A MULTIREGIONAL POPULATION PROJECTION FRAMEWORK THAT INCORPORATES BOTH MIGRATION AND RESIDENTIAL MOBILITY STREAMS: APPLICATION TO METROPOLITAN CITY-SUBURB REDISTRIBUTION

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The ways in which our society may have to adapt and respond to changes induced by energy shortages, environmental ceilings, and food insufficiencies has been the subject of much analysis and debate during the past decade. In all of this flurry of concern with perceived limits to growth, however, insufficient attention has been accorded to the effects of a variable that may overshadow all of the rest in importance: changing population dynamics and lifestyles, and their socioeconomic impacts.

Explosive population growth in the less developed countries and population stabilization in the more developed nations have created unprecedented social issues and problems. The future societal ramifications of changing age compositions, patterns of family formation and dissolution, movements from one region to another, health status and demands for care, and participation in the labor force will be profound.

Population projections provide an estimate of what some of these future changes might be. In this paper, William Frey extends the multiregional population projection model used at IIASA to include the dynamics of intraregional population redistribution, thus incorporating not only the migration of people between regions but also their movements within regions.

A list of related publications appears at the end of this paper.

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ABSTRACT

This paper introduces a population projection framework that incorporates both interregional migration and intraregional residential mobility streams to project future population sizes both across and within regions in a manner that is consistent with existing migration theory. The paper presents a general matrix model of the framework, shows how its parameters can be estimated from fixed interval census migration data, and discusses how the framework can be employed to "update" population projections when recent, more limited data sets become available. These features of the framework are demonstrated with intrametropolitan central city-suburb projections for selected US SMSAs over the period, 1970-2020.
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1. INTRODUCTION

This paper introduces a multiregional population projection framework that extends the existing methodology in order to project intraregional redistribution across community populations that are subject to change due to both interregional migration and intraregional residential mobility streams. It presents a general matrix model of the framework, indicates how the framework's rates and populations at-risk can be computed from fixed interval census or survey migration data, and shows how the framework can be employed to "update" population projections when recent, more limited data sets become available. The framework's capabilities are then illustrated with application to a specific intraregional redistribution context—central city-suburban redistribution within US metropolitan areas. Central city-suburban projections to the year 2020 are produced for three selected Standard Metropolitan Statistical Areas (SMSAs) based on 1970 US Census migration data and "updated" on the basis of subsequently available survey migration tabulations.

The framework presented here is predicated on the assumption that a multiregional projection methodology is of greatest value when the regions employed in the analysis reflect "origins" and
"destinations" that are consistent with the movement process itself. For example, previous research has shown that internal migration is motivated largely by economic considerations so that individual migrants and their families tend to be responsive to "pushes" and "pulls" of entire labor market areas (Lowry 1966; Lansing and Mueller 1967; Greenwood 1975, 1981). For this reason, nationwide labor market area regionalization schemes such as the Metropolitan Economic Labor Areas in the United Kingdom, the Bureau of Economic Analysis Areas in the United States and the sets of Functional Urban Regions that have recently been defined for many European countries (Hall and Hay 1980), constitute appropriate regional schemes for undertaking multiregional population projections in these countries, using the methodology specified by Rogers (1975), Willekens and Rogers (1978) and others. The interregional \(i \to j\) migration streams in these analyses will be consistent with the structure of internal migration processes. They will also facilitate more theoretically valid simulations and updates of the projections than would be possible if a more arbitrary regionalization scheme were employed.

The principle of defining regional schemes to be consistent with mobility processes underlies the projection framework presented here. This framework focuses on both inter- and intra-regional projections—that are generated by both migration and residential mobility streams. While the scholarly literature on population movement shows migration and residential mobility to be distinct from each other in many respects—individual motivation, frequency of occurrence, subgroup selectivity, etc. (Morrison 1972; Long 1973; Speare, Goldstein and Frey 1975; Goodman 1978)—they are also distinct in terms of geographic scope. Unlike migration which, by virtue of its job-relatedness, tends to occur over long distances and between labor markets, the term "residential mobility" is used to characterize mover adjustments to changing requirements for housing, neighborhood amenities, public services and other attributes of local communities that lie within each labor market area. This distinction
is made in the framework which treats interregional (or inter-labor market) movement as migration, and intraregional movement between communities within a single labor market as residential mobility. The latter communities are, therefore, subject to population change due to both interregional migration and intraregional residential mobility streams.¹

This framework extends the multiregional methodology advanced by Rogers (1975) and Willekens and Rogers (1978) by producing population projections for communities within labor market regions as well as across labor market regions through the introduction of a second "layer" of areas. Although it would be possible to generate community population projections with the existing methodology by simply extending the first "layer" of regions into more states, this practice would run counter to mobility literature which makes a clear distinction between migration and residential mobility components of community population change. The projection framework introduced here produces projections both across and within regions in a manner that is consistent with the underlying migration and residential mobility processes.

Four sections of this paper follow. Section 2 provides a nontechnical overview of the migration and residential mobility processes that underlie the projection framework using the example of city-suburb redistribution within a metropolitan area. Section 3 presents a detailed explanation of the projection methodology providing, first, equations that designate populations at risk and rates specific to the projection of intrametropolitan central city-suburban redistribution. This is followed by a matrix model specification for the general process of projecting populations within ℓ subregions of n regions and a discussion of rate computation and "updating" strategies. In section 4, the framework is applied to the projection of central city-suburban population change for three US SMSAs based on rates calculated from 1970 US Census migration data as well as to an update of these projections based on more current estimates for some of the rates from survey data. A brief conclusion follows as section 5.
2. INTRAREGIONAL REDISTRIBUTION: THE CASE OF A METROPOLITAN AREAS'S CENTRAL CITY AND SUBURBS

The migration and residential mobility processes that are incorporated into the projection framework advanced below can be portrayed for the case of central city-suburban redistribution in a single metropolitan area. Assuming that the metropolitan area of interest constitutes a self-contained labor market region within a nationwide system of labor market regions, movement-induced population change for the entire metropolitan area results from the two interregional migration streams:

I. out-migration from the metropolitan area to the rest of the country

II. in-migration to the metropolitan area from the rest of the country

where I actually pertains to the sum of interregional migration streams that lead from the metropolitan area to other labor markets in the country, and II actually pertains to the sum of those streams which lead from other labor market areas to the metropolitan area.

However, movement-induced population change for only the central city portion of the metropolitan area is the result of two interregional migration stream components:

IA. out-migration from the metropolitan area's central city to the rest of the country

IIA. in-migration to the metropolitan area's central city from the rest of the country

and two intraregional residential mobility streams:

III. intrametropolitan residential mobility from the central city to the suburbs

IV. intrametropolitan residential mobility from the suburbs to the central city

Comparable migration stream components IB and IIB (defined by replacing the term "suburbs" for "central city" in the IA and IIA stream definitions) in addition to residential mobility streams III and IV are, likewise, responsible for population change in the suburban (residual, noncentral) portion of the metropolitan area.
The utility of distinguishing the migration stream from the residential mobility stream components of intrametropolitan population change is clearly demonstrated in Table 1 which contrasts the experiences of three US SMSAs—Detroit, Atlanta, and Houston—that differ significantly in the levels of metropolitan-wide net in-migration sustained over the 1965-70 period. Here the 1965-70 net movement figures for their central cities and suburbs are decomposed into net movement attributable to interregional migration streams and net movement attributable to intraregional residential mobility streams.

