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AN EXTENSION OF THE HÄFELE-MANNE MODEL FOR ASSESSING STRATEGIES FOR A TRANSITION FROM FOSSIL FUEL TO NUCLEAR AND SOLAR ALTERNATIVES

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An Extension of the Häfele-Manne Model for Assessing Strategies for a Transition from Fossil Fuel to Nuclear and Solar Alternatives

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Abstract

This paper reports on an extension of the Häfele-Manne model that assesses energy supply strategies for a transition from fossil fuel to nuclear and solar alternatives, and illustrates several optimized strategies. The expanded model solves the problem of how the electricity, petroleum-and-gas, and hydrogen produced by eight possible energy supplying alternatives (two fossil, three nuclear, two solar and one auxiliary) can be allocated to each of the three demand sectors (residential and commercial; industrial; transport) over a 100-year planning horizon, by using a ten-year period formulation. Relevant data for calculation are based on the Aerospace Corporation study for solar technologies, the NASA Systems Design Institute study for hydrogen technologies, and the Häfele-Manne study for fossil fuels and nuclear technologies. Since there are some uncertainties about these data, sensitivity analyses were carried out on the capital cost of solar power stations and on the fuel cost of coal.

I. Introduction

Häfele and Manne [3] built a linear programming model for finding an optimal strategy for a transition from fossil to nuclear fuels. Specifically, they solved the following optimization

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problem: minimize the sum of the present value of costs incurred over a planning horizon, subject to constraints on:

a) limited reserves of petroleum and gas;

b) limited reserves of low-cost uranium;

c) limited industrial capacity for construction of nuclear reactors;

d) limited financial resources available to the energy supplying sector; and

e) minimum requirements of the two secondary energy demands, i.e., electric and non-electric energy.

The energy supply alternatives considered in the original model are:

a) for electricity;

- coal-fired steam generating plant;
- light water moderated reactor (LWR); and
- liquid metal fast breeder reactor (FBR);

b) for non-electric energy;

- petroleum and gas;
- hydrogen from thermochemical water splitting by process heat of high-temperature gas-cooled reactor (HTGR); and
- hydrogen produced by electrolysis.

The model determined a cost-minimal timing of the shift to nuclear technologies (i.e., LWR and FBR for electric demands, and HTGR-hydrogen for non-electric energy demands) from the present situation which supposes that coal provides all the primary energy for generating electricity, and petroleum and gas cover all the non-electric demands.

The purpose of this paper is to extend the original model so as to optimize strategies for a transition not only to nuclear but also to solar technologies. This investigation analyzes the problem of how optimal timing is achieved if we take into account the possibilities of introducing solar as well as nuclear technologies. There are various schemes to convert solar power into useful energy; each of them is under way in the form of R & D efforts aimed at proving the economic feasibility. These schemes include:

a) solar thermal electric conversion system with central tower receiver;

b) ocean-based thermal gradient conversion system;

c) photovoltaic conversion system; and

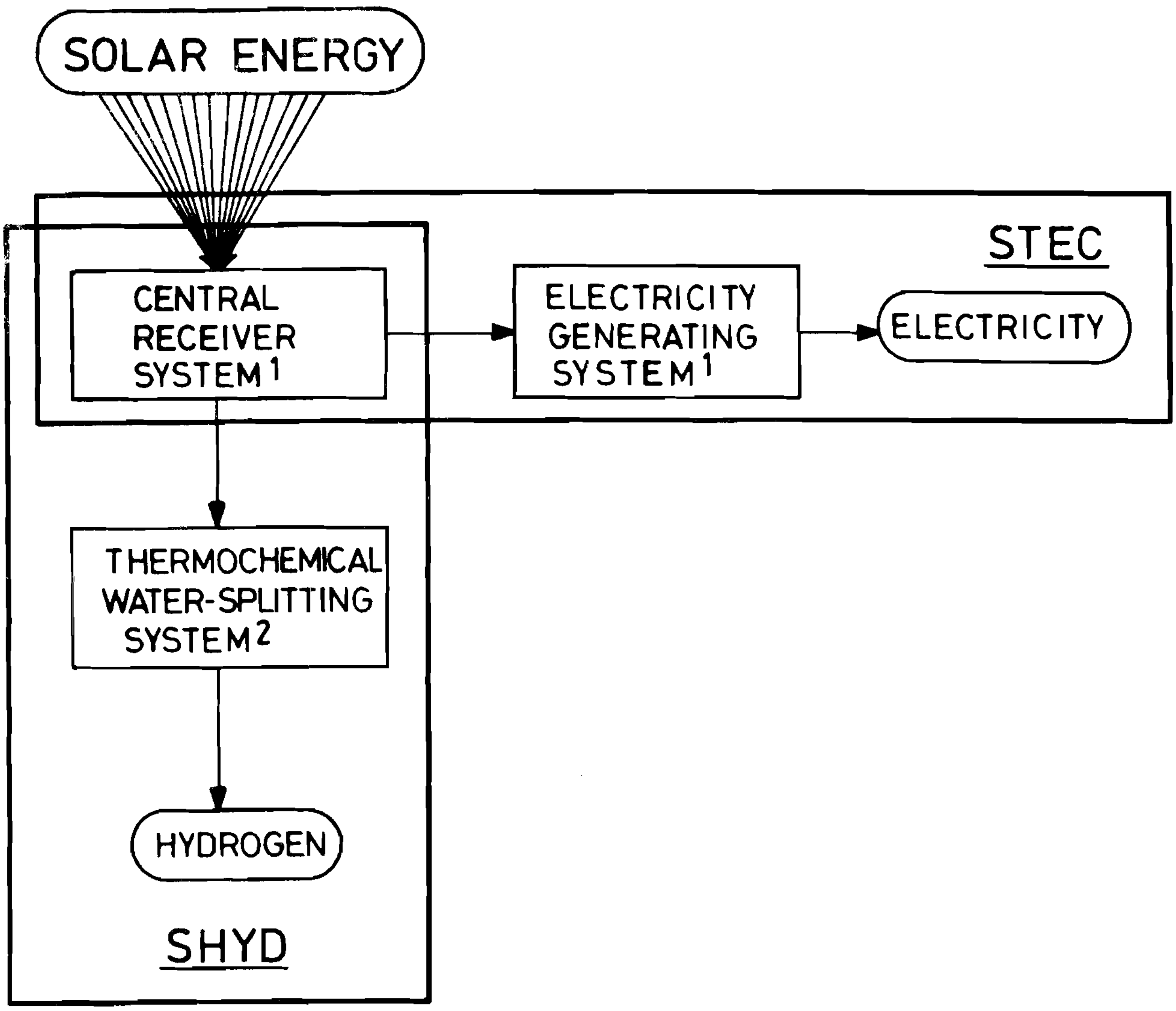
d) hydrogen through thermochemical water splitting by solar energy.

Weingart [11] recently reviewed these schemes, showing that there are still many uncertainties regarding the economic feasibility of these technologies.

This paper does not intend to draw a general conclusion on solar technology assessment; it illustrates an example of optimal transition strategies from fossil fuel to nuclear and/or solar technologies. In addition to the energy supplying technologies considered in the original model, the solar thermal electric conversion system [1] and the hydrogen production system of thermochemical water splitting by solar energy are taken as reference solar technologies (Figure 1). The energy supplying technologies treated in the expanded model are given below in Table 1.

Table 1. Energy-supplying technologies.

Types of Supply Technologies	Electric Uses	Non-Electric Uses
Fossil	(1) coal steam generating plant	(2) petroleum and gas
Nuclear	(3) LWR (4) FBR	(5) hydrogen from HTGR
Solar	(6) solar thermal electric conversion system	(7) solar hydrogen
Auxiliary		(8) electrolytic hydrogen



STEC: Solar Thermal Electric Conversion System

SHYD: Hydrogen Production System by Solar Thermochemical Process

[1]Source: Aerospace Corporation [1]

[2]Source: NASA-ASEE [8]

Figure 1. Reference solar technologies.

The original model made several suppositions:

a) coal is used only for producing electricity and the manner of converting coal into the hydrocarbon that could be used as is, is neglected;

b) both petroleum and natural gas are used only for non-electric energy demand, and oil and gas electric power plants are excluded; and

c) petroleum and gas can be aggregated to one energy supply sector.

The expanded model will also make all of these suppositions since the main purpose of expanding the model is to introduce solar technologies as energy supplying alternatives, not to treat in detail fossil fuel technologies.

Another feature of the expanded model is the classification of energy demand sectors. In the original model, the macroscopic classification was done in order to emphasize an energy supply side rather than a demand side, and to avoid the complication of model building. The original model has two demand categories: electric and non-electric energy; it treats a problem of primary energy allocation in secondary forms.

In solar technology assessment, the economic feasibility is significantly dependent on the load duration curve, since any solar electric conversion system needs to be equipped with a controlled energy storage subsystem that takes into account the time spectra difference between insolation and load duration patterns. A systems analysis study of the solar thermal electric conversion system [1] concluded that, relatively speaking, compared with fossil fuel, the solar system is more economic for intermediate peak load than it is for base load. Thus an energy model for assessing solar economics should take into account the difference between base load and intermediate peak load electricity. Accordingly, the expanded model divides electric energy demand into two categories: base load and intermediate peak loads.

A load duration pattern depends on the type of energy end use, e.g. space heating, air conditioning, water heating, ground transportation, air transportation, steel production, petrochemistry. Therefore, one needs first to assign load duration curves for each of the end use categories, and then the categories whose load duration curves are not significantly different can be aggregated. Categorization might be made by coordinating the effort of model building with the expected accuracy of mathematical formulations and numerical solutions. The demand categories of the expanded model are shown in Table 2.

Table 2. Demand Categories (Expanded Model)

Demand Sectors	Electric Uses	Non-Electric Uses
Residential and Commercial	(1) base load	(3) other than electricity
	(2) intermediate peak load	
Industrial	(4) base load	(6) other than electricity
	(5) intermediate peak load	
Transportation		(7) all

The expanded model is a linear programming model for optimizing an allocation of the energy produced by the eight technologies (Table 1) to each of the seven demand categories (Table 2), over a given planning horizon. Constraints to be considered in this optimization problem--i.e., resource availability and nuclear fuel cycle balance--are treated in accordance with the original model. Figure 2 is the schematic description of the problem, illustrating the conceptual framework of the energy supply/demand system considered in this examination.

II. The Expanded Model

A. Supply Alternative Characterization

The original model characterizes a supply alternative under the following presumptions:

(1) A time differential equation of an energy production activity of each of the technologies can be approximated by a three-year time-step difference equation; a full power operation throughout the entire thirty-year service life is assumed.

(2) To represent limitation on the availability and the rate of adoption of new technologies, upper bounds are imposed upon the annual construction rates of nuclear power plant capacity.

(3) Natural resource availability (e.g., of coal, petroleum and gas, and natural uranium) are fixed exogenously to

the cumulative sums of each resource consumption. The original model supposes that the availability of fossil fuels is independent of costs; however, the availability of natural uranium depends on costs. Specifically, two grades of natural uranium, i.e., low cost and high cost, are defined in the original model in such a way that, compared with the length of the planning horizon, the reserve of low cost uranium is limited while the reserve of high cost uranium is unlimited.

(4) Man-made resource availability (e.g., of plutonium and uranium-233) are determined endogenously by nuclear fuel cycle equations that correspond to the reactor configuration defined in the original model. That is, the FBR produces not only plutonium but also uranium-233, thus meeting demands for both plutonium and uranium-233 for an initial inventory of the FBR and an annual replacement of the HTGR, respectively.

The four above presumptions play an important role in the original model since they constitute a mathematical framework of the energy model. The expanded model makes only minor changes to the mathematical framework of the original model; these changes are as follows:

(1) A ten-year period formulation is used in place of a three-year period so that even the inclusion of solar technologies in the expanded model and the disaggregation of demand sectors might bring about a reasonable length of computing time. In addition, the equation is rewritten in such a way that each of the energy supplying technologies does not necessarily produce full power, and can infact operate below the level of full power if regarded as obsolescent.[1] Figure 3 shows how to formulate the ten-year period equation for an energy production activity with a flexible power level. It is supposed that all the plants constructed during a ten-year period will start operating at the middle of that period, and that therefore the average capacity during that period is one half of the full power level. This supposition is used to make the simplified scheme of plant installation with a thirty-year service life as shown in Figure 3. For a fixed power level operation, there is no distinction between this plant installation scheme and the corresponding plant operation scheme. However, for a flexible power level operation, the operating factor is determined endogenously, and the operation scheme should be different from the installation

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[1]Konno and Srinivasan [5] have reported on the effect of flexible power level operation on an optimal solution.

scheme. Figure 3 shows five different operation schemes as sequential series, in order to illustrate that the energy production activity of each of the technologies at any time is expressed by the sum of activities of four plants of different age.

(2) In the expanded model, upper bounds are imposed upon the annual construction rates of solar power plant capacity and of nuclear power plant capacity. The mathematical description is the same as that used in the original model: an upper bound is fixed a priori by using two parameters, one for the possible starting introductory year and the other for the maximum limit of increment of construction rate.

(3) The third presumption is concerned with the mathematical treatment of natural energy resource availability. Both the original and the expanded models take into account one grade of each of the fossil fuels and two grades of natural uranium, depending on costs.

(4) In the expanded model, the nuclear fuel cycle equations are reformulated for each of the nuclear fuels--i.e., natural uranium, plutonium and uranium-233--by means of the four-phase refueling scheme as shown in Figure 4. Special attention is paid to time lags for fuel preparation and reprocessing, and to the relationship of a refueling scheme and the operation scheme as shown in Figure 3. Because of the lag-times, there are non-stationary fuel flows at the beginning stage (I) and at the end stage (IV). Since the operation scheme is not identical with the installation scheme, a distinction should be drawn between the fuel flow that is related to installed capacity and the fuel flow that is related to used capacity. Figure 4 gives details of these fuel flows.

B. Demand Projections

In the original model, energy demand projections are made in terms of a secondary energy form--electric or non-electric energy--taking into account three different scenarios called model societies 1, 2, and 3. In societies 1 and 2, the demands are exogenous and the difference between them is that society 1 assumes that the demands will be saturated, and that society 2 assumes that the demands will continue to increase at a constant rate. In the case of society 3, the demands are endogenously determined on the assumption that market demands are the outcome of a utility maximizing process.

A part of the expanded model for assessing energy demand projections is built so that the model societies 1 and 2 can be applied even for the more disaggregated demand sectors--residential-and-commercial, industrial and transport.

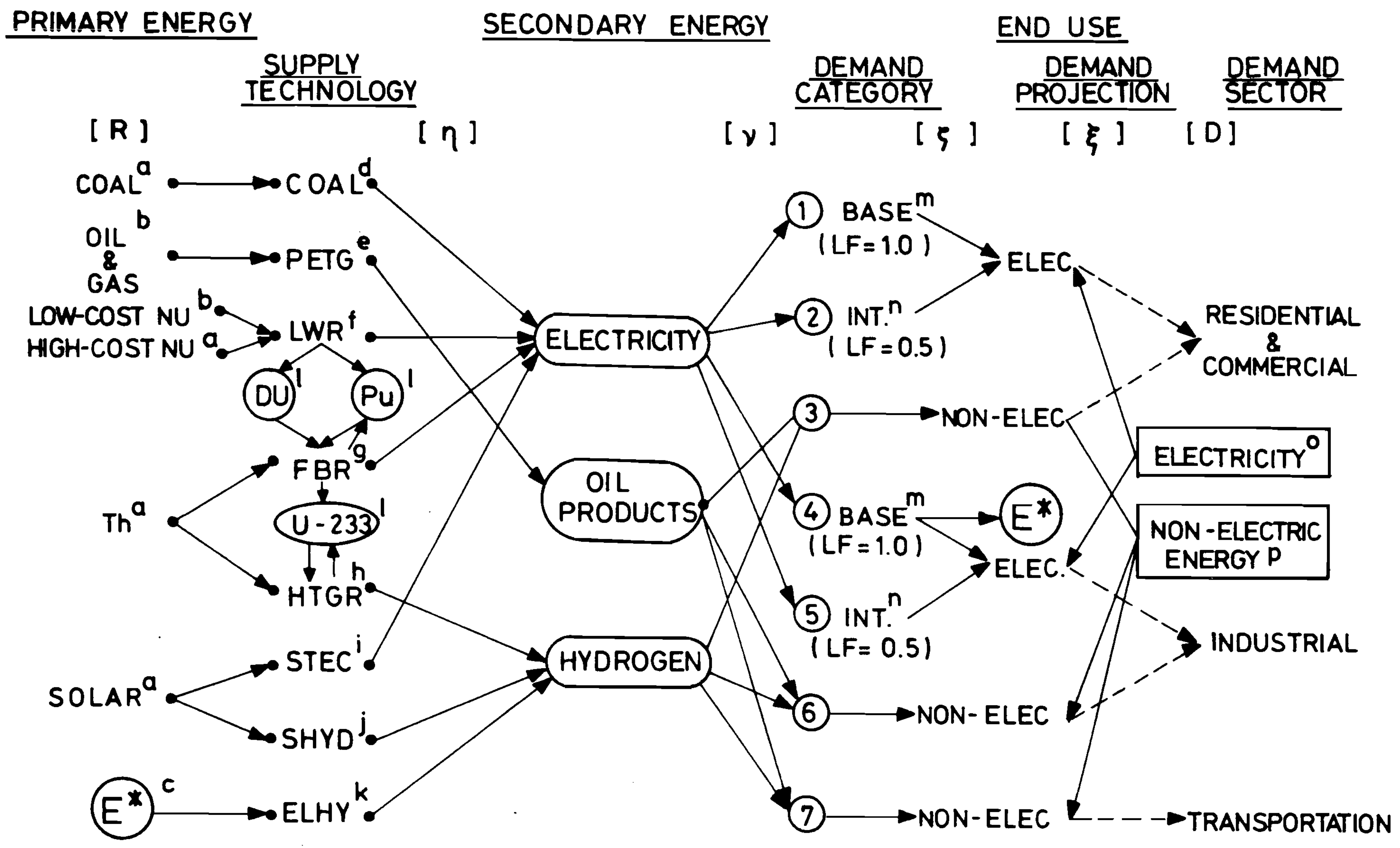


Figure 2. Schematic description of energy allocation problem.
(For notes see following page.)

Notes for Figure 2.

a. Virtually unlimited reserves.

b. Limited reserves.

c. Electricity for electrolytic hydrogen is determined endogenously as one part of base electricity of industrial use.

d. Coal steam generating power plant.

e. Petroleum and gas refinery plant.

f. Light water reactor power plant with enrichment and reprocessing.

g. Fast breeder reactor power plant with reprocessing.

h. Thermochemical hydrogen production plant with high-temperature gas-cooled reactor.

i. Solar thermal electric conversion plant by central receiver system.

j. Thermochemical hydrogen production plant with central receiver system of solar energy.

k. Hydrogen production plant by electrolysis.

l. Availability of man-made nuclear resource is determined endogenously by nuclear fuel cycle equation.

m. Base load electricity, load factor, L = 1.0.

n. Intermediate load electricity, load factor, LF = 0.5.

o. Total demand of electricity is given by Häfele-Manne model society. Electric use for transportation is neglected.

p. Total demand of non-electric energy is given by the Häfele-Manne model society.

[R] = Resource availability.

[η] = Energy supply efficiency.

[ν] = Fuel utilization factor.

[ζ] = Load duration factor.

[ξ] = Electricity or non-electric energy allocation factor into each demand sector.

[D] = Demand constraints.

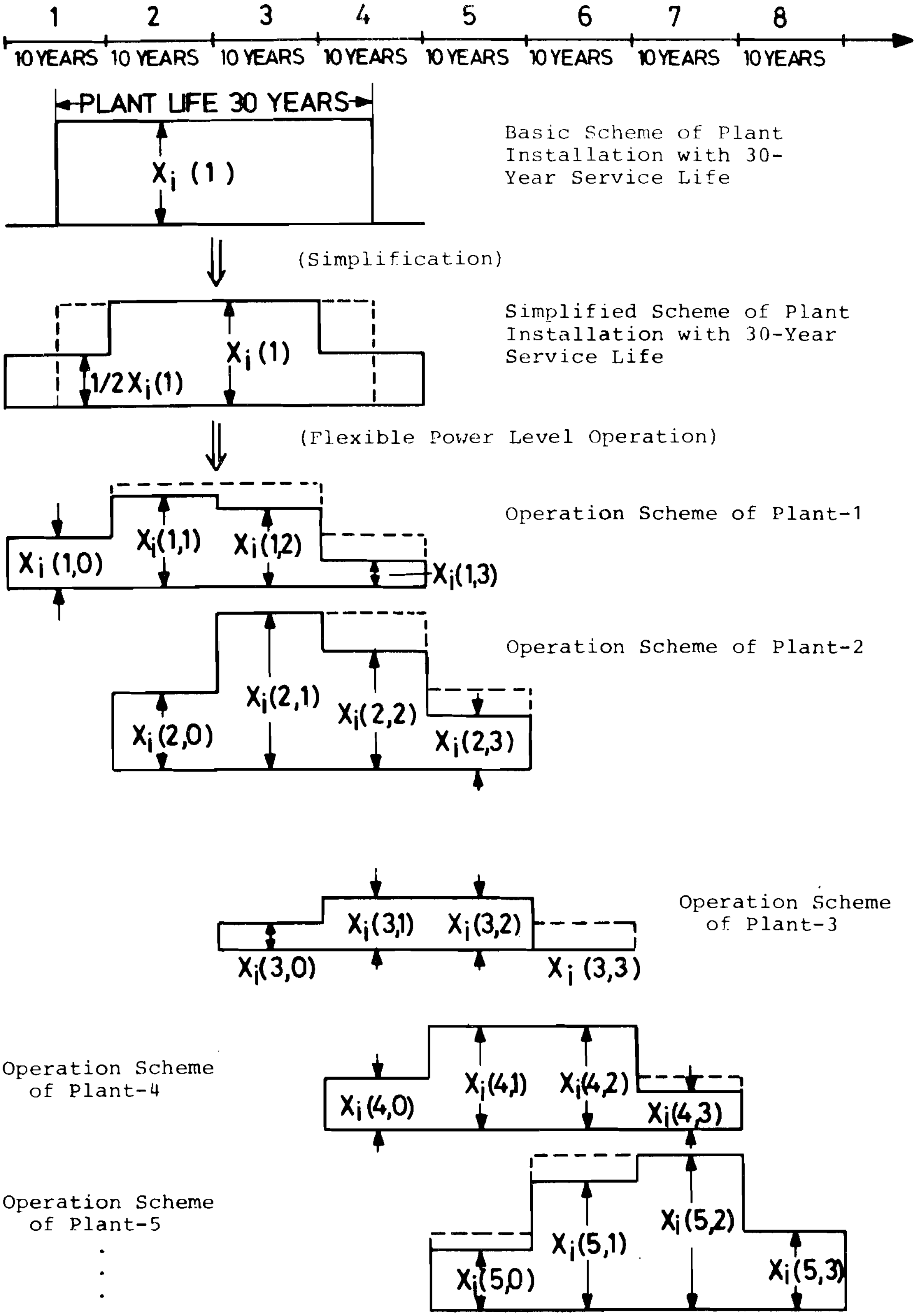


Figure 3. Four-phase scheme of plant installation and operation with flexible power level and fixed service life. (For notes see following page.)

Notes for Figure 3.

$X_i(h)$: Installed capacity level of technology i which is constructed at time-step h.

$x_i(h,\tau)$: Used capacity level of technology i which is constructed at time-step h and whose age is τ decades.

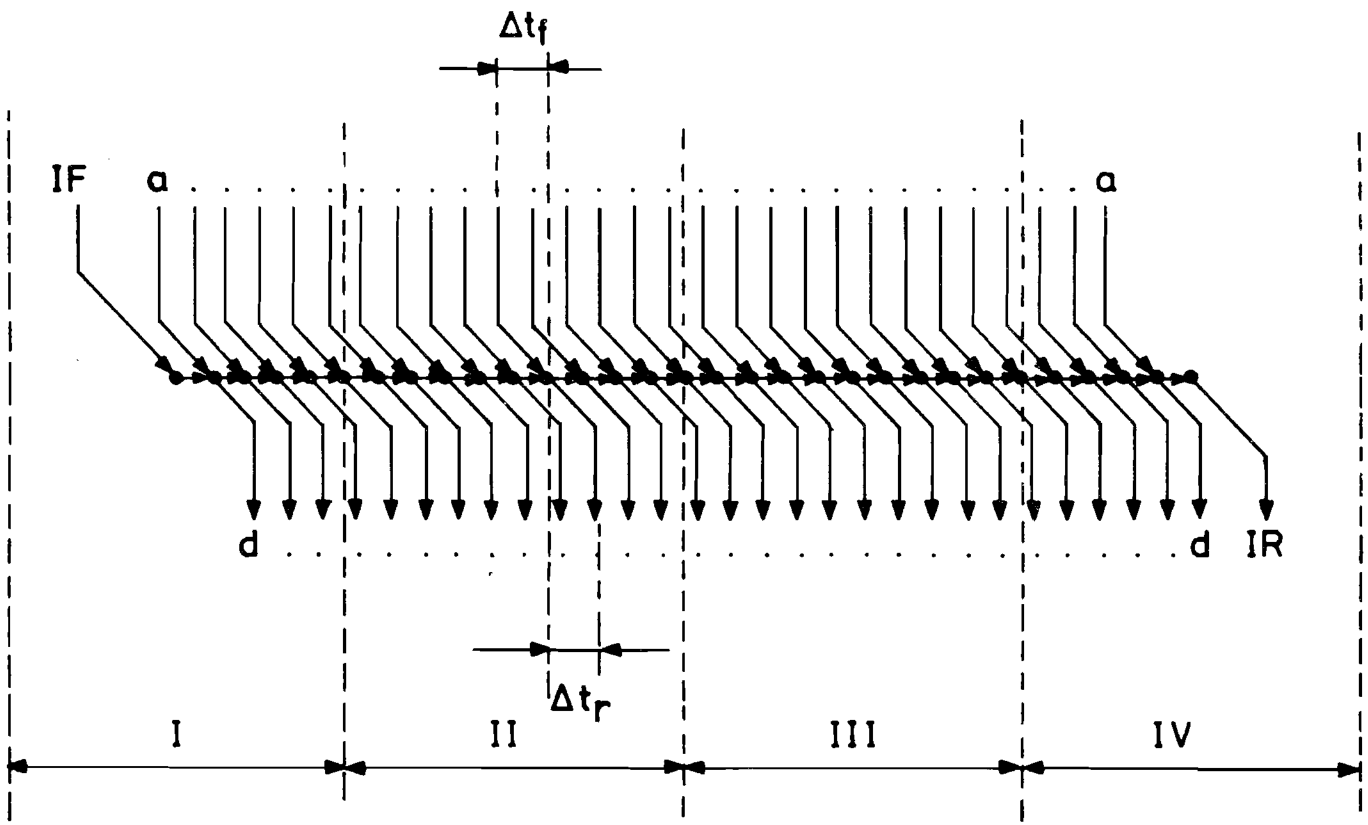
$$X_i(h,\tau) \leq \theta\ (\tau) \cdot X_i(h)$$

where,

$$\theta(\tau) = \begin{matrix} 0.5 \text{ for } \tau = 0,3 \\ 1.0 \text{ for } \tau = 1,2 \end{matrix} \quad .$$

$U_i(h)$: Production activity of technology i at time-step h.

$$U_i(h) = \sum_{\tau=0}^{3} x_i(h-\tau,\tau) \quad .$$



	I	II	III	IV
Fuel Input :	$IF \cdot X_i(h)$ + $(4 + \Delta t_f) \cdot a \cdot X_i(h,0)$	$10 \cdot a \cdot X_i(h,1)$	$10 \cdot a \cdot X_i(h,2)$	$(5 - \Delta t_f) \cdot a \cdot X_i(h,3)$
Fuel Output:	$(5 - \Delta t_r) \cdot d \cdot X_i(h,0)$	$10 \cdot d \cdot X_i(h,1)$	$10 \cdot d \cdot X_i(h,2)$	$IR \cdot X_i(h)$ + $(4 + \Delta t_r) \cdot d \cdot X_i(h,3)$

Figure 4. Four-phase refueling scheme of nuclear reactor with thirty-year plant life. (For notes see following page.)

Notes for Figure 4.

IF = Initial Inventory Requirement

a = Annual Replacement Requirement

d = Annual Recovery

IR = Final Inventory Retirement

Δt_f = Lag-time for Transportation, Enrichment and Fabrication

Δt_r = Lag-time for Cooling and Reprocessing

I = First 10 years

II = Second 10 years

III = Third 10 years

IV = Fourth 10 years

Two additional parameters are defined: an electric or a non-electric energy allocation factor, and a load duration factor. An electric or non-electric energy allocation factor allocates the energy demands projected by model societies 1 or 2 to each of the three demand sectors at each of the points of time. Thus an energy allocation factor must be assigned for each of the six energy flows: electricity to three sectors, and non-electric energy to three sectors. A load duration factor is concerned with the share between base and intermediate load electricity, and must be assigned a priori to each of the demand sectors at each of the points of time. By making use of these two factors, the energy demands for each of the seven categories (Table 2) can be given consistent with the projections of the model societies 1 or 2.

The energy demands fixed in the above-mentioned manner will be provided by the previously defined four types of supply technologies--fossil, nuclear, solar and auxiliary. This linkage between supply and demand is represented by a supply/demand balance equation for each of the demand categories. An additional parameter associated with inter-fuel substitutability for the same end use must be taken into account in the formulation of the equation. Using a ground-transportation purpose as an example, the model considers two alternatives: oil and hydrogen. In the case of oil, a car with a gasoline driven engine is used. In the case of hydrogen, a hydrogen combustion engine must be developed practically. Combustion engines have different efficiency rates, and a BTU of each of the fuels (oil and hydrogen) yields a different horsepower that is useful. Inter-fuel substitutability depends on how energy is used in each of the end uses. Thus the supply/demand balance equation must include efficiencies of each of the energy uses, called the fuel utilization factor.

III. Input Data Preparation

Natural resource availability. Table 3 gives the value of natural resource availability that is used for computation. Since this examination illustrates an optimal strategy for the transition to nuclear and/or solar technologies (as compared to the optimal strategy for only nuclear technology shown by the original model) all of the values on resource availability assessment are the same as those in the original model.

The maximum available amount of coal is not considered, and the amount of petroleum and gas is treated only optionally. The method used to assess the availability of low-cost natural uranium is unchanged. Hence, low-cost uranium at \$15/lb of U_3O_8 can be used up to the limit 2.0×10^6 metric tonnes of U. High cost uranium at \$50/lb of U_3O_8 is unlimited.

Table 3. Natural resource availability.

1. <u>Coal</u> (10^{18} BTU)

$$RA_{COAL} = \infty$$

2. <u>Petroleum and Gas</u> (years in terms of 1970 US annual consumption rate)

$$RA_{PETG} = \left.\begin{array}{l} 40\ (2.250 \times 10^{18}\ \text{BTU}) \\ 60\ (3.375 \times 10^{18}\ \text{BTU}) \\ 80\ (4.500 \times 10^{18}\ \text{BTU}) \\ 100\ (5.625 \times 10^{18}\ \text{BTU}) \end{array}\right\} \text{(Optional)}$$

3. <u>Natural Uranium</u> (metric ton of U)

$RA_{NULC} = 2.0 \times 10^6$ for \$15/lb of U_3O_8

$RA_{NUHC} = \infty$ for \$50/lb of U_3O_8

Upper bounds on annual construction rates of nuclear and solar plants. As regards nuclear reactors, the data are provided by the original model. Only a small change is made so that the data fit the ten-year period formulation. As for solar technologies, it is more difficult to assess the value because of less industrial experience. Therefore, in this paper a provisional assessment is made so that the upper bounds of the solar thermal electric conversion system and of the solar hydrogen system will be equal to the upper bounds of the FBR and of the HTGR hydrogen, respectively. Figure 5 shows the upper bounds assumed here, and compares them with the corresponding maximum permissible installed capacity.

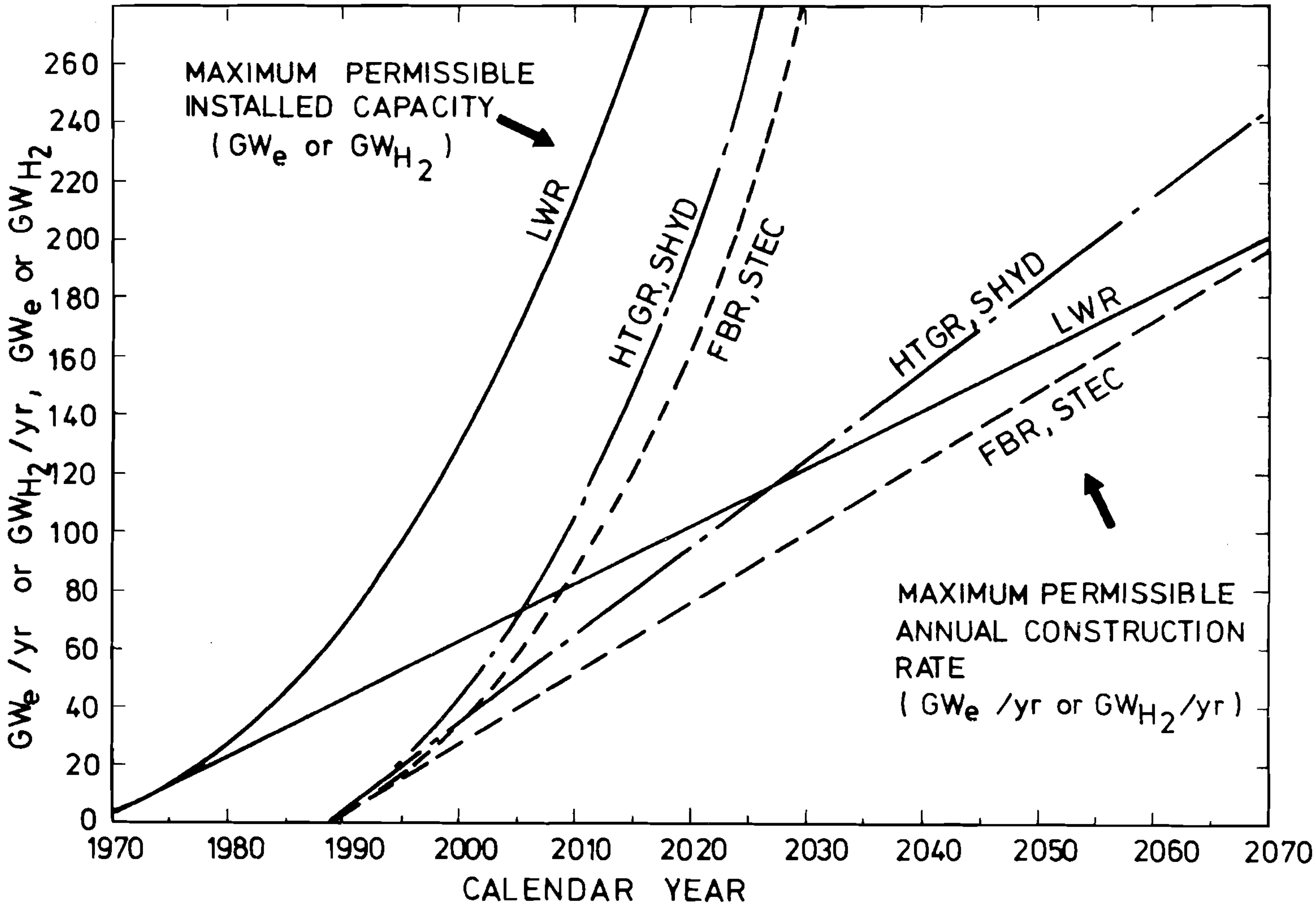
Reactor data. As stated previously, nuclear fuel cycle equations are rewritten in the expanded model in accordance with the ten-year period formulation. Therefore, relevant reactor data are resettled so that they may be used for the revised formulation. Table 4 provides the data built in the expanded model, and the footnotes to the table state how to prepare these data. The relationship between the data and the simplified four-phase refueling scheme shown in Figure 4 is given in Appendix A.

Energy supply efficiency. The value of energy supply efficiency that has been selected is shown in Table 5. As far as fossil fuel, nuclear and auxiliary technologies are concerned, the values fixed in the original model are used in the expanded model without any changes. In the case of the solar thermal electric conversion system, the efficiency assessed in [1] is taken unchanged. In the case of the solar hydrogen, the value in Table 5 is obtained by multiplying the efficiency of the thermochemical water-splitting system fixed by [3] with the efficiency of the central receiver system assessed in [1].

Electric or non-electric energy allocation factor. This energy allocation factor should be assessed by demand projections for each of the demand sectors, taking into account the total demands for electric and non-electric energy fixed by the model society. It has been found in [10] that the demand study of Hoffman [4] is useful for this purpose. While the total amounts of electric and non-electric energy demands projected by [4] are not equal to the amounts fixed by the model society, relative values of energy demands allocated to each of the demand sectors by [4] can be applied to the model society.

Based on this comparison, the electric or non-electric energy allocation factor may be assigned as shown in Figure 6, where model society 1 is taken as an example.

Load duration factor. This factor should be assessed by an electricity load duration curve. However, it is difficult to predict a load duration curve over a long planning horizon. Therefore, in this examination, an example of the monthly demand pattern predicted in [2] is taken as input data. The corresponding curve is shown in Figure 7.



STEC: Solar Thermal Electric Conversion System

SHYD: Solar Hydrogen by Water Splitting

Figure 5. Upper bounds on annual construction rate of nuclear and solar power plants.

Table 4. Relevant reactor data.

Relevant Reactor Data			Nuclear Reactor	LWR	FBR	HTGR
Initial Inventory Requirement	Natural Uranium,	NU	IF_{NU} (ton/GW_e)	500[a]	0	540[a]
	Fissile Plutonium,	PU	IF_{PU} (ton/GW_e)	0	2.00[a]	0
	Uranium-233,	U3	IF_{U3} (ton/GW_e)	0	0	0
Annual Replacement Requirement	Natural Uranium,	NU	a_{NU} (ton/$GW_e \cdot$ yr)	210[b]	0	0
	Fissile Plutonium,	PU	a_{PU} (ton/$GW_e \cdot$ yr)	0	.70[b]	0
	Uranium-233,	U3	a_{U3} (ton/$GW_e \cdot$ yr)	0	0	.48[b]
Annual Recovery	Natural Uranium,	NU	d_{NU} (ton/$GW_e \cdot$ yr)	30[b]	0	0
	Fissile Plutonium,	PU	d_{PU} (ton/$GW_e \cdot$ yr)	.17[a]	.86[b]	0
	Uranium-233,	U3	d_{U3} (ton/$GW_e \cdot$ yr)	0	(.29)[d]	.19[d]
Final Inventory Retirement	Natural Uranium,	NU	IR_{NU} (ton/GW_e)	100[c]	0	0
	Fissile Plutonium,	U	IR_{PU} (ton/GW_e)	.34[c]	2.32[c]	0
	Uranium-233,	U3	IR_{U3} (ton/GW_e)	0	(.44)[c]	1.35[d]
Lag-Time for Preparation of Fed Fuels			Δt_f (years)	1.5[e]	.75[e]	.75[e]
Lag-Time for Reprocessing of Spent Fuels			Δt_r (years)	1.5[e]	1.25[e]	1.25[e]

[a] W. Häfele and A. Manne [3].

[b] Three-batch refueling. Net annual requirement is the same as [3].

[c] Uniform fuel "burnup" and three-batch refueling are assumed.

[d] Nuclear News (February, 1973).

[e] WASH-1139 (1974).

Table 5. Energy supply efficiency.

Supply Technology, i	Efficiency, η_i
1 = COAL	.40[a]
2 = PETG	1.00[b]
3 = LWR	.33[a]
4 = FBR	.40[a]
5 = HTGR (HTR)	.50[c] (.40[a])
6 = STEC	.19[d]
7 = SHYD	.35[e]
8 = ELHY	.80[f]

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[a] Thermal efficiency of producing electricity [3].

[b] Efficiency of refinery [3].

[c] BTU of produced hydrogen/BTU of consumed fuel [3].

[d] BTU of generated electricity/BTU of collected solar energy [1].

[e] BTU of produced hydrogen/BTU of collected solar energy [3], [1].

[f] BTU of produced hydrogen/BTU of used electricity [3].

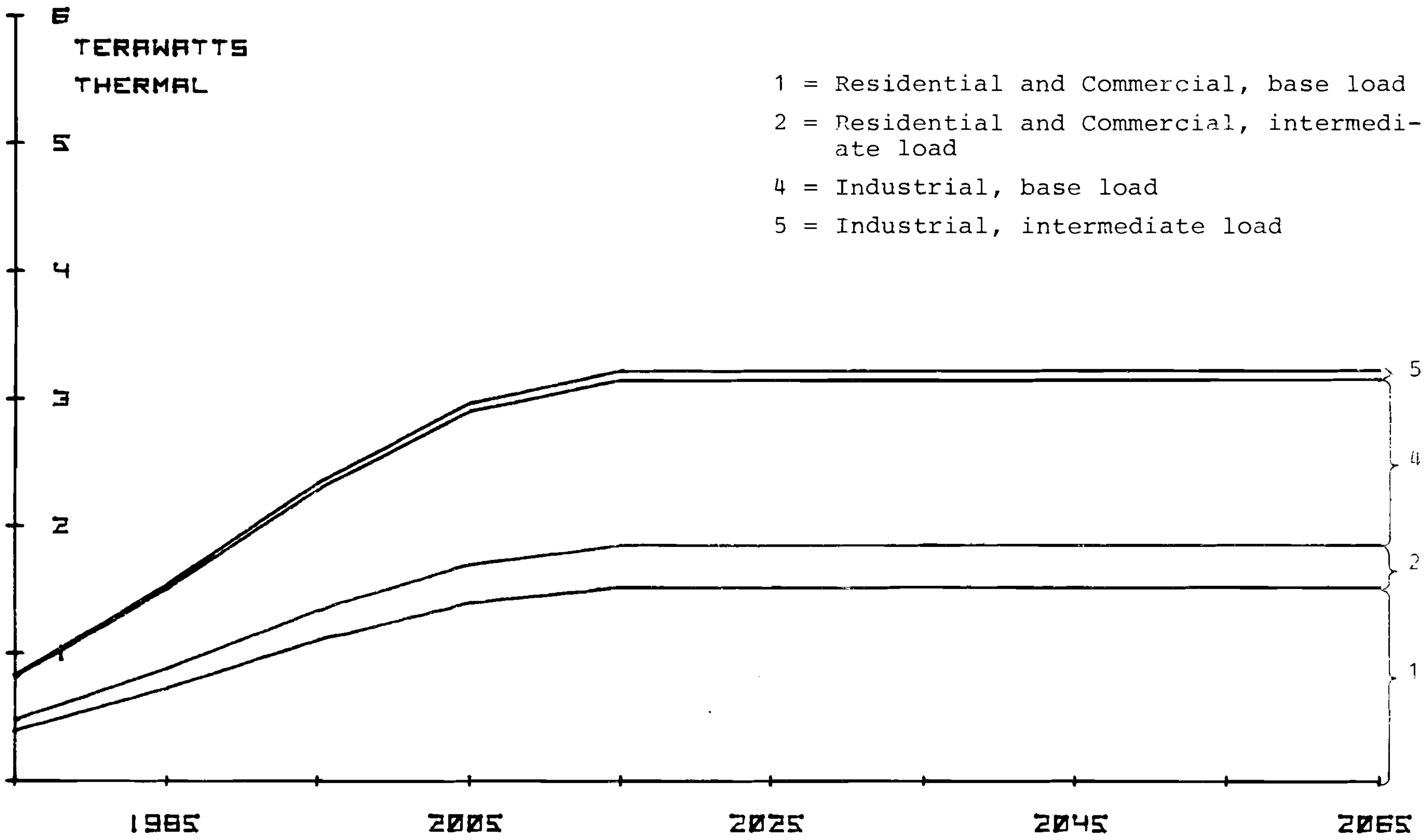

Figure 6a. Model society 1: electric energy demand.

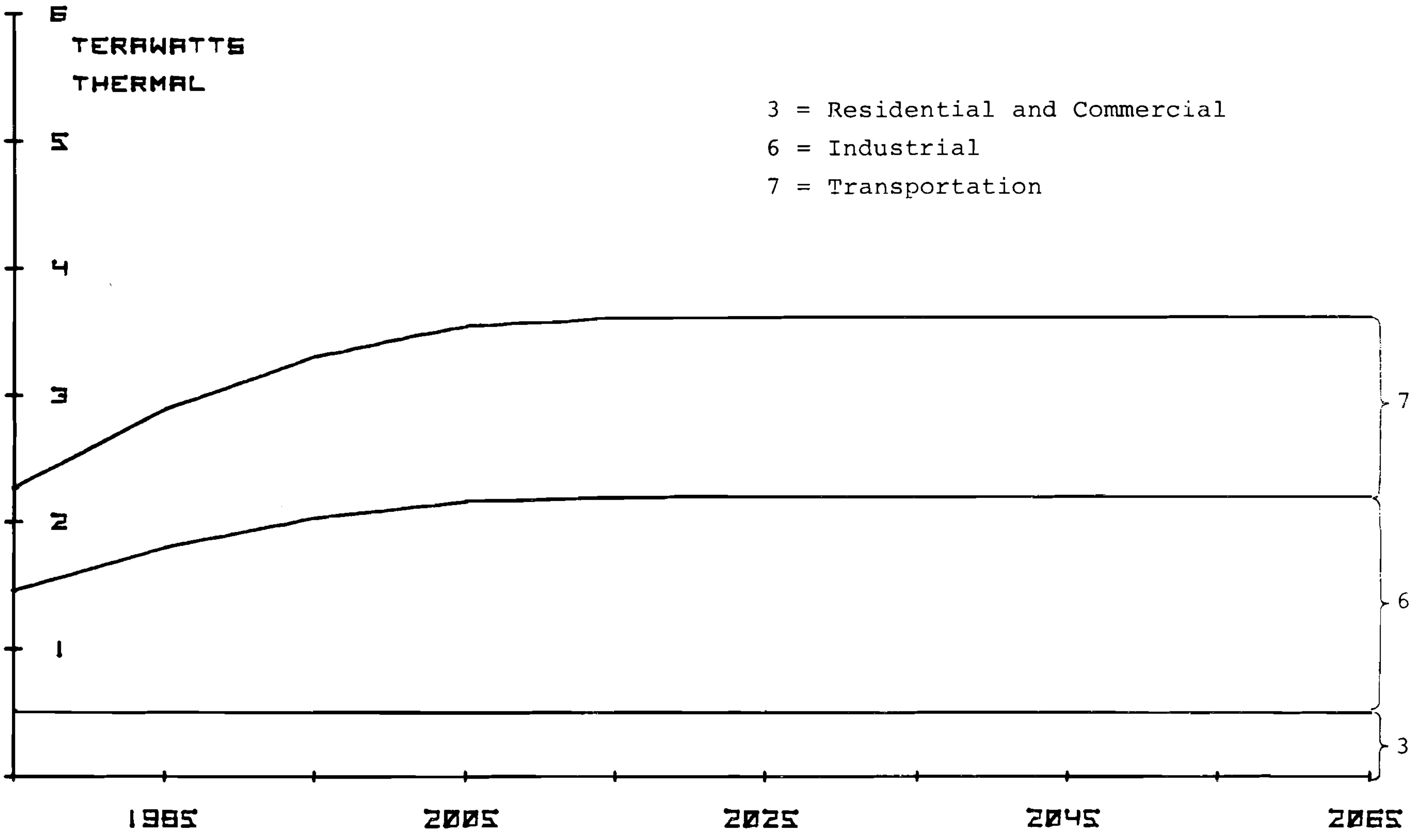

Figure 6b. Model society 1: non-electric energy demand.

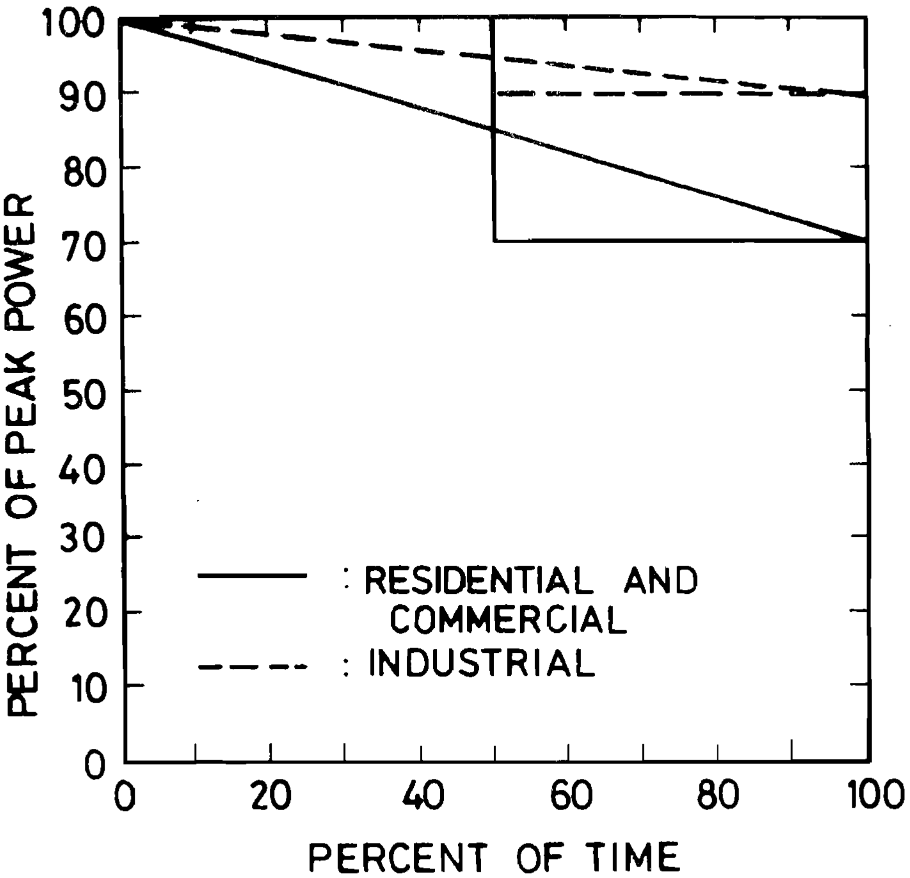

Figure 7. Approximated load duration curve for each demand sector.

Fuel utilization factor. As investigated in [9], the energy utilization factor has a significant effect on an optimal solution, since a static cost ranking of each of the supply alternatives depends remarkably on the value of this factor. As far as the present problem is concerned, a comparison of the hydrogen utilization factor with the use factor of oil products is most crucial, since (it is supposed) oil products can be replaced only by hydrogen and not by electricity.

It was assumed in the original model that 1 BTU of hydrogen can be replaced by 1.5 BTU of oil products, averaged over all types of end uses. That is, 1.5 = hydrogen utilization factor. In the expanded model, the value of this factor must be assigned for each of the demand sectors; data shown in Table 6 have been chosen as input according to Marchetti [6].

Cost coefficients. Coefficients of the objective function must be prepared. Since the mathematical form of the objective function in the expanded model is the same as that used in the original model, the cost coefficients for the supply alternatives of the original model can also be used in the expanded model. However, the assessment of the capital cost for intermediate electricity should take into account the load factor 0.5. The assessment of the current cost for the petroleum and gas alternative should take into account a different sort of oil product for each of the demand sectors.

Table 7 gives the cost data of each of the electric supplying alternatives, showing that the capital cost for intermediate load is twice that for base load, since the load factor of intermediate electricity is 0.5. The reason why the energy delivery cost is not considered is that the delivery cost is the same for each of the alternatives as far as the same demand category is concerned.

For the solar thermal conversion technology, the data are assessed based upon [1]. According to static cost comparison, the FBR is the cheapest technology, and the LWR is the second cheapest. While the coal-fired steam generating plant is expensive, the solar electricity is even more expensive. The present supply alternative, i.e., coal--can probably be replaced by the FBR and the LWR because of their low energy production costs. However, it is unlikely that the solar alternative makes any contribution.

The above cost estimates are accompanied by uncertainties since they are involved with the assessment of future technology. Therefore, in this examination, some sensitivity analysis on the cost data will be done, especially concerning the current cost of coal and the capital cost of the solar thermal electric conversion system.

The cost data of each of the non-electric supplying alternatives are shown in Table 8. The energy delivery cost as well

Table 6. Fuel utilization factor.

	Residential and Commercial	Industrial	Transportation
Electricity			
ν_{ELEC}	1.0	1.0	-
Oil Product			
ν_{PETG}	1.0	1.0	1.0
Hydrogen[a]			
ν_{HYDR}	1.2	1.5	2.0

[a]C. Marchetti [6].

Table 7. Cost data for each of the electricity supply alternatives.

		Current Cost ($\$10^9/TW_{th} \cdot$ year)				Capital Cost ($\$10^9/TW_{th}$)			Total Energy Cost ($\$10^6 BTU_e$)		
		i	E.P.	E.D.[c]	Total	E.P.	E.D.[c]	Total	E.P.	E.D.[c]	Total
Base Load	R & C (RCEBL)	COAL[a]	30.0			192			4.58		
		LWR[a]	5.8			200			3.21		
		FBR[a]	3.5			264			3.15		
		STEC[b]	2.2			245			5.96		
	Ind. (INEBL)	COAL[a]	30.0			192			4.58		
		LWR[a]	5.8			200			3.21		
		FBR[a]	3.5			264			3.15		
		STEC[b]	2.2			245			5.96		
Intermediate Peak Load	R & C (RCEIP)	COAL[a]	30.0			384			6.66		
		LWR[a]	5.8			400			5.79		
		FBR[a]	3.5			528			6.01		
		STEC[b]	2.2			354			8.46		
	Ind. (INEIP)	COAL[a]	30.0			384			6.66		
		LWR[a]	5.8			400			5.79		
		FBR[a]	3.5			528			6.01		
		STEC[b]	2.2			354			8.46		

[a] W. Häfele and A. Manne [3].

[b] The Aerospace Corporation [1].

[c] The model supposes that the energy delivery costs (E.D.) of all electricity supply alternatives are the same, as far as the same demand category is concerned. Namely, the energy delivery costs are not relevant in the model.

Table 8. Cost data for each of the non-electric energy supply alternatives.

		Current Cost ($10^9/TW_{th}$ · year			Capital Cost ($\$10^9/TW_{th}$)			Total Energy Cost ($\$/10^6 BTU_{PETG}$ or H_2)			Static Cost Comparison ($\$/10^6$BTU of PETG equivalent)
		E.P.	E.D.	Total	E.P.	E.D.	Total	E.P.	E.D.	Total	
Residential and Commercial (RCN)	PETG	48.6[a]	25.8[e]	74.4	-	-	-	1.62	.86	2.48	2.48
	HTGR	7.0[b]	27.3[d]	34.3	220[b]	-	220	2.37	1.82	4.19	3.49
	SHYD	4.0[c]	19.1[d]	23.1	270[c]	-	270	3.72	1.82	5.54	4.62
	ELHY	-	14.2[d]	14.2	20[b]	-	20	4.35	1.82	6.17	5.14
Industrial (INN)	PETG	37.9[a]	2.4[d]	40.3	-	-	-	1.26	.08	1.34	1.34
	HTGR	7.0[b]	8.4[d]	15.4	220[b]	-	220	2.37	.56	2.93	1.95
	SHYD	4.0[c]	5.9[d]	9.9	270[c]	-	270	3.72	.56	4.28	2.85
	ELHY	-	4.4[d]	4.4	20[b]	-	20	4.35	.56	4.91	3.27
Transport (TRN)	PETG	63.4[a]	28.5[d]	91.9	-	-	-	2.11	.95	3.06	3.06
	HTGR	7.0[b]	33.3[e]	40.3	220[b]	156[e]	376	2.37	3.54	5.91	2.95
	SHYD	4.0[c]	23.3[e]	27.3	270[c]	109[e]	379	3.72	3.54	7.26	3.63
	ELHY	-	17.6[e]	17.6	20[b]	82[e]	102	4.35[f]	3.54	7.89	3.95

[a] W. Häfele/A. Manne [3] and K. Hoffman [4].

[b] W. Häfele/A. Manne [3]

[c] The Aerospace Corporation [1] and NASA/ASEE [8].

[d] K. Hoffman [4].

[e] NASA-ASEE [8].

[f] The use of electricity from the LWR is assumed.

as the energy production cost have been estimated because the energy delivery cost clearly depends on whether the type of fuel used is oil or hydrogen.

Methods for estimating delivery cost are based on [4], in part with the aid of [8]. As regards oil:

- fuel oil is the oil product for residential and commercial use, and is transported in small quantities;
- residual oil (in large quantities) is for industrial use; and
- gasoline (in small quantities) is for transportation.

As for hydrogen, small quantity delivery is presumed for residential and commercial purposes, and small quantities of liquefied hydrogen for transportation purposes.

As regards the solar hydrogen, the cost data in Table 8 corresponds to the hydrogen production system that combines the central receiver system specified by [1] and the thermochemical water-splitting system specified by [8].

Two facts are worth noting. First, a static cost ranking is obviously dependent upon the value of the hydrogen utilization factor. Second, among the three hydrogen alternatives, the HTGR hydrogen is the cheapest. The cost differences between the solar hydrogen and the electrolytic hydrogen are slight.

IV. Calculation Results

A. Base Cases

Base case is that which is specified numerically by the input data discussed in the previous section. The only parameter that is evaluated optionally is the petroleum and gas reserve availability. According to the terminology of the Häfele-Manne model, the base cases to be examined here are denoted by B-1.40, 1.60, 1.80 and 1.100; where

1 = Model society 1,

and

40,60,80,100 = years of petroleum and gas availability level, 1970 annual consumption rate of the most developed country.

Figures 8 to 11 illustrate the curves of optimal energy production activities over time for each of the base cases. The figures are displayed in terms of the aggregated demand categories (electric and non-electric energy). The infrastructures of optimal solutions for each of the demand categories are given in Appendix B.

During the period when petroleum and gas production decreases, hydrogen comes in to take its place. Most of the hydrogen is produced by the HTGR, although some is produced by electrolysis; however, the solar hydrogen appears to play a limited intermediate role for only B-1.40 and 1.60.

There are three explanations why the electrolytic hydrogen is more often used than the solar hydrogen (in spite of the former's higher static cost given in Table 8). First, the HTGR is so inexpensive that it replaces the petroleum and gas regardless of the other hydrogens. The part of energy demand that the HTGR cannot supply (in part because of the upper limits set on the construction rate and because of the coupling effect of the FBR on uranium-233 availability) must be supplied by the electrolytic hydrogen and/or by the solar hydrogen. Second, the contributions of the electrolytic and/or solar hydrogens are optimized by the dynamic cost comparison, and not by the static cost comparison. The dynamic cost of the electrolytic hydrogen is determined by both the capital cost given in Table 8, and the marginal cost of the electricity required for the electrolysis, i.e. the shadow price of the base load electricity for industrial purposes. The dynamic cost of the solar hydrogen is determined by the energy production and delivery costs given in Table 8, and by the investment loss that would have to be paid if the already-constructed plant were under-utilized (or operating at less than full power level). This type of investment loss will be high because of the high capital cost. Third, as can be seen from Figures 8 to 11, if the solar hydrogen plant met all the reamining demand that the HTGR and the petroleum and gas were unable to supply, it would inevitably be under-utilized, and the dynamic cost of the solar hydrogen would be higher than that of the electrolytic hydrogen.

In the original model, the size of the petroleum and gas reserves only served to prolong the period of use of the petroleum gas. However, in the new model the greater the petroleum and gas reserves, the smoother the transition to the hydrogens. In other words, in the original model, the velocity of the shift from the petroleum and gas to hydrogen is not remarkably dependent on the petroleum and gas reserves. There are two reasons for this. First, the original model supposes that the operating power level is fixed by the corresponding installed capacity. Second, the original model considers the macroscopic demand classification, and does not consider individual end uses. Since the expanded model revised these aspects, the solution, as could have been expected, changed. Appendix Figures B-3, B-6, B-9, and B-12 show the results brought about by the revision to the original model. From these figures it may be seen that the shift from petroleum and gas to hydrogen

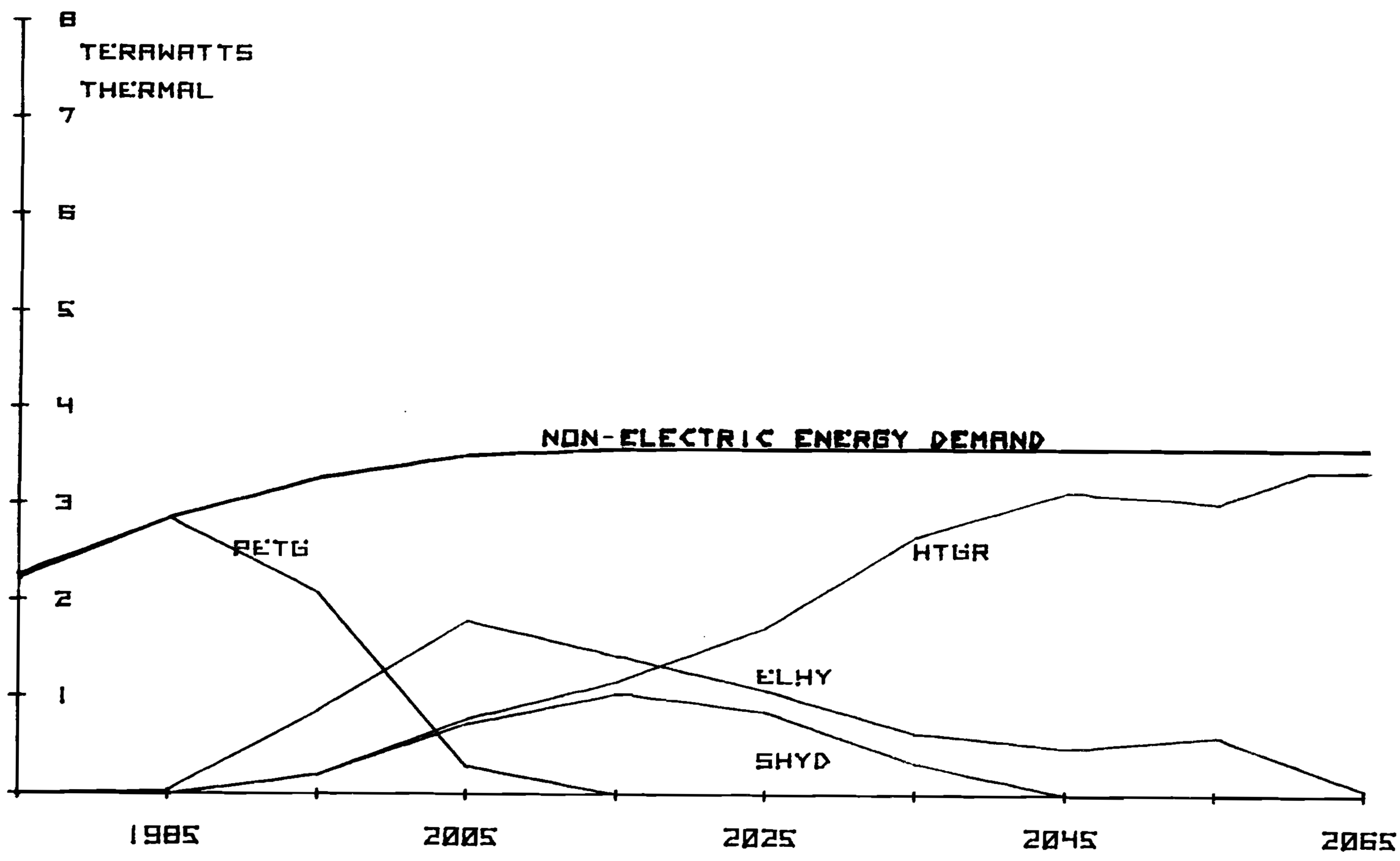

Figure 8a. Model society 1.40: non-electric energy demands and supplies.

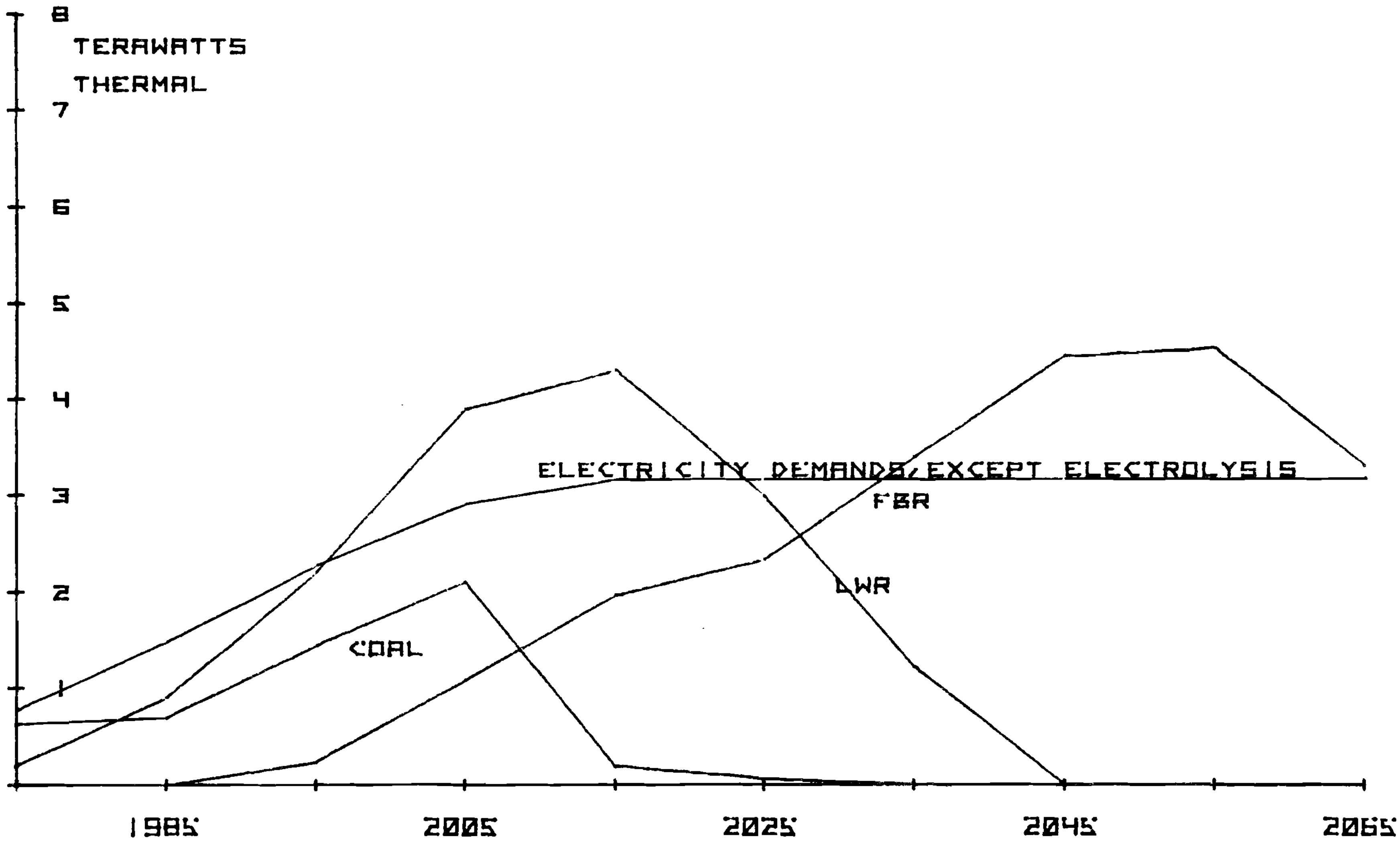

Figure 8b. Model society 1.40: electric energy demands and supplies.

