shallow reed ponds near river mouths. The word tertiary refers here to only phosphorous removal--not complete advanced treatment. The concept of removal of phosphorous via reeds involves cutting and removing reeds annually while still green. There are many uncertainties related to this concept, including efficiency of P removal (particularly variability over several years) and cost of constructing and cost of operating reed projects. Therefore, the efficiencies and costs used in the Balaton application are only rough (optimistic) estimates.

The reed ponds are usually, but not always, provided with an upstream sediment basin. The models presented here calculate available phosphorous as the sum of PO_4 - P and 20% of particulate P. The latter term is approximately the difference between total and PO_4 phosphorous (see for example Jolánkai and Somlyódy, 1981). The sediment basins are assumed to remove some fraction of particulate P, while both tertiary treatment and the reed ponds are assumed to remove a fraction of the PO_4 - P.

The sewage and reed projects are modeled individually. The simulation model also includes P removal by management of urban runoff, but only in an aggregated form (total lake sections). The phosphorous loading data used for the Lake Balaton application are completely in accordance with the loading estimate of Jolánkai and Somlyódy (1981).

1. MODEL STRUCTURE ALTERNATIVES

Consider the following possible formulations for lake water quality management models (ST = subject to):

Min:	Cost	
ST:	Nutrient Loading $\leq$ Standard	Problem 1a
or		
Min:	Cost	
ST:	Lake Quality $\geq$ Standard	Problem 1b
Max:	Reduction in Nutrient Loading	
ST:	Cost $\leq$ Budget	Problem 2a
or		
Max:	Lake Quality	
ST:	Cost $\leq$ Budget	Problem 2b
Min:	Deviation from Nutrient Loading Goals	
ST:	Cost $\leq$ Budget	Problem 3a
or		
Min:	Deviation from Lake Quality Goals	
ST:	Cost $\leq$ Budget	Problem 3b

The first two formulations (Problem 1a and 1b) are useful for river basin management problems, where regulatory agencies have set absolute standards for pollution residual loadings or for quality indicators (such as dissolved oxygen), in various reaches of a river, or standards for any point load such as sewer treatment plant effluent. This kind of standard implies an infinite cost for any violation of the standard. Nevertheless such standards became very widespread in the US and other developed countries during the 1970's thereby generating a need for river basin planning models formulated as problem 1a or 1b. Examples of such models which use an AIP algorithm as the optimizing tool include: Liebman and Marks (1968); Finney, et al. (1977); and Ijjas and Kindler (1978). An example of this type of model using a dynamic programming (DP) algorithm is Zukovs and Adams (1980). The latter publication however, employs a penalty cost for violation of the standard (the standard being treatment of 100 percent of sewer flow).

The notion of minimizing cost subject to a set water quality standard has a definite advantage in terms of simplifying an optimization model. Specifically, the level of either loadings or quality indicators (or both), after management actions, are constants rather than variables. This is important, since it eliminates many calculations within the optimization model. Therefore, a very simple AIP model is possible in which the 0,1 variables indicate which level of treatment is required at each management alternative to achieve a known increment of quality improvement, the sum of which will meet the standard. It should be noted, however, that the increments of improvements are usually "known" only via output from a simulation model. The elimination of continuous variables is even more important (necessary) in a DP formulation.

Now consider lake water quality management, specifically eutrophication management rather than river reach quality management. In many aspects the modeling problems are the same as for the river. However, it is much more difficult to set a rational loading standard. The longer term implications of lake nutrient loading, wind/sediment and temperature dynamics and particularly the longer time required for response to management actions make the setting of absolute standards (problem 1a or 1b) very unproductive. Indeed, the setting of a desired standard will most likely produce a problem with no feasible solution. A much more useful formulation in the lake eutrophication setting is either the minimization of deviations from loading or lake quality goals subject to realistic budget constraints (problem 3a or 3b), or which is mathematically essentially equivalent, minimizing loading or maximizing lake quality subject to budget constraints (problem 2a or 2b)--the only difference being the trivial exercise of explicitly calculating the deviation from an array of constants representing goals. That is, the difference between problems 2 and 3 is trivial _if_ the optimization model is capable of handling continuous variables. If an all integer algorithm is to be used, then the formulation of problems 3a or b is not possible. In fact, although formulation 2b is possible for an AIP algorithm, the very cumbersome operations involved in generating the coefficients virtually dictate either an MIP algorithm or a simpler surrogate for lake quality such as weighting the components of the problem 2a objective function in order to simulate the problem 2b objective. The latter approach will be demonstrated in this paper.

The difference between the "a" and "b" formulations is that "a" does not include the submodel which translates the rate of nutrient loading (kg per day of available phosphorous for example) into some indicator of lake quality chlorophyll, biomass or phytoplankton phosphorus for example. Here the latter will be used since it is a state variable of the available lake model (see van Straten 1980; Leonov 1980). If a lake is essentially homogenous in terms of water quality, the problem "a" form of model may be adequate, since minimizing nutrient loading then becomes almost synonymous with maximizing water quality (except for possible differences in weighting of seasons). However, for an application such as Lake Balaton, where there are large gradients in both water quality and water volume among various sections of the lake, the two forms of objective function will produce radically different allocations of management resources (with problem "b" being the proper formulation).

2. SELF CONTAINED OPTIMIZATION MODEL

2.1. Model Description--Problem 3a

A mixed integer programming version of a model in a form appropriate to problem 3a has been developed and applied to the Balaton problem. That application will not be reported here, however, a detailed description of the model and its notation is given in Appendix A. The following description merely summarizes the structure of the model.

The objective function minimizes the weighted sum of deviations from nutrient reduction goals in the ith lake section and the kth season, for all i and k. A discussion of how one might determine relative weighting of such goals (if indeed relative weights need to differ at all) will be presented later.

The constraints include five types as follows:

a. Calculate deviations from goals as: Σ Removal (kg/day) + Dev. from Goal = Σ Initial Loads - Goal.
b. Force load treated by reed projects to zero if the project is not built.
c. Define load reaching each reed project as a function of upstream sewer treatment.
d. Limit P removed by sewer projects to the appropriate fraction of either the P load or capacity of the treatment unit selected, whichever is smaller.
e. Limit total of investment and operating costs to the budget available.

The most natural form of a model which expresses these constraints mathematically involves the product of two variables in the cases of both (b) and (d); however, such variables were separated (a necessary condition for linearity between discrete steps) by using various 0,1 manipulations as explained in the Appendix. Both the sewer and reed type management actions are represented by 0,1 type variables (don't build, build decisions) while continuous variables are required to represent (1) deviations from goals, (2) load actually treated by sewer projects, and (3) loads reaching the lake edge as impacted by both management activities and river reach effects. The model allows an arbitrary number of seasons, management projects and lake sections.

2.2. Secondary Sewage Treatment

The phosphorous management models presented here are based upon the assumption that the sewer line and secondary treatment locations and capacities will be predetermined exogenously. In the case of Lake Balaton this will apparently be accomplished by a very detailed planning model (SZTAKI 1980). This means that effluent quantities are fixed by considerations such as regionalization related to pipelines and secondary treatment plants (in which economies of scale are substantial) and which therefore can be treated as constants in regard to tertiary projects (in which scale effects are very minor). If one wished to include secondary treatment alternatives in this model it would be conceptually very simple to add a new set of integer variables and related costs and to use constraints that require secondary treatment capacity to be not less than phosphorous removal capacity. This would ensure that no tertiary unit is constructed without a related secondary plant of appropriate capacity. However, the other additional constraints necessary to make optimal decisions on design of the secondary treatment would produce a totally intractable model. Since tertiary treatment cannot be accomplished without previous secondary treatment of all effluents, and since secondary treatment is an order of magnitude more costly than phosphorous removal, there is a very low probability that tertiary unit decisions could change the optimal configuration of secondary projects. It is therefore very logical to decompose the overall problem into separate secondary and tertiary models.

2.3. Investment Timing Considerations

The model presented previously does not include investment timing decisions related to future growth in phosphorous loads. In the case of Lake Balaton (as well as most other lake environments one can imagine) investment timing considerations are almost irrelevant. The reasons for this are primarily related to: (1) the necessity for tertiary unit capacities to be not less than secondary treatment capacities and, (2) the fact that economies of scale are very minor for tertiary units (thereby almost eliminating any benefit of a temporary excess capacity). The optimal strategy would therefore seem to be to provide treatment for initial period capacities in an optimal way (with either tertiary or reed projects or both) and then simply add tertiary units as needed, to match future additions in secondary capacity. Very short lead times are required for construction and such decisions can best be made as future growth rates are observed rather than guessed at.

However, if some situation occurred where an investment timing model did appear useful (for example, if municipal effluents were not the most important and fastest growing parameter) then the model presented could be easily transformed to add this capability as follows:

1. Add a time subscript (t) to all decision variables;
2. add necessary rows to iterate over t planning periods as well as k seasons;
3. change phosphorous load constants to reflect growth in sewered population in the new rows;
4. change both investment and operating cost coefficients to represent appropriate present worth quantities;
5. add constraints to restrict the implementation of a particular reed project to a single time period (not needed for tertiary facilities since there is no limit on number of units).

For most lakes this would produce a large model which may well be intractable. The computations could be reduced very substantially (for problem 3a) by decomposing the model into small problems (four in the Balaton case) each representing one section of the lake. The model decomposes easily since it already includes separate objective function terms for each lake section (each i). The only portion of the model then requiring modification would be the budget row. A separate budget will be required for each i. The only difficulty with the decomposition is the loss of ability to allocate the total lake budget among lake sections in a guaranteed optimal manner. However, such decomposition works only for the reduced loading objective (3a), not for the lake quality objective (3b) which requires quantification of the interaction between lake sections.

2.4. Deviation from Lake Quality Goals--Problem 3b

In order to transform the previous problem into a form which minimizes deviation from lake quality goals rather than phosphorous loading goals, it is necessary to add to the model a series of equations representing the lake production response to phosphorous loading (and change the objective function to minimize deviation from these new goals). For this purpose lake ecological models (see, e.g., for Lake Balaton, van Straten 1980, Leonov 1980) are required. Such models would describe the temporal and spatial changes in the lake's water quality under given environmental conditions. However, the use of such a detailed model directly in a management optimization framework is not a realistic task (van Straten and Somlyódy 1980). From the point of view of policy making, the day to day quantities are not useable. Rather, the knowledge of some average or typical values (yearly or seasonal averages, yearly or summer peak, etc.) as a function of the nutrient load are required. Within the frame of an ongoing study and with the use of the detailed lake water quality models, Somlyódy and Eloranta (forthcoming) found that this relationship can be modeled satisfactorily by a linear transformation matrix as follows:

$$\bar{C} = \bar{C}_0 + \bar{A}(\Delta\bar{L}) \quad , \tag{1}$$

where $\bar{C}_0$ and $\bar{C}$ are the initial (without management), and final (with management) levels of lake water quality respectively; $\bar{A}$ is a matrix indicating fractional improvement in water quality in the lake sections due to management induced changes in phosphorous loadings in each lake section. (ΔL_{ik} = sum of the first three terms in equation (A1)). Details of this transformation will be demonstrated in connection with the simulation model.

3. ALL INTEGER MODELS

3.1. Maximize Reduction of Nutrient Loading or Maximize Water Quality--Problems 2a and 2b

The previous MIP models (problems 3a and 3b) were presented as examples of phosphorous management model forms which can be operated without accompanying management simulation models. The resulting numerous constraints include many continuous variables representing loadings and goal deviations at various locations which change as a function of the configuration of selected reed, sediment, and tertiary sewer projects. It is possible, however, to construct a combination of simulation and optimization models which provide at least an equivalent amount of information but in a more convenient form and which require much less computational effort for the optimal solution search. Such optimization models will now be presented. The related simulation model will be presented in Section 4.

$$\text{Max:} \quad \sum_{ijk}\sum\sum \Delta L_{ijk}\, W_{ik}\, X_{ij} \tag{2}$$

$$\text{ST:} \quad \sum_{j=*} X_{ij} \leq 1 \qquad (i = 1,\ldots,I) \tag{3}$$

$$\sum_{ij}\sum C_{ij}X_{ij} \leq \text{Budget} \quad . \tag{4}$$

This model maintains the same subscript notation (lake section i, project j, season k) as the previous model but other notation as follows: the X_{ij} are 0,1 decision variables, indicating one or more tertiary sewer projects or a reed project or some combination of both. For example, if a sewer project (A) is located upstream from a potential reed project (B), an X_{ij} variable will be defined for each possible combination of these projects (A only, A + B, B only). The C_{ij} represents the sum of investment and operating costs (present worth). The ΔL_{ijk} are constants representing seasonal changes in available phosphorous load due to operation of project X_{ij}.