The comparison points up the significance of the metropolitan area's migrant attractivity for redistribution across communities within the SMSA. While all three SMSAs sustain city-to-suburb population redistribution due to net residential mobility streams alone, this redistribution is countered in Atlanta and Houston by net migration gains in both central city and suburbs—associated by the strong metropolitan-wide migrant "pull" in these SMSAs. These data support the contention that entire labor market areas constitute appropriate "origins" and "destinations" for interregional migration streams, whereas smaller communities are more likely to serve these roles for local residential mobility streams.

It is useful to view the streams contributing to this redistribution process as occurring in a sequence of two analytically distinct stages. The first stage is named "the interregional exchange" stage and refers to the exchange of interregional migration streams between each pair of labor market areas in the nationwide system of regions. The second stage is named the "intraregional allocation" stage and refers to the cross-community residential mobility streams of the region's residents who were not attracted out of the region in the first stage, as well as the allocation of all in-migrants to the region (generated in the first stage) to common types of destinations within the region. From the perspective of a given metropolitan area, streams I (including IA and IB) and II as defined above, are the results of the interregional exchange.
Table 1. Contributions to central city, suburb and SMSA population change, 1965-70 attributable to net migration and net intrametropolitan residential mobility: Detroit, Atlanta, and Houston SMSAs.

<table>
<thead>
<tr>
<th>Population Size/ Components of Change</th>
<th>Detroit</th>
<th>Atlanta</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970 Population (in 1,000s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central City</td>
<td>1511</td>
<td>497</td>
<td>1231</td>
</tr>
<tr>
<td>Suburbs</td>
<td>2688</td>
<td>893</td>
<td>753</td>
</tr>
<tr>
<td>SMSA</td>
<td>4199</td>
<td>1390</td>
<td>1985</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Detroit</th>
<th>Atlanta</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Migration and Mobility</td>
<td>-12.6</td>
<td>-8.9</td>
<td>-0.7</td>
</tr>
<tr>
<td>Net Migration with Outside SMSA</td>
<td>-2.3</td>
<td>+1.5</td>
<td>+5.1</td>
</tr>
<tr>
<td>Net Mobility within SMSA</td>
<td>-10.3</td>
<td>-10.4</td>
<td>-5.8</td>
</tr>
</tbody>
</table>

^a pertains to internal migration only

SOURCE: 1970 US Census tabulations adjusted for "residence 5 years ago not known".
stage of the process, while streams III and IV, IIA and IIB results from the intraregional allocation stage of the process.

The two-stage process suggests that the streams of interregional in-migrants to communities that are located within a region, should be viewed as the result of both stages. In the case of in-migration to the metropolitan area's central cities and suburbs in streams IIA and IIB, it follows that

\[
\text{IIA} = \text{in-migration to the metropolitan area from the rest of the country (stage one)} \\
\times \text{city destination propensity rate of metropolitan area in-migrants (stage two)}
\]

and

\[
\text{IIB} = \text{in-migration to the metropolitan area from the rest of the country (stage one)} \\
\times \text{suburb destination propensity rate of metropolitan area in-migrants (stage two)}
\]

where the destination propensity rate, in this context\(^3\), indicates the proportion of the metropolitan area's in-migrants that locates in a specific community (central city or suburb) destination. This designation of the two stages is consistent with the premise that the entire region (metropolitan area) represents an appropriate labor market destination for interregional migrants but that within-region communities represent appropriate local destinations for interregional migrants.

The destination propensity rate can also be incorporated into the analysis of the residential mobility streams—although these streams are generated entirely within the second stage of the two stages outlined above. It is useful to view the stream rate of residential movement from community x to community y as the product of: (a) a mobility incidence rate—the proportion of community x's at-risk residents that move anywhere within the region (including within community x) and (2) a destination propensity rate—the proportion of community x-origin movers that locate in community y. This parametrization of the x to y stream rate is motivated by residential mobility decision making literature which suggests that "resident's
decision to move" and "mover's destination choice" are subject to different individual and areal determinants (Rossi 1955; Speare, Goldstein and Frey 1975). Moreover, redistribution analyses which have incorporated the above parametrization (Frey 1978a, 1978b, 1979b, 1980) indicate that the latter destination propensity rates tend to vary more widely across areas, and differently across individual characteristics (e.g., age) than do mobility incidence rates. Incorporating distinct movers' destination propensity rates into the second stage of the redistribution process permits local movers to be allocated to community destinations in the same manner as in-migrants to the region are so allocated.

The redistribution process that affects the metropolitan area example can now be stated as follows: the interregional exchange directs migration streams from the area's central city and suburb portions to other regions at the same time that migrant streams, originating in these regions, descend upon the area. The intraregional allocation stage then produces "pools" of local movers (as determined by each community's mobility incidence rates) and allocates these mover pools and metropolitan in-migrants to community (central city and suburb) destinations through appropriate destination propensity rates.

3. THE PROJECTION FRAMEWORK
3.1 Equations for Central City-Suburban Projections

The relationships that are composed of populations-at-risk and rates necessary to project future central city and suburb sizes, based on the redistribution process discussed in the previous section, will be presented here. We shall, first of all, specify the equations which are used to project the population of an entire metropolitan area (region) \( i \) when that metropolitan area is a part of a nationwide systems of regions \( j = 1, \ldots, n \). Given beginning-of-period \( (t) \) regional population sizes disaggregated by age categories: 0-4, 5-9, \ldots, 60-69, 70 and over, the following relationships compute the end of period \( (t+1) \) regional populations
\[
K_i^{(t+1)}(x+5) = s(x)K_i^{(t)}(x) - s(x)K_i^{(t)}(x) \left[ \sum_{j=1, j \neq i}^n m_{ij}(x) \right] \\
+ \sum_{i=1, j \neq i}^n s(x)K_j^{(t)}(x)m_{ji}(x) \quad (1)
\]

for end-of-period ages 5-9, 10-14, \ldots, 75 and over, and

\[
K_i^{(t+1)}(0) = \sum_{x=10}^{45} \left[ 2.5s(0) \left( f_i(x)K_i^{(t)}(x) \right. \right.
\\
+ f_i(x+5)K_i^{(t+1)}(x+5) \left. \right) \right] \quad (2)
\]

for end-of-period ages 0-4;

where

- \( K_j^{(t)}(x) = \text{total population of region } j \) (\( j = 1, \ldots, n \text{ where one value of } j = 1 \)), ages \( x \) to \( x+4 \) at time \( t \)
- \( m_{ij}(x) = \text{interregional migration rate} \) (proportion of residents of region \( i \), ages \( x \) to \( x+4 \) at time \( t \), and surviving to \( t+1 \), that resides in region \( j \) at time \( t+1 \))
- \( s(x) = \text{survival rate} \) (proportion of the population ages \( x \) to \( x+4 \) at time \( t \), that is alive at time \( t+1 \))
- \( s(0) = \text{survival rate of births} \) (proportion of persons born between time \( t \) and \( t+1 \) that survives to age 0-4 at time \( t+1 \))
- \( f_i(x) = \text{fertility rate} \) (the average annual number of births born to persons age \( x \) to \( x+4 \) in region \( i \))

Equation (1) indicates that the end-of-period metropolitan area \( i \) population for age categories equal to or greater than the period length (5 years) are equivalent to the beginning-of-period population reduced by the sum of all out-migration streams to other regions in the system augmented by the sum of all in-migration streams from other regions in the system. All beginning-of-period migrant and non-migrant populations are "survived" to the end-of-period with age-specific survival rates which, for
convenience of exposition, are assumed constant across regions of migrant categories. The end-of-period metropolitan area i population, as specified in equation (2), is calculated from a knowledge of the beginning and end period populations in the childbearing ages, age-specific fertility rates for metropolitan area i, and the survival rate of births.

