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DESIGN OF AGRICULTURAL DRAINAGE UNDER UNCERTAINTY,1, BACKGROUND AND CHANCE CONSTRAINT APPROACH

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ABSTRACT

The problems of waterlogging and salinity on agricultural lands has led to the installation of agricultural drainage systems. The effect of soil property uncertainty on drainage system design and thus ,drain performance is not implicitly considered by present design procedures. This is the first in a series of papers that will present methods for analyzing the effect of soil property uncertainty on drainage system design. An analysis of the nature of uncertainty and spatial variability in recharge rate and soil permeability is presented. A First Order-Second Moment (FOSM) approach is developed for the Hooghoudt steady-state drainage design equation to provide an estimate of the of the uncertainty of the dewatering zone between the drains as a function of the design variables and the uncertainty in the soil properties. Based upon the FOSM approach , a Chance Constraint model for optimal design of drains is developed which incorporates uncertainty in recharge rate , permeability, dewatering zone with the economics of drain installation to provide the least cost design for given reliabilities of drain performance.

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DESIGN OF AGRICULTURAL DRAINAGE UNDER UNCERTAINTY.1. A CHANCE CONSTRAINT APPROACH

Kenneth M. Strzepek, John L. Wilson and David H. Marks

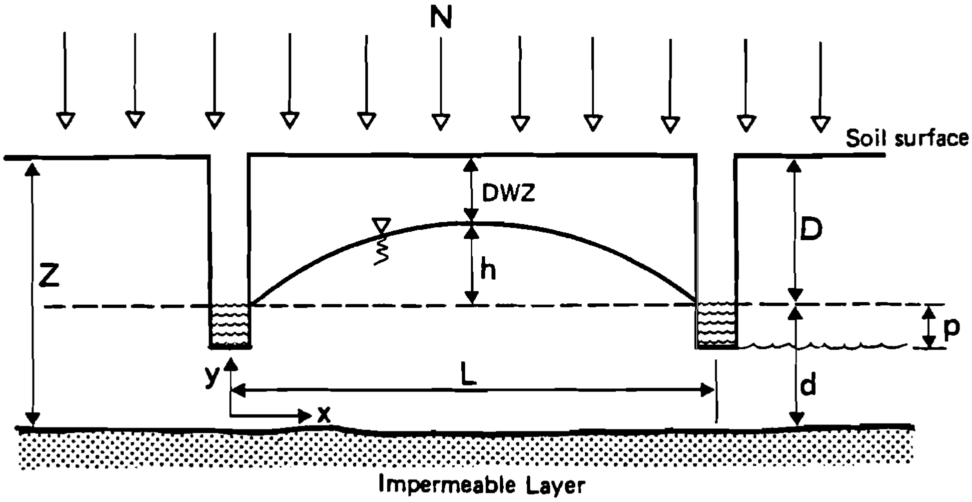
1. Introduction

In the design of drainage systems to alleviate problems of waterlogging and salinity on agricultural lands uncertainty can be found in many aspects of the analysis: the model, the physical properties, the agronomic response,economic coefficients, and institutional factors. The effect of these uncertainties on drain performance is not considered by present design procedures, which also lack any explicit consideration of economic efficiency. This series of papers will present for the first time methods for analyzing the effect of soil property uncertainty on drainage system design. A method to provide a measure of uncertainty in drain performance will then be used to develop a model for the economically optimal design of surface or subsurface drains under steady-state conditions of water application.

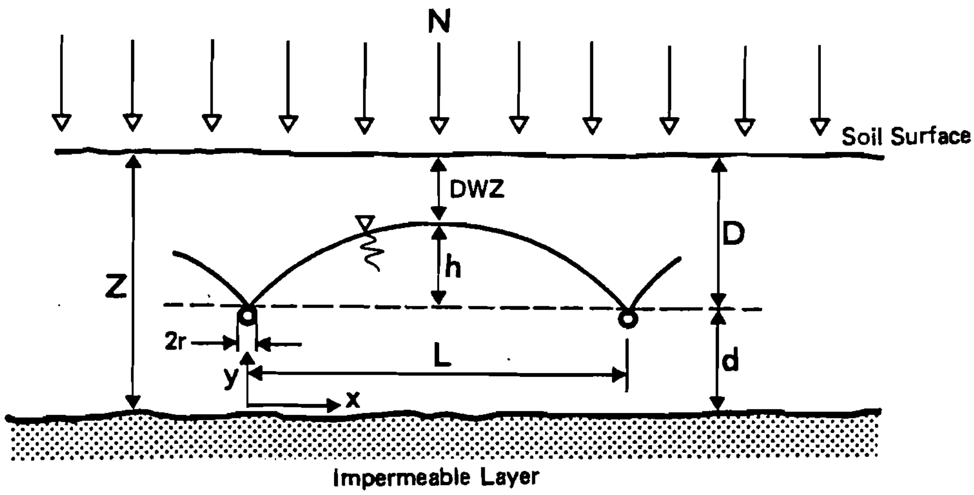
An analysis of the nature of uncertainty and spatial variability in recharge rate (sometimes called drainage rate or drainage coefficient) and soil permeability (hydraulic conductivity) is presented. A First Order-Second Moment (FOSM) approach which is developed for the Hooghoudt steady-state drainage design equation provides an estimate of the first and second moments of the dewatering zone between the drains as a function of the design variables and the uncertainty in the soil properties. Based upon the FOSM approach two methods for optimal design of drains are developed. One of these two methods, a Chance Constraint model which minimizes drainage installation cost subject to a reliability constraint on drain performance, is presented in this paper. In a second paper [Strzepek, Marks, and Wilson, 1982] a Stochastic Programming model which incorporates crop yield functions into optimal drain design is developed, applied, and the two approaches are compared.

2. Uncertainty in Drainage Design

Figures 1a and 1b are representations of the drainage design problem. The objective of installing a drainage system is to control the dewatering zone, DWZ, between the drains. The dewatering zone is a function of the design variables: the spacing between drains,L; the depth to the water level in the ditch,D, and the penetration of the ditch below the water level,P, for surface drains (see Figure 1a) and for sub-surface drains (see Figure 1b) DWZ is a function of the spacing between drains,L, the depth of the drains,D,and their effective radius,r. In both cases the soil properties ,permeability, K, and the recharge rate ,N, and the depth to the impervious layer,Z, effect the dewatering zone The soil properties'



a) SURFACE DRAINS



b) SUBSURFACE DRAINS

Figure 1. Agricultural Drain Design Problem

and the design variables' effect upon the dewatering zone between the drains is related through a model of the physics of groundwater flow. The goal of the designer is to choose a depth, D , and a spacing ,L, that satisfy design criteria for the dewatering zone . These design criteria are usually based upon crop response to various dewatering zones . The possible combinations of D and L which meet this criteria are related to the physical parameters through the model. The D and L that are chosen should maximize the benefits of drainage installations.

A problem that arises in drainage design is that the model formulation and the estimates of the physical parameters contain uncertainty and therefore the performance of the drainage design is itself uncertain. With respect to uncertainty in the physical parameters ,Strzepek,Wilson, and Marks [1982] have shown that drain design is insensitive to the depth to impervious layer for sufficiently large Z. It can also be demonstrated [Strzepek,et al, 1982] that subsurface drainage design is relatively insensitive to the effective drain radius,r, and surface drain design is insensitive to the penetration depth, P, for the range of values encountered in the field. Thus, uncertainty in these parameters are neglected in these papers, although if necessary, they could easily be accounted for . The important parameters are recharge rate ,N, and soil permeability, K. The uncertainty of these parameters effect the design process and will be analyzed in detail. In addition, any mathematical model of drainage flow will be an approximation and thus be uncertain.

2.1. Model Uncertainty

Model uncertainty arises from the assumptions and approximations implicit in the model of the physical system. For the Hooghought Model[Hooghouhgt,1940, Wesseling,1979] presented below these include the assumptions of steady-state flow,essentially horizontal flow except near the drains, constant permeability over the depth of the groundwater system, and a fixed depth to an impervious stratum below. The Hooghought model also assumes spatially uniform permeability and recharge rate. The importance of these last two assumptions is evaluated later in this paper. As already mentioned the assumption of a fixed depth for the impervious layer below is valid for sufficiently large depth .

Steady-state flow is a significant assumption , which is not always applicable. On the contrary, recharge rate is a stochastic variable in both time and space. Sagar[1980] evaluates the effects of stochastic temporal variation of a uniform recharge on the time response of the water table between two drains. Earlier work by Gelhar[1974] and Gelhar et al. [1974] uses spectral methods to examine a similar problem. The present approach can be extended to account for the temporal source of uncertainty.

Another significant assumption is that flow is essentially horizontal except near the drains. A great deal of attention has been focused on this topic in the drainage literature, using deterministic analysis. In fact, the Hooghought Model is a simple modification of the Dupuit Model [see,e.g. Wesseling , 1979] to account for the non-horizontal flow near the drains. The essentially horizontal flow assumption has often been used in previous analyses of the stochastic spatial variability [See,e.g., Gelhar et al , 1974;Freeze,1975; Gelhar,1976;Smith and Freeze,1979;Dettinger and Wilson,1981,1982; Wilson and Dettinger,1982] , but up to this time there has been no definitive evaluation of it for stochastic spatial variation. Gelhar [1974] has demonstrated under what conditions the horizontal flow assumption is valid, for stochastic temporal variation .

The most basic study of model uncertainty appears in Bakr et al, [1978] and Gutjhar et al,[1979]. They show that there is a significant difference between the effects of stochastic permeability variation in one and three dimensions. In one-dimensional flow, zones of low permeability have an exaggerated influence on the flow field. This is of concern here, as the Hooghought Model is one dimensional,[as are the models of Freeze,1975;Gelhar,1976; Smith and Freeze,1979; and Wilson and Dettinger,1981]. This problem is specifically avoided by assuming that the permeability is constant over the vertical. In fact , the permeability of the Hooghought Model actually represents some weighted depth average permeability of the soil. The effects of spatial variations in the two horizontal directions are analyzed below and compared to results presented for the simpler one-dimensional Hooghought Model.

2.2. Parameter Uncertainty

2.2.1. Permeability Uncertainty

Information uncertainty for the soil permeability is due both to the error in taking each individual sample and to the sparseness of the sampling network. Punctual measurements of permeability are usually made via the auger hole method [Beers,1976] or some equivalent technique. These samples are made at a small distance beneath the ground surface(say,2 to 3 meters). They are subsequently assumed to represent the permeability for the entire soil column at that point, unless there is a well defined soil stratification. Coupling this assumption together with the difficulty of accurately repeating an experiment at the "same" point leads to the sample error. This sample error can be estimated through statistical analysis of exhaustive field investigations, or more practically, it can be subjectively estimated on the basis of experience. In either case sample error can be directly taken into account in the estimation techniques introduced below.