When the W_{ik} terms are all set at unity, the model represents problem 2a (maximization of P load reduction). However, as will be demonstrated for the Balaton application, a solution which minimizes loading of P for the entire lake is very different from a solution for maximizing lake quality (the real objective). Therefore, the problem 2a can be converted to 2b by using the W_{ik} as coefficients which weight various P load reductions in proportion to their importance in improving water quality. Although it is theoretically possible to produce W_{ik} which explicitly represent water quality in mg/l of some lake production indicator (as does problem 3b, via equation (1)), the calculations necessary for determining these coefficients are simply prohibitive. The basic difficulty is that lake quality is a function of the residual P load--not P removed (ΔL). On the other hand, the parameter which is functionally related to the X_{ij} decision variables is ΔL--not the residual after management. Therefore, for a model to simulate actual lake quality it is necessary to calculate both ΔL and residual P (as does the MIP model). This is not possible in the AIP framework without generating simulation model output for literally every possible combination of management alternative.

However, as a surrogate for explicit water quality calculations, the ΔL_{ijk} can be weighted as follows:

$$W_{ik} = LP_{i,k}/vol_i \quad , \tag{5}$$

where $LP_{i,k}$ = the initial lake production indicator before management, and vol_i is water volume of lake section i. The volume term is necessary to capture the dilution effect of loads to each lake section. The existing water lake production term indicates that changing the quality of a lake section that already has good quality (low $LP_{i,k}$) is less important (and more difficult) than in a section with poor quality. This derivation of $W_{i,k}$ implies linearity in the ΔL--LP relationship which obviously does not exist. However, over the small ranges of lake quality usually involved, the resulting error should not be serious since what is desired is a relative ordering of project impacts--not actual water quality.

Finally, the $\Sigma_{i=*} X_{ij}$ in equation (3) represents illegal combinations of projects located on a single tributary such as those which duplicate the same project. Definition of an X_{ij} for all mathematically possible combinations could produce a large problem if some watersheds have many individual projects. However, one can avoid this problem by common sense aggregation of some smaller projects and elimination of improbable configurations.

It should be noted that the decision variables do not include an ℓ subscript for capacity as did the sewer projects in problem 3a. This is not required since the simulation model selects a unique capacity for each alternative (the smallest capacity which is $\geq$ the estimated sewage flow) and the proper cost and nutrient removal level for that capacity.

4. SIMULATION MODEL OF MANAGEMENT ALTERNATIVES

4.1. Some Preliminary Truisms

A principal contrast between simplex based optimization models and simulation models is that the former solves the complete system of equations simultaneously (through many iterations but always simultaneously), while the latter solves the system of equations sequentially. The simulation approach therefore is amenable to model conceptualization via a flow chart of sequential tasks, each of which uses as input data, quantities produced during some previous step (hence Figure 1). Optimization models of course accomplish a similar end product via the interdependence among equations, but there the sequential process is related to iteratively changing trial values of decision variables to determine extreme values of some objective rather than performing a physically meaningful sequence of calculations driven by a particular (selected) level of each decision variable--as do most simulation models.

Another optimization/simulation contrast is related to the much greater programming flexibility inherent in the latter. For example, a combination of "if" statements and "do loops" provide much more flexibility than a combination of inequalities and 0,1 "tricks" to make certain parameter values conditional upon given management decisions.

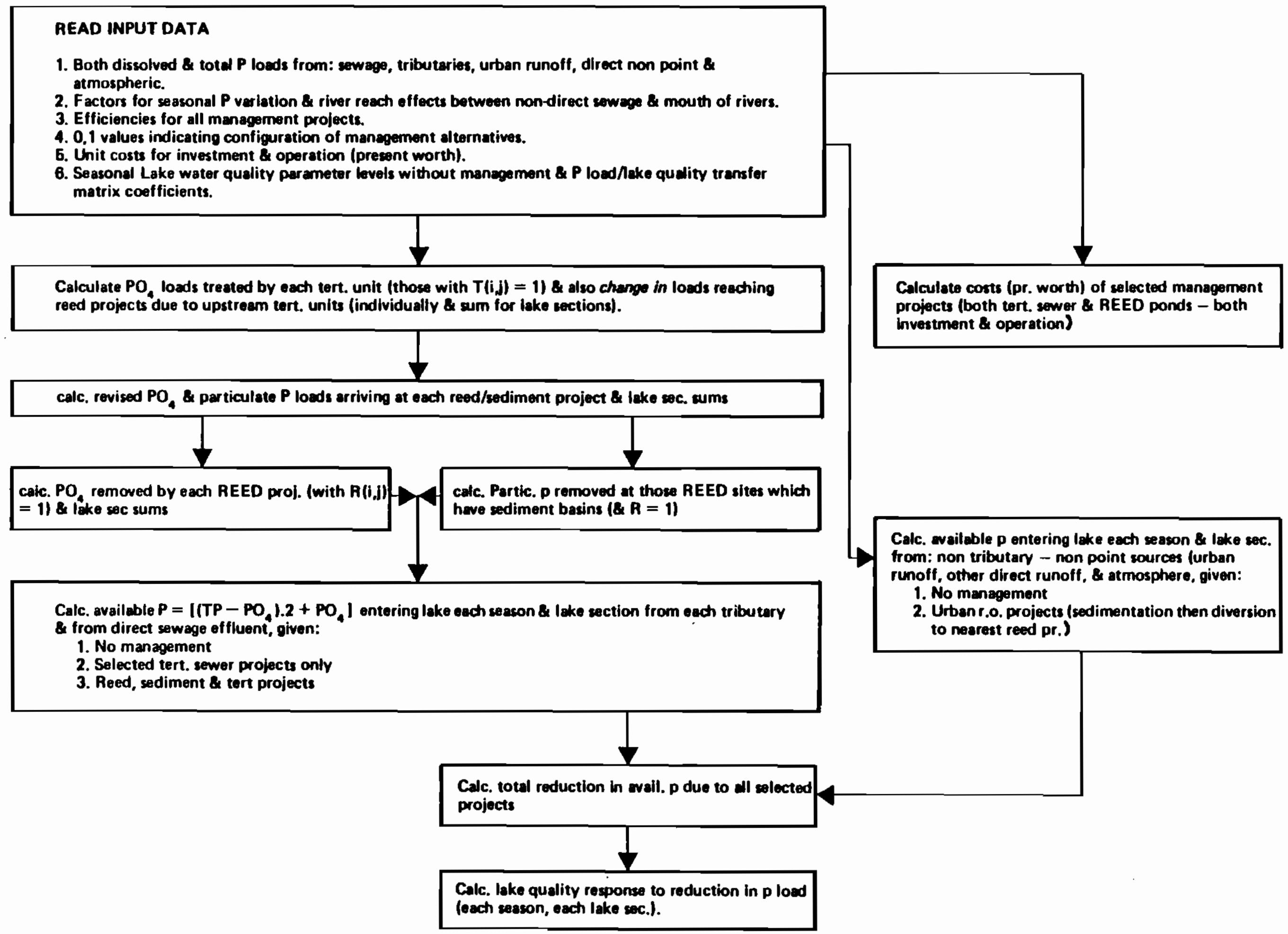

Figure 1. Management simulation model task diagram.

On the other hand, a simulation model run produces output for only a single trial value of decision variables, and therefore can be used to "optimize" only heuristically by many runs (which is to say it cannot optimize at all in the strict mathematical sense). It is not surprising, therefore, that many complex optimization problems are solved by a combination of simulation/optimization models in order to achieve the advantages of both tools. A recent example is Finney, et al. (1977). The usual purpose of the simulation model in such a situation is to provide input data for the optimization model. However, several runs of a simulation model with decision variables varied over an intuitive range of interest can provide important insight regarding the system and therefore can be of great value independent of the optimization model.

4.2. Model Description

The simulation model flow chart is given in Figure 1. There is no attempt to give details such as actual equations in the figure since the complete program (including subscript values for the Lake Balaton application) is given in Appendix B. The program is well documented both in terms of notation definition and comments on the purpose of each series of calculations immediately preceding the calculations.

The model, as presented, handles a problem with four lake sections (see Figure 2), three seasons, 18 potential sewer projects, nine reed projects, and urban runoff projects aggregated by lake sections; however, these quantities and even types of projects can easily be modified.

Results of an example model application will be presented and discussed later.

5. MODEL APPLICATIONS TO THE LAKE BALATON PROBLEM

5.1. MIP Model

The model presented in Section 2 was applied to Lake Balaton as a two season model with 17 potential tertiary sewer projects and nine reed projects. In addition to these 26 integer variables,

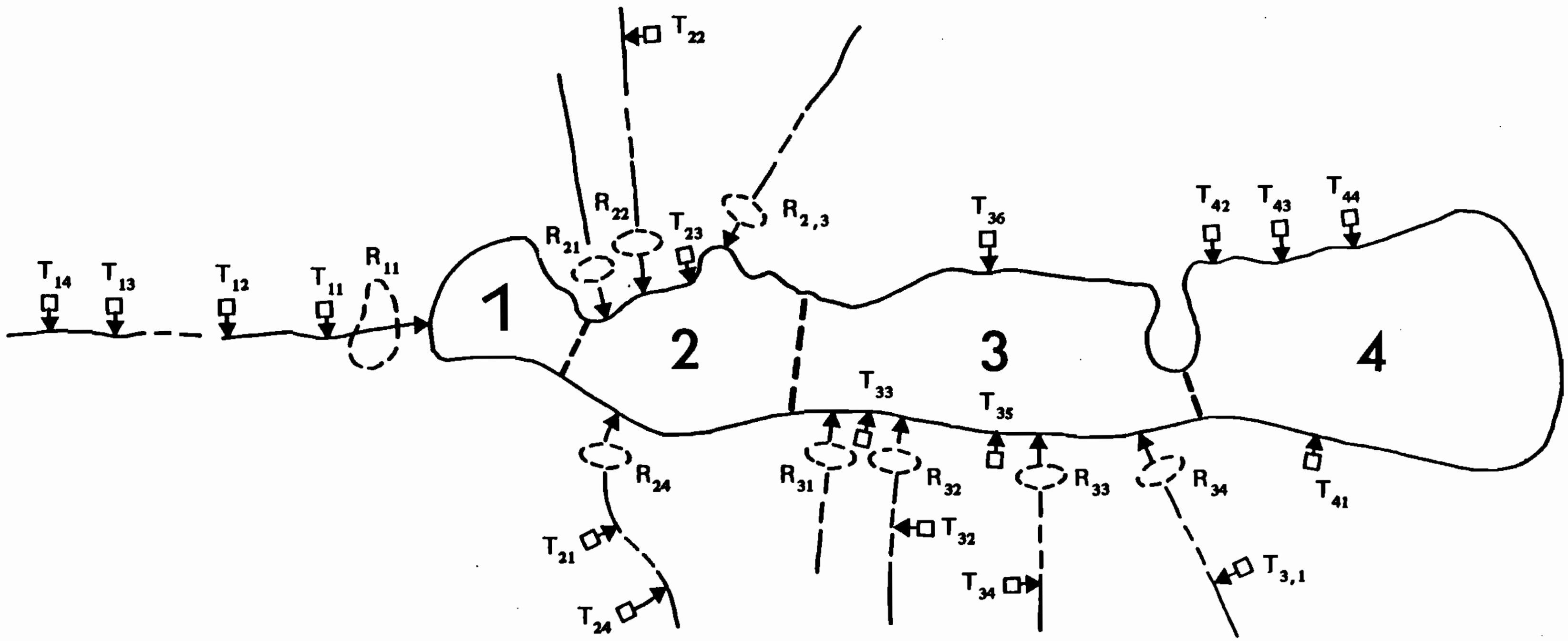

Figure 2. Potential reed (R_{ij}) and tertiary sewer (T_{ij}) project locations.

the model had 56 continuous variables and 96 constraints. It produced solutions for various levels of budget, generally within a few seconds of CPU time, but took over four minutes CPU at one budget level on a CDC system with the APEX-III MIP algorithm.

The model seemed to produce reasonable solutions but results will not be presented here for the following reasons:

1. A more detailed analysis of phosphorous loading data (Jolánkai and Somlyódy 1981) subsequently produced significant changes in the input data, some management project configurations, and also motivated a change in the simulation and AIP model structure from two to three seasons.
2. At the present time, the MIP model has not been modified to include these changes since additional changes are expected when the results of the secondary sewer regionalization model are available.
3. Since the results of the MIP and AIP models will not be directly comparable until both are revised to the same problem structure and data, only the simulation and AIP model results will be reported here. The two latter models are based upon the P loading data in Jolánkai Somlyódy (1981).

5.2. Simulation Model

5.2.1. Loading Data and Management Alternatives

The version of the Balaton problem which was modeled is displayed in Figure 2. Each location marked T_{ij} is both a source of P and a potential tertiary sewer project. Regionalization-type alternatives will be added later upon availability of the secondary sewage treatment study results (SZTAKI 1980). Point sources with avg. $P < 2$ kg/day are not identified as potential projects but are aggregated as a single load in each lake section. Each location marked R_{ij} is both a river tributary source of phosphorous and a potential reed project. Other small tributaries which contribute very minor P loads are aggregated in the model as a single source for each lake section in order to maintain the total estimated loading, but are not subject to management.