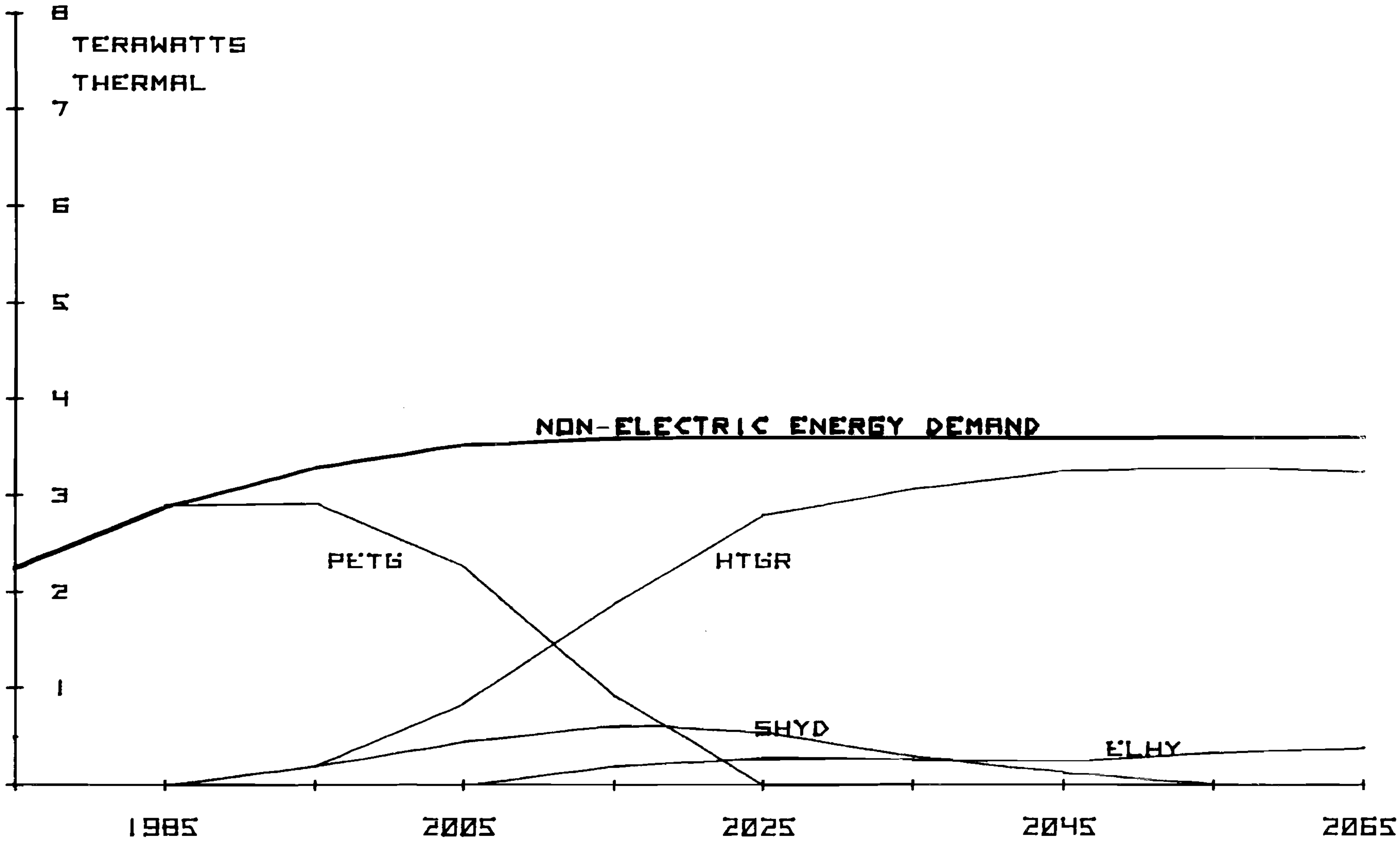

Figure 9a. Model society 1.60: non-electric energy demands and supplies.

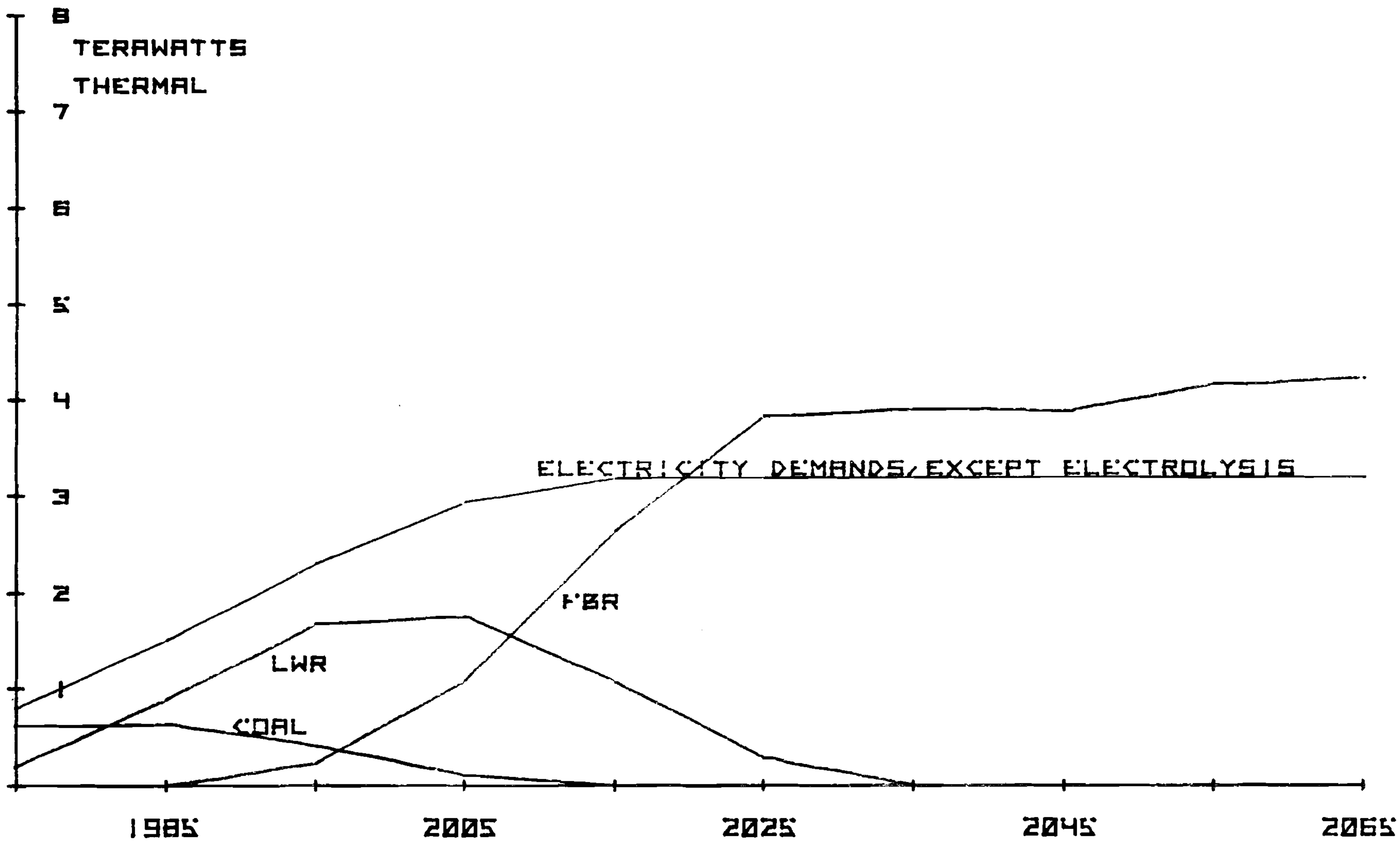

Figure 9b. Model society 1.60: electric energy demands and supplies.

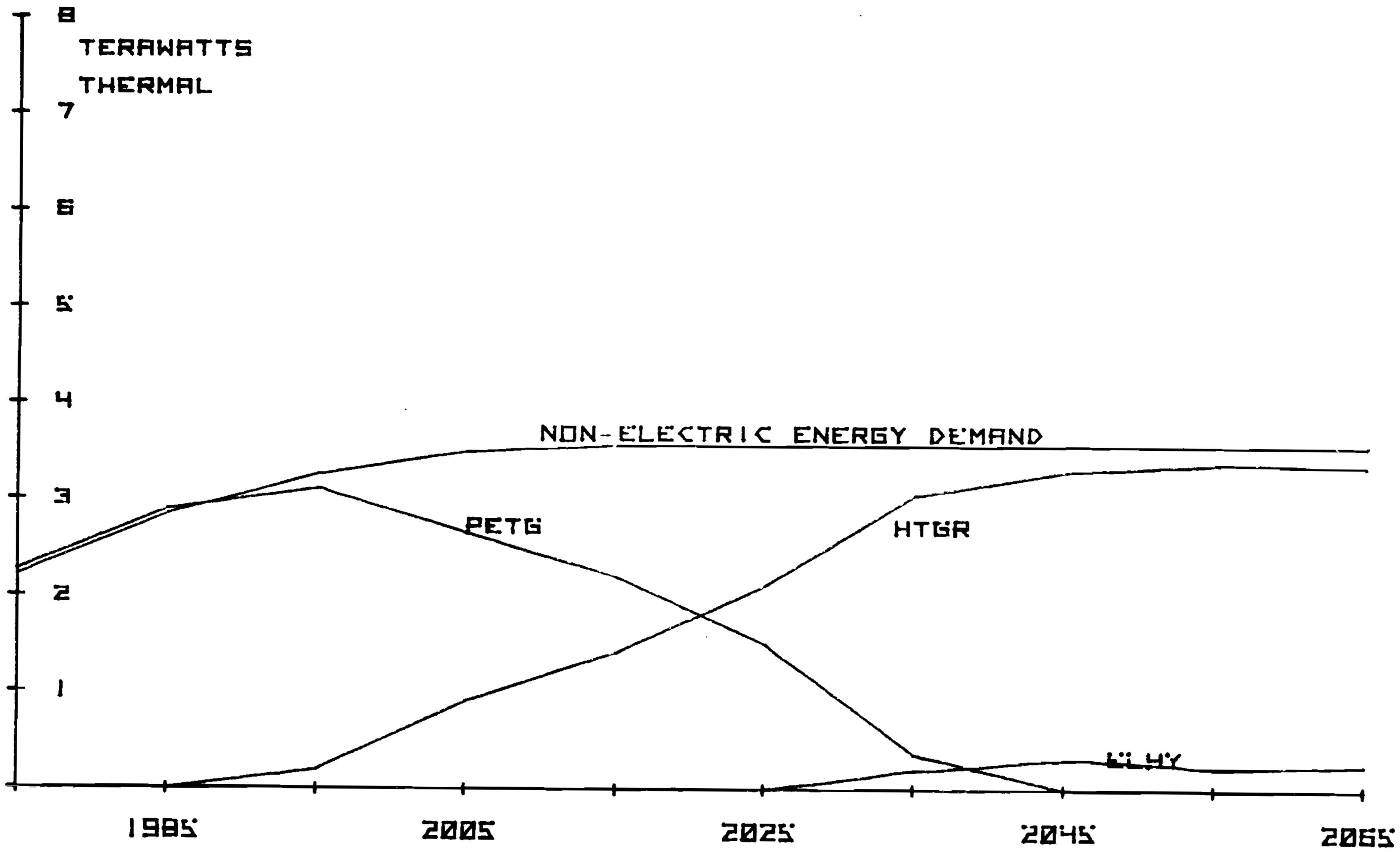

Figure 10a. Model society 1.80: non-electric demands and supplies.

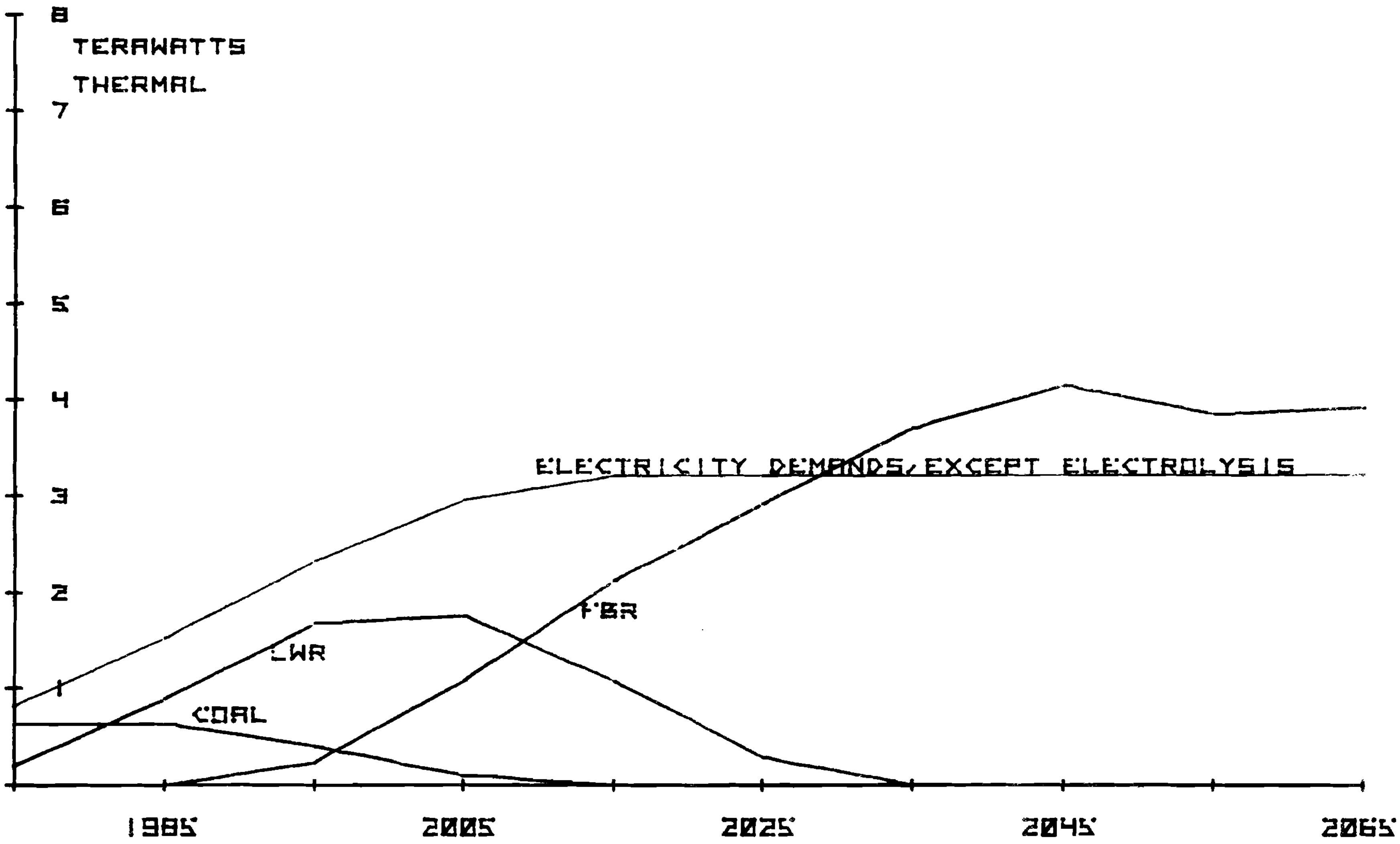

Figure 10b. Model society 1.80: electric energy demands and supplies.

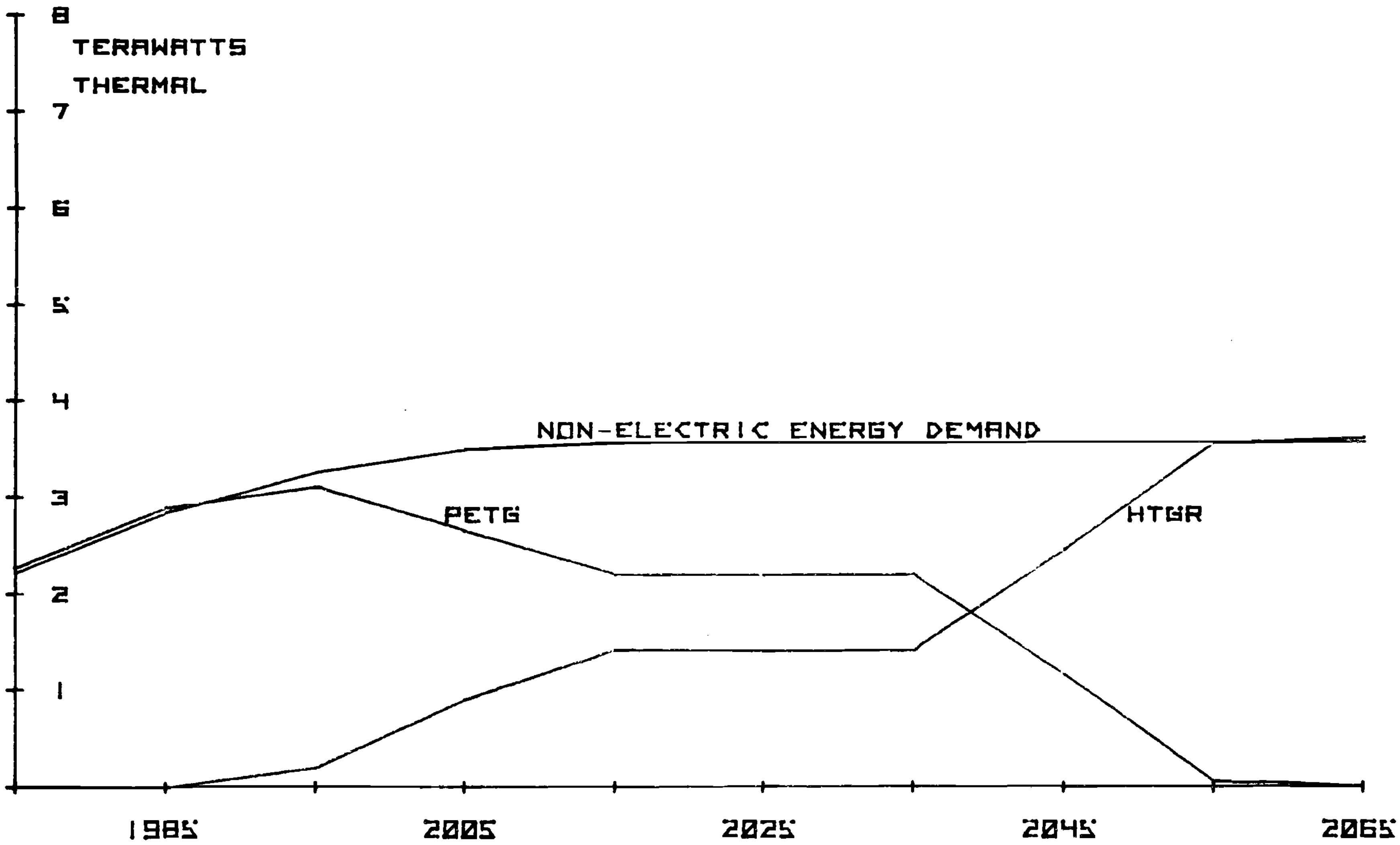

Figure 11a. Model society 1.100: non-electric energy demands and supplies.

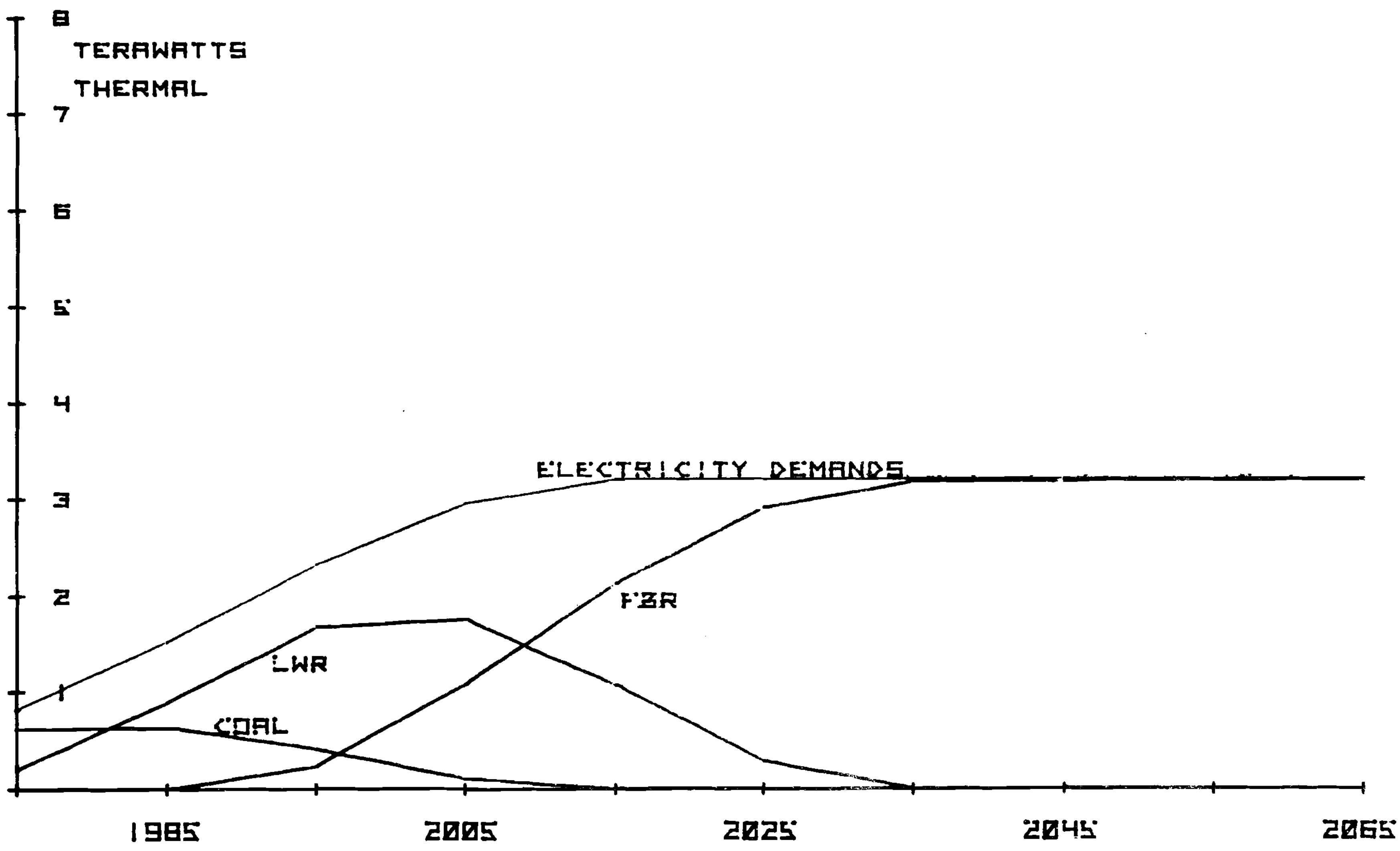

Figure 11b. Model society 1.100: electric energy demands and supplies.

is obviously dependent on the type of demand sector, and the velocity of the shift in some cases is so rapid that the energy production activity of the petroleum and gas is below the level of installed capacity.

The replacement of petroleum and gas by hydrogen begins at the earliest stage for the transportation demand sector, because the energy cost of the petroleum and gas is relatively the highest in this sector (see Table 8). Also, the dropping slope becomes more gentle as the petroleum and gas reserves are more abundant. The period of petroleum and gas activity is the longest for the residential and commercial sector; this may be explained by the comparison of the static cost data shown in Table 8. That the shifting manner of conversion depends on the type of demand sectors is mainly due to the difference in the value of the hydrogen utilization factor among the various sectors.

As regards the solution for electric energy, the following observations can be made on the basis of Figures 8 to 11. First, for B-1.60, 1.80 and 1.100, the manner of phasing out the coal-fired plants and that of introducing and then abandoning the LWR are exactly the same for each of the cases. That is, it is optimal for all of the cases to phase out the coal-fired plants in the year 2015, and to use the LWR as an intermediate technology to the year 2035, when the FBR technology can by itself meet all the electric demands. This will be confirmed by looking at the detailed results shown in Appendix B. Figures B-4c, B-7c and B-10c indicate that in all of the cases the curves of coal and of the LWR for base load electricity are completely unchanged. Moreover, the situation shown in Figures B-5c, B-8c and B-11c for intermediate peak electricity is the same in all of the cases. However, an examination of the curves for each of the demand sectors shows that the curves remain unchanged in some of the cases and change in others. The reason is that there is no difference between the cost data for the residential-and-commercial use and those for the industrial use (see Table 7). Mathematically, an optimal solution for electric energy production activity for individual demand sectors is not unique but can be degenerated. As for electric energy production activity, the model can yield optimal solutions only in terms of base load and intermediate peak load.

A second observation with respect to the solution for electric energy is concerned with the endogenous electricity demand, i.e. the electricity for the electrolytic hydrogen. Apparently, the need for the electrolytic hydrogen increases as the petroleum and gas reserves become scarce; thus there is greater use of electrolytic hydrogen in B-1.40. According to Figure 8b, the need for electrolytic hydrogen arises at such an early stage in the transition that the coal and the LWR must be introduced additionally. This is why the activities of the coal and the LWR in B-1.40 are different from those in other cases. In B-1.60 and 1.80, the electrolytic hydrogen is used at a later stage (see Figures 9a and 10a); hence the FBR can supply the endogenous demand. As for B-1.100, there is no need for the electrolytic hydrogen; Figure 11b indicates that the sum of electric energy activities is equal to the exogenous demand at each of the points of time.

Let us note the value of the objective function so as to make economic comparisons among the base cases. Figure 12 provides the value of the objective function, i.e. the present value of costs minus benefits. However, these values are not meaningful in themselves since they are dominated to a certain extent by a fixed component: the present value of costs incurred during the initial ten-year to twenty-year period when there are virtually no technological choices to be made. If, for example, we compare the value from the original model[2] with that in Figure 12, the value in Figure 12 is more than twice that of the original model. An explanation of this difference is as follows. The cost of the remaining fossil fuel plants (that have been constructed for supplying the energy requirements before the beginning of the planning horizon) was excluded in the original model. This was done because (it was supposed) those fossil fuel plants will require a full-power operation for a fixed thirty-year service life. However, in the expanded model this cost has been included as a component of the objective function because (it is supposed) all the plants to be considered during the planning horizon can operate with a flexible-power level. A rough estimate of this cost is $1,000 billion, which corresponds to approximately 50 percent of the total of the value given in Figure 12.

The difference in the values for each of the base cases is a meaningful measure for knowing the relative benefits obtained from additional petroleum and gas reserves. The difference in the values may also be seen from Figure 12: if instead of forty years' worth of petroleum and gas availability there were sixty, eighty or 100 years; these additional reserves would have a present value of $113, $126 or $128 billion, respectively. Based on these results, the difference between B-1.80 and 1.100 appears to be so slight that it is not necessary to do a computer run for another case with more than 100 years of petroleum and gas reserves.

Another observation based on the results of the base cases concerns the marginal costs of the constraints considered in the model. The shadow price of petroleum and gas that is represented in Figure 13, is given in current not present value. Adding these values to the energy cost given in Table 8, one could assess the price of petroleum and gas (see Table 9). Obviously, the shadow price is the highest for B-1.40 and decreases rapidly with the scarcity of the reserves. Table 9 also shows that even for B-1.40, the royalty is only 10%, 30% and 200% of the total, in the years 1970, 1980 and 2000, respectively. This can be attributable to the fact that the HTGR hydrogen cost is set in the model to be sufficiently low to take its place.

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[2] See [3], p. 42.

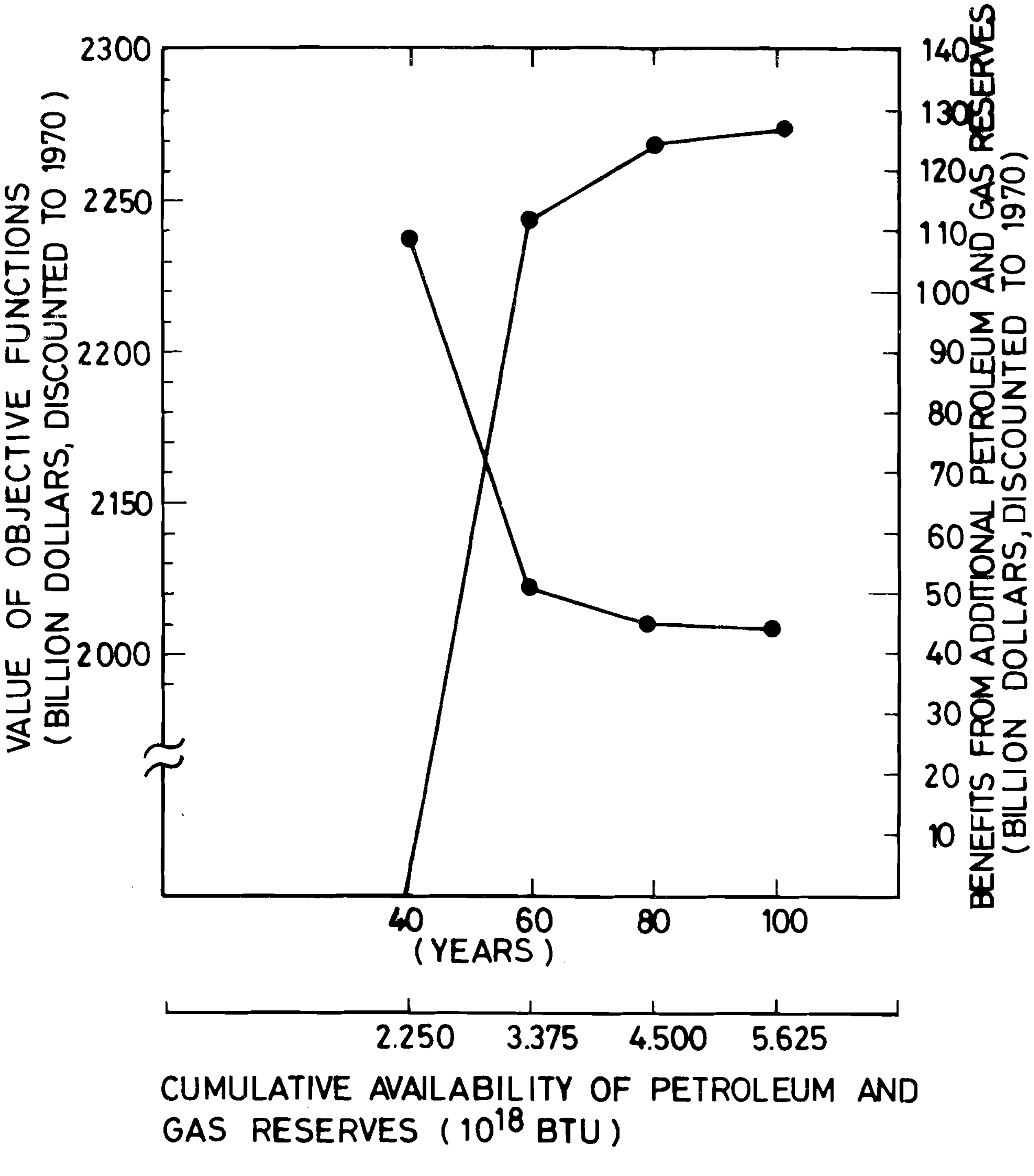

Figure 12. Benefits from additional petroleum and gas reserves (40 + 20, + 40, + 60 years).

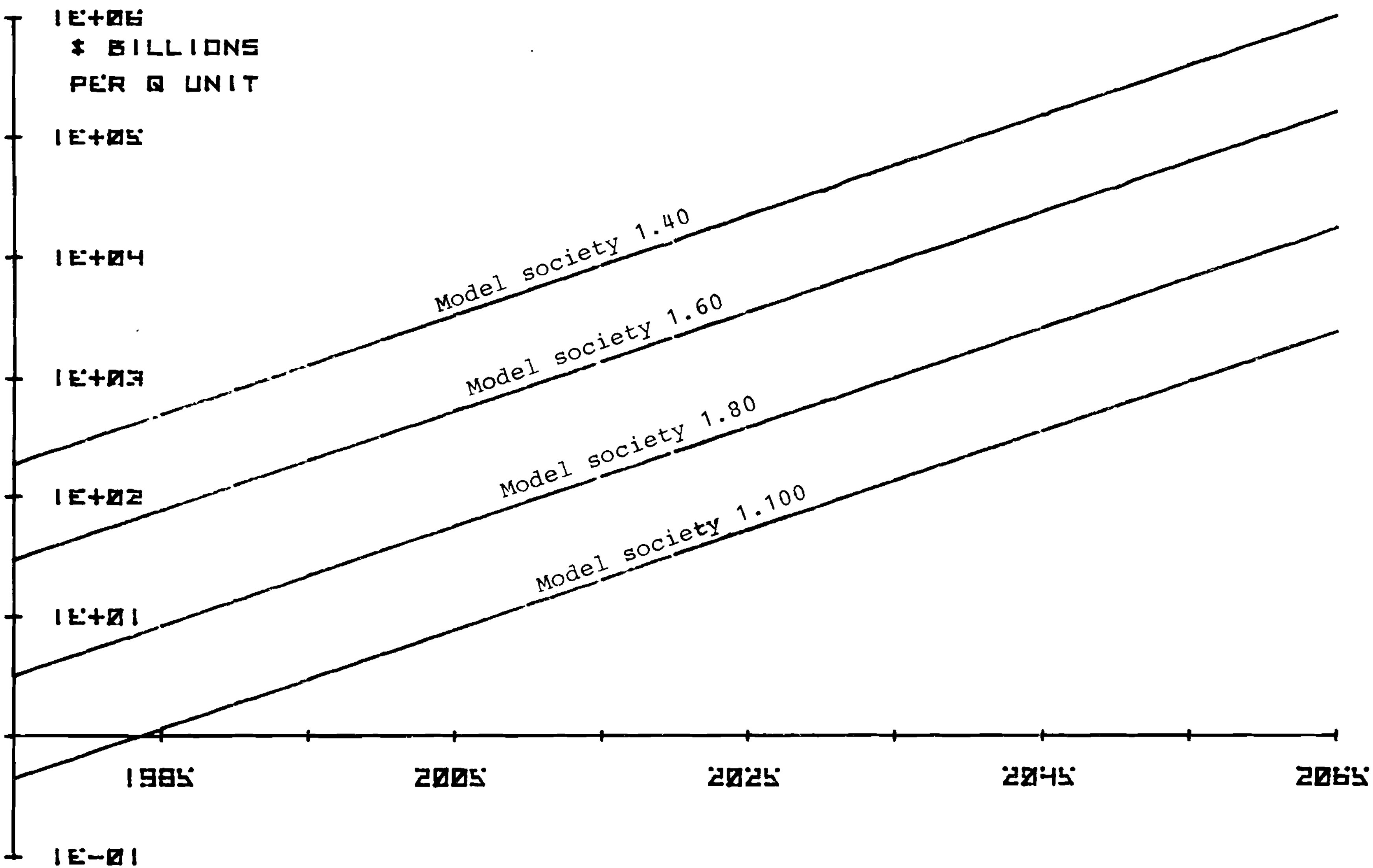

Figure 13. Shadow price of petroleum and gas, in terms of current value.

Table 9. Price of petroleum and gas[a]. ($/barrel, 1974 price)

	YEAR 1970 (US$)	1980 (US$)	2000 (US$)	2030 (US$)
B-1.40	11.14	12.95	29.84	(356.16)[b]
B-1.60	10.18	10.46	13.13	64.58
B-1.80	10.02	10.05	10.34	15.94
B-1.100	10.00	10.01	10.05	10.82

[a]Shadow price plus $10/barrel, which was used in the original model to estimate the current annual cost of the petroleum and gas.

[b]The reserves are already exhausted and therefore the value is of no practical meaning.

The shadow price of plutonium and uranium-233 stockpile constraints yields an index of the abundance or the scarcity of this man-made resource. The cost data given in Tables 7 and 8 exclude the credit or royalty of plutonium and uranium-233 that is usually included in an estimate of power costs of nuclear reactors. The plutonium credit for LWR power costs, for instance, is usually estimated between $8 and $12 per g of plutonium. In the expanded model, however, this sort of price is determined endogenously. As far as the base cases are concerned, there is always a positive stockpile of plutonium (see Figure 14); thus the credit or royalty of plutonium is nil. On the other hand, the constraints on the uranium-233 stockpile are binding at many points of time in any of the base cases (see Figure 15); hence the incremental value of uranium-233 is assessed as illustrated in Figure 16.

The reason for this difference between plutonium and uranium-233 is as follows. The scarcity of petroleum and gas reserves leads to a strong incentive to introduce the HTGR as much as possible which means that the uranium-233 produced by the FBR is used to the most by the HTGR. On the other hand, the amount of plutonium that is required for an initial inventory of the FBR for the early introductory years has been supplied by the LWR. When the LWR disappears, the electric energy demand reaches the saturation level; and thus the plutonium balance can be kept sufficient by the fact that the FBR provides sufficient plutonium for itself.

Figure 16 indicates that to the year 2005, there will be extensive price changes depending on petroleum and gas availability. During this period, however, the market for uranium-233 will not yet be established (recall the upper bounds on reactor construction rates of the FBR and HTGR shown in Figure 5). Thus the shadow price is virtually of less meaning than it will be at the later period. Table 10 gives the static costs of the FBR and the HTGR, including the credit and the royalty of uranium-233; the contribution of the uranium-233 price is about fifteen percent and thirteen percent to the total costs for the FBR and the HTGR, respectively.

Finally, let us consider the price of electric and non-electric energy. The energy demands for individual demand categories are supplied by the mixture of energy supplying alternatives chosen at each of the points of time by the criterion of cost-minimand over the whole planning horizon. The price of energy for each of the demand categories varies with time as well as with petroleum and gas availability. Table 11 gives the values for each of the representative years, indicating that prices become less stable as petroleum and gas become more scarce. This is because the need for the HTGR and for the electrolytic hydrogen increases with the scarcity of petroleum and gas. This brings about an unstable mixture of energy supplying alternatives since both the HTGR and the electrolytic hydrogen alternatives are dependent on other alternatives.

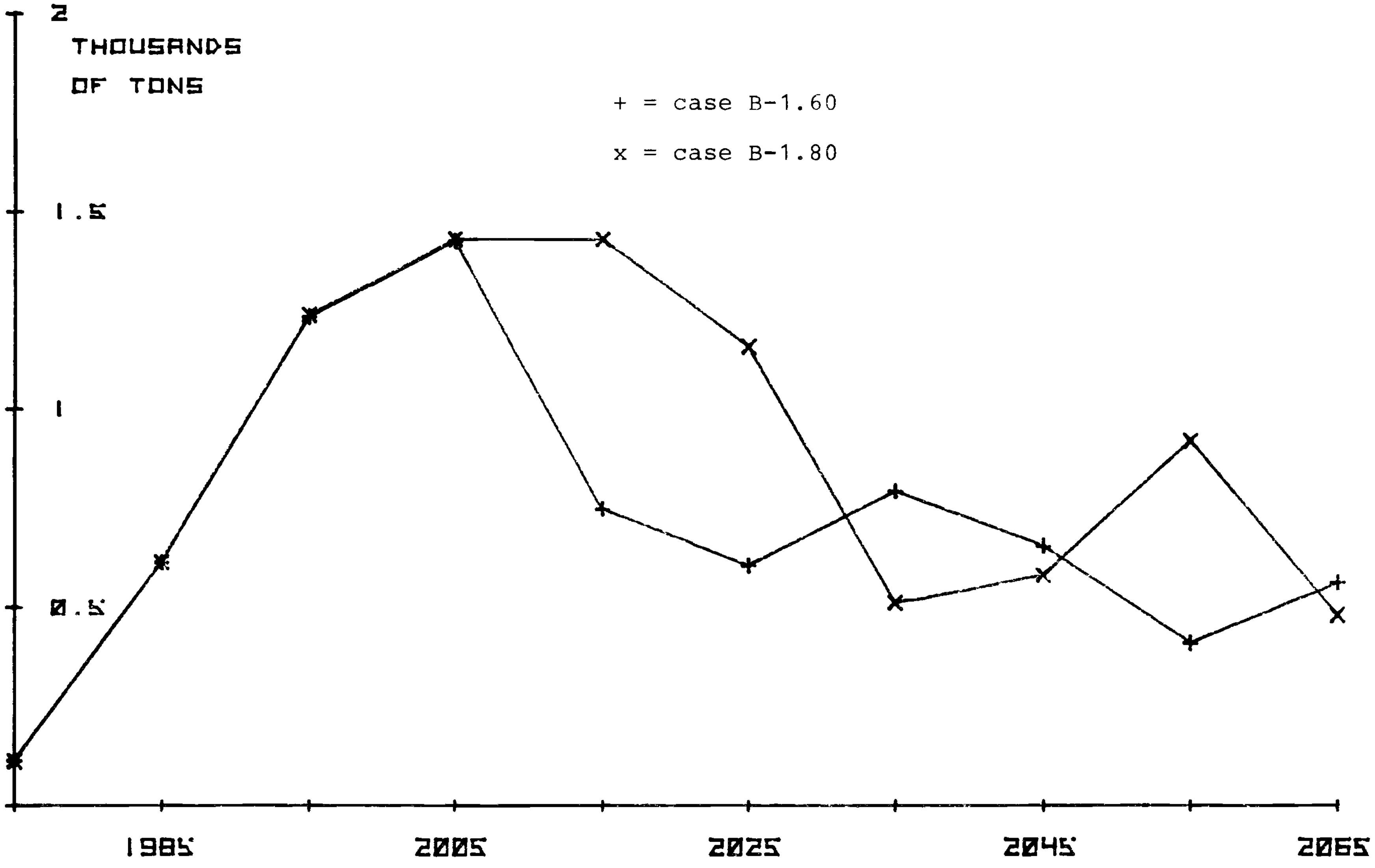

Figure 14. Plutonium stockpile for cases B-1.60 and 1.80.

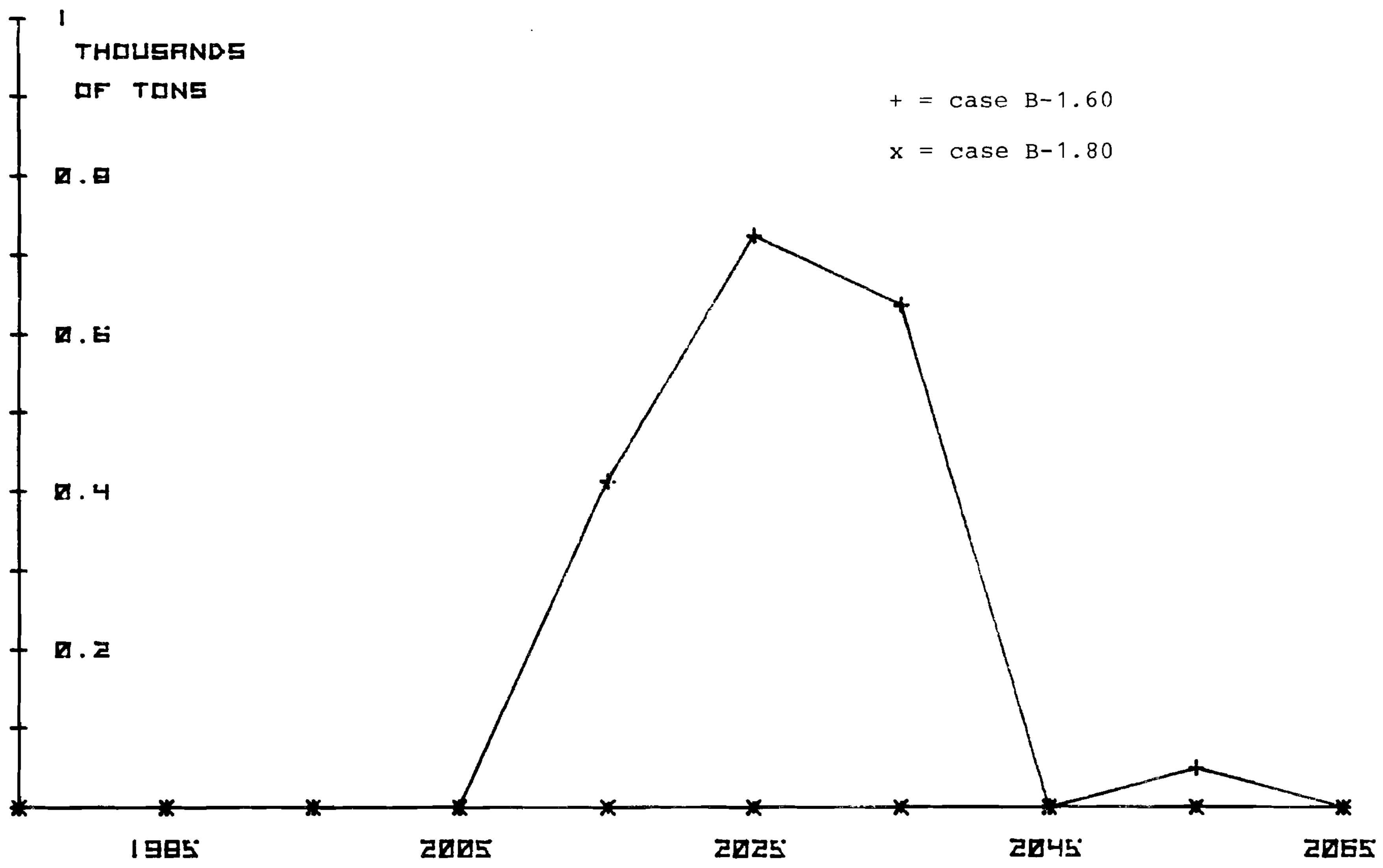

Figure 15. Uranium-233 stockpile for cases B-1.60 and 1.80.

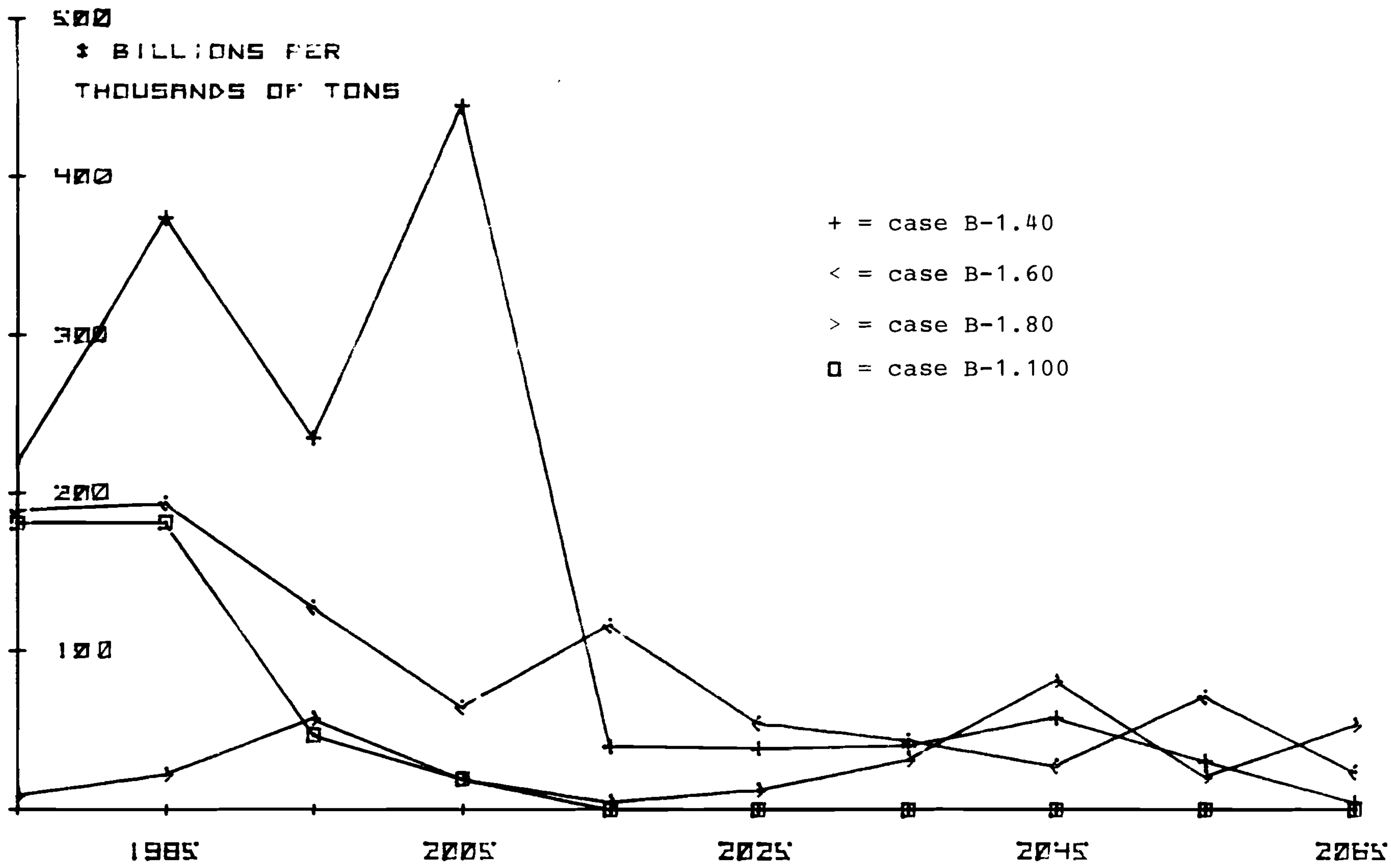

Figure 16. Shadow price of uranium-233, in terms of current value.

Table 10. The effect of static price of uranium-233 on energy production costs of HTGR and FBR. (1974 price)

	HTGR[a] ($\$/10^6 BTU_{H_2}$)			FBR[b] ($\$/10^6 BTU_e$)		
	E.P.[c]	U233[d]	Total	E.P.[c]	U233[e]	Total
B-1.40	2.29	0.17	2.46	3.15	0.22	2.93
B-1.60	2.29	0.46	2.75	3.15	0.57	2.58
B-1.80	2.29	0.27	2.56	3.15	0.34	2.81
B-1.100	2.29	0.00	2.29	3.15	0.00	3.15

[a]Energy production cost in Table 7 plus royalty of uranium-233.

[b]Energy production cost in Table 7 minus credit of uranium-233.

[c]Energy production cost.

[d]Royalty averaged over the period 2010 to 2070.

[e]Credit averaged over the period 2010 to 2070.

Table 11. Shadow prices of electric and non-electric energy (1974 price).

			1970 (US$)	1980 (US$)	2000 (US$)	2030 (US$)
B-1.40	Elec.[a]	Base	4.42	3.27	4.09	2.18
		Intermed.	5.62	5.51	5.97	4.56
	Non-e.[b]	R&C	2.53	2.97	7.10	3.44
		Ind.	1.50	1.94	5.20	1.99
		Transport	3.06	3.50	4.39	3.70
B-1.60	Elec.[a]	Base	5.20	2.39	2.27	1.98
		Intermed.	6.34	4.57	4.55	4.36
	Non-e.[b]	R&C	2.30	2.37	3.02	3.54
		Ind.	1.26	1.33	1.98	2.05
		Transport	2.83	2.90	3.55	3.24
B-1.80	Elec.[a]	Base	5.21	2.34	2.27	2.18
		Intermed.	6.35	4.52	4.55	4.56
	Non-e[b]	R&C	2.26	2.27	2.34	3.70
		Ind.	1.23	1.23	1.30	2.49
		Transport	2.79	2.60	2.87	2.95
B-1.100	Elec.[a]	Base	5.20	2.36	2.27	2.64
		Intermed.	6.35	4.54	4.55	5.02
	Non-e.[b]	R&C	2.26	2.26	2.27	2.45
		Ind.	1.22	1.22	1.23	1.42
		Transport	2.79	2.79	2.80	2.70

[a] $\$/10^6$ BTU of electricity.

[b] $\$/10^6$ BTU of petroleum-gas equivalent.

B. Sensitivity Analyses

As far as the base cases are concerned, the solar technologies do not make a main contribution to energy supply because of their high costs. The cost of solar hydrogen is slightly lower than that of electrolytic hydrogen. Nevertheless, the electrolytic hydrogen plays a greater role than the solar hydrogen since the extremely high capital cost of the solar technology prevents it from being used as an auxiliary or intermediate part of the energy supply. As stated before, there are many uncertainties about the cost estimates of solar technologies; hence sensitivity analyses on the capital costs were carried out.

Method used to select the values of capital costs of the solar technologies for the sensitivity analysis is as follows. First, the values for the solar thermal electric conversion system were selected: \$200 (\$1,053), \$150 (\$789), \$100 (\$526) \$50 (\$263) per KWth (KWe), compared with the base case, \$245/KWth (\$1,289/KWe). Then, the values for the solar hydrogen corresponding to these values were estimated on the assumption that the above capital cost reduction for the solar thermal electric conversion system is due to a technological improvement in the central receiver system. A technological improvement of this sort is applicable also for the solar hydrogen (recall the delineation of the solar hydrogen in Figure 1). The estimated values are \$231, \$187, \$143 and \$99 per KWth; let these four cases be denoted by S1, S2, S3 and S4, respectively (see Table 12).

Figure 17 is an aggregated representation of the calculation results of an energy supplying contribution of the solar hydrogen. It is natural that the contribution may increase with the lower capital cost. To clarify the reason for this increase in Figure 17, it is necessary to make a cost comparison of the alternatives. First, it is apparent from the static cost comparison displayed in Figure 17 that the solar hydrogen for S1 is still more expensive than the HTGR, and the petroleum and gas. Thus the effect of the change from the case B to S1 is in large part due to an economic cost comparison of the solar hydrogen and the electrolytic hydrogen. As stated before, the difference between the two is so subtle that both are used intersupplementarily to meet an auxiliary or intermediate demand, especially in the case of B-1.40 and 1.60. It follows that the change from the case B to S1 results in a small replacement of the electrolytic hydrogen by the solar hydrogen. Second, the influence of the changes from the S1 to S2 and further to S3 is significantly different. The break-even value of the solar hydrogen cost that yields the equivalence of the HTGR hydrogen cost is between S2 and S3. For this reason, in S2, the solar hydrogen begins to take the place of the HTGR hydrogen; in S3 the HTGR hydrogen almost disappears. Third, Figure 17 indicates that the change from S3 to S4 has little effect on the solar hydrogen contribution. The reason for this is that the capital cost reduction from S3 to S4 does not affect the cost ranking between the petroleum and gas, the solar hydrogen and the HTGR hydrogen. This implies that the optimal

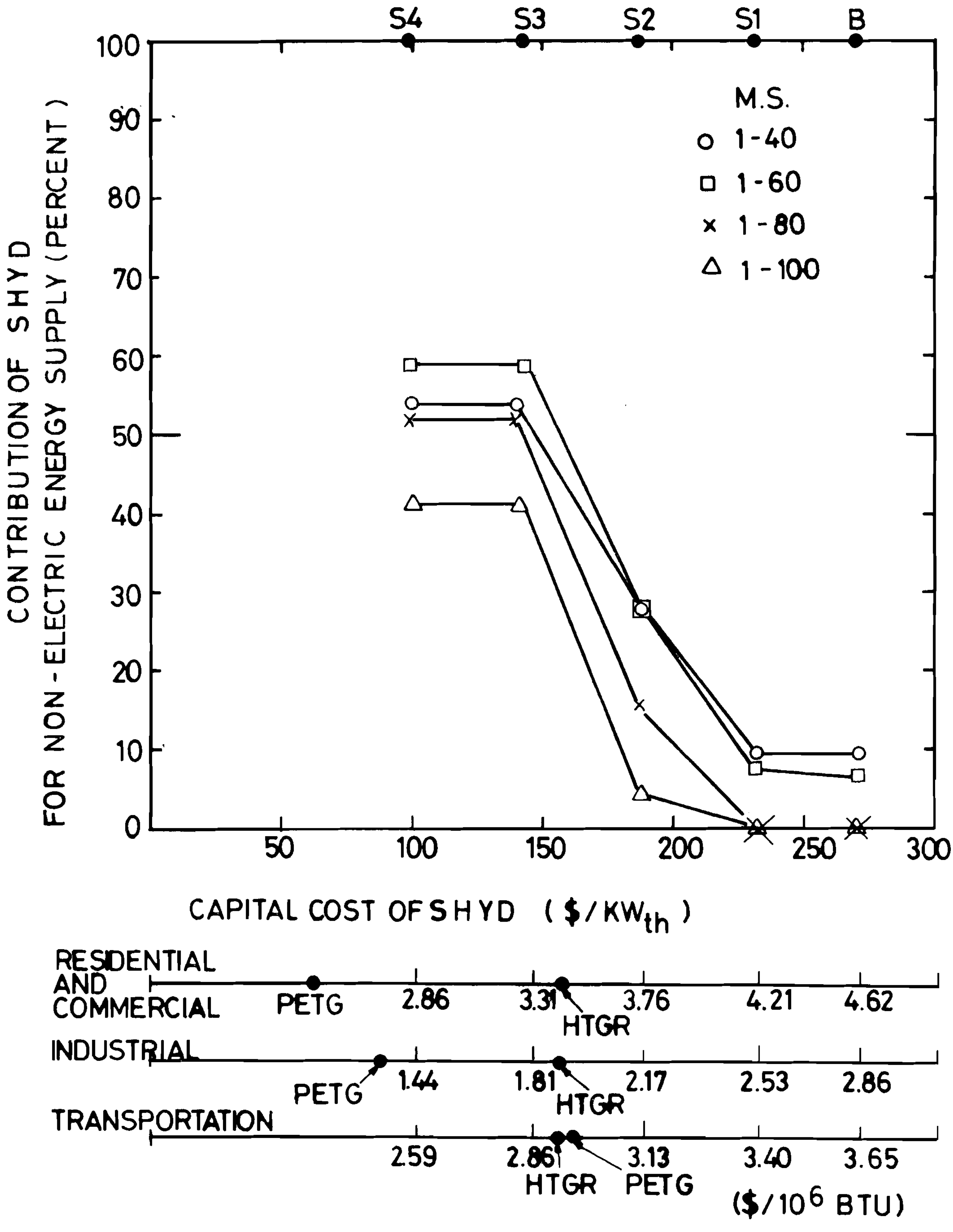

Figure 17. Contribution of solar hydrogen (SHYD) for non-electric energy supply versus capital cost of SHYD.

choice between the three alternatives in S4 must be the same as the choice in S3, if the dynamically variable cost components have no influence. According to the calculated results for S3 and S4, these variable cost components have to transform the energy allocation of the petroleum and gas and the solar hydrogen into each of the demand sectors but not to change the total amount of energy supply of each of the technologies.

The calculated results of an energy supplying contribution of the solar thermal electric conversion system are shown in Figure 18. The following may be observed from Figure 18. First, the introduction of the solar thermal electric conversion system begins at S2, where the energy production cost of the solar thermal electric conversion system is about the same as the FBR power cost. As may be seen from Table 12, a greater part of the contribution is caused by the energy supply for the intermediate peak load. Second, since in S3 the solar cost is less than the nuclear cost, solar electricity is required in place of nuclear not only for the intermediate peak load but also for the base load. Third, the price of the solar electricity in S4 is low enough to use its electricity for the electrolytic hydrogen, if necessary. This is the case for S4-1.40 and 1.60.

Another point worth noting as regards Figures 17 and 18 is the relationship between the petroleum and gas availability and the energy supplying contribution of the solar technology. Using intuition, we could expect that the solar contribution increases as the petroleum and gas reserves decrease. This intuitive observation would be correct if there were no coupling between electric and non-electric energy supplying alternatives. In the model, however, there are several coupling relations including the constraints on the uranium-233 stockpile, and the endogenous demand of electricity for the electrolytic hydrogen. The fact that the solar contribution decreases as the petroleum and gas reserves decrease may be explained by the following: provided the HTGR hydrogen is used for supplementing the scarcity of the petroleum and gas, the FBR must be additionally constructed in order to produce the corresponding additional uranium-233. The FBR obviously produces some electricity; the electric energy demand that remains (to be supplied by the solar thermal electric conversion system) decreases.

The above coupling relations have an interaction with the energy costs of all of the alternatives. Thus, the relationship between the petroleum and gas availability and an energy supplying contribution of the solar technology is not monotonous (see Figure 19). The above-mentioned interpretation will be more easily understood with the aid of Appendix C that gives the timing of energy production activities of the solar technologies, corresponding to each of the points mentioned of Figures 17 and 18.

Figure 20 estimates the benefits of a technological improvement on solar power plants. A comparison of the base case and S1 shows that for S1 a \$45/KWth (\$237/KWe) reduction of the solar thermal electric conversion system results in cost savings of only \$3 to \$5 billion, because of the small solar contribution. For all

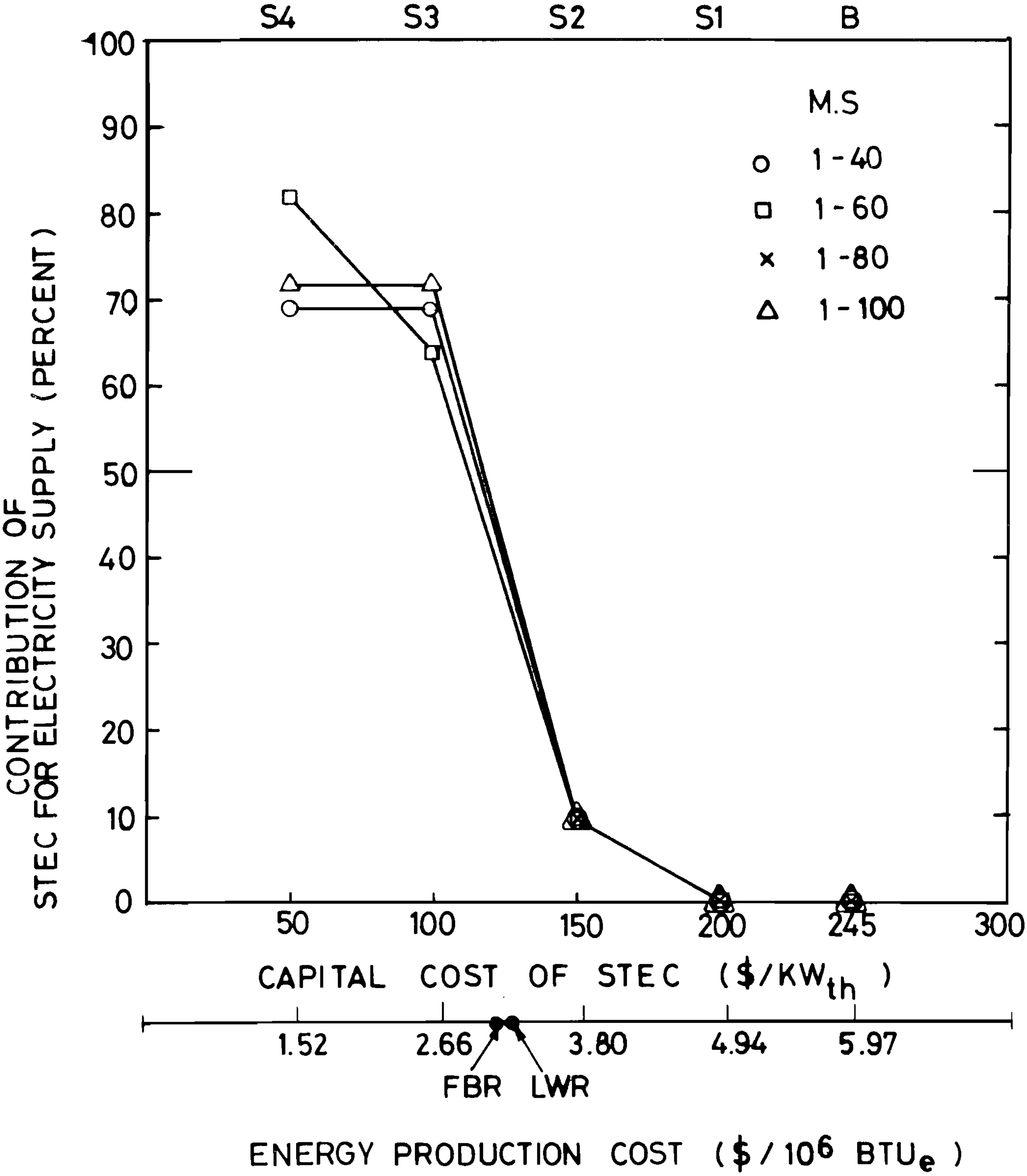

Figure 18. Contribution of solar thermal electric conversion system (STEC) for electricity supply versus capital cost of STEC.

Table 12. Cost data for sensitivity analysis on STEC (S1, S2, S3, S4).

		Current ($\$10^9/TW_{th}Y$)	Capital ($\$10^9/TW_{th}$)					Static Total Energy Cost ($\$/10^6$ BTU_e or PETG equivalent)				
		B.S1∿S4	B	S1	S2	S3	S4	B	S1	S2	S3	S4
Base Load	COAL LWR FBR STEC	30.0 5.8 3.5 2.2	192 200 264 245	200	150	100	50	4.58 3.21 3.15 5.96	4.95	3.81	2.67	1.53
Intermed. Load	COAL FWR FBR STEC	30.0 5.8 3.5 2.2	384 400 528 354	300	225	150	75	6.66 5.79 6.01 8.46	7.23	5.52	3.81	2.10
Residential and Commercial	PETG HTGR SHYD ELHY	74.4 34.3 29.1 14.4	- 220 270 20	231	187	143	99	2.48 3.49 4.62 5.14	4.21	3.76	3.31	2.86
Industrial	PETG HTGR SHYD ELHY	40.3 15.4 9.9 4.4	- 220 270 20	231	187	143	99	1.34 1.95 2.85 3.27	2.53	2.17	1.81	1.45
Transport	PETG HTGR SHYD ELHY	91.9 40.3 27.3 17.6	- 376 379 102	340	296	252	208	3.06 2.96 3.63 3.95	3.40	3.13	2.86	2.59

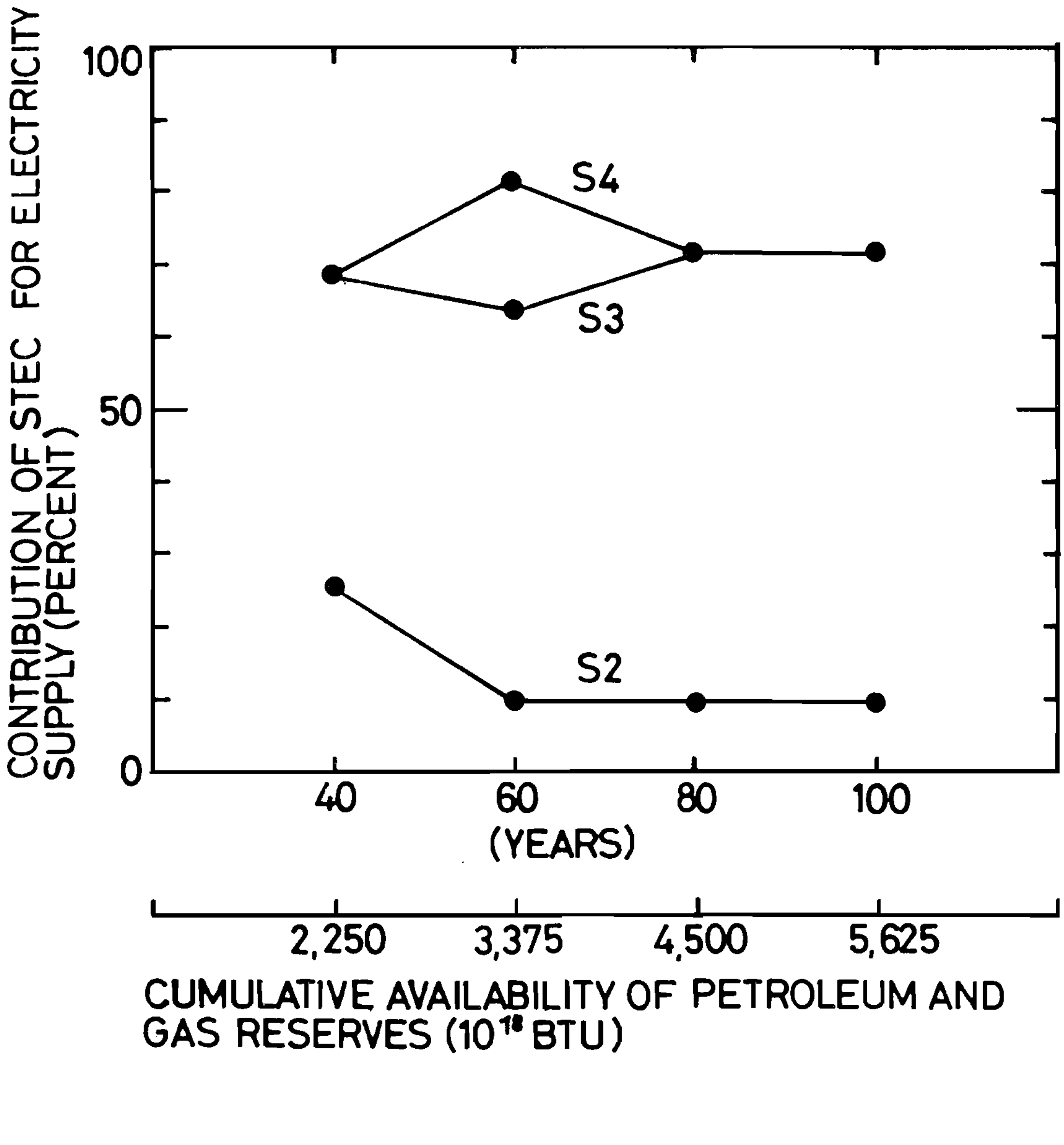

Figure 19. Contribution of STEC to electricity supply versus cumulative availability of petroleum and gas reserves.

