ANALYSIS OF SURFACE RUNOFF

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June 1982
CP-82-33

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PREFACE

During 1979-1981, erosion processes were the central theme of the Environmental Problems of Agriculture task of the Resources & Environment Area of IIASA. At present there are several ways of approaching this problem. Some of them include consideration of the physical processes which constitute the erosion process. Soil erosion and pollution of water resources are very closely connected to surface runoff from agricultural fields, therefore mathematical models of surface runoff can calculate both negative consequences. This is why the analysis of surface runoff is important.

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ABSTRACT

A mathematical model of surface runoff is presented which is of use in building a model of erosion processes. The method used for deriving the conceptual model of surface runoff is based on the mathematical expression of the basic laws of movement of water--the equation of continuity and the equation of motion. Both equations form a system of nonlinear partial differential equations with two unknown functions expressing the depth and velocity of the movement of water along the slope, in dependence on their location on the slope, and time. The input variables of the model are the intensity and direction of the impinging raindrops, the intensity of infiltration and the physical characteristics of the slope (gradient, length and properties of soil surface). Extensive laboratory experiments have been carried out to determine the functional dependence of tangential stress on the depth and rate of runoff from different types of soil surfaces.

Further, the conceptual model of surface runoff has been simplified to a kinematic one by using a simple relation between depth and rate of surface runoff instead of the equation of motion. Two empirical parameters of this relation have been determined by using data from the above mentioned laboratory experiments during calibration of the kinematic model. The kinematic model is recommended because of its simplicity with regard to simulation of the surface runoff formation from individual slopes within the watershed.

The model is a multipurpose one. It may be used either for hydrological purposes (simulation of surface runoff characteristics) or for soil conservation purposes. The model outputs are surface characteristics (depth, velocity, rate). It is possible by comparing the surface runoff velocity with the critical non-scouring velocity for given field conditions to determine the critical slope length which is the basis for planning efficient soil conservation measures.

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INTRODUCTION

The basic approach to the analysis of the conditions necessary for efficient agriculture is to study the relations between soil, water, vegetation, and the atmosphere. An important part of such studies is the analysis of surface runoff, especially with respect to the transport processes caused by flowing water. These processes, namely transportation of substances detached by erosion (soil particles, mineral fertilizers, pesticides, etc.) cause soil degradation on one side and pollution of water resources on the other. The consequences of soil erosion on agricultural production are more serious, therefore, the Resources and Environment Area of IIASA focused its attention on soil erosion problems, both on the regional and global scale. One of the basic problems in this field is the analysis of surface runoff.

A number of solutions of these models have been offered (Clarke, 1973). Their applicability depends mainly on how they are in accordance with the natural laws of the movement of water or to the extent and reliability of the data base used. For evaluation of various hydrologic models for determination of the amount and course of the surface runoff and erosion models using surface runoff as one of the most important input data for further calculations, it is useful to analyze the surface runoff from the viewpoint of physical laws and evaluate whether the models used for various purposes correspond to these unchangeable natural laws.

1. ANALYSIS OF SURFACE RUNOFF

The overland flow of water is based on the physical laws of its movement. The laws on the conservation of matter and momentum apply to water running off the slope. From these relations, the equation of continuity and the equation of movement arise.

1.1 Equation of Continuity

The first stage of the overland movement of water - sheet surface runoff - may be studied in the Cartesian co-ordinate system (Figure 1).

For mathematical expression of the basic relations we assume that

- the surface of the slope is a plane forming angle α
- the length of the slope is unlimited
- the intensity of rainfall is even on the whole slope and is only a function of time

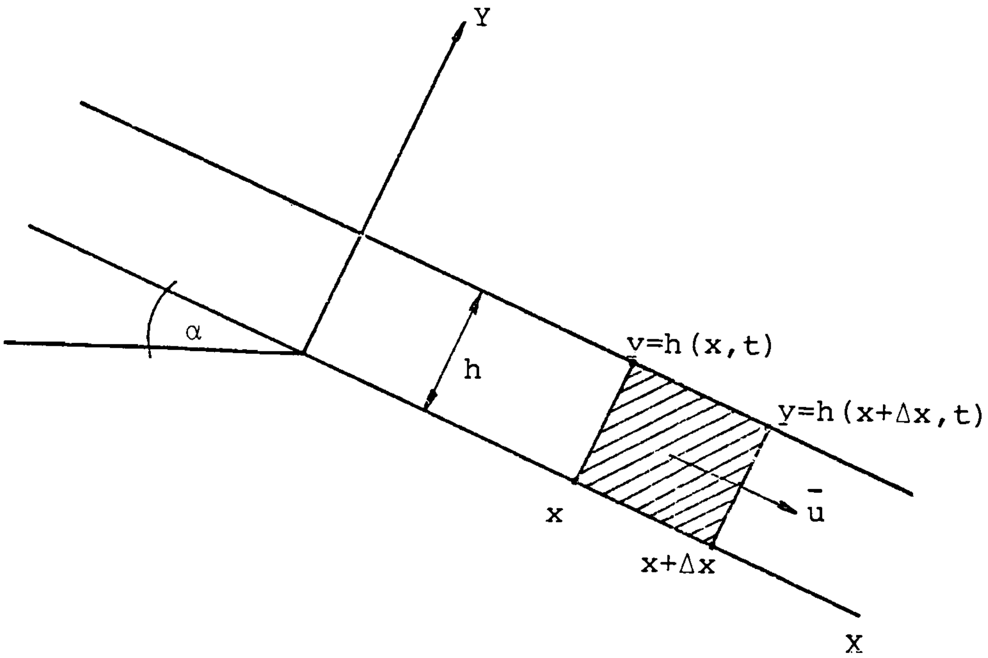

Figure 1. Diagram illustrating Equation of Continuity

- the infiltration rate of water into the soil is only a function of time for given soil conditions

In Figure 1:

X is the coordinate axis in the direction of the surface runoff (Cartesian coordinates were used)

Y - the coordinate axis perpendicular to the direction of runoff

α - the gradient of the slope

h - the height of the surface runoff (it is the function of position and time h (x,t))

$\bar{u}$ - the average velocity of surface runoff in the direction of the X-axis

r(t) - the intensity of rainfall

i(t) - the intensity of infiltration.

The flow velocity of water changes with the change of x,y,t. If $\vec{w}$ is the vector of the velocity of water running down the slope it applies that

$$\vec{w} = [u(x,y,t),\ v(x,y,t)], \tag{1}$$

where

u is the component of water velocity in the direction of X

v - the component of water velocity in the direction of Y; analyzing the movement of water in the direction of X, the component v may be neglected.

From the law on the conservation of mass, it follows that in section $\langle x, x+\Delta x\rangle$ the difference between the water flowing in and flowing out equals the increment of its volume. The increment is either positive or negative depending on which of the two components, i.e. the inflow or the outflow, is the prevalent factor.

The amount of water which flows into the considered section at a time interval $\langle t, t+\Delta t\rangle$ may be expressed by the integral

$$\int_{t}^{t+\Delta t} \left[\int_{0}^{h(x,\bar{t})} u(x,y,\bar{t})\,dy \right] d\bar{t} \quad . \tag{2}$$

At the same time interval the runoff from the same section is

$$\int_{t}^{t+\Delta t} \left[\int_{0}^{h(x+\Delta x,\bar{t})} u(x+\Delta x,y,\bar{t})\,dy \right] d\bar{t} \quad . \tag{3}$$

The amount of water in the section <x,x + Δx> will increase by precipitation

$$\int_{t}^{t+\Delta t} \Delta\, x\, r(\bar{t})\, d\bar{t} \quad , \tag{4}$$

and will be reduced by infiltration of water into the soil by

$$\int_{t}^{t+\Delta t} \Delta\, x\, i\, (\bar{t})\, d\bar{t} \quad . \tag{5}$$

The volume of water in the section <x,x + Δx> in time t is

$$\int_{x}^{x+\Delta x} h\, (\bar{x},t)\, d\bar{x} \quad , \tag{6}$$

and anologically in time t + Δt

$$\int_{x}^{x+\Delta x} h(\bar{x},t + \Delta t)\, d\bar{x} \quad . \tag{7}$$

According to the law on the conservation of matter the volume of water in the section <x,x + Δx> is

$$\int_{t}^{t+\Delta t} \{ \int_{o}^{h(x,\bar{t})} u(x,y,\bar{t})dy - \int_{o}^{h(x+\Delta x,\bar{t})} u(x+\Delta x,y,\bar{t})dy + \Delta x[r(\bar{t}) - i(\bar{t})]\}d\bar{t} =$$

$$= \int_{x}^{x+\Delta x} [h(\bar{x},t+\Delta t) - h(\bar{x},t)]d\bar{x} \quad . \tag{8}$$