Information uncertainty due to data sparseness is closely related to the issue of spatial variability, and for that reason spatial variability is addressed next. Consider an agricultural field located on the Embabe Drain in the Nile Delta of Egypt(See Figure 2). The field is approximately 1500 feddans (1 feddan=0.4 hectare ≅ 1 acre) in area and has 101 two meter deep auger hole permeability tests taken on an almost regular grid of 200 meter spacing, as shown in Figure 3. The permeability values range from 0.01 to 0.45 meters/day, and assuming independence, are distributed lognormally,at the 85% significance level,(see Figure 4), which is typical for this parameter[see Freeze,1975]. Figure 3 is a contour plot of the data. Spatial variability, such as that presented in these figures, may have large scale trends (or drifts) as well as smaller scale stochastic fluctuations. The slowly varying large scale trends can be identified by trend[eg,Davis,1973] or drift[eg,David,1978;Delhomme,1978] analysis, while the covariogram[David,1978;Delhomme,1978] describes the higher frequency variability of the parameter. In the case where the parameter variability has a finite variance ,the covariogram $\gamma[u]$ is related to the

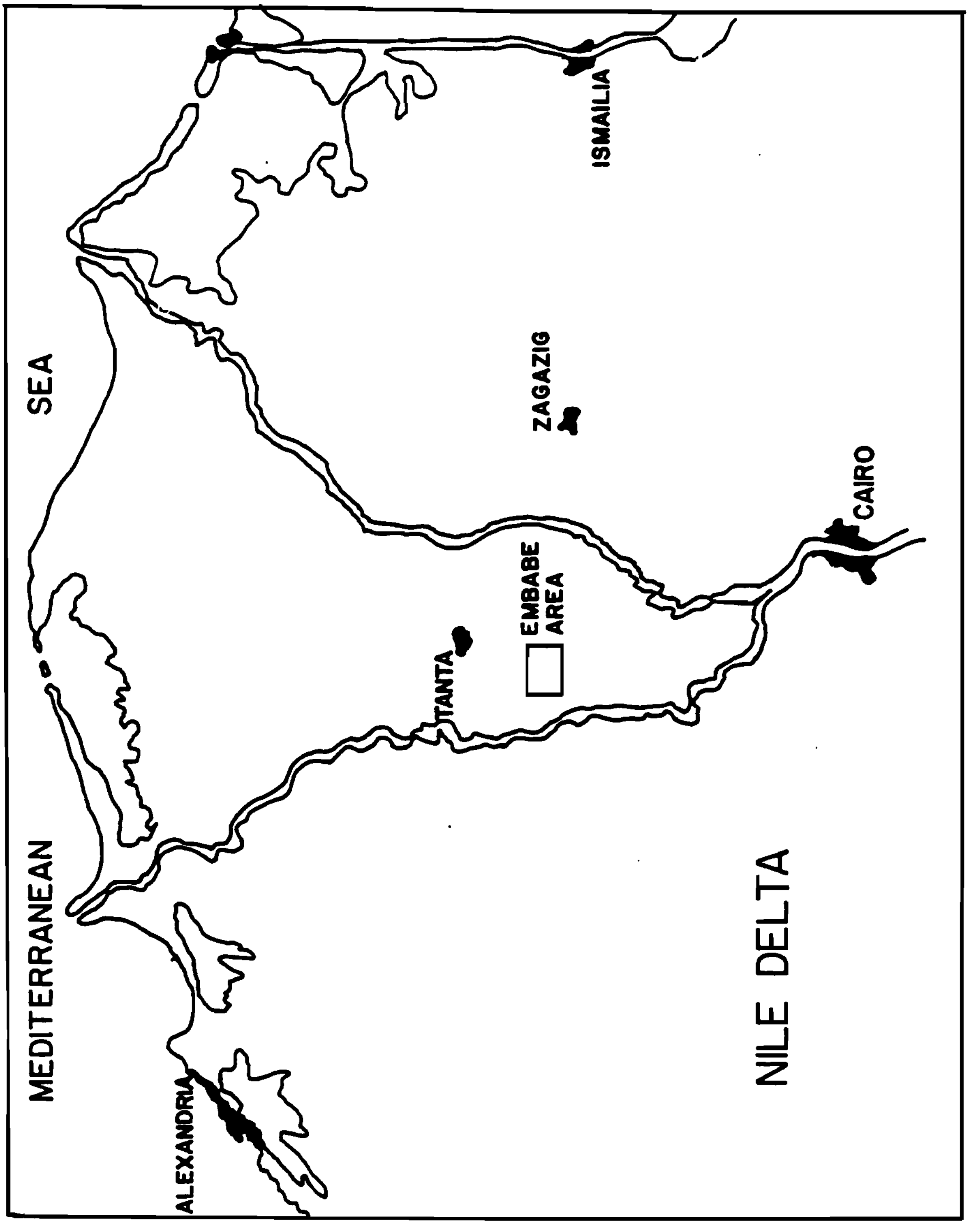


Figure 2. Location of Embabe Case Study Region

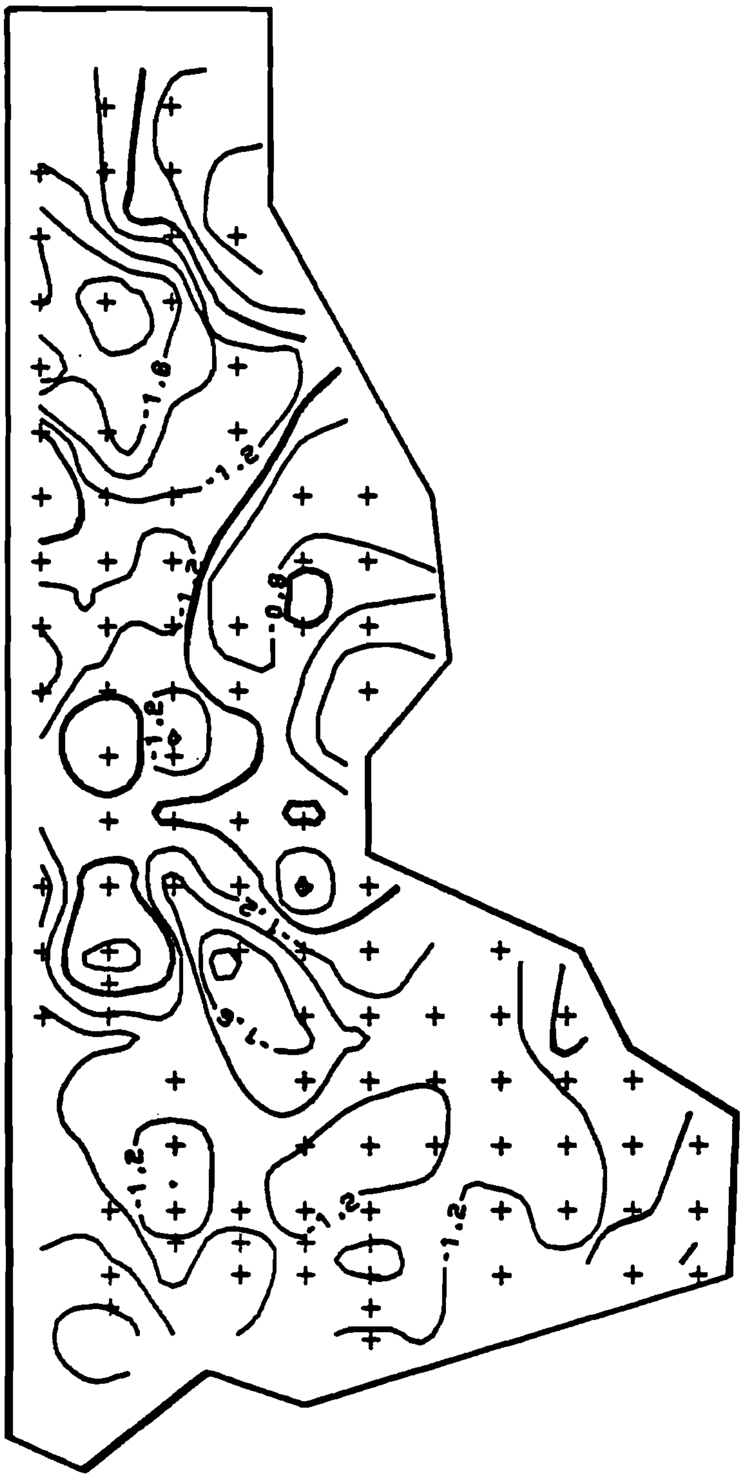


Figure 3. Embabe Study Area and Contour of *Ln* of Permeability Data

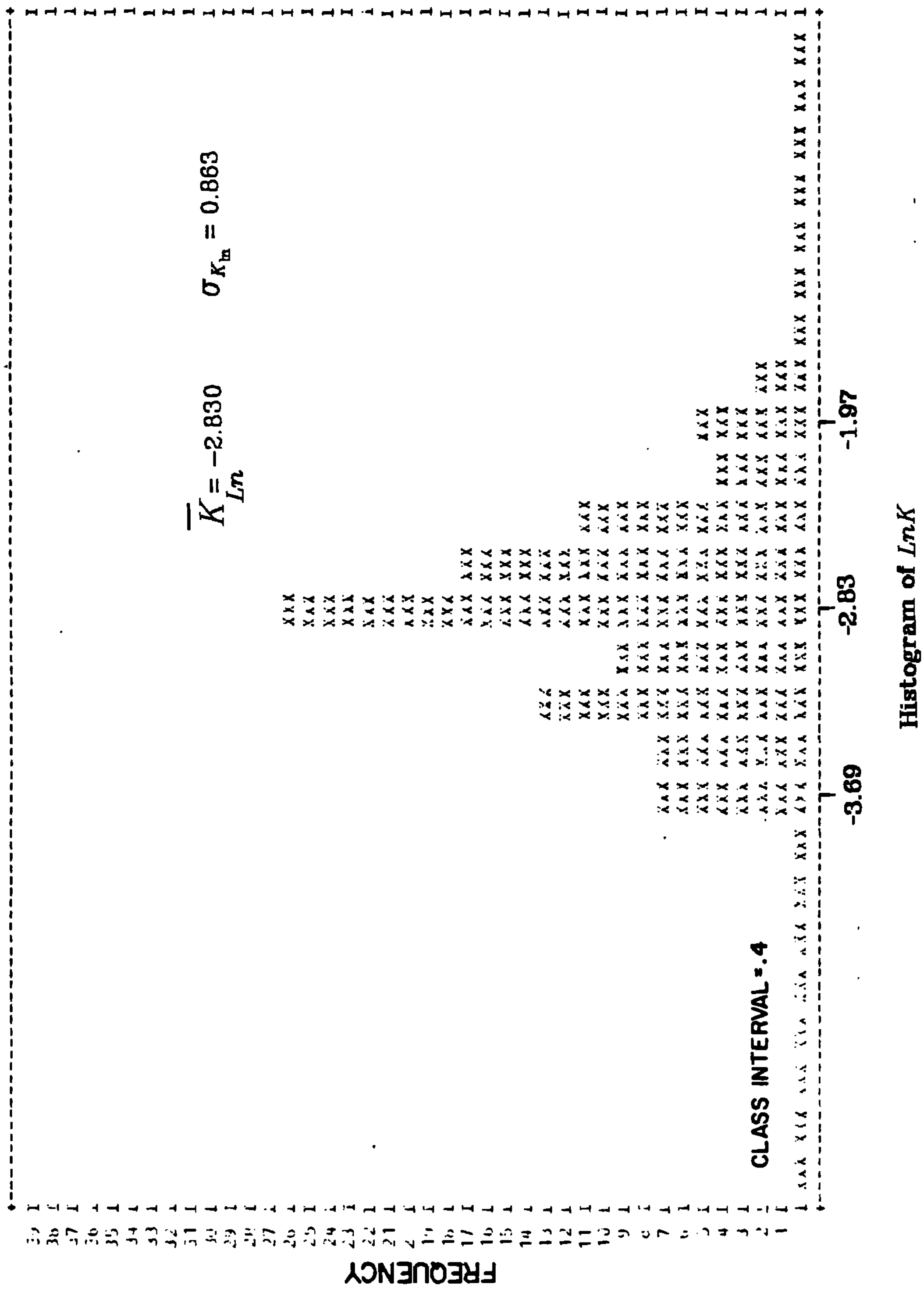


Figure 4. Histogram of *Ln* of Permeability Samples

covariance function by:

$$\gamma[u] = Var[0] - Cov[u] \quad (1)$$

where the $Cov[u]$ is the covariance of the parameter over the distance u [assuming isotropic] , $Var[0]$ is the point variance , and $gamma[u]$ is the covariogram over u .