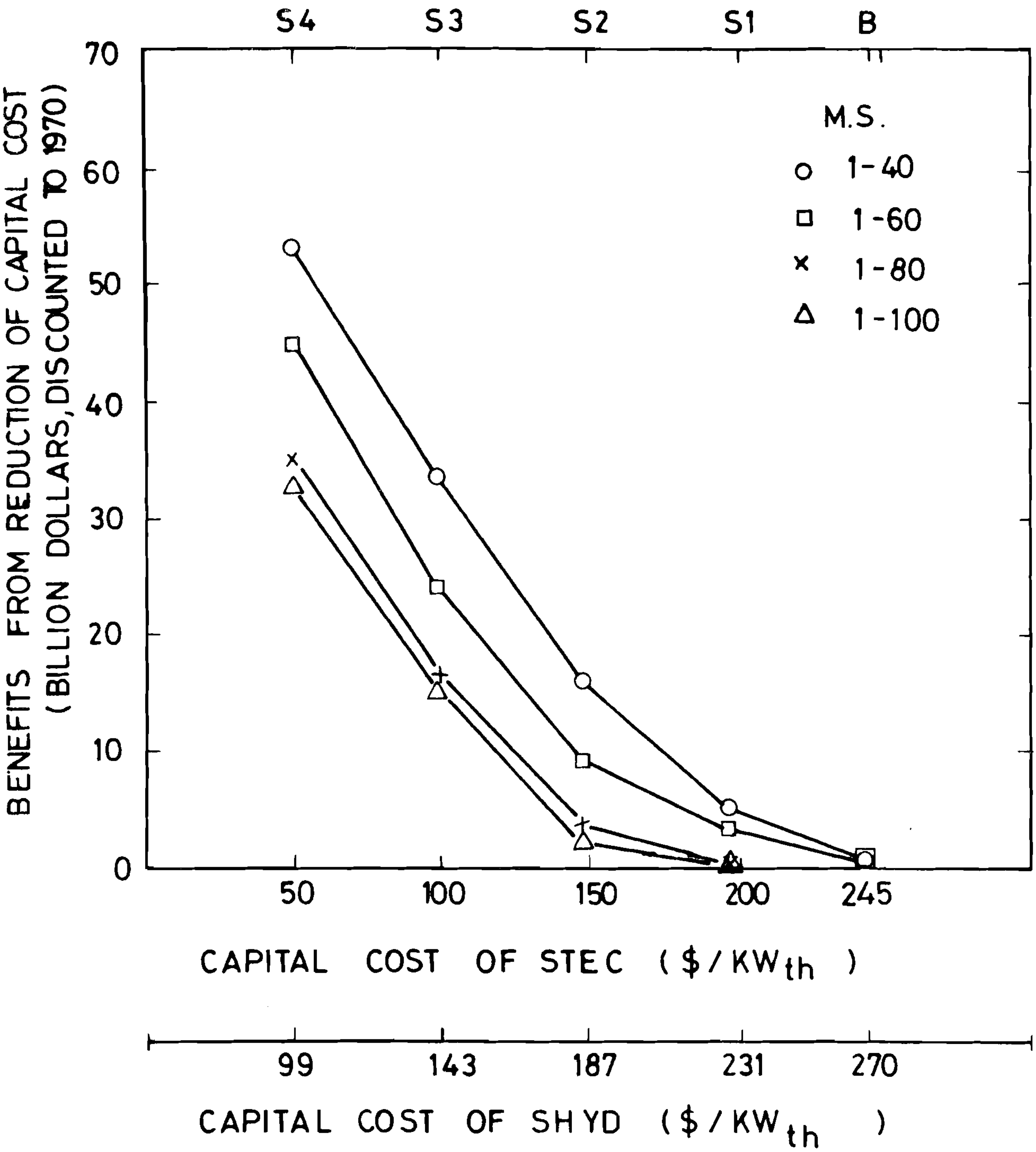

Figure 20. Benefits from reduction of capital cost of solar technologies (STEC: solar thermal electric conversion system, SHYD: solar hydrogen).

of the other cases, the benefits increase gradually, and in S4, for instance, the cost savings are between \$33 and \$53 billion. The cost savings resulting from a technological improvement are to be considered an index of the technological assessment.

Finally, let us look at the results of another sensitivity analysis of the value of the current annual cost of a coal-fired electricity generating plant. The values that were chosen for this purpose are given in Table 13: \$1.00, \$0.83, \$0.67, \$0.50, and \$0.33 per 10^6 BTU, for the base case, and for cases C1, C2, C3 and C4, respectively.

Figure 21 provides the energy supplying contribution of the coal as a function of the current annual cost. It follows from Figure 21 that:

a) the situation in the model society 1.40 is significantly different from that in other societies, since the reserves of petroleum and gas are so scarce that even the coal electricity that is statically more expensive than the LWR and the FBR is useful for the electrolytic hydrogen; and

b) the situations in the model societies 1.60, 1.80 and 1.100, are the same since the reserves of the petroleum and gas are so abundant that the FBR can provide almost all of the endogenous electricity demand (see Figures 9, 10 and 11). Thus, there is less need to introduce a larger number of coal plants in order to supplement the scarcity of the petroleum and gas.

The contribution of coal is still below fifty percent of the total contribution, even though the static power cost of the coal is lower than the costs of the LWR and the FBR. This is because of the inevitable need to introduce the HTGR that replaces the petroleum and gas, and the need to install the FBR that produces uranium-233. Therefore, the ratio of the installed capacities of the FBR and the HTGR will be stationary in later years (see Appendix D).

Calculations of the benefits from the above mentioned reductions of the current annual cost of the coal are shown in Figure 22. For C4 that yields a coal power cost which is less than a nuclear power cost, the resulting cost savings for the model society 1.40 and for all of the other societies are \$170 and \$110 billion, respectively. One reason why the benefits of the coal cost reduction is higher than those of the solar cost reduction is that the model assumes the unlimited reserves of coal. If C4, for instance, is compared with the base case, the coal consumption of C4 through the year 2050 is three to ten and one-half times more than that of the base case because of the low cost of coal (see Table 14). In actual fact, coal reserves are limited, depending on the cost. Provided availability is assigned to each of the grades of coal, the model would give a different, perhaps lower, value to the benefit of coal cost reduction.

Table 13. Cost data for sensitivity analysis on coal (C1, C2, C3, C4).

		Current ($\$10^9/TW_{th}$Year)					Capital ($\$10^9/TW_{th}$)	Static Total Energy Cost ($\$/10^6 BTU_e$)				
		B	C1	C2	C3	C4	B.C1∿C4	B	C1	C2	C3	C4
Base Load	COAL	30.0	25.0	20.0	15.0	10.0	192	4.58	4.16	3.75	3.33	2.91
	LWR	5.8					200	3.21				
	FBR	3.5					264	3.15				
	STEC	2.2					245	5.96				
Intermed. Load	COAL	30.0	25.0	20.0	15.0	10.0	384	6.66	6.24	5.83	5.41	4.99
	LWR	5.8					400	5.79				
	FBR	3.5					528	6.01				
	STEC	2.2					354	8.46				

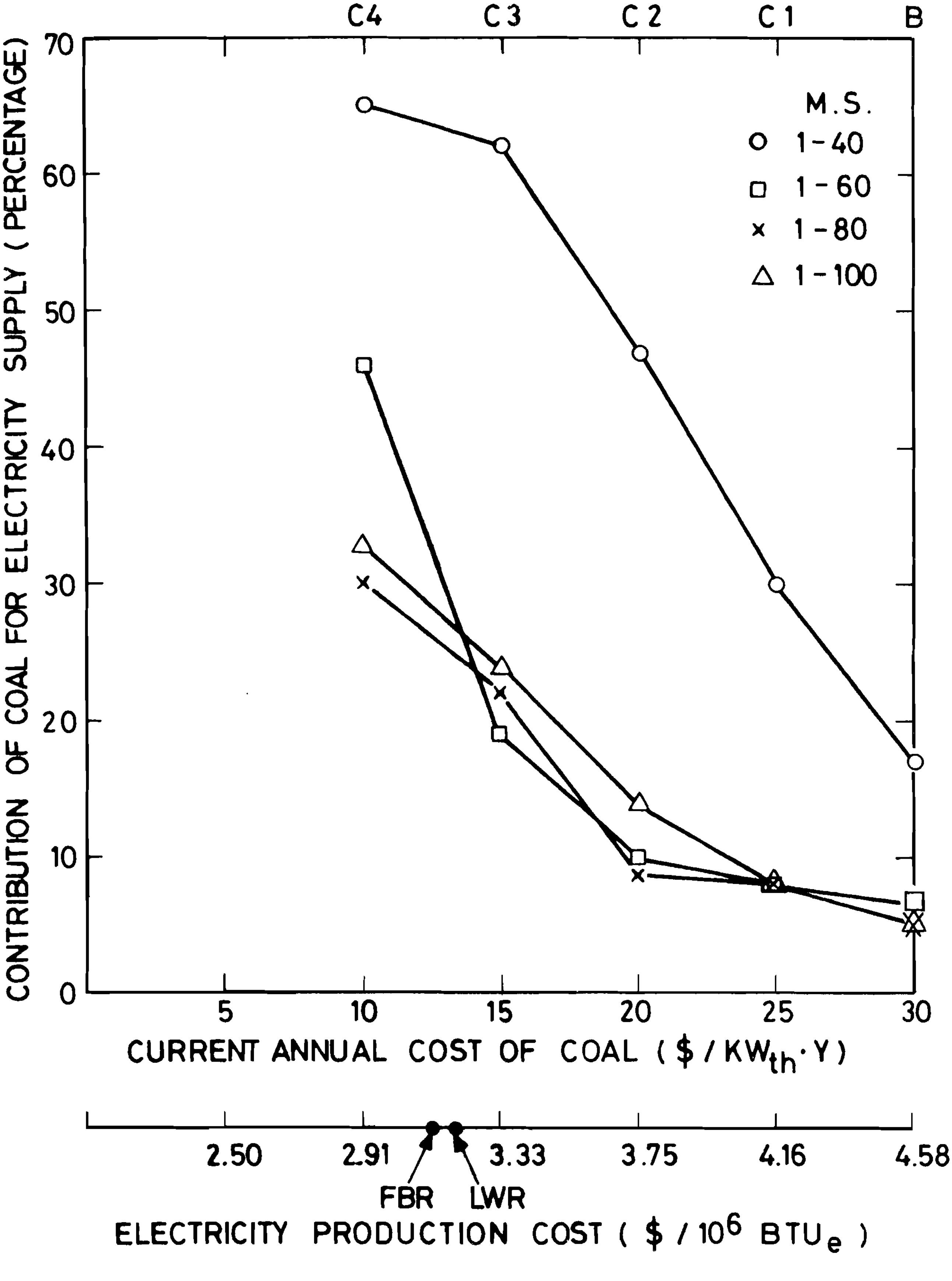

Figure 21. Contribution of coal to electricity supply versus current annual cost of coal.

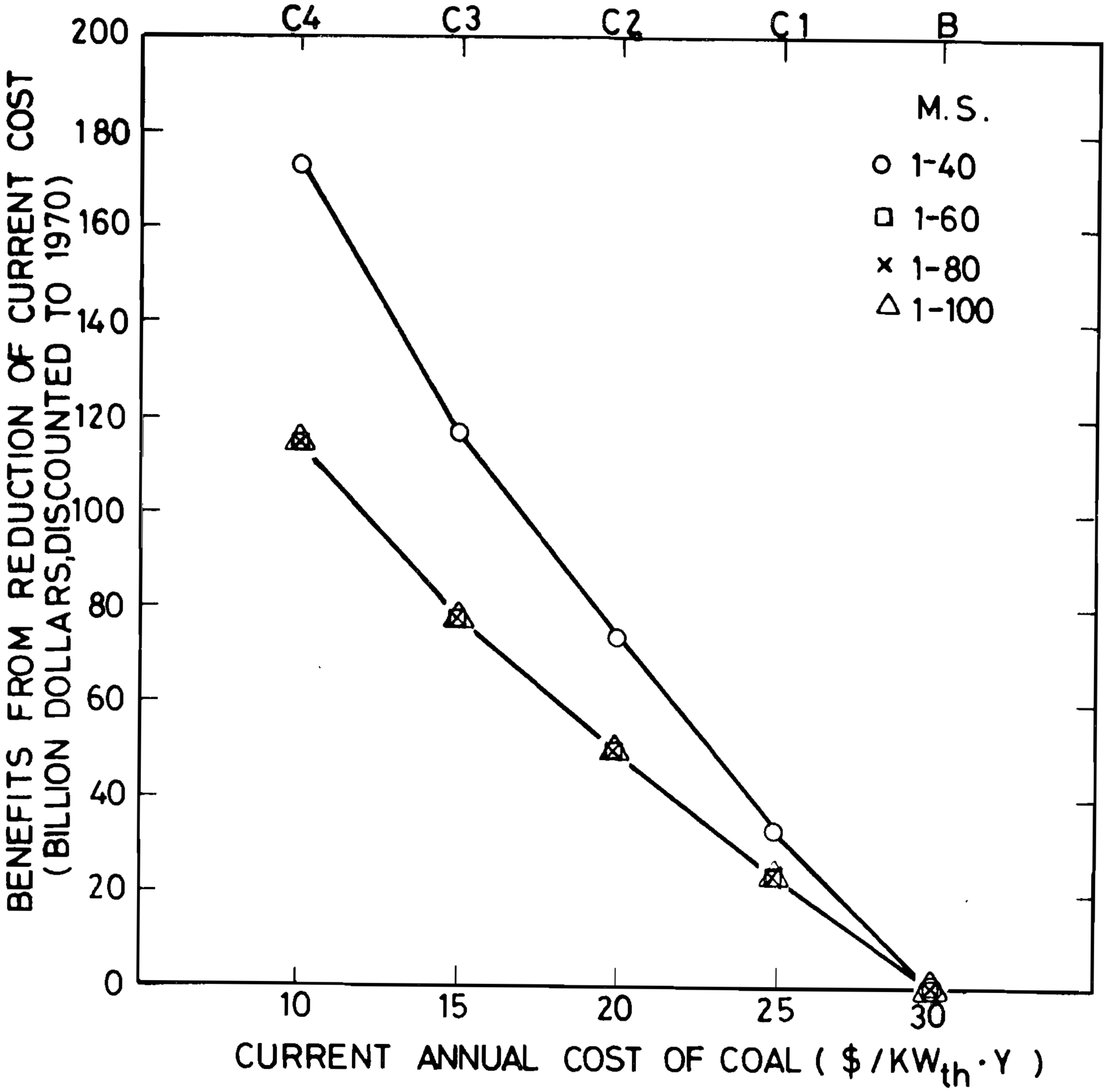

Figure 22. Benefits from reduction of current annual cost of coal-fired plant.

Table 14. Cumulative consumption of natural resources.

		1-40			1-60			1-80			1-100		
		PETG (Q)	COAL (Q)	NU (10^6t)	PETG (Q)	COAL (Q)	NU (10^6t)	PETG (Q)	COAL (Q)	NU (10^6t)	PETG (Q)	COAL (Q)	NU (10^6t)
S4	2000	2.1	0.7	2.5	2.4	0.4	1.8	2.4	0.4	1.8	2.4	0.4	1.8
	2050	2.3	1.1	6.0	3.4	0.4	2.9	4.5	0.4	2.9	5.6	0.4	2.9
S3	2000	2.2	0.7	2.5	2.4	0.4	1.8	2.4	0.4	1.8	2.4	0.4	1.8
	2050	2.3	1.3	6.5	3.4	0.4	3.1	4.5	0.4	2.9	5.6	0.4	2.9
S2	2000	2.2	0.6	2.5	2.4	0.4	2.0	2.4	0.4	2.0	2.4	0.4	2.0
	2050	2.3	1.1	8.0	3.4	0.4	4.6	4.5	0.4	4.4	5.6	0.4	4.5
S1	2000	2.2	0.7	2.5	2.4	0.4	2.1	2.5	0.4	2.1	2.5	0.4	2.1
	2050	2.3	1.3	11.1	3.4	0.4	5.3	4.5	0.4	4.9	5.6	0.4	5.0
Base Case B	2000	2.2	0.7	2.5	2.4	0.4	2.1	2.5	0.4	2.1	2.5	0.4	2.1
	2050	2.3	1.3	12.2	3.4	0.4	5.8	4.5	0.4	5.8	5.6	0.4	5.3
C1	2000	2.1	0.8	2.5	2.1	0.8	2.5	2.1	0.8	2.5	2.5	0.4	2.0
	2050	2.3	2.1	7.0	2.3	2.1	7.0	2.3	2.1	7.0	5.6	0.5	4.8
C2	2000	2.1	0.9	2.0	2.1	0.9	2.0	2.1	0.9	2.0	2.5	0.5	1.7
	2050	2.3	3.2	4.6	2.3	3.2	4.6	2.3	3.2	4.6	5.6	1.0	3.7
C3	2000	2.1	1.2	1.3	2.1	1.2	1.3	2.1	1.2	1.3	2.5	0.6	1.4
	2050	2.3	4.2	3.0	2.3	4.2	3.0	2.3	4.2	3.0	5.6	1.5	3.1
C4	2000	2.0	1.5	0.8	2.0	1.5	0.8	2.0	1.5	0.8	2.5	1.0	0.5
	2050	2.3	4.2	3.2	2.3	4.2	3.2	2.3	4.2	3.2	5.6	2.0	2.7

Notes:

PETG = Petroleum-gas, in Q; = 10^{18}BTU.

COAL = Coal, in Q; = 10^{18}BTU.

NU = Natural Uranium in 10^6t; = 10^6metric tonnes of U.

V. Concluding Remarks

This paper illustrates selected strategies for a transition from fossil fuel technologies to nuclear and solar technologies, by using a linear programming model that extends the Häfele-Manne model. The results of this investigation of solar technology assessment are as follows:

(1) The target value of the capital cost of the solar thermal electric conversion system is approximately \$800/KWe (case S2) in 1974 US dollars. Provided solar electricity is used for the base load, the capital cost of solar power is higher than that of nuclear power. However, for the intermediate load, the difference in the costs of nuclear and solar power disappears. The reason is that, despite the high capital cost, the solar power station is more economic for the intermediate load than it is for base load, mainly due to the lesser requirement of the number of heliostats. In order to make solar cost more competitive with nuclear cost for base load, it is necessary to further reduce the capital cost of the solar thermal electric conversion system by about \$200/KWe.

(2) Solar hydrogen is competitive with the electrolytic hydrogen in the base case itself. If the process heat from the solar power station, the capital cost of which is \$800/KWe, can be used for producing hydrogen (as is the case for S2), then the hydrogen production cost is almost equal to the HTGR cost. Thus, the capital cost of a solar power station, i.e., \$800/KWe, is also the rough target value for the solar hydrogen that would replace the nuclear hydrogen.

(3) The cost savings obtained from reaching this target cost are \$2 billion to \$3 billion if there are abundant petroleum and gas reserves (80 and 100 years respectively), and \$9 billion to \$16 billion if there are scarce petroleum and gas reserves (forty and sixty years, respectively).

In general, hydrogen technologies play an important role in the expanded model since hydrogen is the only fuel that can take the place of petroleum and gas. The above observation--that there is a significant difference in the benefits depending on the petroleum and gas availability--is because the cost savings result more from the introduction of solar hydrogen in place of electrolytic hydrogen than from the use of the solar electricity for the intermediate load. In other words, strategies for the transition from the petroleum and gas basis vary greatly with the choice of the HTGR, solar and electrolytic hydrogen. Therefore, a cost estimate on hydrogen technology is one of the most crucial parammeters in the model. For the expanded model to yield some detailed results, it is necessary to not only improve the accuracy of cost estimation but also to take into account other alternatives, such as an advanced type of hydrocarbon and biogas fuel.

The model is also concerned with the effect of the coal cost on different energy supply strategies. The break-even value of the current annual cost of a coal-fired generating plant (that yields a zero difference between nuclear power costs and coal power costs) is approximately 3.7 mills/KWeH. The solution indicates that $80 billion are expected to be the cost savings. This figure is provisional, based on the availability of more than 3.5×10^{18} BTU of the petroleum and gas reserves, and more than 3.0×10^{18} BTU of the coal that costs $\$0.4/10^6$ BTU (this corresponds to the above break-even value).

In fact, however, resource availability depends on cost. To assess the eonomic feasibility of coal technologies, one must take into account this cost dependency, i.e. the different grades of coal resource. In addition, petroleum and gas should be regarded as different resources; however, the expanded model supposes that they can be aggregated to be the same. Thereafter, the pace of transition from one alternative to another will be more smooth.[3]

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[3]See, for example, [7].

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APPENDIX A

Mathematical Description of the Expanded Model

A-1: Notation on Indices, Variables and Parameters.

A-2: Supply Alternatives Characterization.

A-3: Demand Projections.

A-4: Objective Function-Cost Minimand.

Appendix A-1: Notation

Indices

h: index for time-step (ten years) of planning horizon;

h = 1,...,H; H: number of decades for planning horizon,

h = 1:	1971	to	1980
h = 2:	1981	to	1990
.	.	.	.
.	.	.	.
.	.	.	.
h = H:	1971 + 10(H - 1)	to	1980 + 10(H - 1).

i: index for energy supplying alternatives;

i = 1,...,I; I: number of alternatives.

Fossil:

i = 1: COAL (coal steam generating electricity);

i = 2: PETG (petroleum and gas in form of secondary energy other than electricity).

Nuclear:

i = 3: LWR (LWR electric generation);

i = 4: FBR (FBR electric generation);

i = 5: HTGR (hydrogen production from thermochemical water splitting by process heat of HTGR).

Solar:

i = 6: STEC (solar thermal electric conversion);

i = 7: OBSE (ocean-based solar electricity);†

i = 8: PVSE (photovoltaic solar electricity);†

i = 9: SBIO (biogas production from solar power);†

i = 10: SHYD (hydrogen production from solar power).

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†As regards the calculation results reported here, they are excluded because of the lack of data.

Auxiliary:

$i = 11$: ELHY (hydrogen production from electrolysis).

j: index for energy resource, natural and man-made;

$j = 1,\ldots,J$; J: number of energy resources.

Natural Resources:

$j = 1$: COAL (coal and lignite);

$j = 2$: PETG (petroleum and natural gas);

$j = 3$: NATU (natural uranium, low and high costs).

Man-Made Resources:

$j = 4$: PLUT (fissile plutonium);

$j = 5$: U233 (uranium-233).

k: index for energy utilization path from alternative, i to end use category, ℓ;

$k = k(i,\ell)$; $k(1,1)$, $k(1,2),\ldots,k(I,L)$;

$k(1,1)$: from alternative, COAL to category-1;

$k(1,2)$: from alternative, COAL to category-2;

.
.
.

$k(I,L)$: from alternative, ELHY to category-L.

ℓ: index for end use category;

$\ell = 1,\ldots,L$; L: number of end use category;

$\ell = 1$: RCEBL (residential and commercial uses, base load electricity);

$\ell = 2$: RCEIP (residential and commercial uses, intermediate and peak load electricity);

$\ell = 3$: RCN (residential and commercial uses, non-electric energy);

$\ell = 4$: INEBL (industrial use, base load electricity);

$\ell = 5$: INEIP (industrial use, intermediate and peak load electricity);

$\ell = 6$: INN (industrial use, non-electric energy);

$\ell = 7$: TRN (transport use, non-electric energy).

Variables

Endogenous

DQ_i^h : Installed energy production capacity of alternative i, introduced at time-step h (TW_{th}/year);

DP_i^h : Energy production capacity of alternative i, actually used out of DQ_i^h (TW_{th}/year);

$DQ_{k(i,\ell)}^h$: Installed energy production capacity introduced for path $k(i,\ell)$ at time-step h (TW_{th}/year);

$DP_{k(i,\ell)}^h$: Energy production capacity actually used out of $DQ_{k(i,\ell)}^h$ (TW_{th}/year);

$DP_{k(FBPL,\ell)}^h$: Plutonium production capacity used out of $DP_k^h(FBR,\ell)$ (TW_{th}/year);

PC_i^h : Annual energy production activity of alternative i at time-step h (TW_{th});

$PC_{k(i,\ell)}^h$: Annual energy production activity in path $k(i,\ell)$ at time-step h (TW_{th});

DCS_j^h : Annual consumption of resource j (10^{18}BTU/year, 10^6 or 10^3ton/year);

DCS_{NULC}^h : Annual consumption of low-cost ($15/lb) natural uranium ($10^6$ton/year);

DCS_{NUHC}^h : Annual consumption of high-cost ($50/lb) natural uranium ($10^6$ ton/year);

CS_j^h : Cumulative consumption of resource j (10^{18}BTU, 10^6 or 10^3ton); and

CS_{NULC}^h : Cumulative consumption of low-cost ($15/lb) natural uranium ($10^6$ton).

Exogenous

DM^h_{ELEC} : Annual demand of electric energy (TW_e • year/year);

DM^h_{NONE} : Annual demand of non-electric energy (TW_{non-e} • year/year);

DM^h_{RCE} : Annual electricity demand of residential and commercial use (TW_e • year/year);

DM^h_{INE} : Annual electricity demand of industrial use (TW_e • year/year); and

DM^h_ℓ : Annual energy demand of category ℓ ($TW_{e,\text{ or none}}$ • year/year).

Parameters

η_i : Energy supply efficiency of alternative i (-);

η_{HTR} : Thermal efficiency of high-temperature gas cooled reactor (-);

$\delta F_{j,i}$: Initial inventory of nuclear fuel j of reactor i (10^6 or 10^3ton/TW_e);

$\delta R_{j,i}$: Retired inventory of nuclear fuel j of reactor i (10^6 or 10^3ton/TW_e);

$\alpha F_{j,i}$: Annual requirement of nuclear fuel j of reactor i (10^6 or 10^3ton/TW_e • year);

$\alpha R_{j,i}$: Annual recovery of nuclear fuel j of reactor i (10^6 or 10^3ton/TW_e • year);

RA_j : Natural energy resource availability (years or 10^6ton);

$\overline{DMELEC}^h$: Total annual demand of electric energy ($TW_{th,\text{ LWR equivalent}}$ • year/year);

$\overline{DMNONE}^h$: Total annual demand of non-electric energy ($TW_{th,\text{ PETG equivalent}}$ • year/year);

$\xi^h_{RCE \text{ or } INE}$: Electricity allocation factor for each demand sector, residential and commercial or industrial (-);

ξ^h_ℓ : Non-electric energy allocation factor for each demand sector, ℓ = RCN, INN, or TRN (-);

ζ_ℓ : Share of intermediate peak electricity; ℓ = RCEIP, INEIP (-).

ρ_ℓ : Load duration factor of each demand category (-);

$\nu_{i(\ell)}$: Fuel utilization factor of non-electric energy $i(\ell)$;

$Cur_{k(i,\ell)}$: Current cost of energy utilization path k (i,ℓ) ($\$10^9/TW_{th} \cdot$ year);

$Cap_{k(i,\ell)}$: Capital cost of energy utilization path k (i,ℓ) ($\$10^9/TW_{th}$);

r : Annual discount rate $\cdot$ ($year^{-1}$);

τ : Lag time between commissioning and full power operation (year);

λ_i : Upper bound on annual solar technology introduction rate (TW_{th}/year);

θ_i : R&D time-interval up to commercial introduction of solar technology (year);

$UBDQ^h_i$: Upper bound on annual introduction rate of alternative i (TW_{th}/year); and

ε_ℓ : 1970 annual increasing rate of energy demand of demand category ℓ.

Appendix A-2: Supply Alternatives Characterization

Annual Energy Production Activity $PC^h_{k(i,\ell)}$ (TW_{th})

$$PC^h_{k(i,\ell)} = 5.0 * DP^{h-3}_{k(i,\ell)} + 10.0 * DP^{h-2}_{k(i,\ell)} + 10.0 * DP^{h-1}_{k(i,\ell)} + 5.0 * DP^h_{k(i,\ell)} \quad ; \quad \forall_h, \forall_k \quad , \qquad (A1)^{\ddagger}$$

where,

$$DP^h_{k(i,\ell)} \leq DQ^h_{k(i,\ell)} \quad ; \quad \forall_h, \forall_k \quad , \qquad (A2)^{\ddagger}$$

and

$$DP^h_i = \sum_{\ell=1}^{L} DP^h_{k(i,\ell)} \quad ; \quad \forall_h, \forall_i \quad ,$$

$$PC^h_i = 5.0 * DP^{h-3}_i + 10.0 * DP^{h-2}_i + 10.0 * DP^{h-1}_i + 5.0 * DP^h_i \quad ; \quad \forall_h, \forall_i \quad .$$

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‡As regards fossil fuel and solar alternatives, these equations can be combined such that $PC^h \leq 5DQ^{h-3} + 10DQ^{h-2} + 10DQ^{h-1} + 5DQ^h$. In case of nuclear alternatives, however, these should be, in principle, separated because of the complicatedness of the nuclear fuel cycle, as will be shown later. Furthermore, the PC activity should be expressed also in terms of the age of plants (Figure 3). However, these considerations require a great number of variables; therefore, in the present version of the program, the most simplified form was taken; the combined equations mentioned above were used not only for the fossil and solar but also for the nuclear, after confirming the fact that <u>all</u> nuclear plants are not underutilized but used to <u>full</u> power level because of their high capital costs.

Initial Condition on $DQ^{h}_{k(i,\ell)}$ (TW_{th}/year)

For i = 1 (COAL)

$$(1+\varepsilon_\ell)^{30} * DQ^{-3}_{k(1,\ell)} = (1+\varepsilon_\ell)^{20} * DQ^{-2}_{k(1,\ell)}$$

$$= (1+\varepsilon_\ell)^{10} * DQ^{-1}_{k(1,\ell)} = DQ^{0}_{k(1,\ell)} \quad ,$$

$$PC^{0}_{k(1,\ell)} = \frac{1}{\eta_{COAL}} * DM^{0}_{\ell} \quad ;$$

$$\ell = 1\ (RCEBL),\ 2\ (RCEIP),\ 4\ (INEBL),\ 5\ (INEIP) \quad .$$

For i = 2 (PETG)

$$(1+\varepsilon_\ell)^{30} * DQ^{-3}_{k(2,\ell)} = (1+\varepsilon_\ell)^{20} * DQ^{-2}_{k(2,\ell)}$$

$$= (1+\varepsilon_\ell)^{10} * DQ^{-1}_{k(2,\ell)}$$

$$= DQ^{0}_{k(2,\ell)} \quad ,$$

$$PC^{0}_{k(2,\ell)} = DM^{0}_{\ell} \quad ; \quad \ell = 3\ (RCN),\ 6\ (INN),\ 7\ (TRN) \quad .$$

For all the other i's

$$DQ^{-2}_{k(i,\ell)} = DQ^{-1}_{k(i,\ell)} = DQ^{0}_{k(i,\ell)} = 0 \quad ; \quad \forall_\ell \quad .$$

Upper Bounds on DQ^{h}_{i} (TW_{th}/year)

$$DQ^{h}_{i} \leq UBDQ^{h}_{i} \quad ; \quad \forall_h, \forall_i \quad .$$

For i = 1 (COAL), 2 (PETG)

$$UBDQ^{h}_{i} = \infty \quad ; \quad \forall_h \quad .$$

For $\underline{i = 3\ (LWR)}$

$$UBDQ^h_{LWR} = .04 + .06 * (h - 1) \quad ; \quad \forall_h \quad .$$

For $\underline{i = 4\ (FBR),\ 5\ (HTGR)}$

$$UBDQ^h_i = .04 + .06 * (h - 3) \quad ; \quad h \geq 3$$

$$= 0 \quad ; \quad h = 1,2 \quad .$$

For $\underline{i = 6\ (STEC),\ 7\ (OBSE),\ 8\ (PVSE)}$

$$UBDQ^h_i = \frac{\eta_{FBR}}{\eta_i} * [.04 + \frac{.06}{\lambda_i} * (h - \frac{\theta_i}{10})] \quad ; \quad h \geq \frac{\theta_i}{10}$$

$$= 0 \quad ; \quad h < \frac{\theta_i}{10} \quad .$$

For $\underline{i = 9\ (SBIO),\ 10\ (SHYD)}$

$$UBDQ^h_i = \frac{\eta_{HTGR}}{\eta_i} * [.04 + \frac{.06}{\lambda_i} * (h - \frac{\theta_i}{10})] \quad ; \quad h \geq \frac{\theta_i}{10}$$

$$= 0 \quad ; \quad h < \frac{\theta_i}{10} \quad .$$

For $\underline{i = 11\ (ELHY)}$

$$UBDQ^h_{ELHY} = \infty \quad .$$

<u>Net Annual Consumption of Resource j, DCS^h_j</u>

<u>Natural Resources</u>

For <u>j = 1 (COAL), (10^{18} BTU/year)</u>

$$DCS^h_{COAL} = .03 * PC^h_{COAL} \quad ; \quad \forall_h \quad .$$

For j = 2 (PETG), (10^{18} BTU/year)

$$DCS^h_{PETG} = .03 * PC^h_{PETG} \quad ; \quad \forall_h \quad .$$

For j = 3 (NATU), (10^6 tons of NU/year)

$$\begin{aligned} DCS^h_{NATU} = & \sum_{\ell}^{1,2,4,5} \Bigg[\eta_{LWR} * \Bigg\{ \delta R_{NU,LWR} * DQ^{h-3}_{k(3,\ell)} + \delta F_{NU,LWR} \\ & * DQ^h_{k(3,\ell)} + \alpha^e_{NU,k(3,\ell)} * DP^{h-3}_{k(3,\ell)} + \alpha^s_{NU,k(3,\ell)} \\ & * DP^{h-2}_{k(3,\ell)} + \alpha^s_{NU,k(3,\ell)} * DP^{h-1}_{k(3,\ell)} + \alpha^b_{NU,k(3,\ell)} \\ & * DP^h_{k(3,\ell)} \Bigg\} \Bigg] + \sum_{\ell}^{3,6,7} \Bigg[\eta_{HTR} * \Bigg\{ \delta R_{NU,HTR} \\ & * DP^{h-3}_{k(5,\ell)} + \delta F_{NU,HTR} * DQ^h_{k(5,\ell)} + \alpha^e_{NU,k(5,\ell)} \\ & * DP^{h-3}_{k(5,\ell)} + \alpha^s_{NU,k(5,\ell)} * DP^{h-2}_{k(5,\ell)} + \alpha^s_{NU,k(5,\ell)} \\ & * DP^{h-1}_{k(5,\ell)} + \alpha^b_{NU,k(5,\ell)} * DP^h_{k(5,\ell)} \Bigg\} \Bigg] \quad ; \quad \forall_h \quad , \end{aligned} \tag{A3}$$ ||

|| Provided that all nuclear reactors are not underutilized but used with full power operation, nuclear fuel cycle equations, A3, A4 and A5 can be simplified by using DQ and PC instead of DP. As stated in the footnote on page 71, those simplified forms are used in the present version. The relations between the coefficients of Equations A3, A4, A5, and the reactor data in Table 4 are as below:

$$\alpha^b = a(4 + \Delta t_f) - d(5 - \Delta t_r) \quad ,$$

$$\alpha^s = a - d \quad ,$$

$$\alpha^e = a(5 - \Delta t_f) - d(4 + \Delta t_r) - \delta_r \quad ,$$

$$\alpha B^b = (a - d)(5 - \Delta t_r) \quad ,$$

$$\alpha B^s = a - d \quad ,$$

$$\alpha B^e = (a - d)\ (4 + \Delta t_r) \quad ,$$

$$\delta F = IF$$

$$\delta R = - IR + \delta_r$$

(Note: δ_r = the component of the nuclear fuel recovered from retired core, the amount of which is dependent on the operational scheme during plant life.)

and

$$DCS^{h}_{NATU} = DCS^{h}_{NULC} + DCS^{h}_{NUHC} \quad ; \quad \forall_h \quad .$$

Man-Made Resources

For j = 4 (PLUT), (10^3tons of Pu(f)/year)

$$DCS^{h}_{PLUT} = \sum_{\ell}^{1,2,4,5} \Bigg[\eta_{LWR} * \Bigg\{ \alpha^{e}_{PU,k(3,\ell)} * DP^{h-3}_{k(3,\ell)}$$

$$+ \alpha^{s}_{PU,k(3,\ell)} * DP^{h-2}_{k(3,\ell)} + \alpha^{s}_{PU,k(3,\ell)} * DP^{h-1}_{k(3,\ell)}$$

$$+ \alpha^{b}_{PU,k(3,\ell)} * DP^{h}_{k(3,\ell)} \Bigg\}$$

$$+ \eta_{FBR} * \Bigg\{ \delta R_{PU,FBR} * DQ^{h-3}_{k(4,\ell)} + \delta F_{PU,FBR}$$

$$* DQ^{h}_{k(4,\ell)} + \alpha^{e}_{PU,k(4,\ell)} * DP^{h-3}_{k(4,\ell)} + \alpha^{s}_{PU,k(4,\ell)}$$

$$* DP^{h-2}_{k(4,\ell)} + \alpha^{s}_{PU,k(4,\ell)} * DP^{h-1}_{k(4,\ell)} + \alpha^{b}_{PU,k(4,\ell)}$$

$$* DP^{h}_{k(4,\ell)} + \alpha B^{e}_{PU,k(4,\ell)}$$

$$* \left(DP^{h-3}_{k(FBPL,\ell)} - DP^{h-3}_{k(4,\ell)} \right) + \alpha B^{s}_{PU,k(4,\ell)}$$

$$* \left(DP^{h-2}_{k(FBPL,\ell)} - DP^{h-2}_{k(4,\ell)} \right) + \alpha B^{s}_{PU,k(4,\ell)}$$

$$* \left(DP^{h-1}_{k(FBPL,\ell)} - DP^{h-1}_{k(4,\ell)} \right) + \alpha B^{b}_{PU,k(4,\ell)}$$

$$* \left(DP^{h}_{k(FBPL,\ell)} - DP^{h}_{k(4,\ell)} \right) \Bigg\} \Bigg] \quad ; \quad \forall_h \quad . \qquad (A4)^{\S}$$

§ See footnote || on page 74.

where

$$DP^{-2}_{k(FBPL,\ell)} = DP^{-1}_{k(FBPL,\ell)} = DP^{0}_{k(FBPL,\ell)} = 0 \quad ; \quad \forall_\ell \quad ,$$

$$DP^{h}_{k(FBPL,\ell)} \leq DP^{h}_{k(FBR,\ell)} \quad ; \quad \forall_h, \forall_\ell \quad .$$

For $\underline{j = 5\ (U233),\ (10^3 \text{tons of } {}^{233}U/\text{year})}$

$$\begin{aligned}
DCS^{h}_{U233} = \sum_{\ell}^{3,6,7} \Bigg[\eta_{HTR} * \Bigg\{ & \delta R_{U3,HTR} * DQ^{h-3}_{k(5,\ell)} + \delta F_{U3,HTR} \\
& * DQ^{h}_{k(5,\ell)} + \alpha^{e}_{U3,k(5,\ell)} * DP^{h-3}_{k(5,\ell)} + \alpha^{s}_{U3,k(5,\ell)} \\
& * DP^{h-2}_{k(5,\ell)} + \alpha^{s}_{U3,k(5,\ell)} * DP^{h-1}_{k(5,\ell)} + \alpha^{b}_{U3,k(5,\ell)} \\
& * DP^{h}_{k(5,\ell)} \Bigg\} \Bigg] \\
& + \sum_{\ell}^{1,2,4,5} \Bigg[\eta_{FBR} * \Bigg\{ \alpha B^{e}_{U3,k(4,\ell)} \\
& * \left(DP^{h-3}_{k(4,\ell)} - DP^{h-3}_{k(FBPL,\ell)} \right) + \alpha B^{s}_{U3,k(4,\ell)} \\
& * \left(DP^{h-2}_{k(4,\ell)} - DP^{h-2}_{k(FBPL,\ell)} \right) + \alpha B^{s}_{U3,k(4,\ell)} \\
& * \left(DP^{h-1}_{k(4,\ell)} - DP^{h-1}_{k(FBPL,\ell)} \right) + \alpha B^{b}_{U3,k(4,\ell)} \\
& * \left(DP^{h}_{k(4,\ell)} - DP^{h}_{k(FBPL,\ell)} \right) \Bigg\} \Bigg] \quad ; \quad \forall_h \quad . \qquad (A5)^{\P}
\end{aligned}$$

¶ See footnote || on page 74.

Upper Bounds on Cumulative Resource Extraction, CS_j^h :

Natural Resources

For j = 1 (COAL), (10^{18} BTU)

$$CS_{COAL}^{H} = 10 * \sum_{h=1}^{H} DCS_{COAL}^{h}$$

$$\leq RA_{COAL}(\text{years}) * .625(TW_{th}) * .03\left(\frac{10^{18}\text{BTU}}{TW_{th}\ \text{year}}\right) \quad .$$

For j = 2 (PETG), (10^{18} BTU)

$$CS_{PETG}^{H} = 10 * \sum_{h=1}^{H} DCS_{PETG}^{h}$$

$$\leq RA_{PETG}(\text{years}) * 1.875(TW_{th}) * .03\left(\frac{10^{18}\text{BTU}}{TW_{th}\ \text{year}}\right) \quad .$$

For j = 3 (NATU), (10^{6} tons of NU)

$$CS_{NATU}^{h} - CS_{NUHC}^{h} = CS_{NULC}^{h} = 10 * \sum_{h'=1}^{h} DCS_{NULC}^{h'}$$

$$= 10 * \sum_{h'=1}^{h} (DCS_{NATU}^{h'} - DCS_{NUHC}^{h'})$$

$$\leq RA_{NULC}\ (10^{6}\text{tons of NU}) \quad ; \quad \forall_h \quad .$$

Man-Made Resources

For j = 4 (PLUT), (10^{3} tons of PU(f))

$$CS_{PLUT}^{h} = 10 * \sum_{h'=1}^{h} DCS_{PLUT}^{h'}$$

$$\leq 0.0\ (10^{3}\text{tons of Pu(f)}) \quad ; \quad \forall_h \quad .$$

For $j = 5$ (U233), (10^3tons of ^{233}U)

$$CS^{h}_{U233} = 10 * \sum_{h'=1}^{h} DCS^{h'}_{U233}$$

$$\leq 0.0 \ (10^3 \text{tons of } ^{233}U) \quad ; \quad \forall_h \ .$$

Upper Bounds on Cumulative Resource Extraction, CS_j^h :

Natural Resources

For j = 1 (COAL), (10^{18}BTU)

$$CS_{COAL}^{H} = 10 * \sum_{h=1}^{H} DCS_{COAL}^{h}$$

$$\leq RA_{COAL}(\text{years}) * .625(TW_{th}) * .03\left(\frac{10^{18}\,BTU}{TW_{th}\ \text{year}}\right) \quad .$$

For j = 2 (PETG), (10^{18}BTU)

$$CS_{PETG}^{H} = 10 * \sum_{h=1}^{H} DCS_{PETG}^{h}$$

$$\leq RA_{PETG}(\text{years}) * 1.875(TW_{th}) * .03\left(\frac{10^{18}\,BTU}{TW_{th}\ \text{year}}\right) \quad .$$

For j = 3 (NATU), (10^{6}tons of NU)

$$CS_{NATU}^{h} - CS_{NUHC}^{h} = CS_{NULC}^{h} = 10 * \sum_{h'=1}^{h} DCS_{NULC}^{h'}$$

$$= 10 * \sum_{h'=1}^{h} (DCS_{NATU}^{h'} - DCS_{NUHC}^{h'})$$

$$\leq RA_{NULC}\ (10^{6}\text{tons of NU}) \quad ; \quad \forall_h \quad .$$

Man-Made Resources

For j = 4 (PLUT), (10^{3}tons of PU(f))

$$CS_{PLUT}^{h} = 10 * \sum_{h'=1}^{h} DCS_{PLUT}^{h'}$$

$$\leq 0.0\ (10^{3}\text{tons of Pu(f)}) \quad ; \quad \forall_h \quad .$$

For j = 5 (U233), (10^3tons of ^{233}U)

$$CS^{h}_{U233} = 10 * \sum_{h'=1}^{h} DCS^{h'}_{U233}$$

$$\leq 0.0 \ (10^3 \text{tons of } {}^{233}\text{U}) \quad ; \quad \forall_h \ .$$

Appendix A-3: Demand Projections

Electricity

Secondary Energy, ($TW_e \cdot$ year)

$$DM^h_{ELEC} = \eta_{LWR} * \overline{DMELEC}^h \quad ; \quad \forall_h \quad .$$

End Use, ($TW_e \cdot$ year)

$$DM^h_{RCE} = \xi^h_{RCE} * DM^h_{ELEC} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{RCEBL} = \rho_{RCEBL} * (1 - \zeta_{RCEIP}) * DM^h_{RCE} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{RCEIP} = \rho_{RCEIP} * \zeta_{RCEIP} * DM^h_{RCE} \quad ; \quad \forall_h \quad .$$

$$DM^h_{INE} = \xi^h_{INE} * DM^h_{ELEC} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{INEBL} = \rho_{INEBL} * (1 - \zeta_{INEIP}) * DM^h_{INE} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{INEIP} = \rho_{INEIP} * \zeta_{INEIP} * DM^h_{INE} \quad ; \quad \forall_h \quad .$$

Non-Electric Energy

Secondary Energy, ($TW_{non-e} \cdot$ year)

$$DM^h_{NONE} = \overline{DMNONE}^h \quad ; \quad \forall_h \quad .$$

End Use, ($TW_{non-e} \cdot$ year)

$$DM^h_{RCN} = \rho_{RCN} * \xi^h_{RCN} * DM^h_{NONE} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{INN} = \rho_{INN} * \xi^h_{INN} * DM^h_{NONE} \quad ; \quad \forall_h \quad ,$$

$$DM^h_{TRN} = \rho_{TRN} * \xi^h_{TRN} * DM^h_{NONE} \quad ; \quad \forall_h \quad .$$

Supply/Demand Balance

Electricity, ($TW_e \cdot$ year)

$$\sum_{i}^{1,3,4,6,7,8} \eta_i * PC^h_{k(i,\ell)} \geq DM^h_\ell \quad ;$$

$$\forall_h \quad , \quad \ell = 1 \text{ (RCEBL)}, \ 2 \text{ (RCEIP)}, \ 5 \text{ (INEIP)}.$$

$$\sum_{i}^{1,3,4,6,7,8} \eta_i * PC^h_{k(i,4)} \geq DM^h_4 + \sum_{\ell}^{3,6,7} \eta_{LWR}$$

$$* \ PC^h_{k(11,\ell)} \ ; \quad \forall_h \quad , \quad \ell = 4 \text{ (INEBL)} \quad .$$

Non-Electric Energy, ($TW_{non-e} \cdot$ year)

$$PC^h_{k(2,\ell)} + \frac{\nu_{HYDR(\ell)}}{\nu_{PETG(\ell)}} * \Big[\eta_{HTGR} * PC^h_{k(5,\ell)}$$

$$+ \eta_{SHYD} * PC^h_{k(10,\ell)} + \eta_{ELHY} * \eta_{LWR}$$

$$* \ PC^h_{k(11,\ell)} \Big] + \frac{\nu_{BIOG(\ell)}}{\nu_{PETG(\ell)}} * \eta_{SBIO}$$

$$* \ PC^h_{k(9,\ell)} \geq DM^h_\ell \quad ; \quad \forall_h \quad ,$$

$$\ell = 3 \text{ (RCN)}, \ 6 \text{ (INN)}, \ 7 \text{ (TRN)} \quad .$$

Appendix A-4: Objective Function - Cost Minimand

Present value of costs incurred annually during each decade over the planning horizon, $EC^h_{k(i,\ell)}$ (\$ 10^9/year)

$$EC^h_{k(i,\ell)} = \beta^{10(h-5)} * \Big[cur_{k(i,\ell)} * PC^h_{k(i,\ell)} + \beta^{-\tau} * (1 - TV^h) * cap_{k(i,\ell)} * DP^h_{k(i,\ell)} + cur_{NUHC} * DCS^h_{NUHC} \Big] \quad ; \quad \forall_h, \forall_k \quad ,$$

where,

$$\beta = 1/(1 + r) \quad ,$$

$$TV^h = \begin{cases} \beta^{10(H-h+1)} & ; \ h > H - 3 \\ 0 & ; \ \text{otherwise} \end{cases} \quad .$$

Objective Function, TC^H_K (\$$10^9$)

$$\min_{\left\{ \begin{array}{lll} DP^h_{k(i,\ell)} & ; & \begin{array}{l} 1 \le h \le H \\ 1 \le k \le K \end{array} \\ DCS^h_{NUHC} & ; & 1 \le h \le H \end{array} \right\}} \left[TC^H_K = \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{\ell=1}^{L} 10 * EC^h_{k(i,\ell)} \right] \quad .$$

APPENDIX B

Optimal Solutions of Base Cases, B-1.40, 1.60, 1.80 and 1.100, in Terms of Individual Demand Categories

	Case	Demand Category	
Figure B-1a	B-1.40	Elec., Base Load,	Res. and Comm.
1b			Indus.
1c			Subtotal
2a	B-1.40	Elec., Intermed.,	Res. and Comm.
2b			Indus.
2c			Subtotal
3a	B-1.40	Non-Elec.,	Res. and Comm.
3b			Indus.
3c			Transportation
Figure B-4a	B-1.60	Elec., Base Load,	Res. and Comm.
4b			Indus.
4c			Subtotal
5a	B-1.60	Elec., Intermed.,	Res. and Comm.
5b			Indus.
5c			Subtotal
6a	B-1.60	Non-Elec.,	Res. and Comm.
6b			Indus.
6c			Transportation
Figure B-7a	B-1.80	Elec., Base Load,	Res. and Comm.
7b			Indus.
7c			Subtotal
8a	B-1.80	Elec., Intermed.,	Res. and Comm.
8b			Indus.
8c			Subtotal
9a	B-1.80	Non-Elec.,	Res. and Comm.
9b			Indus.
9c			Subtotal
Figure B-10a	B-1.100	Elec., Base Load,	Res. and Comm.
10b			Indus.
10c			Subtotal
11a	B-1.100	Elec., Intermed.,	Res. and Comm.
11b			Indus.
11c			Subtotal
12a	B-1.100	Non-Elec.,	Res. and Comm.
12b			Indus.
12c			Transportation

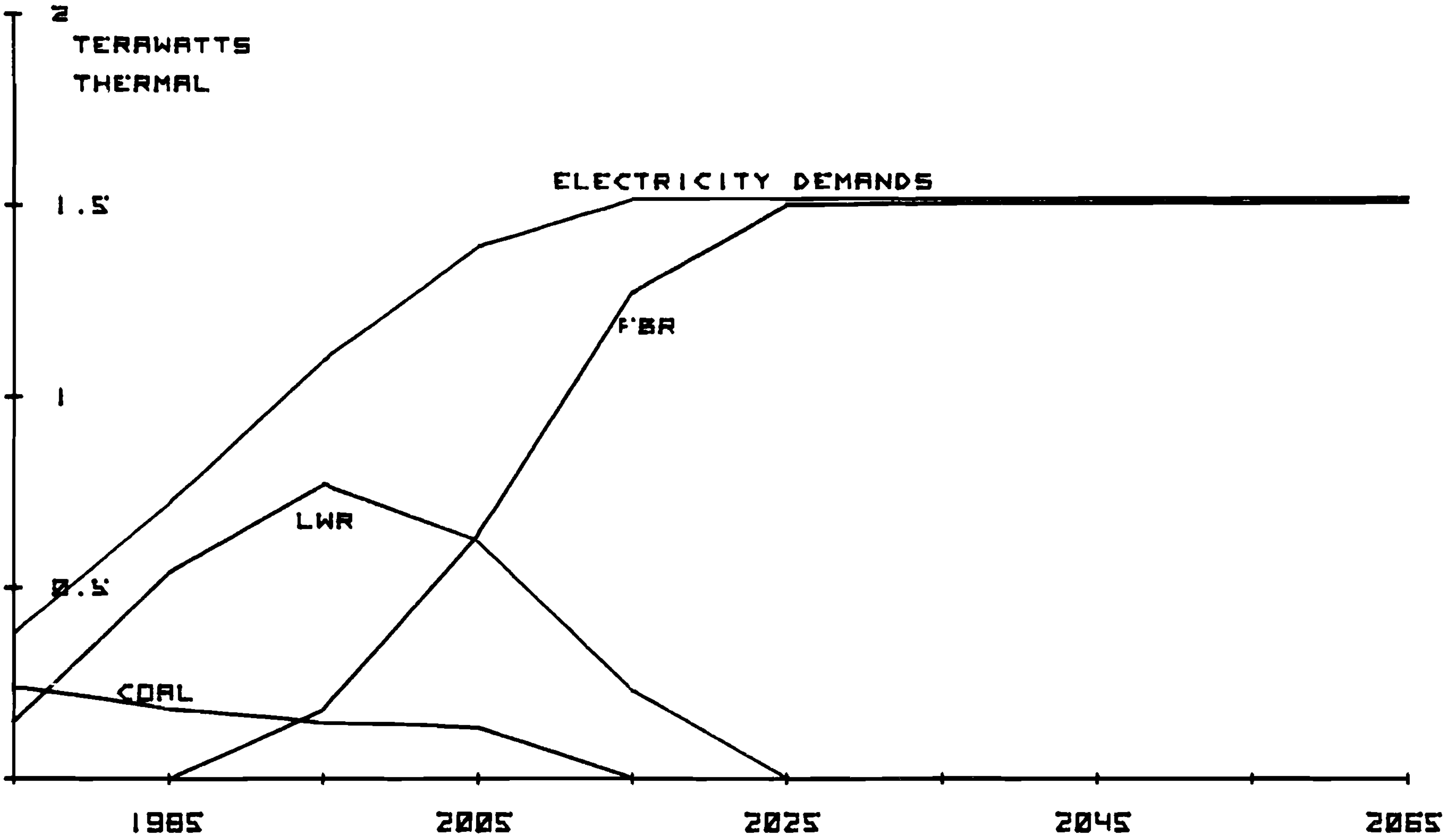

Figure B-1a. Model society 1.40: electric energy demand sector 1.

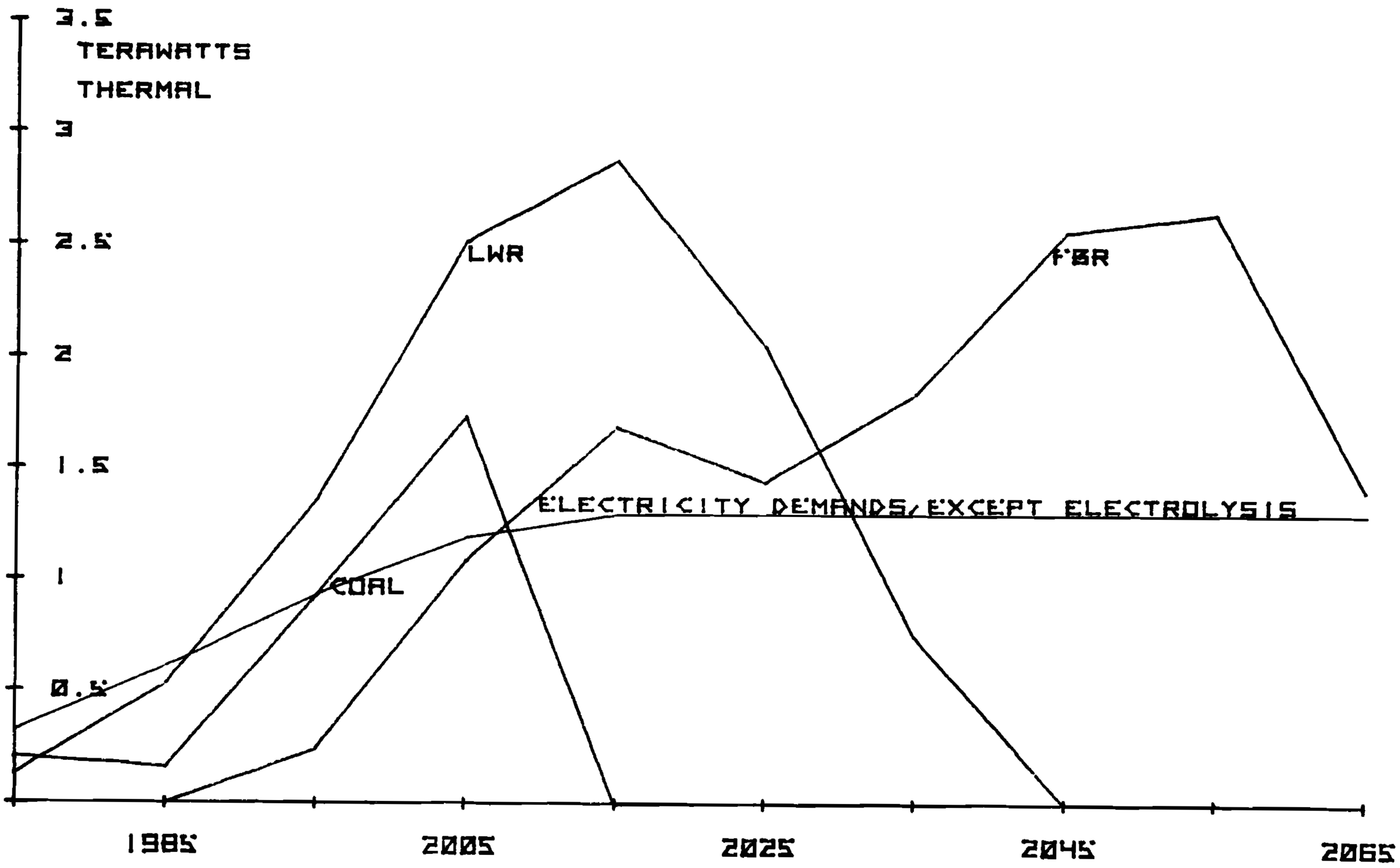

Figure B-1b. Model society 1.40: electric energy demand sector 4.

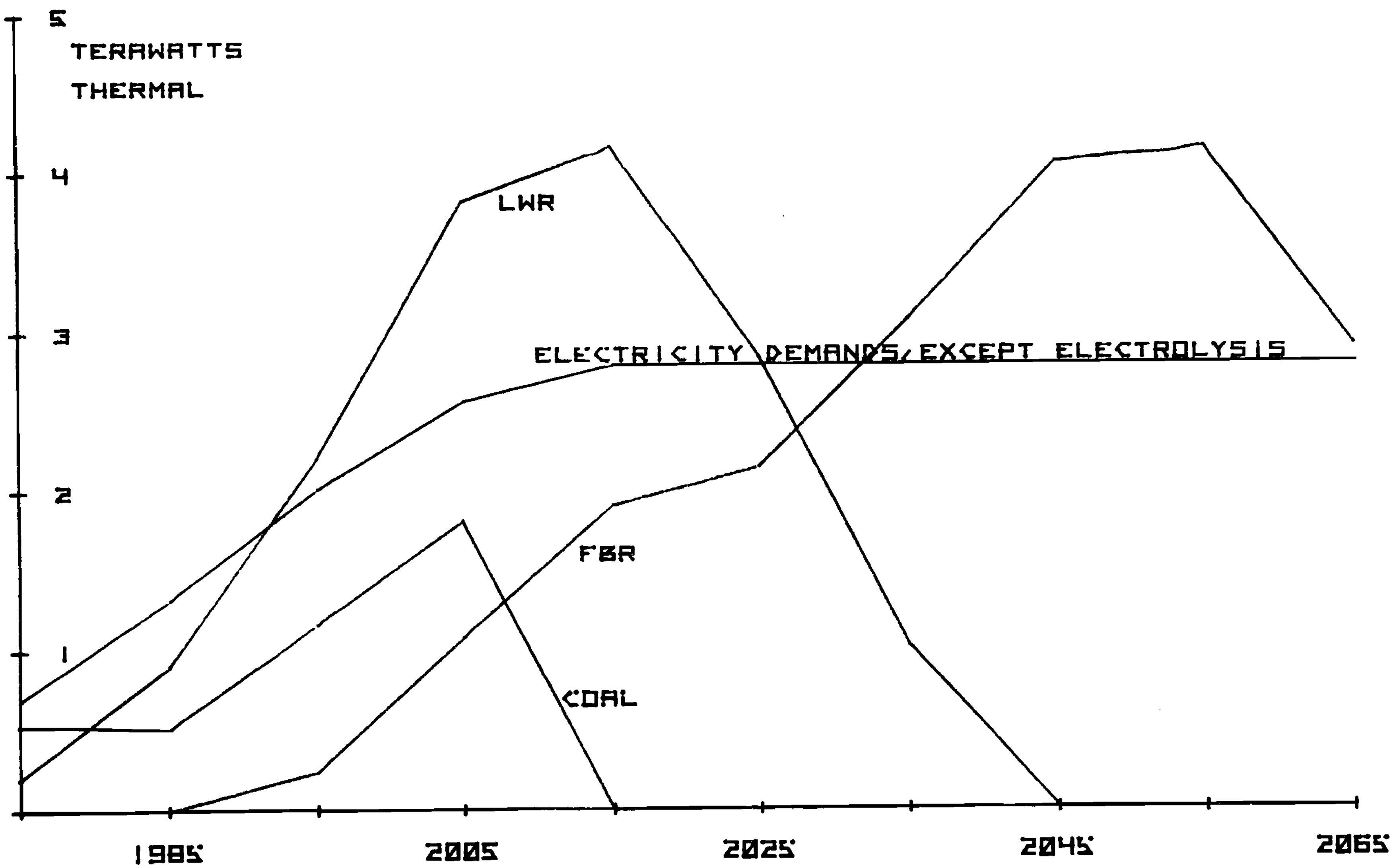

Figure B-1c. Model society 1.40: electric energy (base load) demands and supplies.

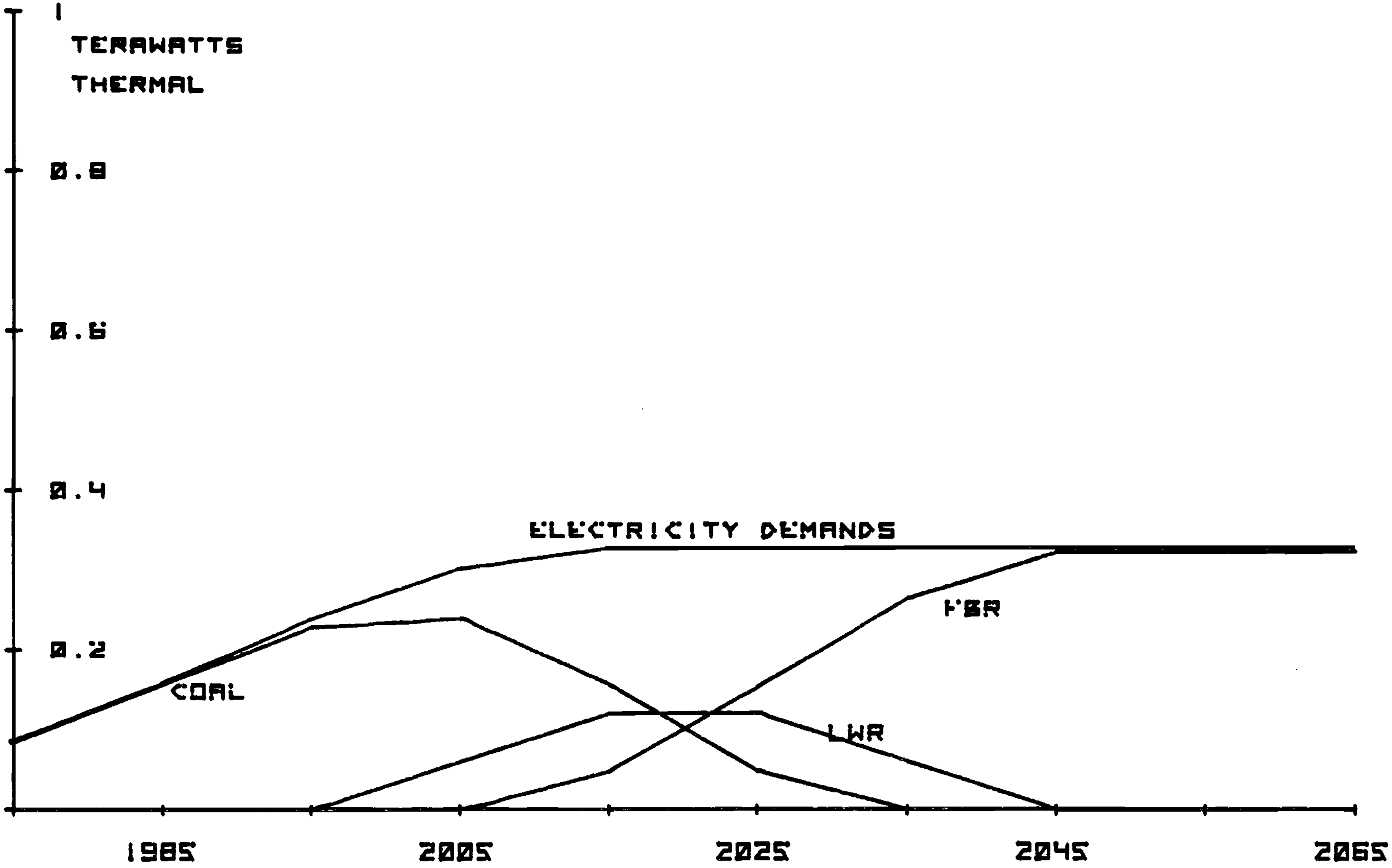

Figure B-2a. Model society 1.40: electric energy demand sector 2.

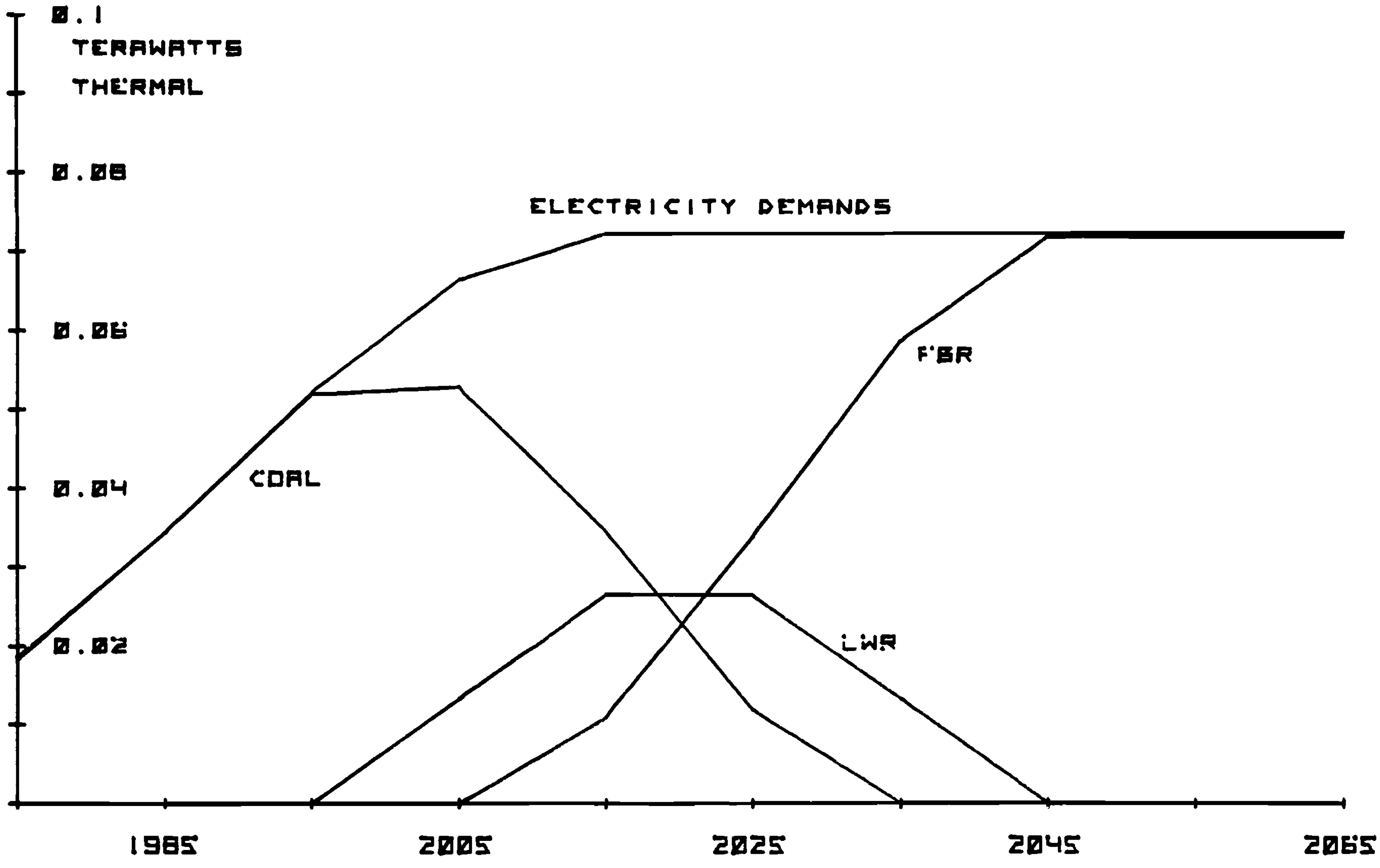

Figure B-2b. Model society 1.40: electric energy demand sector 5.