If we introduce the mean profile velocity of the surface runoff into further calculations

$$\bar{u}(x,t) = \frac{1}{h(x,t)} \int_{o}^{h(x,t)} u(x,y,t)dy \quad , \tag{9}$$

we may simplify equation (8) by

$$\int_{o}^{t+\Delta t}\{h(x,\bar{t})\bar{u}(x,\bar{t}) - h(x+\Delta x,\bar{t})\bar{u}(x+\Delta x,\bar{t}) + \Delta x[r(\bar{t}) - i(\bar{t})]\}\,d\bar{t} =$$

$$= \int_{x}^{x+\Delta x}[h(\bar{x},t + \Delta t) - h(\bar{x},t)]\,d\bar{x} \quad . \tag{10}$$

If we assume that h, $\bar{u}$, v, r, i have continous derivatives of the second order with respect to their corresponding variables, it is possible to write

$$h(x+\Delta x,\bar{t}) = h(x,\bar{t}) + \frac{\partial h}{\partial x}(x,\bar{t})\Delta x + \frac{1}{2}\frac{\partial^2 h}{\partial x^2}(\Theta,\bar{t})\Delta x^2 \quad , \tag{11}$$

$$\bar{u}(x+\Delta x,\bar{t}) = \bar{u}(x,\bar{t}) + \frac{\partial \bar{u}}{\partial x}(x,\bar{t})\Delta x + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}(\Theta,\bar{t})\Delta x^2 \quad , \tag{12}$$

$$h(\bar{x},t+\Delta t) = h(\bar{x},t) + \frac{\partial h}{\partial t}(\bar{x},t)\Delta t + \frac{1}{2}\frac{\partial^2 h}{\partial t^2}(\bar{x},\Theta)\Delta t^2 \quad , \tag{13}$$

where Θ are certain values from section $<x,x + \Delta x>$ and interval $<t,t + \Delta t>$.

Using equations (10), (11), (12), and (13) we may obtain

$$\Delta x \int_{t}^{t+\Delta t}[r(\bar{t}) - i(\bar{t}) - h(x,\bar{t})\frac{\partial \bar{u}}{\partial x}(x,\bar{t}) - \bar{u}(x,\bar{t})\frac{\partial h}{\partial x}(x,\bar{t}) + \ldots]\,d\bar{t} =$$

$$= \Delta t \int_{x}^{x+\Delta x}[\frac{\partial h}{\partial t}(\bar{x},t) + \ldots]\,d\bar{x} \quad . \tag{14}$$

If assumption of continuous derivatives of the second order of the functions mentioned is valid, it exists in the interval $<t,t + \Delta t>$ such a value t_1, and in the section $<x,x + \Delta x>$ such a value x_1, that equation (14) may be written as

$$\Delta x \Delta t[r(t_1)-i(t_1) - h(x,t_1)\frac{\partial \bar{u}}{\partial x}(x,t_1) - \bar{u}(x,t_1)\frac{\partial h}{\partial x}(x,t_1)+ \ldots] =$$

$$= \Delta x\ \Delta t[\frac{\partial h}{\partial t}(x_1,t)+ \ldots] \quad . \qquad (15)$$

By dividing equation (15) by the expression $\Delta\ x\ \Delta t$ and using the limit process for $\Delta x \to 0$ and $\Delta t \to 0$, the equation of continuity is obtained

$$\frac{\partial(\bar{u} \cdot h)}{\partial x}(x,t) + \frac{\partial h}{\partial t}(x,t) = r(t) - i(t) \quad . \qquad (16)$$

1.2 Equation of Movement

The equation of movement of the surface water may be derived from Newton's second law of motion studying the forces which act on water in the elementary section of the investigated slope (Figure 2).

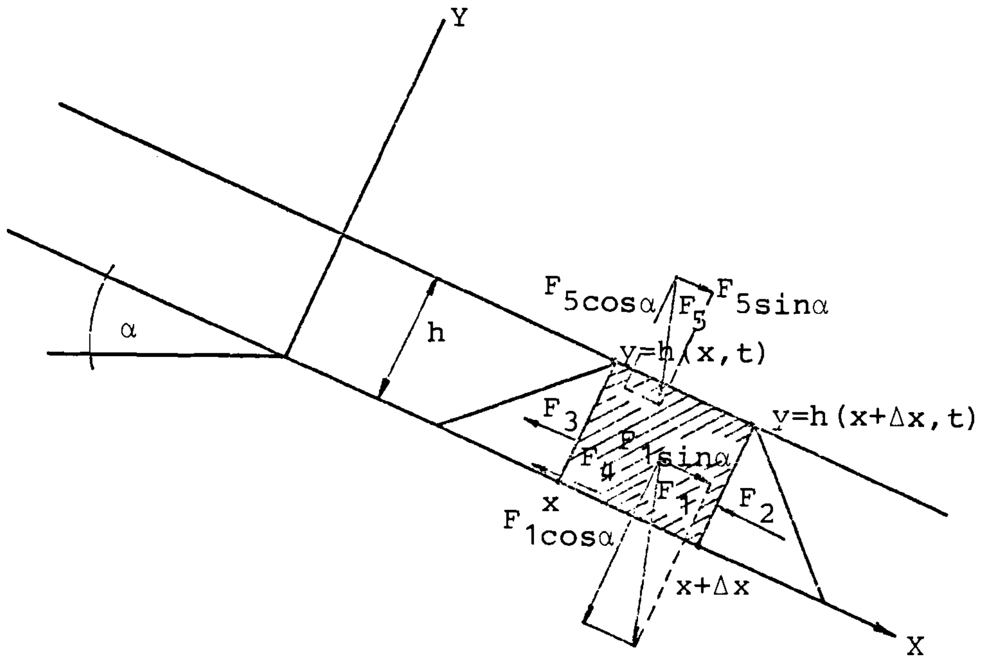

Figure 2. Diagram for Equation of Motion

In Figure 2:

F_1 is the force expressing gravity of water in the elementary section;

F_2 - the pressure force acting on water in the elementary section at distance $x + \Delta x$;

F_3 - the pressure force acting on water in the elementary section at distance x;

F_4 - the friction force;

F_5 - the force coming from water drops impinging on the elementary section.

The acting forces may be expressed as follows:

$$F_1 \sin \alpha = \Delta m g \sin \alpha \quad , \qquad (17)$$

where Δm is the mass of water in the elementary section

$$\Delta m = \rho \int_x^{x+\Delta x} h(\bar{x},t)\, d\bar{x} \quad , \qquad (18)$$

where ρ is the density of water

$$F_1 \sin\alpha = \rho\, g \sin\alpha \int_x^{x+\Delta x} h(\bar{x},t)\, d\bar{x} \quad , \qquad (19)$$

$$F_2 = \int_0^{h(x+\Delta x,t)} p(x+\Delta x,y,t)\, dy \quad , \qquad (20)$$

where p is the total hydrostatic pressure of water, $p(x,y,t)$,

$$F_3 = \int_0^{h(x,t)} p(x,y,t)\, dy \quad . \qquad (21)$$

Expressing the force F_4, we consider that the inner friction of water may be neglected and that the friction of water flowing over the slope surface is the only force acting against its movement. This force is in linear proportionality with the area of friction.

Then

$$F_4 = \Delta x \cdot \tau , \tag{22}$$

where τ is the function expressing tangential stress, $\tau(h,\bar{u})$.

To determine the force F_5 we may use the basic relation for the force impulse (Chow, 1967; Chow and Ten, 1968):

$$F \cdot t = m \cdot v \quad . \tag{23}$$

If the number of raindrops impinging at time interval Δt on the section Δx is <u>n</u>, the mass of every raindrops is <u>m</u> and velocity of the impingement of raindrops v* (Bisal, 1960; Laws, 1941), the time effect of the force acting on the given section may be expressed as

$$F_5 \cdot \Delta t = n \cdot m \cdot v^* \quad . \tag{24}$$

If the unit of volume contains ϑ raindrops

$$n = \vartheta \Delta t \; v^* \; \Delta x \; \cos \alpha \quad , \tag{25}$$

the intensity of impinging raindrops may be expressed as

$$r = \frac{m}{\rho} \vartheta \, v^* \cos \alpha \quad . \tag{26}$$

From equations (24), (25), and (26) it follows that

$$F_5 = \Delta x \, \rho \, r(t) \, v^*(t) \quad . \tag{27}$$

This force acts in time t on section Δx of the surface in a perpendicular direction.