The reed ponds (and related sediment basins) are assumed to be located and sized as suggested in VATI (1979). The tertiary sewer project loads are mostly from municipalities but include some animal farms. The point sources and their average P loads are identified in Table 1. The reed projects and estimated areas and loads are given in Table 2.

In addition to the point loads treatable by sewer projects and the point and non-point loads treatable by reed projects, Jolánkai and Somlyódy (1981) quantify three other types of loads as follows: (1) urban runoff, (2) non-point load flowing directly to the lake, and (3) atmospheric loads. These are included in the simulation model as shown in Table 3.

The river reach effect upon reduction (or occasional increase) in PO_4 - P load is modeled as a ratio of total (unmanaged) loads at upstream sewer project locations to those measured at reed project locations. These data are given in Table 4.

Assumed efficiencies for P removal for various management activities are as follows:

	Seasons			
Facility	Jul-Aug (1)	May,Jun,Sep (2)	Oct-Apr (3)	Type of P
Tertiary Sewer	.90	.90	.90	PO_4
Reed Lake	.95	.85	0	PO_4
Sedimentation	.8	.8	.8	Particulate
Urban R.O.	.8	.8	.8	Particulate
Urban R.O.	.95	.85	0	PO_4

The model includes management alternatives for only the urban runoff component of these loads; however, the total loadings are needed for proper lake response calculations.

Table 1. Tertiary sewer projects.

Locations (T_{ij})	Table 3 Number in Jolánkai & Somlyódy	Source Type*	Avg. P (kg/d)
T11	20	M	34.7
T12	23	ID	81.9
T13	24	ID	3.5
T14	25	ID	8.1
T21	1	ID	28.9
T22	18	ID	15.6
T23	21	D	6.0
T24	27	ID	2.3
T31	2	ID	1.3
T32	3	M	8.5
T33	4	M	21.2
T34	5	M	7.4
T35	8	ID	2.8
T36	17	D	6.2
T41	11	D	38.3
T42	12	D	11.0
T43	13	D	33.2
T44	14	D	3.0
other sec. 1	j = 7 in sim. model		2.6
other sec. 2			1.0
other sec. 3			3.9
other sec. 4			4.3
			331.7

* M = Mixed (fish ponds drained directly or indirectly)
D = Direct to lake (not tributary)
ID = Load to lake via tributary

Table 2. Reed projects.

Location $R_{(i,j)}$	Table 1 no. from Jolánkai & Somlyódy	Reed Area (Hectare)	Sediment Area (Ha)	Avg. Load (kg/d) TP	Avg. Load (kg/d) PO_4
R11	1. Zala	5307	1800	225.0	103.7
R21	2.-4. Les., Vilag., Ketöles	470	30	44.4	21.0
R22	5. Tapolca	145	0	36.0	26.3
R23	6. Egerviz	353	0	7.2	3.4
R24	7. Nyugati-Öv.	1317	249	83.0	32.8
R31	8. ABC-Csatorna	58	8	12.6	2.2
R32	9. Keleti-Bozot	1186	0	23.3	7.2
R33	10. Tetves-Patak	466	110	11.3	1.6
R34	15. Körösh-sed	106	95	7.4	3.0
other sec. 1	j = 7 in sim. program			0	0
other sec. 2				0	0
other sec. 3				13.0	4.0
other sec. 4				10.5	3.1

Table 3. Non-tributary type loads (kg/day average).

Lake Section	Urban R.O.		Non-point		Atmosphere	
	TP	PO_4 - P	TP	PO_4 - P	TP	PO_4 - P
1	12.8	1.5	11.3	1.4	10.3	4.1
2	35.4	4.1	21.0	2.8	41.1	16.4
3	38.5	4.4	32.2	4.6	53.0	21.2
4	73.8	8.5	14.5	1.7	66.7	26.7

Table 4. River reach coefficients.

Lake Section	Tributary $R_{(i,j)}$	Seasons		
		1	2	3
1	R11	0.58	0.71	0.95
2	R21	1.0	1.0	1.0
"	R22	1.0	1.0	1.0
"	R23	1.0	1.0	1.0
"	R24	1.0	1.0	1.0
3	R31	0.41	0.41	0.41
"	R32	1.0	1.0	0.5
"	R33	0	0	1.2
"	R34	1.0	1.0	0.24
"	R35	0.53	0.53	0.53
"	R36	1.0	1.0	1.0
4	R41-R44	1.0	1.0	1.0

The model converts annual loadings to seasonal averages by using the seasonal factors shown in Figure 3. These values are identical to those given by Jolánkai and Somlyódy, except for the mixed sewage pattern where mass balance is maintained but the October fishpond release peak load is necessarily distributed uniformly over season 3. This load is very minor relative to other sources, and therefore did not appear to justify a four-season model.

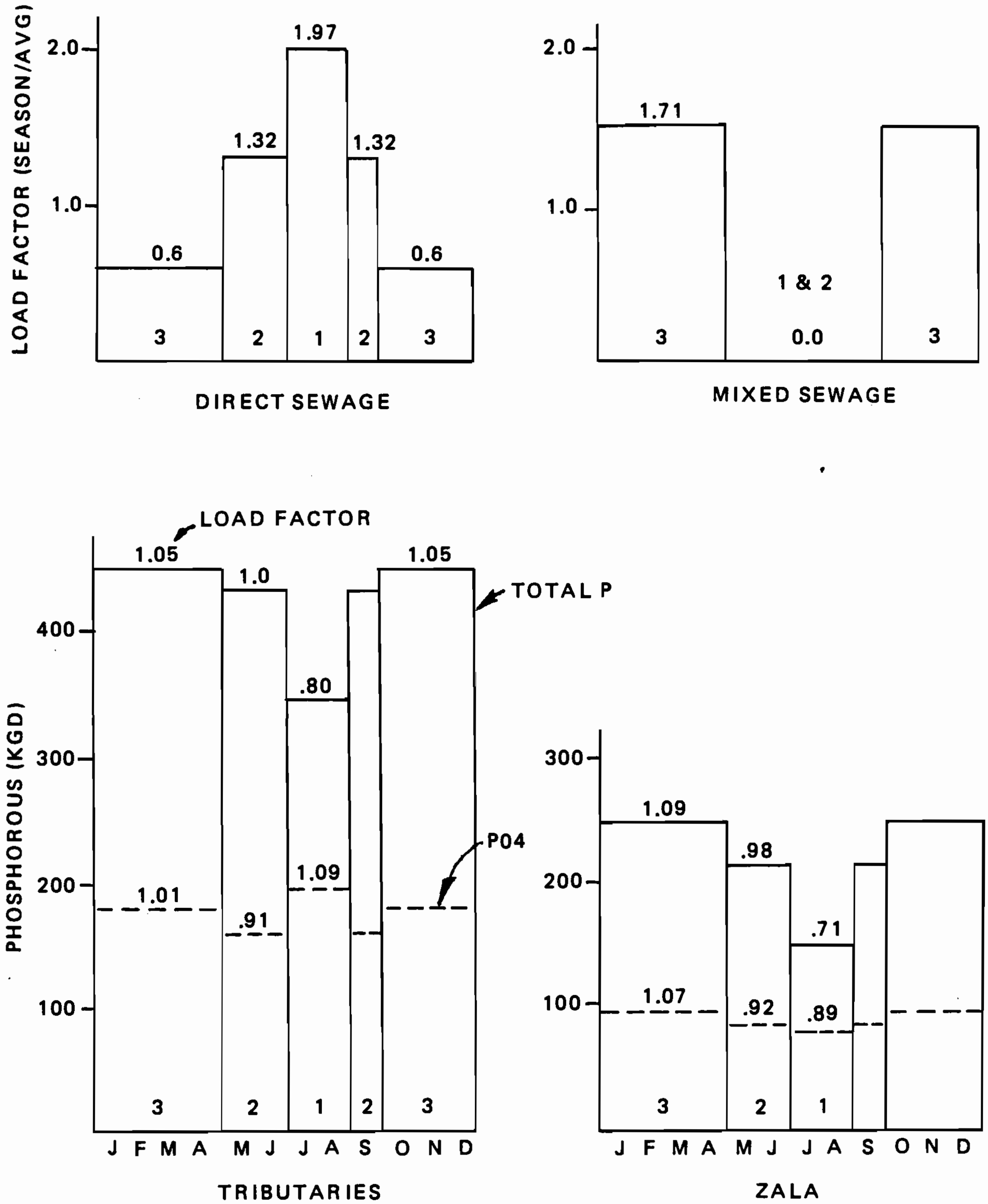

Figure 3. Seasonal loading of TP and PO_4.

The tentative values used for equation (1) to transform the reduction in available P to lake quality response (in terms of phytoplankton phosphorous) were:

$$\begin{pmatrix} C_{1k} \\ C_{2k} \\ C_{3k} \\ C_{4k} \end{pmatrix} = \begin{pmatrix} CO_{1k} \\ CO_{2k} \\ CO_{3k} \\ CO_{4k} \end{pmatrix} + \begin{bmatrix} .7 & .1 & 0 & 0 \\ .15 & 1.0 & .1 & 0 \\ 0 & .15 & 1.0 & .1 \\ 0 & 0 & .15 & 1.0 \end{bmatrix} \begin{pmatrix} \Delta L_{1k} \\ \Delta L_{2k} \\ \Delta L_{3k} \\ \Delta L_{4k} \end{pmatrix}$$

in which in the CO_{ik} values of phytoplankton were:

	k		
i	1	2	3
1	27	23	17
2	15	13	9
3	11	9	7
4	7	6	4

The costs calculated by the model are based upon unit costs as follows (the word "tertiary" as used here refers only to chemical precipitation of phosphorous, not complete advanced treatment). The costs are based upon SZTAKI (1980) and upon personal contacts with staff of various Hungarian agencies.

Tertiary Sewer Treatment:

Capacity (kg/day)	Constant Cost (10^6 Forints)	Operating Cost
≤ 7	.3	70 forints/kg/day
7 to 21	.5	@ 20% discount
21 to 42	1.0	= 350 for./kg/day present worth
42 to 63	1.5	
> 63	2.0	

Reed Projects (including sediment):

Size	Investment Cost	Operating Cost
Large	10^5 for./hectare	$4(10^3)$ for./hec./yr.
Medium	1.5 (10^5)	@ 20% = $2(10^4)$/hec. (present worth)
Small	2.5 (10^5)	

5.2.2. Results of the Simulation Model

The simulation model output can best be analyzed by examining a series of runs, each of which had a different management activity configuration. The parameters of interest (phytoplankton P and P loading changes) will be presented graphically.

Figure 4 shows predicted improvement in phytoplankton P if all sewer projects were built and then incremental improvements were made, if all reed projects were added and finally if urban runoff projects were also added. This truly expensive scenario (build everything) suggests the technically possible upper limits on water quality improvement. Since the urban runoff projects produce only insignificant improvement, it seems clear that:

1. There is no justification for an expensive investigation of possible management of urban runoff projects since this rather optimistic preliminary calculation suggests no significant impact is possible. This is not to say that problems will not be caused by urban runoff. However, such problems require a local solution--not a lake model.
2. There is no point in considering implementation of (or even modeling of) direct non-point source management activities because those P loads are even smaller than from urban runoff. Note, however, that this refers only to direct non-point loads, not tributary non-point loads.