The projection equations (1) and (2) are consistent with multiregional cohort component projection systems advanced previously (Rogers 1975; Rees and Wilson 1977; Willekens and Rogers 1978). Given initial population sizes for all regional populations by 5-year age categories, and values for the rates \( m_{ij}(x), s(x) \) and \( f_i(x) \), equations (1) and (2) can be employed to project population sizes for metropolitan area i (or any other region j in the system) over as many periods as is desired.

The extension of this methodology to project intrametropolitan (intraregional) redistribution across the central city and suburb subregions of a metropolitan area (region) i makes use of equations (3), (4), (5), and (6). Equations (3) and (4) are subregional analogs of equation (1) and compute end-of-period \((t+1)\) city and suburb population sizes of age categories: 5-9, 10-14, ..., 75 and over. Likewise, equations (5) and (6) are subregional analogs of equation (2) and compute end-of-period city and suburb population sizes for the 0-4 age category:

\[
K_{i.c}^{(t+1)}(x+5) = s(x)K_{i.c}^{(t)}(x) - s(x)K_{i.c}^{(t)}(x)m_{i.co}(x)
- s(x)\left[K_{i.c}^{(t)}(x) - K_{i.c}^{(t)}(x)m_{i.co}(x)\right]i.c(x)p_{i.cs}(x)
+ s(x)\left[K_{i.s}^{(t)}(x) - K_{i.s}^{(t)}(x)m_{i.so}(x)\right]i.s(x)p_{i.sc}(x)
+ s(x)K_{i.o}^{(t)}(x)p_{i.oc}(x)
\]

\[
K_{i.s}^{(t+1)}(x+5) = s(x)K_{i.s}^{(t)}(x) - s(x)K_{i.s}^{(t)}(x)m_{i.so}(x)
- s(x)\left[K_{i.s}^{(t)}(x) - K_{i.s}^{(t)}(x)m_{i.so}(x)\right]i.s(x)p_{i.sc}(x)
\]
where \( s(x) \), \( s(0) \), and \( f_i(0) \) are defined as above and

\[
K_{i.c}(t+1)(0) = \sum_{x=10}^{45} \left\{ 2.5s(0) \left[ f_i(x)K_{i.c}(x) + f_i(x+5)K_{i.c}^{(t+1)}(x+5) \right] \right\}
\]

\[
K_{i.s}(t+1)(0) = \sum_{x=10}^{45} \left\{ 2.5s(0) \left[ f_i(x)K_{i.s}(x) + f_i(x+5)K_{i.s}^{(t+1)}(x+5) \right] \right\}
\]

where \( s(x) \), \( s(0) \), and \( f_i(0) \) are defined as above and

\[
K_{i.c}(t)(x) = \text{city population within metropolitan area } i, \text{ age } x \text{ to } x+4 \text{ at time } t
\]

\[
K_{i.s}(t)(x) = \text{suburb population within metropolitan area } i, \text{ age } x \text{ to } x+4 \text{ at time } t
\]

\[
m_{i.co}(x) = \text{out-migration rate for city residents} \text{ (proportion of city residents of metropolitan area } i, \text{ ages } x \text{ to } x+4 \text{ at time } t, \text{ and surviving to time } t+1, \text{ that resides outside of metropolitan area } i \text{ at time } t+1)
\]

\[
m_{i.so}(x) = \text{out-migration rate for suburb residents} \text{ (proportion of suburb residents of metropolitan area } i, \text{ ages } x \text{ to } x+4 \text{ at time } t, \text{ and surviving to time } t+1, \text{ that resides outside of metropolitan area } i \text{ at time } t+1)
\]

\[
s(x)K_{i.o}(t)(x) = \text{surviving in-migrants to metropolitan area } i \text{ (sum of all residents outside of metropolitan area } i, \text{ ages } x \text{ to } x+4 \text{ at time } t, \text{ that survives and resides in metropolitan area } i \text{ at time } t+1)
\]

\[
i_{i.c}(x) = \text{mobility incidence rate for non-migrating city residents} \text{ (proportion of city residents of metropolitan area } i, \text{ ages } x \text{ to } x+4 \text{ at time } t, \text{ surviving to time } t+1 \text{ and not migrating out of the metropolitan area, that resides in a different dwelling unit in metropolitan area } i \text{ at time } t+1)
\]
\( i_{i.s}(x) = \) mobility incidence rate for non-migrating suburb residents (proportion of suburb residents of metropolitan area \( i \), ages \( x \) to \( x+4 \) at time \( t \), surviving to time \( t+1 \) and not migrating out of the metropolitan area, that resides in a different dwelling unit in metropolitan area \( i \), at time \( t+1 \) )

\( p_{i.cs}(x) = \) suburb destination propensity rate for city-origin movers (proportion of city residents of metropolitan area \( i \), ages \( x \) to \( x+4 \) at time \( t \), surviving and residing in a different metropolitan area \( i \) dwelling unit at time \( t+1 \), that resides in the suburbs at time \( t+1 \) )

\( p_{i.sc}(x) = \) city destination propensity rate for suburb origin movers (proportion of suburb residents of metropolitan area \( i \), ages \( x \) to \( x+4 \) at time \( t \), surviving and residing in a different metropolitan area \( i \) dwelling unit at time \( t+1 \), that resides in the city at time \( t+1 \) )

\( p_{i.oc}(x) = \) city destination propensity rate for in-migrants to the metropolitan area (proportion of in-migrants to the metropolitan area \( i \), ages \( x \) to \( x+4 \) at time \( t \), and surviving \( t \) time \( t+1 \), that resides in the city at time \( t+1 \) )

\( p_{i.os}(x) = \) suburb destination propensity rate for in-migrants to the metropolitan area (proportion of in-migrants to the metropolitan area \( i \), ages \( x \) to \( x+4 \) at time \( t \), and surviving to time \( t+1 \), that resides in the suburbs at time \( t+1 \) )

Equation (3) indicates that the end-of-period city population is equal to the survived beginning-of-period city population reduced by out-migrants and city-to-suburb residential movers, and augmented by suburb-to-city residential movers and in-migrants to the SMSA. Similarly, equation (4) indicates the end-of-period suburb population is equal to the survived beginning-of-period suburb population after out-migrants and suburb-to-city movers are removed, and after city-to-suburb movers and SMSA in-migrants are added.

The populations-at-risk and rates can be looked upon in light of the two-stage redistribution process reviewed in the previous section. The "interregional exchange" involves applying out-migration rates \( m_{i.co} \) and \( m_{i.so} \) to the beginning-of-period
city and suburb populations, respectively, in order to produce out-migration streams from the city and suburbs to other regions while in-migration from other regions is represented by the parameter $s(x)K_{i,o}^{(t)}(x)$. In the second "intraregional allocation" stage of the redistribution process, two pools of local residential movers are produced by applying rates of mobility incidence ($i_{i,c}$ and $i_{i,s}$) to those city and suburb residents that did not migrate out of the metropolitan area. To each of these pools (designated as $s(x)\left[K_{i,c}^{(t)}(x) - K_{i,c}^{(t)}(x)m_{i,co}(x)\right]i_{i,c}(x)$ and $s(x)\left[K_{i,s}^{(t)}(x) - K_{i,s}^{(t)}(x)m_{i,so}(x)\right]i_{i,s}(x)$, respectively), and to the surviving in-migrants to the SMSA, appropriate destination propensity rates are applied [$p_{i,cs}(x), p_{i,sc}(x), p_{i,co}(x)$, $p_{i,so}(x)$] in order to allocate these movers and migrants to central city and suburb destinations.