If the forces F_1, F_2, F_3, F_4,F_5 are known, Newton's law of motion

$$\frac{d}{dt}(m\,\vec{u}) = \vec{F} \quad , \tag{28}$$

may be applied for the investigated case as

$$\Delta m\frac{d\bar{u}}{dt} = F_1 \sin\alpha - F_2 + F_3 - F_4 + F_5 \sin\alpha \quad . \tag{29}$$

To get an equation of motion it is convenient to multiply equation (29) by $\frac{1}{\rho\Delta x}$ and determine the limits of the left- and right-side of the equation for $\Delta x \to 0$. But first of all it is necessary to express the total pressure of water in the elementary section. According to Chen Cheng Lung and Ven Te Chow (1968) the total hydrostatic pressure is

$$p^*(x,y,t) = \rho g[h(x,t) - y] \quad . \tag{30}$$

To this, the pressure caused by the impinging raindrops must be added - this pressure is

$$p^{**} = \frac{F_5 \cos\alpha}{\Delta x} \quad , \tag{31}$$

$$p^* + p^{**} = p(x,y,t) \quad . \tag{32}$$

Coming over to the limit for $\Delta x \to 0$ [with (32) in the expressions (20), (21) for F_2,F_3] we get - in a similar way, as in (15), (16) - <u>the equation of motion</u> for the surface runoff in the form

$$h(x,t)\frac{\partial \bar{u}}{\partial t}(x,t) + h(x,t)\bar{u}(x,t)\frac{\partial \bar{u}}{\partial x}(x,t) =$$

$$= g\sin\alpha\ h(x,t) - g\cos\alpha\ h(x,t)\frac{\partial h}{\partial x}(x,t) - g\cos\alpha\ h^{*}(t)\frac{\partial h}{\partial x}(x,t) -$$

$$- \frac{\tau(h,\bar{u})}{\rho} + r(t)v^{*}(t)\sin\alpha \quad . \tag{33}$$

In equation (33)

$$h^{*}(t) = \frac{r(t)v^{*}(t)}{g} \quad . \tag{34}$$

2. A DETERMINISTIC MODEL OF SURFACE RUNOFF

From the equation of continuity (16) it follows by a stationary case that

$$\bar{u}(x)h(x) = (r - i)x \quad , \tag{35}$$

and from this equation

$$h(x) = \frac{(r - i)x}{\bar{u}(x)} \quad . \tag{36}$$

If we introduce (36) for h(x) into the equation of motion (33) we get a differential equation for a stationary case which may be written

$$\frac{d\bar{u}}{dx} = \frac{g\sin\alpha(r-i)x\bar{u} - g\cos\alpha(r-i)^{2}x\bar{u} - g\cos\alpha\ h^{*}(r-i)\bar{u}^{2}}{(r-i)x\bar{u}^{3} - g\cos\alpha(r-i)^{2} - g\cos\alpha\ h^{*}(r-t)x\bar{u}} -$$

$$- \frac{C(r-i)^{\delta}x^{\delta}\bar{u}^{3+\beta-\delta} - r\,v^{*}\sin\alpha\ \bar{u}^{3}}{(r-i)x\bar{u}^{3} - g\cos\alpha(r-i)^{2} - g\cos\alpha h^{*}(r-i)x\bar{u}} \quad , \tag{37}$$

assuming $\frac{\tau(h,\bar{u})}{\rho} = Ch^{\delta}\bar{u}^{\beta}$.

If we determine initial and boundary conditions and the values C, β, δ for this deterministic model we may obtain surface runoff value for any profile of the slope and the total value of runoff in the lowest profile of the slope.

2.1 Initial and Boundary Conditions

Any consideration of initial conditions must proceed from the fact that the investigated surface runoff starts in time $t = 0$. At this point of time, precipitation starts acting on the slope surface and surface runoff is formed. The unknown functions have zero value for all x,

$$h(x,0) = 0 \quad , \tag{38}$$

$$\bar{u}(x,0) = 0 \quad . \tag{39}$$

Determination of the boundary conditions depends on the distance between the investigated section and the water divide of the slope. For an arbitrary distance of this section from the water divide $x_o > x$, it is necessary to determine

$$h(x_o,t) \quad , \tag{40}$$

$$\bar{u}(x_o,t) \quad , \tag{41}$$

in accordance with the conditions affecting the formation of the surface runoff.

2.2 Determination of Values C, δ, β

The values C, δ, β express the relation of tangential stress and the depth and velocity of the water running down the slope.

Till now the influence of tangential stress in the course of the sheet surface runoff has not been determined. Some results gained in the laboratory (Karantounias, 1974; Wakhlu, 1970) are difficult to accept for field conditions. The surface investigated under simulated runoff were artificial (synthetics, glass, rubber, etc.,) and the relation between them and soil surface was not fixed. The criterion of similarity of small laboratory surfaces (mostly 1 m x 1 m) to natural soil surface is not known.

It seems possible to use the results of the laboratory test made by the Institute of Water and Land Reclamation at the Technical University of Prague, where tests were carried out in a tilting hydraulic flume (Holý, 1980; Holý et al., 1981) - 9 m in length, 1.5 m in width - on surfaces made by natural soils with various characteristics (clay, loamy and sandy soils). The runoff was simulated by water coming over a weir.

By a simulated runoff we may neglect the influence of impinging raindrops. From equation (33) we may derive the relation

$$g \sin\alpha \; h(x,t) = \frac{\tau(h,\bar{u})}{\rho} \quad . \tag{42}$$

If $\frac{\tau}{\rho} = \nu$, we get

$$\nu(h,\bar{u}) = g \; h(x,t) \; \sin \alpha \quad . \tag{43}$$

To express the dependence of ν on h and $\bar{u}$, the expression

$$\nu(h,\bar{u}) = C \; h^{\delta} \bar{u}^{\beta} \quad , \tag{44}$$

was used.

In this case, from equation (43) we get

$$g \sin\alpha = C \; h^{\delta-1} \bar{u}^{\beta} \quad . \tag{45}$$

Through measurements, obtained values C,δ,β are introduced in Table 1.

Table 1. Values C,δ,β obtained through measurements

	SOIL		
	Clay	Loamy	Sandy
C	0.01395	0.01676	0.01596
δ	-0.03551	-0.30406	-0.66759
β	1.71434	1.85542	1.98857

Because the values of water discharge Q, and its height h were measured it is possible to write equation (45) in the form

$$g \sin \alpha = C\, Q^{\beta}\, h^{\gamma} , \tag{46}$$

where $\gamma = \delta - 1 - \beta$.

From equation (46) a system of equations was obtained, the solution of which gives the unknown values and therefore also τ for different soils.

3. A KINEMATIC MODEL OF SURFACE RUNOFF

The deterministic model of the surface runoff may be expressed in a simpler way as it is by equations (16) and (33) if we use the equation (42) as

$$\tau = \rho\, g\, h \sin \alpha , \tag{47}$$

for a steady uniform flow of water without the influence of impinging raindrops.

If we take this equation into consideration with the basic equation for tangential stress

$$\tau = f_t \, \rho \, \frac{\bar{u}^2}{2} \quad , \tag{48}$$

where f_t is the coefficient of friction, we get

$$\bar{u} = C' \sqrt{h \sin \alpha} \quad , \tag{49}$$

$$C' = \sqrt{\frac{2g}{f_t}} \quad , \tag{50}$$

where C' is the Chezy coefficient. It may be considered as well as the friction coefficient f_t as the function of Reynold's number and of the roughness of soil surface. If C' is constant we may write $C_1 = C' \sqrt{\sin \alpha}$.

The expression for surface runoff is

$$q = \bar{u} \cdot h \quad . \tag{51}$$

From equations (49) and (51) we get

$$q = C_1 h^{3/2} \quad , \tag{52}$$

which may be written as

$$q = a \, h^b \quad . \tag{53}$$

For the laminar flow

$$f_t = \frac{4}{R} \quad , \tag{54}$$

where R is the Reynold's number,

$$R = \frac{h \, \bar{u}}{\nu} \quad , \tag{55}$$

where ν is the kinematic viscosity of water.

From equations (50) till (55) we get for the laminar flow

$$a = \frac{g \sin\alpha}{2\nu} \; ; \; b = 3 \quad . \tag{56}$$

For the turbulent flow, Manning's formula for determination of the friction coefficient may be used

$$f_t = 0.9 \; g \; n^2 \; h^{-1/3}, \tag{57}$$

where n is Manning's coefficient.

Then we get

$$a = \frac{1.49}{n} \sqrt{\sin \alpha}; \; b = \frac{5}{3} \quad . \tag{58}$$

For natural soil surfaces, R.E. Norton (1938) found out that b equals approximately 2. On the basis of laboratory research with various surfaces and their configurations, V.P. Singh (1975) recommended use of constant value b = 1.5. Singh found the value a variable.