Figure 5 shows that much of the treatment capability of the sewer projects overlaps that of the reed projects, since many large sewer projects are upstream from reed locations. This is

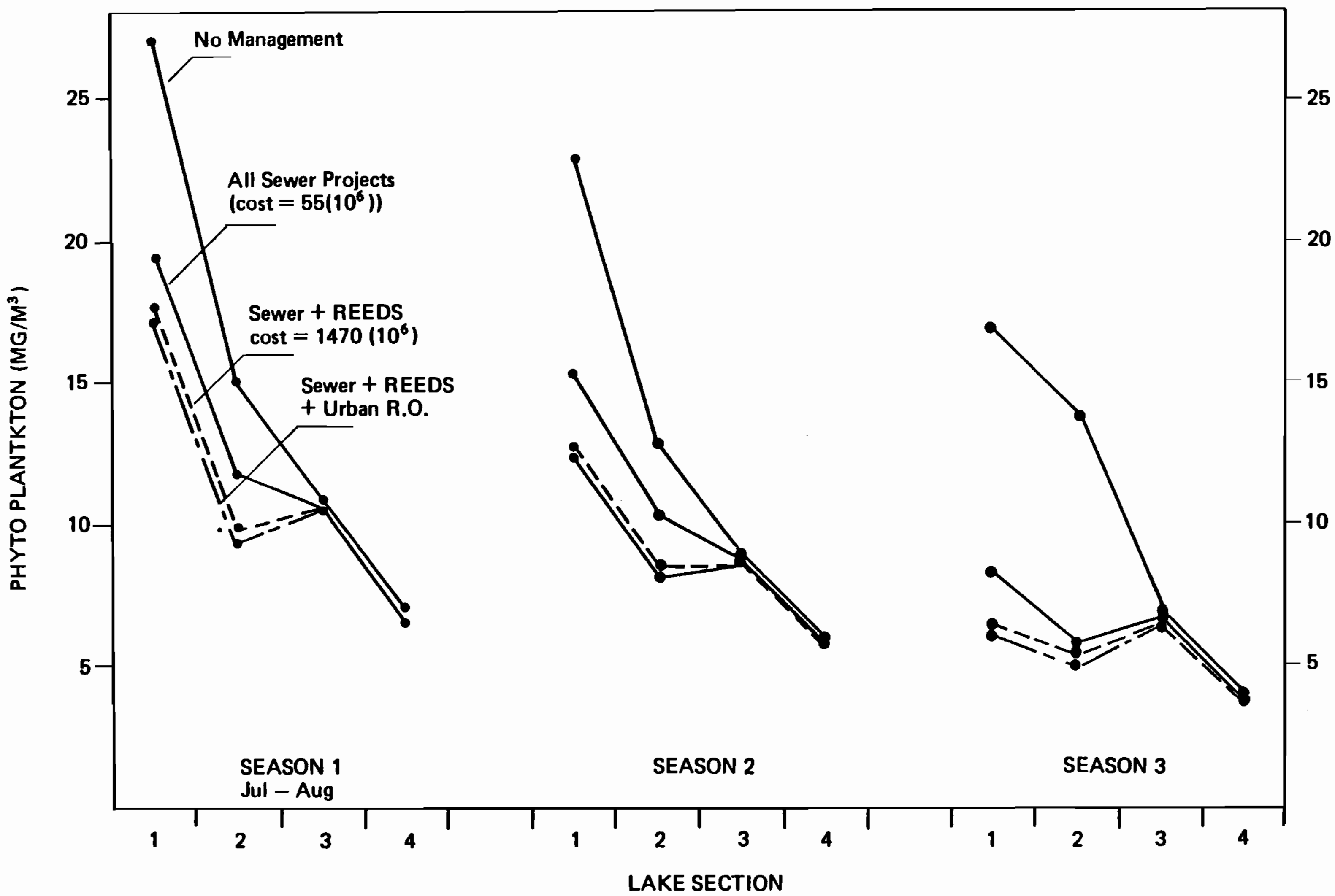

Figure 4. Improvement from 1. sewer projects; 2. reed projects; 3. urban runoff projects.

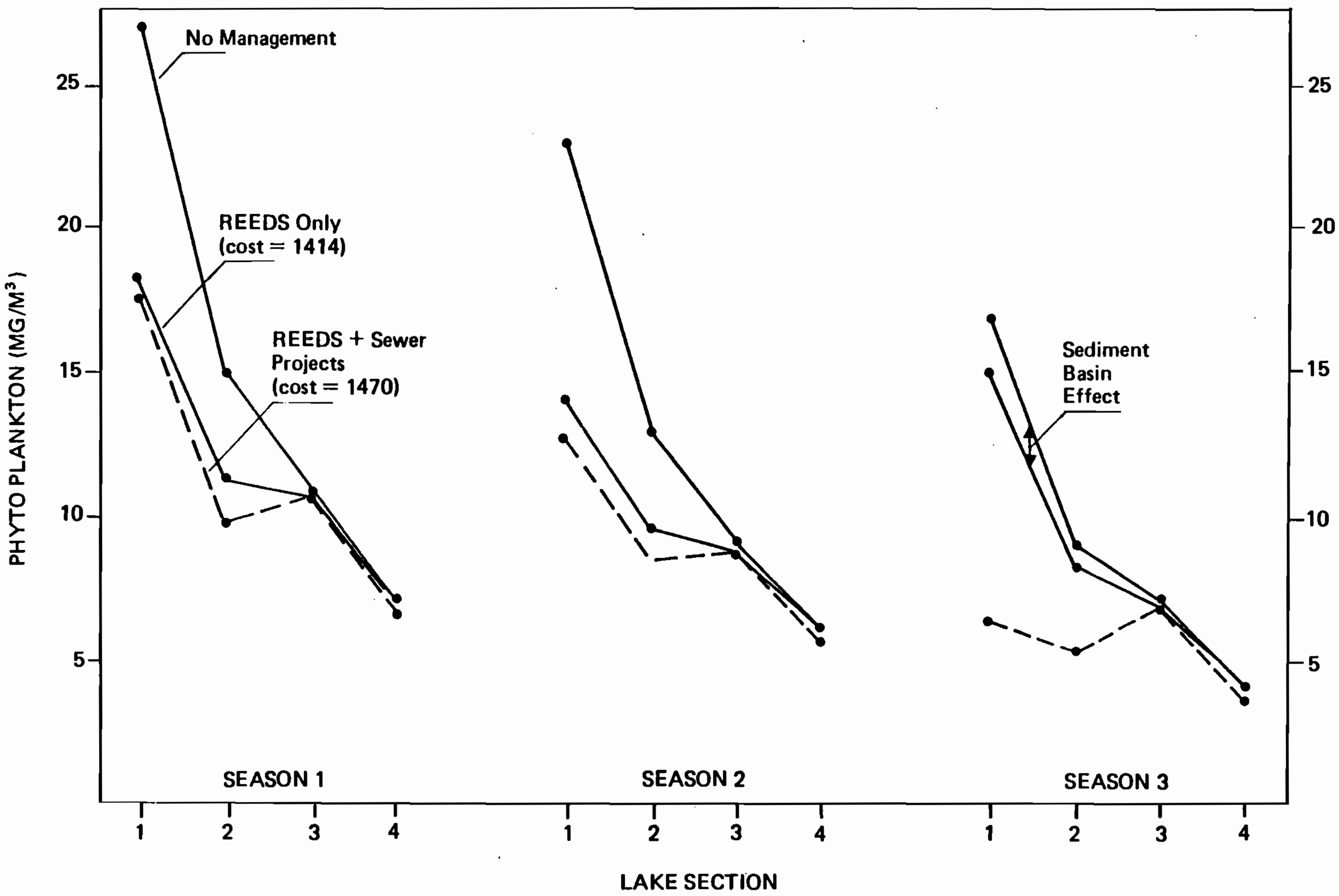

Figure 5. Improvement from 1. reed projects; 2. sewer projects.

not true of season 3, however, (the cold season), when the only effect of reed projects is from the sediment basins.

The most striking aspect of both figures is the insignificant improvement in basins 3 and 4. This is due to two facts: (1) the volumes of basins 3 and 4 are an order of magnitude larger than that of basin 1 and therefore, dampen most management impacts; and (2) the existing water quality in basins 3 and 4 are much better than 1 and 2 and this makes them insensitive to management changes.

Another striking aspect of the figures is that reed projects are more than an order of magnitude more costly than tertiary sewer projects. Therefore, since the reed projects produce only marginally better responses than do sewer projects, the latter appears to be much more cost effective. Recall that this comparison is between reeds and only the phosphorous removal sewer units--not secondary treatment and sewer collection lines. Secondary treatment is assumed to be required in any case, in order to prevent anaerobic conditions; but since it provides very little phosphorous removal, those (sunken) costs are considered to be irrelevant to phosphorous management decisions.

The "build everything" situations portrayed by Figures 4 and 5 are of course far from optimal or even reasonable solutions. Therefore, let us now consider incremental increases in phytoplankton as various management projects are eliminated. Figure 6 indicates that the two largest of four possible sewer projects in section 1 produce essentially all of the lake response (in that critical section). Also, the four largest sewer projects in sections 1 and 2 produce essentially all of the entire lake response at half the cost of constructing all 18 (sewer) projects.

Since the Kis Balaton reed project on the Zala River is already being constructed, an interesting question is: what additional sewer and/or reed projects are necessary to accomplish a large fraction of the total technically feasible improvement? Figure 7 suggests that the Kis Balaton reed lake does a little better than the sewer projects in section 1 (except during

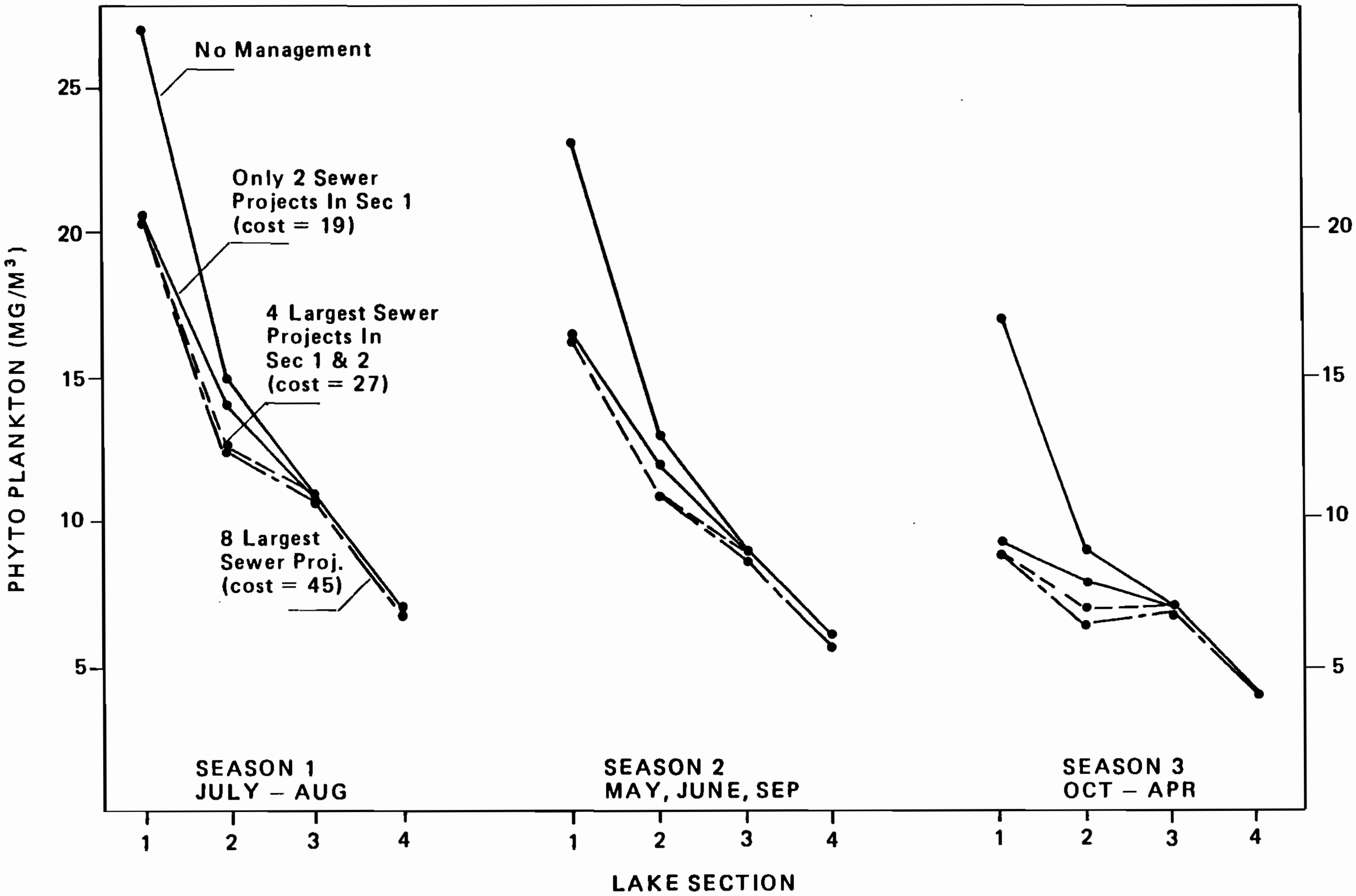

Figure 6. Variation in water quality with number of sewer projects.