Subtracting the large scale trend from the spatial process produces a residual of the process. The residual contains information about the small scale spatial structure, which can be represented by the covariogram [see ,eg, David,1978] . Using automatic(generalized Kriging using BLUEPACK- described in Delfiner[1976], see also Journel and Huijbregts,[1978]), as well as manual[David,1978] drift identifiers it has been found that there is no identifiable drift in the Embabe data. Thus the soil permeability is approximately homogeneous in the mean, which can be subtracted from the sample data to yield the residual.

The covariogram of the residuals of natural log permeability is shown in Figure 5. It has a "nugget effect" [see Delhomme,1978] equal to 0.40. This is probably due to sample error in the auger hole tests , which has been subjectively estimated by local engineers to be "+/- 25%" [Amer,1979], as well as small scale permeability variation not captured by the 200 meter scale grid. The ***range*** of the covariogram, l , is approximately 1000 meters, and the ***sill*** is 0.74. The "best fit" covariogram which is shown , is achieved with a spherical function [David,1978;Delhomme;1978]. The data is second order stationary as indicated by the presence of the ***sill***, which implies a finite variance.

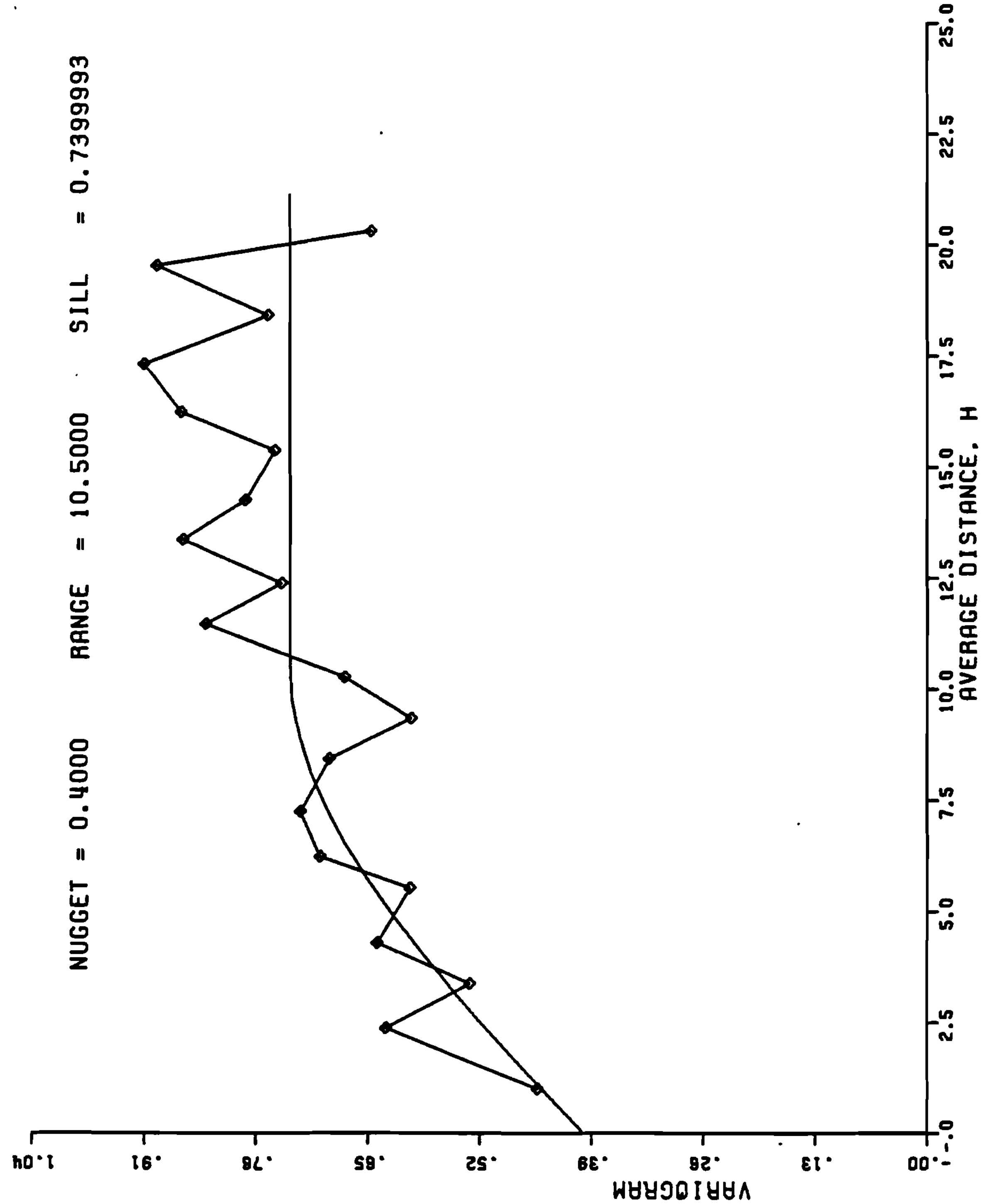


Figure 5. Covariogram of *Ln* of Permeability Data

$$Sill = \gamma_{lnK}(\infty) = Var_{lnK}(0) - Cov_{lnK}(\infty) = Var_{lnK}(0) = \sigma^2_{lnK} = 0.74 \qquad (2)$$

This is almost identical to the value of the variance of ln$K$ calculated from the sample data assuming independence (see figure 4).

The uncertainty intrinsic in the spatial variability of the permeability can be reduced by sampling or using the estimated small scale spatial structure of the fluctuation[as represented by the covariogram] to interpolate between the sampling points. This , of course, is the purpose of Kriging[see, eg,David,1978;Delhomme,1978;Journel and Huijbreghts;1978]. The degree of uncertainty reduction achieved by sampling depends on the degree of spatial correlation and the density of the sampling network. Therefore ,if one wishes to reduce parameter uncertainty in the design procedure, in order to provide more reliable and less expensive designs, then one must increase the sampling network. However, there is a trade-off between the additional information obtained and the cost of sampling .The correlation structure and the sample error may lead to a maximum density of sampling above which the information returned is simply not worth the effort. This subject will be addressed in future papers.

2.2.2. Recharge Rate Uncertainty

Recharge rate varies stochastically in both time and space, although the former will be ignored in this analysis.Perhaps more importantly, it is much more difficult to directly sample recharge than it is to directly sample a property such as permeability. One indirect approach is to infer it through an "inverse solution", but this is unrealistic in the types of design situations faced in the field. A more common indirect approach is to perform a local salt balance at each point in the field at which a permeability test is performed. The water application rate and salt concentration is the estimated from records, then compared to the salt concentration in the water found in the auger hole to yield a crude estimate of recharge rate as a function of space. Even this test is a luxury in many situations, and a uniform recharge rate for the entire field is estimated via "engineering judgement."

For the Embabe drain data 154 samples of salt concentration were made at the auger holes. The values for the salt concentration exhibited a lognormal distribution , at an 85% significance level. There was almost no correlation found between the permeability values and those of the salt values , in fact there was a slight negative correlation of $\rho_{KN} = -.014$ (see Table 1.).

2.3. Uncertainty in Prediction of the Dewatering Zone

The dewatering zone DWZ is the distance from the ground surface to the water table. The crop response to the drainage system depends on the size of this zone. A predictive model is used to estimate DWZ based on estimates of recharge rate ,N, and permeability ,K, for each alternative design drain spacing,L, and depth, D. By assuming that N and$K$ are uniform (constant) between the drains, simple analytical expressions can be used to predict DWZ. If the estimates of the uniform N and K are uncertain, then probabilistic models are employed to account for the uncertainty of the DWZ prediction, and therefore the uncertainty of the crop response. If N and K are assumed to be spatially variable, numerical models are required , with appropriate modifications to handle the stochastic nature of the variables.

The evidence collected from the Embabe area indicates that permeability is correlated over large distances, of the order of five hundred meters or so. The evidence is somewhat ambiguous because of the "nugget effect" observed in the covariogram(Figure 5), which may indicate sample error or simply reflect the fact that the closest data points are still a full 200 meters apart. If the correlation length of K is truly on the order of five hundred meters, then permeability fluctuations between two drains, spaced only 20 to 40 meters apart, will be relatively small. When this is the case it is possible to assume that K is uniform (constant) between the drains, but uncertain. It is uncertain because the samples contain errors and because there may be no direct measurement of K in that particular location, so that K must be inferred from measurements at nearby stations using, for example , Kriging.

No major spatial structure could be identified for the salt concentration data for the Embabe area. In addition ,this is a crude indirect estimate of recharge rate N . Therefore it is assumed the recharge rate N is uniform between the drains , but uncertain.