Equation (16) together with equation (53) makes it possible to determine the unknown functions $\bar{u}$,h. They represent another model of surface runoff in the form

$$h(x,t)\frac{\partial \bar{u}}{\partial x}(x,t) + \bar{u}(x,t)\frac{\partial h}{\partial x}(x,t) + \frac{\partial h}{\partial t}(x,t) = r(t) - i(t) \quad , \tag{59}$$

$$\bar{u}(x,t) - a \; h^{b-1}(x,t) = 0 \quad . \tag{60}$$

This model may be classified as empirical-deterministic. It represents a kinematic description of surface runoff. M.H. Lighthill and G.B. Whitman (1955) who used a similar model for determination of flood waves called it equations of kinematic wave.

Equations (16) and (33) representing the deterministic model may be simplified by solving such surface runoff when only inner and pressure forces are important, and the influence of precipitation and infiltration are negligible. On the basis of equations (16) and (33) we get

$$\frac{\partial h}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + h\frac{\partial \bar{u}}{\partial x} = 0, \text{ and} \tag{61}$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + g \cos \alpha \frac{\partial h}{\partial x} = g \sin \alpha - \frac{\tau}{\rho h} \quad . \tag{62}$$

These are called equations of a dynamic wave. They describe the diffusion of longitudinal waves in shallow water and give a picture not only of sheet surface runoff but of concentrated runoff in shallow, broad streambeds as well.

Woolhiser and Liggett (1967) showed that for most types of flow, the dynamic wave goes down and is negligible in comparison with the kinematic wave. The equations of kinematic wave offer a sufficiently exact description of the problems of surface runoff and are suitable for the investigation of surface runoff processes.

3.1 Calibration of the Kinematic Model of Surface Runoff

The kinematic model has two parameters, a and b, which express the empirical part of the model. When using the model, it is necessary to determine these values.

Laboratory tests in a tilting hydraulic flume (see Holý et al., 1981) showed that parameter a depends on the slope and soil properties. The relation is exponential and may be expressed:

for clay soils $a = 47.497\ I^{0.562}$ (63)

for loamy soils $a = 26.873\ I^{0.613}$ (64)

for sandy soils $a = 25.645\ I^{0.491}$. (65)

All these are characterized by a high coefficient of correlation nearing between 0.9 and 1.

The relation of parameter a and the slope I is represented in Figure 3.

In analyzing the parameter b, it was possible to come to the conclusion that it depends only on the properties of soils and that it is possible to consider it as constant for soils with similar properties.

Parameter b has the following values:

for clay soils $b = 1.585$

for loamy soils $b = 1.726$

for sandy soils $b = 1.859$.

These values correspond with the results of V.P. Singh (1975) who came to the conclusion that they vary between 1.0 and 3.0 and recommends use of the fixed value 1.5.

4. CONCENTRATED SURFACE RUNOFF

The concentrated surface runoff in the individual elements of the hydrographic network originates from the inflow of water into the network from the respective water basin. We may assume that function q_s (s,t) is the surface runoff coming from both sides of the elementary length of the streambed and the cross-section of this streambed in which water movement in an arbitrary point s in time t occurs is P(s,t).

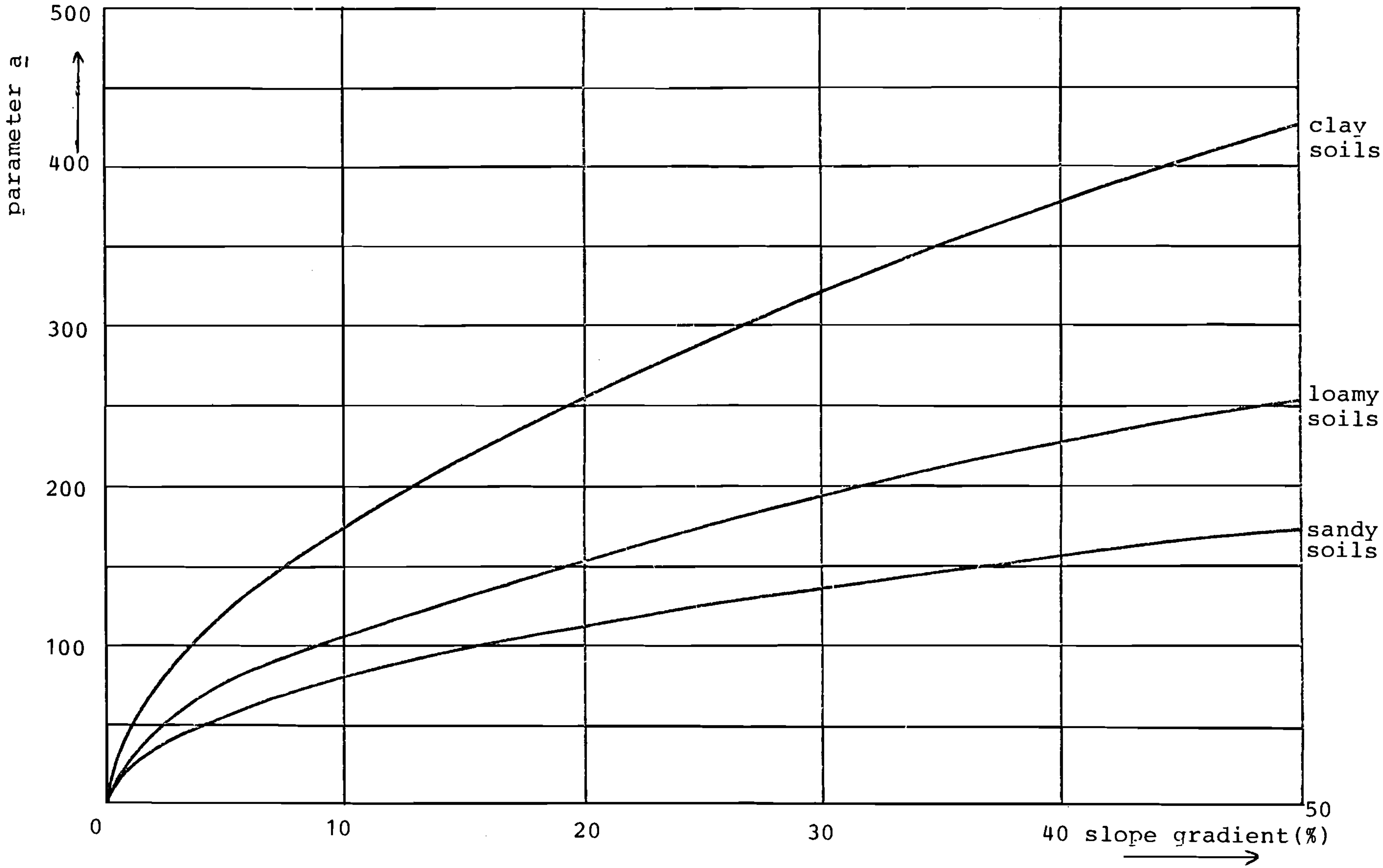

Figure 3. Dependence of Parameter a on Slope Gradient for Basic Soil Types

If we consider an elementary section of the streambed $\langle s, s+\Delta s \rangle$ where $\Delta s > 0$, the volume of water in the section in time $\underline{t}$ is

$$\int_{s}^{s+\Delta s} P(s^*, t)\, ds^* \quad . \tag{66}$$

In time $t + \Delta t$ where $\Delta t > 0$, the volume of water in the elementary section is

$$\int_{s}^{s+\Delta s} P(s^*, t+\Delta t)\, ds^* \quad . \tag{67}$$

If ξ, η are rectangular coordinates on the level of the cross-section of stream flow, the vector of point velocity in (s, ξ, η) is

$$[u_s(s,\xi,\eta,t),\ u_\xi(s,\xi,\eta,t),\ u_\eta(s,\xi,\eta,t)] \tag{68}$$

where

u_s is the component of velocity in direction s

u_ξ - the component of velocity in direction ξ

u_η - the component of velocity in direction η.

Let us define the mean velocity of the waterflow in the streambed at time t by the relation

$$u(s,t) = \frac{1}{P(s,t)} \int_{P(s,t)} u_s(s,\xi,\eta,t)\, d\xi\, d\eta \quad . \tag{69}$$

The discharge of water in the streambed in profile $\underline{s}$ in time $\underline{t}$ is

$$Q(s,t) = P(s,t)\ u(s,t) \quad . \tag{70}$$

During the time interval $\langle t, t + \Delta t\rangle$ the volume

$$\int_{t}^{t+\Delta t} Q\ (s,t^{*})\ d\ t^{*} \quad , \tag{71}$$

$$\int_{t}^{t+\Delta t} Q\ (s+\Delta s,t^{*})dt^{*} \quad , \tag{72}$$

through the section $s + \Delta s$ flows out.