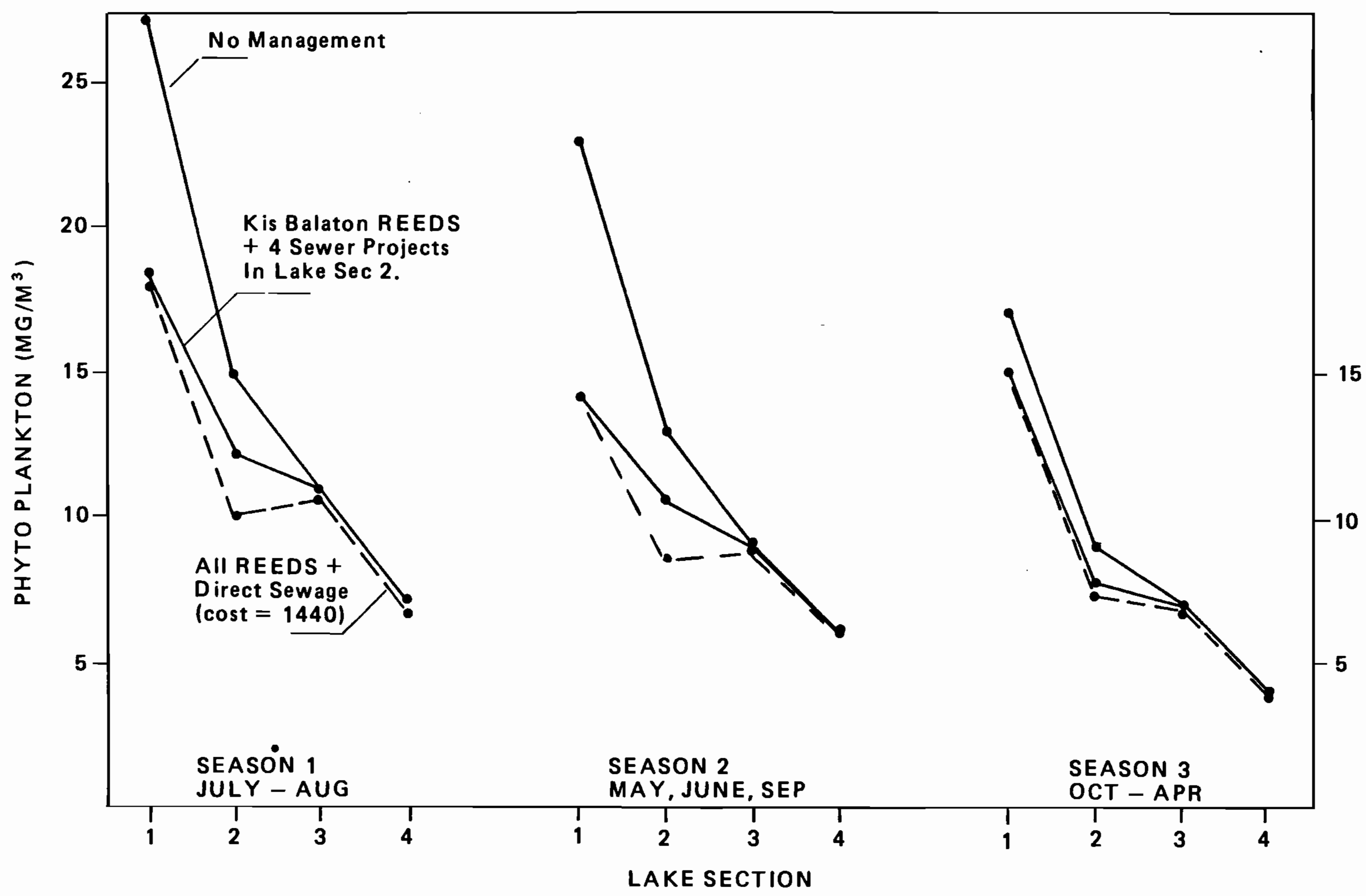

Figure 7. Other configurations.

season 3), but at a much higher cost; also that building all four sewer projects in section 2 will produce about half the response of a combination of the reed projects in section 2 plus the direct sewage load projects.

What solution is optimal, is of course, a question that requires the fixing of a budget and an objective function (and an optimization model). However, from only simulation model results, some conclusions seem apparent:

1. Management activities should be focused on sections 1 and 2 of the lake, with the first priority being the Zala load and any additional budget being allocated to management of section 2.
2. The reed project efficiencies and costs contain a great deal of uncertainty. However, if the rather optimistic efficiencies used here are reasonable, the sewer and reed projects are approximately competitive, in terms of lake response, with the reed projects being slightly better during summer months (but totally ineffective in winter). However, the reed projects appear to cost at least an order of magnitude more than the tertiary sewer units.

In order to show the simulation results in terms of P load reduction rather than lake response, Figures 8 and 9 are included. They include many of the same management options as Figures 4, 5, and 6.

5.3. AIP Model

5.3.1. Input Coefficients

The simulation model was used to develop coefficients for the AIP model. This task required extraction of cost and P reduction quantities for management project combinations represented by the X_{ij} of Problem 2. These combinations are defined in Table 5.

The AIP matrix (Figure 10) has 34 binary variables and only seven constraints. This compact size is possible because it

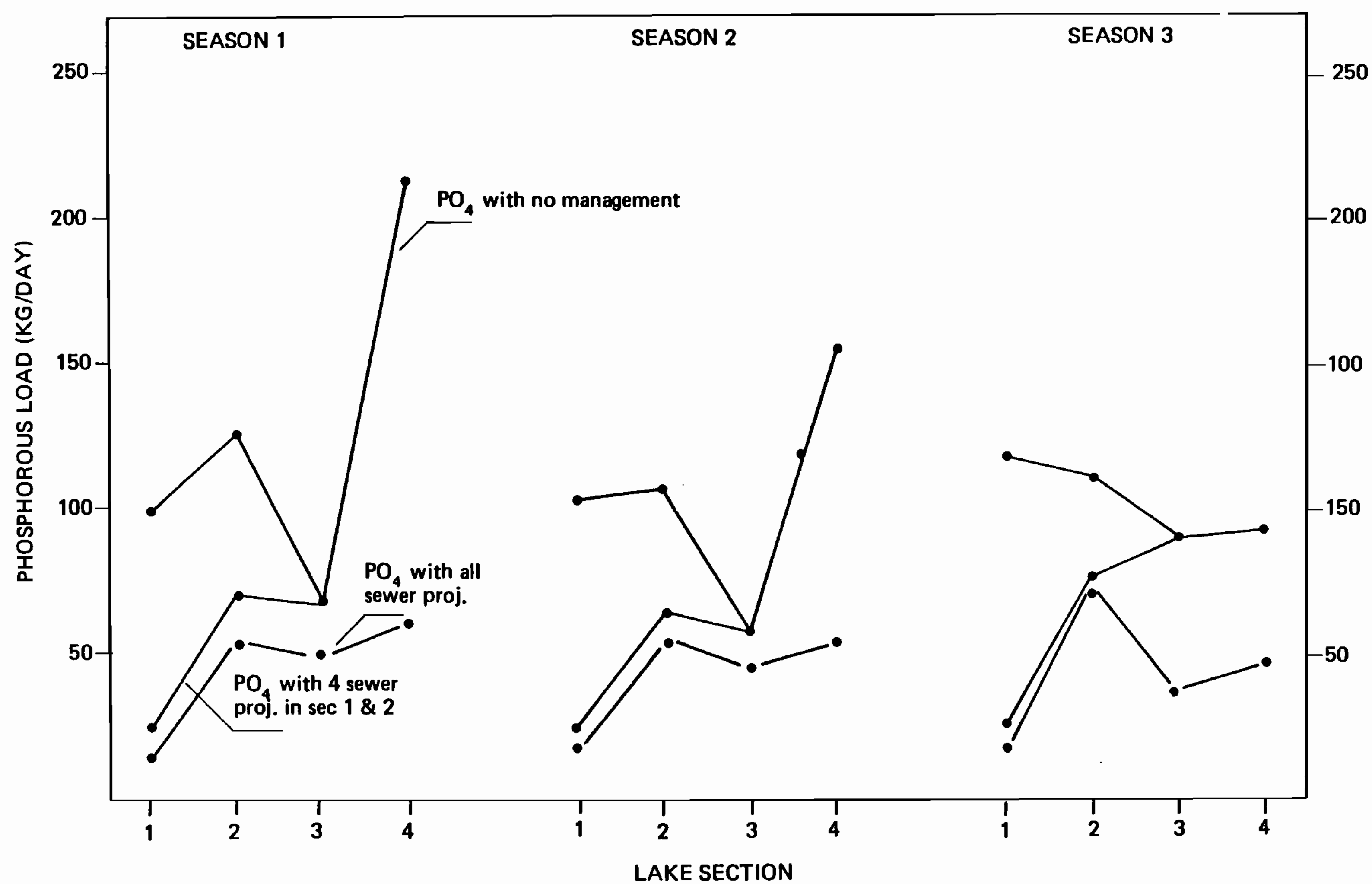

Figure 8. P loading reduction from sewer projects.

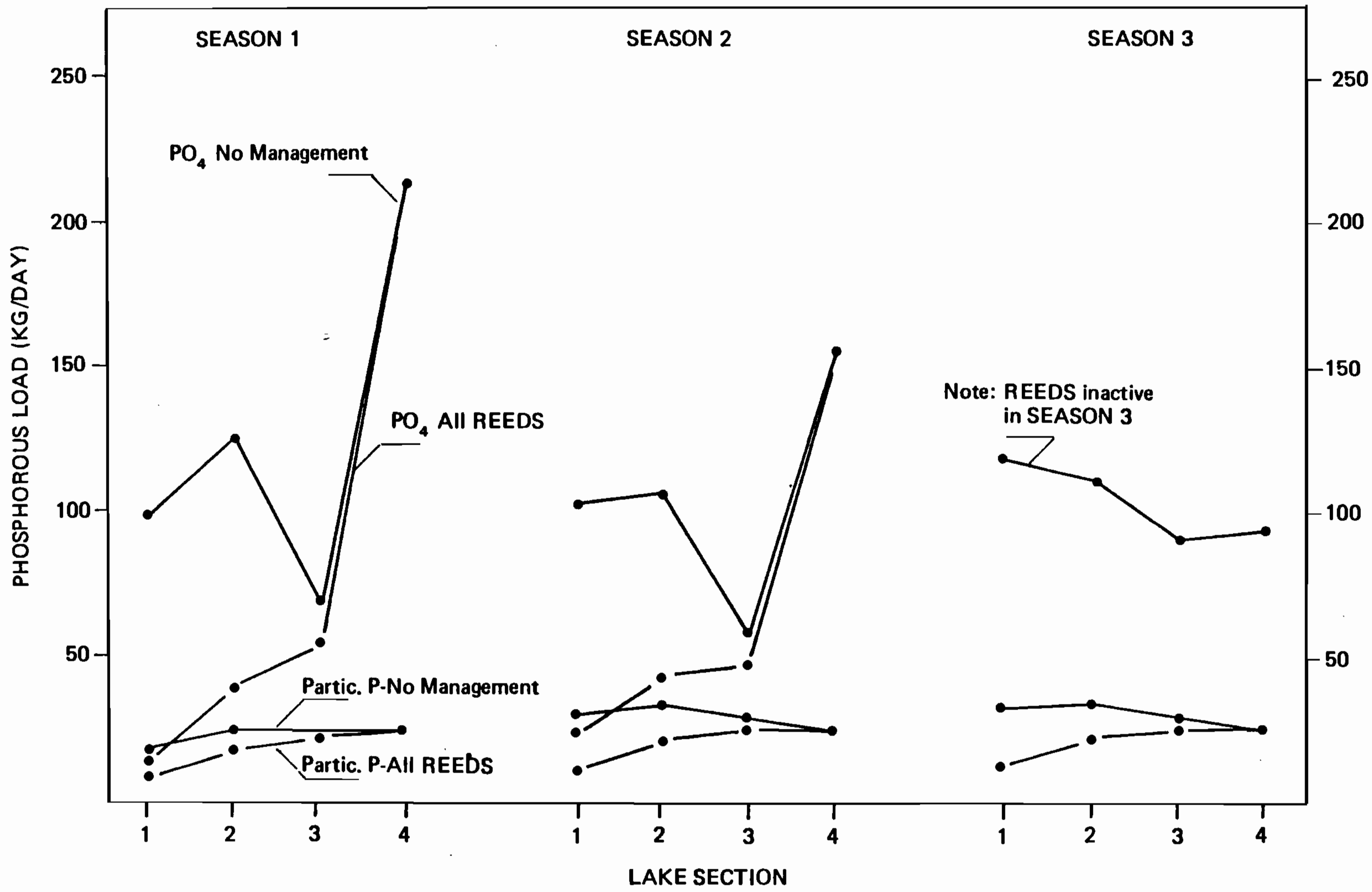

Figure 9. P loading reduction from reed projects.

Table 5. Key to simulation/AIP model project combinations.

Lake Section 1				
X_{ij}	R1,1	T1,1	T1,2	T1,3&4
X11	0	1	1	1
X12	1	1	1	1
X13	0	1	1	0
X14	0	0	1	0
X15	1	1	1	0
X16	1	0	1	0

Lake Section 2								
X_{ij}	R21	R22	R23	R24	T21	T22	T23	T24
X21	1							
X22		1				1		
X23		0				1		
X24		1				0		
X25			1					
X26				1	1			1
X27				1	1			0
X28				1	0			0
X29				0	1			1
X2,10				0	1			0
X2,11							1	

Lake Section 3										
X_{ij}	R31	R32	R33	R34	T31	T32	T33	T34	T35	T36
X31	1									
X32		1				0				
X33		1				1				
X34		0				1				
X35			1					0		
X36			1					1		
X37			0					1		
X38				1	0					
X39				1	1					
X3,10				0	1					
X3,11							1			
X3,12									1	
X3,13										1

Lake Section 4				
X_{ij}	T41	T42	T43	T44
X41	1			
X42		1		
X43			1	
X44				1

Note: For X_{ij} = , the following table values apply:
1 ⇒ project is built
0 ⇒ project is not built
blank ⇒ project is undefined by this X_{ij}.