The first model presented below is based on the assumption of constant , but uncertain N and K between the drains. However, if the correlation length of N and K is somewhat smaller, approaching in magnitude the spacing between the drains, then the spatial variation of these parameters becomes important. The second model examines stochastic spatial variation using a one-dimensional numerical discretization between the drains. A third numerical model has been formulated to examine the more realistic two-dimensional horizontal flow pattern between two drains,from the collector at which they discharge up to the edge of the field. All three models are based on the approximate probabilistic modeling approach called First Order-Second Moment(FOSM) analysis[see Benjamin and Cornell,1970;Dettinger and Wilson,1981, 1982,and Wilson and Dettinger,1982]. All three models focus on predicting the water table ,h,and dewatering zone, DWZ, at the midpoint between drains, because under most conditions the water table will be a maximum at this point and DWZ a minimum. This mid-point is designated by the subscript $L/2$. The models are written in terms of water table height h. The statistics of the predicted dewatering zone DWZ are related to those of the water table height by the expressions in which the over bar presents the expected value.

$$DWZ = D - h \tag{3a}$$

$$\overline{DWZ} = D - \overline{h} \tag{3b}$$

$$\sigma^2_{DWZ} = \sigma^2_h \tag{3c}$$

2.3.1. Uniform but Uncertain Permeability and Recharge.

A model of this situation is given by the Hooghoudt equation, [Hooghoudt, 1940]. From FOSM analysis, [see, for example, Wilson and Dettinger, 1981], the first order expected value of the water table elevation midway between the drains is

$$\begin{aligned}\overline{h}_{L/2} &= f_1(L,d',\overline{N},\overline{K}) \\ &= -d' + \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{1/2}\end{aligned} \tag{4}$$

The first order estimate is identical to the deterministic estimate, with the parameters evaluated at their expected value. The vertical flow near the drains is accounted for by replacing the true depth by an equivalent depth d', which depends on the geometry :L,d, and type and size of drain.

For tile drains, the equivalent depth has been expressed in closed form [USBR, 1978]

$$d' = f_2(L, d, r)$$

$$d' = \begin{cases} \dfrac{d}{1 + \dfrac{d}{L}\left[2.55\ln(\dfrac{d}{r}) - 3.55 - 1.6(\dfrac{d}{L}) + 2(\dfrac{d}{L})^2\right]} & \text{if } 0.0 < \dfrac{d}{l} \leq 0.31 \\ \dfrac{L}{2.55\left[\ln(\dfrac{L}{r}) - 1.15\right]} & \text{if } 0.31 < \dfrac{d}{L} \end{cases} \quad (5)$$

It depends primarily on design parameters, and is not a function of recharge rate N or permeability K. When $d'=d$, the Hooghoudt model becomes a simple Dupuit model. The variance of water table estimate at the midpoint, calculated by FOSM, is [Strzepek et al., 1982; see also Wilson and Dettinger, 1981]

$$\sigma^2_{h_{L/2}} = f_3(L, d', \overline{N}, \sigma_N, \overline{K}, \sigma_K, \rho_{KN}) \quad (6)$$

$$= \left[\frac{L^2}{8\overline{K}}\right]^2 \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{-1} \left[\sigma^2_N - 2\frac{\overline{N}}{\overline{K}}\rho_{KN}\sigma_{KN} + (\frac{\overline{N}}{\overline{K}})^2\sigma^2_K\right]$$

where σ^2_N and σ^2_K are the variances of the estimated values of recharge N and permeability K, and ρ_{KN} is the correlation between N and K. In the Embabe case study, ρ_{KN} is almost zero [$\rho_{KN}=-0.014$]. When K is log normally distributed, with $Y=\log K$ normally distributed, the ratio $\frac{\sigma_K}{\overline{K}}$ in (5) is replaced by σ_Y and the remaining $\overline{K}$ in (3) and (4) represent geometric (logorithmic) averages of the permeability data. The correlation coefficient becomes ρ_{YN} .

Using the data from the Embabe area, (Table 1 with $\sigma_N=0.0004 m/d$), the predicted (3) water table elevation above the drains, and an estimate of its reliability (5) are given in Table 2. The drain spacing in this example is $L=40m$, and the depth to the impervious

Table 1. Field Data for the Embabe Case Study

Properties for	Mean	Standard Deviation
Sample K	0.085m/day	0.082m/day
Sample $Y=lnK$	-2.830	0.863
K calculated from Sampled Y	0.086m/day (geometric mean)	0.090m/day
Sample $W=lnS^1$	3.75	0.815
N calculated from Sampled W	0.0004m/day	0.0006m/day
Subjective Estimates for N	0.0004m/day	0.0004m/day

Sample correlation of N and K : $\rho_{KN} = -0.014$

[1] S = samples of salt concentration

Table 2. Statistics of Water Table Elevation for Uniform but Uncertain Parameters

Uncertain Parameters	Correlation ρ_{KN} or ρ_{YN}	$\bar{h}_{L/2}$ (m)	$\sigma_{h_{L/2}}$ (m)
K,N	0	0.299	0.396
K,N	-0.014	0.299	0.399
K	-	0.299	0.275
N	-	0.299	0.285
Y,N	0	0.299	0.374
Y,N	0	0.396*	0.374

* Second Order Estimate of Expected Value

bottom is $d=3m=d'$ (neglecting vertical flow lead losses). The first order expected value of the water table height at the midpoint is 0.299 meters, assuming K is normally distributed. The standard deviation of this estimate is 0.396 m, neglecting the slight negative correlation between N and K, and 0.395 m accounting for it. In this example, the correlation is unimportant and is ignored below. If only the permeability is uncertain, then the estimated standard deviation drops to 0.275 m, while if only the recharge rate is uncertain, it still drops to an almost identical value, 0.285 m. Recognizing that K is log-normally distributed hardly disturbs the first order estimate of the water table height, but it does decrease the estimated standard deviation by 6%. Because in this example the coefficients of variation of K and N are on the order of one, FOSM may be only approximate, having neglected higher order terms in the relationship between the estimate for h and the moments of K and N.

A second order estimate of the water table height can be found that depends only on the first two moments of K and N. Following the procedure in Benjamin and Cornell [1970], and Wilson and Dettinger [1981], this estimate is

$$\begin{aligned} \overline{h}_{L/2}\Big|_{2^{nd}order} &= f_4(L, d', \overline{N}, \overline{K}, \sigma_Y) \\ &= \overline{h}_{L/2}\Big|_{1^{st}order} + \overline{N}\frac{L^2}{8\overline{K}}\left\{d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right\}^{-1/2} \times \\ &\left[\frac{1}{2} - \overline{N}\frac{L^2}{16\overline{K}}\left\{d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right\}^{-1}\right]\sigma_y^2 \end{aligned} \tag{7}$$

where $\overline{K}$ is log-normally distributed. The importance of this additional term for the example is shown at the bottom of Table 2, where it adds almost a tenth of a meter to the expected height of the water table. The log-normality of the permeability data does not change the reliability of the prediction significantly, but the large coefficients of variation for N and K imply that first order estimates may be non-conservative, as illustrated in the example. In the remaining analyses and designs described in this paper, K will be taken as normal, and only first order estimates of expected water tables height will be made. In practice, log-normality and second order estimates would be the rule.

2.3.2. Spatial Variation in 1-D Between the Drains.

Permeability and recharge may vary between the drains. Assume that the statistics of this stochastic spatial variation are known *a priori*, and are represented in terms of expected values and a covariogram or variance-covariance. If the spatial scale of the fluctuations are large compared to the distance between the drains, then the analytical Hooghoudt model based on uniform but uncertain parameters should accurately represent the uncertain physical system. If, on the other hand, the scale of fluctuation is small compared to the distance between drains, then spatial variability between the drains becomes important and a stochastic , distributed parameter model for the physical response must be used. In most cases, this model will be solved numerically using Monte Carlo Simulation [see, for example, Freeze, 1975, or Smith and Freeze, 1979], or FOSM [see Dettinger and Wilson, 1981,1982]. Consider the drain design explained above with the Hooghoudt model, in which $L=40m$,and$d=d'=3m$. For spatially varying K and N, the groundwater response to this design is described by the Dupuit model

$$\frac{d}{dx}\left[K(h + d)\frac{dh}{dx}\right] = -N \qquad 0\le x \le L \tag{8}$$

with boundary conditions (neglecting the vertical flow under the drains, i.e., d=d'). This model can be transformed to

$$\frac{d}{dx}\left[Kd\frac{\Phi}{dx}\right] = -N \qquad 0\le x \le L \tag{9}$$

where $\Phi=\left[\frac{(h+d)^2}{2}\right]$, which has boundary conditions $\Phi=\frac{d^2}{2}$ at $x=0,L$. Solved on Dettinger and Wilson's [1981] FOSM stochastic numerical model of groundwater flow, the results, in terms of mean and standard deviation of $\Phi_{L/2}$ at the midpoint between the drains are converted to the statistics for $h_{L/2}$ via

$$\bar{h}_{L/2} = (2\bar{\Phi})^{1/2} \tag{10a}$$

$$\sigma_{h_{L/2}} = \frac{\sigma_{\Phi_{L/2}}}{2\bar{\Phi}_{L/2}} \tag{10b}$$

Spatial variation of N and K is somewhat arbitrarily represented by an exponential variogram/variance-covariance. For example, the spatial structure of $\log K$ is described by

$$Cov_{\ln K}(U) = Var_{\ln K}(0)e^{-u/l} = \sigma^2_{\ln K}e^{-u/l} \tag{11a}$$

or

$$\gamma(u) = \sigma^2_{\ln K}(1 - e^{-u/l}) \tag{11b}$$

where l is sometimes referred to as the "correlation length".

Figures 6 plots dimensionless correlation length, l/L, versus $\sigma_{[hL/2]}$, using the data of Table 1 (with σ_N = 0.0004 m/day), for uncertainty in K and N. In both cases, the uncertainty of the water table elevation prediction converges to the value predicted by the uniform parameter model. For $l/L \geq 1$, there is essentially no difference. The first order predicted mean is constant for all l. Thus, the uniform but uncertain model provides an accurate indication of prediction uncertainty, for spatial variation scales on the order or larger than the spacing of the drains.