During the same time interval the volume

$$\int_{t}^{t+\Delta t} \int_{s}^{s+\Delta s} q_s(s^{*},t^{*})ds^{*}dt^{*} \quad , \tag{73}$$

comes from slopes into the same elementary section of the streambed.

Using formulas (66), (67), (71), (72) and (73) we may write

$$\int_{s}^{s+\Delta s} [P(s^{*},t+\Delta t) - P(s^{*},t)]ds^{*} =$$

$$\int_{o}^{t+\Delta t} [Q(s,t^{*}) - Q(s+\Delta s,t^{*}) + \int_{s}^{s+\Delta s} q(s^{*},t^{*})ds^{*}]dt^{*} \quad . \tag{74}$$

If we multiply equation (74) by $\frac{1}{\Delta s \Delta t} \neq 0$ and come over to the limit for $\Delta s \to 0$, $\Delta t \to 0$, and using the assumption that functions P, Q have continuous partial derivatives and function q_s (s,t) is continuous, we get the relation

$$\frac{\partial Q}{\partial s}\ (s,t) + \frac{\partial P}{\partial t}\ (s,t) = q_s\ (s,t) \quad . \tag{75}$$

This equation is a linear partial differential equation of the first order for the unknown functions P and Q. It is therefore not only sufficient to determine the course of the runoff, but also the differential equation formulating the relation between P and Q. Such an equation is the equation of motion for water moving in a streambed. It is given by P.S. Eagleson (1970) in the form

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\cos\Theta\frac{\partial y}{\partial x} = g\sin\Theta - (1 - \frac{2y}{d})\frac{\tau}{\rho y} \qquad (76)$$

where U is the mean velocity of water

Θ - the gradient (slope) of the streambed

d - the width of the streambed.

If we consider the movement of water as uniform and steady, equation (76) may be transformed into

$$\tau = \rho g\,\frac{dy}{d+2y}\sin\Theta \quad , \qquad (77)$$

where $\frac{dy}{d+2y}$ equals approximately the hydraulic radius

$$R_h = \frac{P}{W} \quad . \qquad (78)$$

Using equations (48) and (70) we get

$$Q = \sqrt{\frac{2g\sin\Theta}{W.f_t}}\,P^{3/2} \quad , \qquad (79)$$

which may be written in a simpler form

$$Q = a\,P^b \quad .$$

The values of a, b are again parameters which have to be determined for the investigated streambed. P.S. Eagleson (1970) recommends use of b = 3/2. The value a is relatively changeable and its value must be determined from the analysis of experimental data.

5. VERIFICATION OF THE DETERMINISTIC AND KINEMATIC MODELS OF SURFACE RUNOFF

5.1 The Deterministic Model

In order to verify the deterministic model of surface runoff (equation 37), the characteristics of surface runoff were ascertained on an experimental plot of the Institute on Water and Land Reclamation of the Civil Engineering Faculty at the Technical University, Prague. The computed characteristics of surface runoff were compared with the values measured on the plot. An example is given below of the computation of surface runoff characteristics for the input data:

- intensity of rainfall $r = 0.088$ cm min^{-1}
- duration of rainfall $T = 20$ mins
- infiltration intensity $i = 0.036$ cm min^{-1}
- velocity of impingement of raindrops $v^{*} = 6.4$ m s^{-1}
- values for determination of friction (loamy soil) $C = 1.676 \cdot 10^{-2}$; $\beta = 1.85542$; $\delta = -0.30406$
- slope length $L = 20$ m
- slope gradient $I = 44\%$

The resulting measured soil wash in the given slope length was 5.45 kgm^{-2}, where the erosion rills began to form at a distance of 8-10m from the upper end of the slope.

For a numerical solution of equation (37) to begin with the differential method of the fourth order (the Rung-Kutte fourth order method) was used.

The problem was solved by a computer program using the FORTRAN language (Appendix 1). The results of the solution are summed up in Table 2 and graphically presented in Figure 4.

Table 2. Results of Numerical Solution of Characteristics of Surface Runoff according to:
(A) the conceptual model
(B) kinematic model

Distance from start of slope	Depth of Surface Runoff		Runoff Velocity		Flow Rate	
x(m)	h(mm) A	B	$u(cms^{-1})$ A	B	$q(cm^2.s^{-1})$ A	B
1.0	0.080	0.094	10.917	9.22	0.087	0.087
2.0	0.121	0.140	14.363	12.30	0.174	0.173
3.0	0.155	0.178	16.894	14.60	0.262	0.260
4.0	0.184	0.210	18.968	16.50	0.349	0.347
5.0	0.210	0.239	20.758	18.10	0.436	0.433
6.0	0.234	0.265	22.348	19.60	0.523	0.520
7.0	0.256	0.290	23.791	20.90	0.610	0.607
8.0	0.278	0.314	25.117	22.10	0.697	0.693
9.0	0.298	0.336	26.350	23.20	0.784	0.780
10.0	0.317	0.357	27.504	24.30	0.872	0.867
11.0	0.335	0.377	28.593	25.30	0.959	0.953
12.0	0.353	0.397	29.626	26.20	1.046	1.040
13.0	0.370	0.415	30.609	27.10	1.133	1.130
14.0	0.387	0.434	31.549	28.00	1.220	1.210
15.0	0.403	0.451	32.450	28.80	1.308	1.300
16.0	0.419	0.468	33.317	29.60	1.395	1.390
17.0	0.434	0.485	34.153	30.40	1.482	1.470
18.0	0.449	0.502	34.961	31.10	1.569	1.560
19.0	0.463	0.517	35.742	31.80	1.656	1.650
20.0	0.478	0.533	36.500	32.50	1.743	1.730

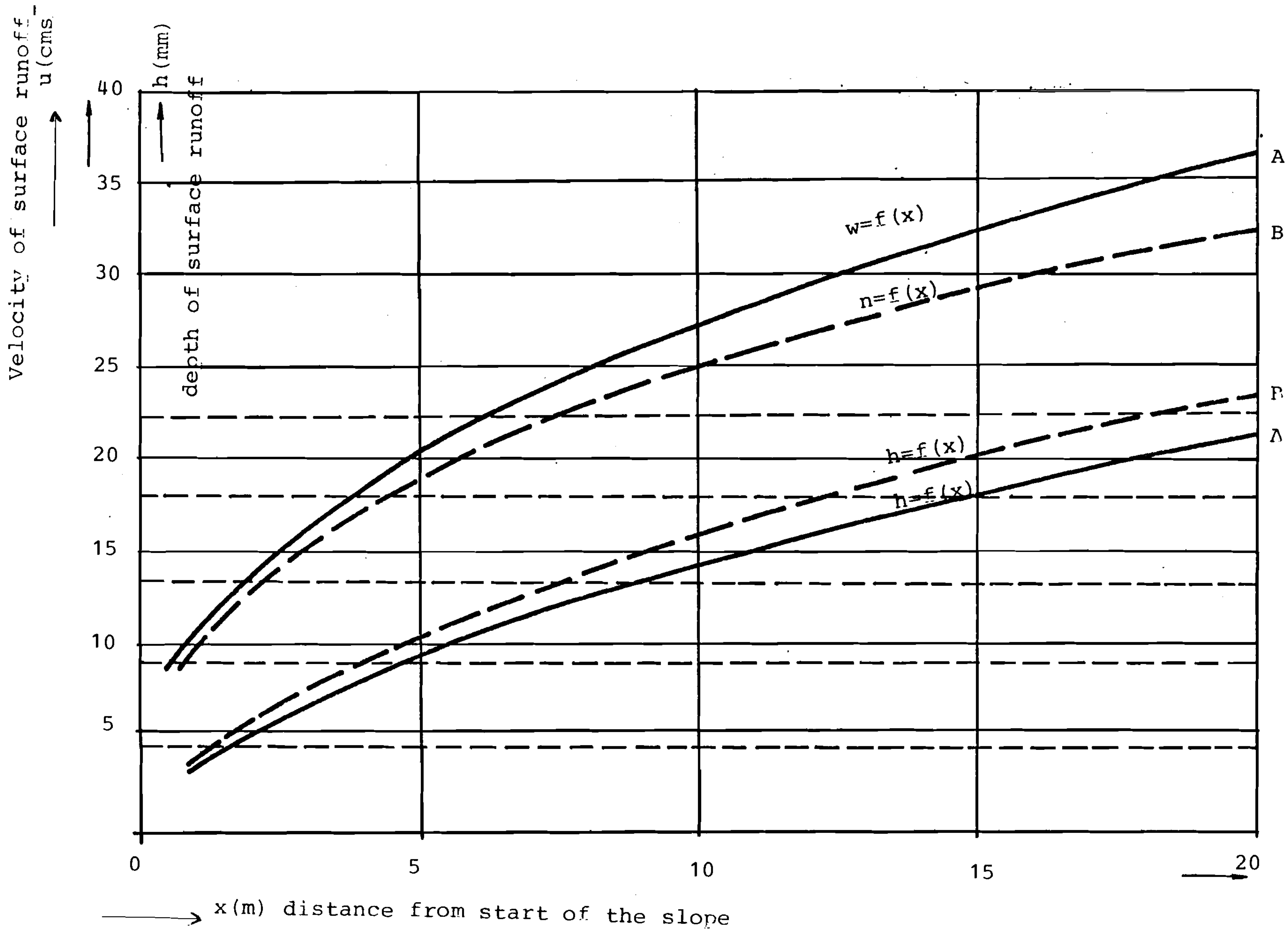

Figure 4. Course of Depth and Velocity of Surface Runoff in Variance with Slope Site
(A) according to conceptual model;
(B) according to kinematic model.