		X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	$X_{2,10}$	$X_{2,11}$	Row Type	RHS
(Prob. 2a)	Z_1 =	270	338	244	151	312	305	48	60	55	47	6.1	127	124	81	86	78	21	N	
(Prob. 2b)	Z_2 =	221	278	199	127	255	254	4.8	5.6	5.3	4.9	.6	11.7	11.4	8	7.8	7.1	2	N	
Illegal combi- nations	Row 1	1	1	1	1	1	1												$\leqslant$	1
	Row 2								1	1	1								$\leqslant$	1
	Row 3												1	1	1	1	1		$\leqslant$	1
(Budget)	Row 7	21	658	19	12.5	656	650	97	42	3	39	59	240	240	235	5.3	4.7	1.3	Cont'd below	

		X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}	X_{38}	X_{39}	$X_{3,10}$	$X_{3,11}$	$X_{3,12}$	$X_{3,13}$	X_{41}	X_{42}	X_{43}	X_{44}	Row Type	RHS
(Prob. 2a)	Z_1 =	8.7	13.1	19.6	6.5	7.2	9.9	2.7	7.4	17.9	10.5	39.1	5.2	22	134	39	116	10.4	N	
(Prob. 2b)	Z_2 =	.4	.66	.9	.23	.33	.43	.1	.36	.87	.5	1.4	.25	1.0	3.1	1.1	2.7	.24	N	
Illegal combi- nations	Row 4		1	1	1														$\leqslant$	1
	Row 5					1	1	1											$\leqslant$	1
	Row 6								1	1	1								$\leqslant$	1
(Budget)	Row 7	17	202	203	1.1	95	96	0.9	32	34	1.4	2.7	.7	1.3	7	2.4	6.3	.7	$\leqslant$	1,000

Note: Z_1 is the objective function for problem 2a (max. reduction in P load);
Z_2 is the objective function for problem 2b (max. quality of lake water).

Figure 10. AIP Matrix.

merely summarizes that portion of the information produced by the simulation model which is needed for the optimization phase. The model structure is such that a unique level of P reduction is associated with each 0-1 variable.

The optimization model was run several times with various budget levels. Both Problems 2a and 2b were solved. All that was needed to convert from Problem 2a (P removal objective) to Problem 2b (lake quality objective) was to change the objective function coefficients by addings the W_{ij} weighting factors as described in Section 3.1. Both objective functions are included in Figure 10.

5.3.2. Results of AIP Optimization

Table 6 displays optimal solutions for six type 2b problems (maximum lake quality objective) and one type 2a problem (maximum P reduction). The budget constraints are varied from 30 to 1100 (10^6 Forints present worth). The table includes results in terms of both the AIP variables (many of which represent combinations of variables) and the individual projects as defined for the simulation model. A budget constraint of 30 allows construction (and operation) of only part of the sewer projects and no reed projects while the 1100 constraint allows construction of almost everything.

The optimal configuration for a 30 million Forint budget (Problem 2b) is to construct tertiary sewer projects for all cities in section 1 (the west end of the lake and 3 out of 4 of the cities in section 2). As the budget is increased to 40, 50 and 60 million, the optimal solutions are to complete all of the sewer projects in section 2 and then to spend the remaining money on sewer projects in sections 3 and 4. The first reed project to enter an optimal solution is in section 3 (a very small one) at a budget constraint of 100. At a budget level of 700 the optimal solution is to construct the very large Kis Balaton project plus all possible sewer projects.

The general pattern of optimal management project configuration is precisely that which was suggested by the simulation

Table 6. Results of AIP model Balaton application.

Optimal Solutions in Terms of Individual Projects:

Type Obj.	Budget Limit	Reed Projects R_{11}	R_{21}	R_{22}	R_{23}	R_{24}	R_{31}	R_{32}	R_{33}	R_{34}	Sewer Projects T_{11}	T_{12}	T_{13}	T_{21}	T_{22}	T_{23}	T_{24}	T_{31}	T_{32}	T_{33}	T_{34}	T_{35}	T_{36}	T_{41}	T_{42}	T_{43}	T_{44}	Budget Used	Opt. Obj. Function	Cost Effectiveness (Obj./Budget Used)
2b	30										1	1	1	1	1		1											29.5	234.1	7.9
2b	40										1	1	1	1	1	1	1	1		1		1	1		1		1	40.0	240.6	6.0
2b	50										1	1	1	1	1	1	1	1		1			1	1		1		49.5	244.8	4.9
2b	60										1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	55.3	246.7	4.5
2b	100						1				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	72.5	247.1	3.4
2b	700	1									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	692.1	303.7	0.44
2b	1100	1	1		1	1	1				1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1099.7	313.3	0.28
2a	30													1	1	1	1		1	1				1	1	1	1	29.8	506.0	--

Optimal Solutions in Terms of AIP Model Variables:

Type Obj.	Budget Limit	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	$X_{2,10}$	$X_{2,11}$	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}	X_{37}	$X_{3,8}$	$X_{3,9}$	$X_{3,10}$	$X_{3,11}$	$X_{3,12}$	$X_{3,13}$	X_{41}	X_{42}	X_{43}	X_{44}
2b	30	1								1						1																			
2b	40	1								1						1		1										1	1	1	1		1		1
2b	50	1								1						1		1										1	1		1	1		1	
2b	60	1								1						1		1				1			1			1	1	1	1	1	1	1	1
2b	100	1								1						1		1	1			1			1			1	1	1	1	1	1	1	1
2b	700		1							1						1		1				1			1			1	1	1	1	1	1	1	1
2b	1100		1					1		1		1	1					1	1			1						1	1	1	1	1	1	1	1
2a	30									1						1		1				1							1			1	1	1	1

model, namely--the most cost effective use of management actions is to begin with sewer (P removal) projects in the west end of the lake where dilution of P is least, existing quality is worst, and P loads are highest. Allocate the budget first in lake section 1, then 2, then 3 and 4, building only sewer projects. Then, if any budget remains, begin building reed projects.

The optimization model adds the capabilities to: (1) Verify general patterns of change in optimal configuration as budget is varied (which seemed intuitively appropriate from the simulation analysis); (2) Determine specific optimal configurations for any particular budget (which because of combinatorial interdependencies are not apparent from the simulation model).

The fact that the AIP model appears to verify the general pattern of results predicted by the simulation model implies that the weighting factors used to convert the problem 2a to 2b are adequate. However, these factors can produce only an approximation of the explicit lake response equations. Therefore, for a more accurate indication of lake quality response, the simulation model should be used for any specific configuration recommended by the optimization model.

The single 2a type problem solution in Table 6 demonstrates the importance of maximizing lake quality rather than simply maximizing reduction in nutrient loading. When the lake response was not modelled, a radically different solution was obtained. The P reduction objective selected all of the sewer projects in lake section 4 and some in 2 and 3, while the lake quality objective selected all of the sewer projects in Section 1, some in 2, and none in 4.

6. SUMMARY AND CONCLUSIONS

6.1. Conclusions Related to the Balaton Application

The management alternatives considered here included phosphorous removal sewer treatment projects, P removal from nonpoint sources by reed lakes, and P removal from urban runoff. The simulation model calculated only insignificant response from the urban runoff projects and therefore they were not included in the optimization model.

The P removal efficiency (and costs) of reed lakes are very uncertain and in fact represent an area of controversy among researchers. Such questions as how (and at what cost) to harvest green reeds (before the nutrients move below the water line); will such harvesting cause the plants to die; how much P is removed by the reeds and how does this quantity vary over time, are still largely unanswered. The approach taken in this model was to be optimistic on all of these issues. The logic behind this decision was that if under the most optimistic estimates (95% PO_4 - P removal during July-August, 85% during other warm months, and only particulate sedimentation during winter) the reed projects are cost effective, then more research to verify actual efficiencies would be justified. However, the opposite appears to be true. Reed projects are not cost effective even under the optimistic assumptions used here. What should be done is to concentrate management effort on chemical treatment of sewer effluents, particularly in the west end of the lake. One possible exception to this negative picture in regard to reed lakes is the Kis Balaton project. The phosphorous load from the Zala River is very large and only part of it is removable by sewer projects. (Even with low reed removal efficiency, the particulate P removal in the sediment basin would be significant.)

6.2. Conclusions Related to Use of Generalized Lake Eutrophication Models

Two somewhat independent approaches were pursued in this study. First, a mixed integer programming model was developed. It has the capability of producing optimal management alternative solutions for any budget constraint. It does this by calculating internally both the phosphorous removal (dissolved and particulate) quantities and lake water quality response seasonally for any given vector of assumed removal efficiencies. The P removal lake response relationship must, of course, be obtained from an exogenous water quality modelling effort. The MIP optimization model performed satisfactorily for the Balaton application but could become very large (resulting in possible computational difficulties) in an application where more than three seasons or more than four lake sections are modelled.

The second type of modelling approach was a simulation model coupled (manually) to an all integer optimization model. The simulation model was designed to perform all of the management alternative--phosphorous removal--lake response calculations. It therefore produced input coefficients for a very compact form of optimization model. The seasonal variations modelled by the simulation model are not explicit in the AIP optimization model because the decisions are only to build or not build particular projects. However, the phosphorous and lake response impacts transferred from the simulation model can be either totals from all seasons or from a particular season.

The simulation/optimization combination appears to be superior to the single MIP model for the following reasons: (1) the simulation model is very useful by itself in answering all sorts of "what if" questions related to a specific management configuration. It runs at very low computer cost and produces output in a convenient format designed specially for this purpose. (2) Since the AIP optimization model contains only a summary of information produced by the simulation model, it is very compact and could handle problems much larger than the Balaton problem at reasonable computer cost. None of the Balaton problems required more than one minute of CPU time. The algorithm used was MIP 370 on an IBM 370 system located in Budapest and accessed via the IIASA network system.

One important aspect of lake eutrophication modelling which was demonstrated by these models is that maximizing the change in lake quality due to reduction in nutrient loading is a very different objective than simply maximizing the reduction of the nutrient loading itself. Unless a lake has nearly equal volume of water and pre-management water quality in each of the model sections, the distinction between the two objectives (the a and b type objectives of this report) is important. If the nearly equal conditions do exist, this implies that the model need not divide the lake into sections and in that case the two objectives become synonymous. In the more typical case where lake section volumes differ significantly, an objective which minimizes nutrient loading per unit of volume for each lake section would be a great improvement over one which ignores nutrient dilution. This would represent a compromise between the type a and b objectives which is very easy to model since it does not require a sophisticated lake eutrophication model--only knowledge of lake geometry and nutrient loading.

APPENDIX A

A. SELF CONTAINED OPTIMIZATION MODEL

A.1. Deviation from P Loading Goals--Problem 3a

A.1.1. Notation

A.1.1.1. Subscripts:

i = section of lake,

j = management project location (either reed or tertiary project),

k = season,

ℓ = tertiary unit capacity indicator.

A.1.1.2. Decision Variables:

$T_{ij\ell}$ = integer variable indicating number of tertiary sewer treatment units constructed at location j, of capacity ℓ and discharging into lake section i,

R_{ij} = 0,1 variable indicating construction of a reed project (including related sediment basin),

D_{ik} = continuous variable indicating deviation from phosphorous reduction goal for lake section i during season k (in kg/day),

LT_{ijk} = available P load actually treated by tertiary units,

LR_{ijk} = continuous variable indicating dissolved P load at mouth of river j as reduced by tertiary units and by natural river action before reed project (the load entering reed project j).

A.1.1.3. Constants:

GL_{ik} = goal indicating desired level of available P,

LD_{ijk} = dissolved P load entering lake section i from either point load j or river j (average) during season k without any management action,

LP_{ijk} = particulate P load entering lake section i from jth river,

ET_{ijk} = efficiency of tertiary treatment (fraction of plant capacity that is removed),

ERS_{ijk} = efficiency of particulate P removal by sediment basin associated with reed project j,

ER_{ijk} = efficiency of dissolved P removal by reed project j during season k,

$\alpha_{i\ell}$ = weighting factor indicating relative importance of P load reduction goals among lake sections and/or seasons,

β_{ijk} = factor which transforms upstream dissolved P load to river mouth quantity (river reach natural P removal addition effect),

γ = fraction of particulate P which becomes available P,

∂_{ℓ} = capacity of tertiary treatment unit ℓ in kg/day of phosphorous,

$C_{ij\ell}$ = capital cost of tertiary project at location ij of capacity ℓ.

C_{ijk} = present worth of future operating costs of project ij during season k (either tertiary or reed project) per kg/day of phosphorous.