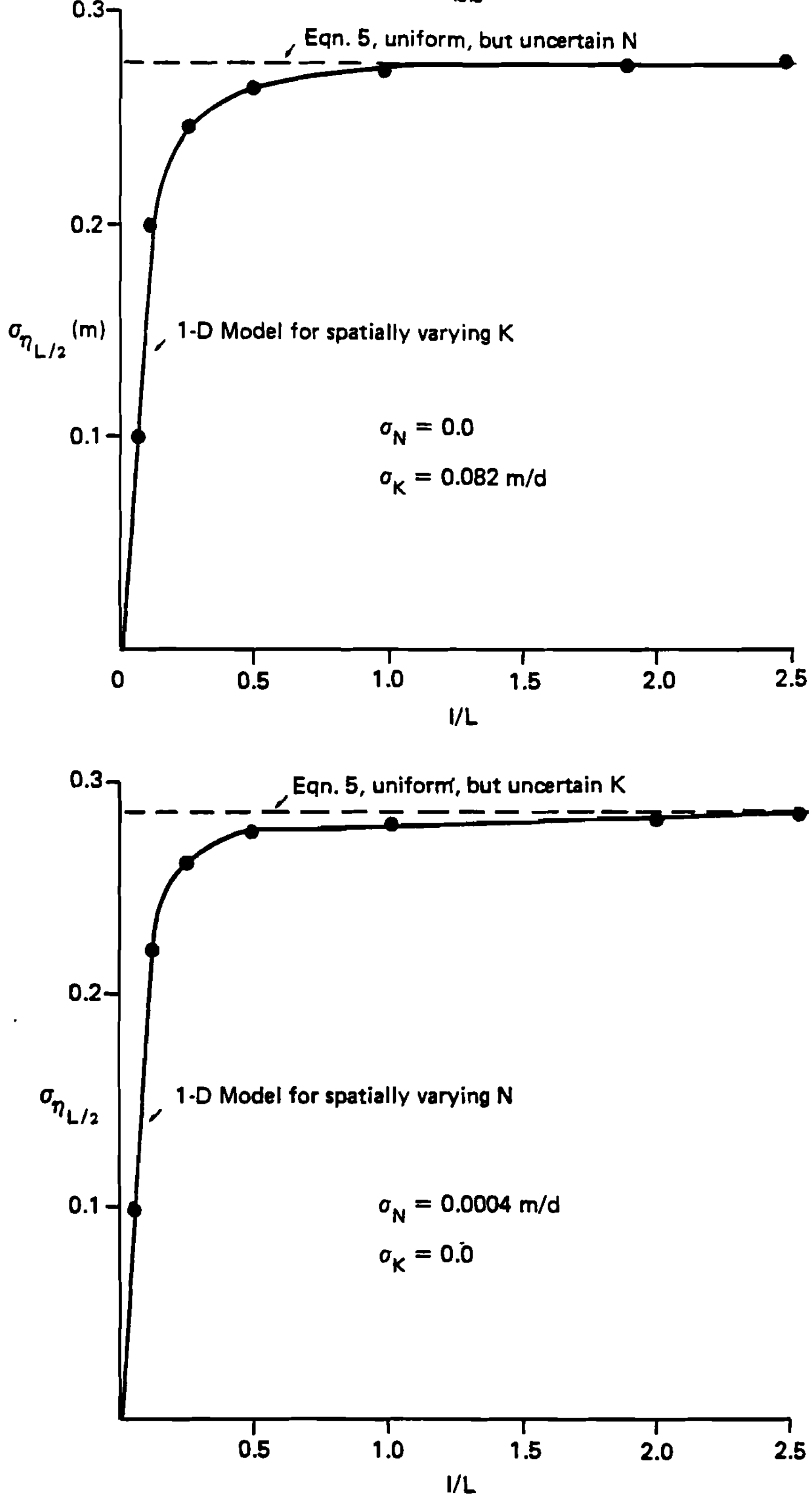


Figure 6. $\sigma_{h_{L/2}}$ versus correlation length of *K&N*

2.3.3. Spatial Variation in 2-D between the Drains and Collector.

Figure 7 is a plan of a section of a drainage project, bounded by drains to the left and right, by a collector at the top and the edge of the drained field below. Although it is not strictly correct for spatial stochastic systems, assume that the top and bottom boundaries are exact "no flow" boundaries of symmetry. Following the assumptions of the previous case, the groundwater flow in the field, for spatially variable K and N is described by

$$\frac{d}{dx}\left[K(h+d)\frac{dh}{dx}\right]+\frac{d}{dy}\left[K(h+d)\frac{dh}{dy}\right]=-N \qquad \begin{matrix} 0\le x\le L \\ 0\le y\le B \end{matrix} \tag{12}$$

equation with boundary conditions

$$(h-d)\frac{dh}{dx}=0 \qquad 0\le x\le L, y=0,B \tag{13a}$$

$$h=0 \qquad 0\le y\le B\ ,\ x=0,L \tag{13b}$$

In the transformed state with variable Φ , this becomes

$$K\frac{d^2\Phi^2}{dx^2}+K\frac{d^2\Phi^2}{dy^2}=-N \tag{14}$$

with boundaries $\Phi=\frac{d^2}{2}$ on the drains and $\frac{d\Phi}{dy}=0$ at the collector and at the lower edge of the drained field. Modeling this situation using Dettinger and Wilson's [1981] FOSM stochastic numerical model yields identical results to the previous models for the first order expected value of the water table. The sensitivity of the water table uncertainty in the middle of the field $[x=L/2, y=B/2]$ to permeability correlation is shown in Figure 8. In this multi-dimensional case, permeability variation results in a reduction of the water table uncertainty because water is now able to flow around areas of low permeability. Nevertheless, the predicted uncertainty converges to the value found for uniform but uncertain parameters

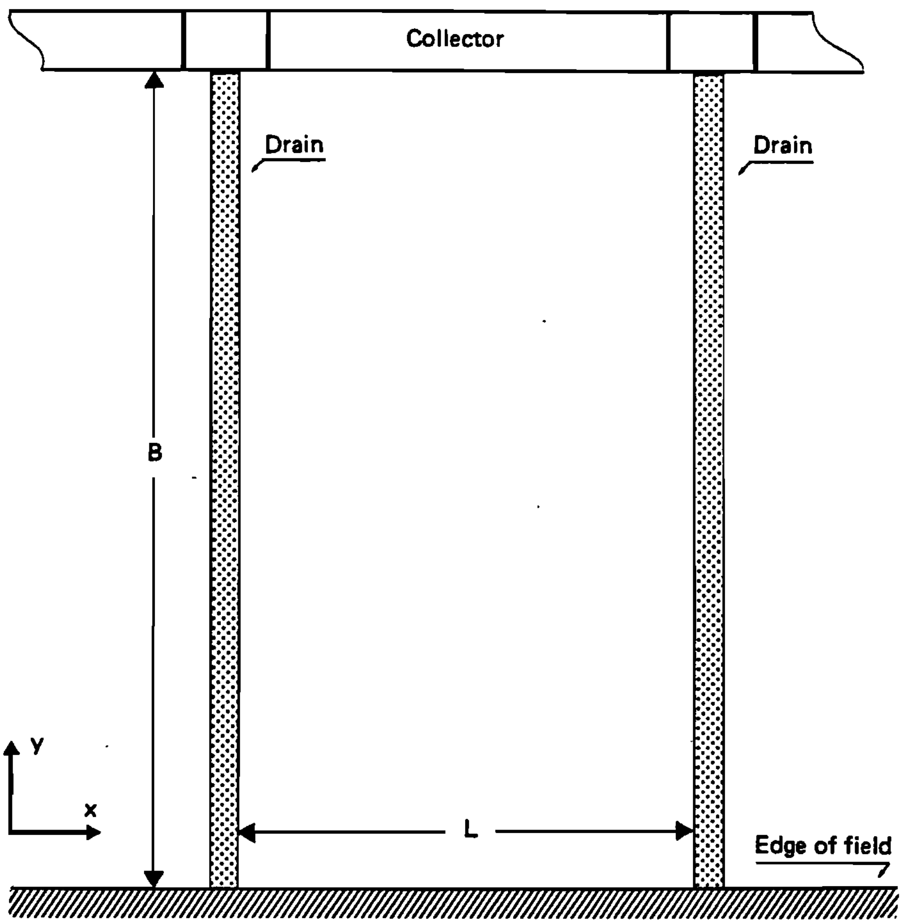


Figure 7. Plan View of Drain Field

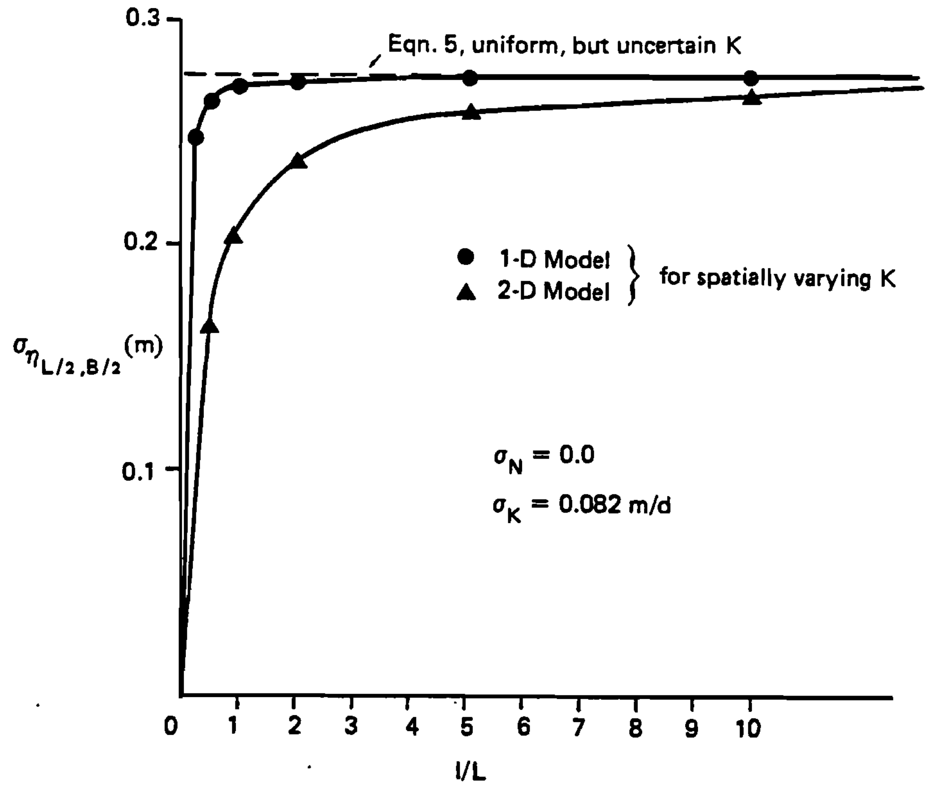


Figure 8. $\sigma_{h_{L/2}}$ versus Correlation length K in 2-D

for $l \,/\, L > 5$, once again demonstrating that for sufficiently large correlation length the simple uniform model can be reasonably employed.

2.3.4. The PDF of h and DWZ.

The FOSM models used above to examine the uncertainty of water table predictions are, by definition, second moment models. They provide estimates of the first two moments of the probability density function (PDF) of h and DWZ via (2). However, the drainage design depends on the full PDF, not solely on its moments, when the decision is based on reliability, as in this paper, or expected loss, as in Strzepek et al., [1982]. For small water table standard deviation relative to the water table height above the drains, the PDF of h or DWZ is normal. This has been demonstrated by full distributional Monte Carlo simulations for similar problems [see, for example, Freeze, 1975; Smith and Freeze, 1979], which show that the farther from the boundaries (drains) one gets, the more normal the distribution. For larger relative variance of the water table prediction, due to increasing variance of K or N, the distribution on h or DWZ becomes skewed. Since the water table cannot rise above the groundsurface, and if we presume it will not fall below the drains (steady-state), then it is clear that the true distribution on h or DWZ is finite, $0 \leq h \leq D$, and $0 \leq DWZ \leq D$, but with various shapes depending on the position between the drains and the expected height and variance of the water table elevation.

A finite distribution that would allow for varying shapes of h would be the β distribution. Further experiments need to be performed to confirm the validity of the β distribution for the pdf of h. The results of the FOSM analysis provide $\bar{h}$ and σ_h which can be directly used to estimate the β distribution. However, in this series of papers to demonstrate the procedures, h will be assumed to be normally distributed which is true for small values of $\frac{\sigma_N}{\bar{N}}$ and $\frac{\sigma_K}{\bar{K}}$.