Relationships (3) and (4) indicate how the two-stage redistribution process affects central city and suburb change within metropolitan area $i$. The "interregional exchange" also involves linking migration streams into and out of metropolitan area $i$ with other regions in the multiregional system. The linkage between equations (3) and (4) and the standard multiregional projection equation [(1) above] which incorporates interregional migration streams $m_{ij}(x)$, is made through equations (7) and (8):

$$s(x)K_{i,o}^{(t)}(x) = \sum_{j=1}^{n} s(x)K_{j}^{(t)}m_{ji}(x) \quad (7)$$

$$m_{i,co}(x) = m_{i,so}(x) = \sum_{j=1}^{n} m_{ij}(x) \quad (8)$$

Equation (7) indicates that the term $s(x)K_{i,o}^{(t)}(x)$ in equations (3) and (4) is equivalent to the final term in equation (1)—the survived sum of in-migration streams from all other regions in the system. Equation (8) makes the assumption that age-specific metropolitan out-migration rates for both city and suburb residents are equivalent to metropolitan-wide out-migration rates.
This assumption is consistent with the view that the metropolitan area rather than the city or suburb represents the appropriate "origin" for interregional migration. The assumption made in relationship (8) also reduces the complexity of the data that are required to estimate the various in- and out-migration rates (to be discussed below).

Additional note should be taken of the conditionalities associated with intrametropolitan residential mobility in equations (3) and (4). As specified, mobility incidence rates, \( i_{i.c} \) and \( i_{i.s} \), are conditional on not migrating out of the metropolitan area during the period. Because only one movement transition can be recorded over the period, it is assumed that a residential move is not substitutable for a migratory move. Hence, an individual is only "at-risk" to move locally if an interregional migration is not undertaken. This assumption also simplifies the data requirements for estimation, as will be discussed below.

The foregoing equations (1) through (8) constitute the methodology for projecting city-suburb redistribution within a single metropolitan area that is part of a nationwide system of regions. Given initial population sizes for the metropolitan area's city and suburbs (in addition to those for other regions in the system) by 5-year age categories, and given values for the rates \( i_{i.c}(x) \), \( i_{i.s}(x) \), \( p_{i.cs}(x) \), \( p_{i.sc}(x) \), \( p_{i.os}(x) \) [in addition to those for rates \( m_{i,j}(x) \), \( s(x) \), and \( s(0) \)], these equations can be employed to project metropolitan area i city and suburb population sizes over as many periods as desired. The above specification follows from the two-stage redistribution process discussed in the previous section of the paper, and is consistent with the conventional interregional population projection methodology [as designated in equations (1) and (2) only] if relationships (7) and (8) can be assumed.
3.2 General Matrix Model of the Projection Framework

The above set of relationships can be specified in a matrix model of the projection framework that is general to \(L\) subregions within \(n\) regions. If one begins with

\[
\{ \bar{K}^t(x) \} = \begin{bmatrix}
\{ \bar{K}_1^t(x) \} \\
\{ \bar{K}_2^t(x) \} \\
\vdots \\
\{ \bar{K}_n^t(x) \}
\end{bmatrix}
\]

and

\[
\{ \bar{K}_i^t(x) \} = \begin{bmatrix}
K_{i.1}^t(x) \\
K_{i.2}^t(x) \\
\vdots \\
K_{i.\ell}^t(x)
\end{bmatrix}
\]

where

\[
\{ \bar{K}^t(x) \} = \text{column vector of population totals for } n \text{ regions and their subregions, for ages } x \text{ to } x+4
\]

\[
\{ \bar{K}_i^t(x) \} = \text{column vector of subregional populations of region } i, \text{ for ages } x \text{ to } x+4 \text{ with elements } K_{i.a}^t(x) \text{ (where } a = 1, \ldots, \ell) \text{ and } K_{i.0}^t(x)
\]

\[
K_{i.a}^t(x) = \text{population of region } i, \text{ subregion } a, \text{ ages } x \text{ to } x+4 \text{ at time } t
\]

\[
K_{i.0}^t(x) = \text{in-migrants to region } i \text{ between time } t \text{ and } t+1, \text{ ages } x \text{ to } x+4 \text{ at time } t \text{ (initially assigned a 0 value in the projection process)}
\]

then the equation projecting end-of-period populations from beginning-of-period populations in age classes 0-4, 5-9, ..., 70 and over is
\[(\bar{X}(t+1)(x+5)) = [p(x)i(x) + [I - i(x)]][\bar{m}(x)s(x)]\bar{X}(t)(x) \] (9)

where

\[s(x) = \text{survival rate expressed in scalar form}\]

\[\bar{m}(x) = (l+1)n \times (l+1)n \text{ matrix of interregional migration rates (in terms of rates } m_{ij} \text{ as illustrated below)}\]

\[i(x) = (l+1)n \times (l+1)n \text{ matrix of intraregional mobility incidence rates [in terms of the rates } i_{ia}(x) \text{ as illustrated below]}\]

\[p(x) = (l+1)n \times (l+1)n \text{ matrix of destination propensity rates for intraregional movers and interregional in-migrants [in terms of rates } p_{i.ab}(x) \text{ and rates } p_{i.ob}(x) \text{ as illustrated below]}\]

\[I = (l+1)n \times (l+1)n \text{ identity matrix with 1 in each diagonal element, 0 in all other elements}\]

When it is assumed that \(n = 2\) regions, each with \(l = 2\) sub-regions, the elements of \(\bar{m}(x), i(x),\) and \(p(x)\) can be specified as:

\[
\bar{m}(x) = \begin{bmatrix}
1 & \sum_{j \neq 1} m_{1j}(x) & 0 & 0 & 0 \\
0 & 1 - \sum_{j \neq 1} m_{1j}(x) & 0 & 0 & 0 \\
0 & 0 & m_{21}(x) & m_{21}(x) & 0 \\
0 & 0 & 0 & 1 - \sum_{j \neq 2} m_{2j}(x) & 0 \\
m_{12}(x) & m_{12}(x) & 0 & 0 & 0
\end{bmatrix}
\]

where

\[m_{ij}(x) = \text{interregional migration rates (proportion of residents in region } i, \text{ ages } x \text{ to } x+4 \text{ at time } t \text{ and surviving to time } t+1, \text{ that resides in region } j \text{ at time } t+1)\]
\[ i(x) = \begin{bmatrix}
  i_{1.1}(x) & 0 & 0 & 0 \\
  0 & i_{1.2}(x) & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & i_{2.1}(x) \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & i_{2.2}(x) \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

where

\[ i_{1.a}(x) = \text{mobility incidence rate for subregion a residents} \]
(proportion of residents of region \( i \) and sub-region \( a \), ages \( x \) to \( x+4 \) at time \( t \), surviving to time \( t+1 \) and not migrating out of the region, that resides in a different dwelling unit in region \( i \) at time \( t+1 \))