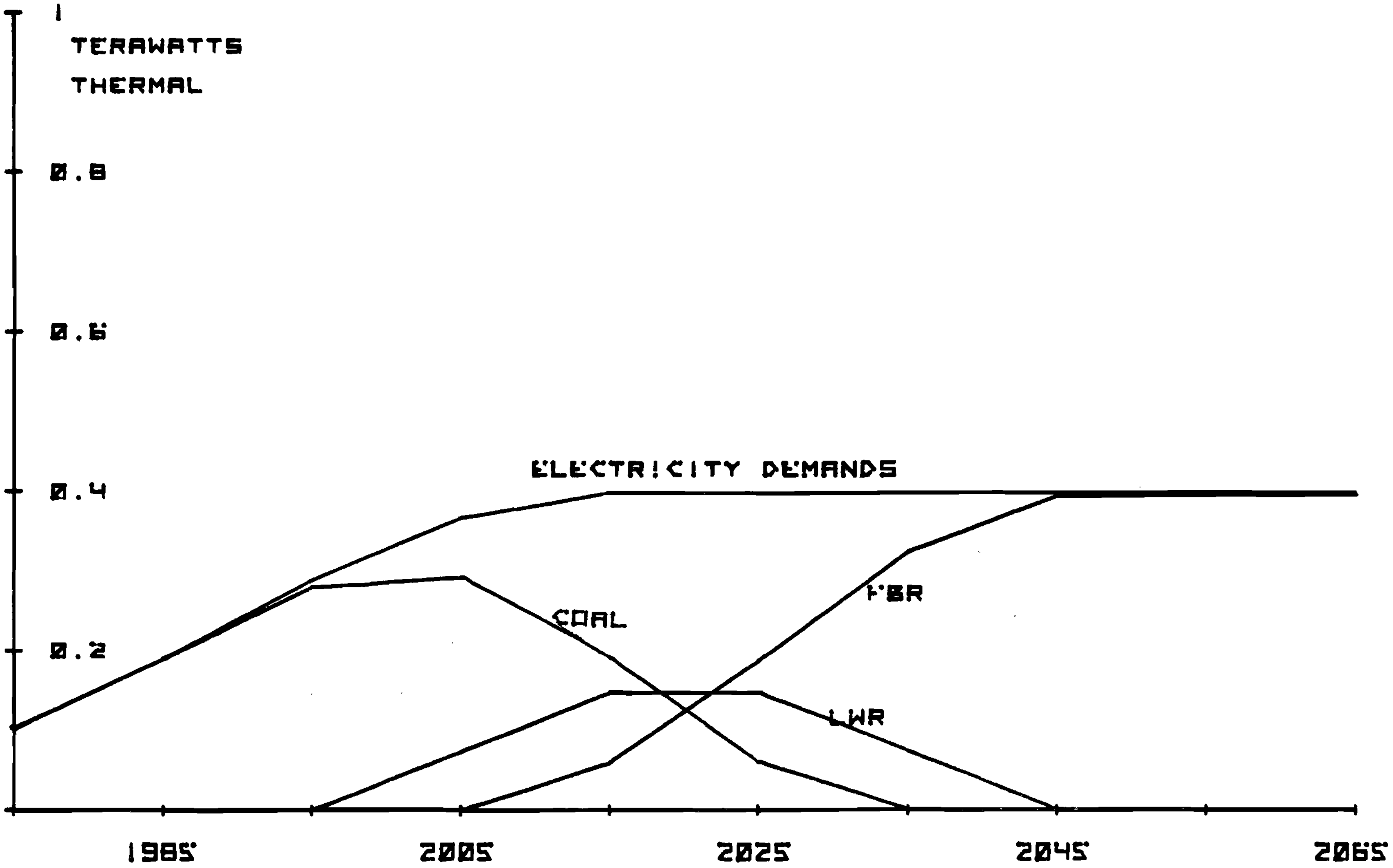

Figure B-2c. Model society 1.40: electric energy (peak load) demands and supplies.

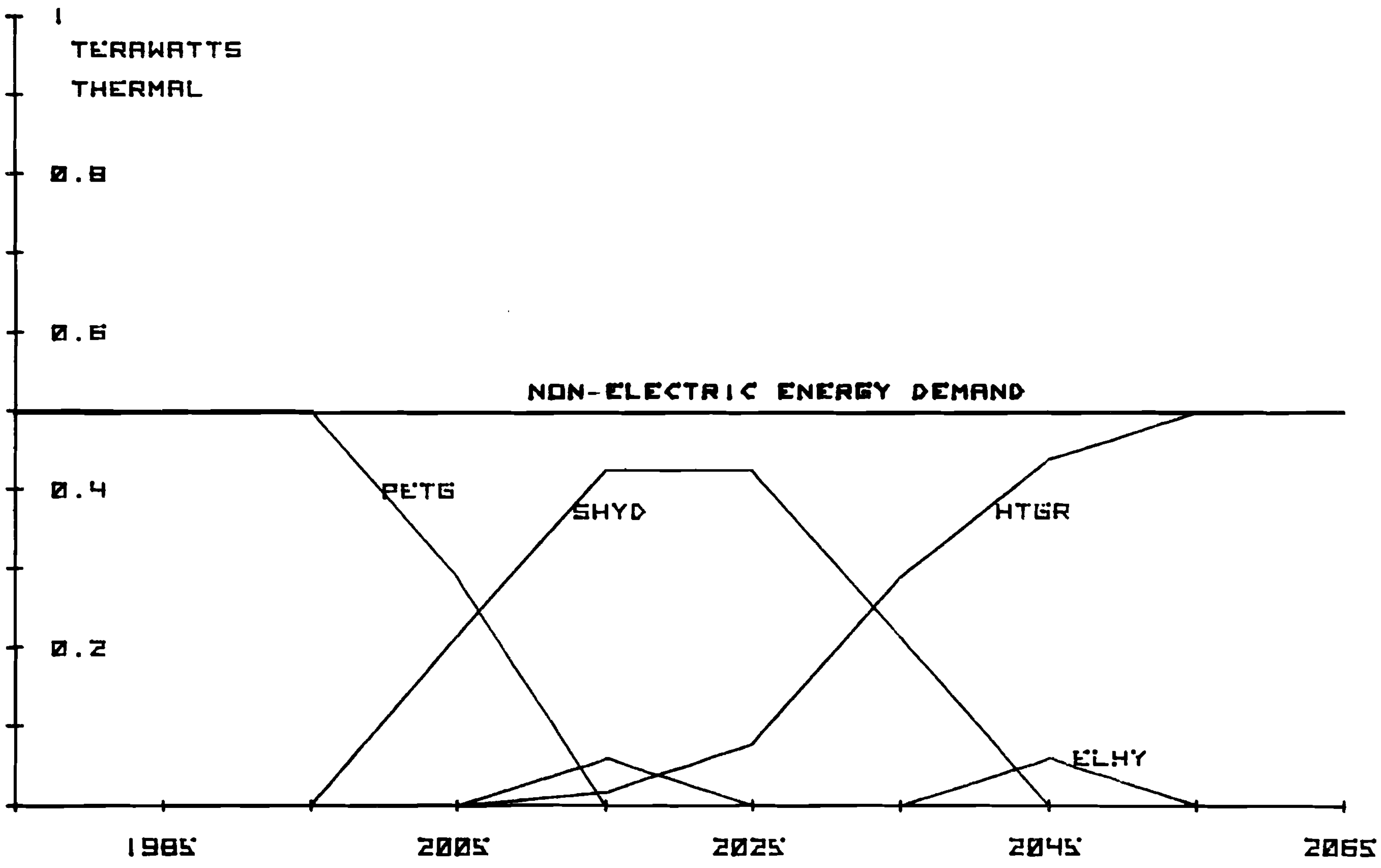

Figure B-3a. Model society 1.40: non-electric energy demand sector 3.

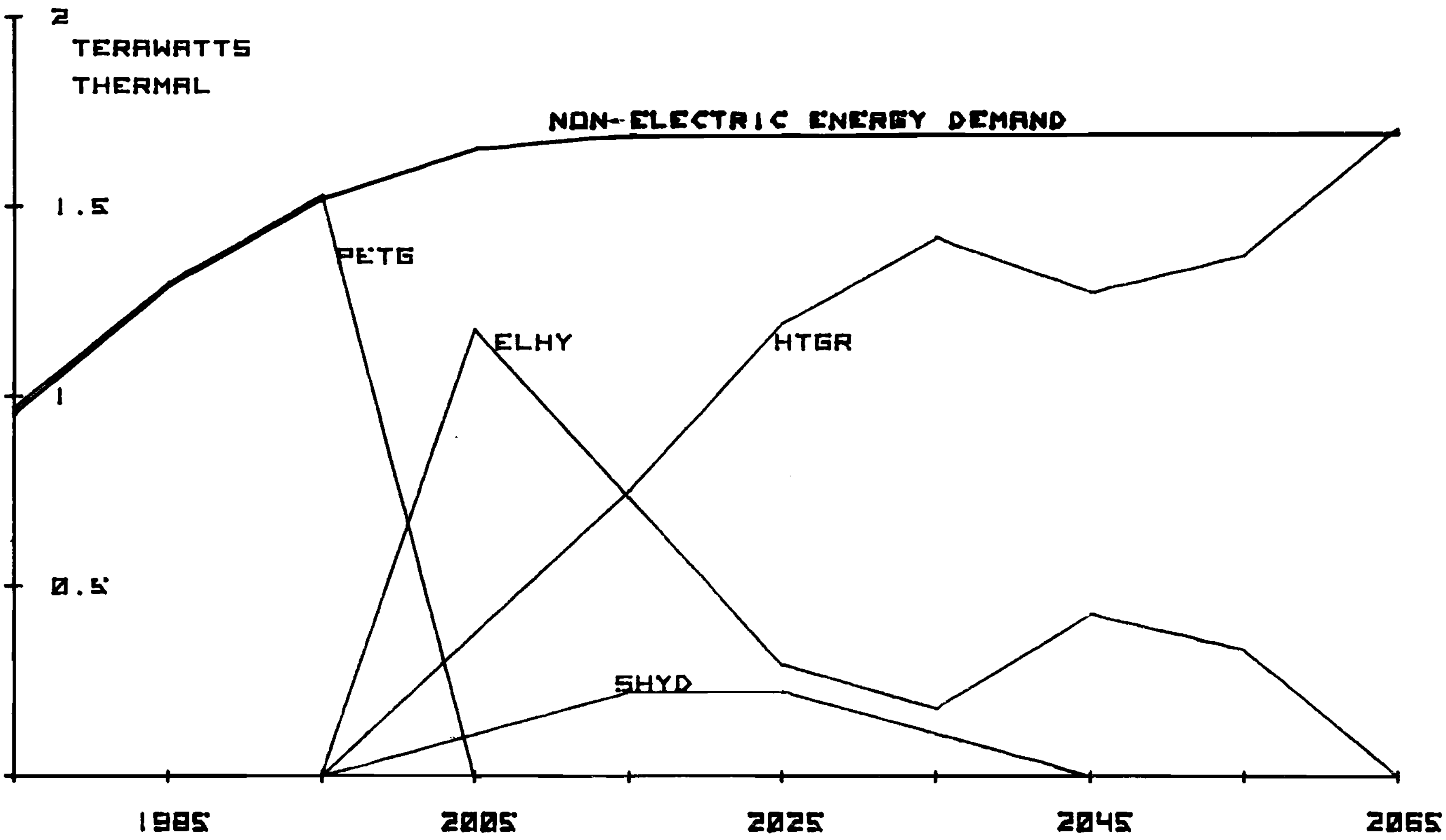

Figure B-3b. Model society 1.40: non-electric energy demand sector 6.

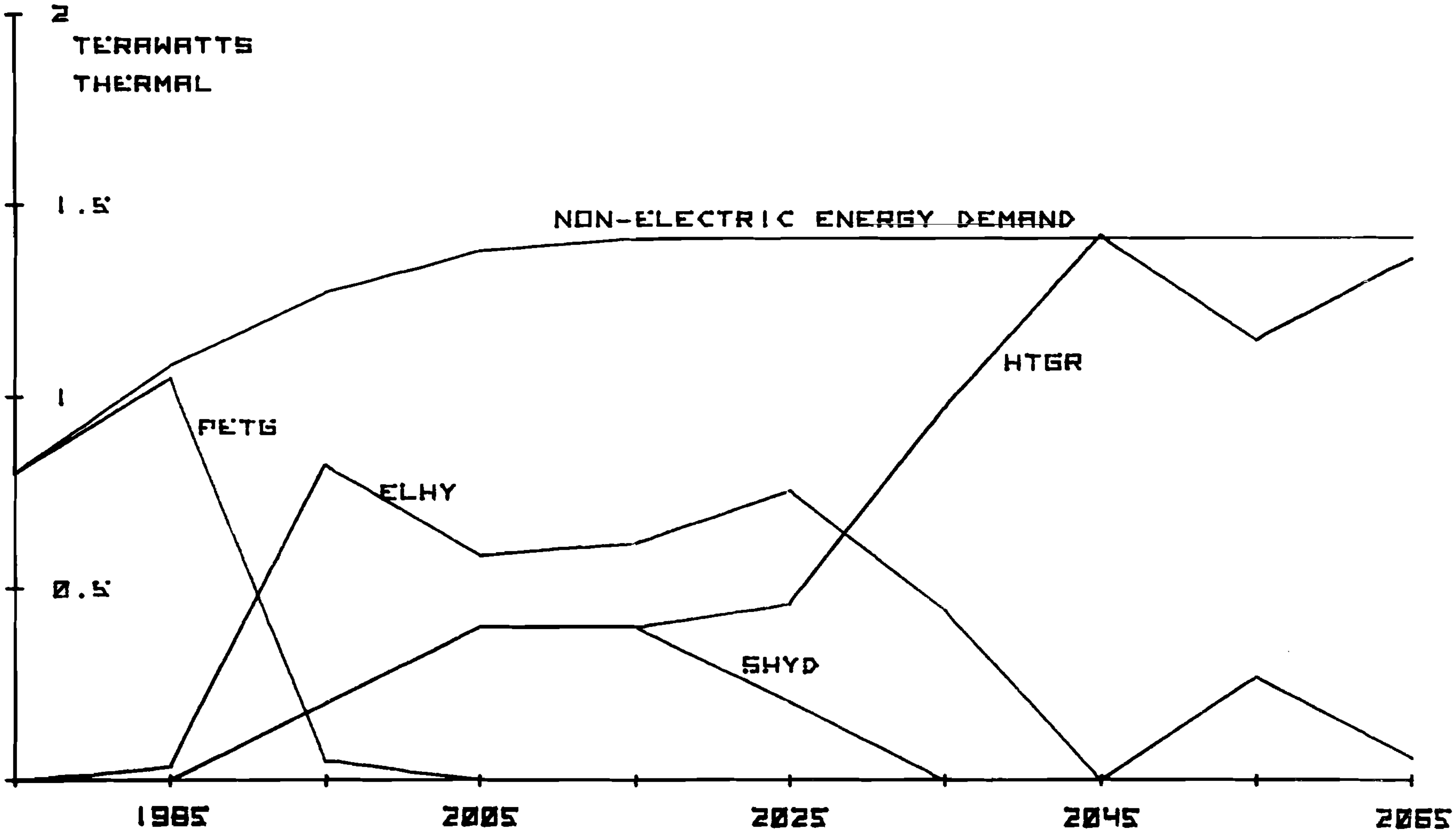

Figure B-3c. Model society 1.40: non-electric energy demand sector 7.

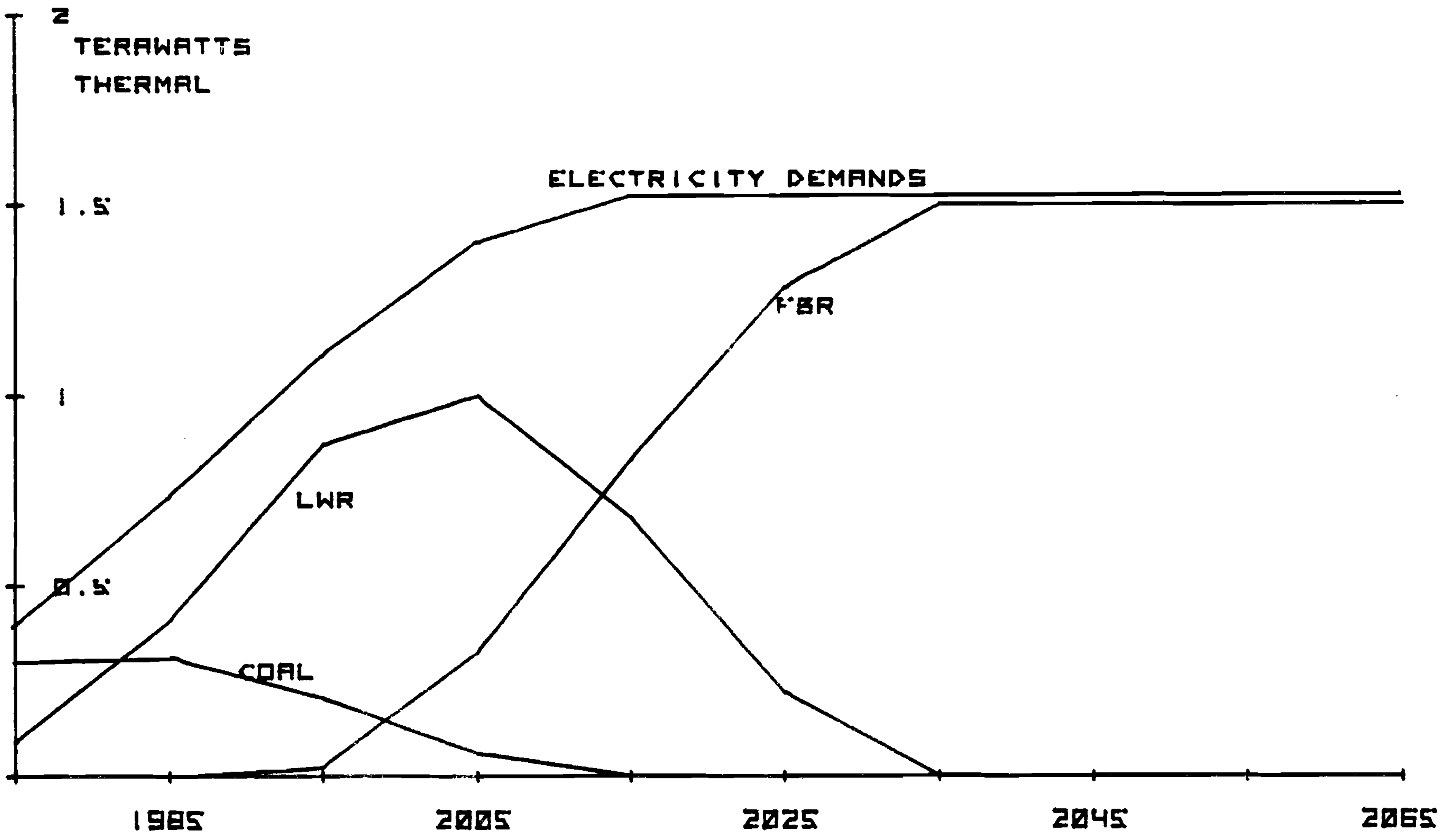

Figure B-4a. Model society 1.60: electric energy demand sector 1.

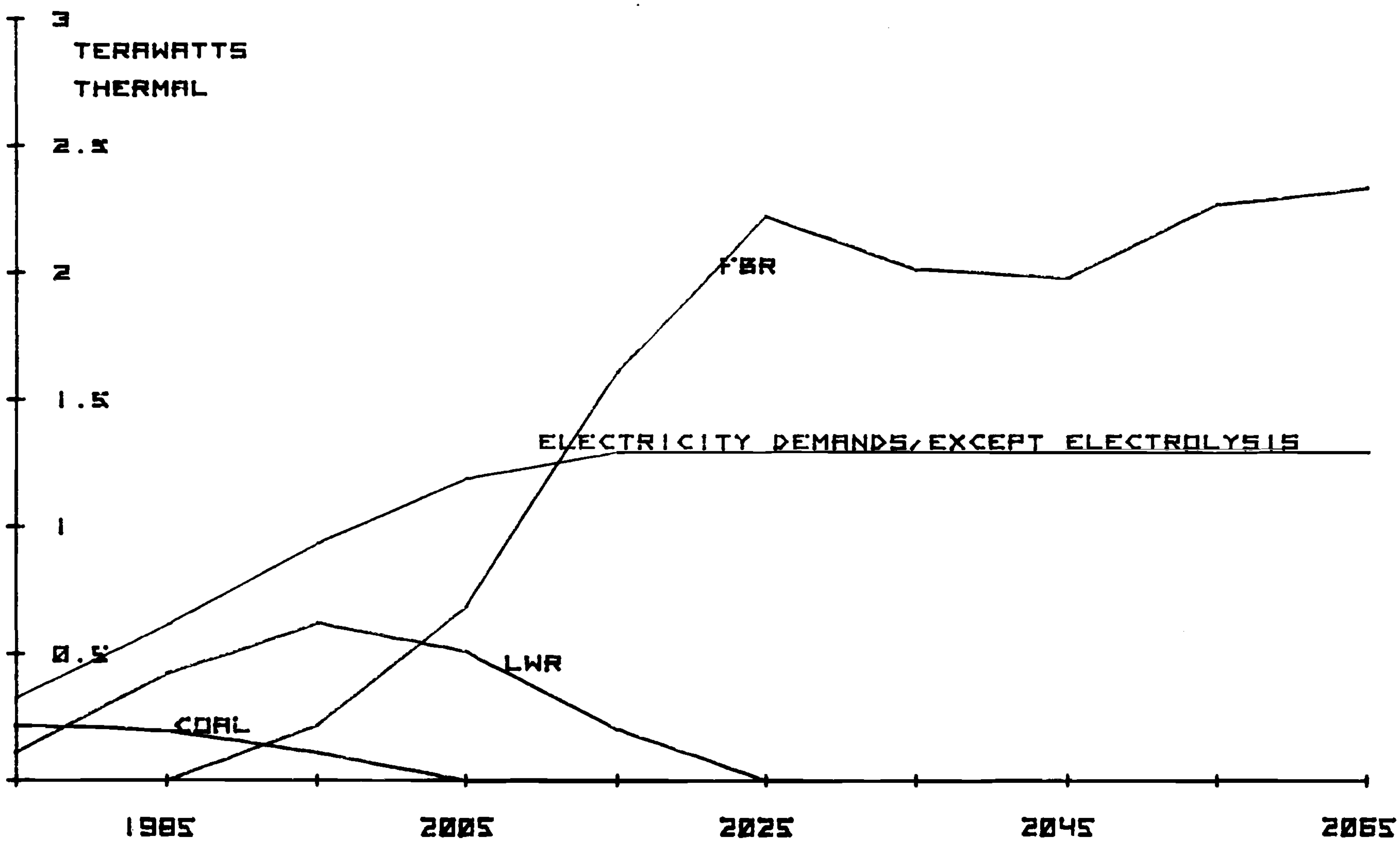

Figure B-4b. Model society 1.60: electric energy demand sector 4.

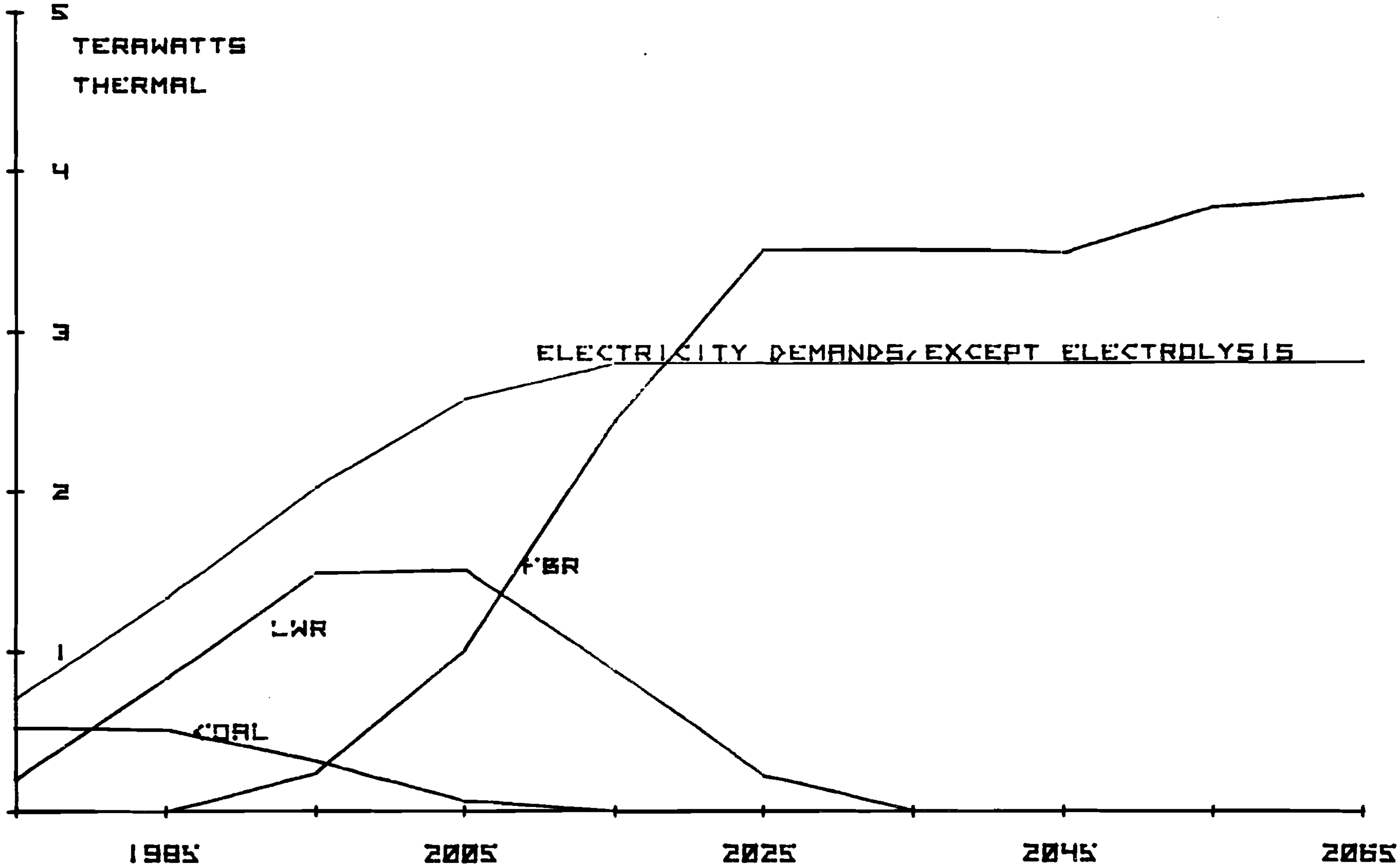

Figure B-4c. Model society 1.60: electric energy (base load) demands and supplies.

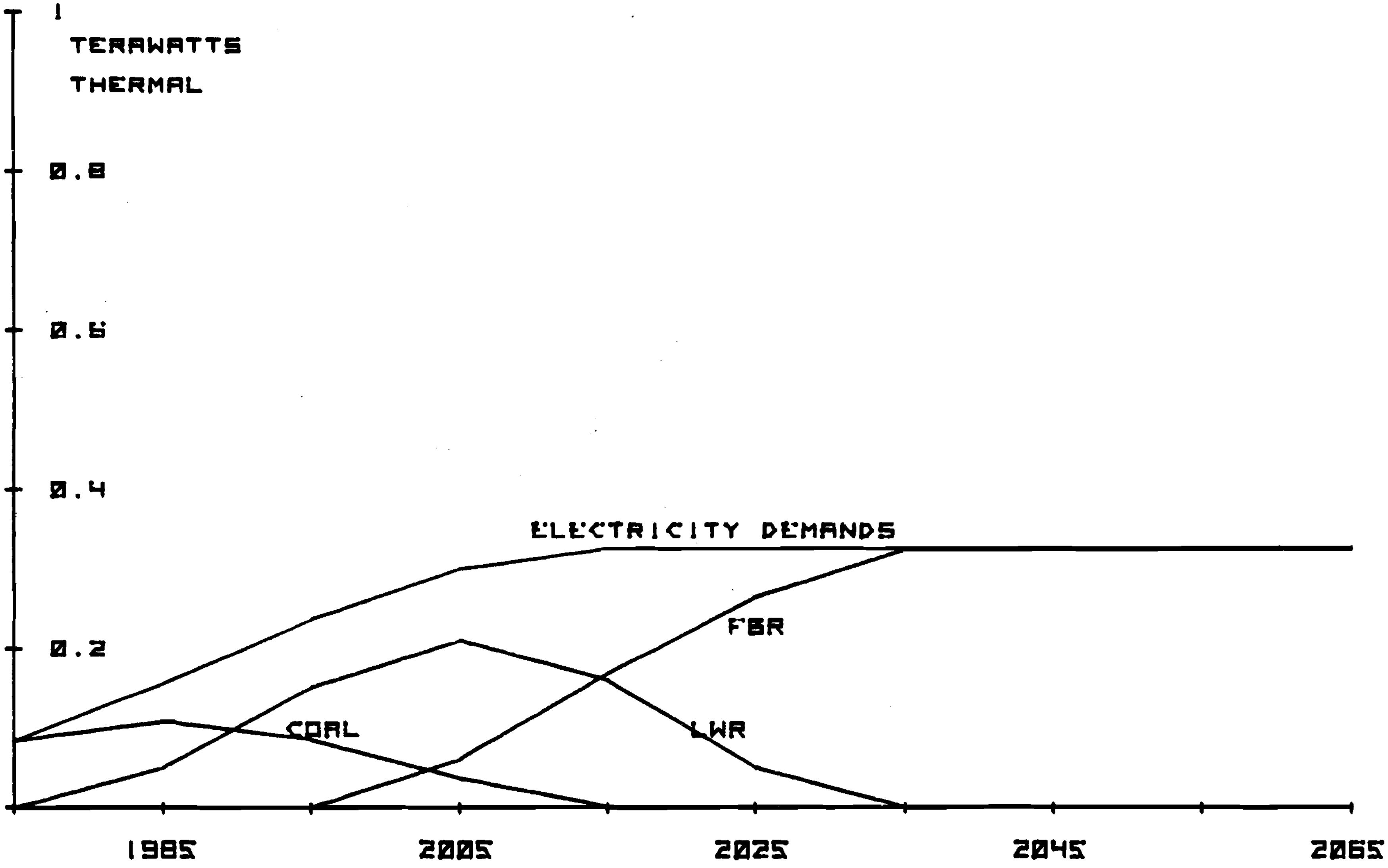

Figure B-5a. Model society 1.60: electric energy demand sector 2.

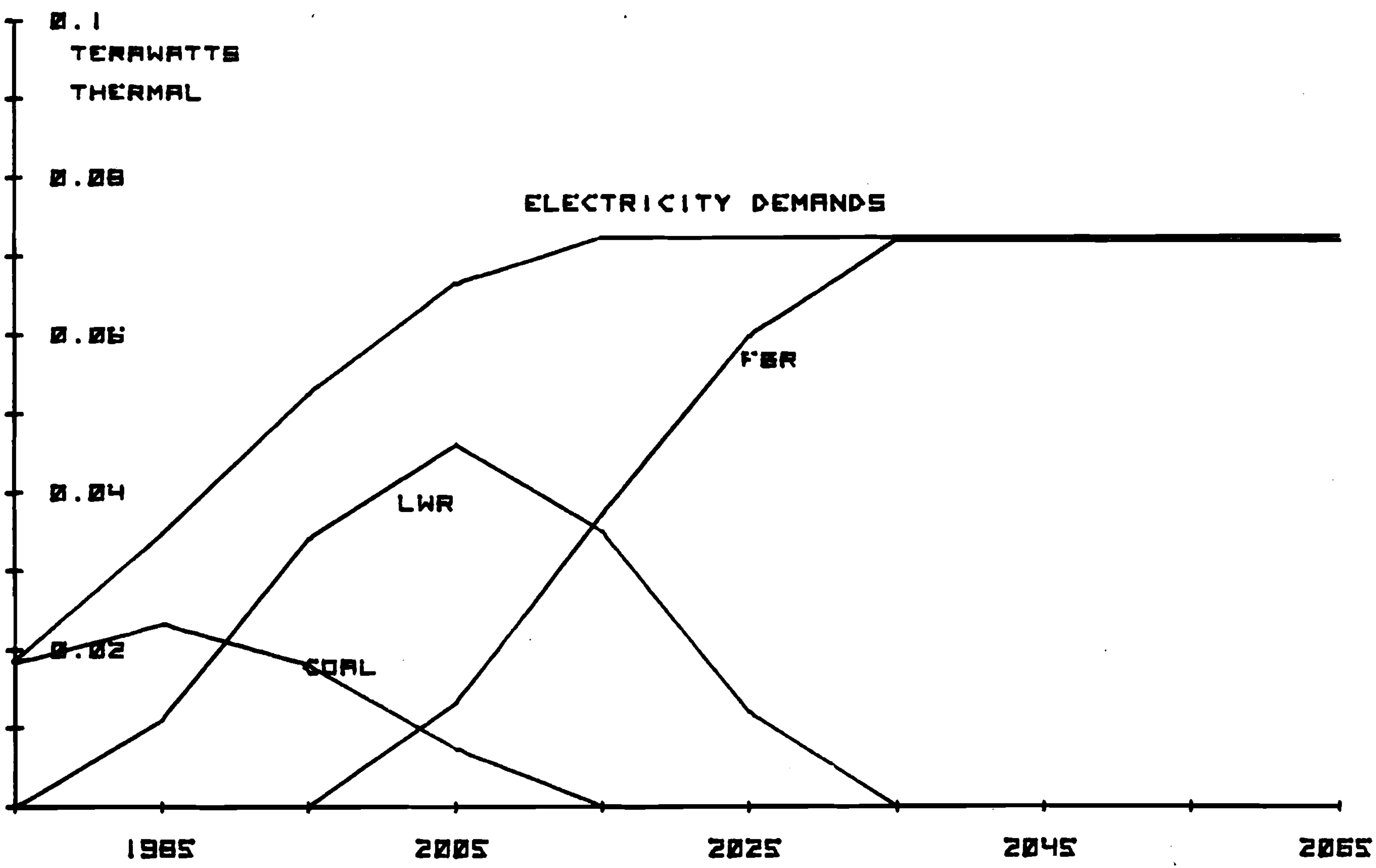

Figure B-5b. Model society 1.60: electric energy demand sector 5.

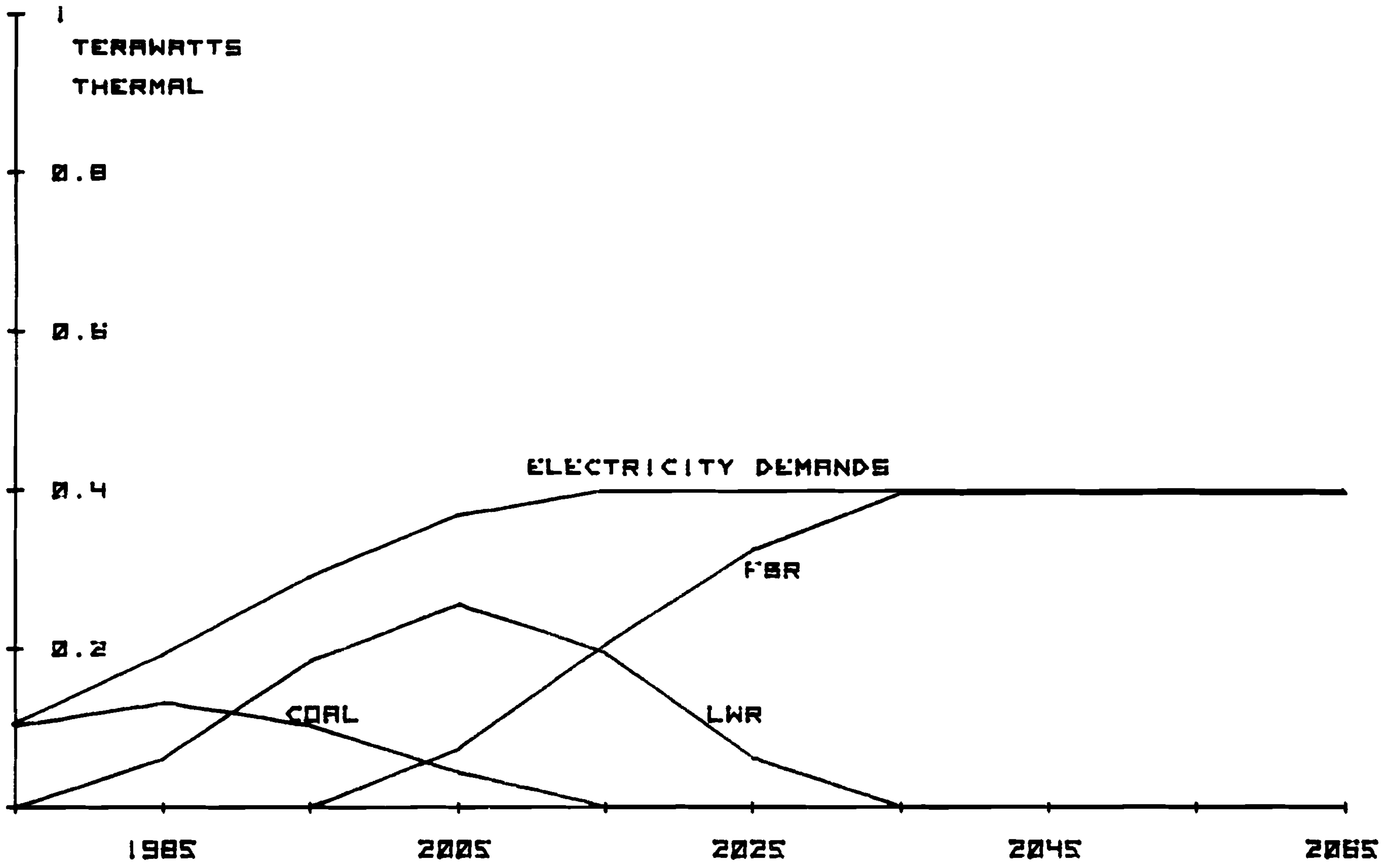

Figure B-5c. Model society 1.60: electric energy (peak load) demands and supplies.

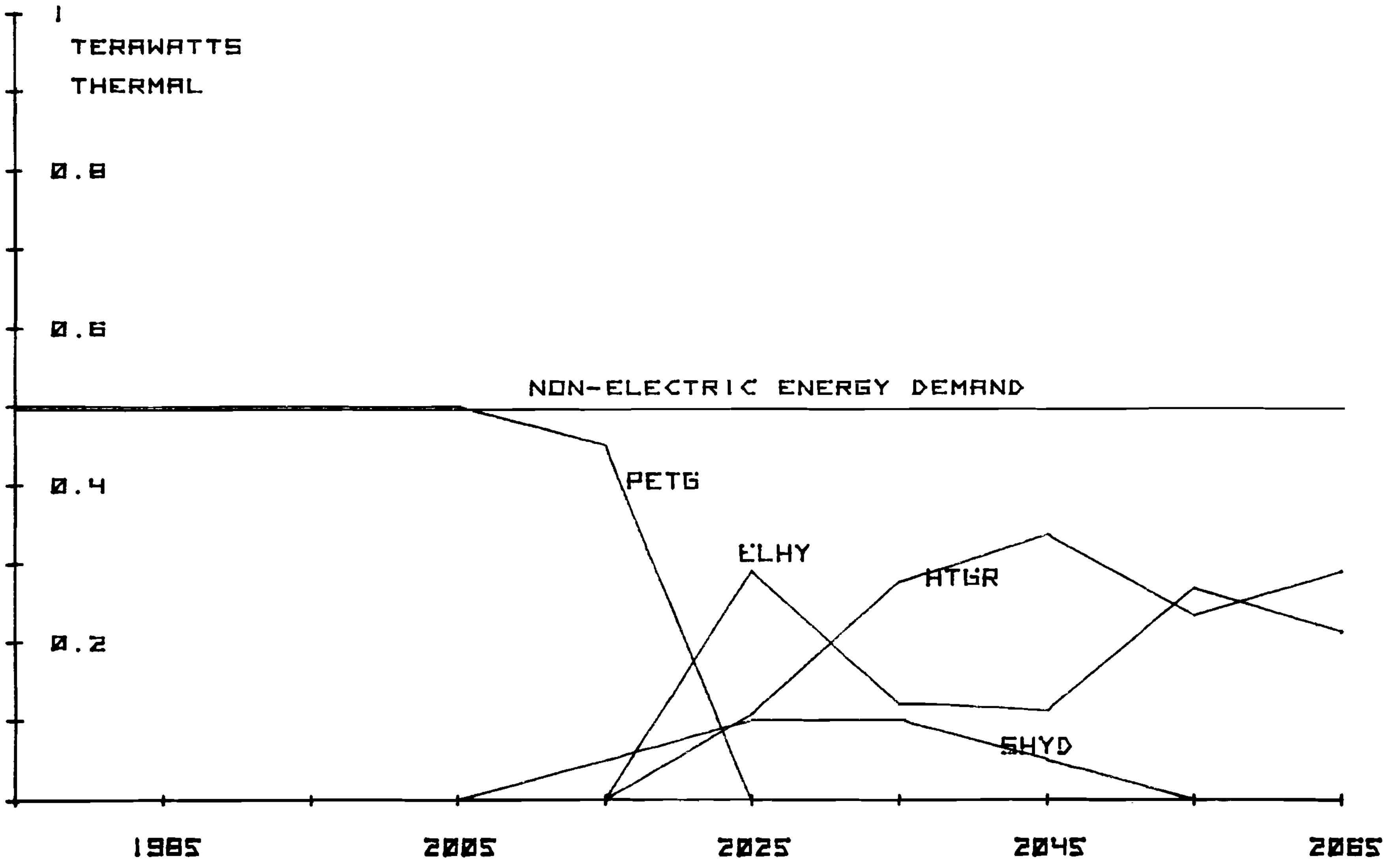

Figure B-6a. Model society 1.60: non-electric energy demand sector 3.

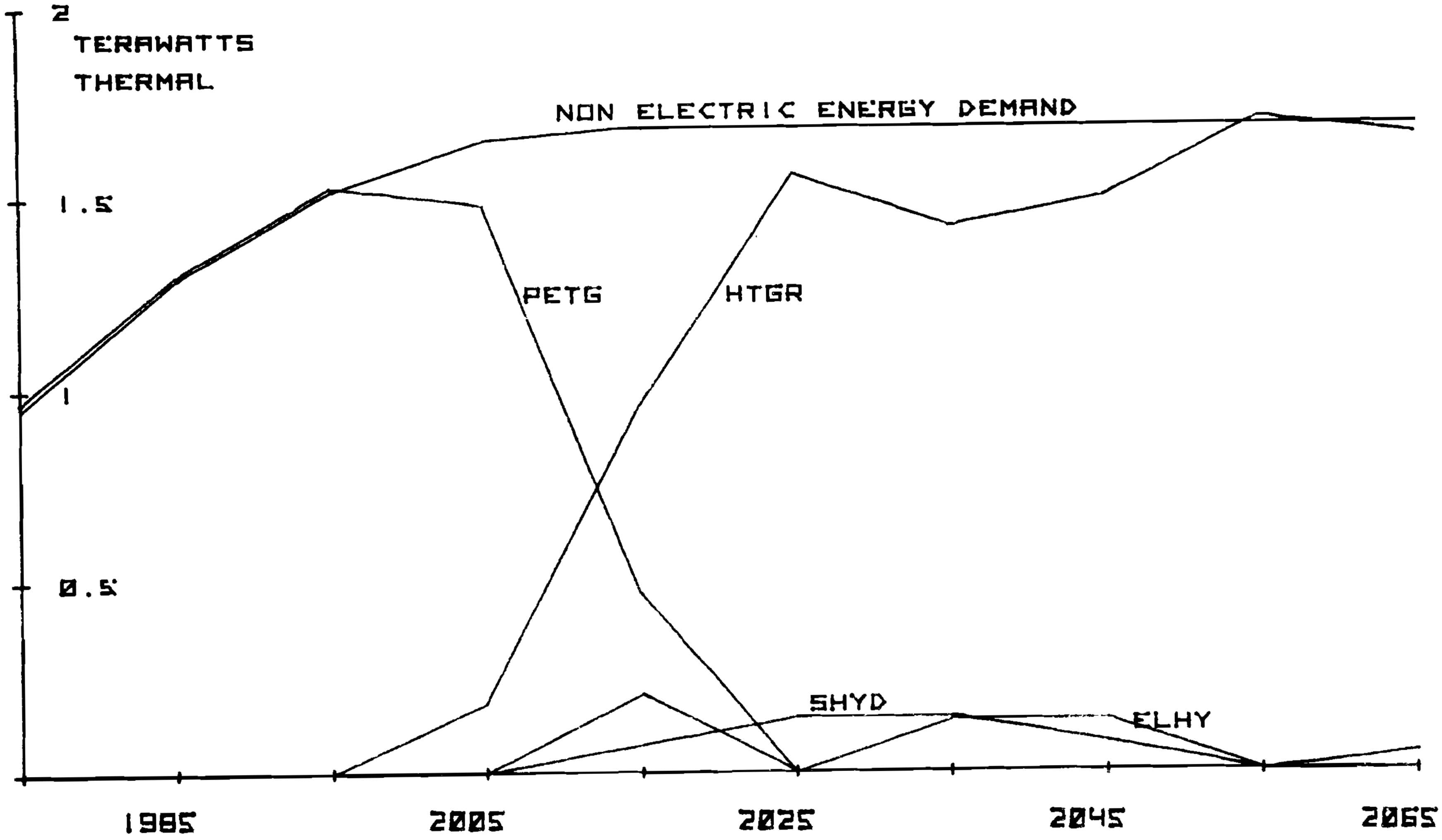

Figure B-6b. Model society 1.60: non-electric energy demand sector 6.

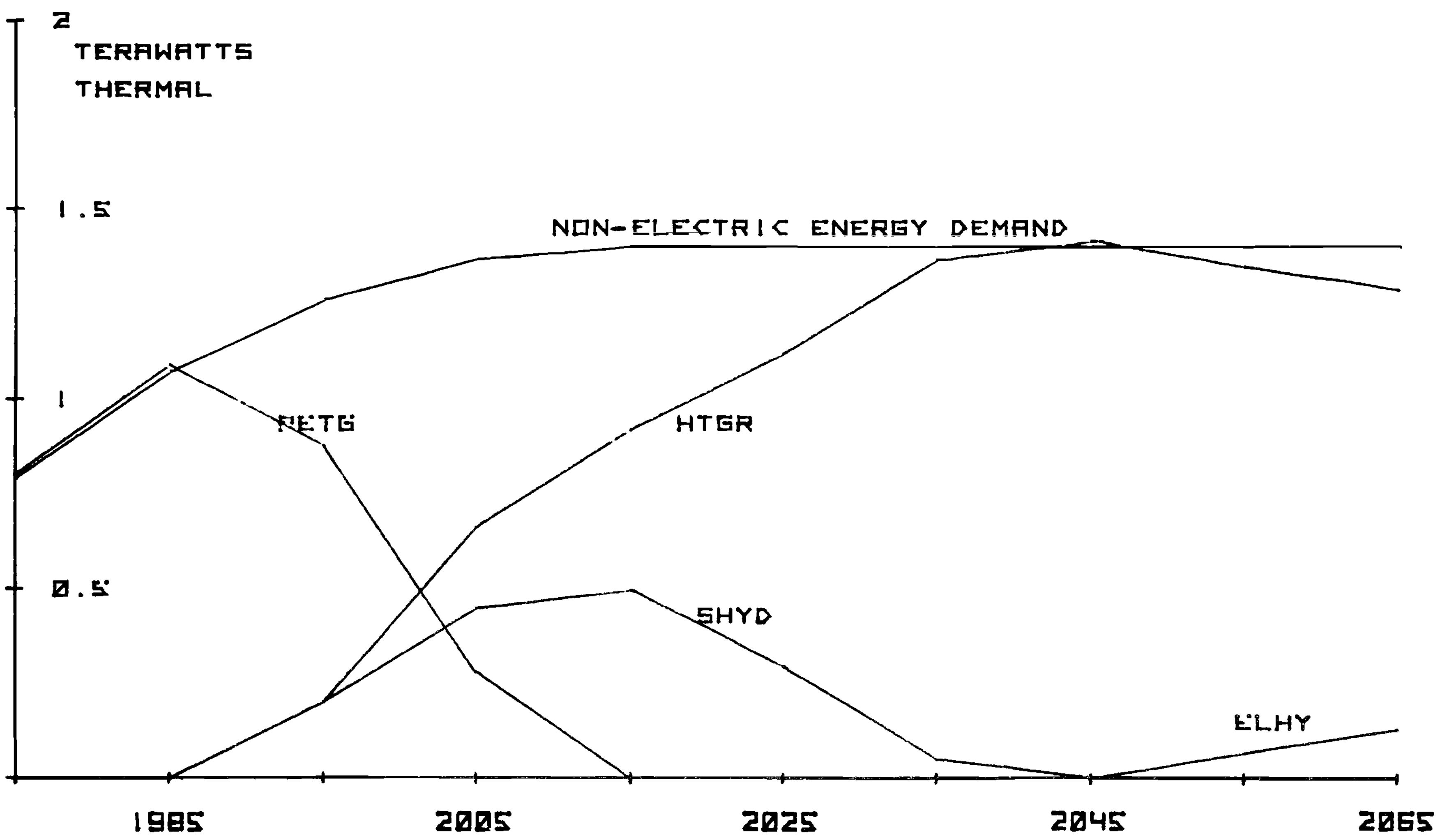

Figure B-6c. Model society 1.60: non-electric energy demand sector 7.

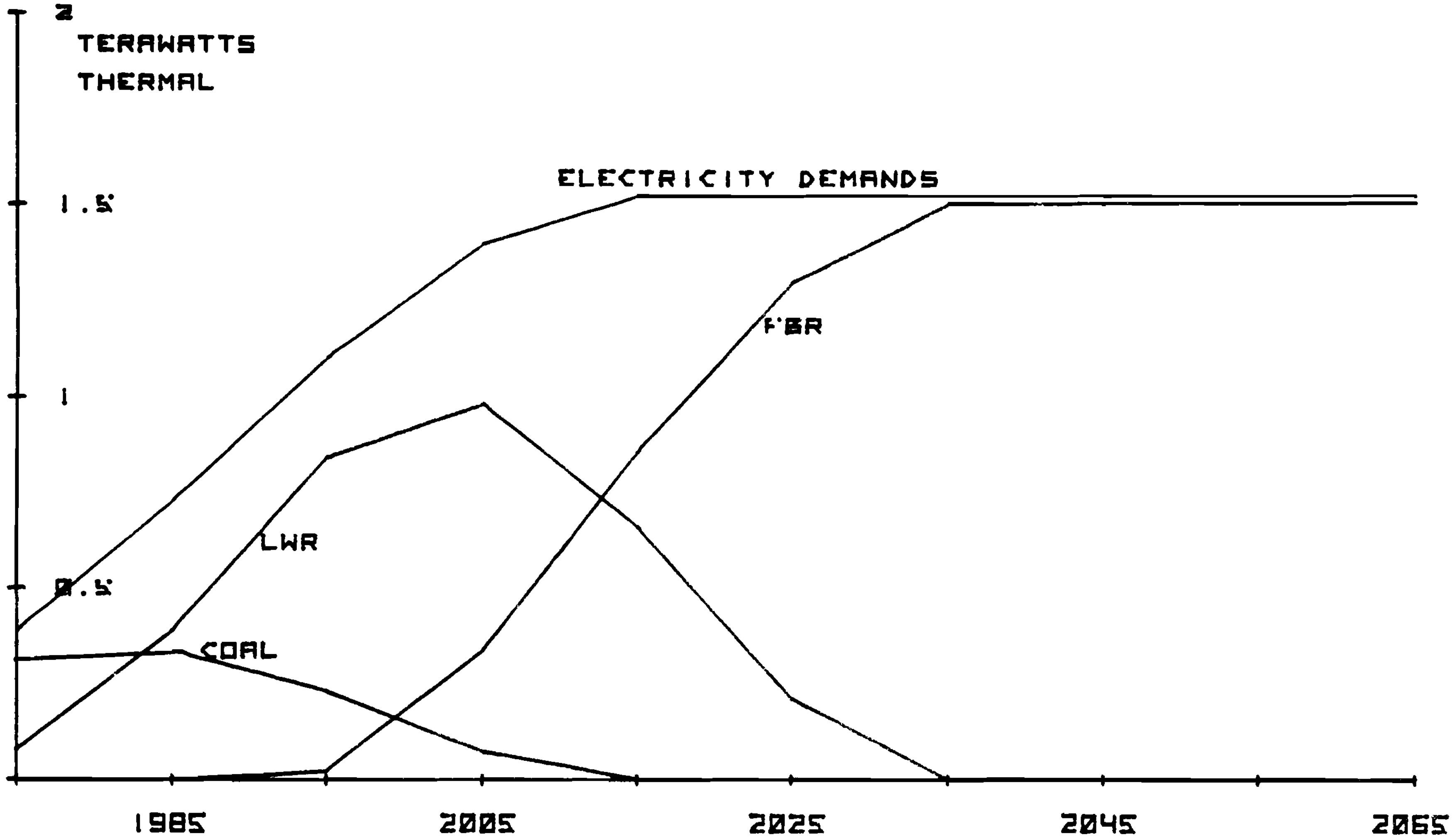

Figure B-7a. Model society 1.80: electric energy demand sector 1.

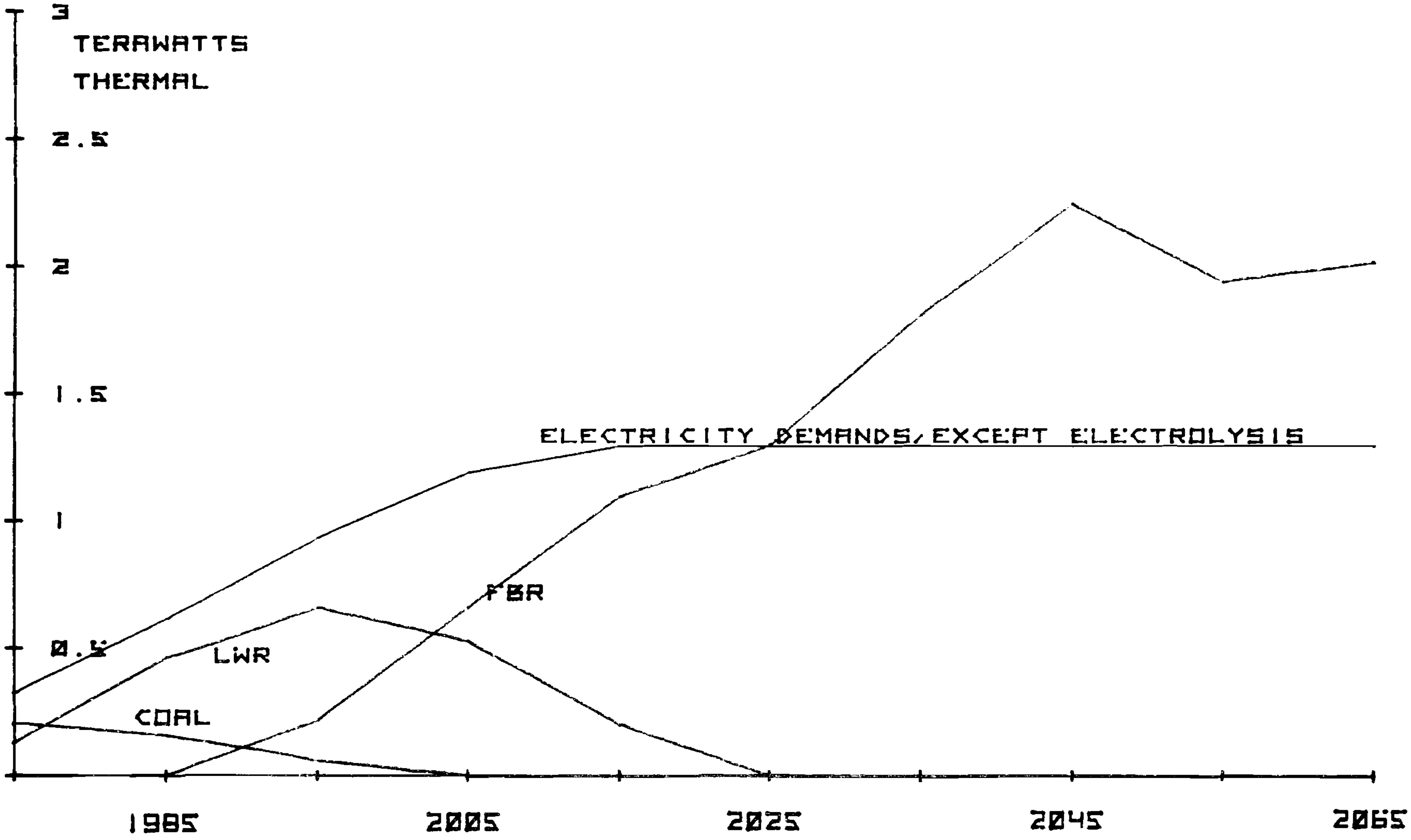

Figure B-7b. Model society 1.80: electric energy demand sector 4.

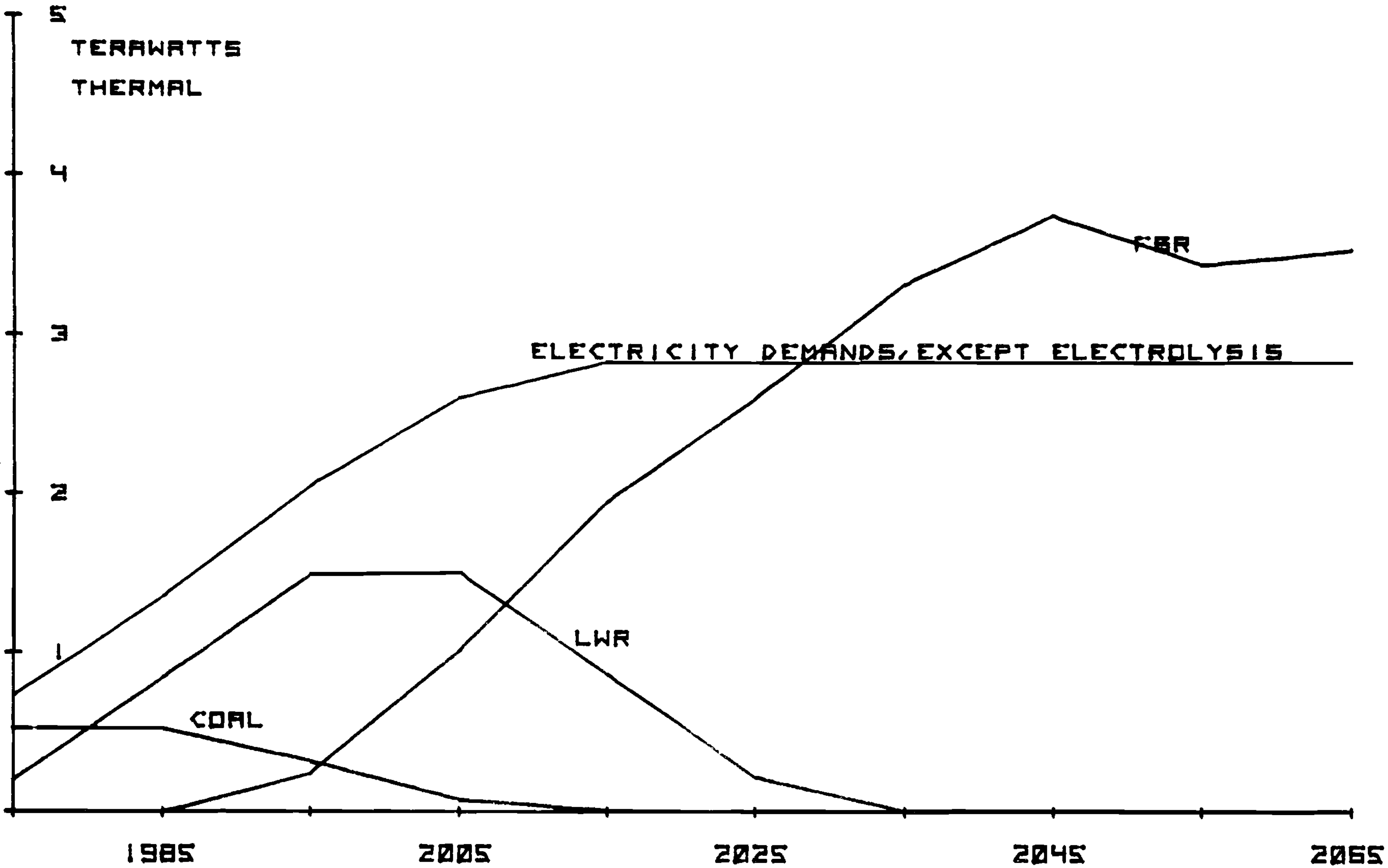

Figure B-7c. Model society 1.80: electric energy (base load) demands and supplies.

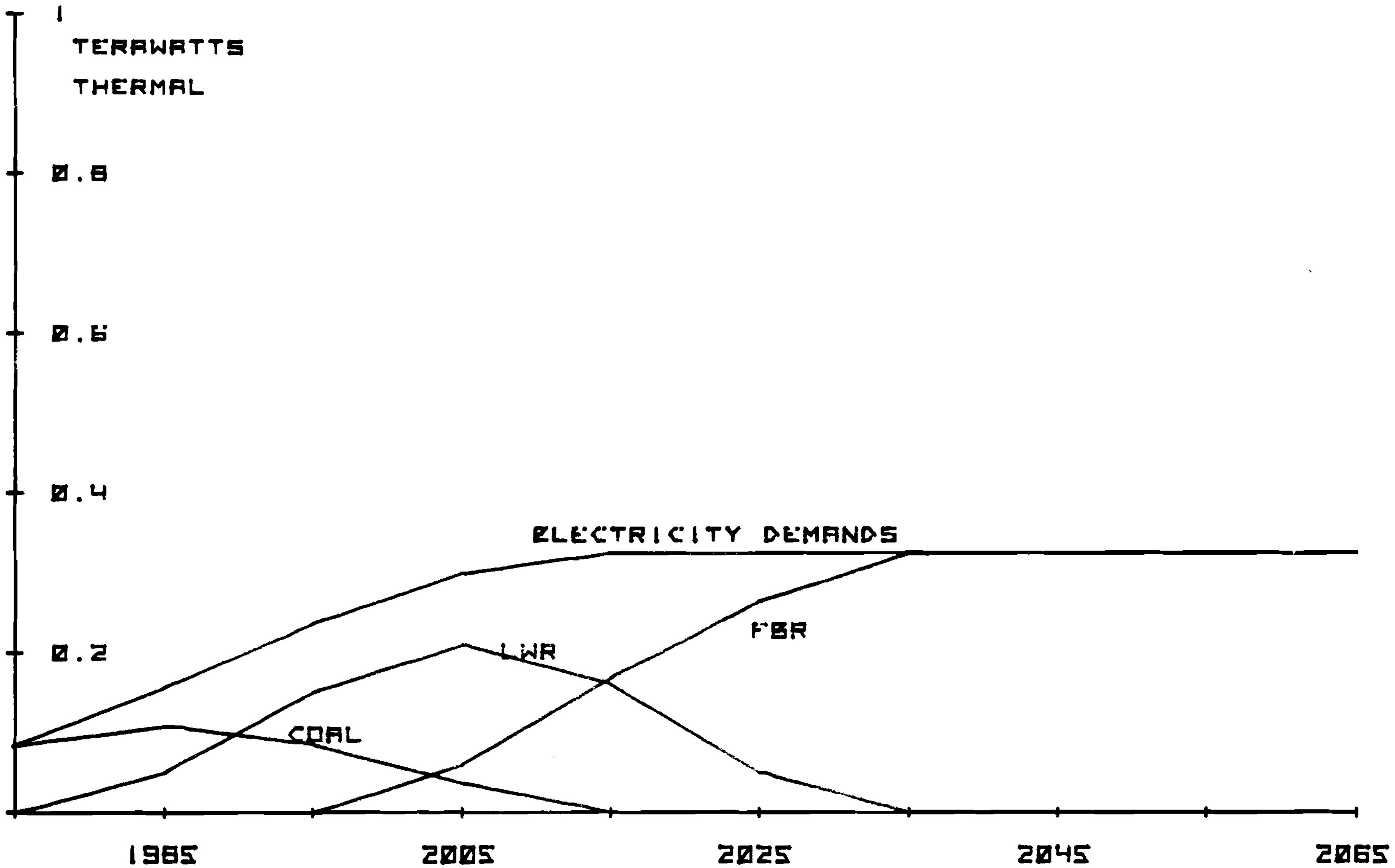

Figure B-8a. Model society 1.80: electric energy demand sector 2.

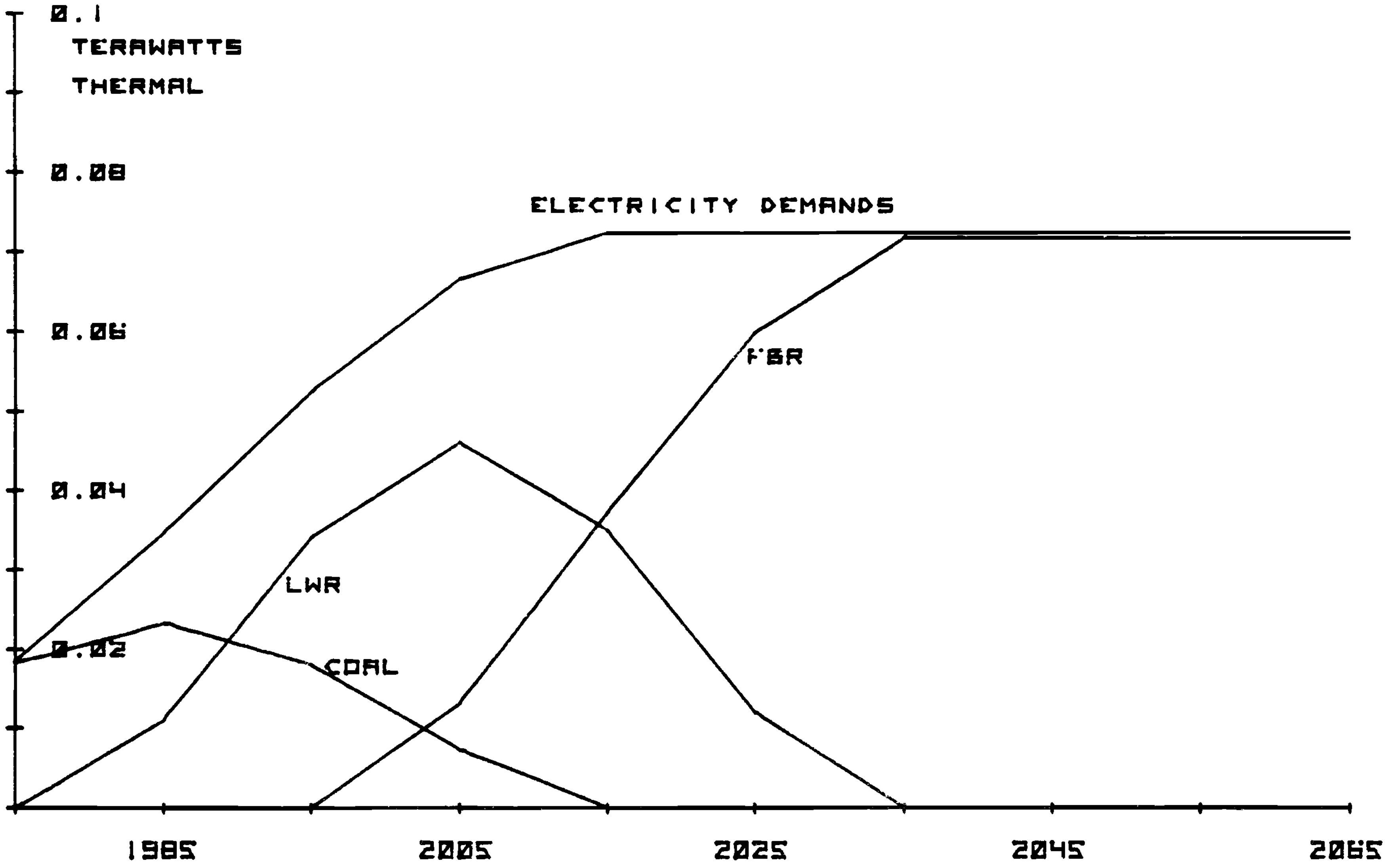

Figure B-8b. Model society 1.80: electric energy demand sector 5.

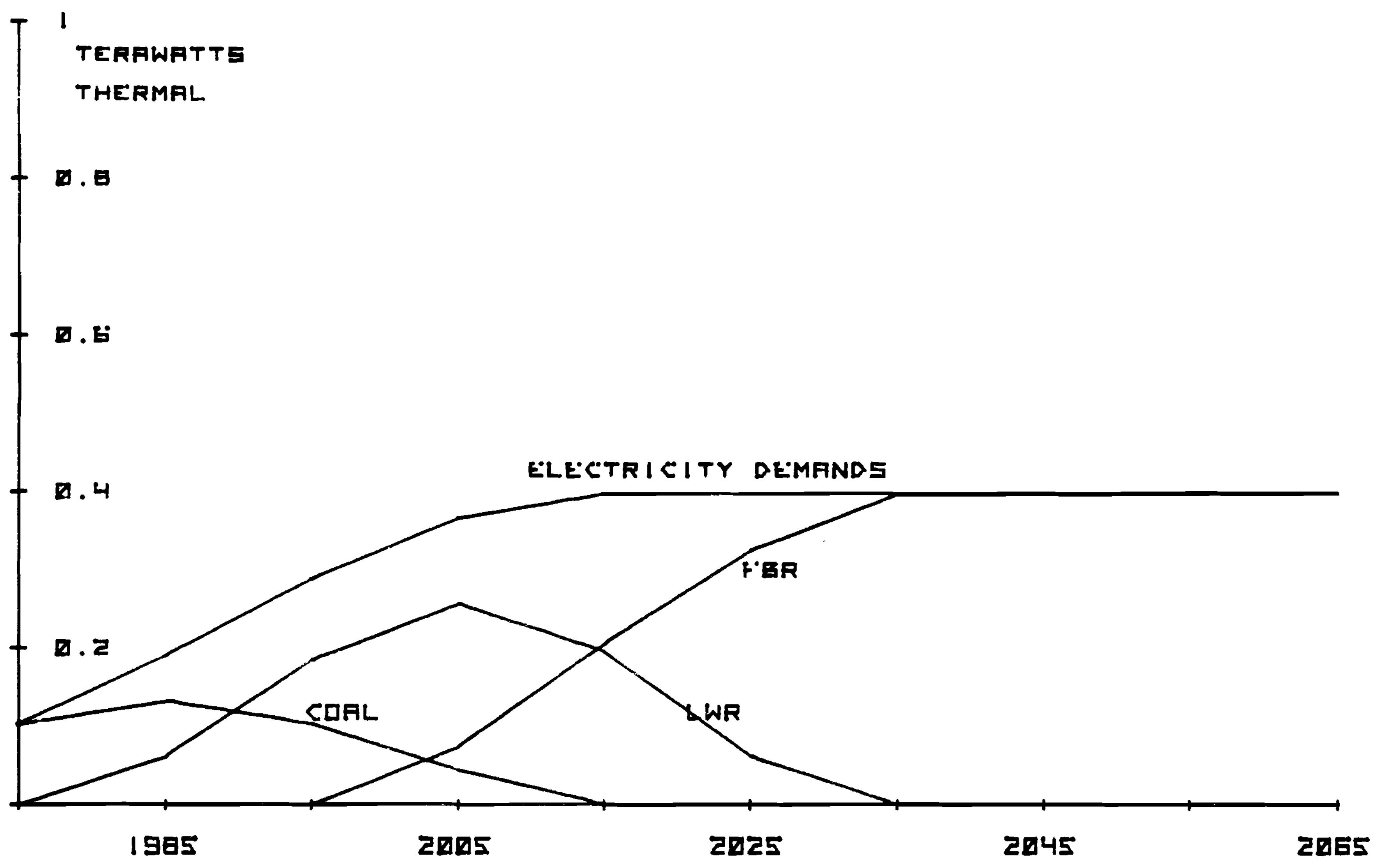

Figure B-8c. Model society 1.80: electric energy (peak load) demands and supplies.