A.1.2. Objective Function

Minimize deviation from P removal goals (Problem 3a)

$$\text{Min} \sum_{i}\sum \alpha_{ik} D_{ik} \quad .$$

A.1.3. Constraints

a. Calculate deviation from goals:

Σ Removal (kg/day) + Dev. from Goal
= Σ Initial Loads* − Goal

as follows:

$$\beta_{ijk}(ET) \sum_{j}^{\square} LT_{ijk} + \sum_{j}^{\Delta} ERS_{ijk}(\gamma)(LP_{ijk})R_{ij} + \sum_{j}^{\Delta} (ER_{ijk})LR_{ijk}$$

$$+ D_{ik} = \beta_{ijk} \sum_{j}^{o} LD_{ijk} + (\gamma) \sum^{\Delta} LP_{ijk} - GL_{ik} \quad ,$$

$$(i = 1,2,\ldots,I) \quad ; \quad (k = 1,2,\ldots,K) \quad , \qquad \text{(A1)}$$

where

$\square$ = j = all municipal point load locations
Δ = j = all reed project locations
o = j = all j except upstream municipal projects which discharge into a river which has a potential downstream reed project.†

b. Force load treated by reed project to zero if $R_{ij} = 0$. Note: cannot use (LR) R = 0 because both are variables; and cannot use (LD) R = 0 because LD arriving downstream = f (tertiary units built), therefore $\neq$ constant. So use:

$$LR_{ijk} \leq (R_{ij})LD_{ijk} \quad , \qquad \text{(redundant if } R = 1) \qquad \text{(A2)}$$

k = 1,2 (reeds do not operate in winter, therefore, R_{ij3} is deleted); i = 1,...,K) j = Δ (each j with reed project).

---

*Loads transformed to river mouth quantity levels.

†The j associated with β's are each river reach j while the Σ_i associated with LD and LT are the related upstream municipal j locations.

c. Definition of load reaching reed projects.

$$LR_{ijk} \leq LD_{ijk} - (\beta_{ijk}) ET \sum_{j} LT_{ijk} \quad , \qquad \begin{array}{l}(i = 1,\ldots,K)\\ (j = \text{each river})^{2}\\ (k = 1,2 \text{ only})\end{array} \qquad \text{(A3)}$$

Note: The $\leq$ formulation in equation (A3) works because the objective function will force LR as large as possible (= is no good because we want LR = 0 if R = 0).

d. Limit P removed by tertiary units to load (LD) or the ℓth unit capacity--whichever is smaller.

$$LT_{ijk} \leq \partial_{\ell} T_{ij\ell} \quad , \qquad \begin{array}{l}(i = 1,\ldots,K)\\ (\text{each } j \; \Sigma \; \square)\\ (k = 1,\ldots,K)\\ (\ell = 1,\ldots,L)\end{array} \qquad \text{(A4)}$$

$$LT_{ijk} \leq LD_{ijk} \quad , \qquad \begin{array}{l}(i = 1,\ldots,K)\\ (\text{each } j \; \Sigma \; \square)\\ (k = 1,\ldots,K)\end{array} \qquad \text{(A5)}$$

e. Budget constraint.

$$\Sigma \text{ Costs} \leq \text{Budget}$$

$$\sum_{ij\ell} C_{ij\ell} T_{ij\ell} + \sum_{ij} C_{ij} R_{ij} + \sum_{ijk} C_{ijk} LT_{ijk}$$