For loamy soils, an admissible maximum velocity of surface runoff in respect to erosion processes is given at v_{cr} = 30 cms^{-1} (cited by Holý, 1980). In the case examined, surface runoff achieved this velocity at a distance of 12.4 m from the upper end of the slope. At a distance of 8-10m the values of the calculated velocity range between 25-27.5 cms^{-1}. The calculated value of the surface runoff (135.2 m^3ha^{-1}) and the measured value (130.7 m^3ha^{-1}) are very close. This indicates a good applicability of the deterministic model of surface runoff.

5.2 The Kinematic Model

The kinematic model of surface runoff expressed by equations (59) and (60) allows the determination of the characteristics of surface runoff of water from the slopes, i.e., the depth and velocity of the surface runoff at an arbitrary point of slope and at an arbitrary moment from the start of rainfall and it allows the determination, at an arbitrary point of the slope, of the runoff hydrogram, i.e., the chronological line of runoff, which is closely linked with erosion processes.

The calculation of the runoff hydrogram is divided into three time intervals. The first interval expresses the ascending runoff line from the start of the runoff to the moment of the attainment of maximum runoff, the second time interval determines the constant value of the maximum runoff wave and the third interval describes the descent of the runoff (Figure 5).

By substituting equation (60) with equation (59) we obtain a partial differential equation for the determination of the runoff hydrogram which may be written in the form

$$a.b.h^{b-1} \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = r-1 \quad . \qquad (80)$$

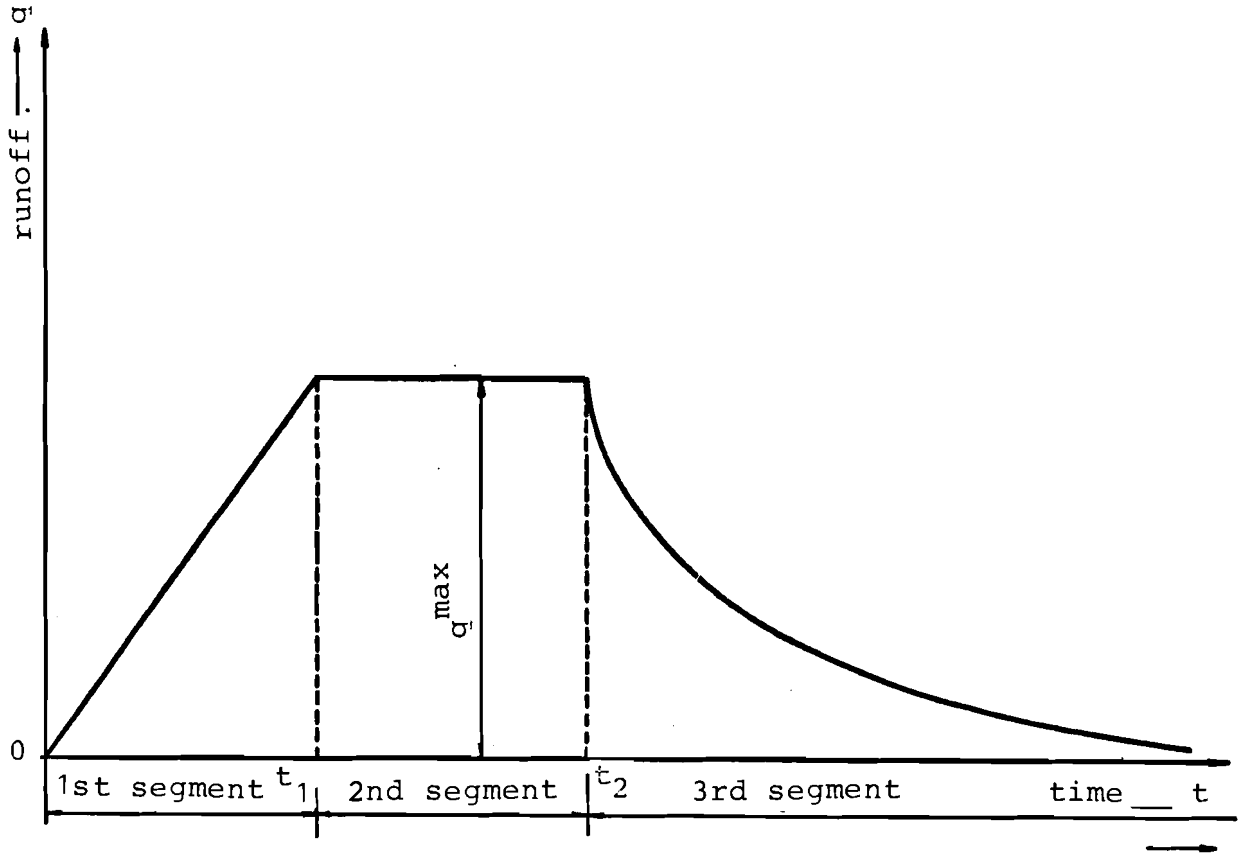

Figure 5. Runoff Hydrogram

The boundary condition is

$$h(0,t) = 0 \qquad \text{for } t \in \langle 0,\infty) \quad , \tag{81}$$

the initial condition is

$$h(x,0) = 0 \qquad \text{for } x \in \langle 0,L\rangle \quad , \tag{82}$$

L is the length of the slope.

Another precondition of the solution is that effective rainfall throughout its duration has constant intensity for which it applies

$$r(t) - i(t) = r_o \qquad \text{for } t \in \langle 0,t\rangle \tag{83}$$

and

$$r(t) - i(t) = 0 \qquad \text{for } t > t_2 \quad , \tag{84}$$

where t_2 is the moment of cessation of rainfall.

Duration of the rainfall is such that it will allow formation of maximum surface runoff in all profiles of the slope which may be expressed by the relation

$$t_2 > r_o \, \frac{1-b}{b} \left(\frac{L}{a}\right)^{1/b} \quad . \tag{85}$$

An analytical solution of equation (80) with inclusion of conditions (81) and (82) is given for

- the first time interval of the runoff hydrogram (ascending line) by the relation

$$h(x,t) = r_o \cdot t \qquad \text{for } t \in \langle 0, t_1 \rangle \tag{86}$$

- the second time interval of the runoff hydrogram (runoff maximum) by the relation

$$h(x,t) = \left(\frac{r_o \cdot x}{a}\right)^{1/b} \qquad \text{for } t \in \langle t_1, t_2 \rangle \tag{87}$$

- the third time interval of the runoff hydrogram (descending line) by the implicit function

$$\frac{a}{r_o} h^b(x,t) + ab(t-t_2)h^{b-1}(x,t) - x = 0 \quad , \tag{88}$$

for $t \in \langle t_2, \infty)$, where $t_1 = r_o \frac{1-b}{b} \left(\frac{x}{a}\right)^{1/b}$,

t_2 is the moment of rainfall cessation.

The auxiliary equations (85), (86), and (87) may be used to determine the time course of the runoff line for an arbitrary profile of the slope (runoff hydrogram).

The verification of the kinematic model was made by means of a computer program written in BASIC language (Appendix 2) for the same input data used for the computation of the deterministic model.

The calculated characteristics of the surface runoff, i.e., the depth and velocity of surface runoff dependent on the distance from the water divide, are shown in Figure 4 and presented in Table 2. The dependence is plotted for time t = 25 mins. i.e., for the moment of the cessation of rainfall when maximum runoff is initiated in all profiles of the slope.

5.3 Discussion

A comparison of the results of computation of the characteristics of surface runoff applying the deterministic and kinematic models showed good agreement of the computer values in both cases (Table 2, Figure 4). An advantage of the kinematic model is the simplicity of the computation, combined with adequate accuracy of the results obtained.