and

\[ p(x) = \begin{bmatrix}
  1 - p_{1.12}(x) & p_{1.21}(x) & p_{1.02}(x) & 0 & 0 & 0 \\
  p_{1.12}(x) & 1 - p_{1.21}(x) & p_{1.02}(x) & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 - p_{2.12}(x) & p_{2.21}(x) & p_{2.02}(x) \\
  0 & 0 & 0 & p_{2.12}(x) & 1 - p_{2.21}(x) & p_{2.02}(x) \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

where

\[ p_{i.ab}(x) = \text{destination propensity rate for subregion a origin movers} \]
(proportion of residents of region \( i \) and sub-region \( a \), ages \( x \) to \( x+4 \) at time \( t \), surviving and residing in a different region \( i \) dwelling unit at time \( t+1 \), that resides in sub-region \( b \) at time \( t+1 \))
\( p_{i,ob}(x) = \) destination propensity rate for in-migrants to region \( i \) (proportion of in-migrants to region \( i \), ages \( x \) to \( x+4 \) at time \( t \) and surviving to time \( t+1 \), that resides in subregion \( b \) at time \( t+1 \))

Equation (9) can now be viewed in terms of the two-stage redistribution process discussed earlier. The "interregional exchange" stage of the process is represented by the factor, \( \bar{m}(x)s(x) \), which redistributes migrants from one region to another. The "intraregional allocation" stage can be viewed as the sum of two factors: \( [I - \bar{r}(x)] \) which identifies subregional residents that do not undertake a residential move and reside in the same dwelling unit at the end of the period; and \( p(x)\bar{r}(x) \) which both identifies residential movers among the subregional population and redistributes those movers as well as regional in-migrants to subregional destinations at the end of the period. This specification of the destination propensity rate matrix \([p(x)]\) treats the allocation to subregions of residential movers and regional in-migrants as like processes and is consistent with the view that these mover and migrant groups are influenced by the same subareal attractions in their "choice of destination" within the region.

The second of two relationships which comprise the projection process projects end-of-period population totals for the 0-4 age class:

\[
\begin{align*}
\bar{R}(t+1)(0) &= \sum_{x=10}^{45} 2.5s(0)\left[ \bar{F}(x)\bar{R}(t)(x) + \bar{F}(x)\bar{R}(t+1)(x+5) \right] \\
\end{align*}
\]

(10)

where

\( s(0) = \) survival rate of births expressed in scalar terms [as in equations (2), (5), and (6)]

\( \bar{F}(x) = (k+1)n \times (k+1)n \) matrix of fertility rates [specified below in terms of elements \( f_i(x) \)]

When it is assumed that the subregions of each region will exhibit the same fertility rates as the region, the \( \bar{F}(x) \) matrix for an illustrative \( n = 2 \) region model is specified as follows:
where

\[ f_i(x) = \textit{fertility rate} \ (\text{the average annual number of births to persons age } x \text{ to } x+4 \text{ in region } i) \]

The reader should note that while the framework outlined in relationships (9) and (10) can handle up to \( \ell \) subregions within each region, the number of subregions can vary across regions and there need not be any subregions in one or more regions. In the former instance, only relevant subareas should be given initial year (\( t = 1 \)) population sizes in submatrix \( \{ \tilde{K}_i(t)(x) \} \) for the region, with all other \( \tilde{K}_i(x) \) elements given a 0 value. In the latter instance, the total region's initial year population should be inserted in the \( K_i(0)(x) \) element, with all other elements given a 0 value. For both instances, appropriate changes need to be made within the \( \tilde{m}(x) \), \( \tilde{p}(x) \) and \( \tilde{i}(x) \) matrices. Taken together, relationships (9) and (10) constitute a more general model of the two-stage inter- and intraregional projection process than was specified for the particular example of intrametropolitan city-suburban redistribution earlier in this section. Because the end-of-period matrix \( \tilde{K}(t+1)(x) \) for ages 5-9, 10-14, ..., represents the beginning-of-period matrix \( \tilde{K}(t)(x) \) for the subsequent projection period, these relationships can produce projected population sizes for \( \ell \) subregions within \( n \) regions for any desired number of periods.
3.3 Rate Calculation and Data Considerations

An important feature of the two-stage projection process is its relatively parsimonious data requirements for estimation of mobility rates. If the conventional "single stage" multi-regional methodology were adapted to accommodate projections of \( l \) subregions within \( n \) regions, the number of new "regions" would simply be expanded to \( ln \) and it would be necessary to compile a nationwide origin-destination matrix of \( ln \times ln \) movement flows in order to estimate the projection framework's movement rates.

The two-stage model requires only a nationwide origin-destination matrix of \( n \times n \) flows, and an \( \ell \times \ell \) origin-destination matrix for each region (or for those regions where a subregion projection is desired). In a nation of 5 regions with 2 subregions each, the former methodology would require a \( 10 \times 10 \) nationwide flow matrix, while the latter methodology would require a \( 5 \times 5 \) nationwide matrix and a \( 2 \times 2 \) matrix for each of the 5 subregions. The latter, more compact nationwide flow matrix is advantageous for rate estimation because it is likely to yield far fewer sparsely populated flows than would be the case with the full-scale nationwide subregion to subregion matrix.

The basic migration and mobility parameters that are required for matrix relationship (9) [or for equations (1), (3), (4), (7), and (8) in the specific city-suburb example] are: \( m_{ij} \) for origin and destination regions \( i \) and \( j = 1,2,\ldots,n; \ i_{i.a}, \Pi_{ab}, \) and \( \Pi_{i.ob} \) for up to \( a \) and \( b = 1,2,\ldots,\ell \) subregions within one or more of the \( n \) regions. Assuming that the period \( t \) to \( t+1 \) is equal to the age category interval (5 years in this case), all of these rates can be estimated from the following fixed interval migration tabulations that are available from a census:

**Tabulation A.** Nationwide population aged five and above, cross tabulated by region of residence, region of residence 5 years ago, and 5-year age groups

**Tabulation B.** Regional population (for each region of interest), aged five and above, cross tabulated by residence in same or different
The rates are computed as follows:

\[ m_{ij}(x) = \frac{\text{Region } j \text{ residents, aged } x+5 \text{ to } x+9 \text{ at census, who resided in region } i, 5 \text{ years ago}}{\text{All national residents aged } x+5 \text{ to } x+9 \text{ at census, who resided in region } i, 5 \text{ years ago}} \]

\[ i_{i.a}(x) = \frac{\text{All region } i \text{ residents, aged } x+5 \text{ to } x+9 \text{ at census, who lived in a different dwelling unit located in subregion } a \text{ of that region, 5 years ago}}{\text{All region } i \text{ residents, aged } x+5 \text{ to } x+9 \text{ at census, who resided in the same or different dwelling unit in subregion } a \text{ of that region, 5 years ago}} \]

\[ i_{i.a}(x) = \frac{\text{Subregion } b, \text{ region } i \text{ residents, aged } x+5 \text{ to } x+9 \text{ at census, who lived in a different dwelling unit located in subregion } a \text{ of that region, 5 years ago}}{\text{All region } i \text{ residents, aged } x+5 \text{ to } x+9 \text{ at census, who lived in a different dwelling unit located outside the region } i, 5 \text{ years ago}} \]

The survival and fertility parameters \( s(x) \) and \( f_{i}(x) \) required for matrix relationships (9) and (10) [or equations (2), (5), and (6) in the specific city-suburb example] can be computed in a more straightforward fashion with available vital statistics data and census tabulations, using standard techniques (Shryock and Siegel 1971; Rogers 1975).