$$+ \sum_{ijk} C_{ijk} \leq \text{Budget} \qquad \text{(A6)}$$

APPENDIX B: SIMULATION MODEL PROGRAM LISTING

```
C Notation
c    ers(i,j)=eff.of particulate p removal by sediment basin j
c      (=.8 except where no sed. basin is possible.
c   t(i,j)=0,1 decision indicating construction of sewer project j.
c   r(i,j)=0,1 decisiion indicating const.of reed project and*
c     related sediment basin.
c   er(ijk)= effic. of po4 removal by reed project j (=.95,.85,.0
c     during season k =1,2,3.
c   lt(ijk)=po4 load actually treated by tertiary sewer project i,j
c   lr(ijk)=po4 load arriving at reed project i,j.
c   lp(ijk)=tp-po4 load  "     " sediment basin i,j.
c   ccr(i,j) and cct(i,j)=construction cost of reed and tertiary
c     project resp. in millions of forints.
c   pwopr(ij) and pwopt(ij) = operating cost present worth of
c     reed and tert. project resp. in millions of forints
c     assuming discount at 20%.
c   pwr(ij) and pwt(ij) = total pr. worth of reed or tert.
c     proj. resp. (const+pw of op cost).
c   tpwopr=total pw of op cost of reed projects.
c   tccr=total const. cost of reed+sed projects.
c   tpwopt=total pw of op cost of tertiary projects.
c   tcct=total const cost of tert. projects.
c   tc=total cost
c   ls(ij)=annual avg po4 load (untreated) at city j in
c     lake sec i (j=7 is sum of all small loads where no
c     project is planned.
c   lnpt(i,j)= annual avg tp load (untreated) entering lake sec i from
c       trib. j (j=5 is sum of loads from small trib.s where no projis planned
c   lnpp4(ij)=annual avg po4 load (untreated) entering lake sec i from
c     trib j
c   lut(i)=annual avg tp from direct urban runoff to sec i.
c   lup4(i)=annual avg po4 from urban runoff.
c   lrt(i)=annual avg tp from direct rural (NON-trib) runoff to sec i.
```

```
c   lrp4(i)=annual avg po4 from direct rural.
c   latt(i)=annual avg tp atmospheric load to sec (i).
c   latp4(i)= "     "  po4     "      "     "
c   ftbst(i,k)=season/avg factor for total p trib loads season k.
c   ftbsp4(i,k)=season/avg factor for po4 trib loads
c   fssl(ijk)=season/avg factor for sewer loads.
c   b(ijk)=factor indicating treatment effect of river reach on po4 load
c     from sewage in *trib* j, season k.
c   lsd(ij)=direct ls(ij)
c   phpo(ik)=phyto-plankton levels befor management
c   resp(ij)=coef. of p load/lake production transformation equations
c   et=tert. unit removal efficiency
c   ur=flag for construction of urban run off projects (ur=1)
c   sumt(ik)=change in po4 load during season k due to tertiary projects
c     built in lake sec i.
c   sums(i,k)=change in .2*(tp-po4)during season k due to sediment basins
c     in sec i.
c   sumr(ik)= change in po4 during season k due to reed projects in
c      sec i.
c   ltt(ijk)=reduction in p load by tert. units at river mouth-not
c      tert. location.
c   lrsec(ik)=po4 load arriving at reed projects for entire section i.
c   delr(ijk)=change inavailable p load due to reed project(ij).
c   slnpt(ik)=total p entering lake sec. i from tributaries with no pr.
c   slnpp4(ik)= "  po4  "       "    "              "             "   "
c   ssdir(ik)=total direct load of po4 with no pr.
c   cost notation: (all in millions of forints)
c      ccr(ij)=const cost of reed pr (ij).
c      pwopr(ij)=pr. worth of op. cost-reed pr.(ij).
c      pwr(ij)=  "    "      total cost "   "
c      tpwopr=total pr.w. all reed pr.
c      pwopt(ijl)=pr.w. of op cost-tert. pr.(ijl)
c      cct(ij)=const. cost tert. pr(ij).
c      pwt=total pr. w. tert. pr.(ij).
c      tpwopt=total pr.w. of all tert. pr.
c      tcost=pr.w. of all reed plus tert. pr.
c   parta(ik)=20% of partic. p. load with no management
c   po4a(ik)=po4 load with no management
c   availa(ik)=total avail load with no management
c   partb,po4b,availb=loads to lake with only selected tert. pr. operating.
c   partc,po4c,availc=loads to lake with selected reed & tert. pr. operating.
c   partd,po4d,availd=loads to lake with reed, tert.,& urban run off pr. op.
c   ll(ik)=avail load reduction due to reed + tert. pr.
c   php(ik)=phyto-plankton levels after management.
c
c
c
c  begin program
      real ls,lnpp4,lnpt,lt,ll,latp4,latt,lrp4,lrt,lup4,lut,lrsec,
     ?lr,lp,lsd,ltt
      integer t,r
      dimension ls(4,7),fssl(4,7,3),t(4,7),b(4,7,3),lnpp4(4,5)
     1,ftbsp4(4,3),ftbst(4,3),r(4,5),ers(4,5),er(4,5,3)
     1,lnpt(4,5),lp(4,5,3),lt(4,7,3),sumt(4,3),x(1,1,3),
     1lr(4,5,3),lrsec(4,3),sums(4,3),sumr(4,3),slnpt(4,3),
     1slnpp4(4,3),ssdir(4,3),ccr(4,5),pwopr(4,5),pwr(4,5),
     1cct(4,6),pwopt(4,6,3),parta(4,3),lut(4),lup4(4),lrt(4)
```

```
     1,lrp4(4),latt(4),latp4(4),po4a(4,3),availa(4,3),partb(
     14,3),po4b(4,3),partc(4,3),po4c(4,3),availb(4,3),availc(4,3)
     1,phpo(4,3),php(4,3),ll(4,3),lsd(4,7),ltt(4,7,3)
     1,resp(3,4),partd(4,3),po4d(4,3),delr(4,5,3)
      data lp/60*0.0/,lt/84*0.0/,sumt/12*0.0/,x/3*0.0/,
     ?lr/60*0.0/,lrsec/12*0.0/,sums/12*0.0/,sumr/12*0.0/,slnpt/12*0.0/,
     ?slnpp4/12*0.0/,ssdir/12*0.0/,ccr/20*0.0/,pwopr/20*0.0/,
     ?pwr/20*0.0/,
     ?cct/24*0.0/,pwopt/72*0.0/,parta/12*0.0/,lut/4*0.0/,lup4/4*0.0/,
     ?lrt/4*0.0/,
     ?lrp4/4*0.0/,latt/4*0.0/,latp4/4*0.0/,po4a/12*0.0/,
     ?availa/12*0.0/,partb/12*0.0/,
     ?po4b/12*0.0/,partc/12*0.0/,po4c/12*0.0/,availb/12*0.0/,
     ?availc/12*0.0/,php/12*0.0/,ll/12*0.0/
       read(1,1000,end=900)((ls(i,j),i=1,4),j=1,7)
 1000 format(4f9.3)
       read(1,1000,end=900)((lsd(i,j),i=1,4),j=1,7)
      read(1,1001,end=900)((t(i,j),i=1,4),j=1,7)
 1001 format(4i3)
      read(1,1000,end=900)((lnpp4(i,j),i=1,4),j=1,5)
      read(1,1000,end=900)((lnpt(i,j),i=1,4),j=1,5)
      read(1,1001,end=900)((r(i,j),   i=1,4),j=1,5)
      read(1,1002,end=900)(((fssl(i,j,k),k=1,3),j=1,7),i=1,4)
 1002 format(3f9.3)
      read(1,1002,end=900)(((b(i,j,k),k=1,3),j=1,7),i=1,4)
      read(1,1000,end=900)((ftbsp4(i,k),i=1,4),k=1,3)
      read(1,1000,end=900)((ftbst(i,k),i=1,4),k=1,3)
      read(1,1000,end=900)((ers(i,j),  i=1,4),j=1,5)
      read(1,1002,end=900)(((er(i,j,k),k=1,3),j=1,5),i=1,4)
      read(1,1000,end=900)((phpo(i,k), i=1,4),k=1,3)
      read(1,1000,end=900)((ccr(i,j), i=1,4),j=1,4)
      read(1,1000,end=900)((pwopr(i,j), i=1,4),j=1,4)
      read(1,1010,end=900)(lut(i),i=1,4)
      read(1,1010,end=900)(lup4(i),i=1,4)
      read(1,1010,end=900)(lrt(i),i=1,4)
      read(1,1010,end=900)(lrp4(i),i=1,4)
      read(1,1010,end=900)(latt(i),i=1,4)
      read(1,1010,end=900)(latp4(i),i=1,4)
 1010 format(4f9.2)
      read(1,1002,end=900)((resp(i,j),i=1,3),j=1,4)
      read(1,2995,end=900)et,ur
 2995 format(2f9.2)
c   write out most of input data for convenience in interpreting results
      write(6,14)((ls(i,j),i=1,4),j=1,7)
   14 format(1h ,'ls='/7(4f10.3/))
      write(6,14)((lsd(i,j),i=1,4),j=1,7)
      write(6,9)((t(i,j),i=1,4),j=1,7)
    9 format(1h ,'t='/7(4i3/))
      write(6,4)((lnpp4(i,j),i=1,4),j=1,5)
    4 format(1h ,'ls='/5(4f10.3/))
      write(6,4)((lnpt(i,j),i=1,4),j=1,5)
      write(6,3)((r(i,j),i=1,4),j=1,5)
    3 format(1h ,'r='/5(4i3/))
      write(6,5) (((fssl(i,j,k),k=1,3),j=1,7),i=1,4)
    5 format(1h ,'fssl='/28(3f10.3/))
      write(6,5)(((b(i,j,k),k=1,3),j=1,7),i=1,4)
      write(6,6)((ftbsp4(i,k),i=1,4),k=1,3)
```

```
    6 format(1h ,'ftbsp4='/3(4f10.3/))
      write(6,6)((ftbst(i,k),i=1,4),k=1,3)
      write(6,4)((ers(i,j),i=1,4),j=1,5)
      write(6,7)(((er(i,j,k),k=1,3),j=1,5),i=1,4)
    7 format(1h ,'er='/20(3f10.3/))
      write(6,6)((phpo(i,k),i=1,4),k=1,3)
      write(6,8)((ccr(i,j),i=1,4),j=1,4)
      write(6,8)((pwopr(i,j),i=1,4),j=1,4)
    8 format(1h ,'ccr='/4(4f10.3/))
 2997 format(1h ,'resp'/4(3f10.3/))
      write(6,2997)((resp(i,j),i=1,3),j=1,4)
      write(6,2994) et,ur
 2994 format(1h ,'et=',f5.2,5x,'ur=',f5.2)
c calculate p load treated by tert. units as total po4 load if project j
c   is built. then calculate change in po4 reaching lake (not change
c   at project location) due to tert. projects.
      do 30 i=1,4
      n=4
      if(i.eq.3)go to 10
      go to 12
   10 n=6
   12 do 20 k=1,3
      do 15 j=1,n
      lt(i,j,k)=ls(i,j)*fssl(i,j,k)*t(i,j)
      ltt(i,j,k)=lt(i,j,k)*et*b(i,j,k)
   15 sumt(i,k)=et*b(i,j,k)*lt(i,j,k)+sumt(i,k)
   20 continue
   30 continue
 3000 format(1h ,'lt=')
      write(6,3000)
 4000 format(1h,/28(3f10.3/))
      write (6,4000)(((lt(i,j,k),k=1,3),j=1,7),i=1,4)
      write(6,2999)
 2999 format(1h ,'ltt=')
      write (6,4000)(((ltt(i,j,k),k=1,3),j=1,7),i=1,4)

 3001 format(1h ,'sumt=')
      write(6,3001)
 4001 format(1h,/3(4f12.2/))
      write(6,4001)((sumt(i,k),i=1,4),k=1,3)
c define loads arriving at reed project j including corrections
c   for removal by upstream tert. projects *l*.
      do 120 k=1,3
 5002 format(1h,f10.2/)
      lr(1,1,k)=lnpp4(1,1)*ftbsp4(1,k)-sumt(1,k)
  120 continue
c  some reed projects have no upstream tert. projects.
      do 130 k=1,3
      do 125 j=1,3
  125 lr(2,j,k)=lnpp4(2,j)*ftbsp4(2,k)
c  correct proj. 2,2
      lr(2,2,k)=lr(2,2,k)-ltt(2,2,k)
  130 continue
c  reed project 2,4
      do 150 k=1,3
      lr(2,4,k)=lnpp4(2,4)*ftbsp4(2,k)-  et*(b(2,1,k)*
```

```
     1lt(2,1,k)-b(2,4,k)*lt(2,4,k))
      lr(3,1,k)=lnpp4(3,1)*ftbsp4(3,k)
      lr(3,2,k)=lnpp4(3,2)*ftbsp4(3,k)- et*b(3,2,k)*lt(3,2,k)
      lr(3,3,k)=lnpp4(3,3)*ftbsp4(3,k)- et*b(3,4,k)*lt(3,4,k)
  150 lr(3,4,k)=lnpp4(3,4)*ftbsp4(3,k)- et*b(3,1,k)*lt(3,1,k)
 3002 format(1h ,'lr=')
      write(6,3002)
 4002 format(1h,/20(3f10.3/))
      write (6,4002)(((lr(i,j,k),k=1,3),j=1,5),i=1,4)
c calculate corrected po4 loads to each lake sec (lrsec)
      do 200 i=1,3
      do 190  k=1,3
      n=4
      if(i.eq.1)go to 179
      go to 177
  179 n=1
  177 continue
      do 185 j=1,n
      lrsec(i,k)=lrsec(i,k)+lr(i,j,k)
  185 continue
  190 continue
  200 continue
 3003 format(1h ,'lrsec')
      write(6,3003)
      write(6,4001)((lrsec(i,k),i=1,4),k=1,3)
c calculate change in tp-po4 due to sediment projects (sums) and
c   in po4 due to both reed projects (sumr) and tert. projects (sumt)
      do 60 i=1,4
      n=4
      if(i.eq.1)go to 36
      go to 38
   36 n=1
   38 continue
      do 50 k=1,3
      do 45 j=1,n
      lp(i,j,k)=lnpt(i,j)*ftbst(i,k)-lnpp4(i,j)*ftbsp4(i,k)
      sums(i,k)=sums(i,k)+ers(i,j)*lp(i,j,k)*r(i,j)*.2
      sumr(i,k)=sumr(i,k)+er(i,j,k)*lr(i,j,k)*r(i,j)
c  calc change in load due to each reed pr. given tert. projects.
      delr(i,j,k)=er(i,j,k)*lr(i,j,k)+ers(i,j)*lp(i,j,k)*.2
   45 continue
   50 continue
   60 continue
      write(6,201)
      write(6,4002)(((delr(i,j,k),k=1,3),j=1,5),i=1,4)
  201 format(1h ,'delr=')
c  calculate total load entering lake from trib.s with no projects (each
c    season and lake sec.
      do 70 i=1,4
      do 72 k=1,3
      do 74 j=1,5
      slnpt(i,k)=slnpt(i,k)+lnpt(i,j)*ftbst(i,k)
      slnpp4(i,k)=slnpp4(i,k)+lnpp4(i,j)*ftbsp4(i,k)
   74 continue
   72 continue
   70 continue
```

```
3004     format(1h ,'lp=')
      write(6,3004)
      write(6,4002)
      write (6,4002)(((lp(i,j,k),k=1,3),j=1,5),i=1,4)
3005  format(1h ,'sums=')
      write(6,3005)
      write(6,4001)((sums(i,k),i=1,4),k=1,3)
3006  format(1h ,'sumr=')
      write(6,3006)
      write(6,4001)((sumr(i,k),i=1,4),k=1,3)
3007   format(1h ,'slnpt=')
      write(6,3007)
      write(6,4001)((slnpt(i,k),i=1,4),k=1,3)
3008       format(1h ,'slnpp4=')
      write(6,3008)
      write(6,4001)((slnpp4(i,k),i=1,4),k=1,3)
c calc. total load from direct sewage with no projects
      do 100 i=2,4
      do 95 k=1,3
      do 90 j=1,7
   90 ssdir(i,k)=ssdir(i,k)+lsd(i,j)*fssl(i,j,k)
   95 continue
  100 continue
3009       format(1h ,'ssdir =')
      write(6,3009)
      write(6,4001)((ssdir(i,k),i=1,4),k=1,3)
c calculate cost of reed projects
      do 600 i=1,4
      n=4
      if(i.eq.1)go to 591
      go to 589
  591 n=1
  589 continue
      do 595 j=1,n
      ccr(i,j)=ccr(i,j)*r(i,j)
      pwopr(i,j)=pwopr(i,j)*r(i,j)
      pwr(i,j)=ccr(i,j)+pwopr(i,j)
      tpwopr=tpwopr+pwopr(i,j)
      tccr=tccr+ccr(i,j)
  595 continue
c  calculate tert. costs
      n=4
      if(i.eq.3)go to 480
      go to 475
  480 n=6
  475 do 490 j=1,n
      if(lt(i,j,1).le.0)cct(i,j)=0.
      if(lt(i,j,1).gt.0.)cct(i,j)=0.3
      if(lt(i,j,1).gt.7)cct(i,j)=0.5
      if(lt(i,j,1).gt.21)cct(i,j)=1.0
      if(lt(i,j,1).gt.42)cct(i,j)=1.5
      if(lt(i,j,1).gt.63)cct(i,j)=2.0
      if(lt(i,j,1).gt.90.)cct(i,j)=9999
      tcct=tcct+cct(i,j)
      pwopt(i,j,1)=350.*lt(i,j,1)*62./1000000.
      pwopt(i,j,2)=350.*lt(i,j,2)*91./1000000.
      pwopt(i,j,3)=350.*lt(i,j,3)*212./1000000.
```

```
      pwt(i,j)=pwopt(i,j,1)+pwopt(i,j,2)+pwopt(i,j,3)+cct(i,j)
      do 505 k=1,3
      tpwopt=tpwopt+pwopt(i,j,k)
  505 continue
  490 continue
  600 continue
      tcost=tcct+tpwopt+tccr+tpwopr
 3010      format(1h ,'ccr   =')
      write(6,3010)
 4003 format(1h,/5(4f12.1/))
      write(6,4003)((ccr(i,j),i=1,4),j=1,5)
 3011      format(1h ,'pwopr =')
      write(6,3011)
      write(6,4003)((pwopr(i,j),i=1,4),j=1,5)
 3012      format(1h ,'cost(total)of reed projects')
      write(6,3012)
      write(6,4003)((pwr(i,j),i=1,4),j=1,5)
      write(6,2991)
 2991 format(1h ,'cost(total) of tertiary projects')
      write(6,4005)((pwt(i,j),i=1,4),j=1,6)
 3013      format(1h ,'tccr=')
          write(6,3013)
 4004 format(1h,f12.0)
       write(6,4004)tccr
 3014      format(1h ,'tpwopr=')
          write(6,3014)
       write(6,4004)tpwopr
 3015      format(1h ,'cct=')
          write(6,3015)
 4005  format(1h,/6(4f12.1/))
       write(6,4005)((cct(i,j),i=1,4),j=1,6)
 3016      format(1h ,'pwopt=')
          write(6,3016)
 4006   format(1h,/24(3f10.3/))
       write(6,4006)(((pwopt(i,j,k),k=1,3),j=1,6),i=1,4)
 3017      format(1h ,'tcct=')
          write(6,3017)
          write(6,4004)tcct
 3018             format(1h ,'tpwopt=')
          write(6,3018)
          write(6,4004)tpwopt
 3019      format(1h ,'tcost=')
          write(6,3019)
          write(6,4004)tcost

c  calculate available loads entering each lake sec.
      do 700 i=1,4
      do 690 k=1,3
c available loads with no projects operating
      parta(i,k)=.2*((slnpt(i,k)-slnpp4(i,k))+(lut(i)-lup4(i))+
     1(lrt(i)-lrp4(i))+(latt(i)-latp4(i)))
      po4a(i,k)=ssdir(i,k)+slnpp4(i,k)+lup4(i)+lrp4(i)+latp4(i)
      availa(i,k)=parta(i,k)+po4a(i,k)
c  loads with only tert. projects operating
      partb(i,k)=parta(i,k)
```

```
      po4b(i,k)=po4a(i,k)-sumt(i,k)
c  loads with both tert. and reeds operating
      partc(i,k)=partb(i,k)-sums(i,k)
      po4c(i,k)=po4b(i,k)-sumr(i,k)
      availb(i,k)=partb(i,k)+po4b(i,k)
      availc(i,k)=partc(i,k)+po4c(i,k)
c  calculate effect of urban runoff projects
      if(ur.eq.0)go to 690
      partd(i,k)=partc(i,k)-.2*.75*(lut(i)-lup4(i))
      po4d(i,k)=po4c(i,k)-.37*lup4(i)
      po4d(4,k)=po4c(4,k)
      availc(i,k)=partd(i,k)+po4d(i,k)
 2993 format(1h ,'urban ro projects constructed')
      write(6,2993)
  690 continue
  700 continue
      do 720 k=1,3
      ll(1,k)=(availa(1,k)-availc(1,k))/82
      ll(2,k)=(availa(2,k)-availc(2,k))/413
      ll(3,k)=(availa(3,k)-availc(3,k))/600
      ll(4,k)=(availa(4,k)-availc(4,k))/802
      php(1,k)=phpo(1,k)-resp(1,1)*ll(1,k)-resp(1,2)*ll(2,k)
      php(2,k)=phpo(2,k)-resp(2,1)*ll(1,k)-resp(2,2)*ll(2,k)
     ?     -resp(2,3)*ll(3,k)
      php(3,k)=phpo(3,k)-resp(3,1)*ll(2,k)-resp(3,2)*ll(3,k)
     ?     -resp(3,3)*ll(4,k)
      php(4,k)=phpo(4,k)-resp(4,1)*ll(3,k)-resp(4,2)*ll(4,k)
   13 format(3x,f12.2)
  720 continue
        goto 99999
  900 write(6,1003)
 1003 format('you have less records than you should have')
99999   continue
 3020 format(1h ,'parta',40x,'partb',40x,'partc')
      write(6,3020)
      write(6,1004)((parta(i,k),i=1,4),(partb(i,k),i=1,4),
     ?(partc(i,k),i=1,4),k=1,3)
 1004 format('',4f10.3,3x,4f10.3,3x,4f10.3)
 3021 format(1h ,'po4a',40x,'po4b',40x,'po4c')
      write(6,3021)
      write(6,1004)((po4a(i,k),i=1,4),(po4b(i,k),i=1,4),
     ?(po4c(i,k),i=1,4),k=1,3)
 3022 format(1h ,'availa',40x,'availb',40x,'availc')
       write(6,3022)
      write(6,1004)((availa(i,k),i=1,4),(availb(i,k),i=1,4),
     ?(availc(i,k),i=1,4),k=1,3)
 3023 format(1h 'php')
      write(6,3023)
      write(6,4001)((php(i,k),i=1,4),k=1,3)
      stop 'well done'
      end
```

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