Next to the research plot, the feasibility of results for surface runoff from precipitation according to the given model was assessed by comparing the data obtained using the model with data obtained by a number of specialists from measurements of surface runoff from different slopes in various field investigations.

Mircchulava (1970) sums up the results obtained by measurements carried out by M.D. Koberskiy, M.K. Machavariyan, N.I. Manilov, L. Sheklein, M.K. Daraseliy and A.P. Skaposhnikov in different parts of the USSR. The data are given in Table 3.

The characteristics of surface runoff were determined by a kinematic model for identical input. Table 3 shows that the results obtained by computation of the characteristics of surface runoff applying the kinematic model are relatively good and in agreement with data obtained by field measurements.

Table 3. Comparison of Characteristics of Surface Runoff obtained in Field Measurements with Values determined from Mathematical Model

Research Plot No.	Length (m)	Gradient (%)	Rainfall Intensity ($cm\ min^{-1}$)	Rainfall duration (min)	Coefficient of Runoff	Runoff (ls^{-1})	Characteristics of Surface Runoff: Field Measurements Depth (mm)	Field Measurements Velocity ($m.s^{-1}$)	Kinematic Model Depth (mm)	Kinematic Model Velocity ($m.s^{-1}$)
1	20	33.33	0.16	30	0.03	0.016	0.10	0.16	0.15	0.11
2	4	31.11	0.10	60	0.12	0.008	0.08	0.10	0.16	0.08
3	10	22.22	0.17	20	0.01	0.004	0.03	0.14	0.07	0.05
4	10	22.22	0.17	20	0.01	0.003	0.03	0.10	0.06	0.05
5	4	20.00	0.05	60	0.20	0.007	0.06	0.12	0.10	0.07
6	50	15.55	0.10	58	0.04	0.003	0.02	0.12	0.03	0.11
7	4	13.33	0.10	60	0.25	0.017	0.17	0.10	0.21	0.08
8	4	13.33	0.05	60	0.20	0.007	0.06	0.12	0.12	0.06
9	20	12.06	0.01	179	0.11	0.005	0.05	0.10	0.10	0.05
10	4	6.66	0.10	60	0.46	0.030	0.25	0.12	0.38	0.08
11	40	6.66	0.05	60	0.17	0.057	0.48	0.12	0.55	0.10

6. APPLICATION OF THE KINEMATIC MODEL IN A WATERSHED

In studying the geometry of a watershed we may come to the conclusion that besides the slopes which may be approximated by a plane we have the convergent slopes which may be best approximated by part of a cone surface (Figure 6). They are usually close to the watershed in its upper part.

Research of the surface runoff from a convergent slope was carried out by Singh (1975) under laboratory conditions. Its aim was to test the respective mathematical models of runoff, especially the kinematic model based on equation (53) and on equation of continuity in the form

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t) - i(t) + \frac{q}{L-x} \quad , \tag{89}$$

where L is the distance of the upper end of the convergent slope from its middle part (Figure 6).

V.P. Singh (1975) obtained data on the characteristics of surface runoff for 50 different geometric configurations of convergent slopes. From these data he determined parameters a, b. He came to the conclusion that parameter b is relatively stable while parameter a is very sensitive to the characteristics of rainfall and composition of the slope.

The values of parameter b fluctuated between 1 and 3, and Singh (1975) recommended use of a fixed value b = 1.5 and suggested the model be considered as a one-parameter model, with a single parameter a.

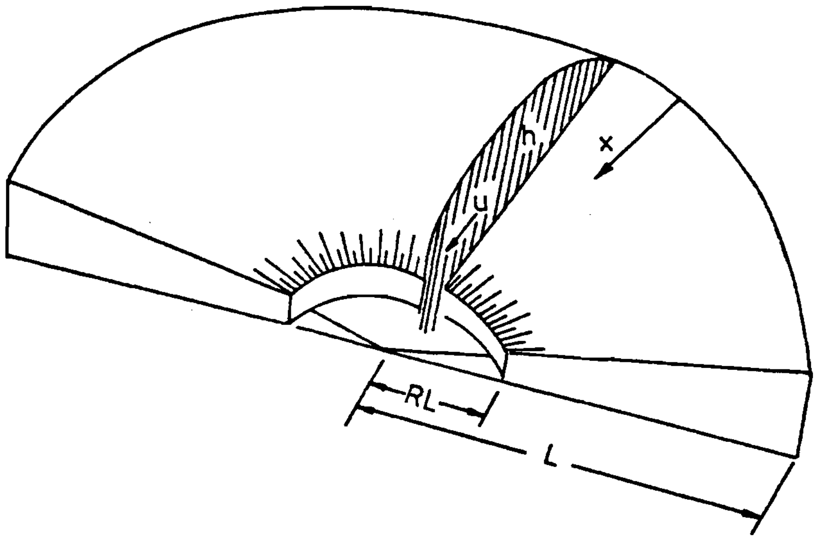

Figure 6. Convergent Slope

The mathematical model of the surface runoff from the convergent slope expressed by equations (53) and (89) may be written as a single partial differential equation

$$\frac{\partial h}{\partial t} + abh^{b-1} \frac{\partial h}{\partial x} = r-i \frac{ah^b}{L-x} \quad , \qquad (90)$$

with one unknown function f(x,t); the function q may be obtained from equation (53).

For the solution of equation (90) a numerical scheme

$$\frac{h_{i,j+1} - h_{i,j}}{\Delta t} + abh_{i,j}^{b-1} \frac{h_{i,j} - h_{i-1,j}}{\Delta x} = r_j - i_j + \frac{ah_{i,j}}{L-i\Delta x} \qquad (91)$$

was used.

In Appendix 3, the BASIC language for solution of equation (91), for example,

$$r(t) - i(t) = \begin{cases} 5.10^4 (1+\sin\frac{\Pi t}{120}) (cms^{-1}) \text{ for } t \in <0,180> \text{ (s)} \\ 0 \text{ for } t > 180 \text{ (s)} \end{cases}$$

was used. As input data, values a = 286.67, b = 1.6787, and L = 23 m were used.

APPENDIX 1

```
REAL K(4)
DIMENSION A(3), B(4), D(5), W(4), X(4), Z(4)
F(S,Y) = (A1xSxY-A2xS-A3xY-A4xSxxATxYxxBT+A5xYxY)/(RxSxYxY-
A2xSxS/Y-A3xS)
A(1) = 5
A(2) = 5
A(3) = 1
W(1) = 1./6.
W(2) = 1./3.
W(3) = W(2)
W(4) = W(1)
B(1) = 3./8.
B(2) = 37./24.
B(3) = 59/24.
B(4) = 55./24.
G = 980.665
READ(1,1)RD, VSAK, VK, SA, CT, AT, BT
FORMAT (7F10.6)
R = RD-VSAK
RE = RD/60
R = R/60
VK = VKx100.
READ(1,2)XP, YP, H, DT, XK
FORMAT (5F10.4)
```

```
XP = 100.xXP
DT = 100.xDT
XK = 100.xXK
T1 = XP+DT
WRITE (3,15)
FORMAT (///5X, 32HPRUBEH RYCHLOSTI ODTEKAJICI VODY////)
WRITE (3,17)
FORMAT (4X, 4HX(M), 4X, 5HH(CM), 7X, 17HV(CM/S) Q(CM/S)/)
Q = RxXP
HL = Q/YP
TISX = XP/100.
WRITE(3,16)TISX, HL, YP, Q
A1 = GxRxSA
CA = SQRT(1-SAxSA)
A2 = GxRxRxCA
A3 = RxCAxRDxVK
BT = BT-AT+2.
A4 = CTxRxxAT
A5 = RDxVKxSA
Z(1) = YP
DO 3 I = 1,4
X(I) = XP+HxFLOAT(I- )
DO 4 J = 1,3
K91) = F(X(J),Z(J))
DO 5 I = 1,3
Y = Z(J)+A(I)xK(I)xH
V = X(J)+A(I)xH
K(I+1) = F(V,Y)
Z(J+1) = Z(J)
DO 6 I = 1,4
Z(J+1) = Z(J+1)+W(I)xK(I)xH
CONTINUE
DO 7 I = 1,4
D(I) = F(X(I),Z(I))
V = X(4)
Y = Z(4)
S = 0.
DO 8 I = 1,4
S = S+D(I)xB(I)
```

```
Y = Y+HxS
V = V+H
D(5) = F(V,Y)
DO 9I = 1,4
D(I) = D(I=1)
IF(V.LT.t1)GO TO 10
T1 = T1+DT
Q = RxV
HL = Q/Y
TISX = V/100.
WRITE (3,16)TISX, HL, Y, Q
FORMAT (/3X, F5.1, 3X, F6.3, 3X, F11.6, 3X, F7.2)
IF(T1.LE.XK)GO TO 10
STOP
END
```