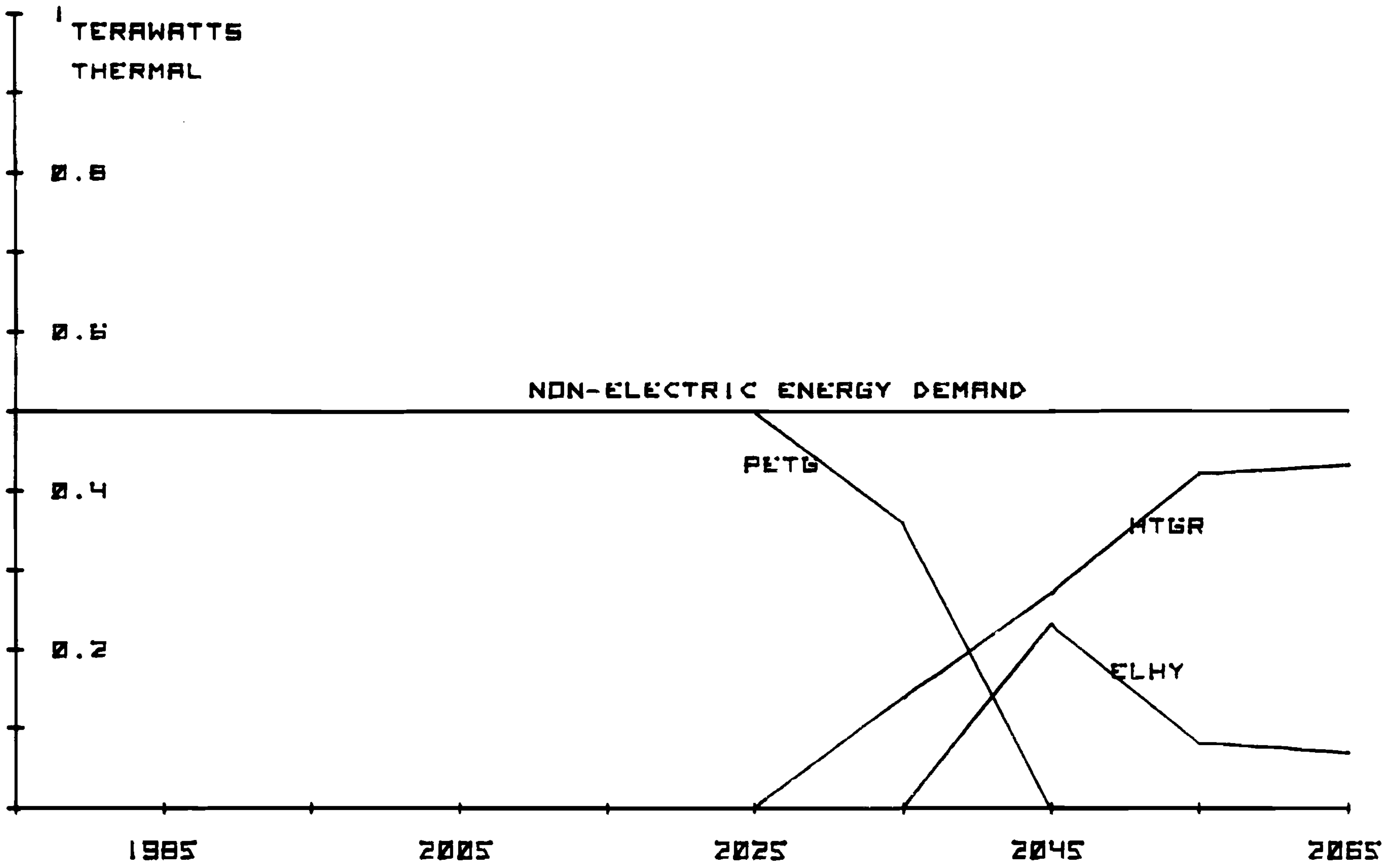

Figure B-9a. Model society 1.80: non-electric energy demand sector 3.

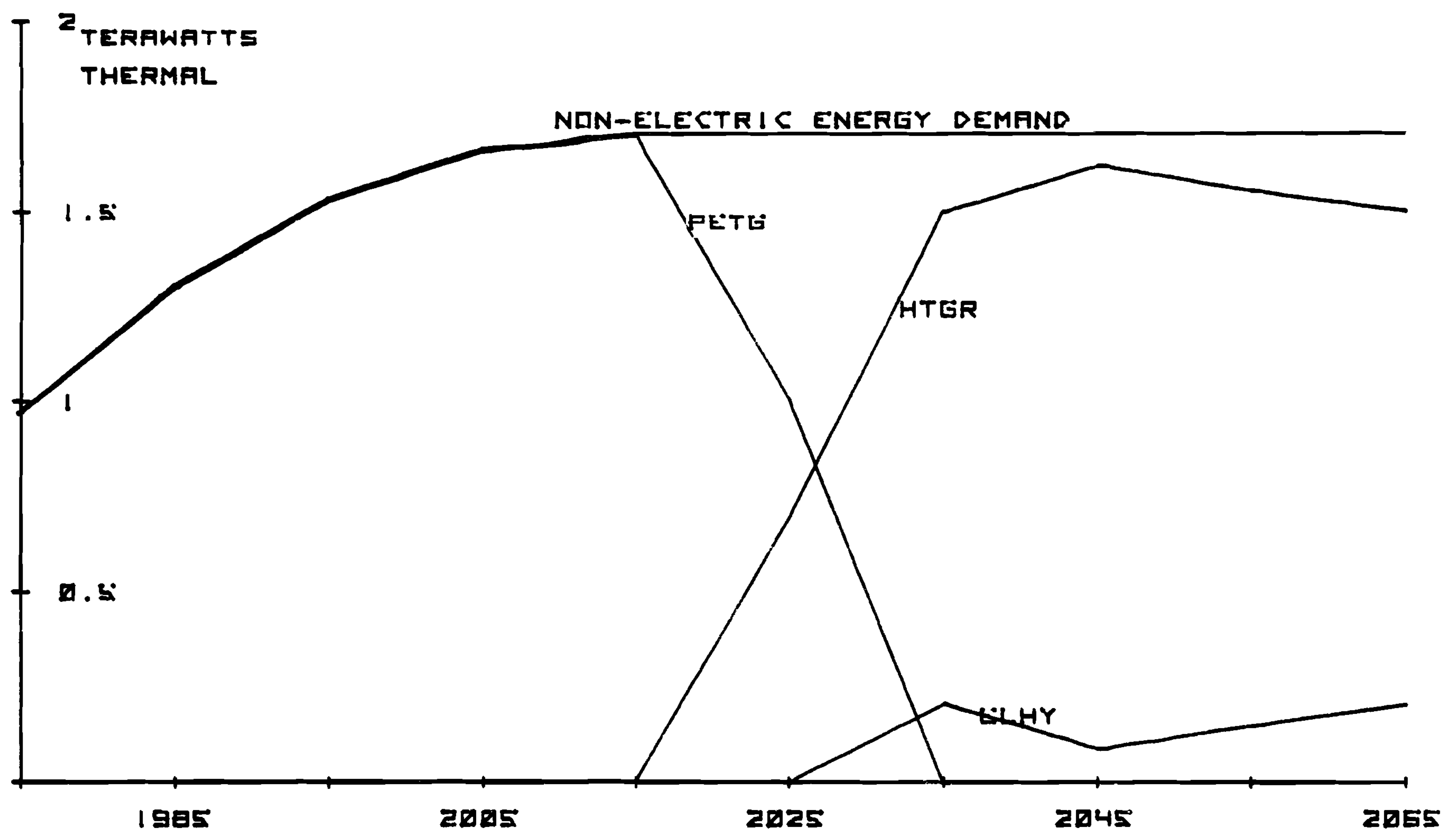

Figure B-9b. Model society 1.80: non-electric energy demand sector 6.

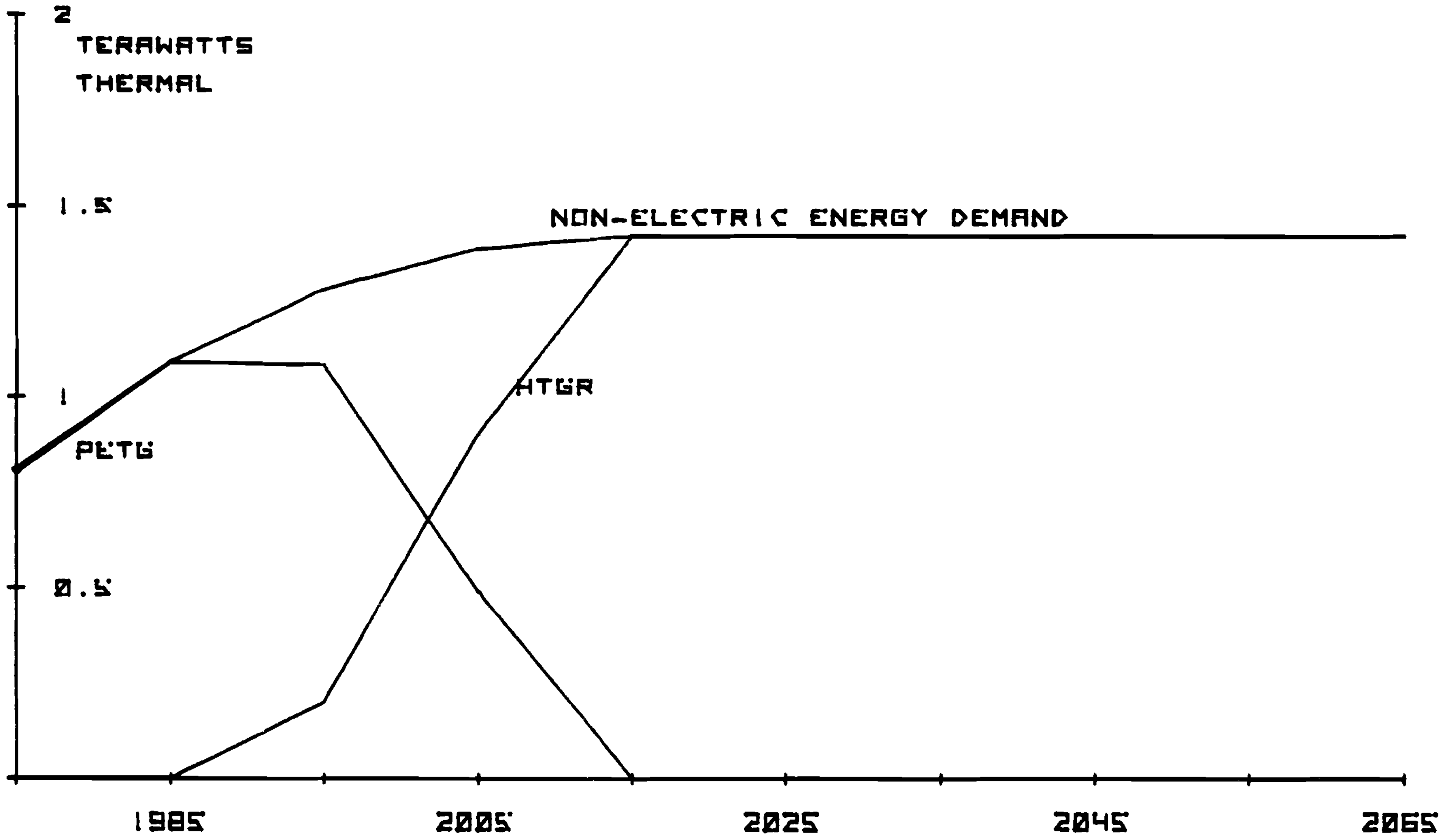

Figure B-9c. Model society 1.80: non-electric energy demand sector 7.

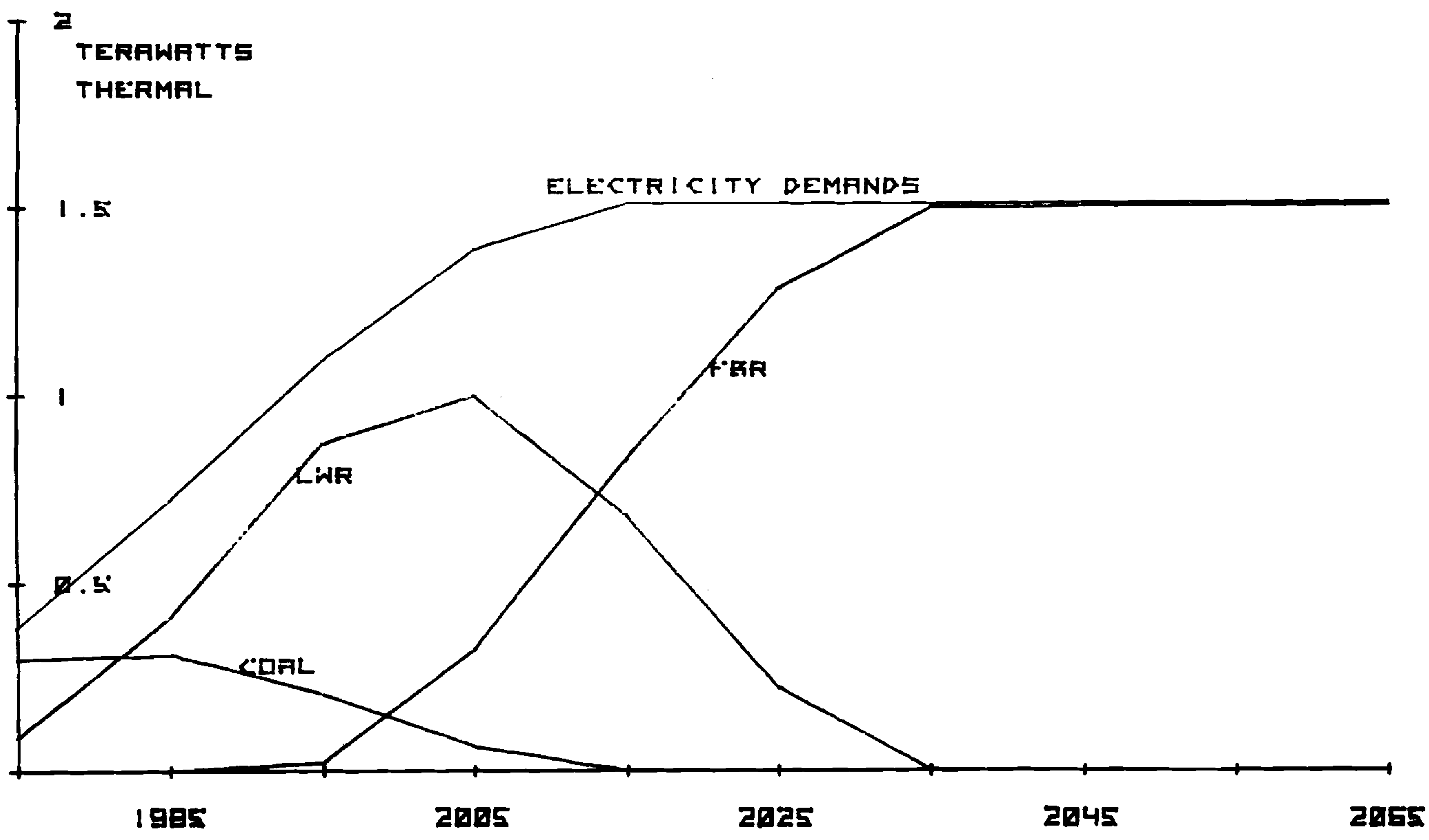

Figure B-10a. Model society 1.100: electric energy demand sector 1.

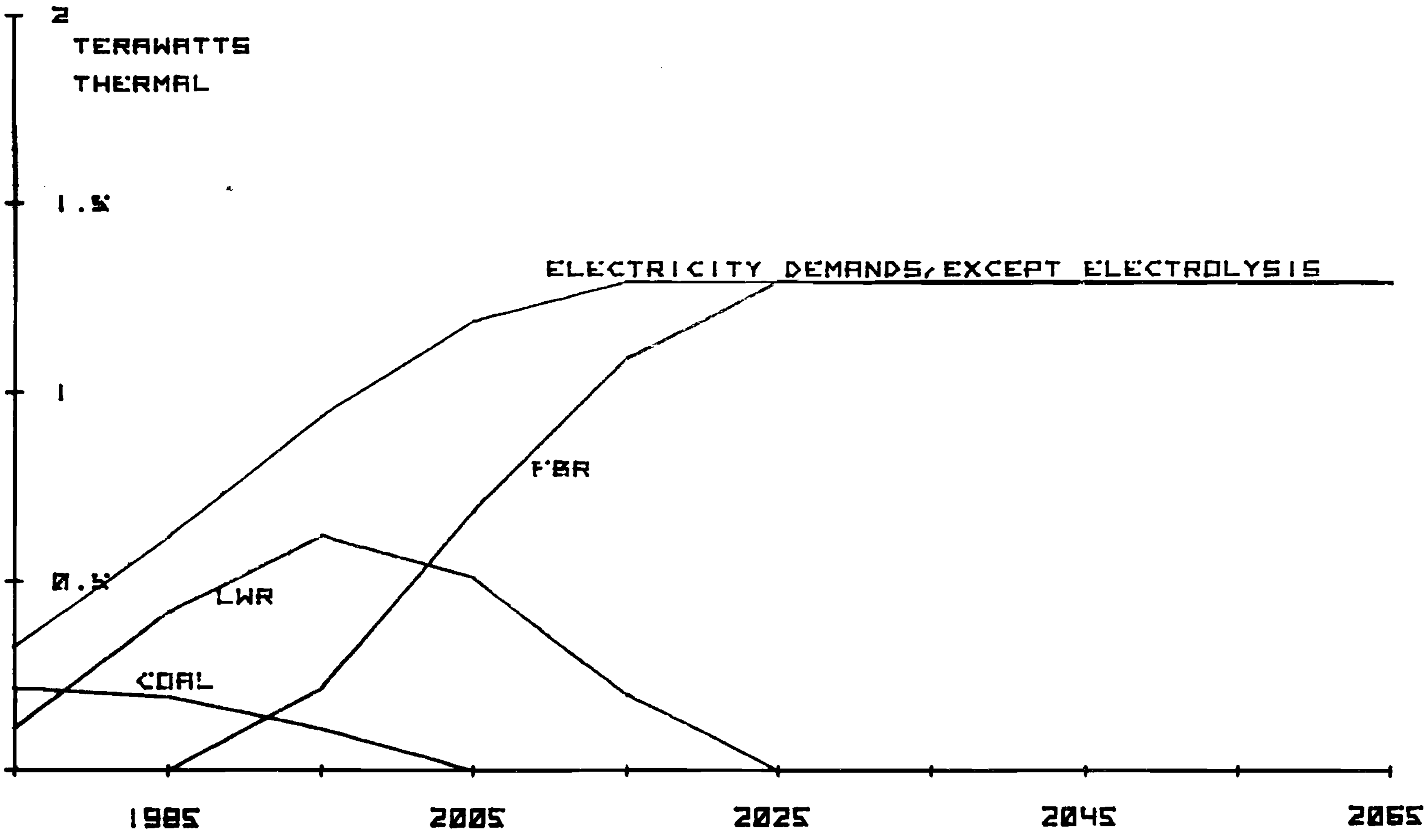

Figure B-10b. Model society 1.100: electric energy demand sector 4.

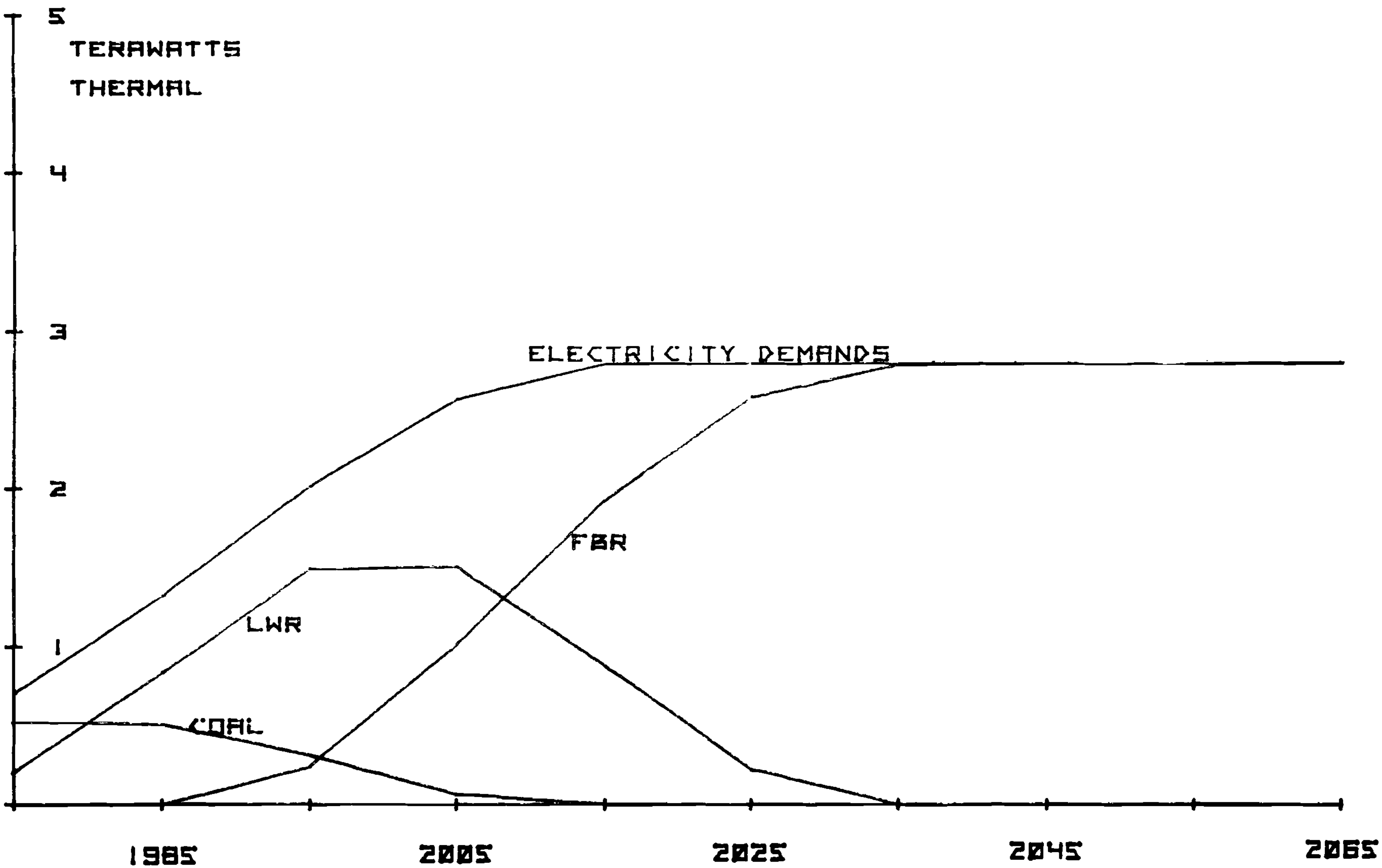

Figure B-10c. Model society 1.100: electric energy (base load) demands and supplies.

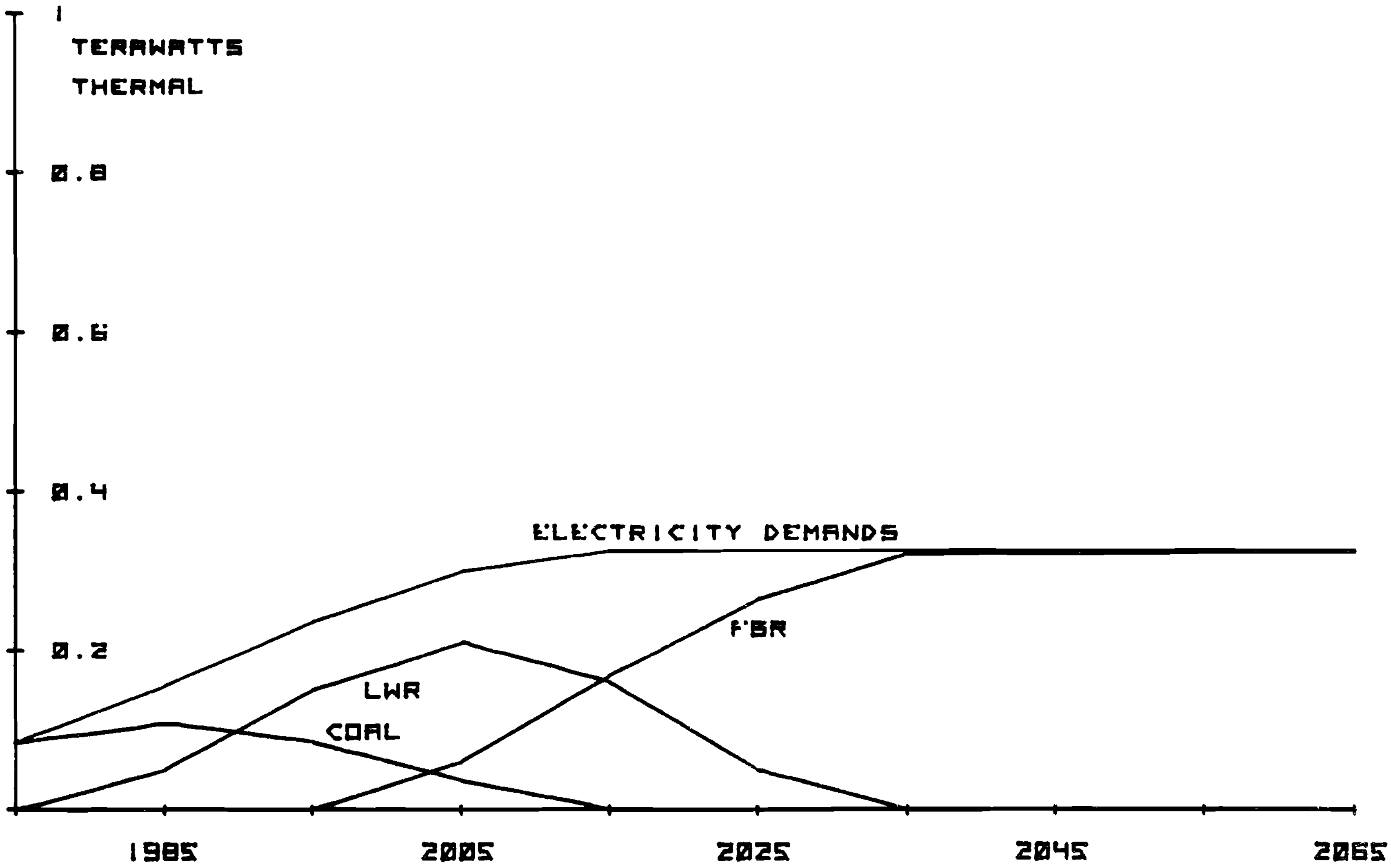

Figure B-11a. Model society 1.100: electric energy demand sector 2.

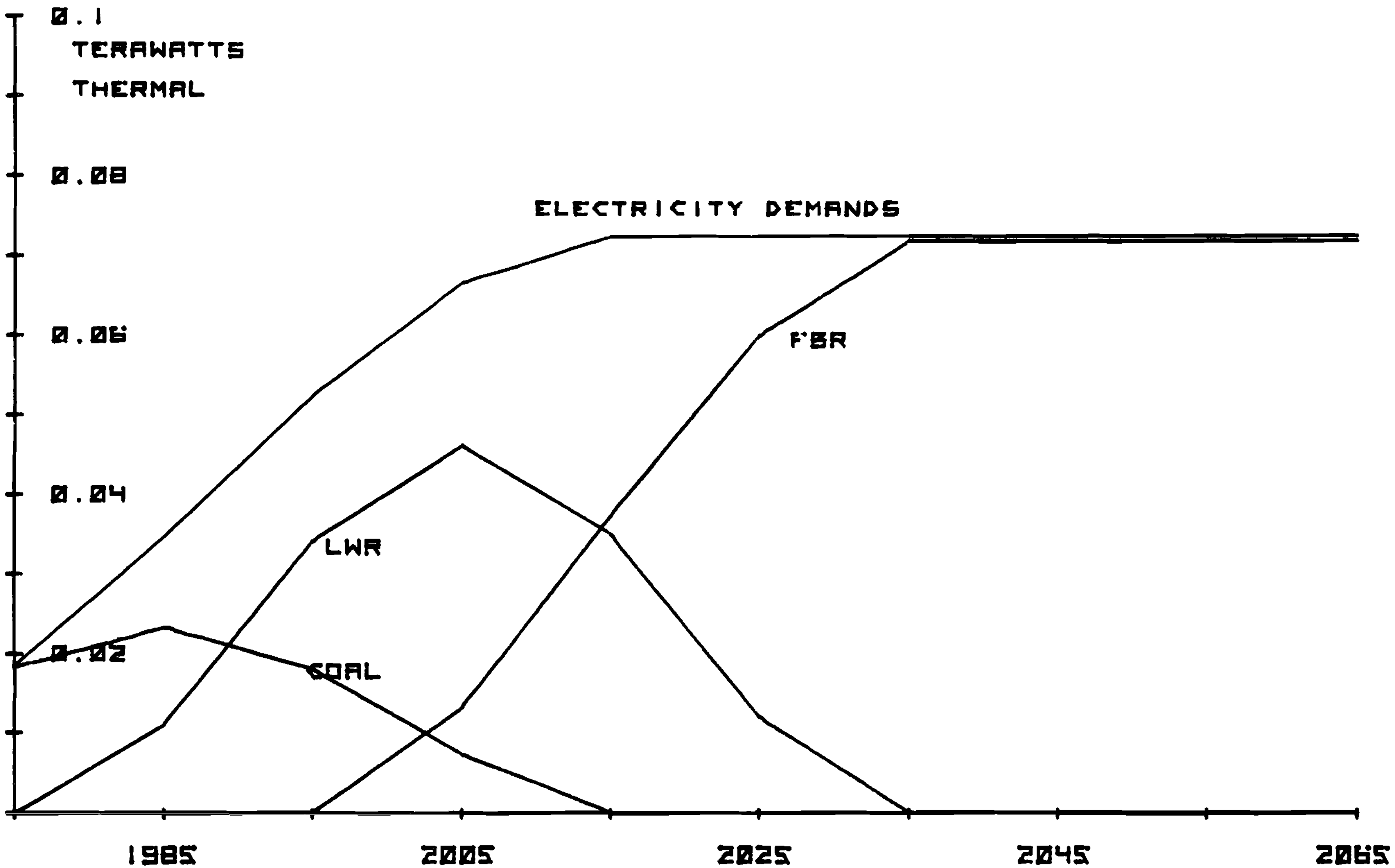

Figure B-11b. Model society 1.100: electric energy demand sector 5.

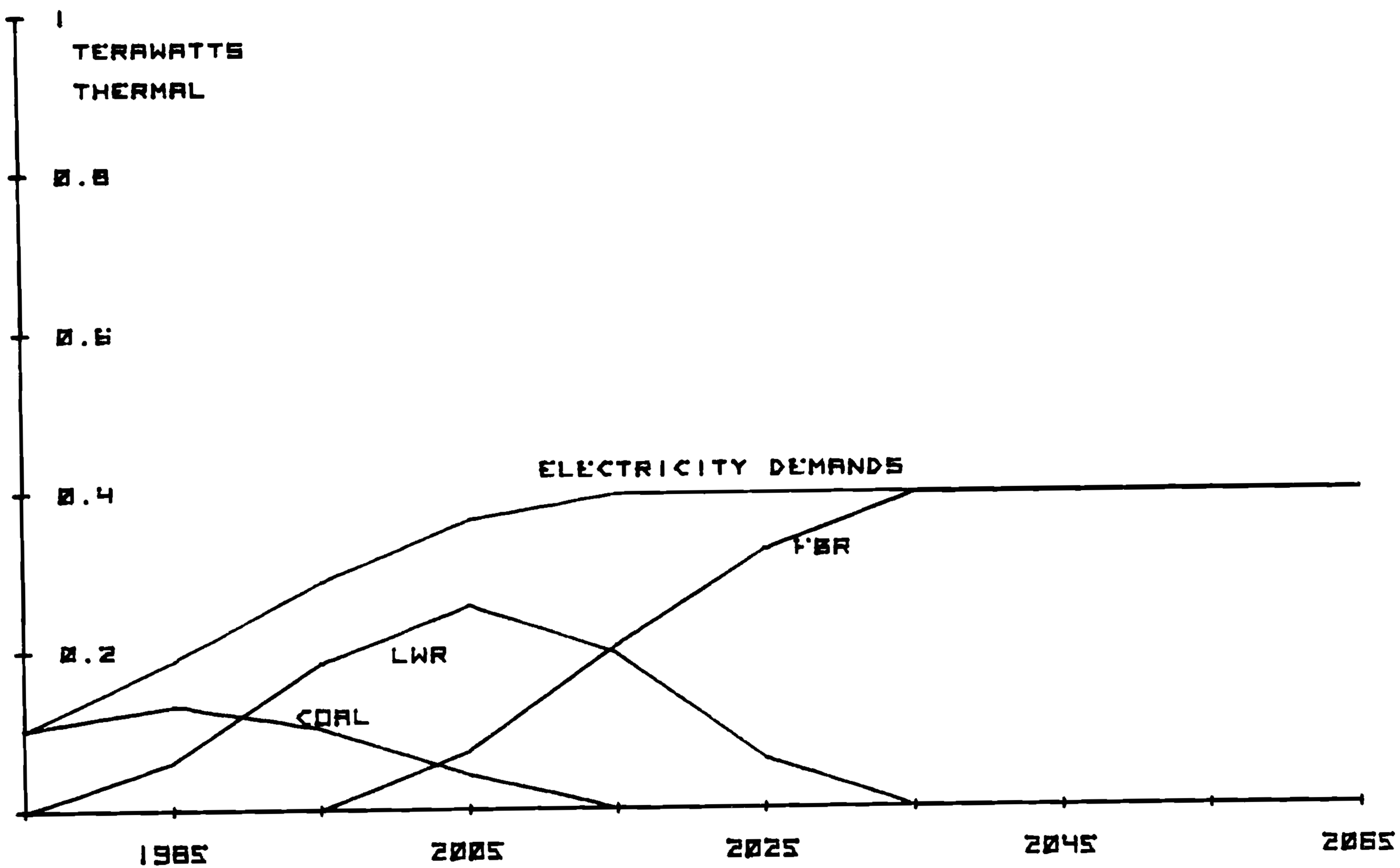

Figure B-11c. Model society 1.100: electric energy (peak load) demands and supplies.

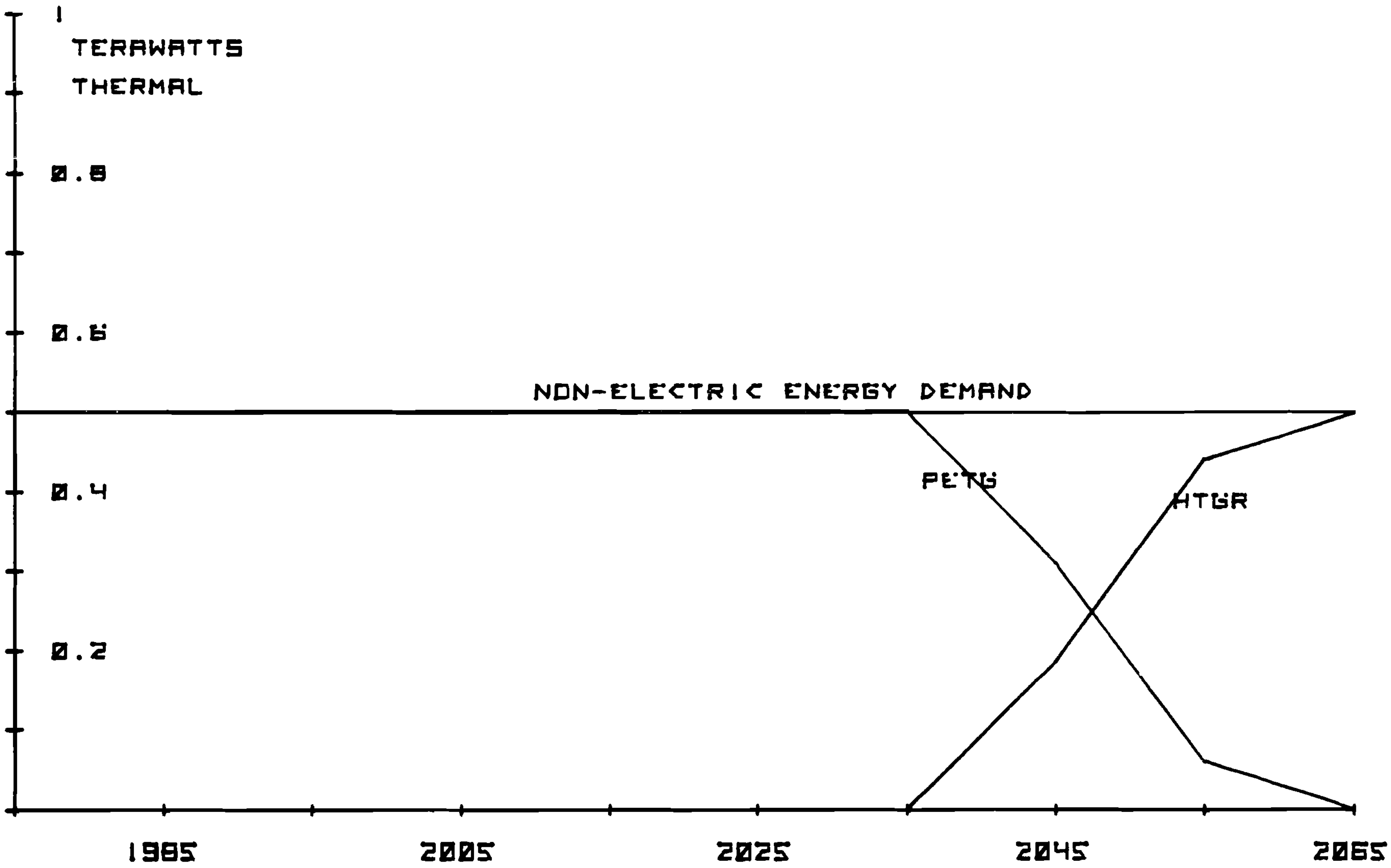

Figure B-12a. Model society 1.100: non-electric energy demand sector 3.

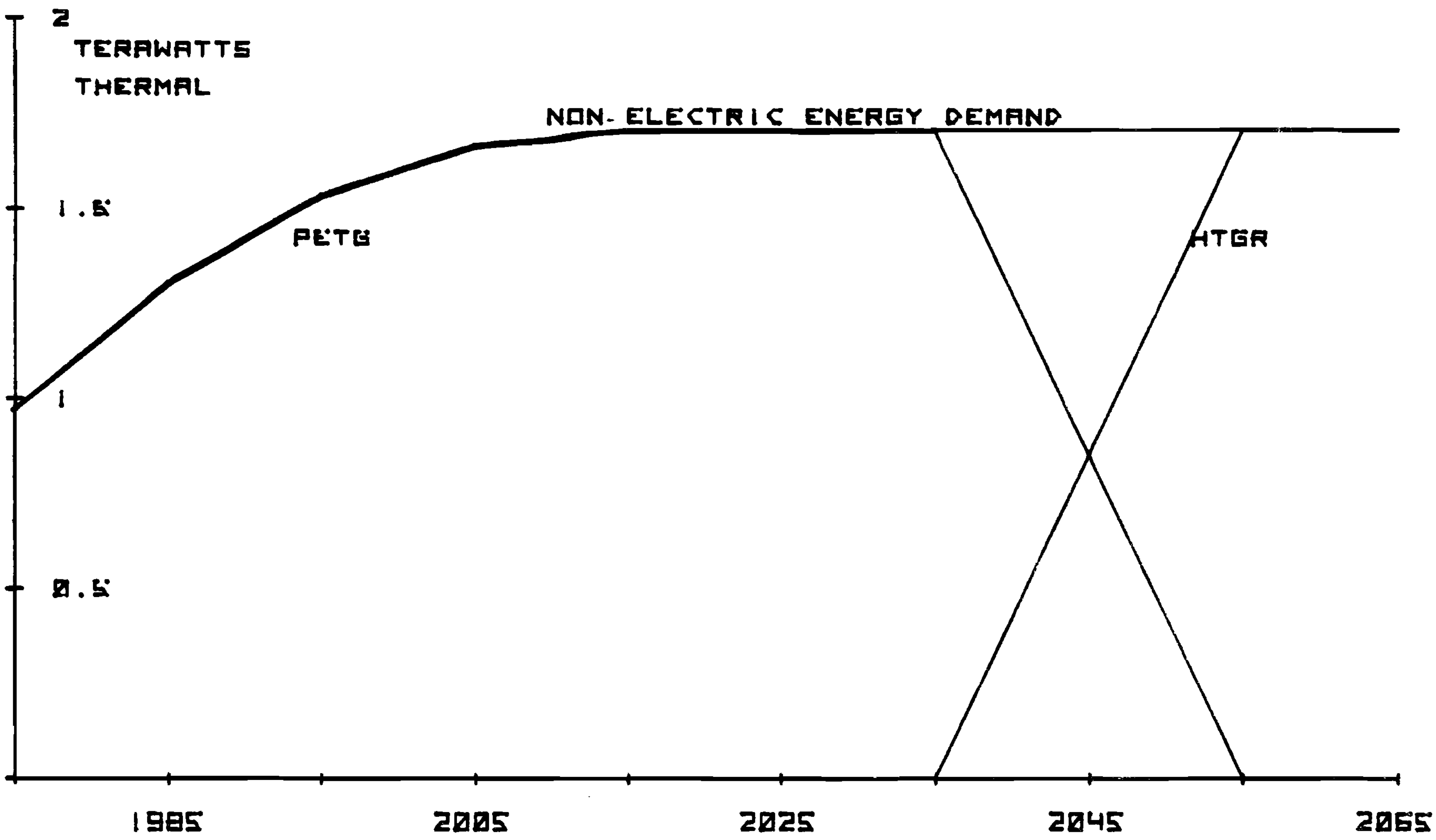

Figure B-12b. Model society 1.100: non-electric energy demand sector 6.

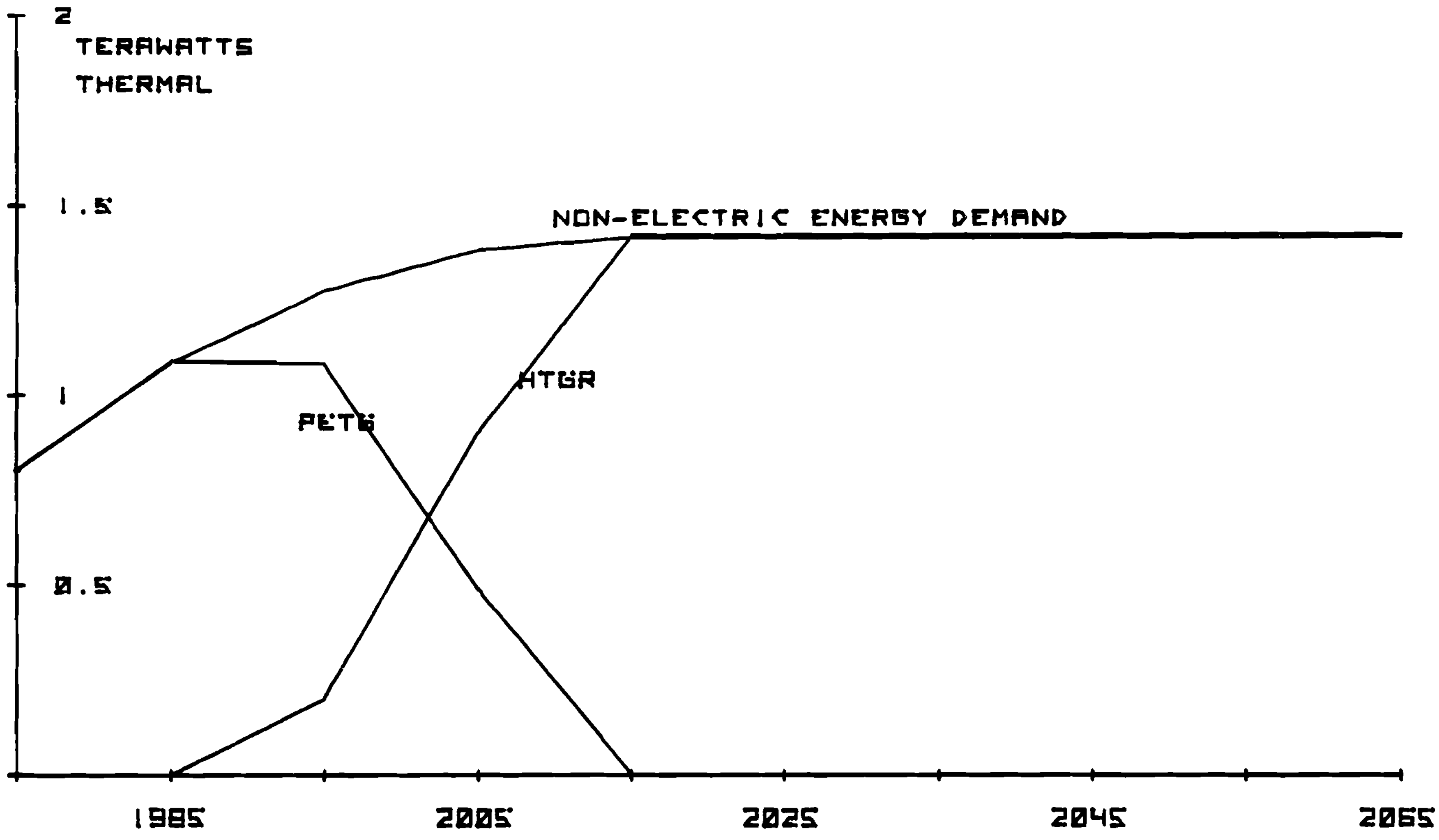

Figure D-12c. Model society 1.100: non-electric energy demand sector 7.

APPENDIX C

Results of Sensitivity Analysis on Capital Costs of Solar Power Plants

(1) Contribution of Solar Alternatives for Energy Supply, Cases: B, S1, S2, S3, S4.

	Case	Energy, Supplying Form
Figure C-1a	1.40	Electricity
1b		Non-Electric Energy
2a	1.60	Electricity
2b		Non-Electric Energy
3a	1.80	Electricity
3b		Non-Electric Energy
4a	1.100	Electricity
4b		Non-Electric Energy

(2) Optimal Energy Production Activities of Each of the Alternatives for Each of the Demand Categories, in Case S2.

	Case	Demand Category	
Figure C-5a	1.40	All Electricity	
5b		All Non-Electric Energy	
6a		Elec., Base Load,	Res. and Comm.
6b			Indus.
6c			Subtotal
7a		Elec., Intermed.,	Res. and Comm.
7b			Indus.
7c			Subtotal
8a		Non-Elec.,	Res. and Comm.
8b			Indus.
8c			Transportation
Figure C-9a	1.80	All Electricity	
9b		All Non-Electric Energy	
10a		Elec., Base Load,	Res. and Comm.
10b			Indus.
10c			Subtotal
11a		Elec., Intermed.,	Res. and Comm.
11b			Indus.
11c			Subtotal
12a		Non-Elec.,	Res. and Comm.
12b			Indus.
12c			Transportation

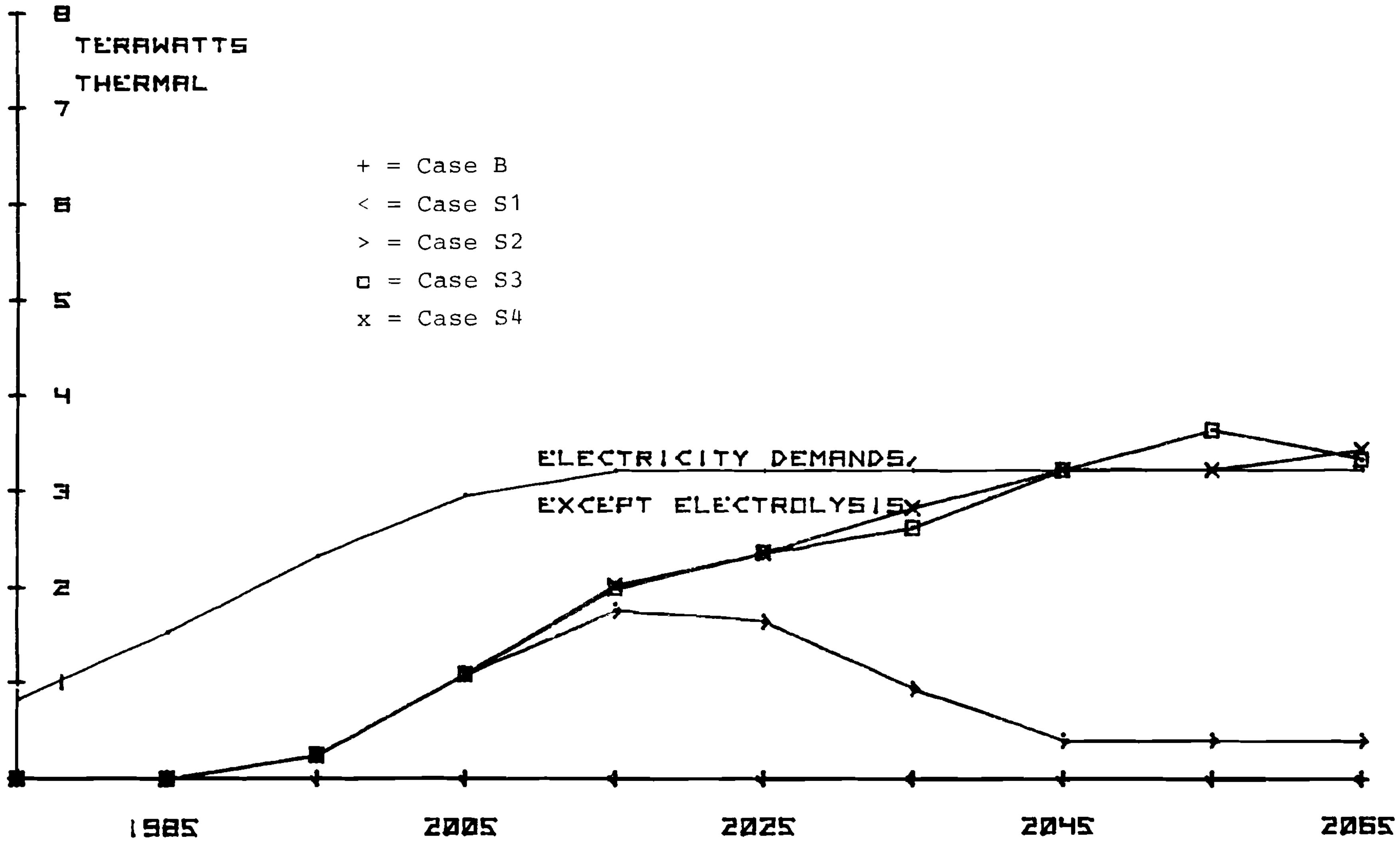

Figure C-1a. Model society 1.40: solar electricity.

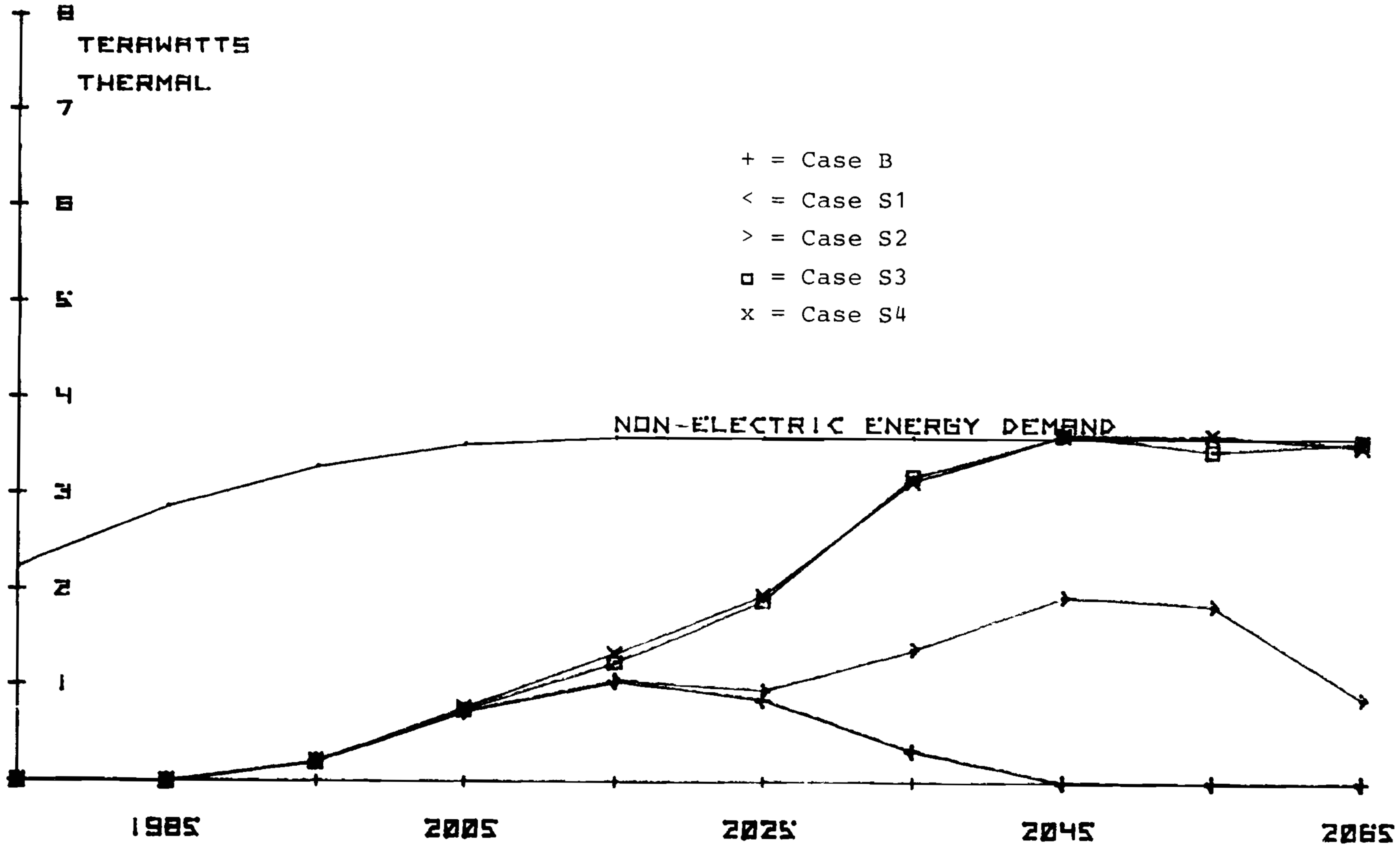

Figure C-1b. Model society 1.40: solar hydrogen of total non-electric energy.

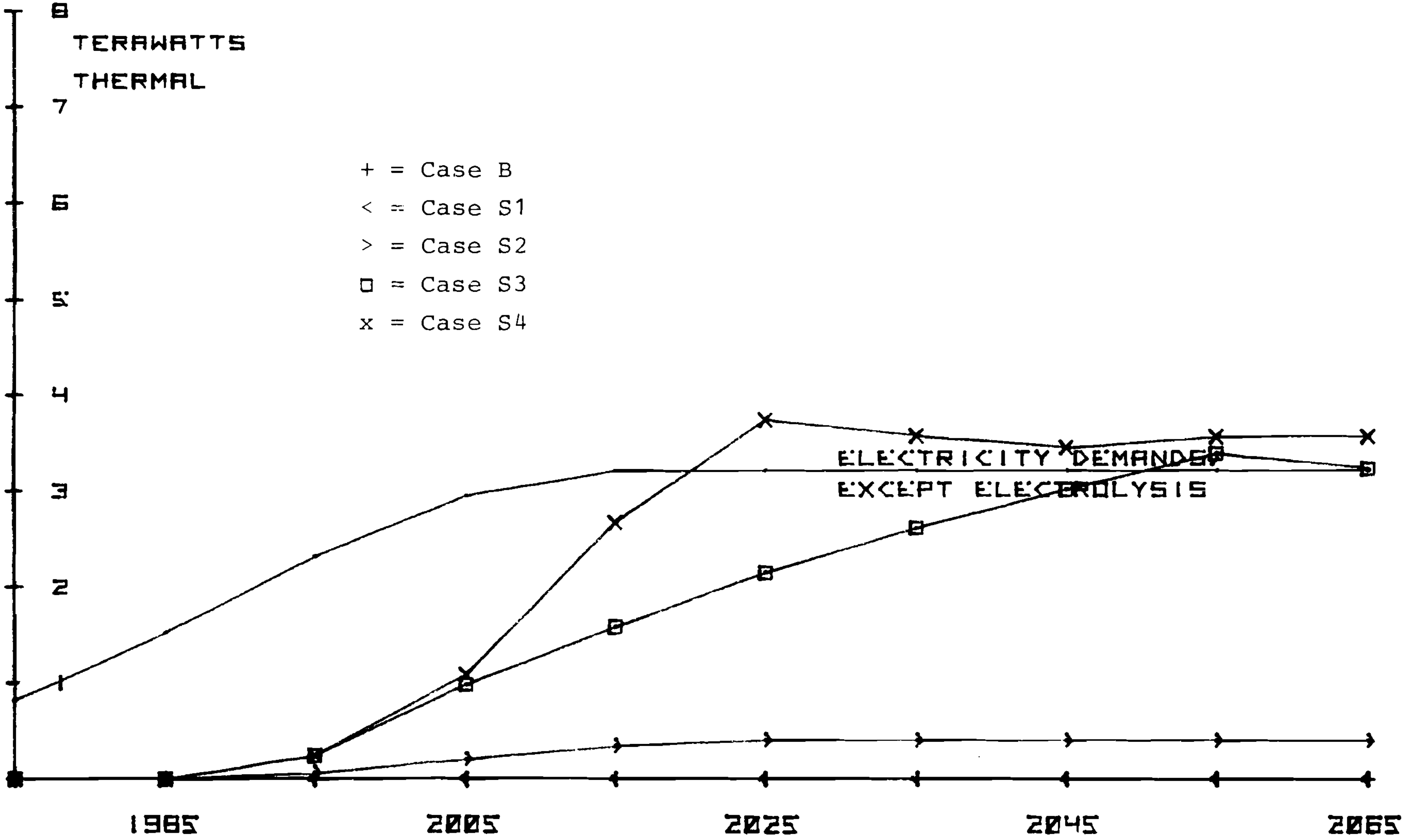

Figure C-2a. Model society 1.60: solar electricity.

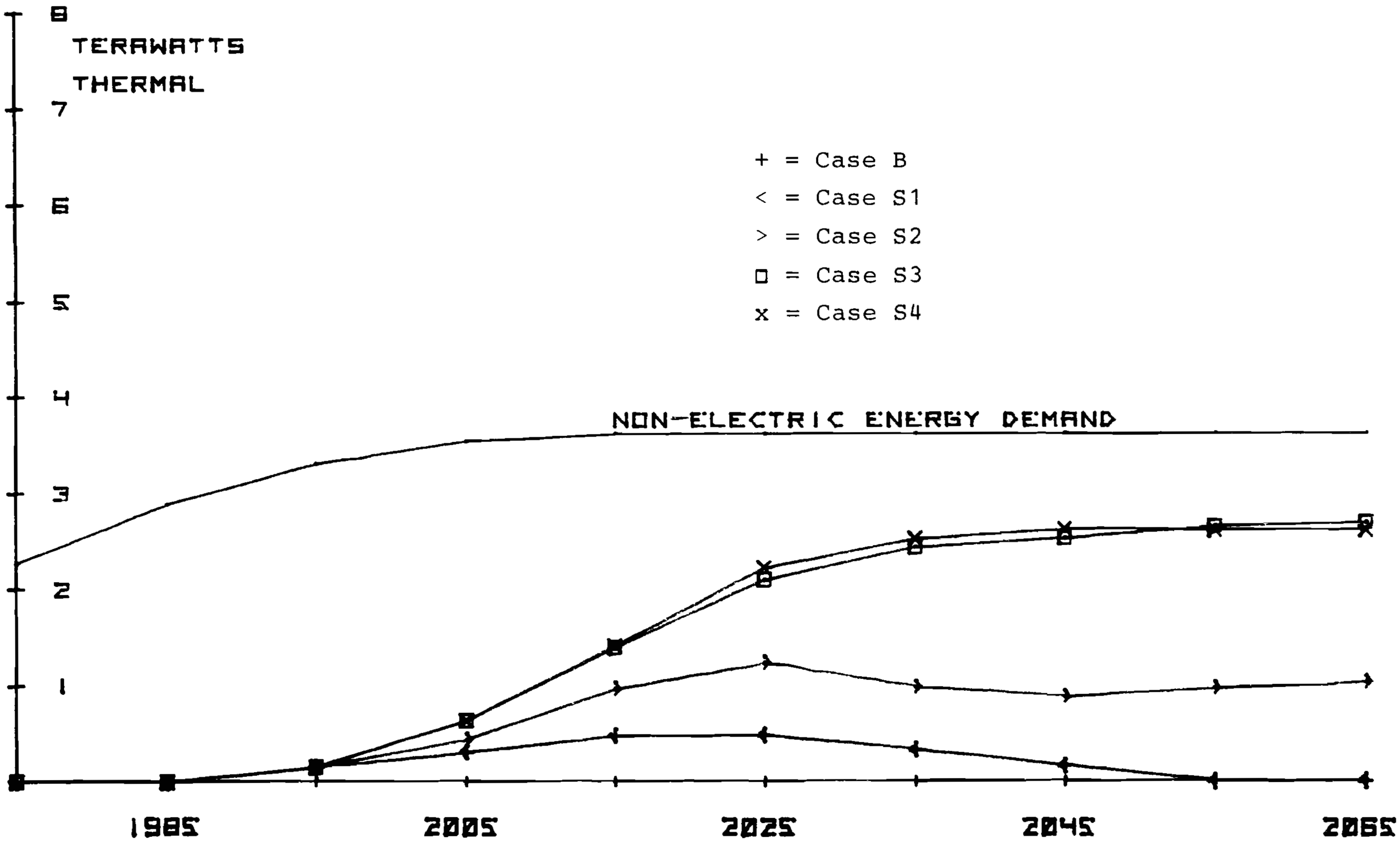

Figure C-2b. Model society 1.60: solar hydrogen of total non-electric energy.

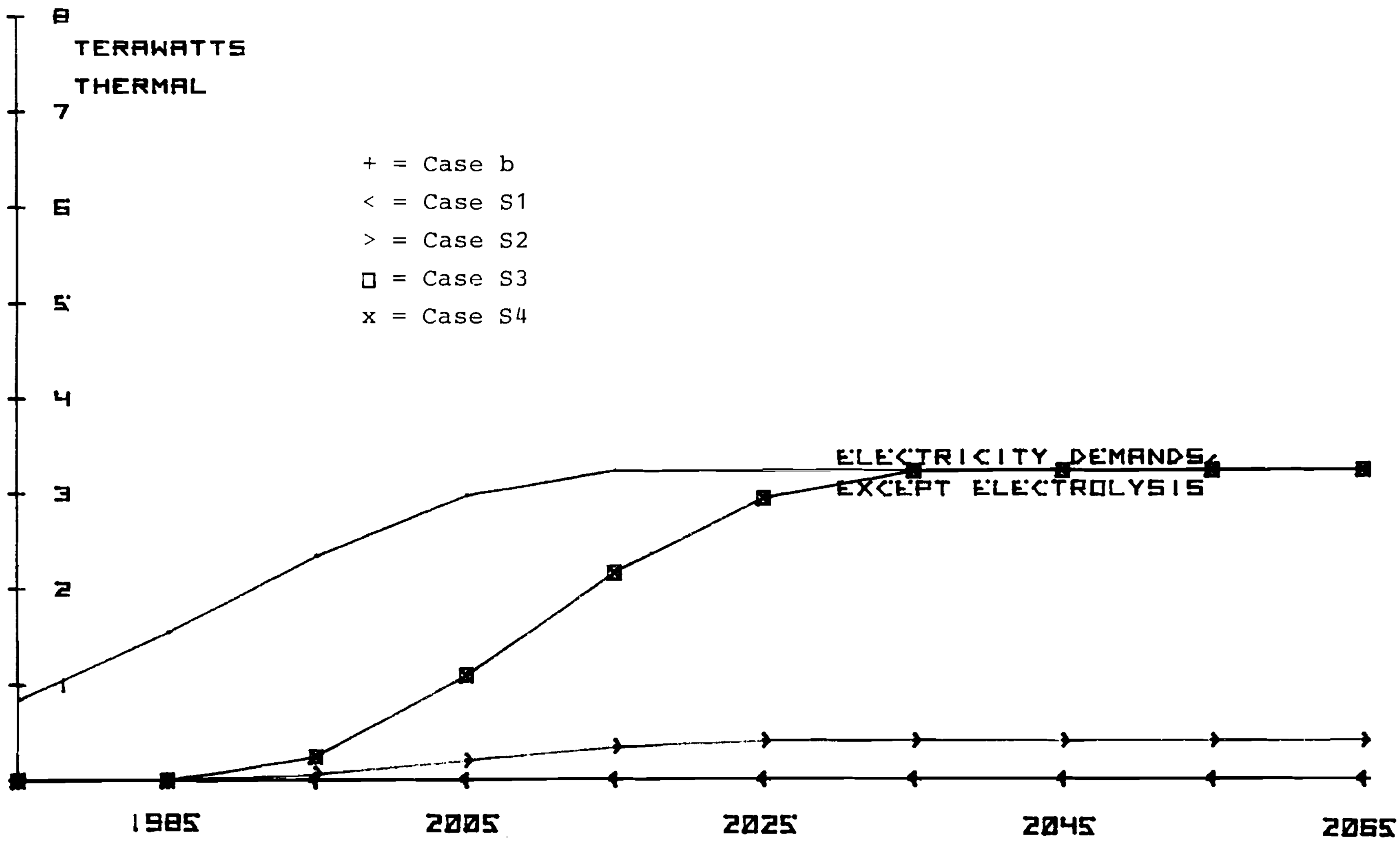

Figure C-3a. Model society 1.80: solar electricity.