Notice that only the nationwide Tabulation A is necessary to compute the \( m_{ij}(x) \) interregional migration rates needed to construct matrix \( m_{ij}(x) \) in equation (9). Only region-specific
Tabulations B are necessary to compute the incidence rates $i_{i.a}(x)$ and propensity rates $p_{i.ab}(x)$ and $p_{i.ob}(x)$ needed for matrix $i(x)$ and $p(x)$. It should now be clear why movement rate estimation becomes simplified when it is assumed that (1) all subregional residents in a given region exhibit the same age-specific out-migration rates [as in equation (8) in section 3.1, or in $\tilde{m}(x)$ in section 3.2]; and (2) intraregional mobility incidence rates are conditional on not migrating out of the region [as defined in equations (3) and (4) in section 3.1; and in matrices $\tilde{i}(x)$ of section 3.2]. If assumption (1) were not made, then it would be necessary to tabulate a nationwide $\ell n \times n$ origin-destination migration matrix to compute all $m_{ij}(x)$. Likewise, if assumption (2) were not made, the same matrix—in addition to Tabulation B—would be necessary to compute all $i_{i.b}(x)$.

An important feature of this projection framework is its capability to produce "updated" projections when current, but limited, data become available. For example, assume that equations (9) and (10) were employed to produce intra- and interregional projections on the basis of fixed interval migration Tabulations A and B that were available with the past census. Several years after the census is taken, a comprehensive survey of residents in one region $i$ becomes available, which includes appropriate information to compile a current Tabulation B. This allows the researcher to produce an "updated" projection of subregions within region $i$ based on the same interregional migration, fertility, and mortality parameters [$\tilde{m}(x)$, $s(x)$, $f_i(x)$] as the last projections, but based on more current intra-regional allocation parameters for region $i$ [$i_{i.a}(x)$, $p_{i.a}(x)$, $p_{i.ob}(x)$].

In this vein, it should be noted from above that the destination propensity rates, $p_{i.ab}(x)$ and $p_{i.ob}(x)$ needed for the $p(x)$ matrix in equation (9) can be computed from a survey of a region's movers. Thus, the availability of a current survey of movers provides the capability of updating past projections if one is willing to assume that the previous $i_{i.a}(x)$ rates, in
addition to the previous $m_{ij}(x)$, $s(x)$ and $f_i(x)$ rates, hold for
the current update. Because age-specific incidence rates tend
to vary less across time and space than destination propensity
rates and because the latter are directly linked to the intra-
regional mover and migrant allocation process (Frey 1978a, 1979a),
an updating of intraregional projections on the basis of current
destination propensity rates constitutes an inexpensive means
of compiling timely projections between censuses.

4. APPLICATION TO THREE US METROPOLITAN AREAS
4.1 Baseline Projections from 1970 Census Data

The projection framework outlined in the previous section
will be employed to project intrametropolitan central city-
suburban redistribution for three large SMSAs—Detroit, Atlanta,
and Houston. The largest US SMSAs are generally recognized to
be self-contained labor market regions, and have been included
as such in both the Bureau of Economic Analysis and State Economic
Area regionalization schemes. The three SMSAs selected for
this application display distinctly different core-periphery
and metropolitan-wide population change patterns over the base
period for the projection (1965-70). Detroit represents a
declining industrial metropolis that has sustained considerable
city loss and core-periphery decentralization; Atlanta is a
growing SMSA, although also undergoing a significant intrametro-
politan city-suburb redistribution; Houston, growing faster than
Atlanta or Detroit, registers moderate growth in its central
city as a consequence of a much less pronounced decentralization
process.

For simplicity of exposition, the inter- and intraregional
projections to be undertaken for each SMSA will be based on a
simple two-region system where one region consists of the SMSA
of interest, and the other region consists of the "rest of the
US". The intraregional projection will then occur within the
SMSA region—across the central city and suburban "subregions"
of the SMSA. This simplified regional system therefore requires
that a separate projection analysis be undertaken for each SMSA.
(A more elaborate analysis would include all national labor market areas—including the three SMSAs—in the regional scheme, and would require only one projection analysis.) The projection process is consistent with equations (1) through (8) which are tailored to the specific case of city-suburb redistribution where there are \( n = 2 \) regions such that \( i = 1 \) for the SMSA of interest and \( i = 2 \) for the rest of the country. Alternatively, the more general specification in relationships (9) and (10) also apply where \( n = 2 \) and \( \ell = 2 \) in region 1, such that \( a \) and \( b \) can take values \( c \) or \( s \) (for central city or suburbs) in the SMSA of interest.

Appropriate fixed interval migration data are available from special tabulations from the 1970 US census and from the US Bureau of the Census (1973). These data make it possible to derive Tabulation A in order to compute the interregional exchange rates \([m_{12}(x) \text{ and } m_{21}(x)]\); and Tabulation B in order to compute the intraregional allocation rates \([i_{i,c}(x), i_{i,s}(x), p_{i,cs}(x), p_{i,sc}(x), p_{i,oc}(x), p_{i,os}(x)]\). The census tabulations were adjusted for mover's unknown residence 5 years prior to the census by allocating "unknowns" to locations of individuals with similar race, age, and socioeconomic characteristics. The tabulations were also adjusted for census underenumeration using measures developed by the US Bureau of the Census (1977a). The 1965-70 migration and residential mobility parameters for the Detroit SMSA are shown in Table 2. In these projections, nationwide age-specific survival rates and nationwide age-specific fertility rates are assumed to hold for all regions and periods \([s(x), f_1(x)]\). The former were compiled from the US Department of Health Education and Welfare (1975) and the latter were taken from the US Bureau of the Census (1977a).

Table 3 displays total (age-aggregated) rates associated with "the interregional exchange" and "intraregional allocation" redistribution stages for each SMSA. These make clear that in the exchange with other regions, Detroit fares less well than either Atlanta or Houston—by suffering a net out-migration to the rest of the country. In the intraregional allocation stage,
Table 2. Migration and residential mobility parameters for Detroit SMSA, based on 1965-70 period.

<table>
<thead>
<tr>
<th>Age Category at Beginning of Period (x to x+1)</th>
<th>SMSA Out-migration Rate</th>
<th>Surviving SMSA In-Migrants</th>
<th>Mobility Incidence Rates of:</th>
<th>Destination Propensity Rates of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n ( \frac{m_{11}(x)}{j=1, j\neq 1} )</td>
<td>n ( \frac{k(x)}{j=1, j\neq 1} ) ( m_{1j}(x) )</td>
<td>( i_{1c}(x) )</td>
<td>( p_{1cs}(x) )</td>
</tr>
<tr>
<td>0-4</td>
<td>.1054</td>
<td>45888</td>
<td>.5910</td>
<td>.3165</td>
</tr>
<tr>
<td>5-9</td>
<td>.0820</td>
<td>31505</td>
<td>.4749</td>
<td>.2956</td>
</tr>
<tr>
<td>10-14</td>
<td>.1264</td>
<td>24915</td>
<td>.4394</td>
<td>.2780</td>
</tr>
<tr>
<td>15-19</td>
<td>.2215</td>
<td>54233</td>
<td>.6509</td>
<td>.2888</td>
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<td>20-24</td>
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<td>61445</td>
<td>.7713</td>
<td>.3808</td>
</tr>
<tr>
<td>25-29</td>
<td>.1267</td>
<td>31351</td>
<td>.6644</td>
<td>.3680</td>
</tr>
<tr>
<td>30-34</td>
<td>.0878</td>
<td>20542</td>
<td>.5372</td>
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<td>35-39</td>
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<td>60-64</td>
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<td>65-69</td>
<td>.0904</td>
<td>2728</td>
<td>.2761</td>
<td>.3060</td>
</tr>
<tr>
<td>70+</td>
<td>.0874</td>
<td>6043</td>
<td>.3084</td>
<td>.3683</td>
</tr>
</tbody>
</table>

SOURCE: 1970 US Census tabulations adjusted for "residence 5 years ago not known" and census underenumeration.
Table 3. Migration and residential mobility parameters for the total populations: Detroit, Houston, and Atlanta SMSAs, based on 1965-70 period.