APPENDIX 2

```
10   DISP "R - I =";
20   INPUT R
30   PRINT "R-I="R"CM/MIN"
40   R=R/60
50   DISP "l=";
60   INPUT L
70   PRINT "L=L"M"
80   L=100xL
90   DISP "SKLON=";
100  INPUT I
110  PRINT "SKLON="I"%
120  A=Ixx0.61327333x26.8734796
130  B=1.7265808
140  DISP "T=";
150  INPUT T
160  PRINT "T="T"MIN"
170  T=60xT
180  IF T<(RxL/A)xx(1/B)/R THEN 140
190  DISP "X=";
200  INPUT X
210  PRINT "X="X"M"
220  WRITE (15,230)X
230  FORMAT 5/,3X,"X=",F4.0, "M",/
240  X = 100xX
250  H = (RxX/A)xx(1/B)
```

```
260  S = H/R
270  Q = AxHxxB
280  U = Q/H
290  M = INT(S/60)
300  WRITE (15,310) M,S-60xM,Q.H,U
310  FORMAT F6.Q,F3.Q,SE12.2
320  M = INT(T/60)
330  WRITE (15,310) M,T-60xM,Q.H,U
340  D = H/10
350  FOR J = 1 TO 9
360  H = H-D
370  S = T+(X-HxxBxA/R)/Hxx(B-1)xAxB)
380  Q = AxHxxB
390  U = Q/H
400  M = INT(S/60)
410  WRITE (15,310) M,S-60xM,Q,H,U
420  NEXT J
430  GOTO 190
440  END
```

APPENDIX 3

L(M) = 23 DX(CM) = 20 DT(SEC) = 0.1

X =	1	3	5	10	15	20
T = 0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
T = 10	0.00509	0.00574	0.00575	0.00579	0.00587	0.00622
	7.96142	8.63708	8.64667	8.68348	8.76609	9.12147
T = 20	0.00651	0.01227	0.01326	0.01353	0.01413	0.01672
	9.40494	14.45840	15.24335	15.45502	15.91677	17.84429
T = 30	0.00712	0.01450	0.01982	0.02383	0.02581	0.03415
	9.99419	16.19448	20.02749	22.68840	23.95635	28.97096
T = 40	0.00757	0.01558	0.02179	0.03489	0.04191	0.06068
	10.42390	17.00977	21.35591	29.39365	33.28670	42.79710
T = 50	0.00787	0.01632	0.02298	0.03840	0.05753	0.09744
	10.70047	17.55309	22.13830	31.37207	41.27270	59.02096
T = 60	0.00801	0.01671	0.02366	0.04004	0.06158	0.11789
	10.82476	17.83749	22.58014	32.27364	43.22388	67.16746
T = 70	0.00798	0.01676	0.02383	0.04077	0.06333	0.12269
	10.79815	17.86889	22.69480	32.66946	44.05477	69.01151
T = 80	0.00779	0.01646	0.02352	0.04063	0.06366	0.12434
	10.62294	17.65393	22.49392	32.59403	44.21194	69.64240
T = 90	0.00745	0.01584	0.02275	0.03968	0.06269	0.12334
	10.30256	17.20077	21.99043	32.07498	43.75009	69.25947
T = 100	0.00696	0.01493	0.02156	0.03799	0.06052	0.11994
	9.84173	16.51979	21.19973	31.14131	42.71809	67.95722
T = 110	0.00635	0.01375	0.01999	0.03565	0.05731	0.11444
	9.24661	15.62406	20.14066	29.82588	41.16711	65.82695

T = 120	0.00563	0.01236	0.01811	0.03277	0.05323	0.10719
	8.52501	14.53007	18.83654	28.16736	39.15443	62.96568
T = 130	0.00484	0.01080	0.01600	0.02947	0.04848	0.09856
	7.68687	13.25880	17.31660	26.21238	36.74642	59.48163
T = 140	0.00399	0.00914	0.01374	0.02591	0.04328	0.08900
	6.74526	11.83773	15.61831	24.01811	34.02159	55.49867
T = 150	0.00313	0.00745	0.01144	0.02224	0.03787	0.07894
	5.71883	10.30449	13.79132	21.65605	31.07413	51.16033
T = 160	0.00230	0.00582	0.00921	0.01865	0.03251	0.06887
	4.63791	8.71413	11.90419	19.21732	28.01819	46.63375
T = 170	0.00156	0.00435	0.00718	0.01533	0.02747	0.05926
	3.56049	7.15305	10.05584	16.82006	24.99281	42.11301
T = 180	0.00099	0.00316	0.00550	0.01247	0.02302	0.05058
	2.61456	5.76430	8.39320	14.61834	22.16666	37.82096
T = 190	0.00065	0.00235	0.00427	0.01020	0.01935	0.04318
	1.97658	4.70772	7.06880	12.76090	19.69915	33.97097
T = 200	0.00047	0.00180	0.00339	0.00846	0.01638	0.03704
	1.56899	3.93196	6.04445	11.23217	17.59786	30.61464
T = 210	0.00035	0.00142	0.00275	0.00710	0.01400	0.03199
	1.29358	3.35135	5.24389	9.97247	15.81613	27.71572
T = 220	0.00027	0.00115	0.00228	0.00603	0.01207	0.02784
	1.09750	2.90703	4.60951	8.92895	14.30419	25.21812
T = 230	0.00022	0.00096	0.00192	0.00518	0.01051	0.02441
	0.95172	2.55941	4.09921	8.05803	13.01641	23.06369
T = 240	0.00019	0.00081	0.00164	0.00450	0.00922	0.02156
	0.83945	2.28181	3.68262	7.32498	11.91374	21.19909
T = 250	0.00016	0.00069	0.00141	0.00395	0.00816	0.01917
	0.75052	2.00598	3.33775	6.70255	10.96376	19.57807
T = 260	0.00014	0.00060	0.00124	0.00350	0.00727	0.01716
	0.67843	1.86923	3.04856	6.16948	10.14008	18.16163
T = 270	0.00012	0.00053	0.00109	0.00312	0.00653	0.01546
	0.61884	1.71256	2.80321	5.70916	9.42130	16.91735
T = 280	0.00010	0.00047	0.00098	0.00280	0.00589	0.01400
	0.56880	1.57945	2.59284	5.30858	8.79012	15.81847
T = 290	0.00009	0.00042	0.00088	0.00253	0.00535	0.01275
	0.52619	1.46507	2.41074	4.95745	8.23255	14.84291
T = 300	0.00008	0.00038	0.00079	0.00230	0.00488	0.01166
	0.48949	1.36582	2.25176	4.64760	7.73720	13.97248
T = 310	0.00008	0.00034	0.00072	0.00211	0.00448	0.01072
	0.45755	1.27893	2.11187	4.37247	7.29479	13.19213
T = 320	0.00007	0.00031	0.00066	0.00193	0.00412	0.00989
	0.42951	1.20227	1.98792	4.12677	6.89767	12.48935
T = 330	0.00006	0.00029	0.00061	0.00178	0.00381	0.00915
	0.40469	1.13416	1.87739	3.90620	6.53956	11.85373
T = 340	0.00006	0.00027	0.00056	0.00165	0.00354	0.00850
	0.38257	1.07326	1.77826	3.70721	6.21520	11.27655

T = 350	0.00005	0.00025	0.00052	0.00153	0.00329	0.00793
	0.36274	1.01849	1.68889	3.52688	5.92024	10.75044
T = 360	0.00005	0.00023	0.00048	0.00143	0.00307	0.00741
	0.34486	0.96899	1.60793	3.36278	5.65097	10.26919
T = 370	0.00005	0.00021	0.00045	0.00134	0.00288	0.00694
	0.32866	0.92403	1.53426	3.21287	5.40431	9.82751
T = 380	0.00004	0.00020	0.00042	0.00125	0.00270	0.00653
	0.31390	0.88303	1.46695	3.07542	5.17760	9.42088
T = 390	0.00004	0.00019	0.00040	0.00118	0.00254	0.00615
	0.30042	0.84549	1.40523	2.94899	4.96859	9.04543
T = 400	0.00004	0.00018	0.00037	0.00111	0.00240	0.00580
	0.28804	0.81098	1.34843	2.83233	4.77533	8.69781
T = 410	0.00004	0.00017	0.00035	0.00105	0.00227	0.00549
	0.27664	0.77917	1.29599	2.72436	4.59616	8.37513

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