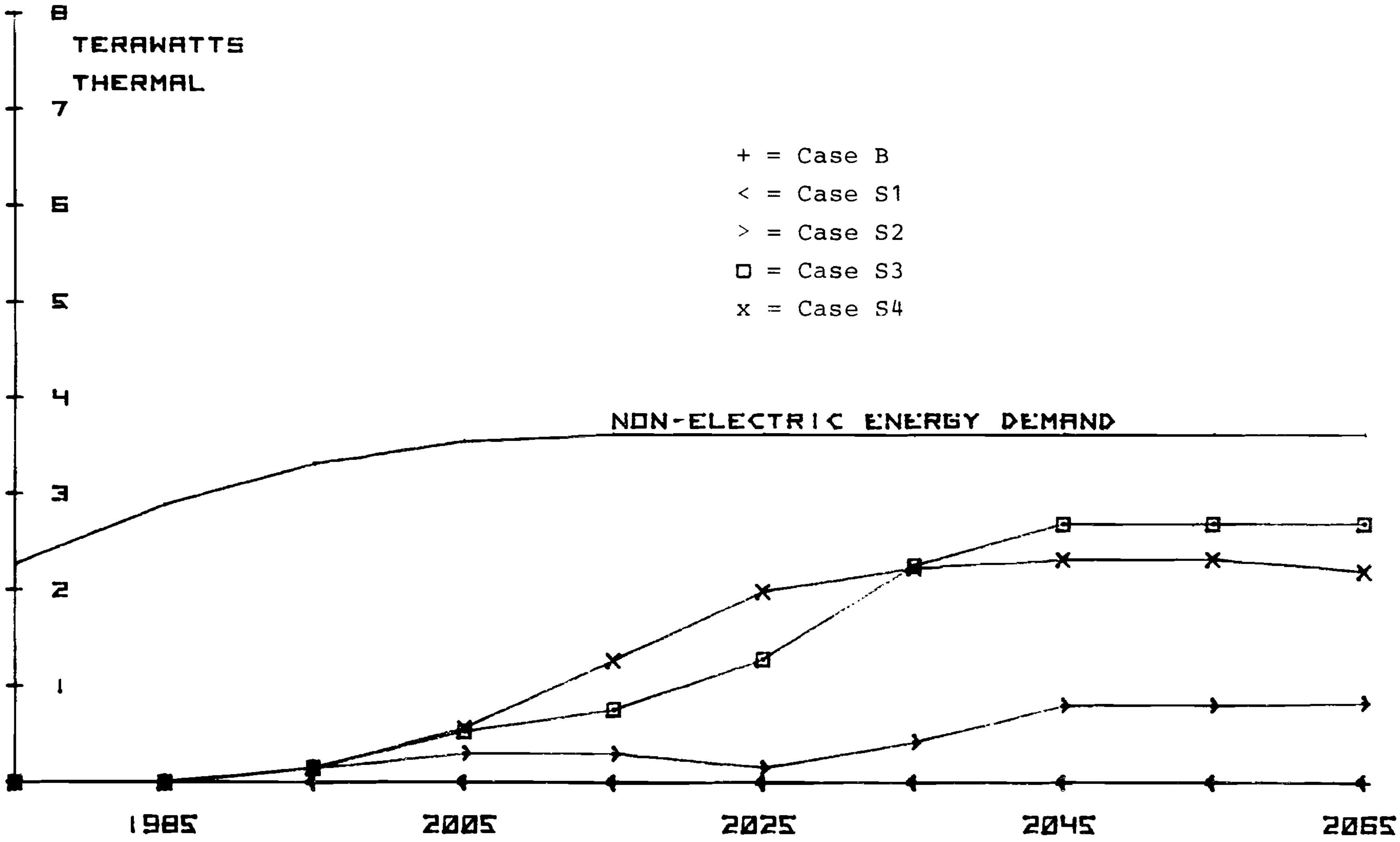

Figure C-3b. Model society 1.80: solar hydrogen of total non-electric energy.

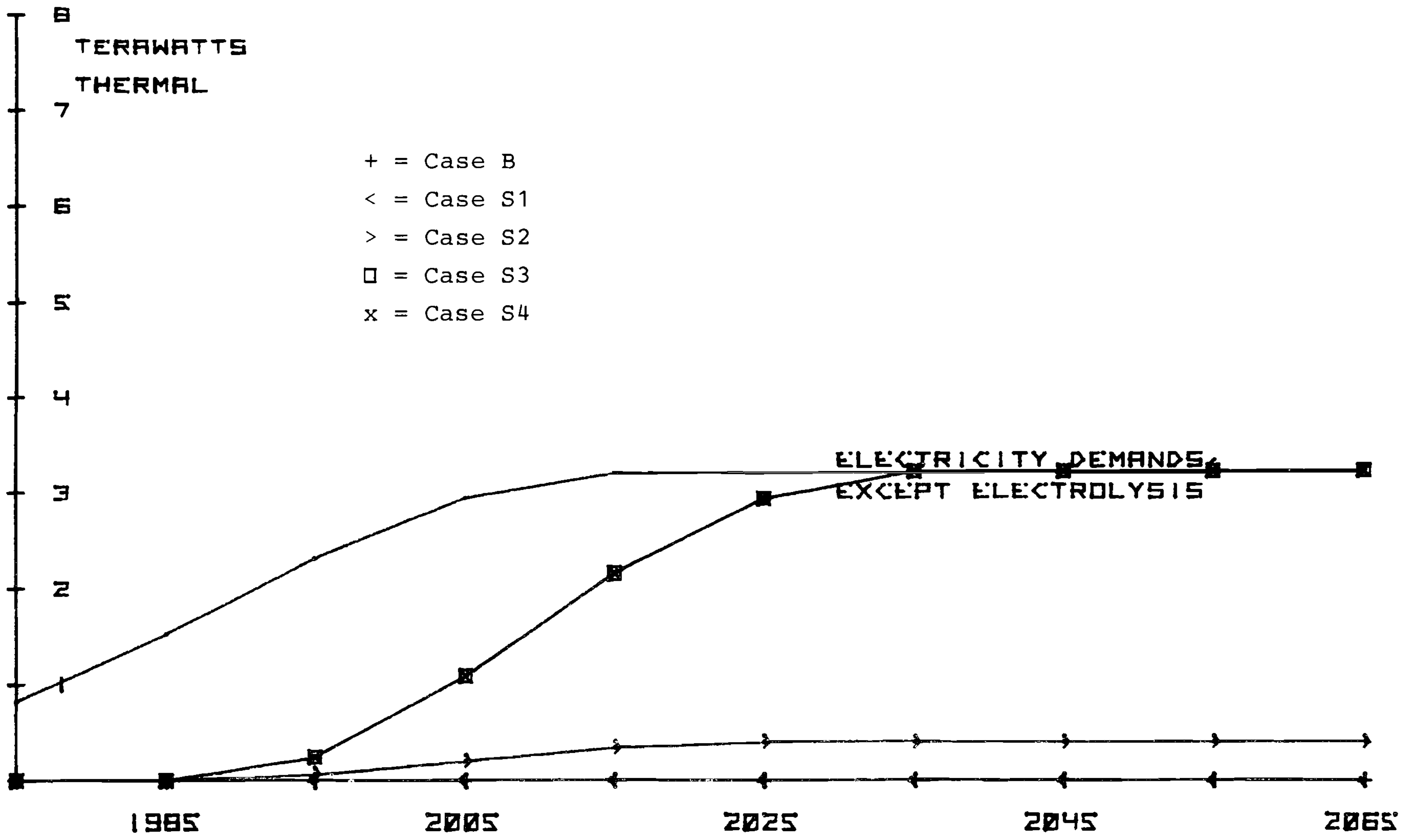

Figure C-4a. Model society 1.100: solar electricity.

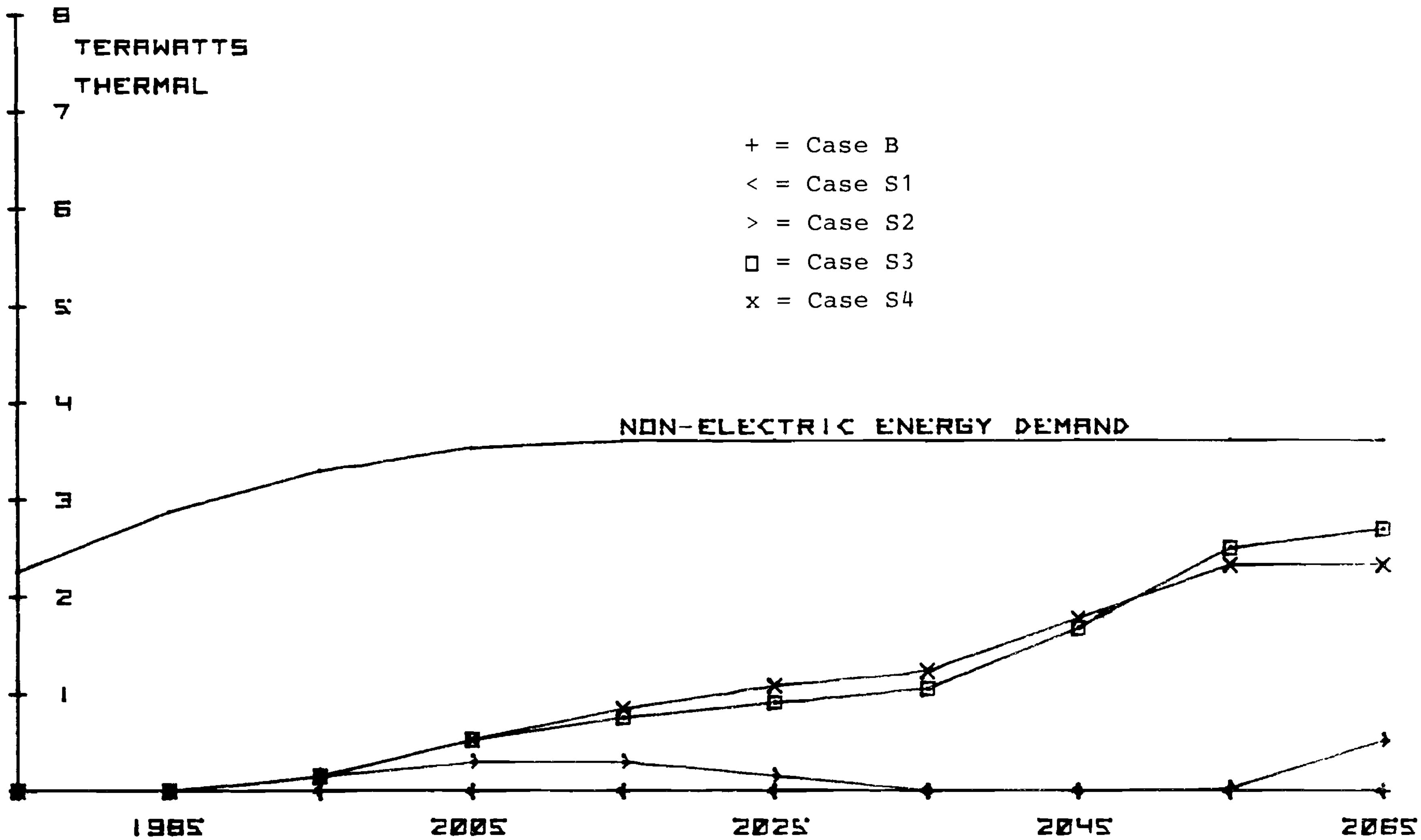

Figure C-4b. Model society 1.100: solar hydrogen of total non-electric energy.

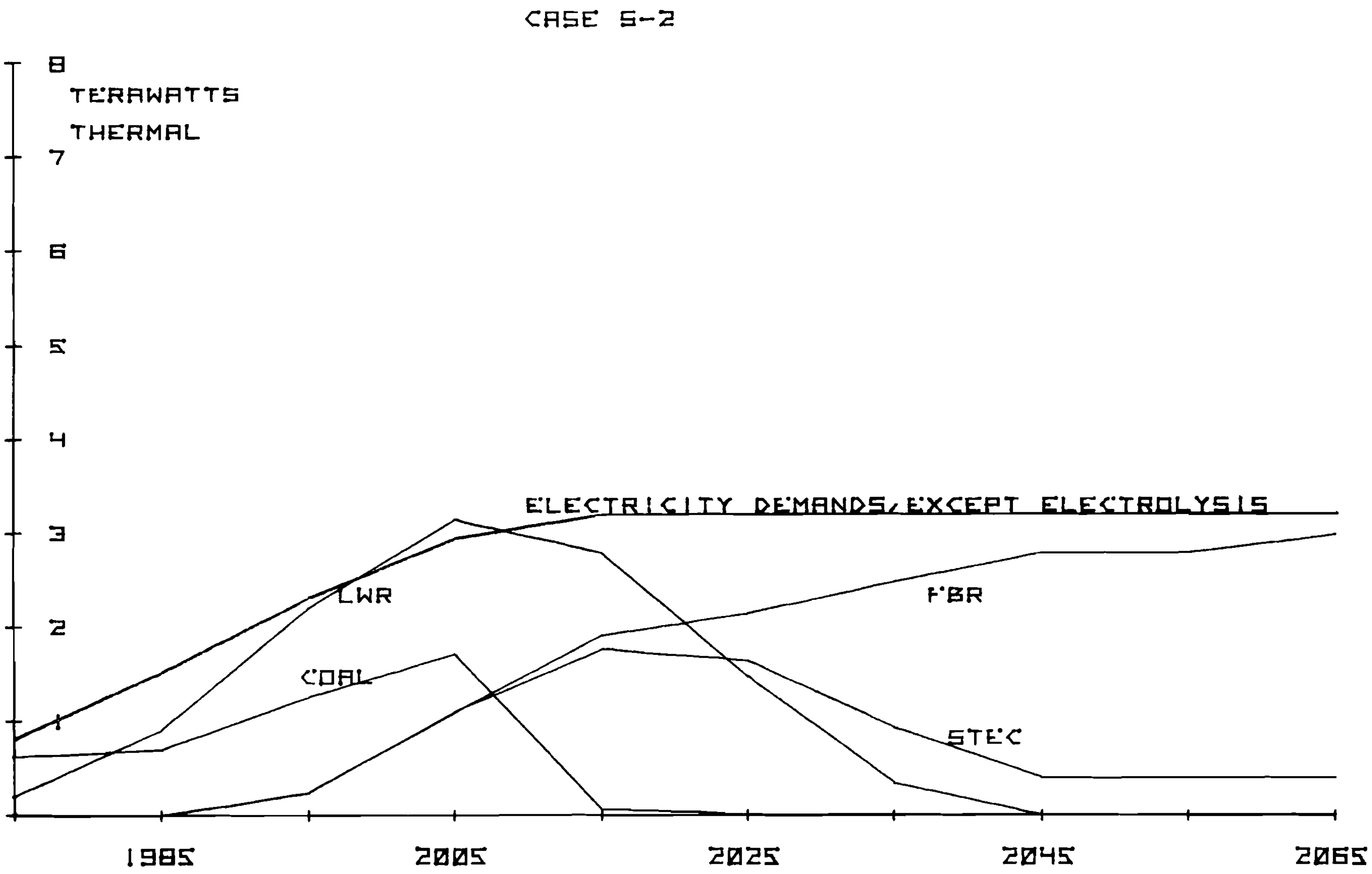

Figure C-5a. Model society 1.40: electric energy demands and supplies.

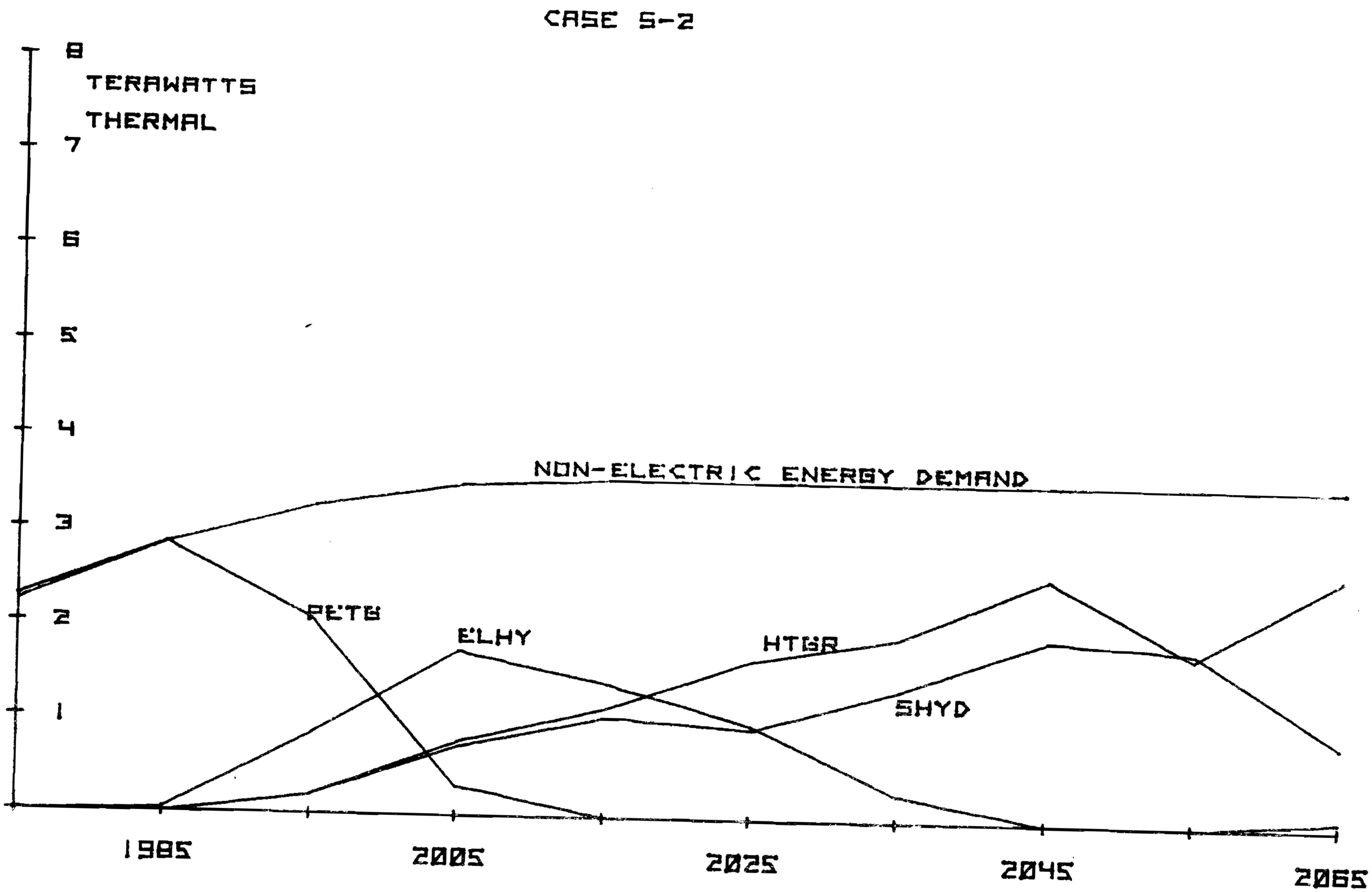

Figure C-5b. Model society 1.40: non-electric energy demands and supplies.

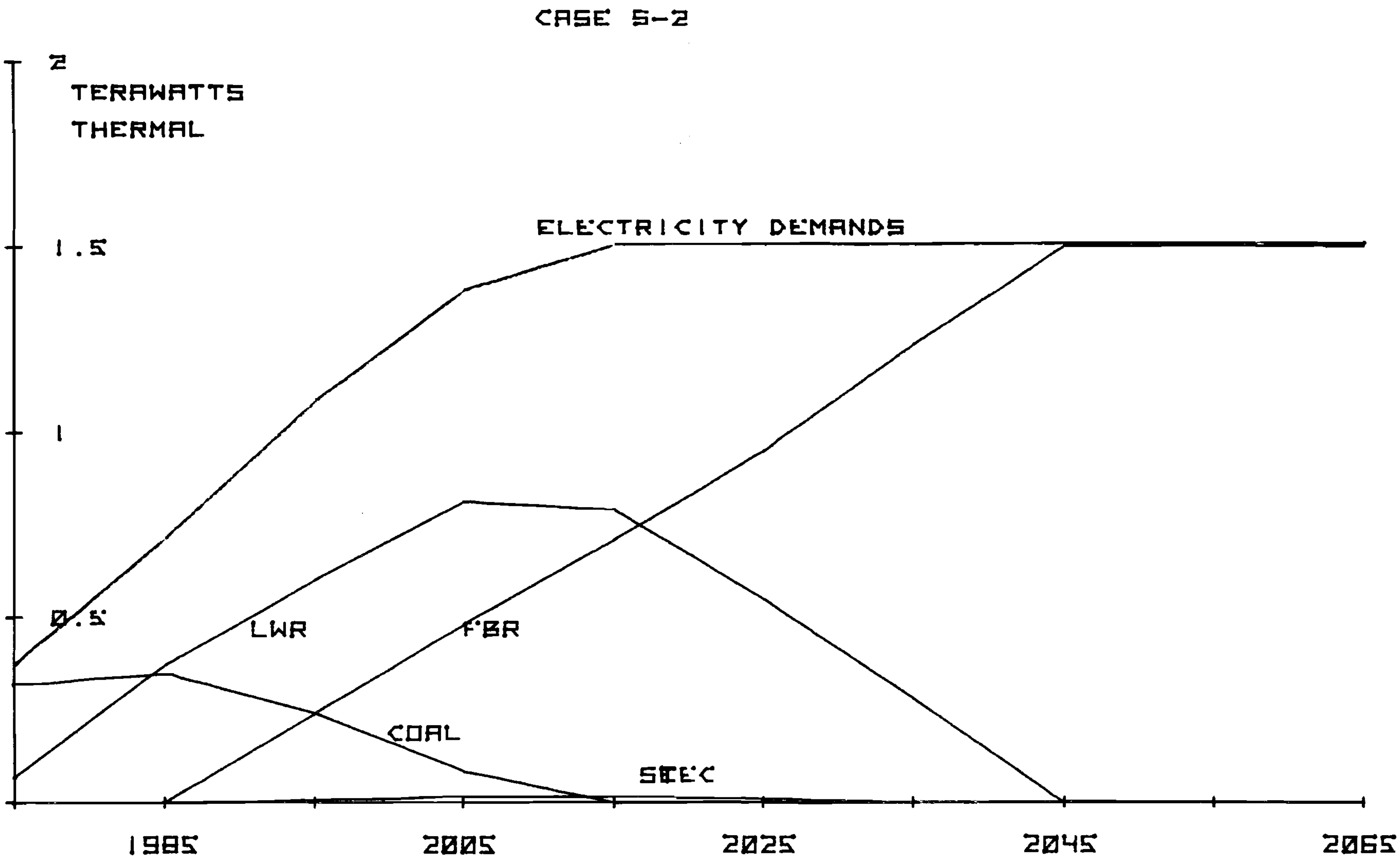

Figure C-6a. Model society 1.40: electric energy demand sector 1.

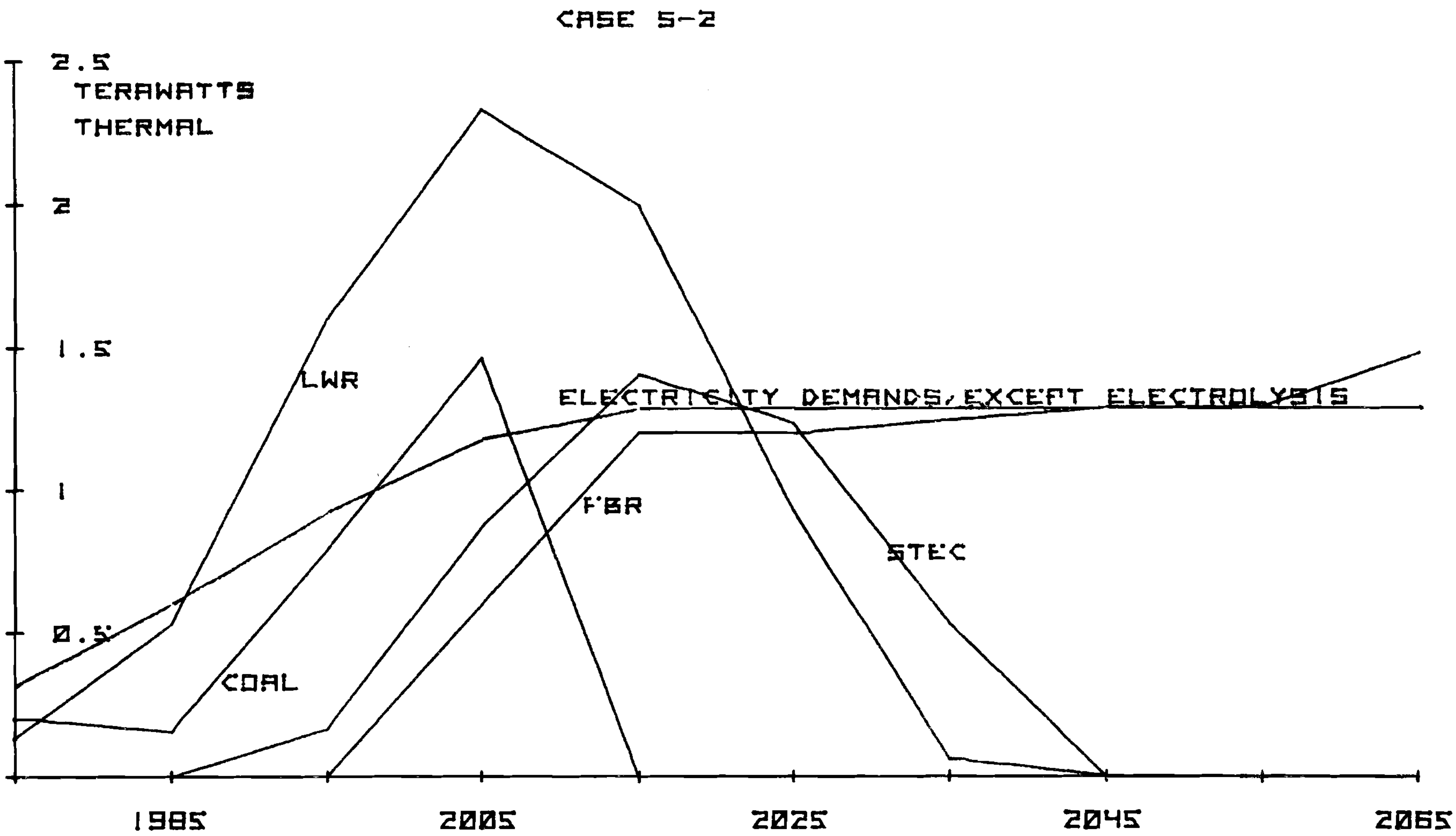

Figure C-6b. Model society 1.40: electric energy demand sector 4.

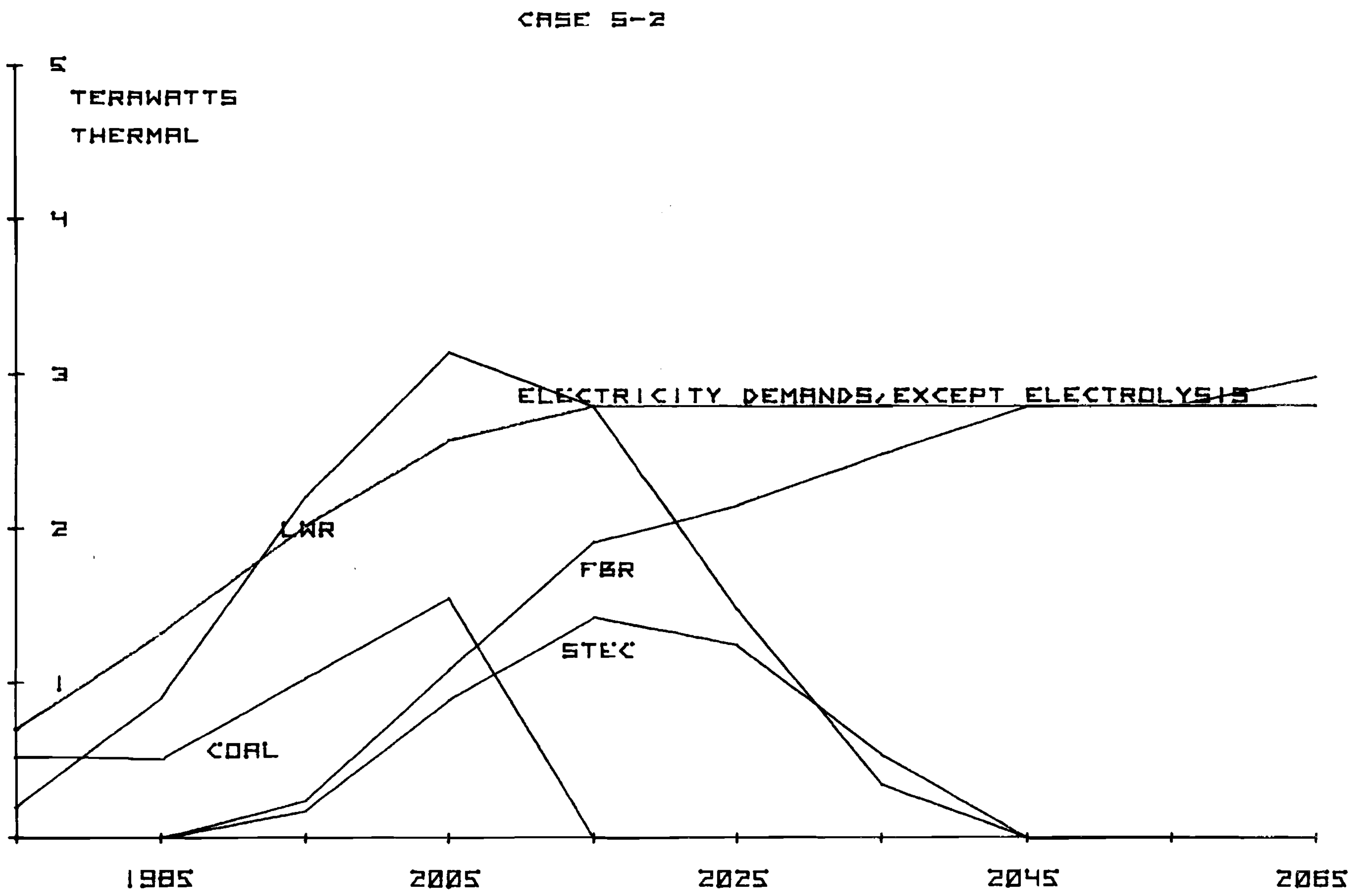

Figure C-6c. Model society 1.40: electric energy (base load) demands and supplies.

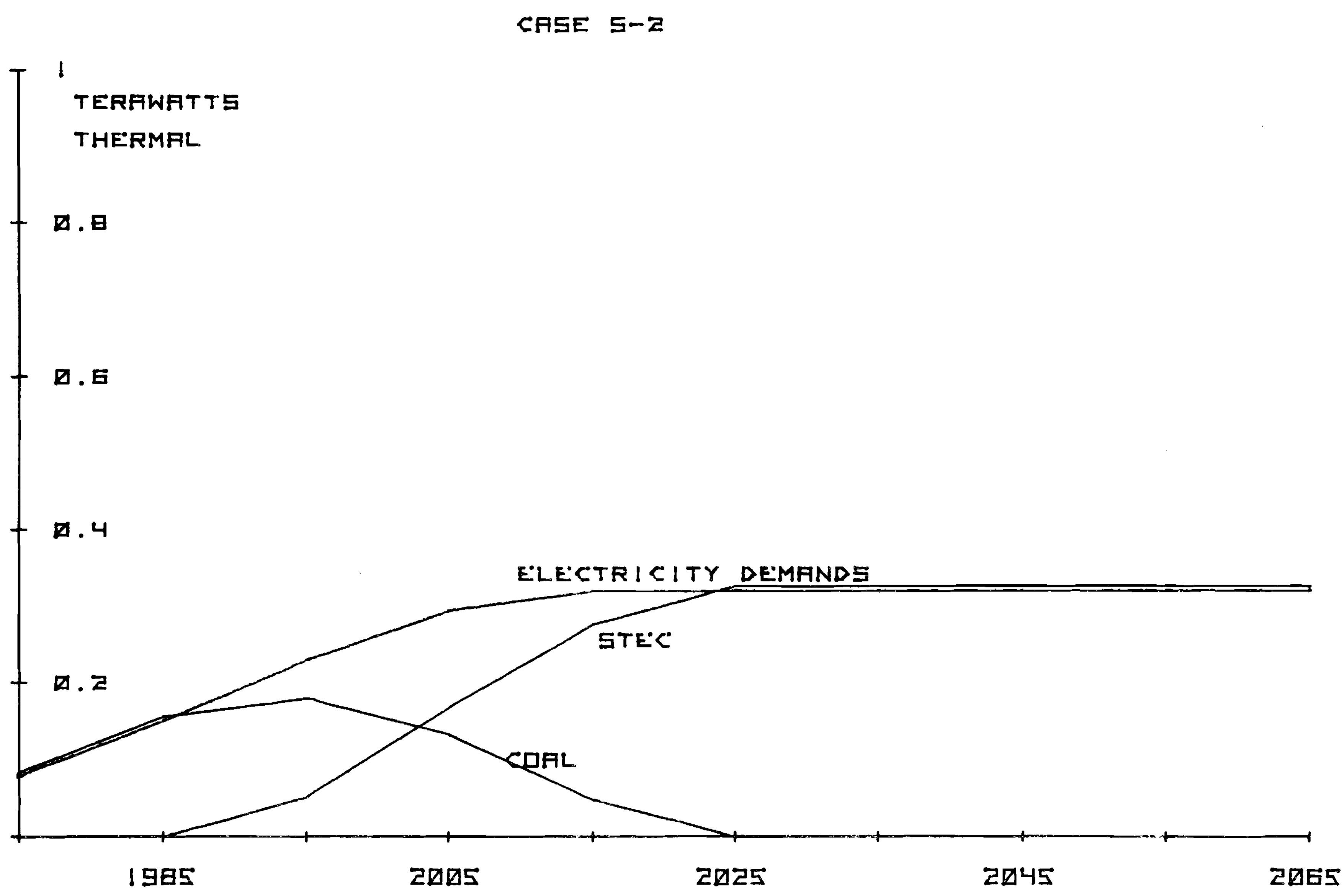

Figure C-7a. Model society 1.40: electric energy demand sector 2.

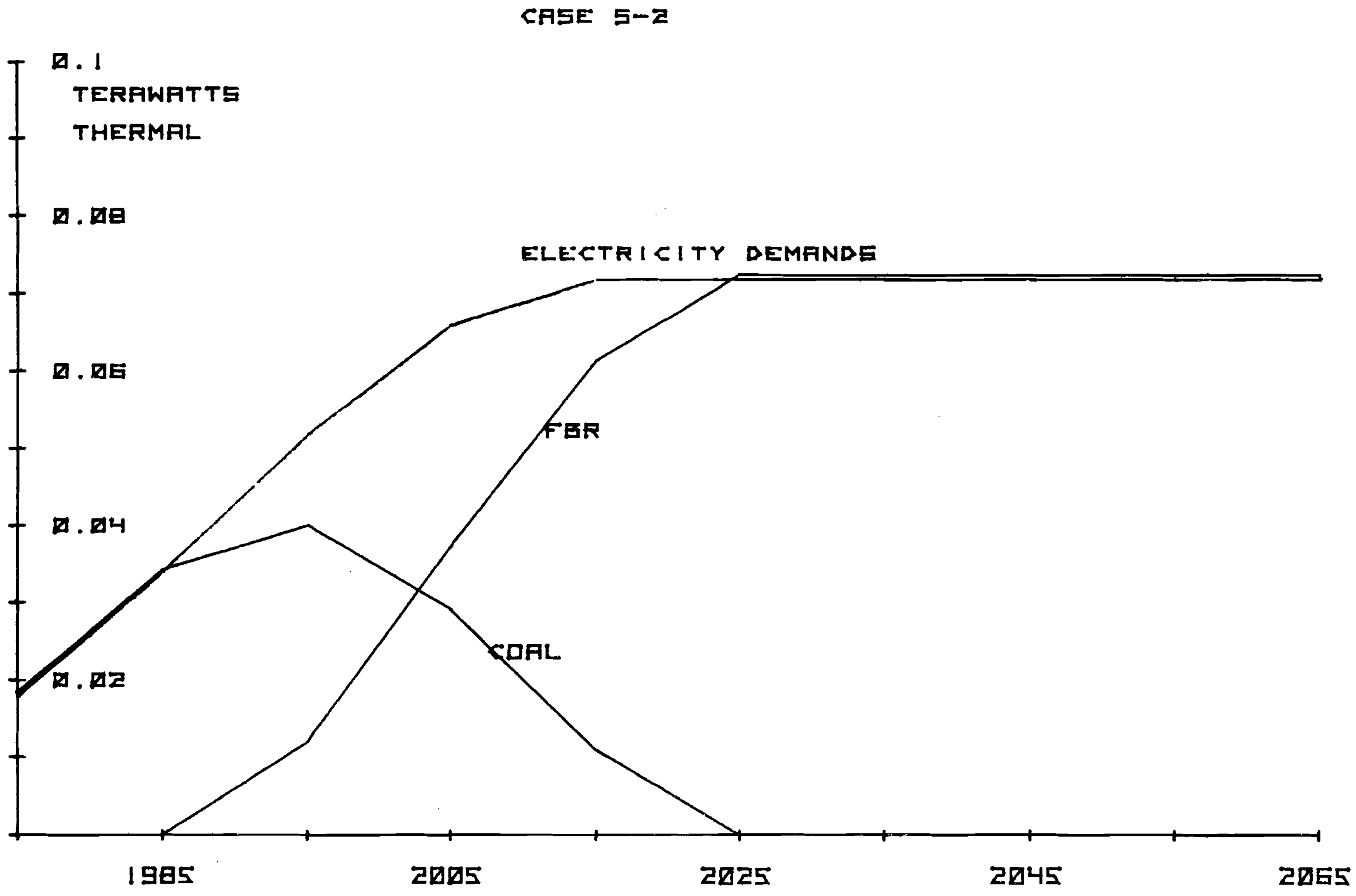

Figure C-7b. Model society 1.40: electric energy demand sector 5.

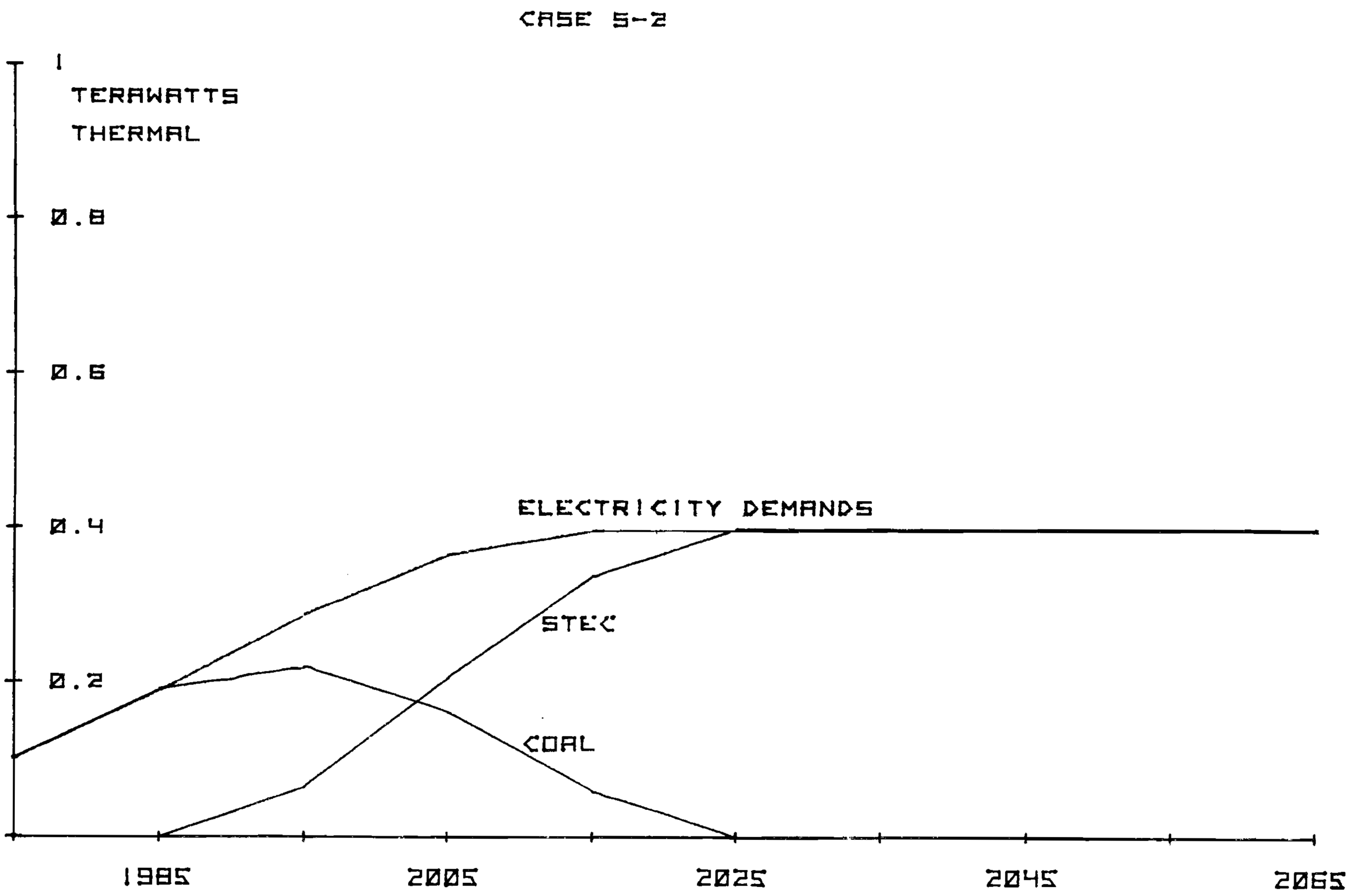

Figure C-7c. Model society 1.40: electric energy (peak load) demands and supplies.

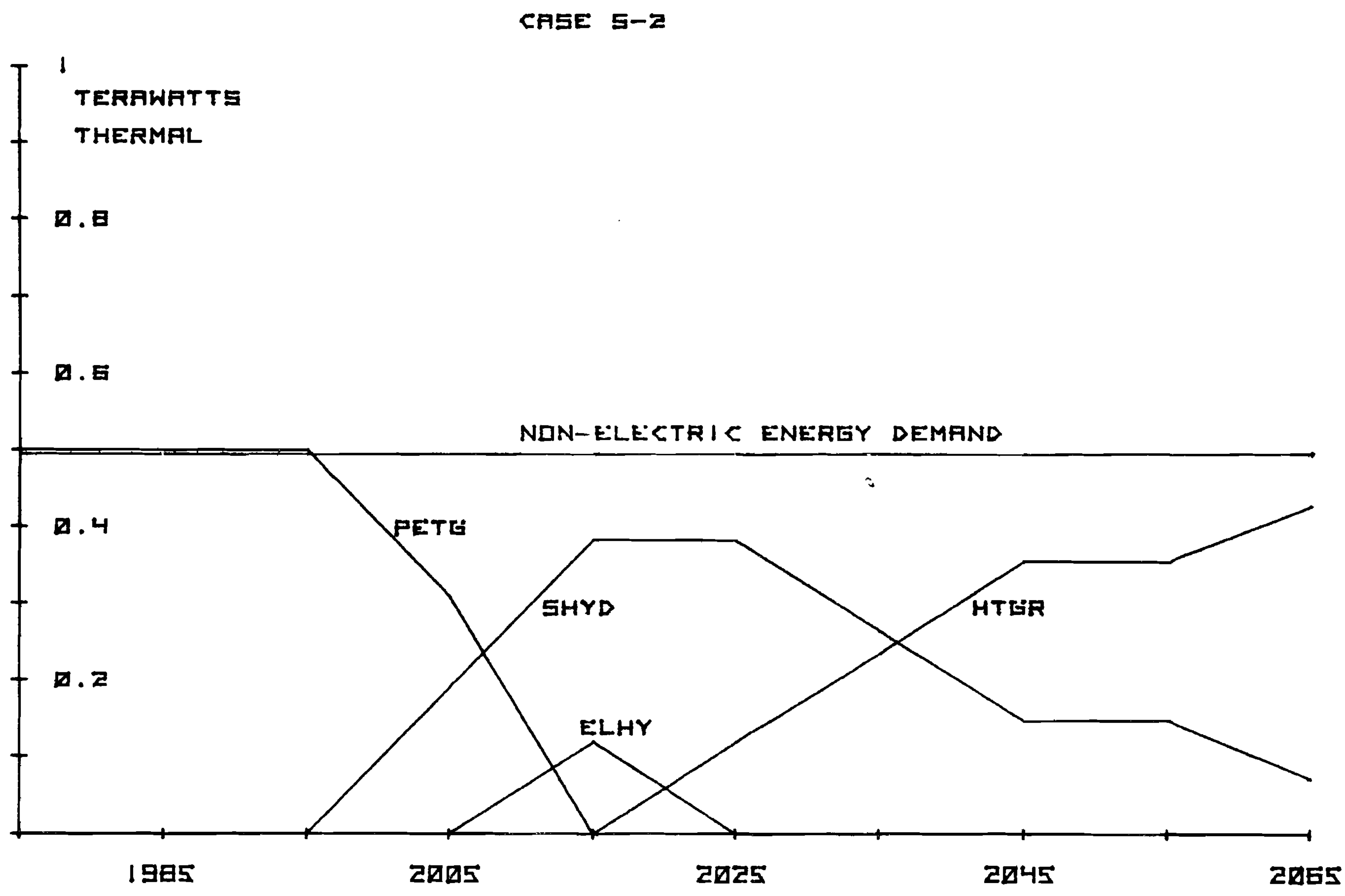

Figure C-8a. Model society 1.40: non-electric energy demand sector 3.

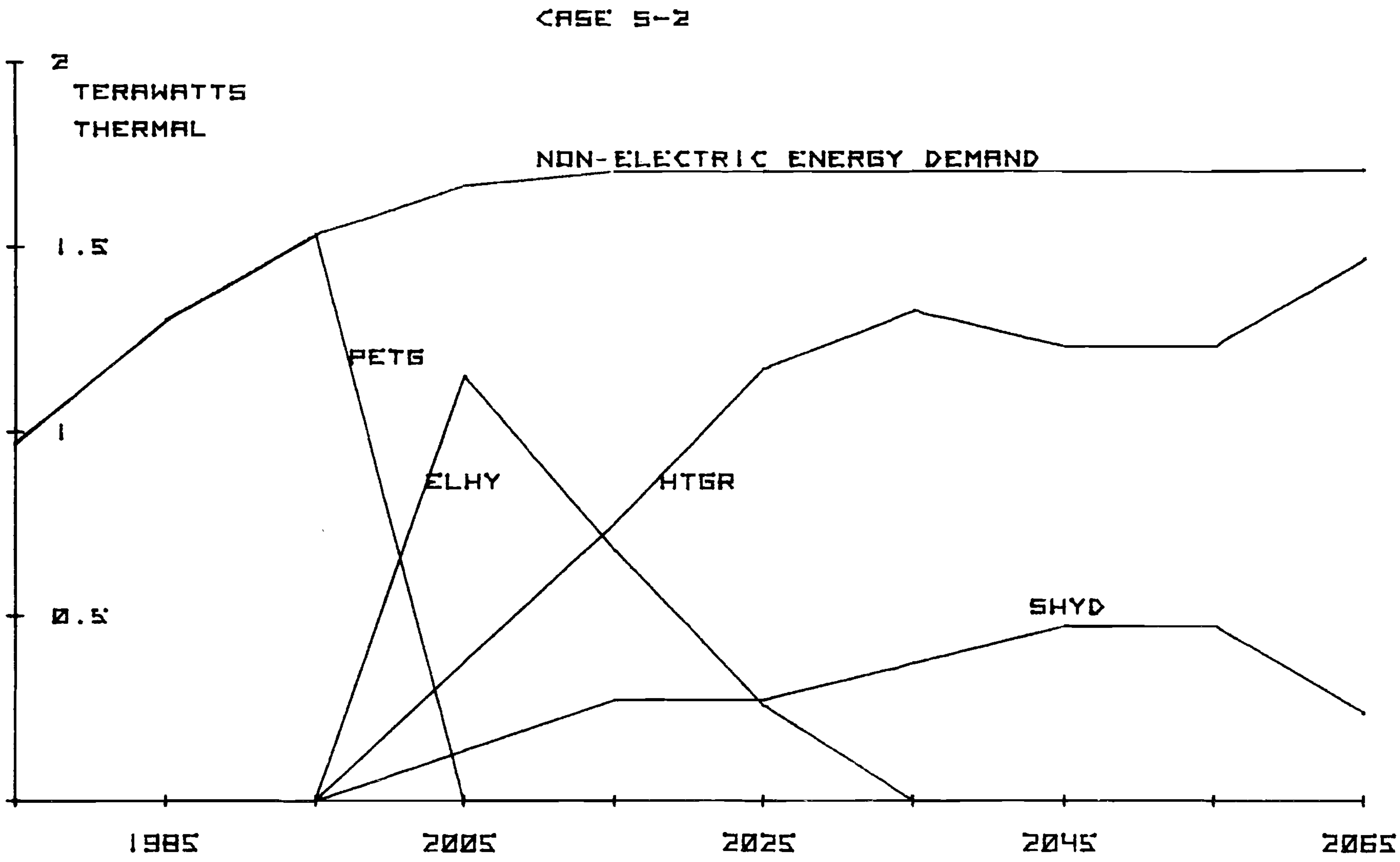

Figure C-8b. Model society 1.40: non-electric energy demand sector 8.

CASE 5-2

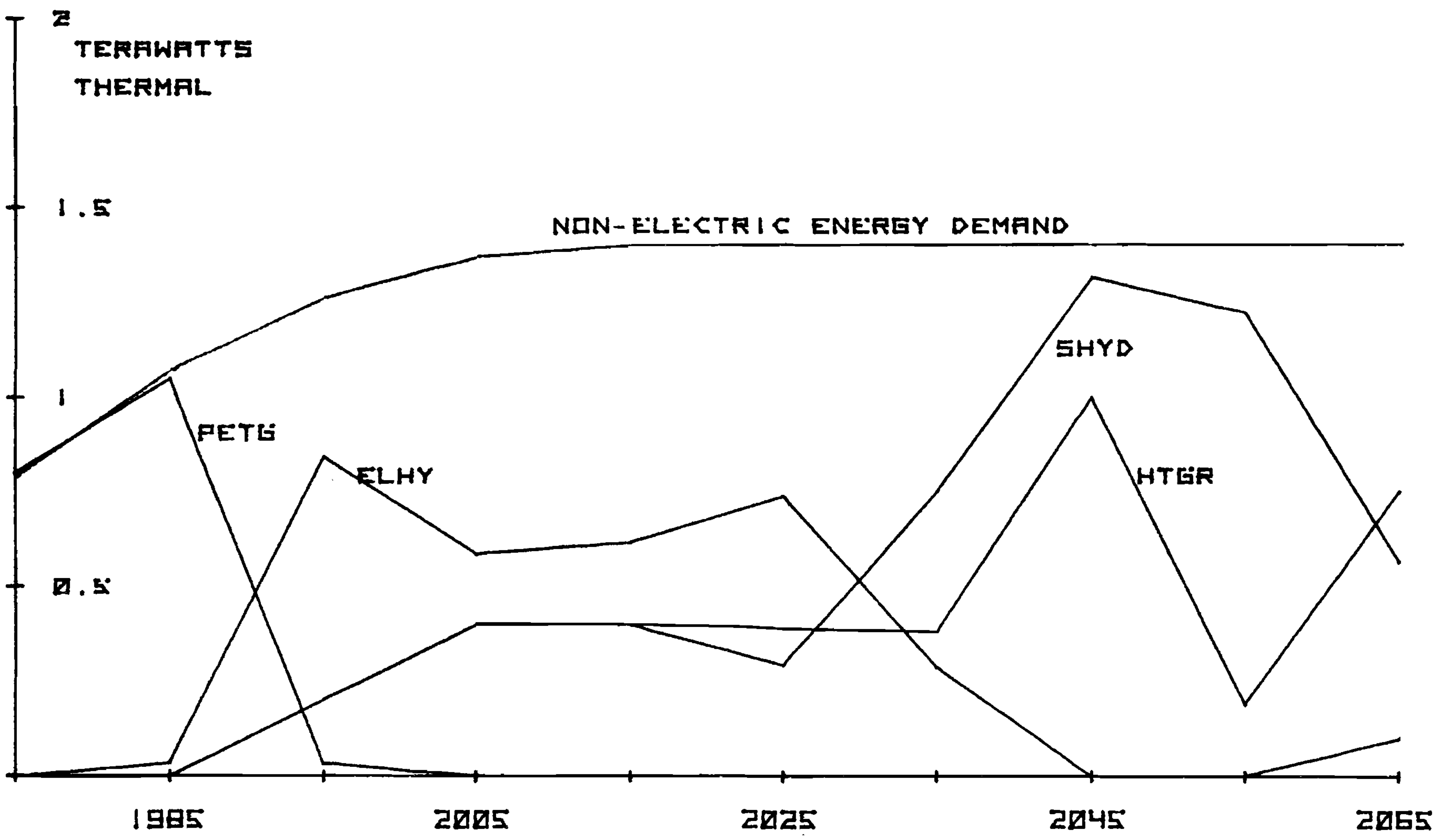

Figure C-8c. Model society 1.40: non-electric energy demand sector 7.

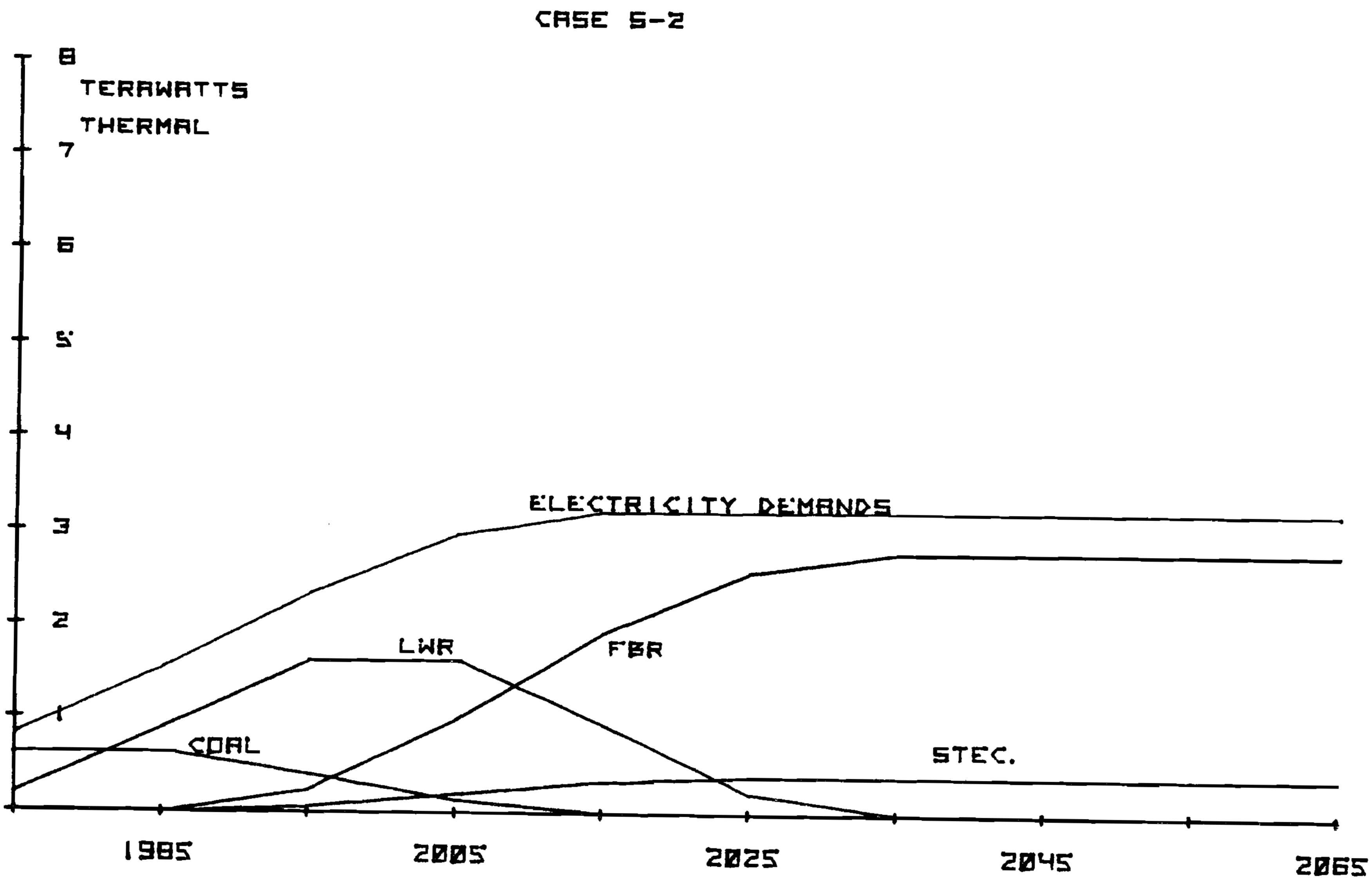

Figure C-9a. Model society 1.80: electric energy demands and supplies.

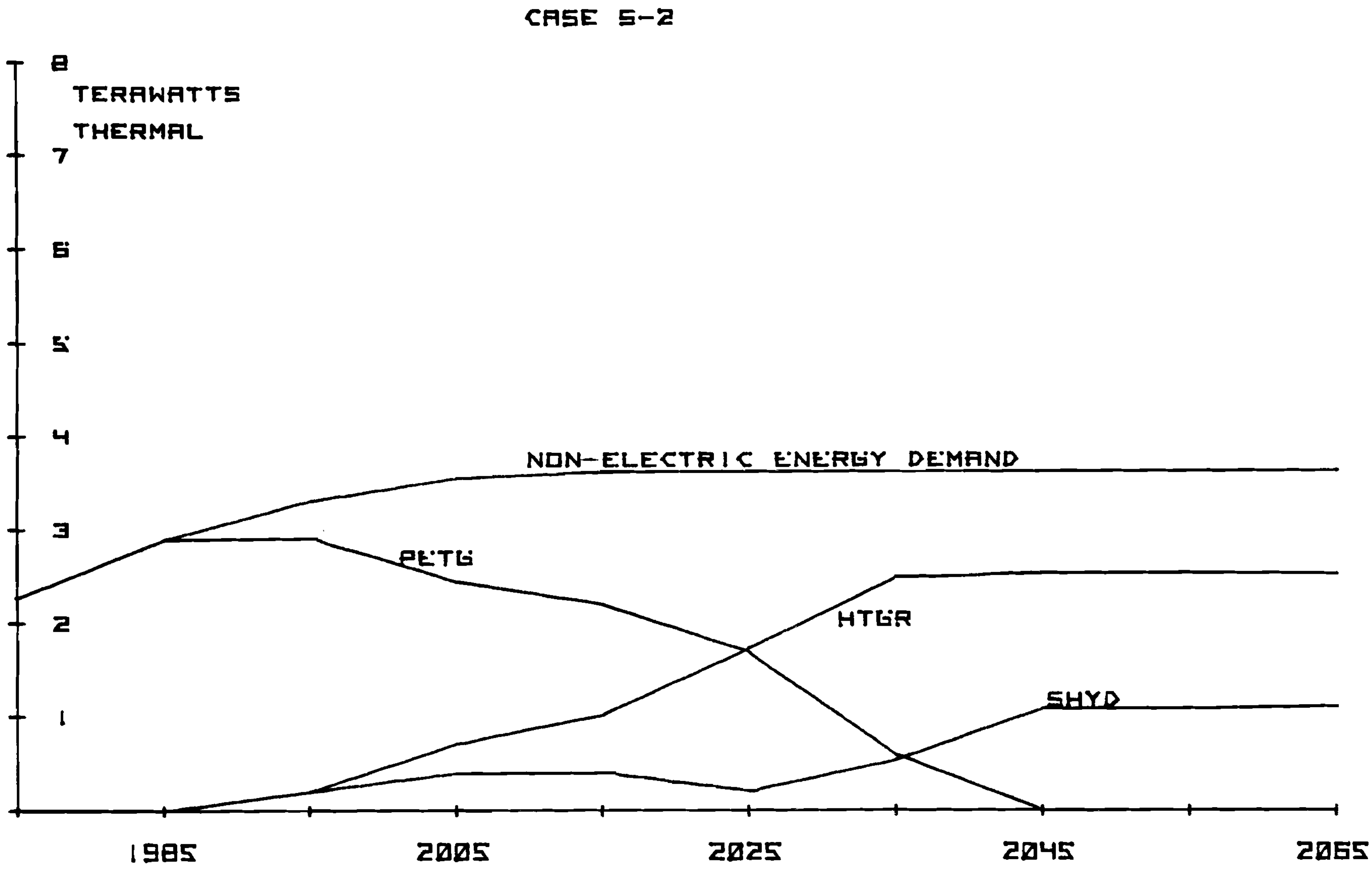

Figure C-9b. Model society 1.80: non-electric energy demands and supplies.

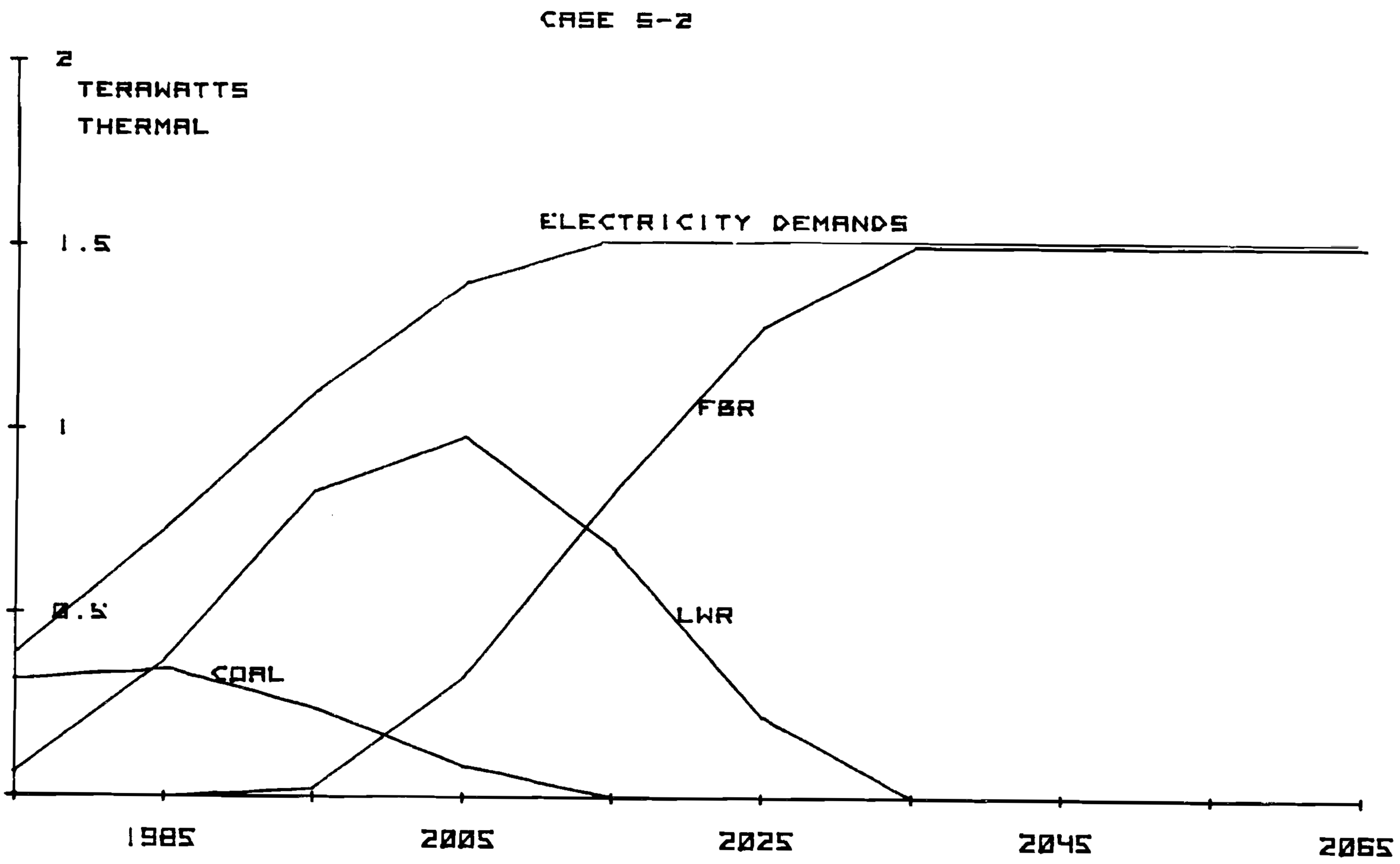

Figure C-10a. Model society 1.80: electric energy demand sector 1.

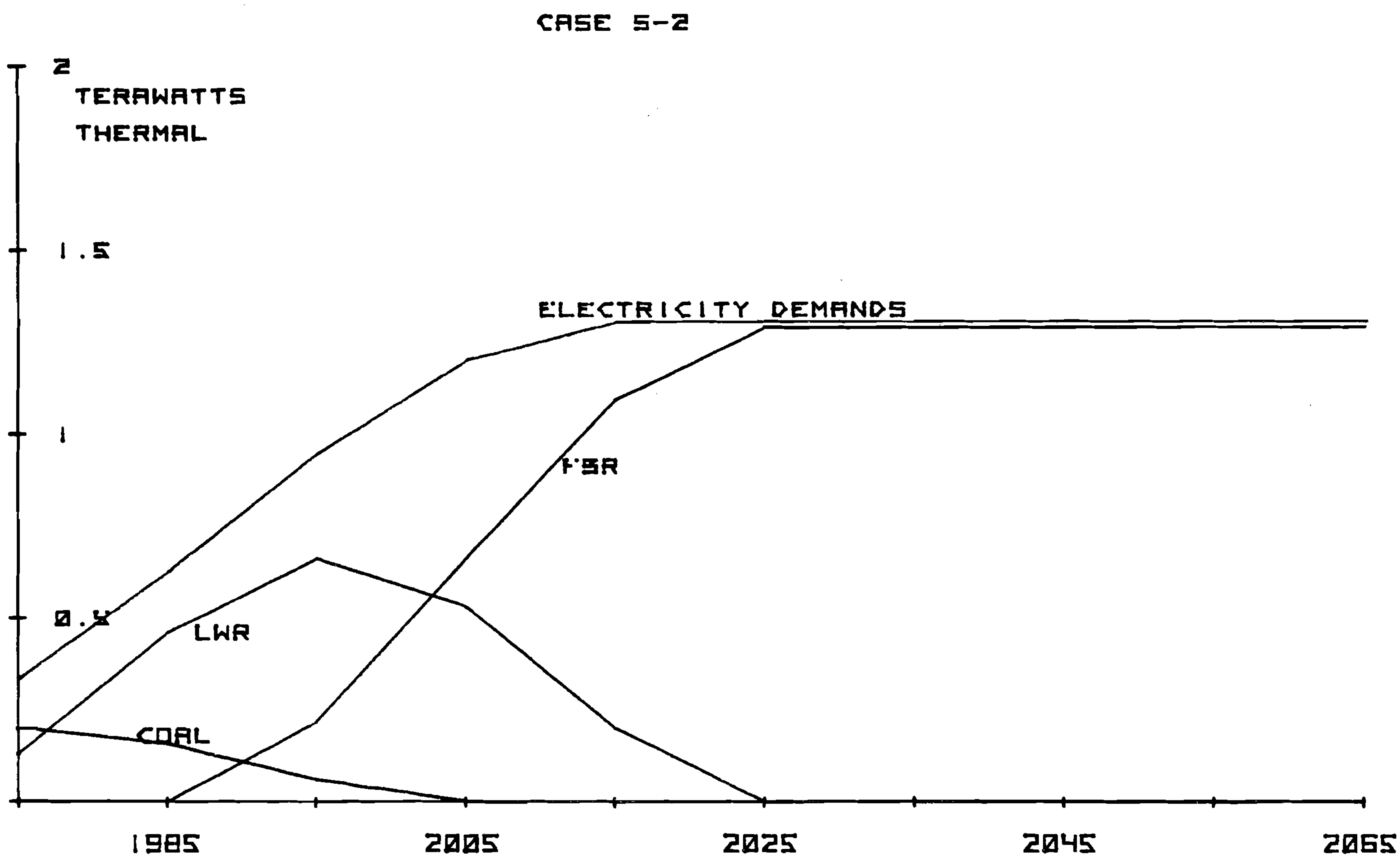

Figure C-10b. Model society 1.80: electric energy demand sector 4.