<table>
<thead>
<tr>
<th></th>
<th>Detroit</th>
<th>Atlanta</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interregional Exchange Stage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMSA out-migration rate</td>
<td>.1055</td>
<td>.1583</td>
<td>.1334</td>
</tr>
<tr>
<td>Surviving in-migrants to SMSA (100s) &amp; (as a percentage of initial population)</td>
<td>3279 (0.0823)</td>
<td>2769 (0.2300)</td>
<td>3574 (0.2105)</td>
</tr>
<tr>
<td><strong>Intraregional Allocation Stage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mobility incidence rate for city residents</td>
<td>.4677</td>
<td>.5305</td>
<td>.4937</td>
</tr>
<tr>
<td>Mobility incidence rate for suburb residents</td>
<td>.3229</td>
<td>.4143</td>
<td>.3625</td>
</tr>
<tr>
<td>Suburb destination propensity rate for city-origin movers</td>
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<td>.3512</td>
<td>.2310</td>
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<tr>
<td>City destination propensity rate for suburb origin movers</td>
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<tr>
<td>City destination propensity rate for SMSA in-migrants</td>
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<td>Suburb destination propensity rate for SMSA in-migrants</td>
<td>.6519</td>
<td>.7244</td>
<td>.3966</td>
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</table>

**SOURCE:** 1970 US Census tabulations adjusted for "residence 5 years ago not known" and census underenumeration.
however, Detroit and Atlanta are most alike. While mobility incidence rates are fairly similar for all three SMSAs, it is clear that the Detroit and Atlanta destination propensity rates will bring about a greater city-to-suburb allocation of movers and in-migrants within those SMSAs than will be the case in Houston.

The results of the projection process for each SMSA are shown in Table 4. The projections for individual SMSA population sizes are consistent with the interregional exchange stage rates that generate the projections. Detroit's SMSA population grew the least—34 percent over the 50-year period; while Atlanta and Houston increased their 1970 populations by 109 and 115 percent, respectively.

With respect to intrametropolitan redistribution, the data in Table 4 show Detroit's share of the SMSA population to decrease from 37 percent to 24 percent over the 50-year period; and to sustain a projected absolute decline of 11 percent of its 1970 population. Atlanta's central city share of the SMSA population undergoes a decrease of similar magnitude—36 percent to 25 percent, but manages to enjoy a projected population gain of 43 percent of its 1970 size. The projected city-suburban decentralization process is much less accentuated in the Houston SMSA. Here, the central city retains the majority share of the SMSA's population throughout the projection interval—declining slightly from 62 percent to 52 percent. The city's projected population gain over the period is 79 percent of the 1970 population size.

Table 5 provides insights into how the migration, residential mobility, and natural increase components of change contribute to each SMSA's city-suburb redistribution process over the 50-year projection period. The data parallel those presented for the base period in Table 1. Again, each SMSA undergoes a significant projected city-to-suburb redistribution as a result of the intrametropolitan residential mobility streams. However, this redistribution is "cushioned" in Atlanta and Houston as a result of net in-migration to the SMSA as a whole—and to both city and suburb subregions. The data show clearly that the
<table>
<thead>
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<tr>
<td>Total</td>
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<td>4899</td>
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<td>Population Share</td>
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SOURCE: Projection equations (1) through (8) in text; with all input populations and rates from 1970 US Census tabulations adjusted for "residence 5 years ago not known" and census underenumeration.
Table 5. Contributions to projected central city, suburb, and SMSA population change, 1970-2020 attributable to natural increase, net migration, and net intrametropolitan residential mobility: Detroit, Atlanta, and Houston SMSAs.

<table>
<thead>
<tr>
<th>Projected Population Size (in 1,000s)</th>
<th>Detroit</th>
<th>Atlanta</th>
<th>Houston</th>
</tr>
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<tr>
<td>Central City</td>
<td>Suburbs</td>
<td>SMSA</td>
<td>Central City</td>
</tr>
<tr>
<td>Projected 2020 Population Size</td>
<td>1407</td>
<td>4390</td>
<td>5797</td>
</tr>
</tbody>
</table>


- **Natural Increase**: +43.5, +38.3, +39.6, +46.7, +41.4, +42.7, +43.8, +36.0, +40.1
- **Net Migration and Mobility**: -56.0, -0.9, -14.2, -16.6, +18.0, +9.4, +0.2, +27.4, +13.4
- **Net Migration with Outside SMSA**: +4.2, -20.2, -14.2, +10.7, +8.9, +9.4, +25.2, +0.5, +13.4
- **Net Mobility within SMSA**: -60.2, +19.3, —, -27.3, +9.1, —, -25.0, +26.9, —

The contribution to 1970-2020 population change attributable to each component (i.e., natural increase, net migration, net mobility) is calculated by summing that component's contribution to population change during each period, 1970-1975, ..., 2015-2020, over the 10 5-year periods of the projection span. This sum of period contribution is then expressed as a percentage of the projected 2020 population size of the appropriate area (i.e., central city, suburb, SMSA).

SOURCE: Projection equations (1) through (8) in text; with all input populations and rates from US Census tabulations adjusted for "residence 5 years ago not known" and census underenumeration.
prospects of long-term population gains for all subregions in a labor market area are enhanced when the labor market, as a whole, sustains a constant net in-migration vis-à-vis other labor markets.

4.2 "Updating" the Projections with Post-Census Survey Data

As indicated in section 3.3, the projection framework advanced here provides the capability for updating projections when recent, more limited mobility tabulations become available for single regions in the regional system. Large-scale post-1970 surveys of movers in the Detroit, Atlanta, and Houston SMSAs provide the opportunity to perform updates to the "baseline" 1970 census-based projections presented above. These updated projections will assume the same rates for interregional migration, mobility incidence, survival, and fertility as did the baseline projections. However, their destination propensity rates \( p_{i,sc}', p_{i,sc}, p_{i,oc}', \) and \( p_{i,os} \) will be calculated from the survey data collected in the late 1970s. The survey tabulations that are used to estimate the late 1970s destination propensity rates are compiled from the metropolitan area-wide Annual Housing Surveys undertaken in the Atlanta, Houston, and Detroit SMSAs in 1975, 1976, and 1977, respectively (as discussed in US Bureau of the Census 1977b, 1978, and 1980). Approximately 15,000 households are interviewed in each SMSA survey which ascertains the number and ages of household members, if the household (head) had changed residence over the previous year, and its city-suburb or outside SMSA location of previous residence. The post-1970 destination propensity rates used in updating each SMSA's 1970 census-based projections were calculated from a tabulation of mover household members. 9

Figure 1 provides some indication of how age-specific destination propensity rates for the late 1970s, to be used in the updated projections, differ from those for the late 1960s. Because of the limited sample size of the Annual Housing Survey, it is necessary to collapse age categories into end-of-period values: 5-14, 15-24, 25-34, 35-44, 45-54, and 55 and over.
I. Suburb-destination propensity rates for city-origin movers.