3. Optimal Design of Agricultural Drains.

The goal of agricultural drainage is the establishment or maintenance of soil water conditions for the optimal utilization of agricultural lands. The dewatering zone that is created by the drainage system should provide for optimal crop production given that all other factors are as assumed . In present design procedures, a design dewatering zone is selected, a drain depth is fixed due to institutional or hydraulic considerations and then the design equation is solved for the drain spacing that achieves the desired dewatering zone, assuming the system to be deterministic. The issue of determining the optimal combination of depth and spacing that meets the design criteria rather than fixing the depth *a priori* is seldom addressed. Given a cost function for tile drainage, the selection of the optimal design can be cast into a Mathematical Programming Problem (MPP) which would determine the depth and spacing that minimize the cost of achieving the desired drainage performance as defined by the drainage equation and the deterministic parameters.

Since the physical parameters of the drainage equation are not deterministic, the performance of the drains becomes uncertain. This translates into uncertainty in achieving the desired soil water conditions upon which the drainage benefits are estimated. The problem then becomes how to design economically efficient drainage systems when there is uncertainty in meeting the design criteria. Mathematical Programming under uncertainty was developed to address this type of issue. Two main approaches can be identified. The first, "Chance Constraint" Programming, was presented by Charnes and Cooper [1959], and is based upon the concept of reliability in system performance. The second approach, "Stochastic Programming", uses an economic response of the system output, together with the probability distribution of the system output, to determine an expected system response. This approach was developed by Dantzig [1955]. This paper presents a "Chance Constraint" approach to optimal drainage design given uncertainty in system performance and no information about the economic response of crops to drainage. In the second paper in this series, [Strzepek, Marks, and

Wilson, 1982], a "Stochastic Programming" approach based upon empirical data of the crop response to drainage is developed, and a comparison of the two approaches and the implications of each are presented.

3.1. Chance Constraint Model

The optimal design of drains is made more complex by the fact that the design criteria for the soil water condition, the dewatering zone, cannot be met with certainty. Instead, there is a probability distribution describing the depth of the dewatering zone, *DWZ*. The question becomes how to account for this uncertainty in the drainage design process. The performance criteria for drain performance is a specified value of the dewatering zone, which represents the optimal condition for crop production. Many times there is no information about the response of crops due to variations from this optimum. In these cases, it is assumed that the design *DWZ* represents some threshold value. For *DWZ* depths greater than this design value, the yield is assumed constant at the optimal value, while for smaller *DWZ* there is a decrease in benefits. Strzepek et al., [1982], will show that this is seldom the case and that this assumption can lead to poor results. However, if no data exists on crop response, then an approach based upon this single value must be developed.

In the drain design MPP, there is a constraint that requires the dewatering zone midway between the drains to achieve a certain design value. This constraint can be met with a specified reliability, and thus a certain probability, α . A generalized "chance constraint" is defined as

$$P_r\left\{a\ x \le b\right\} \ge \alpha \tag{15}$$

which states that the constraint $ax \le b$ must be met with a probability α , when x represents the decision variables, a represents the coefficients, and b represents the resource limitations. The "chance constraint" can be transformed into a deterministic equivalent constraint when b is a random variable with known probability distribution. From the properties of the distribution of b , a value of b that satisfies the condition that α % of the distribution will be less than the value b^{α} can be found. The deterministic equivalent constraint becomes

$$a\, x \leq b^{\alpha} \tag{16}$$

which satisfies the "chance constraint".

To this point, all discussion has been applicable to the analysis of both surface and sub-surface drain design. For clarity, the remainder of the paper focuses on the tile drains. All developments presented can easily be modified to address the analysis of surface drainage design.

In the drainage design MPP, the chance constraint on the drainage performance as defined by the Hooghoudt equation is

$$P_r\left\{DWZ(D,L) \geq DWZ^*\right\} = P_r\left\{D+d' - \left[d'^2 + \frac{N^2}{4K}\right]^{1/2} \geq DWZ^*\right\} \geq \alpha \tag{17}$$

where DWZ* is the design value of the dewatering zone. The chance constraint must now be transformed into a deterministic equivalent. It was shown above that the dewatering zone can be assumed normally distributed with a mean and variance defined by a FOSM analysis of the Hooghoudt equation, when certain conditions on the uncertainty in the output parameters are met. A property of the normal distribution is that a random variable X will exceed a certain value x with a probability α when the mean of X minus "A" times the standard deviation is equivalent to x, where "A" is a function of α defined by the standardized normal distribution. With this property of the first and second moments of the dewatering zone constraint, (17) can be transformed into the following deterministic equivalent constraint

$$D+d' - \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{1/2} + A \times$$
$$\left\{\left[\frac{L^2}{8\overline{K}}\right]^2 \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{-1/2} \left[\sigma_N^2 - 2\frac{\overline{N}}{\overline{K}}\rho_{KN}\sigma_{KN} + (\frac{\overline{N}}{\overline{K}})^2\sigma_K^2\right]^{1/2}\right\} \geq DWZ^* \quad (18a)$$
$$\overline{DWZ} + A \times \sigma_{DWZ} \geq DWZ^* \quad (18b)$$

where d' is the Hoodhoudt equivalent depth (5) which must be included in the constraint set. To complete the Chance Constraint formulation of the tile drainage MPP, the complete constraint set must be defined. The depth of the drain will be constrained to be less than the maximum gravity flow in the main drainage system. Finally, the depth and spacing must not be less than zero.

The objective function for the Chance Constraint MPP for tile drainage design is to minimize the cost of drain installation. This cost is related to the number of tiles needed , which is a function of the drain spacing, and the cost per meter of laying the tiles ,which is a function of the laying machine, the depth of the drains, labor costs, fuel cost etc. These functions can vary from nation to nation, or from region to region. Christopher and Winger [1975], have developed generalized cost functions for three types of drain laying machines, based upon US Bureau of Reclamation drainage projects. El Berry [1979] has developed a detailed cost function for tile drainage installation in the Nile Delta in Egypt. The general form of the El Berry function is

$$COST(D,L) = \frac{c_1}{L}\left\{c_2 D^{c_3} + c_4\right\} \quad (19)$$

where $c_1, c_2, c_3, \text{and} c_4$ are coefficients specific to the region and the type of machine used. This paper will look at a case study of tile drain design under uncertainty in the Nile Delta using the El Berry function for the Embabe region.

A Chance Constraint MPP for tile drainage design can be formulated as follows

$$MIN \quad CapitalCost = COST(D,L) \tag{20}$$

Subject to:

$$\overline{DWZ} + A \times \sigma_{DWZ} \geq DWZ^{*} \tag{21a}$$
$$\overline{DWZ} = D - \overline{h} \tag{21b}$$
$$\sigma^2_{DWZ} = \sigma^2_h \tag{21c}$$
$$\overline{h}_{L/2} = f_1(L,d',\overline{N},\overline{K}) \tag{21d}$$
$$d' = f_2(L,d,r) \tag{21e}$$
$$\sigma^2_{h_{L/2}} = f_3(L,d',\overline{N},\sigma_N,\overline{K},\sigma_K,\rho_{KN}) \tag{21f}$$
$$D \leq D_{\max} \tag{21g}$$
$$d = Z - D \tag{21h}$$
$$D,L \geq 0.0 \tag{21i}$$

The MPP for tile drain design described above has a non-linear objective function and a non-linear constraint set. The objective function is a convex function and the constraint set defines a convex region for the Egyptian case study conditions. These two properties are necessary and sufficient conditions for obtaining a globally optimal solution to a minimization problem. An algorithm, [Wismer and Chattery, 1978], using Newton's method to solve for L in the implicit non-linear boundary to the constraint set, and a one-dimensional golden section search over D was used to find the optimal drain design for each reliability and dewatering zone chosen.

If the conditions on the uncertainty of the input parameters exist as outlined above, then the Chance Constraint MPP for drainage design is a possible tool when there is no information on the crop response function. The model can be used in many ways to aid decision makers and designers in decisions effecting drainage design under uncertainty. The next section presents an application of the model to tile drainage design for the Embabe case study region in Egypt.

3.2. Case Study Application

The results of the analysis of the uncertainty for the soil parameters in the Embabe region are used as a data base for an application of the Chance Constraint approach to drainage design.

Table 3. Parameters for Drain Design MPP

I. Physical Parameters

$\hat{N} = 0.0004 m / day \quad \sigma_N = 0.0006 m / day$

$\hat{K} = 0.085 m / day \quad \sigma_K = 0.082 m / day$

$DWZ^* = 1.0 m$

$D_{max} = 2.0 m$

II. Objective Function Parameters

$c_1 = 52.2$

$c_2 = 1.646$

$c_3 = 0.365$

$c_4 = 55.892$

Table 3 lists the statistics for the input parameters N and K as well as the design parameters; $D_{\max} = 2.0m$ and $DWZ^* = 1.0m$ for Egyptian clover. With these values and a choice of a design reliability, the constraint set is defined. Table 3 also lists the parameters for the El Berry cost function in the Embabe region of the Nile Delta. The function provides for the cost per feddan in Egyptian pounds (1 L.E. = 1.5 U.S. Dollar) of tile drain installation using a Hoes drain laying machine [El Berry,1979].

For each reliability of the depth to the water table midway between the drains, a new optimal tile drain design is found. Curve 1 in Figure 9 is a plot of the cost per feddan of the optimal solution for a given reliability. The results show that the greater the reliability, the greater the cost of the design. As the reliability of the design dewatering zone increases, more and more of the probability distribution of the dewatering zone must be greater than the design value. This is accomplished by increasing the value of the mean dewatering zone or reducing the variance, both of which require more costly designs. This curve could also be viewed in economic terms as a supply curve for reliability on a fixed dewatering zone. If the drainage project budget was limited and a maximum investment per feddan was determined, then a decision maker could determine the optimal reliability available with that investment.