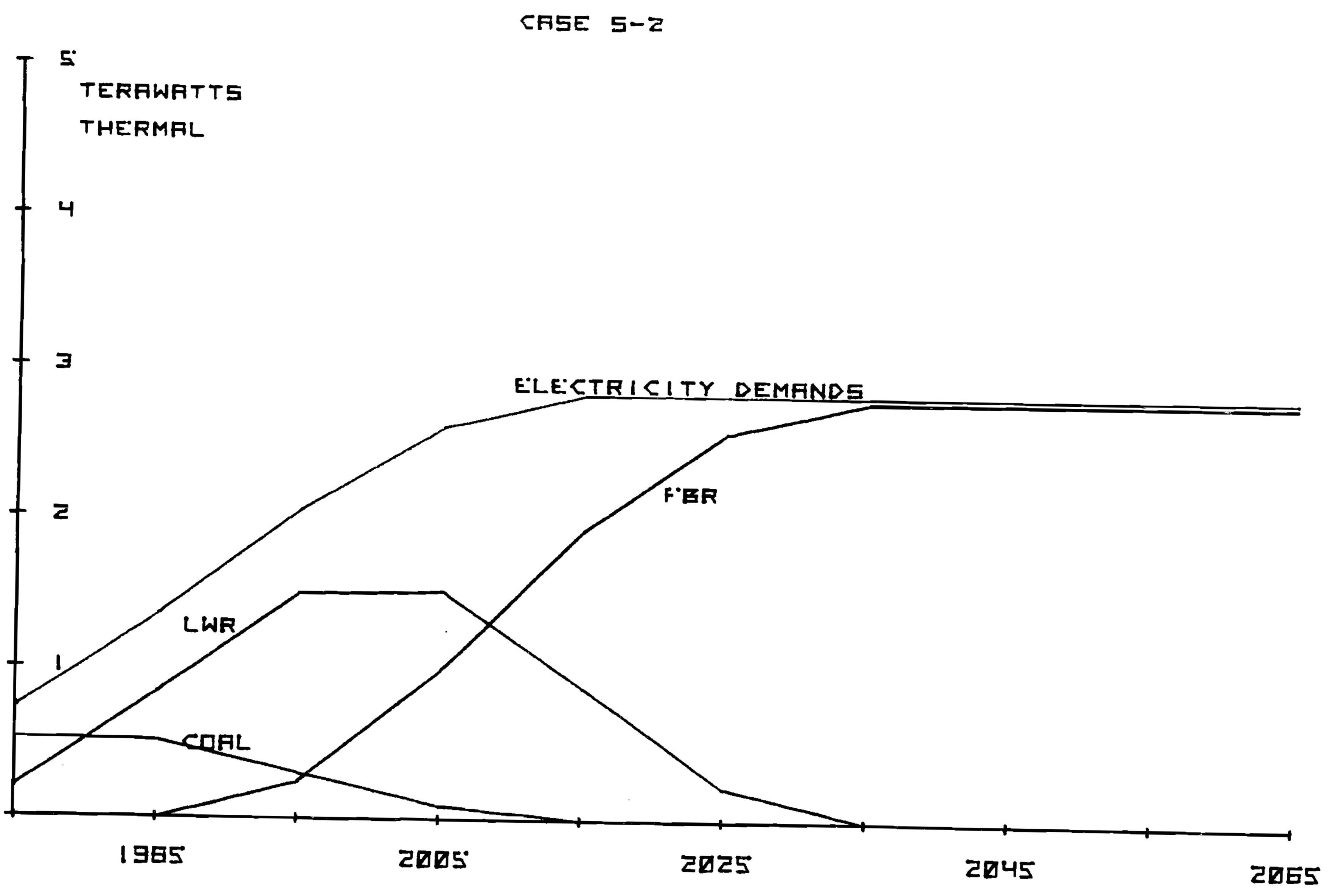

Figure C-10c. Model society 1.80: electric energy (base load) demands and supplies.

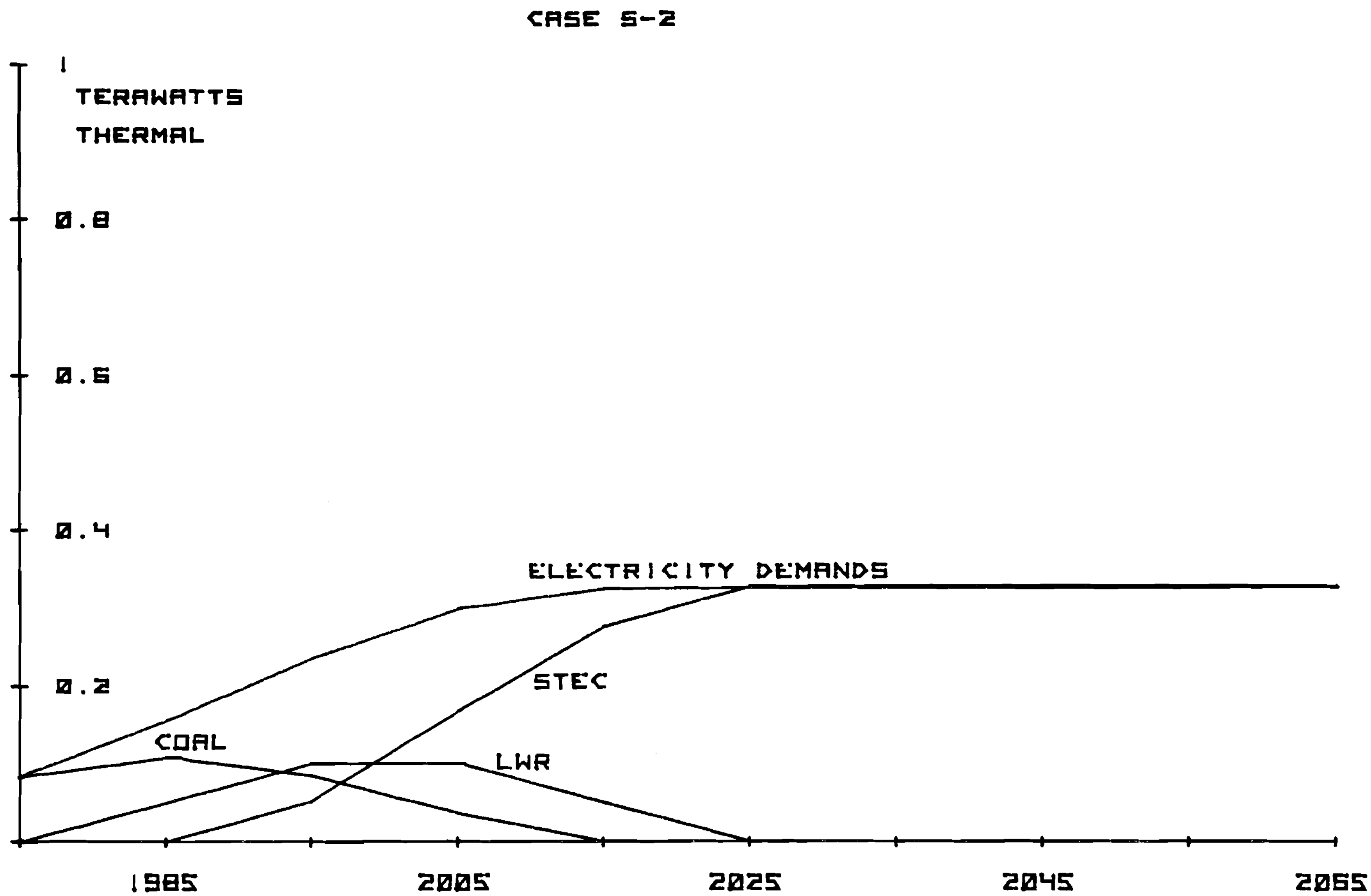

Figure C-11a. Model society 1.80: electric energy demand sector 2.

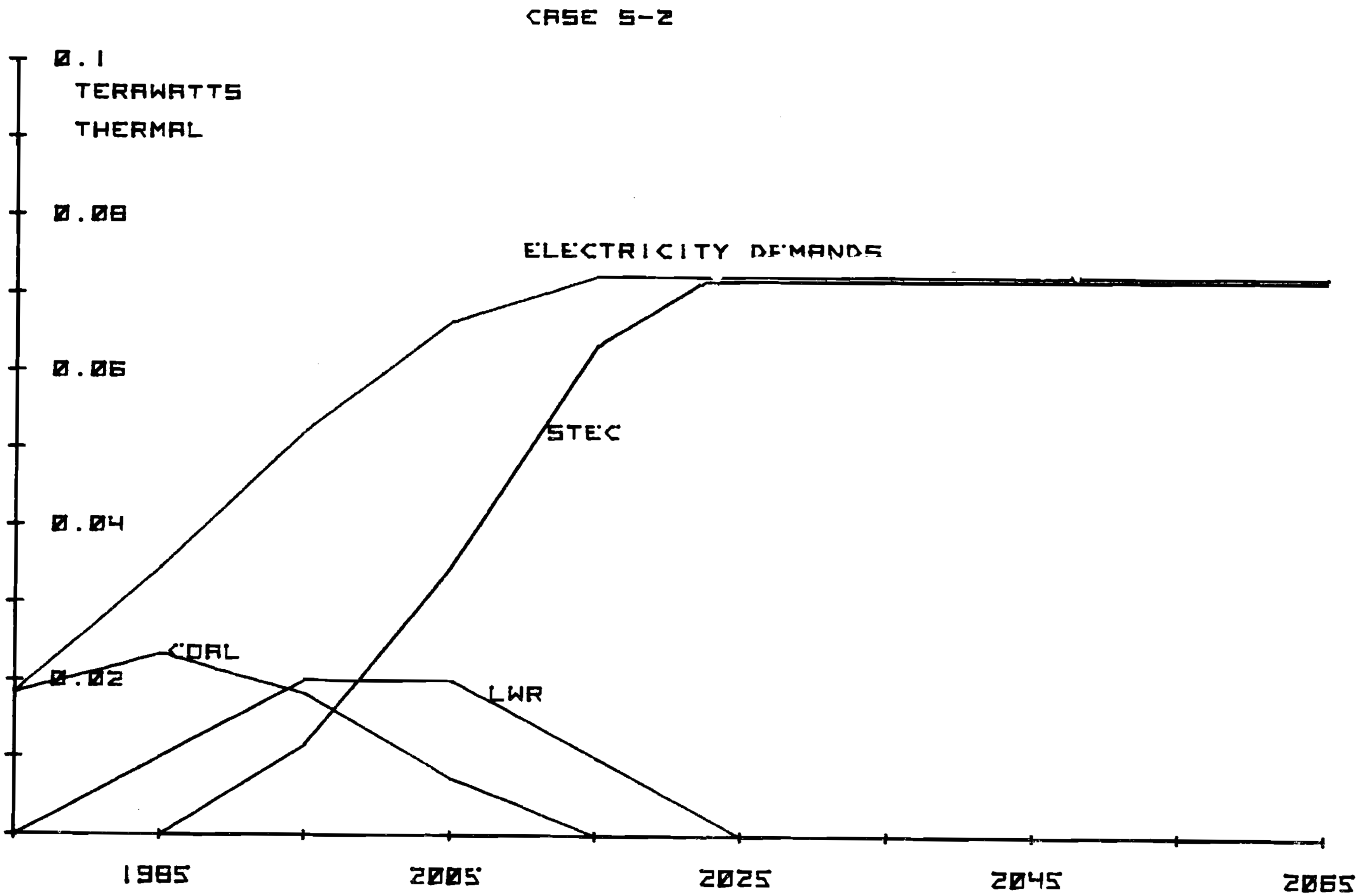

Figure C-11b. Model society 1.80: electric energy demand sector 5.

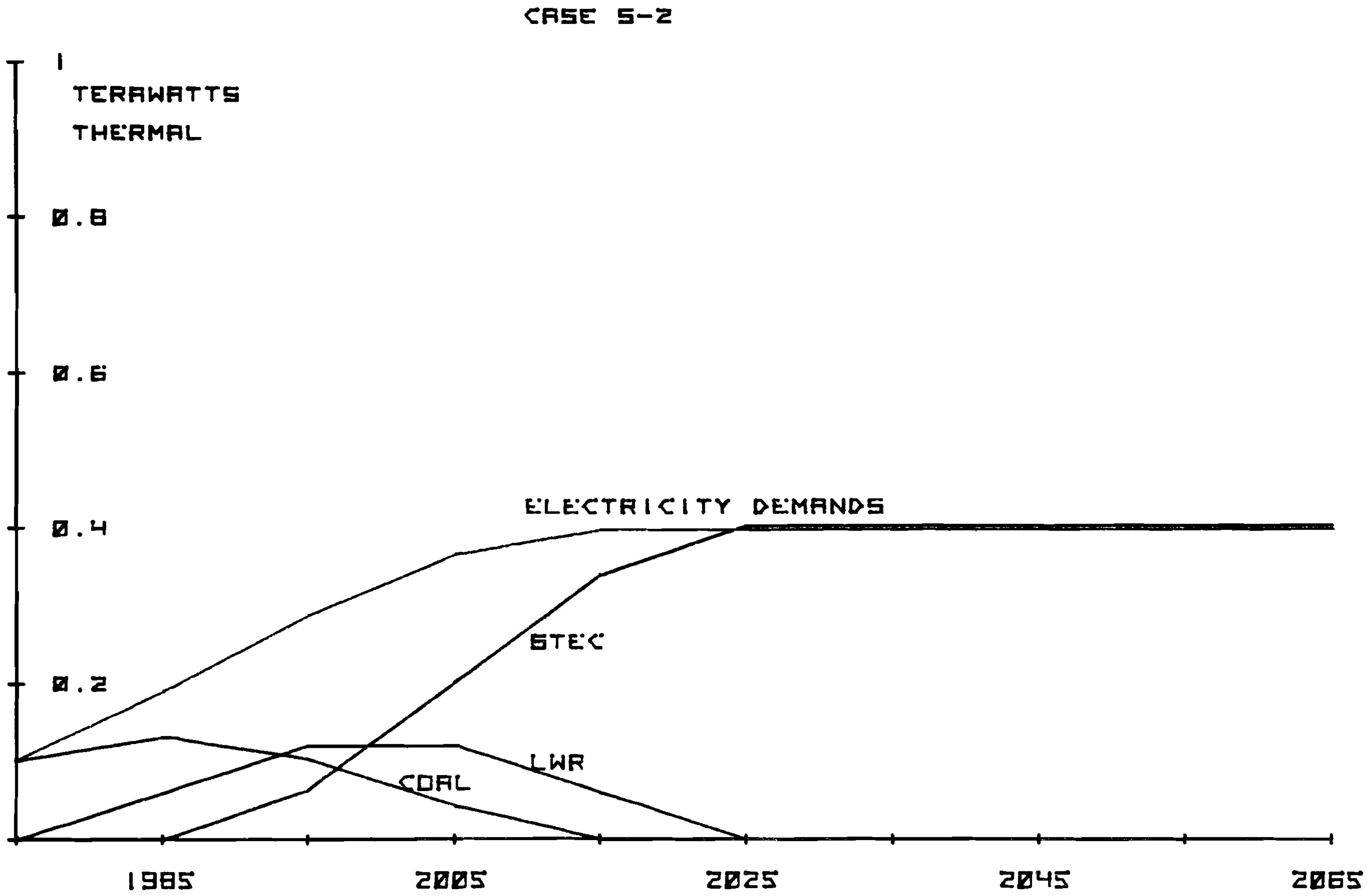

Figure C-11c. Model society 1.80: electric energy (peak load) demands and supplies.

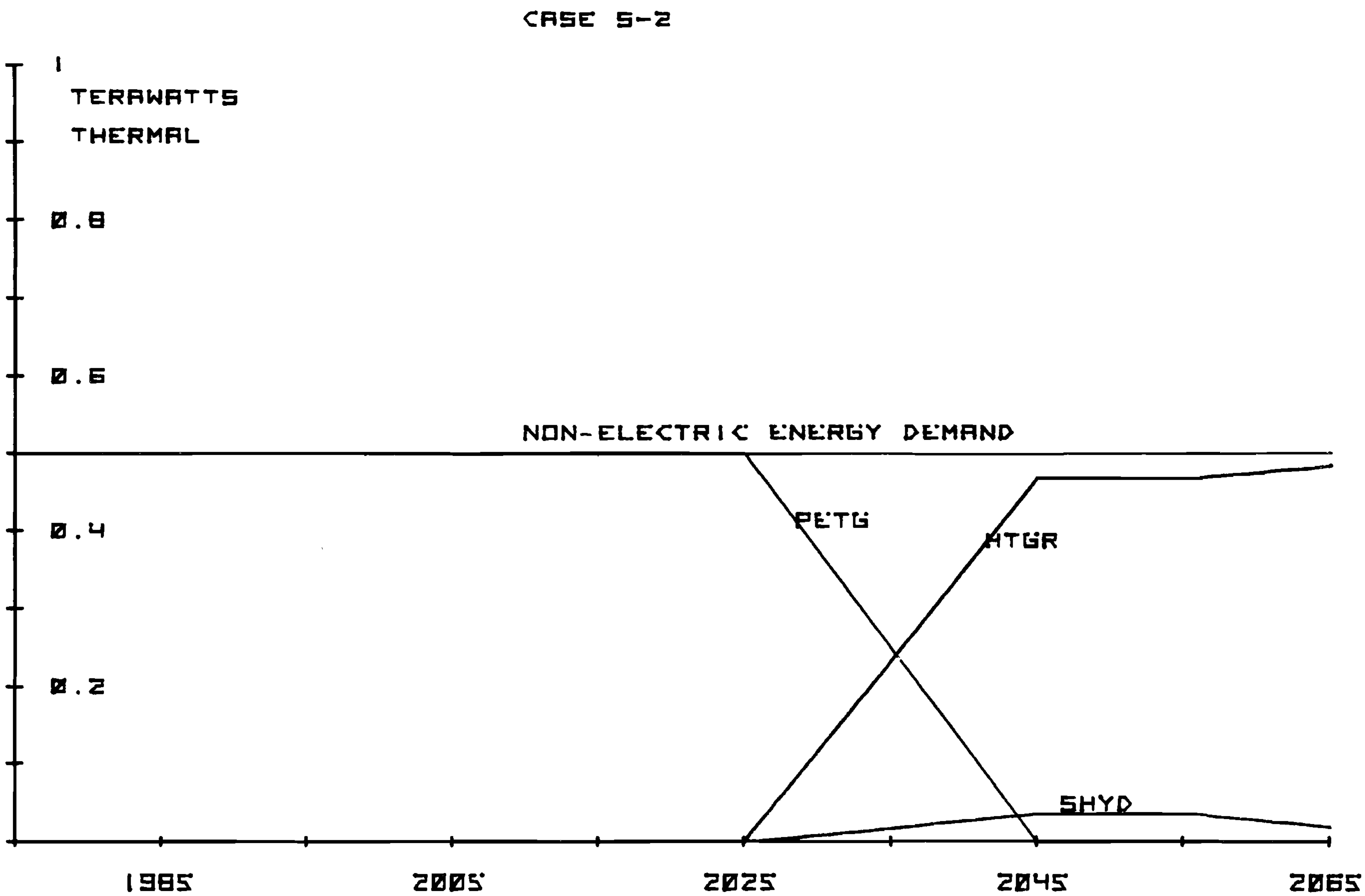

Figure C-12a. Model society 1.80: non-electric energy demand sector 3.

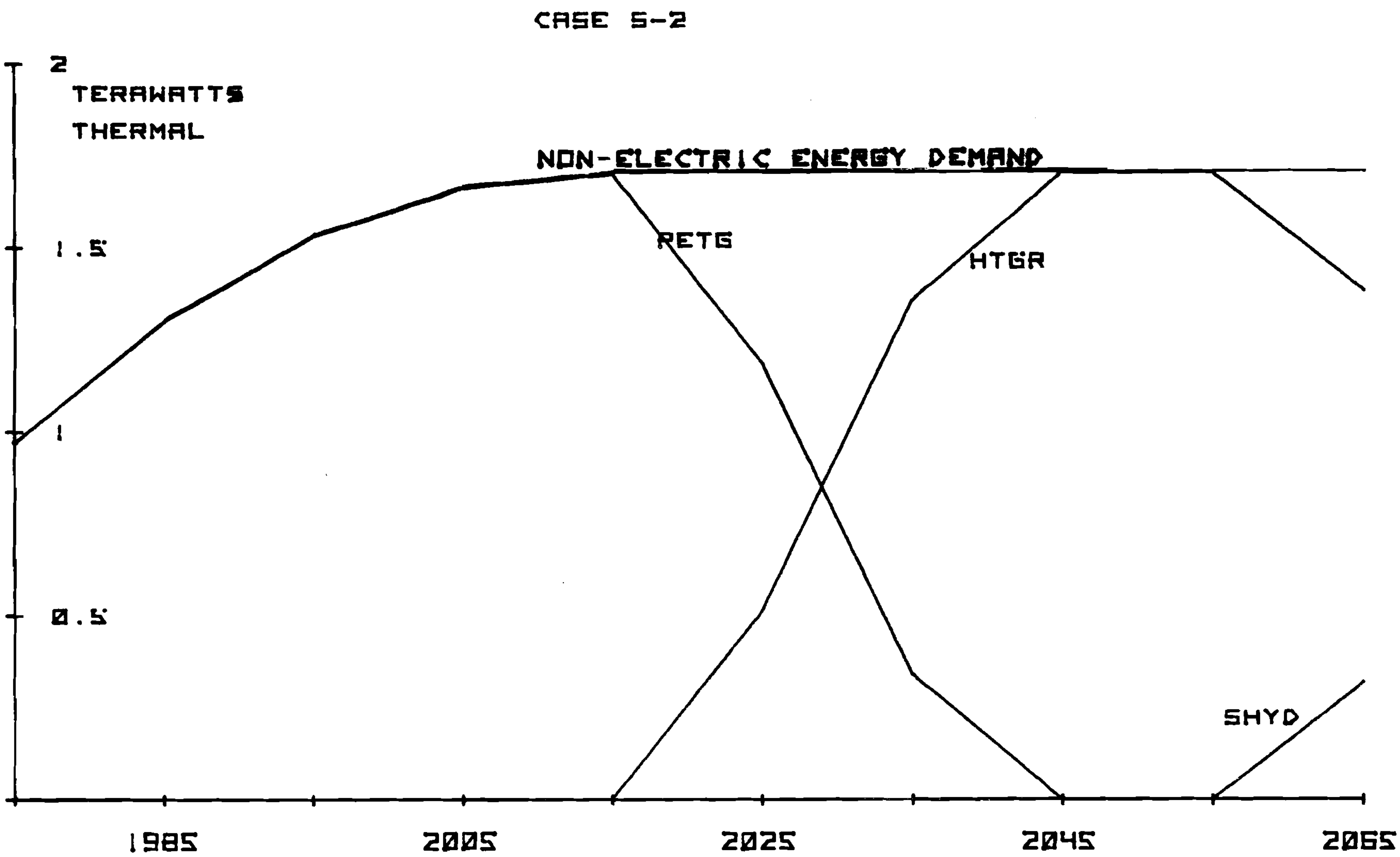

Figure C-12b. Model society 1.80: non-electric energy demand sector 6.

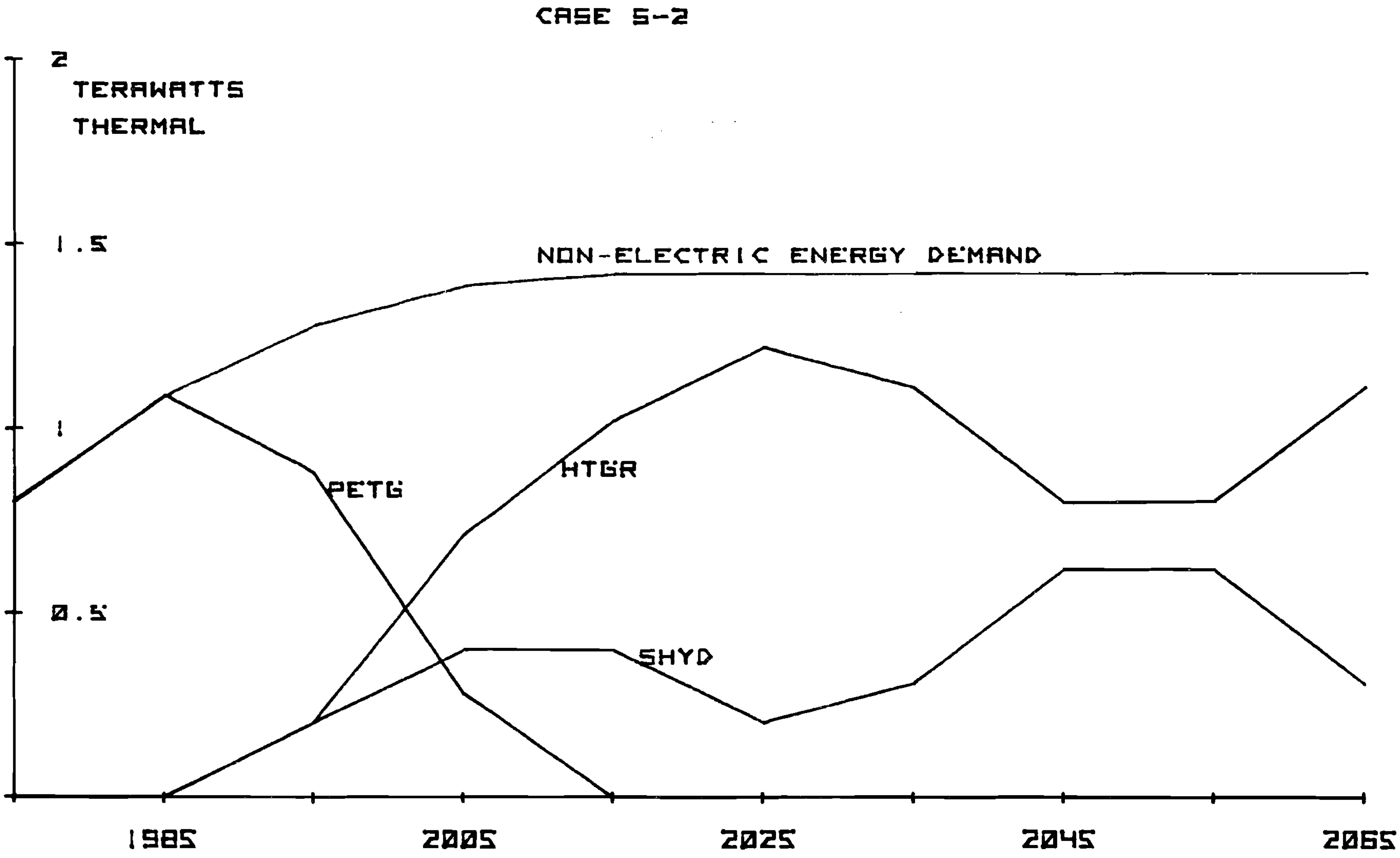

Figure C-12c. Model society 1.80: non-electric energy demand sector 7.

APPENDIX D

Results of Sensitivity Analysis on Fuel Costs of Coal-Fired Power Plants

(1) Contribution of Coal Alternatives for Electricity Supply, Cases: B, C1, C2, C3, C4.

	Case
Figure D-1	1.40
D-2	1.60
D-3	1.80
D-4	1.100

(2) Optimal Energy Production Activities of Each of the Alternatives for Each of the Demand Categories, in Case C3.

	Case	Demand Category	
Figure D-5a	1.40	All Electricity	
5b		All Non-Electric Energy	
6a		Elec., Base Load,	Res. and Comm.
6b			Indus.
6c			Subtotal
7a		Elec., Intermed.,	Res. and Comm.
7b			Indus.
7c			Subtotal
8a		Non-Elec.,	Res. and Comm.
8b			Indus.
8c			Transportation
Figure D-9a	1.80	All Electricity	
9b		All Non-Electric Energy	
10a		Elec., Base Load,	Res. and Comm.
10b			Indus.
10c			Subtotal
11a		Elec., Intermed.,	Res. and Comm.
11b			Indus.
11c			Subtotal
12a		Non-Elec.,	Res. and Comm.
12b			Indus.
12c			Transportation

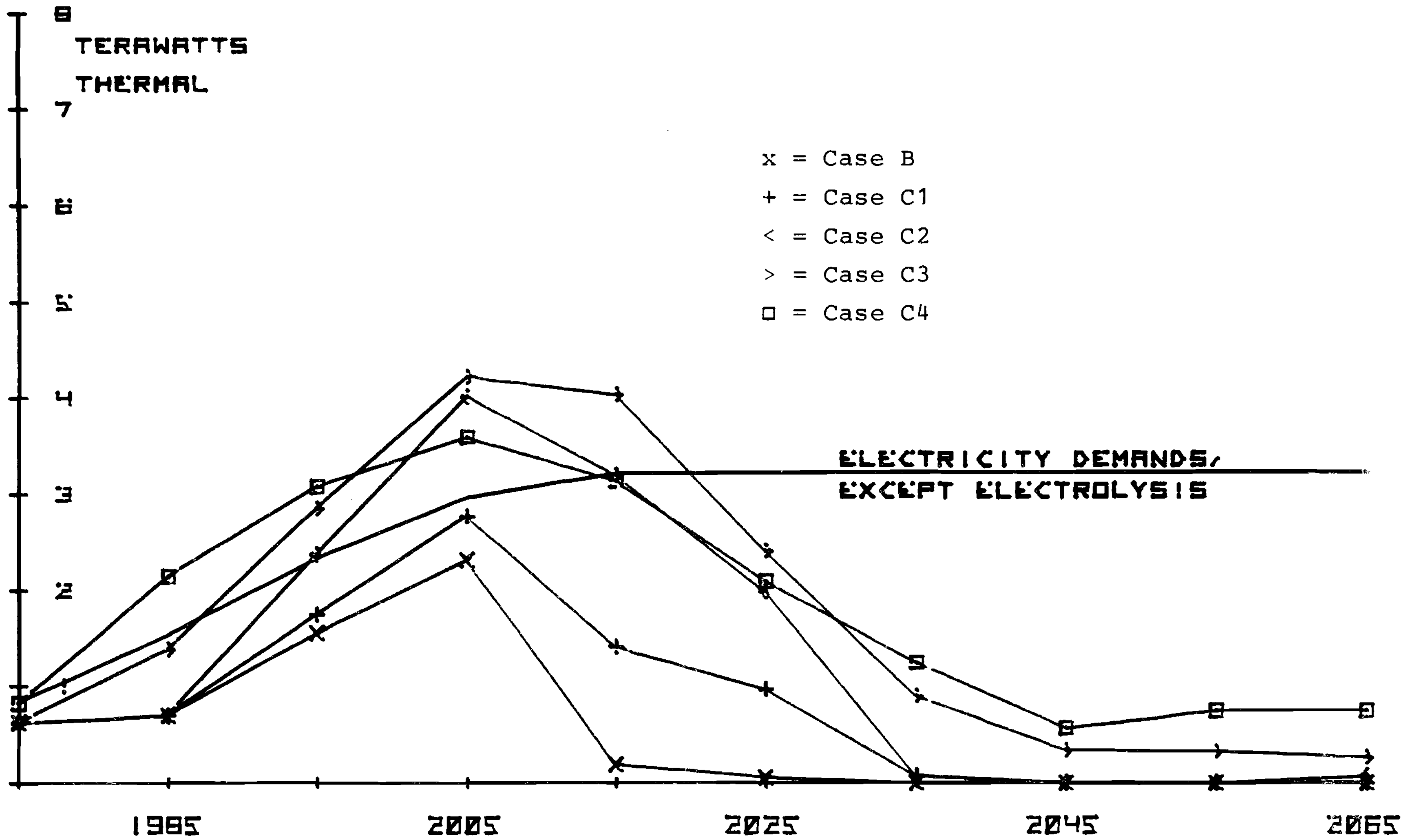

Figure D-1. Model society 1.40: coal plant electricity.

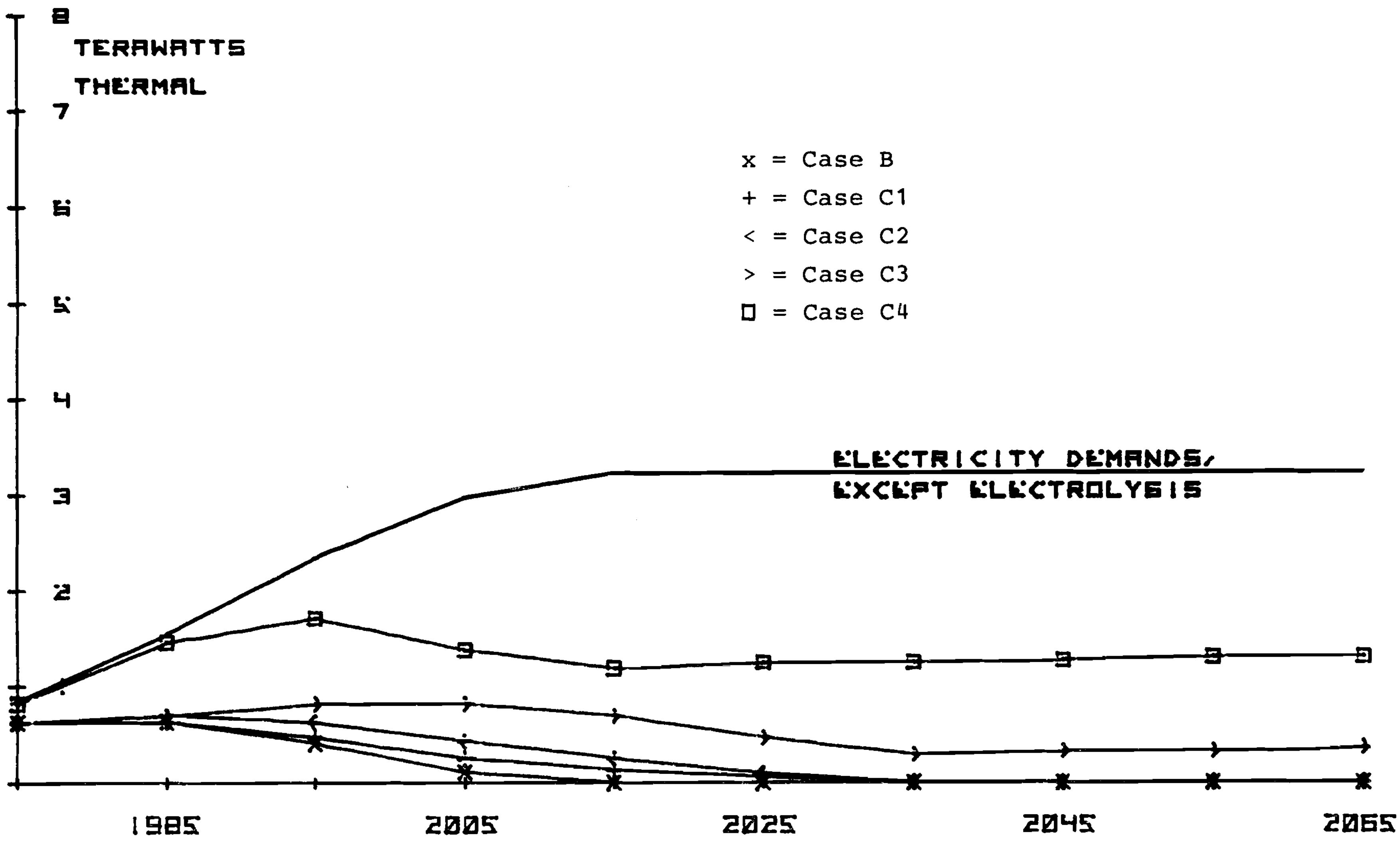

Figure D-2. Model society 1.60: coal plant electricity.

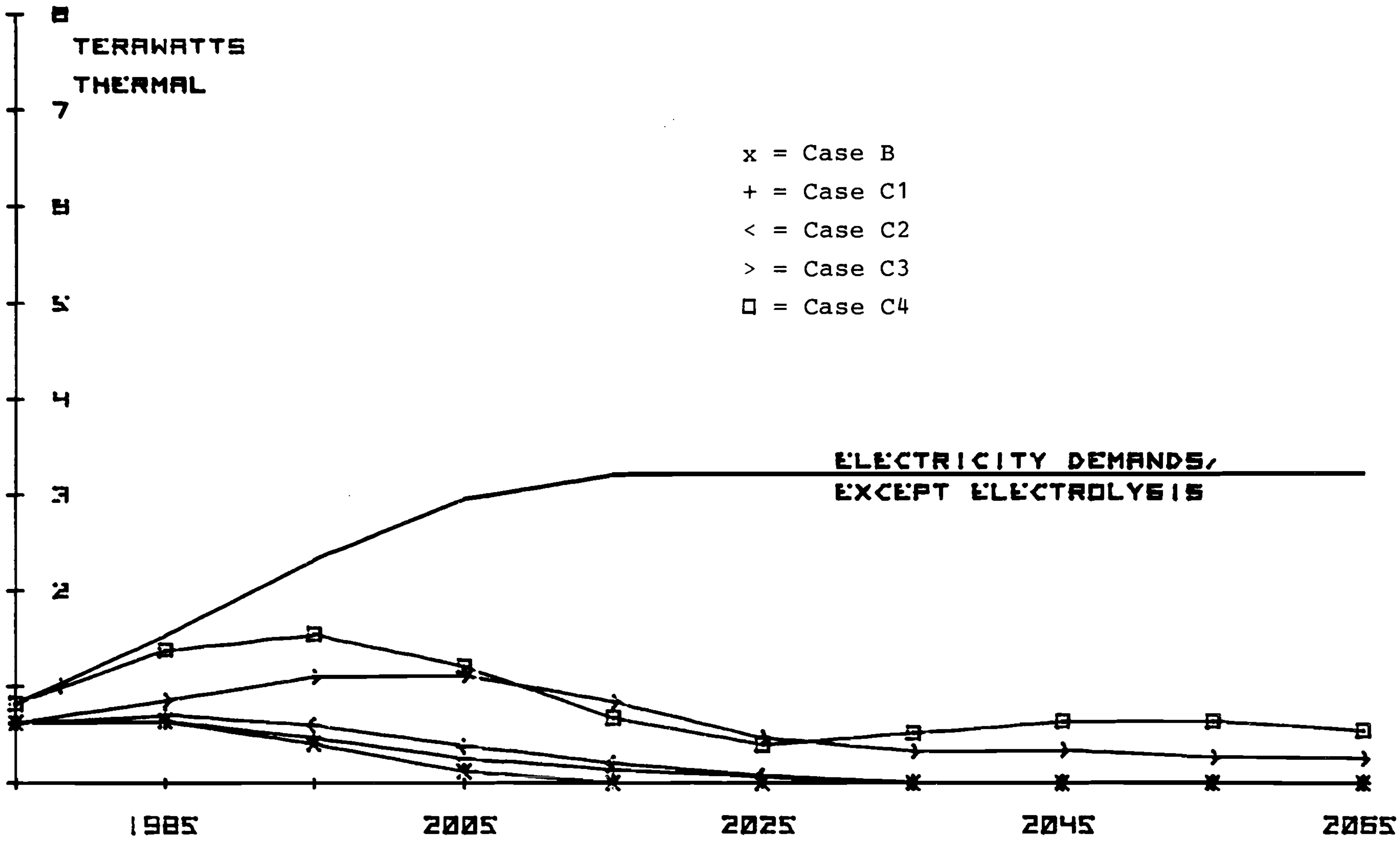

Figure D-3. Model society 1.80: coal plant electricity.

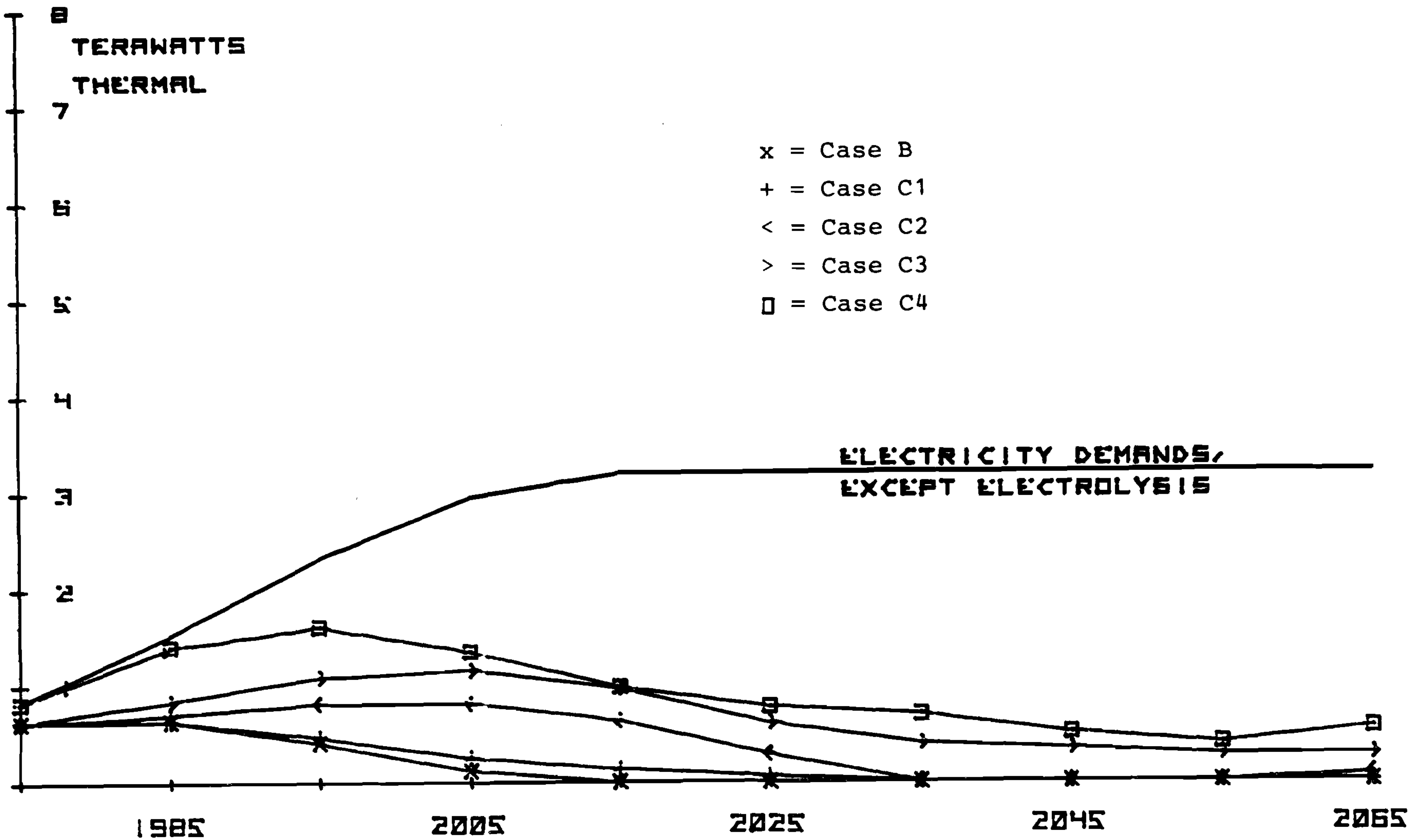

Figure D-4. Model society 1.100: coal plant electricity.

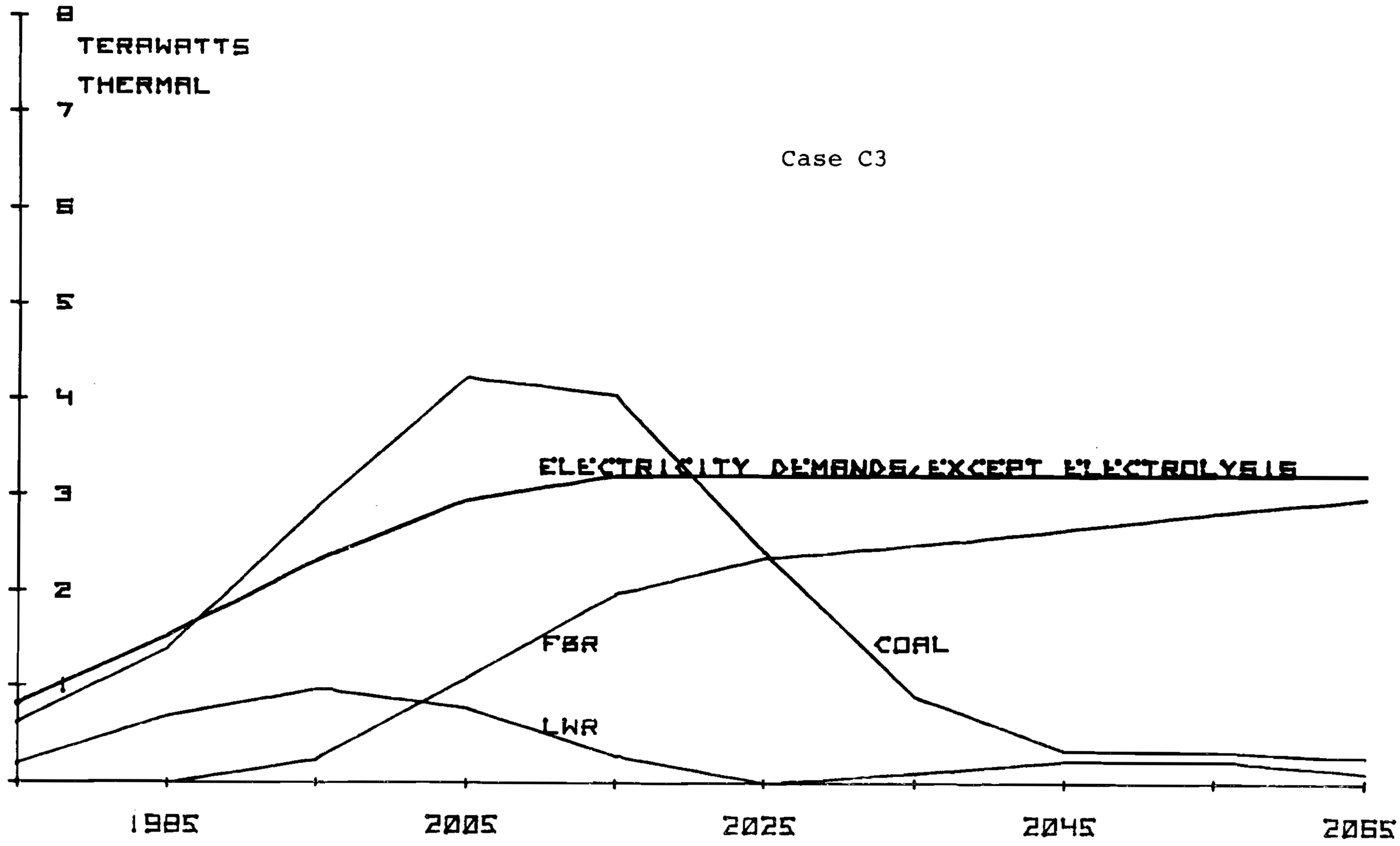

Figure D-5a. Model society 1.40: electric energy demands and supplies.

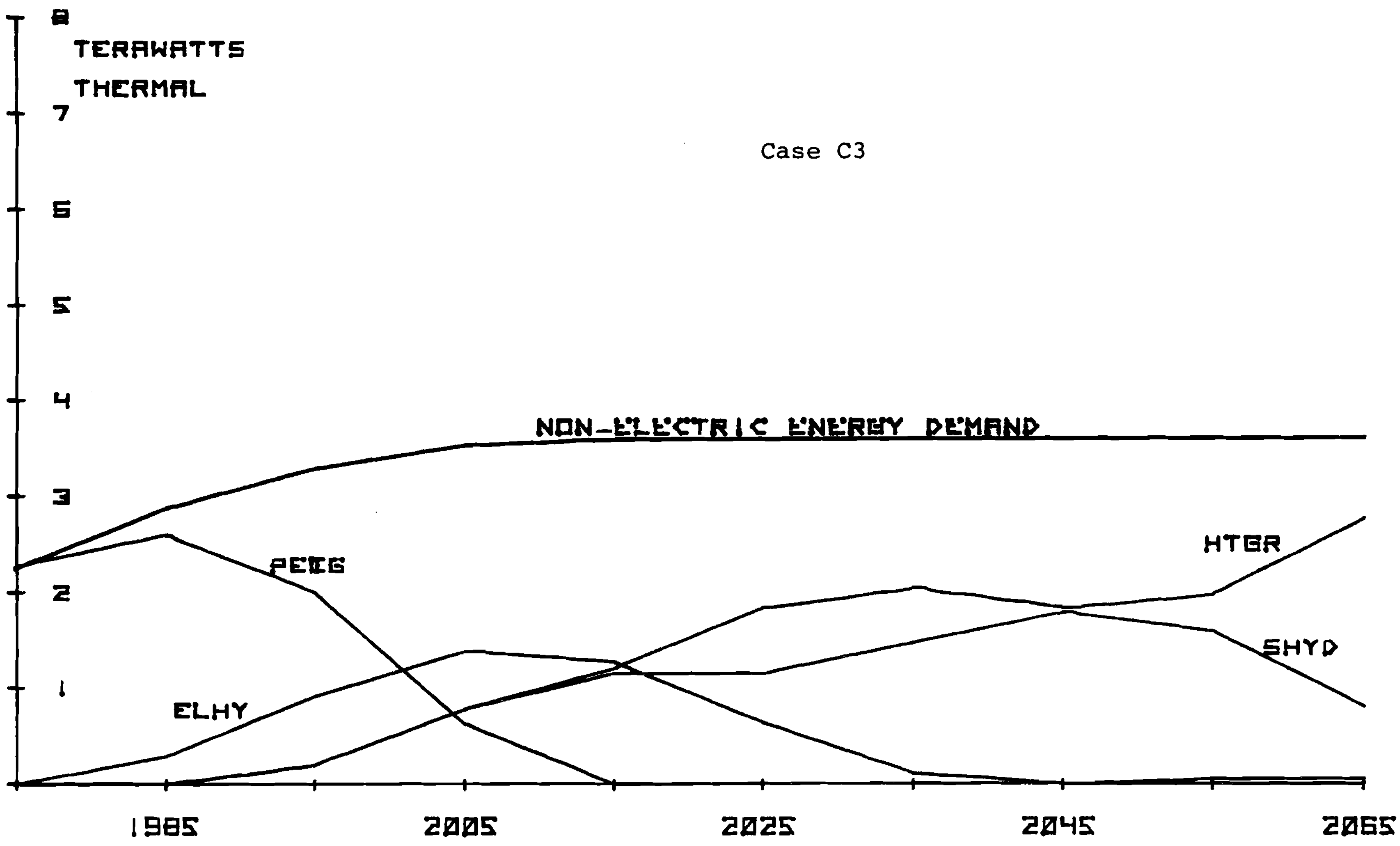

Figure D-5b. Model society 1.40: non-electric energy demands and supplies.

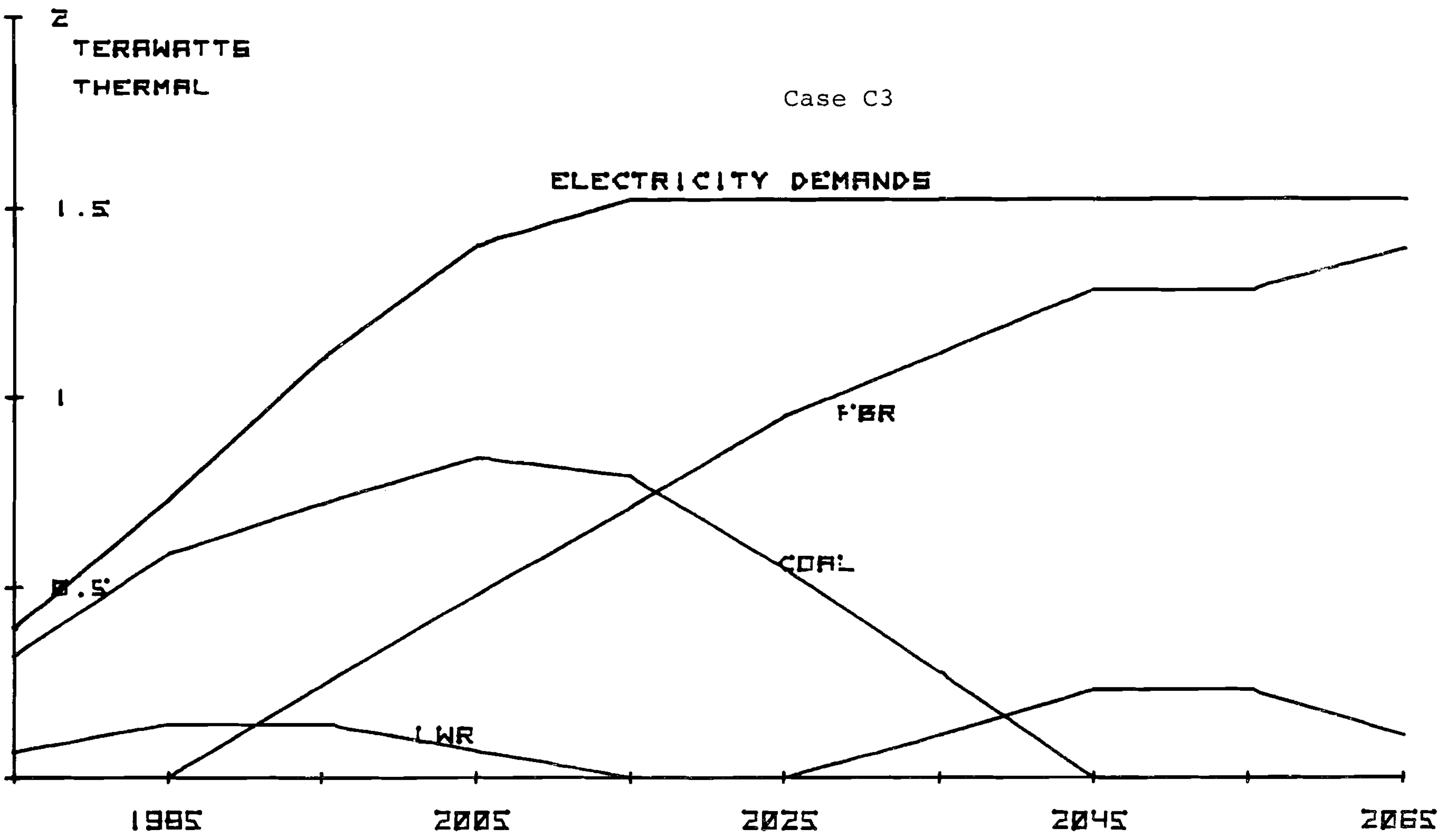

Figure D-6a. Model society 1.40: electric energy demand sector 1.

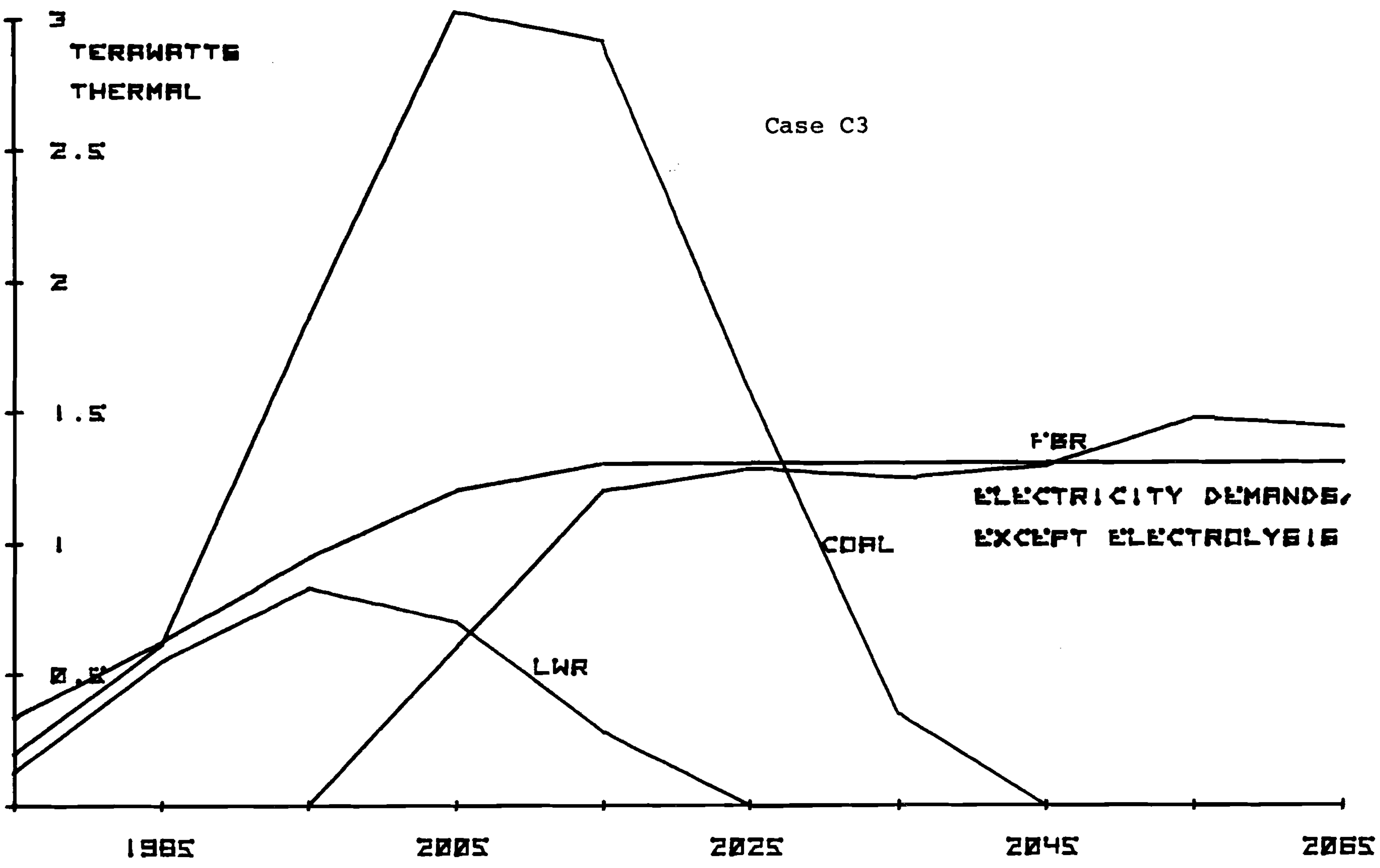

Figure D-6b. Model society 1.40: electric energy demand sector 4.

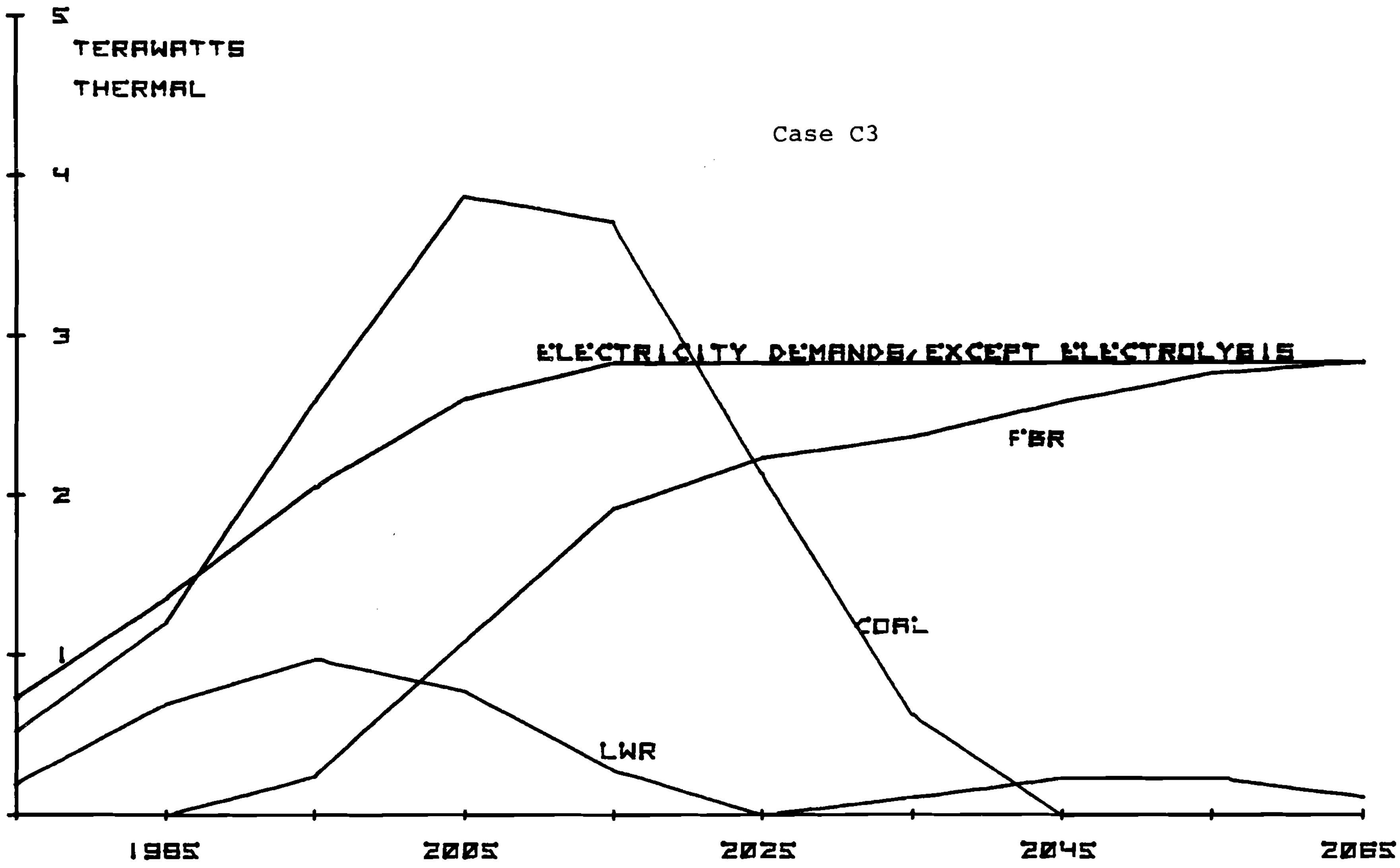

Figure D-6c. Model society 1.40: electric energy (base load) demands and supplies.

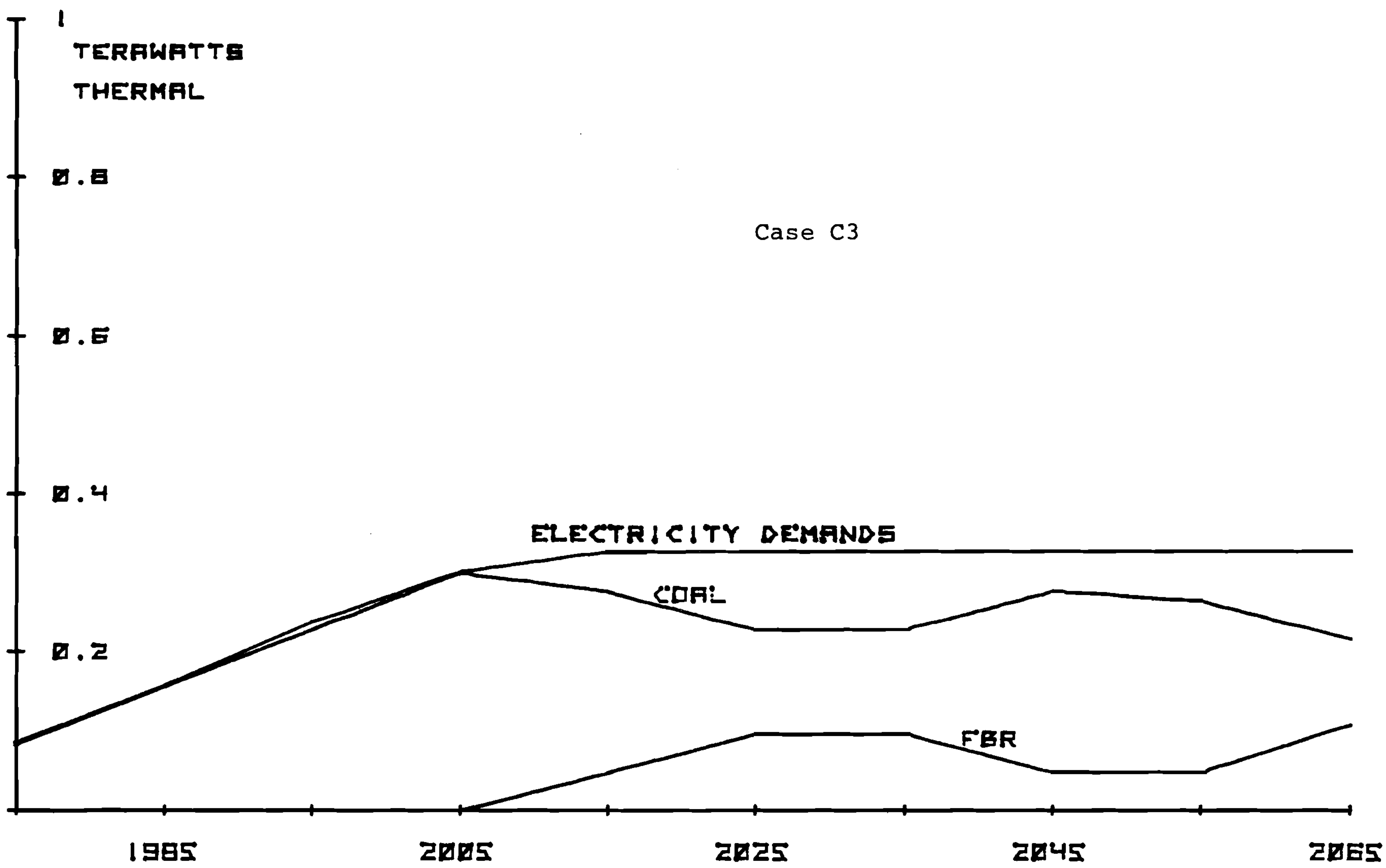

Figure D-7a. Model society 1.40: electric energy demand sector 2.

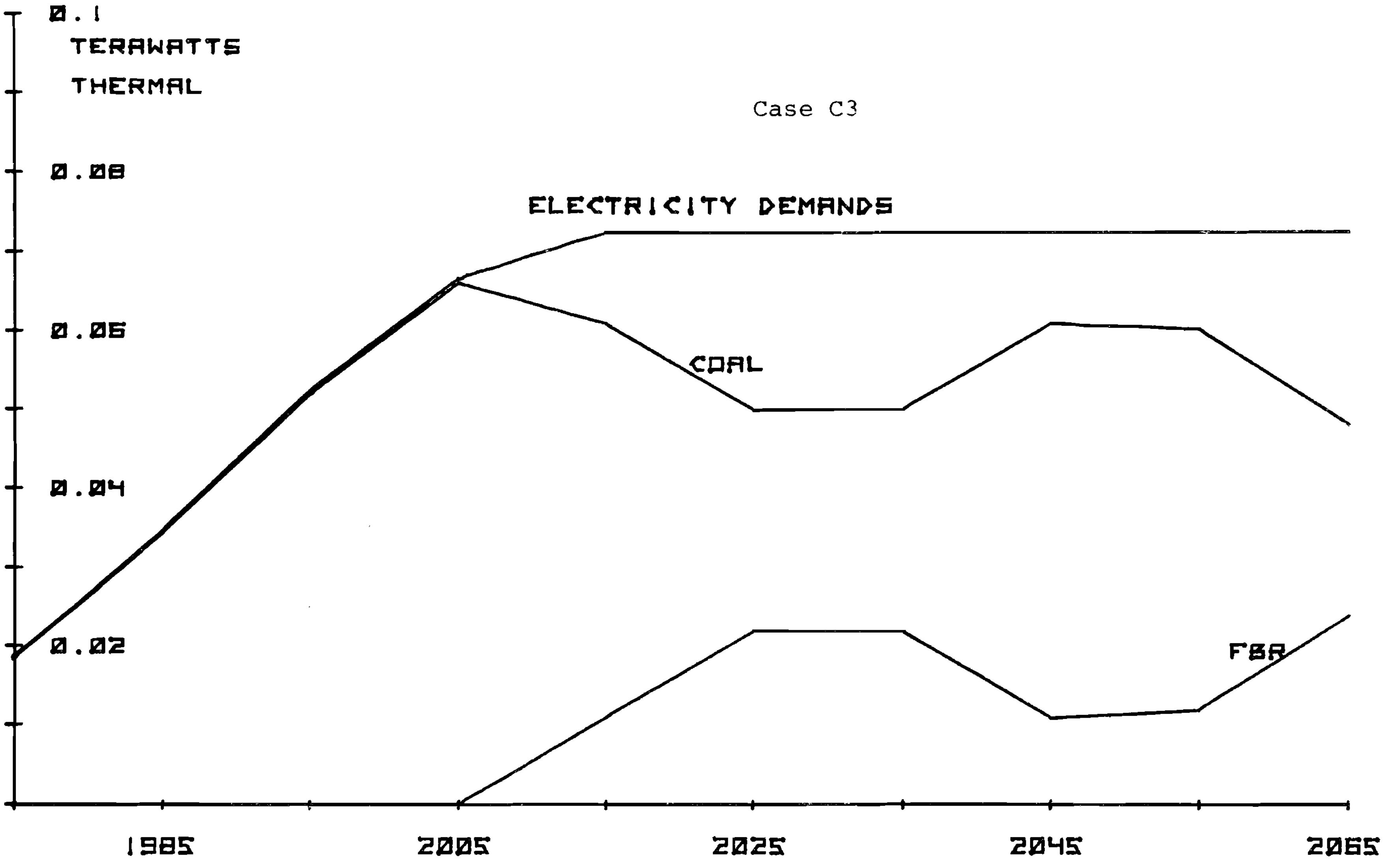

Figure D-7b. Model society 1.40: electric energy demand sector 5.