II. City-destination propensity rates for suburb-origin movers.

Figure 1. Mover and in-migrant destination propensity rates, late 1960s and late 1970s.
III. City-destination propensity rates for SMSA in-migrants.

Figure 1. Continued.
Both late 1970s and late 1960s rates are presented in this manner to facilitate comparisons. In general, there is a tendency toward increased city-to-suburb redistribution. All three SMSAs show lower city destination propensity rates for both suburban-origin movers and metropolitan in-migrants in the late 1970s than in the late 1960s (Panels II and III). Further, Atlanta shows a significant increase in its suburb destination propensity for city-origin movers (Panel I). The latter tendency is not exhibited for either Detroit or Houston.

The updated intrametropolitan projections for the three SMSAs can be contrasted with the baseline projections in Figure 2. Both sets of projections begin with 1970, and progress through 10 five-year periods to the year 2020. They differ only in the destination propensity rates that are assumed. Hence, these comparisons provide a means of evaluating the long-term redistribution implications of changed late 1970s movers' and migrants' intrametropolitan destination selections, when all other migration, mortality, and fertility assumptions are held constant.

It is clear from the plots that the more recently registered destination propensity rates will provide for a more significant city-to-suburb redistribution of population in all three SMSAs, than would have occurred on the basis of late 1960 rates. The updated projections show Detroit's central city share of SMSA population to fall to 18 percent, as contrasted with the 24 percent share with the baseline projections. The newly projected year 2020 central city share for Atlanta is only 12 percent as contrasted with the previously projected 25 percent share. Houston's central city and suburbs grow rapidly under each projection. However, the "updated" projection no longer shows the central city to dominate the suburbs throughout the projection period. By the year 1990, Houston's suburbs are now projected to overtake the central city.

While the updated projections represent something of a compromise between older projections wherein all rates were calculated from data for the same base period, and the need to produce equally elaborate projections from the current year,
Figure 2. Alternative projections of city and suburb population sizes, 1970-2020 based on assumptions of late 1960s and late 1970s destination propensity rates; Detroit, Atlanta, and Houston SMSAs.
they do constitute a means to assess the aggregate implications of intercensal movement patterns until a more satisfactory data base becomes available with the next census. The "updated" projections above, for example, serve to counter a popularly held view that a significant "return to the city" had occurred in large metropolitan areas since the 1970 census was taken.

5. CONCLUSION

We have introduced in this paper a population projection framework that incorporates both interregional migration and intraregional residential mobility streams to project future population sizes both across and within regions in a manner that is consistent with existing multiregional migration theory. We have also shown how the framework can be operationalized with fixed interval migration data that are commonly available with censuses and surveys. A significant advantage of this framework over the existing multiregional projection methodology is its parsimonious data requirements when both inter- and intraregional projections are desired. It also permits the user to "update" baseline projections when recent, more limited regional survey data become available. These features of the framework were demonstrated through projections of intrametropolitan central city-suburban redistribution for three US SMSAs based on migration data from the 1970 US Census and metropolitan area-wide Annual Housing Surveys undertaken in each SMSA over the 1975-77 period. While this inter/intraregional projection framework can be employed with any regionalization scheme the user desires, it is most consistent with underlying migration and residential mobility processes when the "regions" correspond to self-contained labor market areas such as Standard Metropolitan Areas or Bureau of Economic Analysis Areas in the US, or Metropolitan Economic Labor Areas in the United Kingdom.
1. The operational distinction between migration and residential mobility is not always made on the basis of movement across or within labor market areas. Government statistical agencies often make this distinction on the basis of administrative units. The US Census Bureau, for example, defines migration as movement across a county administrative unit, despite the fact that labor market areas generally consist of groups of counties (US Bureau of the Census 1970).

2. This discussion of the city-suburban redistribution process is consistent with the "analytic framework" we have previously advanced to examine the determinants and migration stream components of city-suburban redistribution within a single migration interval (Frey 1978a, 1979b). The projection methodology presented in section 3 represents an extension of this framework to a more general projection model.

3. We have defined the destination propensity rate (Frey 1978a) as the proportion of migrants or movers of a specified origin that locate in a specified destination. It should be applied to an at-risk population of movers or migrants and should always indicate their location of destination (e.g., the j destination propensity rate of i origin movers).
4. These equations are similar to those employed in Frey's (1978a, 1979a) analytic framework to examine the components of central city-suburban population redistribution in a single interval. In the earlier specification [see equations (7) and (8) in Frey (1978a) or equations (1) and (2) in Frey (1979a)], population totals were represented by the letter P rather than the present K, in-migrants to the metropolitan area were represented by the factor $M_0$ rather than by the present $K_{i,o}(t)$ and there was not an explicit subscript i designation for the metropolitan area of an (x) designation for each age class.

5. If this assumption is not made, then:

$$m_{ij} = \sum_{j=1}^{n} \frac{k(t)(x)m_{i.co} + k(t)(x)m_{i.so}}{k(t)(x) + k(t)(x)}$$

rather than the relationship in equation (8).

6. Some data sources do not distinguish between same and different dwelling unit residences for individuals that do not move across subregion boundaries. This precludes estimation of separate mobility incidence rates and destination propensity rates for residential movers in equation (9). An alternative specification for such data sources is offered in the Appendix of Long and Frey (1982).

7. These constitute alternative regionalizations of the national territory wherein the regions approximate single labor market areas. The 183 Bureau of Economic Analysis Areas, designated by the Bureau of Economic Analysis, approximate self-contained commuting regions based on the nodal functional concept (see discussion in Hall and Hay 1980:3-14). The 510 State Economic Areas designated by the US Census Bureau (1970) represent groups of counties that are homogeneous with respect to social and economic characteristics.
8. The reader will note that these projections differ from those presented for the Pittsburgh and Houston SMSAs in Long and Frey (1982), section 4.2. The latter are not strictly estimated with the closed system inter- and intraregional methodology advanced here in that the in-migration component \[ s(x)K_{i.o}(t)(x) \] was generated by applying observed "in-migration-to-beginning-of-period resident" ratios to the SMSA's age disaggregated population at the beginning of each period. Hence, the resulting SMSA projections are not consistent with projections for a system of regions which lies outside the SMSA boundaries.

9. For each metropolitan area, a tabulation was prepared for members of households whose head moved during the year preceding the survey. The tabulations cross-classified the city and suburb location at the date of the survey by city, suburb, or outside the SMSA locations of previous residence for household members in age classes 5-14, 15-24, 25-34, 35-44, 45-54, and 55+ at the time of the survey. Hence, the destination propensity rates compiled from these data are based on mobility observations over a 1-year (not 5-year) period and pertain to the end-of-period household population (not total population) in each SMSA. In generating the projections, destination propensity rates for 5-year age class multiples (i.e., 5-14) are applied to each 5-year age group in the class (e.g., 5-9 and 10-14).
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