Many times in the design process the drain depth is fixed due to institutional or hydraulic considerations. This adds a new constraint to the model. Curves 2 and 3 in Figure 9 show the results when the drain depth is fixed to 1.75 and 1.5 meters respectively. It is seen that the cost increases for the same reliability when the depth is fixed. This increase

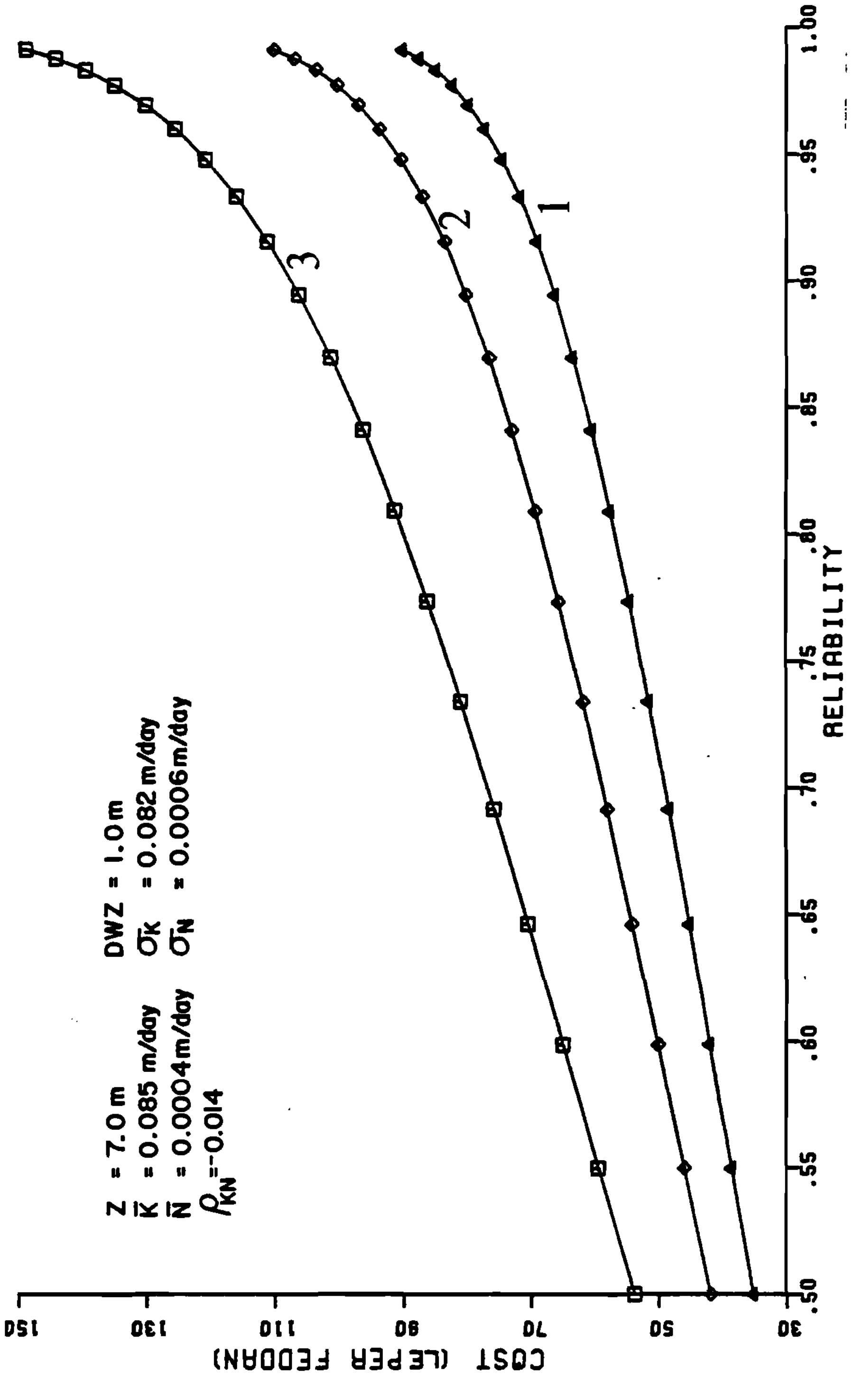


Figure 9. Chance Constraint Results

follows optimization theory, which states that as a minimization problem becomes more constrained, the cost of the optimal solution increases. The effect of *a priori* fixing the drain depth is examined in more detail in Figure 10 for a reliability of 93%. It shows for this case study that as the drain depth increases, the cost of the optimal solution decreases. The decision maker could use this type of result to examine the economic trade-offs between the cost of modifying the drainage system, (especially main drains), in order to allow deeper drains , and the cost saving resulting from installing the drains at a deeper depth.

Figures 11 and 12 are plots of the cost of the optimal solution as a function of uncertainty in the recharge rate, N, and the permeability, K, respectively. In each figure are curves for 63% and 93% reliability which are found by solving a series of models with all factors constant except the single input parameter being analyzed. The results show that the model solution is equally sensitive to the uncertainties of N and K, over the range of values expected for the case study conditions. These results provide a measure of the benefits in reducing the uncertainty about the input parameters. This type of information could be used to aid decision makers in designing data sampling networks for tile drainage.

Figure 13 illustrates the dilemma facing the designer as a result of the shortcomings of the Chance Constraint approach. The Chance Constraint approach is based upon achieving a desired reliability on a single value of the dewatering zone. The question facing the designer is not just what reliability to choose, but also upon what value of the dewatering zone to impose that reliability. In Figure 13 a series of curves reveal this problem. It shows that for a given reliability, the cost increases as the

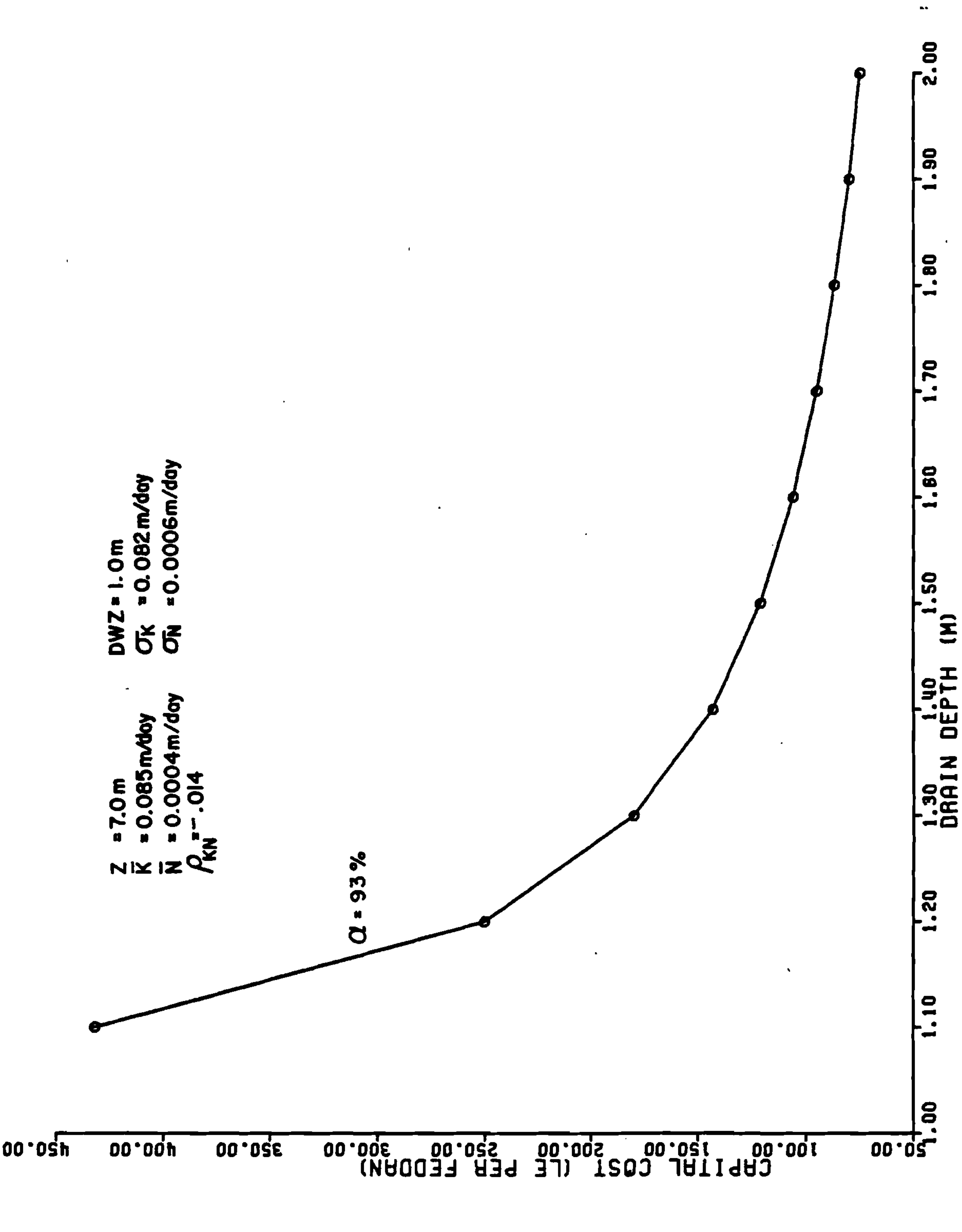


Figure 10. Optimal Solution Sensitivity to Drain Depth

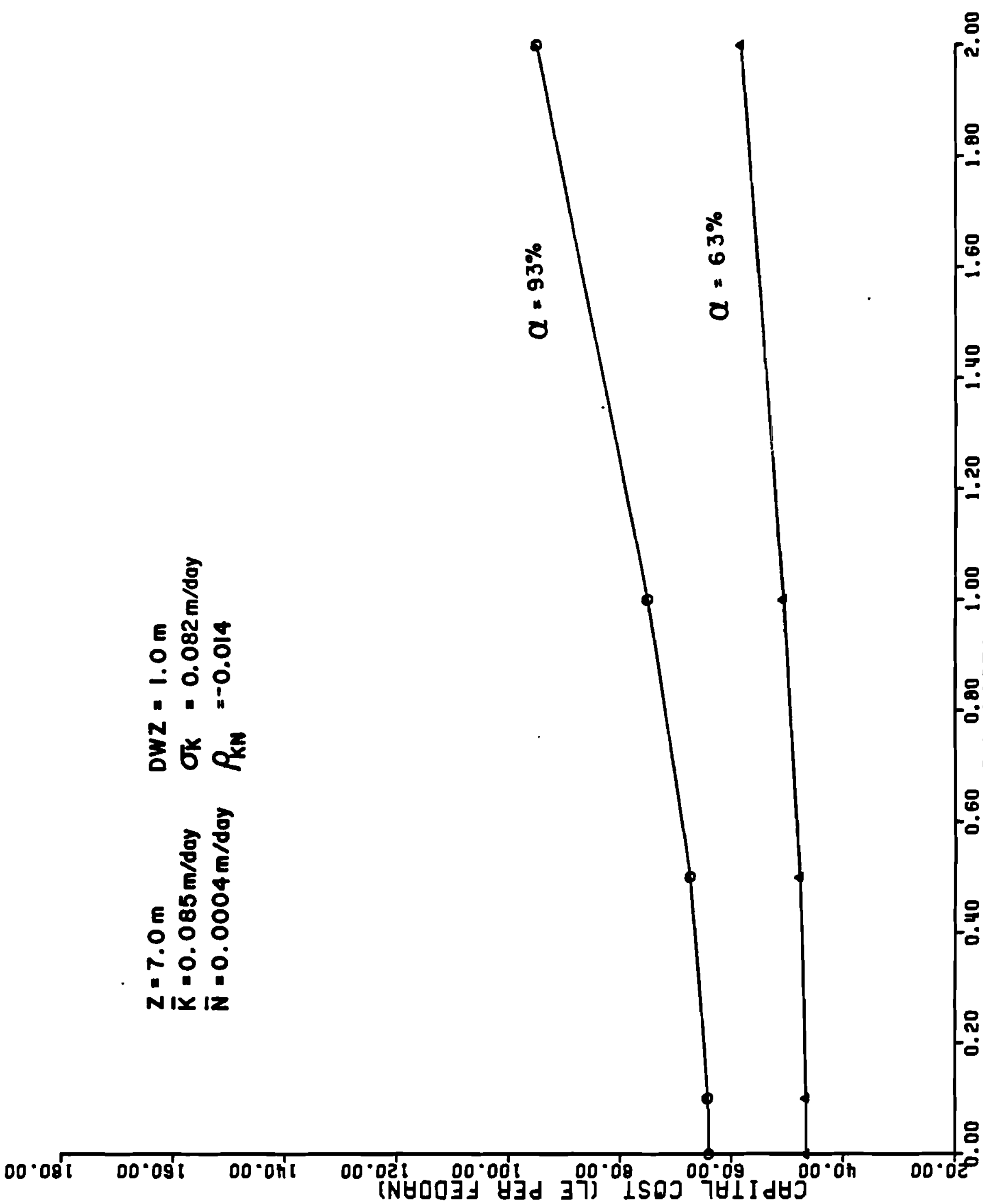


Figure 11. Optimal Solution Sensitivity to $\frac{\sigma_N}{\bar{N}}$

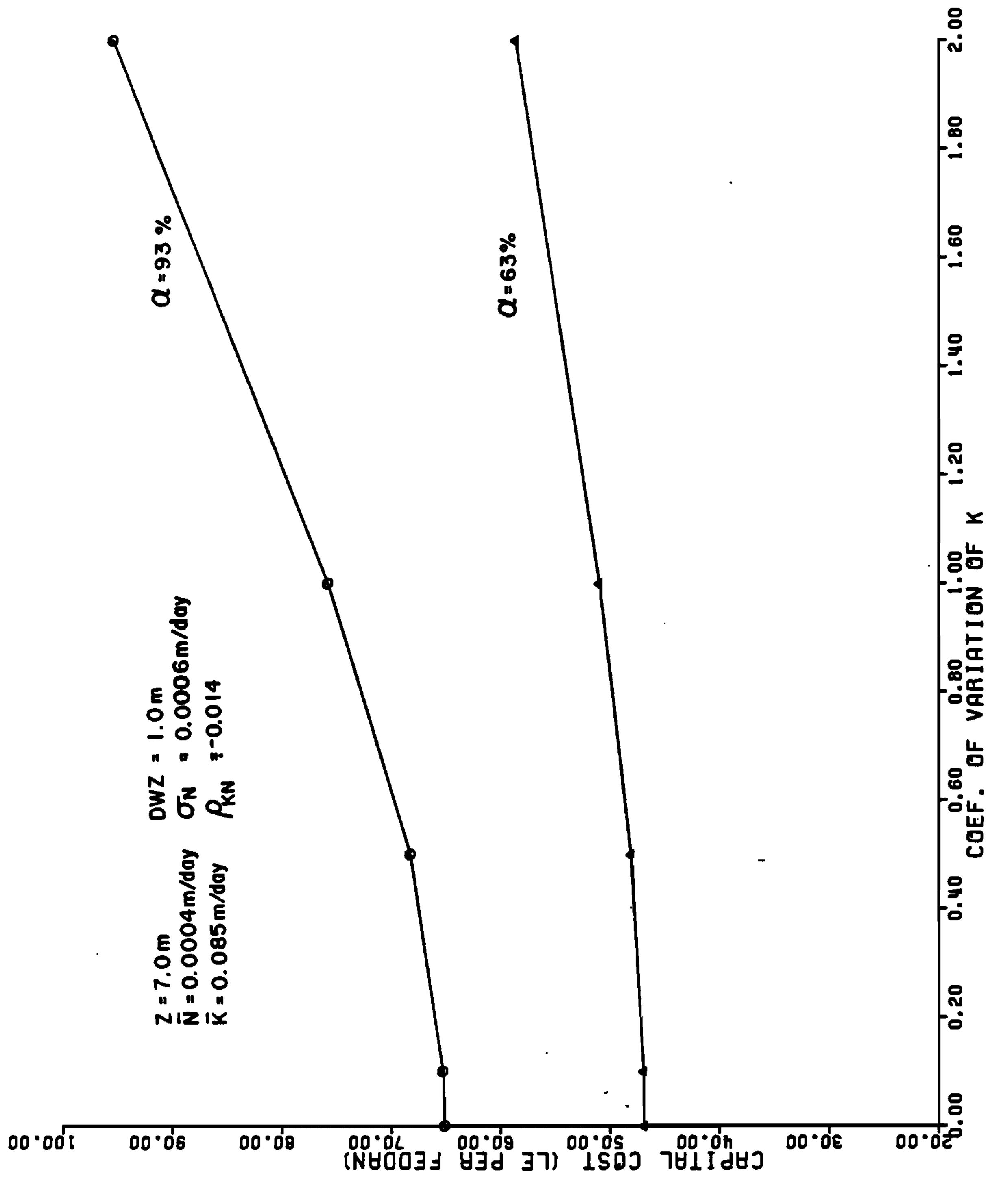


Figure 12. Optimal Solution Sensitivity to $\frac{\sigma_K}{\overline{K}}$

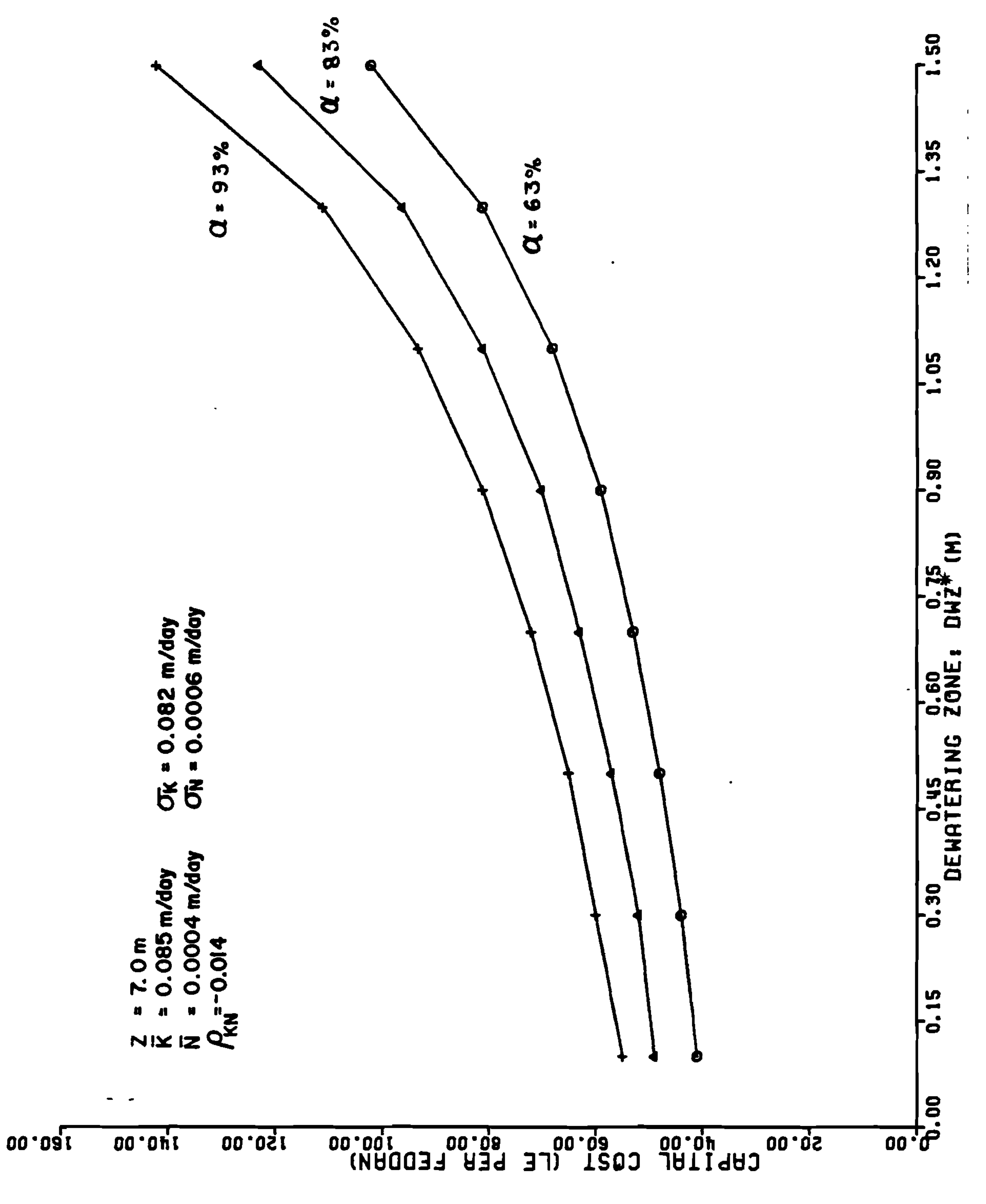


Figure 13. Optimal Solution Sensitivity to DWZ^* & α

design dewatering zone gets larger. The figure also emphasizes that increasing reliability increases cost. It does not show how the benefits to the crop increase with reliability. The decision maker must choose a dewatering zone and a reliability with no information about benefits.

The optimal solution from an economic efficiency viewpoint is to obtain the design in which the marginal benefits of drainage equal the marginal cost of drainage. In the Chance Constraint approach, a judgment decision has to be made as to a design criterion that meets this requirement. The choice of the wrong reliability could result in lost benefits due to over- or under-design. Incorporating information about the economic response of the crop to the output of the drainage system would allow for explicit consideration of the economic efficiency of the crop/drain system.

4. Conclusions

The results of the applications in this paper have shown that the assumptions of deterministic and homogeneous soil properties in drainage design are not valid and uncertainty in these properties must be accounted for in the design process. The uncertainty in soil properties was identified to take two forms, information uncertainty and spatial variability. A First Order-Second Moment analysis of the Hooghoudt steady state drainage equation was performed which allowed for the uncertainty in tile drainage performance to be quantified given data on uncertainty of the soil parameters between two drains. It was shown that spatial variability does exist in the soil properties. It was demonstrated that for the analysis of uncertainty in the dewatering zone midway between two drains that both small and large scale spatial variability could be ignored.

A Chance Constraint programming model for the optimal design of tile drains was developed which minimized the cost of drainage installation while meeting reliability criteria on drain performance. The results showed that the present deterministic approach provides only a 50% reliability on the design performance. It was shown that there are trade-offs between increased reliability of drain performance and the capital costs of drain installation, as well as increased costs due to *a priori* fixing the depth of the tile drains. The Chance Constraint model can be used to provide valuable information for the designer when faced with little data on the response of crops to drainage. However, it is difficult for the designer to choose a reliability for which marginal benefits equal marginal costs. A drawback of the Chance Constraint approach is that it does not take into account the optimal drain design when there is more than one crop being grown on the land being drained.

In the second paper of this series, a stochastic programming model will be presented that will incorporate the crop response function into the optimal design for tile drains. This model will be extended to include the design under a multiple cropping regime, and the results will be compared with those from the Chance Constraint approach.

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