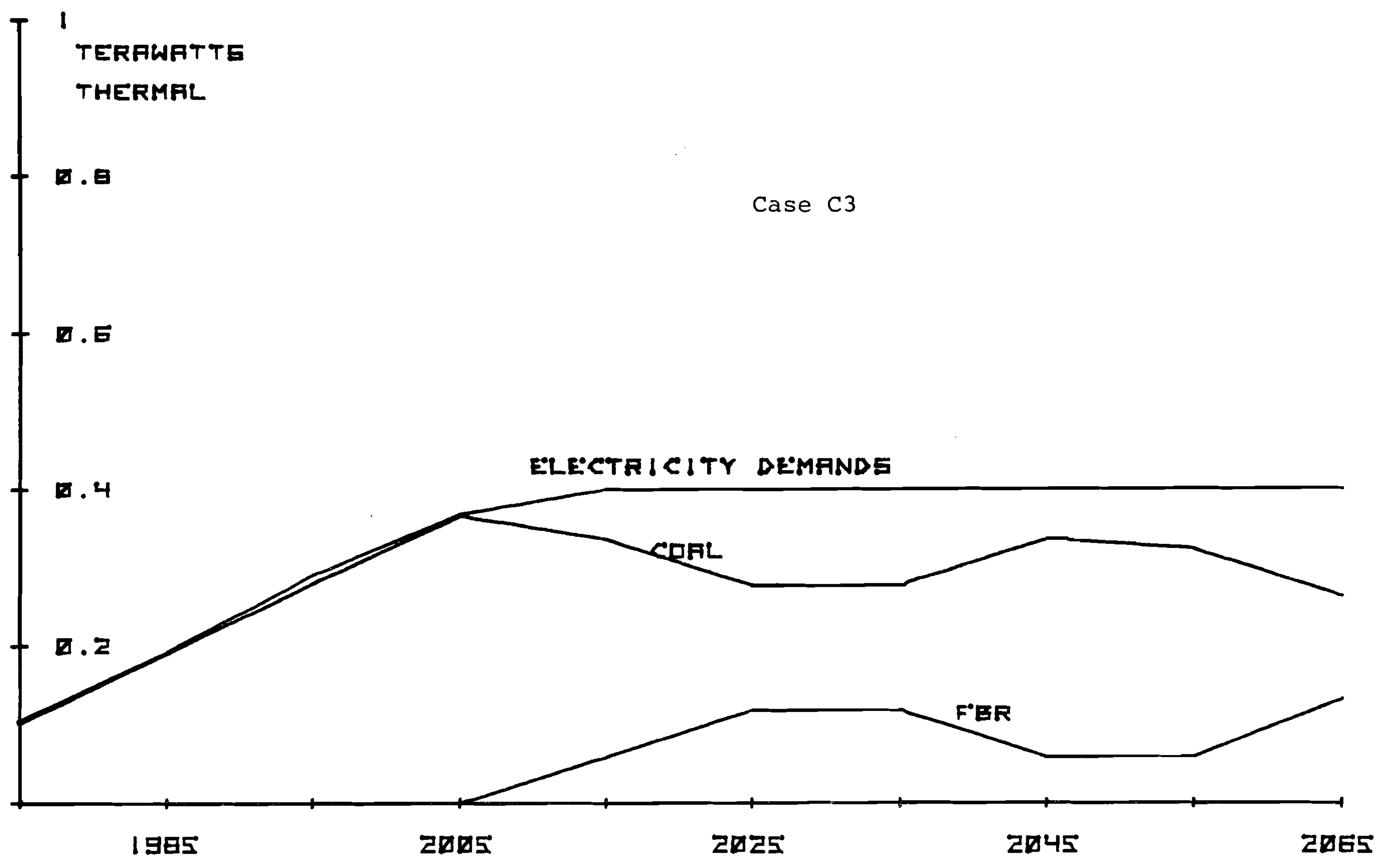

Figure D-7c. Model society 1.40: electric energy (peak load) demands and supplies.

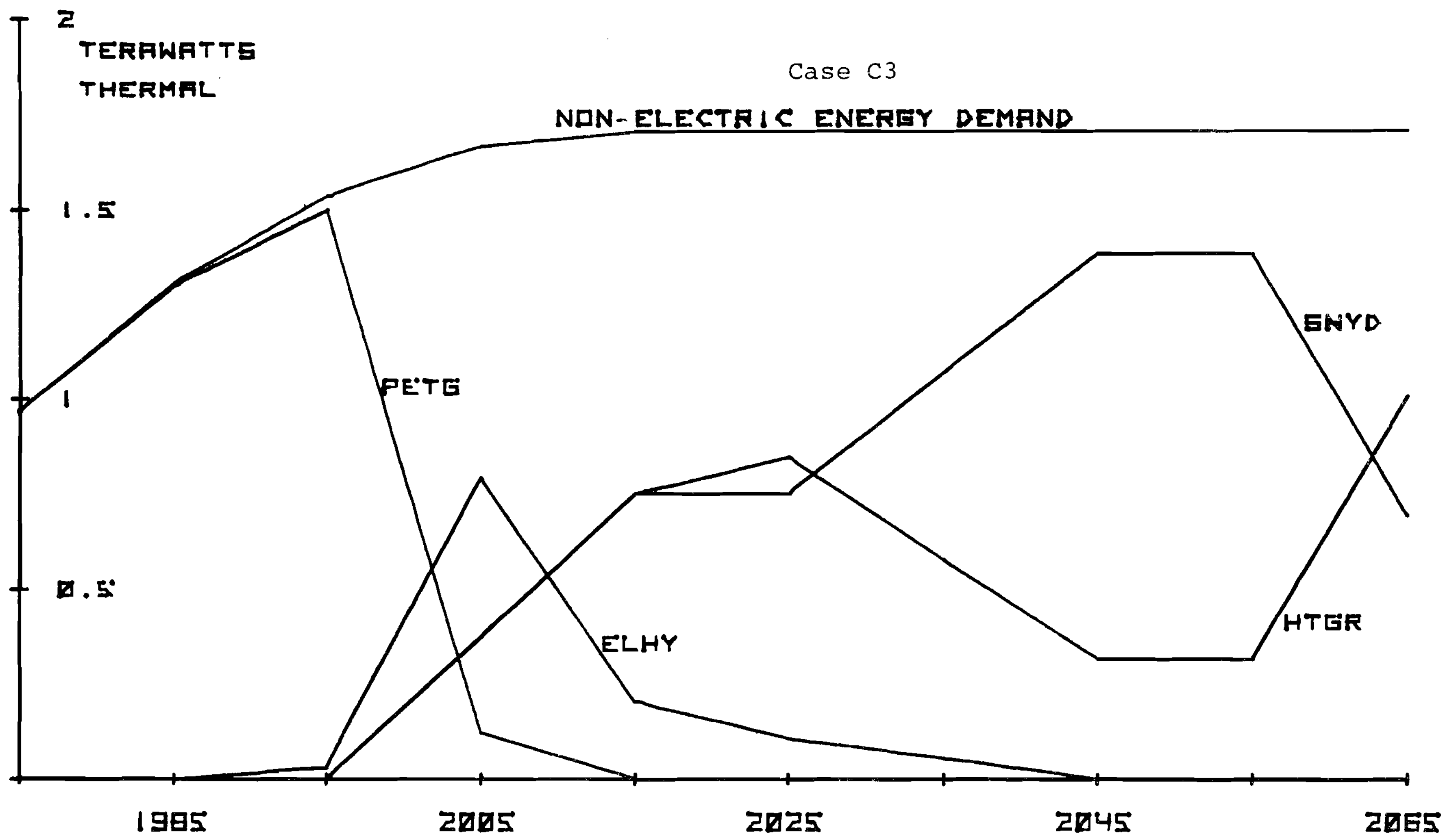

Figure D-8b. Model society 1.40: non-electric energy demand sector 6.

Case C3

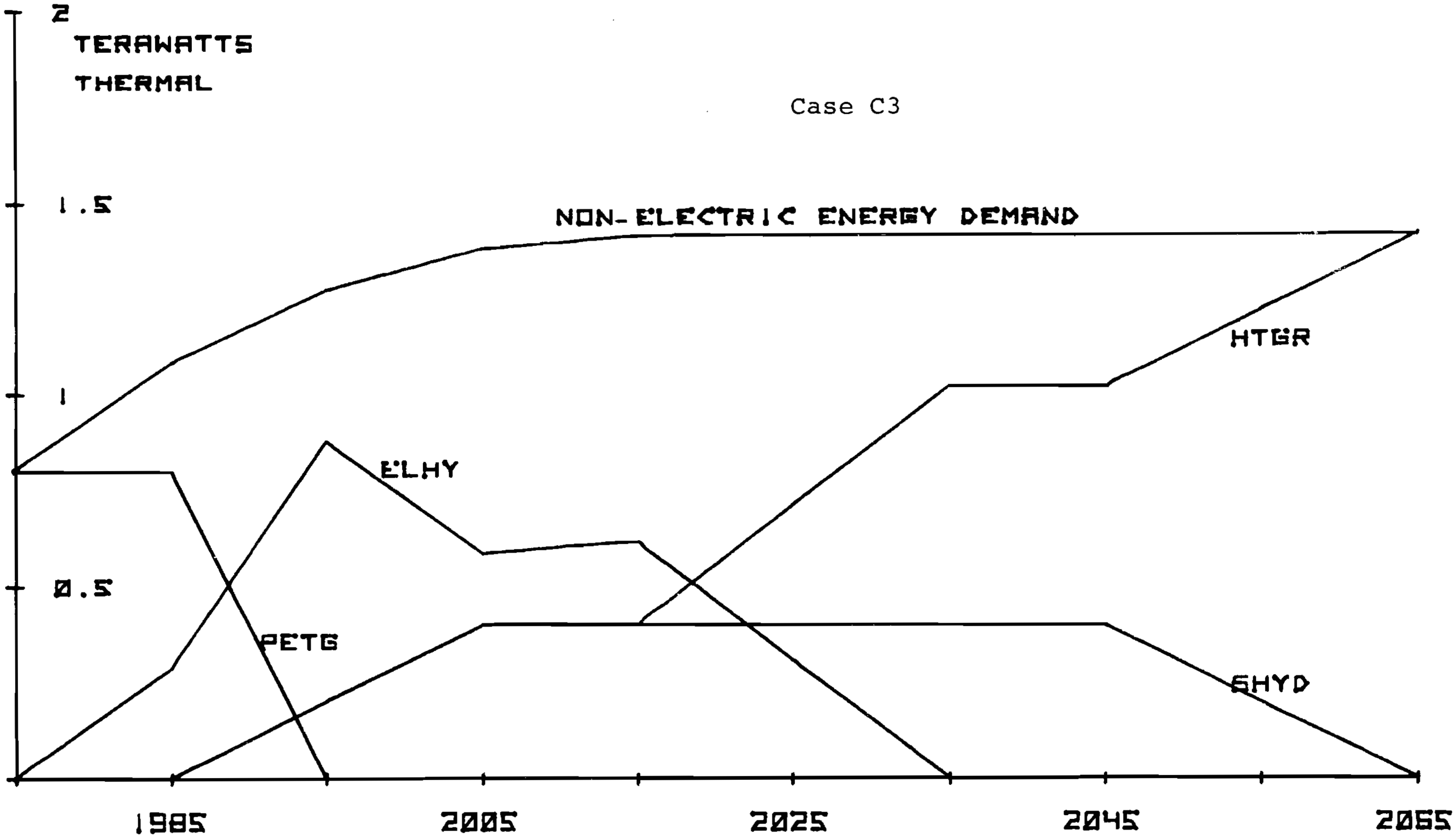

Figure D-8c. Model society 1.40: non-electric energy demand sector 7.

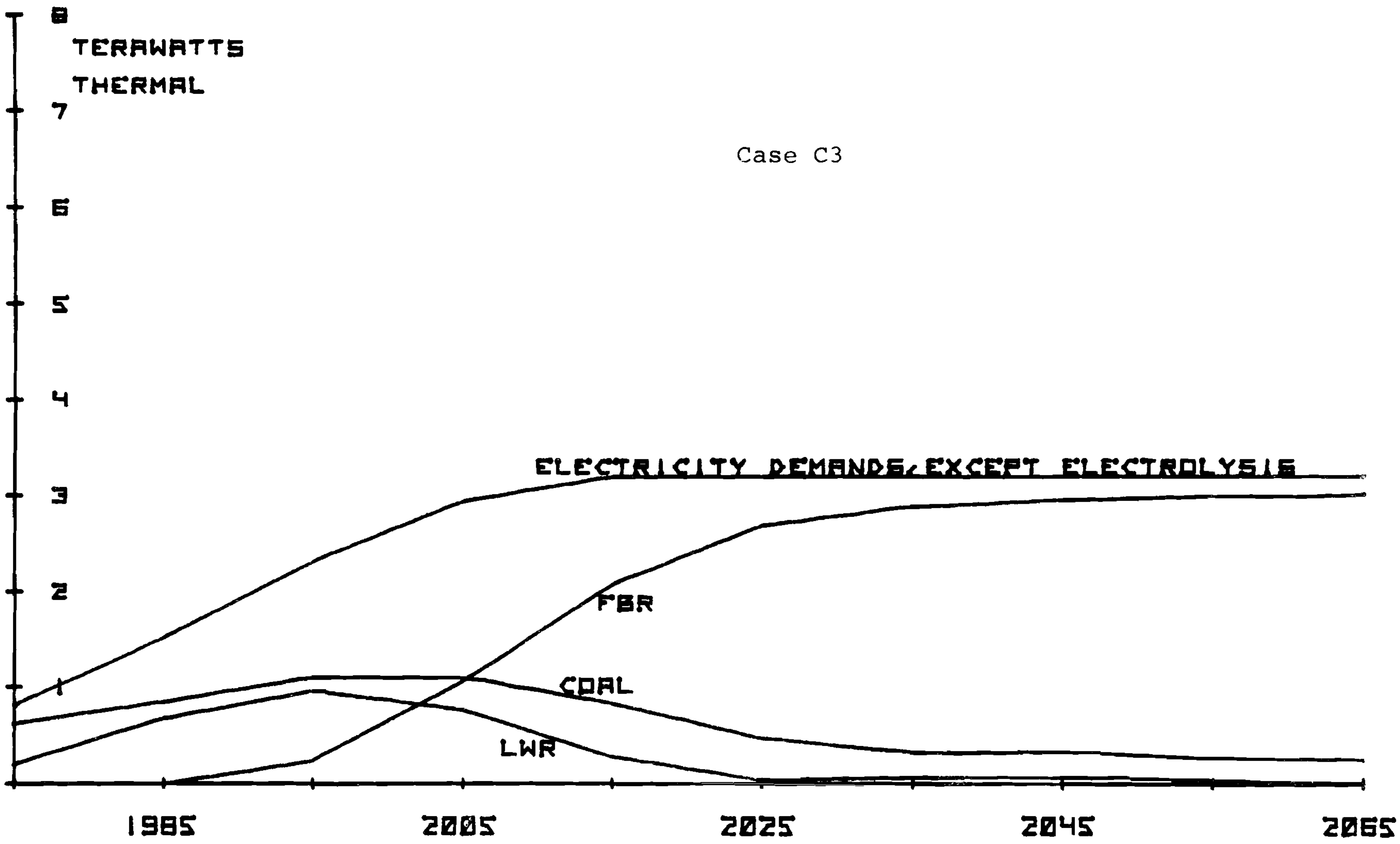

Figure D-9a. Model society 1.80: electric energy demands and supplies.

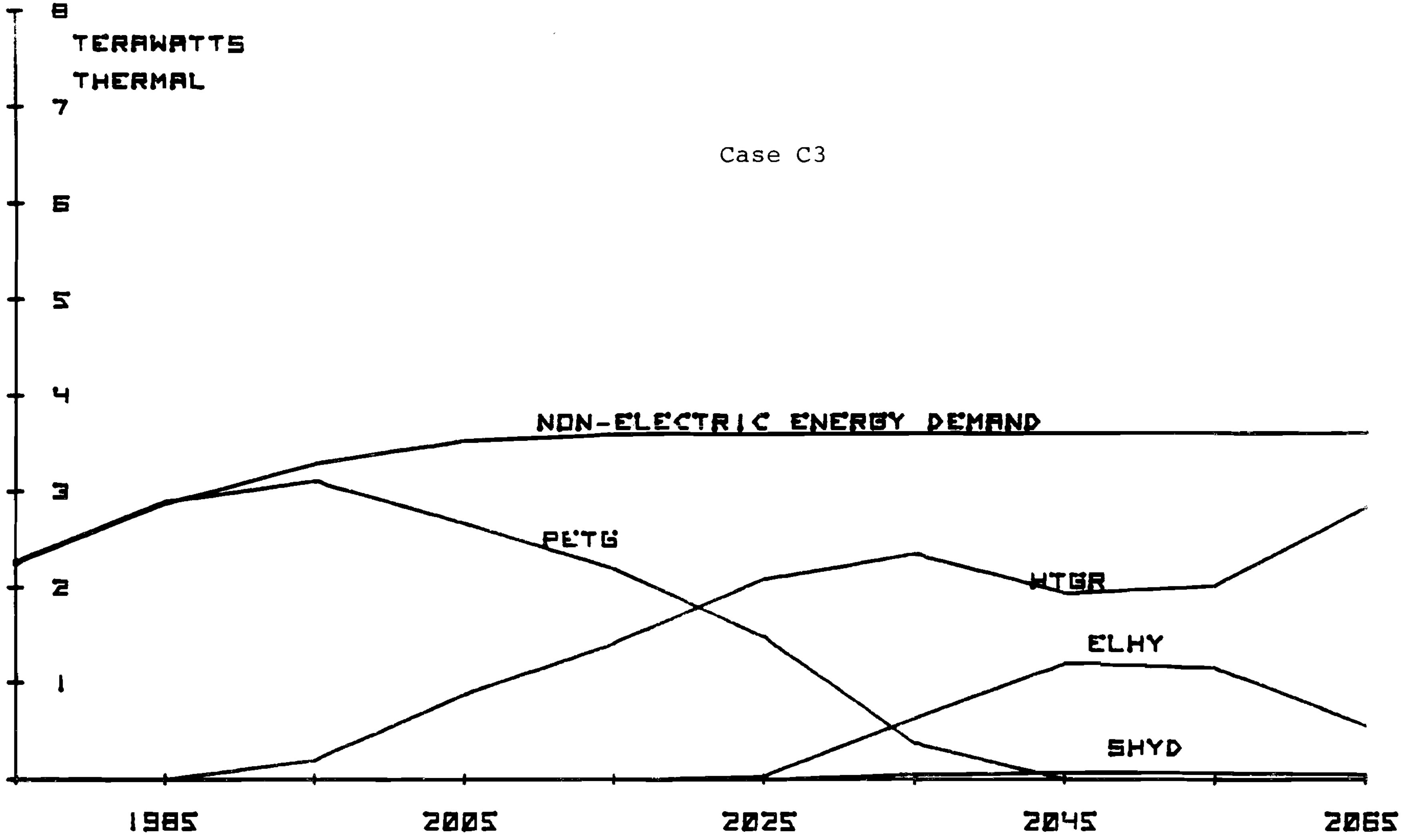

Figure D-9b. Model society 1.80: non-electric energy demands and supplies.

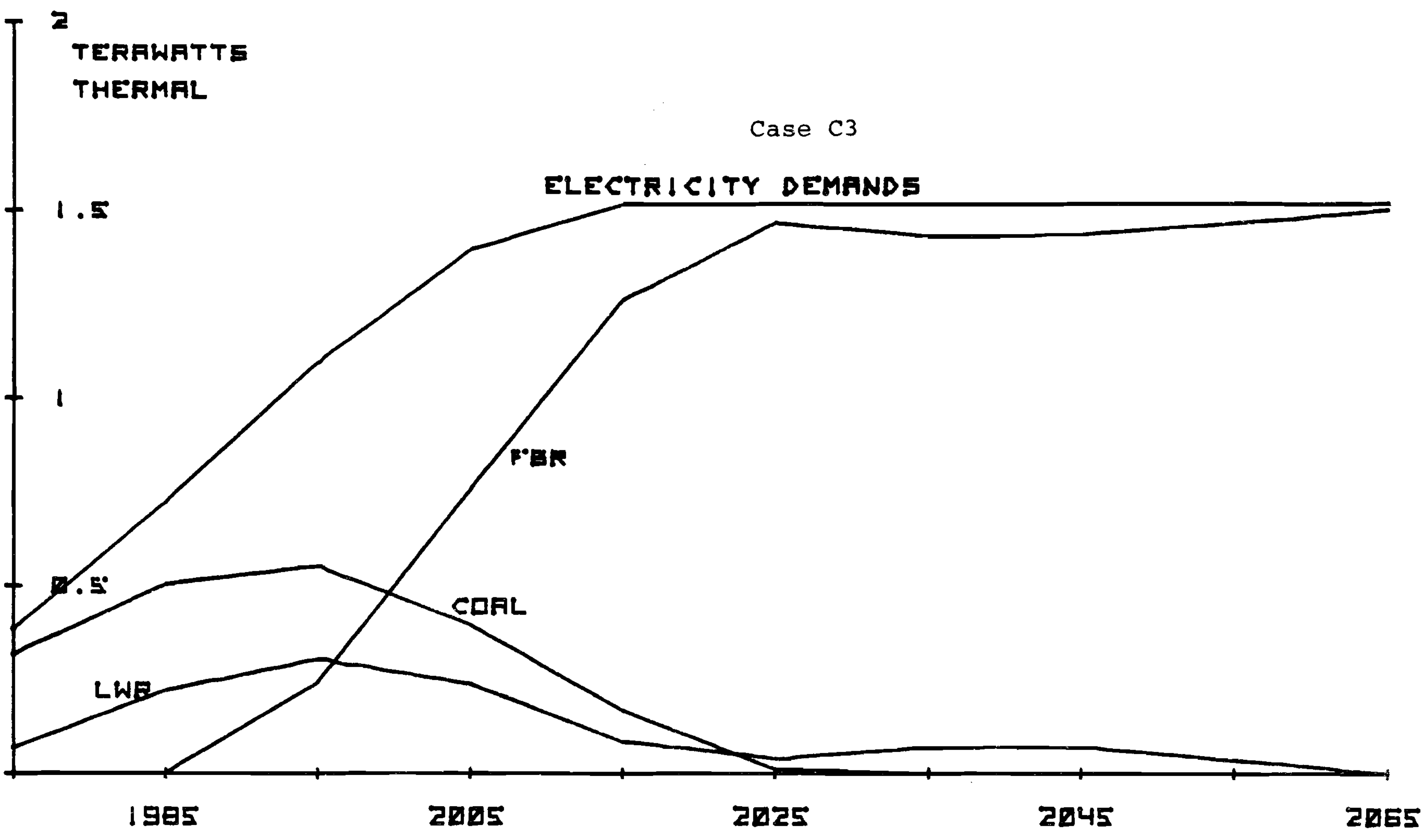

Figure D-10a. Model society 1.80: electric energy demand sector 1.

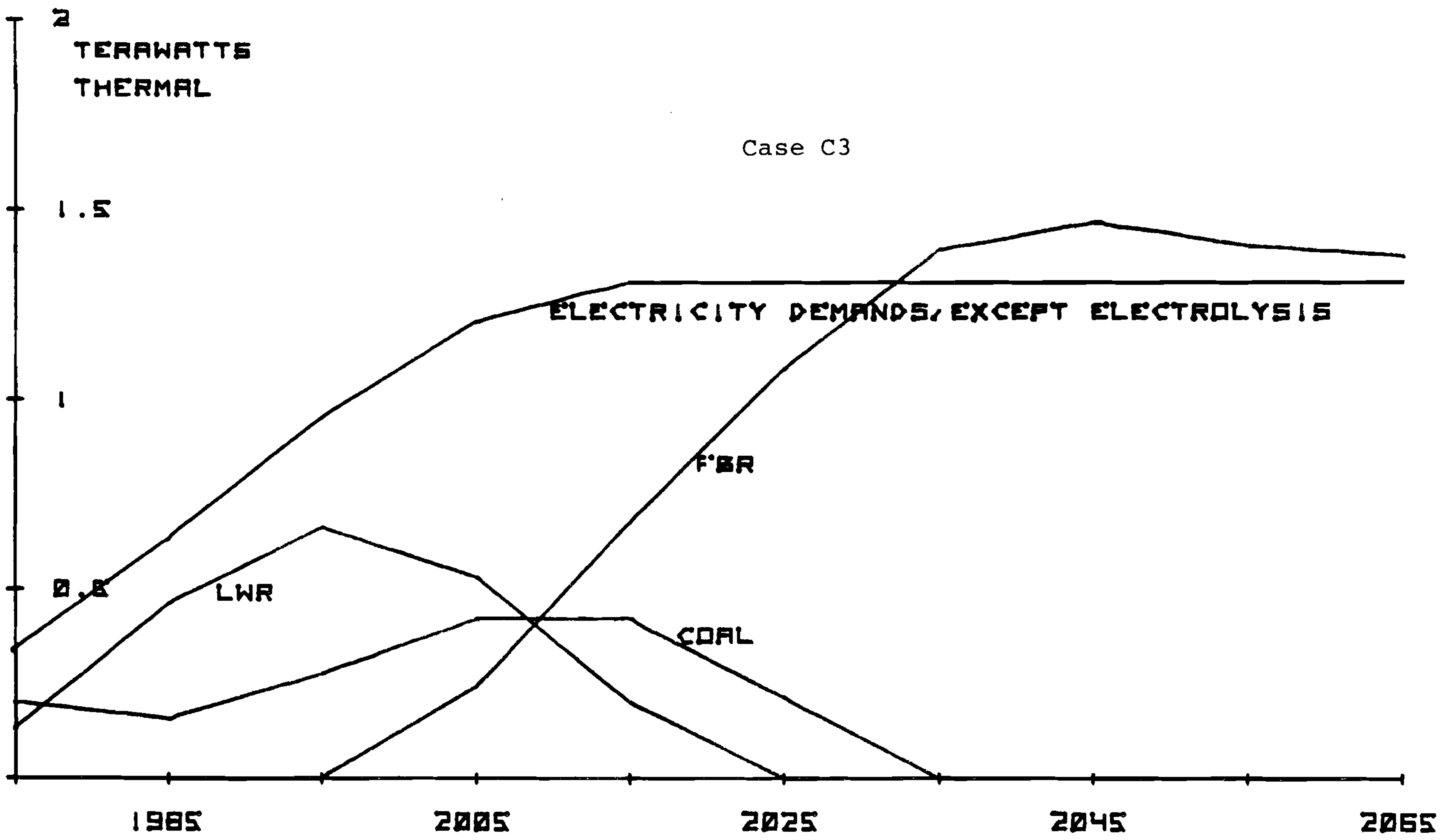

Figure D-10b. Model society 1.80: electric energy demand sector 4.

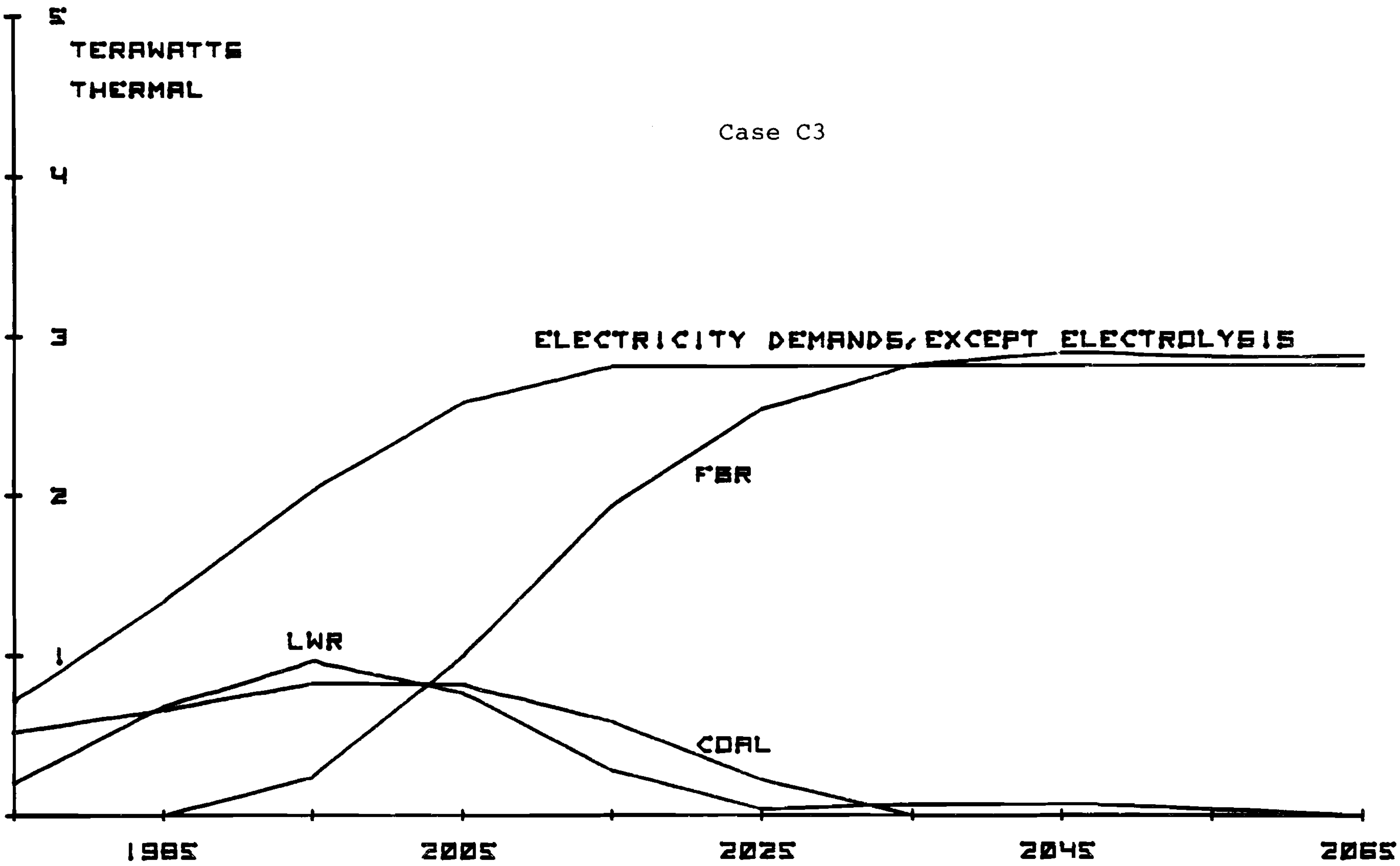

Figure D-10c. Model society 1.80: electric energy (base load) demands and supplies.

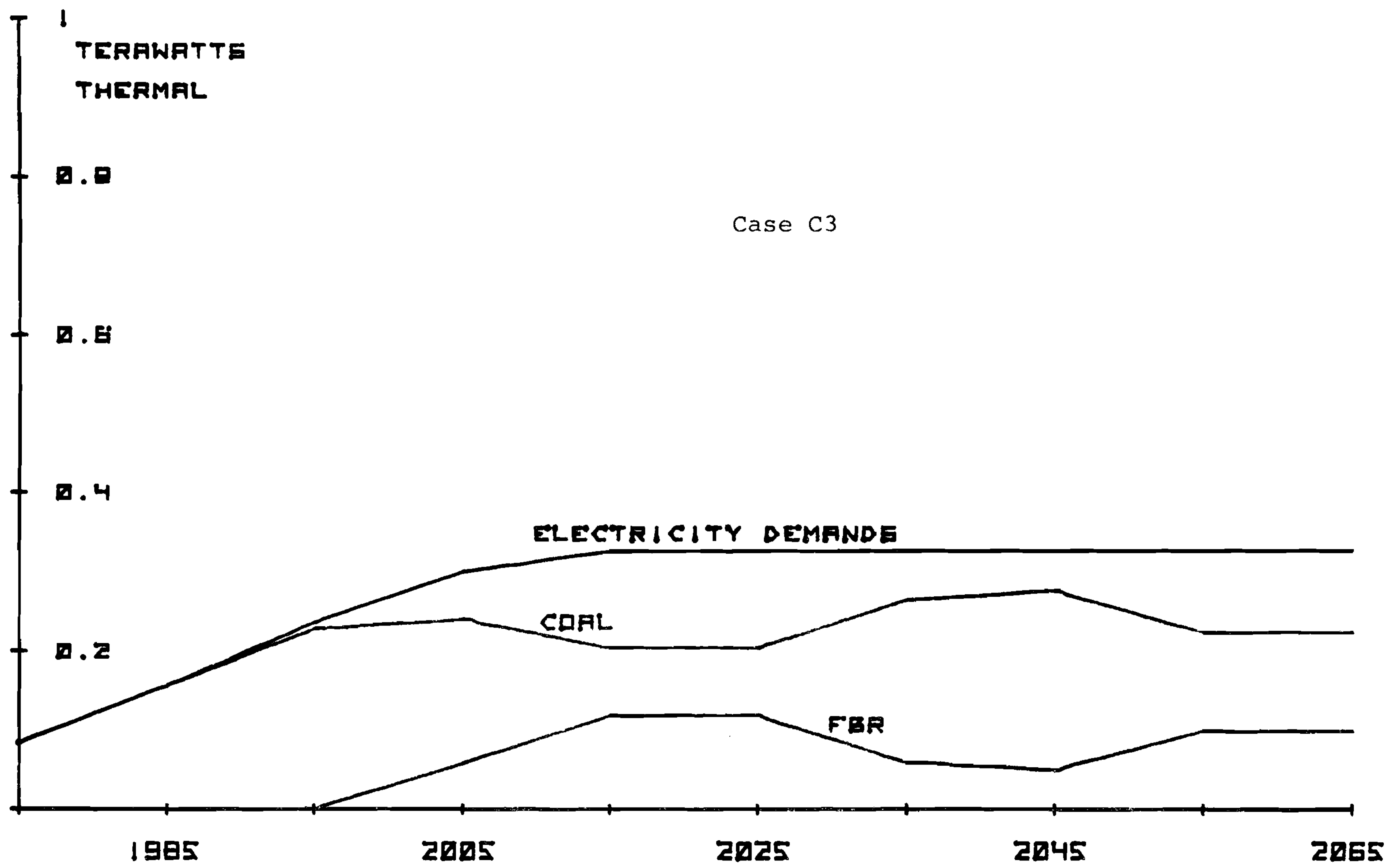

Figure D-11a. Model society 1.80: electric energy demand sector 2.

Case C3

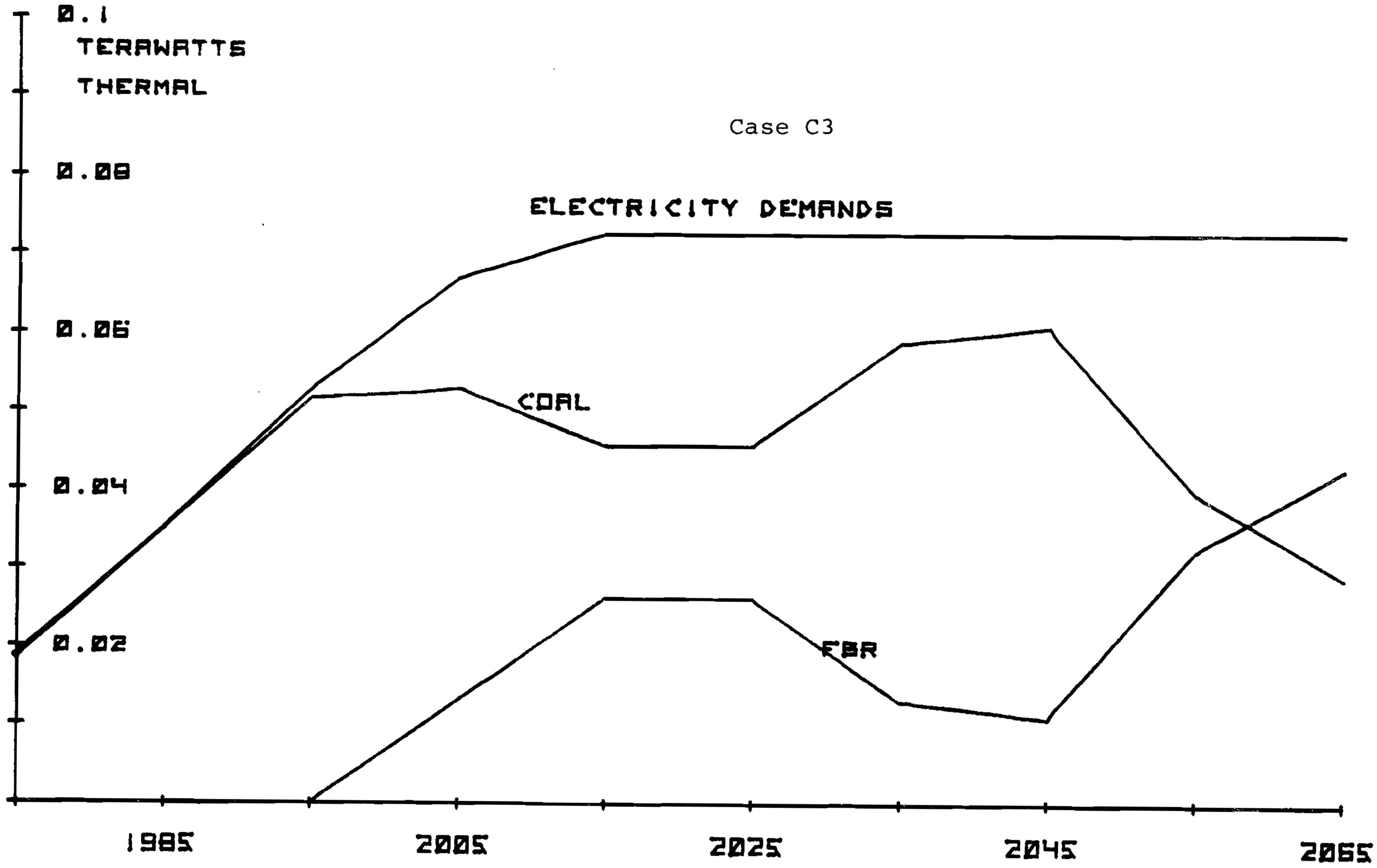

Figure D-11b. Model society 1.80: electric energy demand sector 5.

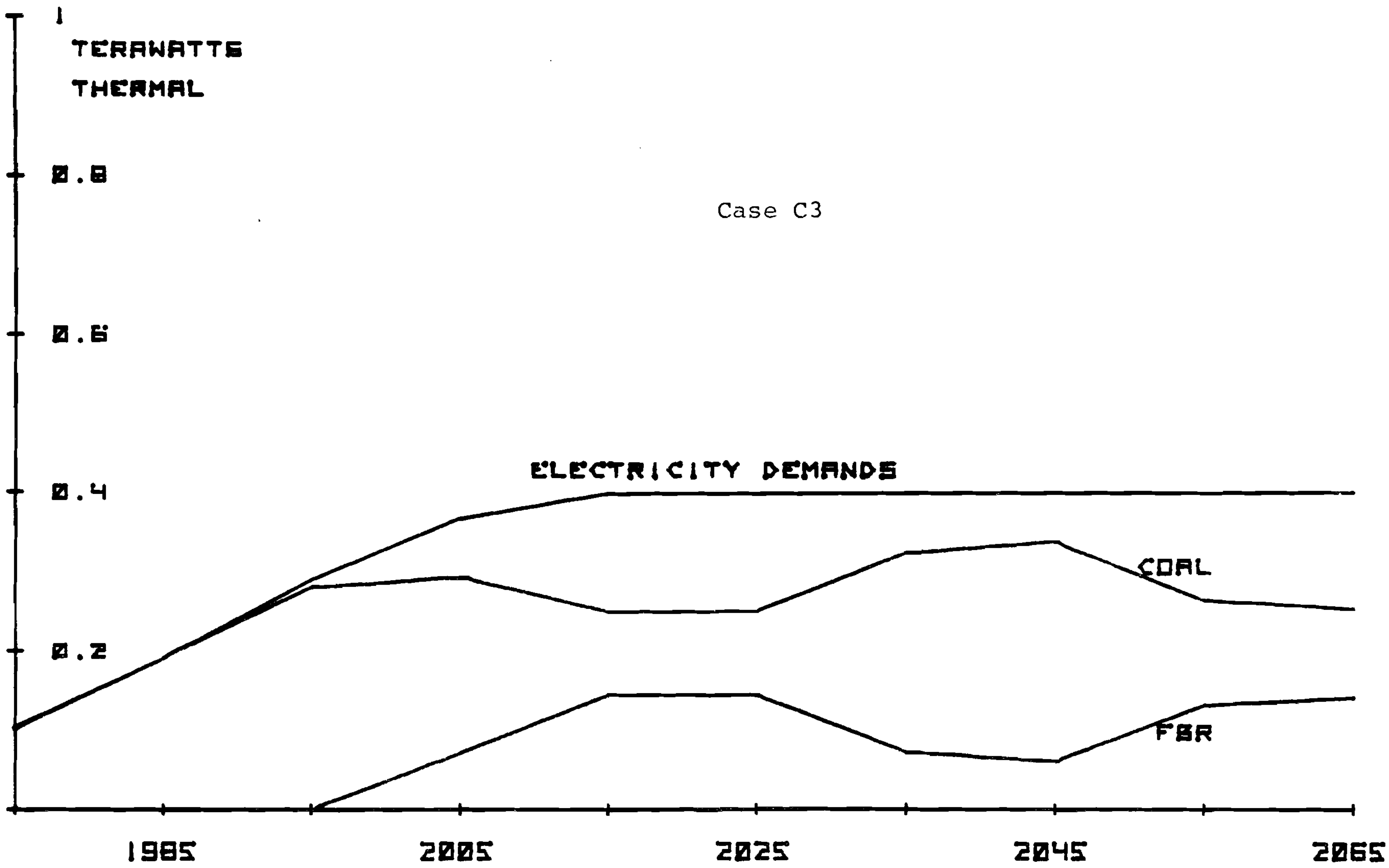

Figure D-11c. Model society 1.80: electric energy (peak load) demands and supplies.

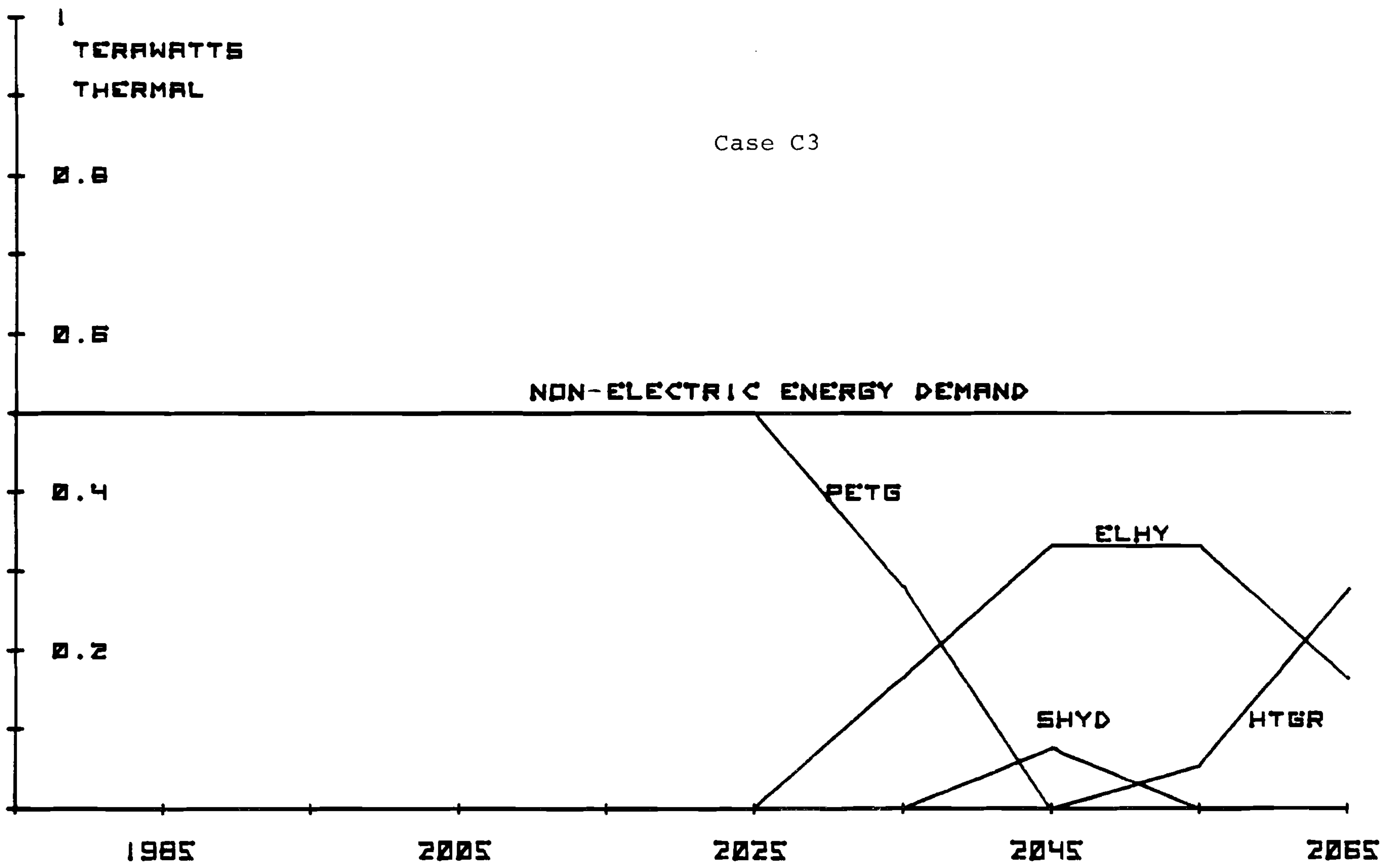

Figure D-12a. Model society 1.80: non-electric energy demand sector 3.

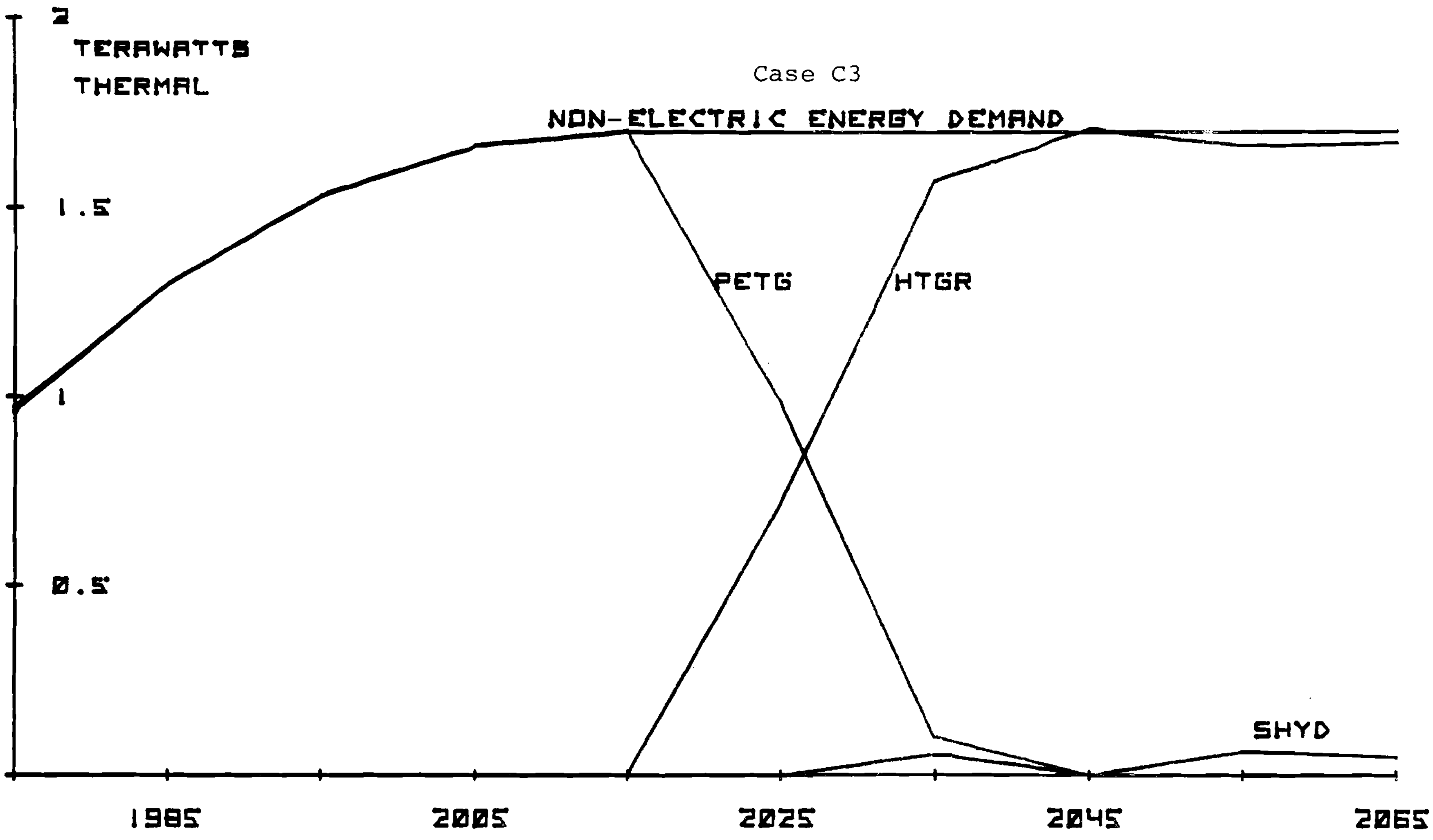

Figure D-12b. Model society 1.80: non-electric energy demand sector 6.

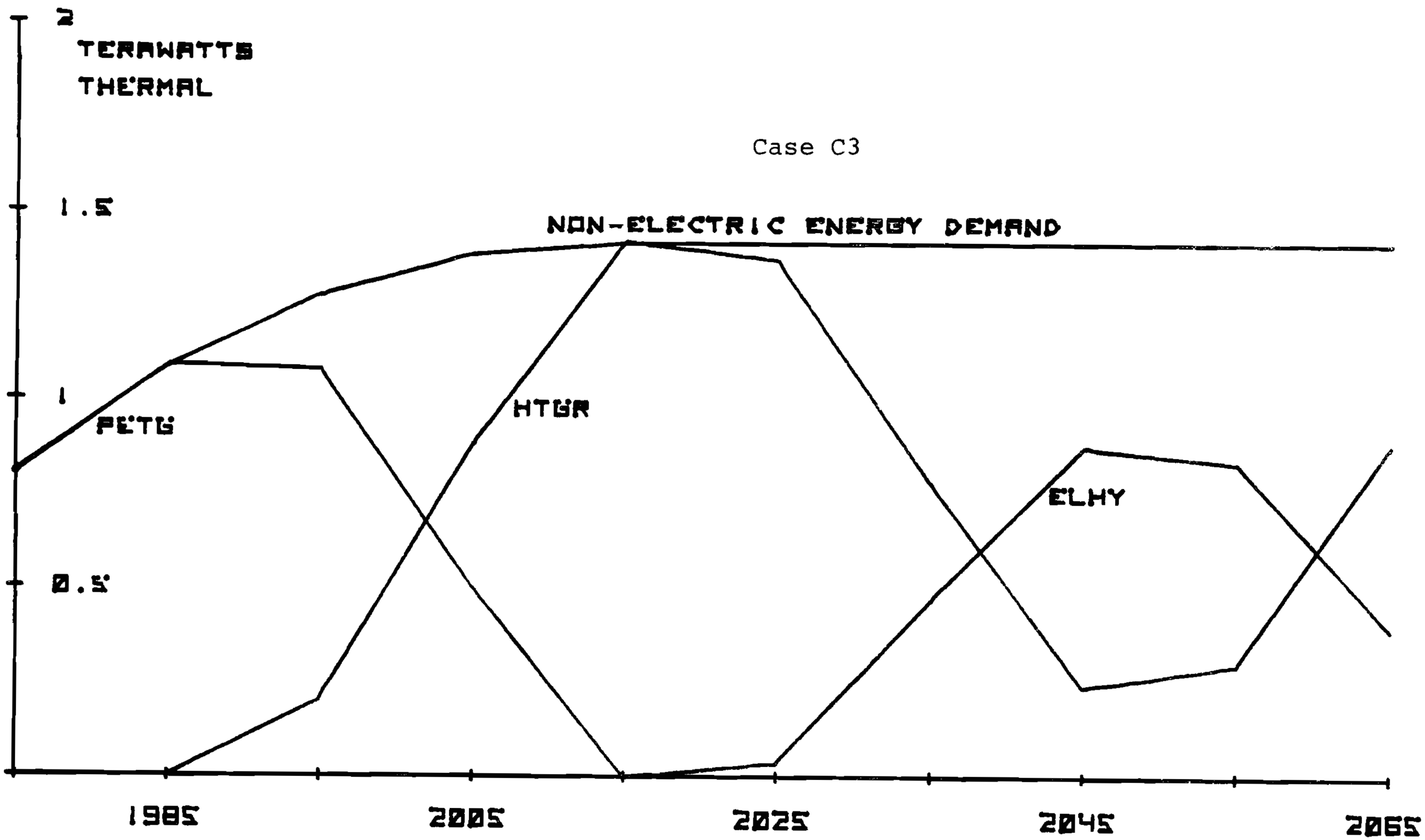

Figure D-12c. Model society 1.80: non-electric energy demand sector 7.

APPENDIX E

Description of the Computer Program Used for the Calculations Reported

Leo Schrattenholzer

1. Introduction and Summary

The purpose of Appendix E is to give a detailed description of the computer program used to calculate the results reported in this paper. This description should enable the reader to run the program, and to do some sensitivity analysis even if he does not want to understand programming details; it also contains further information for a deeper understanding of the program. (The reader who wants only to change some parameters of the model and/or run the program should immediately proceed to Section E-3.2.)

1.1 General Description

The model described in this paper is an LP (Linear-Programming) model consisting mainly of various (linear) constraints imposed on the (non-negative) variables of the model. These constraints can be interpreted as the following matrix vector relation:

$$A \quad x \quad r \quad b \tag{1.1}$$

where:

A = a matrix with one row for each of the constraints, one column for each of the variables, and an entry $a_{i,j} \neq 0$ if the variable j has a non-zero coefficient in the constraint i;

x = the vector of the variables;

r = the vector of relations with entries "$\leq$", "$\geq$", or "=" denoting the type of the corresponding constraint; and

b = the vector of the right hand side values.

The problem is to minimize the vector product:

min cx

subject to the constraints of (1.1), where (1.2)

c = the vector of the cost coefficients.

Constraints and rows are synonyms, as are columns, variables and activities. The vector product in (1.2) is called "objective function."

This LP problem has been solved by using the APEX package on a CDC 6600 computer system; the necessary MPS-formatted input has been generated by a FORTRAN program.

2. Additional Information about the Model

This Section describes the differences between the model as it has been described in the paper, and the final form of the constraints.

2.1 Theory

As has been indicated in Appendix A, (footnote to Equations (A1) and (A2)), it is possible to use the following form of the capacity equations:

$$PC^{h}_{k(i,\ell)} \leq 5 * DQ^{h-3}_{k(i,\ell)} + 10 * DQ^{h-2}_{k(i,\ell)} + 10 * DQ^{h-1}_{k(i,\ell)} + 5 * DQ^{h}_{k(i,\ell)} \tag{2.1}$$

This form has been chosen for the present computer program; the only difference is that the DQ activities have been denoted throughout by DP (mnemonic for $\frac{dPC}{dt}$). The fuel-balance equations have been adapted to the following forms:

$$DCS^{h}_{NATU} = \eta_{LWR} * \left[- \delta R_{NU,LWR} * DP^{h-3}_{3} + \delta A_{NU,LWR} * PC^{h}_{3} + \delta F_{NU,LWR} * DP^{h}_{3} \right]$$

$$+ \eta_{HTR} * \Big[- \delta R_{NU,HTR} * DP_5^{h-3} + \delta A_{NU,HTR} * PC_5^h + \delta F_{NU,HTR} * DP_5^h \Big] \quad ; \quad \forall h \quad . \qquad (2.2)$$

unit: 10^6 tons/yr

$$DCS_{PLUT}^h = \eta_{LWR} * \Big[- \delta R_{PU,LWR} * DP_3^{h-3} + \delta A_{PU,LWR} * PC_3^h + \delta F_{PU,LWR} * DP_3^h \Big] + \eta_{FBR} * \Big[- \delta R_{PU,FBR} * DP_4^{h-3} + \alpha F_{PU,FBR} * PC_4^h - \alpha R_{PU,FBR} * PC_{FBPL}^h + \delta F_{PU,FBR} * DP_4^h \Big] \quad ; \quad \forall h \quad . \qquad (2.3)$$

unit: 10^3 tons/yr

$$DCS_{U233}^h = \eta_{HTR} * \Big[- \delta R_{U3,HTR} * DP_5^{h-3} + \delta A_{U3,HTR} * PC_5^h + \delta F_{U3,HTR} * DP_5^h \Big] + \eta_{FBR} * \left(- \alpha R_{U3,FBR} \right) * \left(PC_4^h - PC_{FBPL}^h \right) \quad ; \quad \forall h \quad . \qquad (2.4)$$

unit: 10^3 tons/yr

$$PC_{FBPL}^h \leq PC_4^h \quad ; \quad \forall h \quad . \qquad (2.5)$$

unit TW-th,
LWR equivalent

(For a definition of the constant factors, see Section 2.3).

The remaining equations given in Appendix A remain unchanged. By solving some equations (that are essentially abbreviations) and by considering the initial conditions, we arrive at the final form of the constraints described below.

2.2 Constraints

A. CS_j^h : j = COAL, PETG, NULC, NATU, PLUT, U233

$h = 10$ for COAL, PETG

$h = 1,\dots,10$ for all other j's . (2.6)

unit: Ω

$$CS_j^{10} : \quad 10 * .03 * \sum_{h'=1}^{10} PC_j^{h'} \leq RXj^{\dagger}$$

j = COAL, PETG .

$$\begin{aligned} CS_{NATU}^{h}: \quad & 10 * \eta_{LWR} * \left[\sum_{h'=1}^{h} \delta F_{NU,LWR} * DP_3^{h'} - \sum_{h'=1}^{h-3} \delta R_{NU,LWR} * DP_3^{h'} + \sum_{h'=1}^{h} \delta A_{NU,LWR} * PC_3^{h'} \right] \\ & + 10 * \eta_{HTR} * \left[\sum_{h'=1}^{h} \delta F_{NU,HTR} * DP_5^{h'} - \sum_{h'=1}^{h-3} \delta R_{NU,HTR} * DP_5^{h'} + \sum_{h'=1}^{h} \delta A_{NU,HTR} * PC_5^{h'} \right] < \infty \; ; \quad \forall h . \end{aligned}$$

(2.7)

unit: 10^6 tons

---

† $RX_{COAL} = RA_{COAL} * .625 * .03$

$RX_{PETG} = RA_{PETG} * 1.875 * .03$

For further information, see page 77.

$$CS^{h}_{NULC}: \left\{ \text{same sum as in } CS^{h}_{NATU} \right\}$$

$$- 10 * \sum_{h'=1}^{h'} ANNUHC^{h'\ddagger} \leq RA_{NULC} \quad \forall h \quad .$$

(2.8)
unit: 10^6tons

$$CS^{h}_{PLUT}: \quad 10 * \eta_{LWR} * \left[\sum_{h'=1}^{h-3} \delta F_{PU,LWR} * DP^{h'}_{3} - \sum_{h'=1}^{h-3} \delta R_{PU,LWR} * DP^{h'}_{3} + \sum_{h'=1}^{h} \delta A_{PU,LWR} * PC^{h'}_{3} \right]$$

(2.9)
unit: 10^3tons

$$+ 10 * \eta_{FBR} * \left[\sum_{h'=1}^{h} \delta F_{PU,FBR} * DP^{h'}_{4} - \sum_{h'=1}^{h-3} \delta R_{PU,FBR} * DP^{h'}_{4} + \sum_{h'=1}^{h} \alpha F_{PU,FBR} * PC^{h'}_{4} - \sum_{h'=1}^{h} \alpha R_{PU,FBR} * PC^{h'}_{FBPL} \right] \leq 0 \quad ; \quad \forall h \quad .$$

$$CS^{h}_{U233}: \quad 10 * \eta_{HTR} * \left[\sum_{h'=1}^{h} \delta F_{U3,HTR} * DP^{h'}_{5} - \sum_{h'=1}^{h-3} \delta R_{U3,HTR} * DP^{h'} + \sum_{h'=1}^{h} \delta R_{U3,HTR} * PC^{h'}_{5} \right]$$

(2.10)
unit: 10^3tons

$$- 10 * \eta_{FBR} * \sum_{h'=1}^{h} \alpha R_{U3,FBR} * \left(PC^{h'}_{4} - PC^{h'}_{FBPL} \right) \leq 0 \quad ;$$

$\forall h$.

$^{\ddagger}DCS^{h}_{NUHC}$ has been renamed to $ANNUHC^{h}$.

B. $DMND^h_\ell$: $h = 1,\ldots,10$

$\ell = 1,\ldots,7$

(2.11)

unit: TW_e

$$\sum_{i\varepsilon\{1,3,4,6\}} \eta i * PC^h_{k(i,\ell)} \geq DM^h_\ell \quad \ell = 1,2,5 \quad ; \quad \forall h \quad .$$

$$\sum_{i\varepsilon\{1,3,4,6\}} \eta i * PC^h_{k(i,4)} \geq DM^h_4$$

(2.12)

unit: TW_e

$$+ \sum_{\ell'\varepsilon\{3,6,7\}} \eta_{LWR}\, PC_{k(11,\ell')} \quad ; \quad \forall h \quad .$$

$$PC^h_{k(2,\ell)} + \frac{\nu HYDR(\ell)}{\nu PETG(\ell)} * \left\{ \eta_{HTGR} * PC^h_{k(5,\ell)} + \eta_{SHYD} \right.$$

$$\left. * PC^h_{k(10,\ell)} + \hat{\eta}_{ELHY}{}^{||} * PC^h_{k(11,\ell)} \right\}$$

(2.13)

unit: TW_{th}

PETG equivalent

$\ell = 3,6,7$;

C. $DIBRGN^h$: $h = 1,\ldots,10$,

(2.14)

unit: TW_{th},

LWR equivalent

$$PCFBR^h - PCFBPL^h \geq 0 \quad .$$

D. $UT^h_{k(i,\ell)}$: $h = 1,\ldots,10$;

k = varying over all possible paths, supplying source $i \rightarrow$ demand sector ℓ .

$$5 * DP^h_{k(i,\ell)} + 10 * DP^{h-1}_{k(i,\ell)} + 10 * DP^{h-2}_{k(i,\ell)} + 5$$

$$* DP^{h-3}_{k(i,\ell)} - PC^h_{k(i,\ell)} \geq 0 \quad .$$

(2.15)

unit: TW_{th} {LWR equiv. for ELEC; PETG equiv. for NELE}

|| For simplicity of programming; $\hat{\eta}_{ELHY}$ means the overall efficiency from thermal demand for electricity to hydrogen ($\hat{\eta}_{ELHY} = \eta_{LWR} * \eta_{ELHY}$) .

E. $AGPC_i^h$; $h = 1,\ldots,10$

$i = 1,\ldots,11$ (excluding 7,8,9) (2.16)

unit: TW_{th}

$$\sum_{\ell\varepsilon\{1,2,4,5\}} PC^h_{k(i,\ell)} - PC^h_i = 0 \quad ; \quad \forall h \quad . \quad i = 1,3,4,6$$

LWR equivalent

$$\sum_{\ell\varepsilon\{3,6,7\}} PC^h_{k(i,\ell)} - PC^h_i = 0 \quad ; \quad \forall h \quad . \quad i = 2,5,10,11 \tag{2.17}$$

unit: TW_{th}

PETG equivalent

F. $AGDP_i^h$: $h = 1,\ldots,10$

$i = 1,\ldots,11$ (excluding 7,8,9) (2.18)

unit: TW_{th}

$$\sum_{\ell\varepsilon\{1,2,4,5\}} DP^h_{k(i,\ell)} - DP^h_i = 0 \quad ; \quad \forall h \quad . \quad i = 1,3,4,6$$

LWR equivalent

$$\sum_{\ell=\{3,6,7\}} DP^h_{k(i,\ell)} - DP^h_i = 0 \quad ; \quad \forall h \quad . \quad i = 2,5,10,11 \tag{2.19}$$

unit: TW_{th}

PETG equivalent

G. Bounds:

Constraints on single variables can be imposed directly by specifying a "bounds set." The bounds set of this model consists of two parts:

i) upper bounds on the DP_i^h variables:

i = LWR, FBR, HTGR, STEC, SHYD

$$DP_i^h \leq UBDP_i^h \quad ; \quad \forall h \quad . \tag{2.20}$$

unit: TW_{th}/yr

ii) Fixing bounds for the $DP^h_{k(i,\ell)}$ variables:

$h = -2, -1, 0$

i = COAL, PETG unit: $TW_{th}/year$

$\ell = 1,\ldots,7$.

These are the initial conditions that have to be specified. For details, see page 72 and Section E 3.2.

H. Objective function:

$$\sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{\ell=1}^{L} 10 * \beta^{10*h-5} * \Big[cur_{k(i,\ell)} * PC^{h}_{k(i,\ell)}$$

$$+ \beta^{-\tau} * (1 - TV^{h}) \qquad (2.21)$$

unit: 10^9\$ (1974)

$$* \; cap_{k(i,\ell)} * DP^{h}_{k(i,\ell)}$$

$$+ \; cur_{NUHC} * ANNUHC^{\S h} \Big] .$$

(For further information, see page 81.

2.3. Data

The parameters of the equations are all defined in the paper. This section will define only the parameters of the (reformulated) fuel balance equations:

		LWR	FBR	HTR	unit
δF	NU	0.50	0.00	0.54	10^6 tons/TW_e
	PU	0.00	2.00	0.00	10^3 tons/TW_e
	U3	0.00	0.00	0.00	10^3 tons/TW_e
αF	NU	0.21	0.00	0.00	10^6 tons/TW_e-a
	PU	0.00	0.70	0.00	10^3 tons/TW_e-a
	U3	0.00	0.00	0.48	10^3 tons/TW_e-a
αR	NU	0.03	0.00	0.00	10^6 tons/TW_e-a
	PU	0.17	0.86	0.00	10^3 tons/TW_e-a
	U3	0.00	0.29	0.19	10^3 tons/TW_e-a

§This activity has been renamed (original name: DCS^{h}_{NUHC}).

		LWR	FBR	HTR	unit
δR	NU	0.10	0.00	0.00	10^6 tons/TW_e
	PU	0.00	2.00	0.00	10^3 tons/TW_e
	U3	0.00	0.00	1.35	10^3 tons/TW_e

$\delta A = \alpha F - \alpha R$

3. The Computer Program

The computer program consists of two parts: the major one i.e., the generation of the input matrix; the minor one, i.e., the call of the LP-routine consisting of only a few statements.

3.1 The Matrix Generation

The matrix has to be created in MPS format, whose main features are: a) it does not require the zero-elements of the matrix and the right-hand-side to be specified; b) the input of the matrix has to be column by column. Therefore, some care is necessary to translate the constraints described above into the actual matrix generation. (For details of the MPS-format see an APEX manual.)

3.2 How to Use the Program

3.2.1 How to Prepare the Input

The following program description by line numbers contains the column "change." An entry "*" means that changing the corresponding value is straightforward and does not require additional changes. There are also data which, on the one hand, are derived from very basic assumptions of the model and are therefore not subject to change in a sensitivity analysis; but these data are also self-explanatory to those who try to understand the details of the program. These variables are not described in detail; only the environment in the program is given, and the "change" column contains an "s."

Line No.	Change	Explanation
8 - 10		APEX requires the input data to be on file TAPE 1
29 - 40	s	Problem output (see Section 3.2.2)

Line No.	Change	Explanation
42	*¶	HX = number of time periods
42		IX = maximum number of supply alternatives
42		JX = number of resource constraints
42		LX = number of demand categories
42		JD = demand sector, which supplies the electricity for the electrolysis
42	*÷	ITAU = τ (see page 70) [years]
45,46	s	Indices for electricity supply paths
47	s	Initial conditions
49 - 54	*	$DM(i,h) = \left\{ \begin{array}{lll} \overline{DM_{ELEC}}^{h} & i = 1 & \text{see page 69} \\ \overline{DM_{NONE}}^{h} & i = 3 & \text{see page 69} \end{array} \right\} [TW_{th}]$ Annual aggregated demand for electricity or non-electric energy, respectively. (These figures are for model society 1.) <u>Note:</u> This is the demand in terms of installed capacity and <u>not</u> the sum of the demands of each sector <u>because</u> of the load factors ρ (ℓ).
54	s	UT constraints
56	*††	NRHS = number of RHS vectors to be generated
57	*††	NBND = number of bounds sets to be generated
59 - 60	*	HP(1,k) = Upper bound for total COAL-consumption for RHS vector k in [Q] (= RXCOAL; see page 179) These 50 Q mean virtually infinity.
61 - 64	*	HP(2,k) = Upper bound for total PETG-consumption for RHS vector k in [Q]. These values correspond to the model societies 1.40, 1.60, 1.80 or 1.100, respectively.

¶Without further changes, HX can only be lowered.

÷Has to be integer at present stage.

††May assume only the values 1,2,3,4 and 5.

Line No.	Change	Explanation
66	*	$ZE(1) = \zeta_{RCEIP}$ (see page 70)
67	*	$ZE(2) = \zeta_{INEIP}$ (see page 70)
68	*	$GR(1) = (1 + \varepsilon_{\ell})$; $\ell = 1,2,4,5$
69	*	$GR(2) = (1 + \varepsilon_{\ell})$; $\ell = 3,6,7$
		GR(i) depends only on the kind of fossil fuel used for demand sector ℓ (COAL for ELEC. PETG for NELE), see page 70.
71 - 74	*	$RHO(L) = \rho(\ell)$ (see page 70).
76 - 87		Break up of demand into sectors.
		$DM(L,H) = DM_h^{\ell}$ (see page 69).
		The following values for the ξ's are used (the ξ's are no variables at the present stage of the program).
		$\xi_{RCE}^{h} = .6$
		$\xi_{INE}^{h} = .4$
		$\xi_{RCN}^{h} = .5/\overline{DMNONE}^{h}$
		$\xi_{INN}^{h} = (1 - \xi_{RCN}^{h}) * 6/11$
		$\xi_{TRN}^{h} = (1 - \xi_{RCN}^{h}) * 5/11$
95 - 102		$DEP(1,h) = DP_{k(i,\ell)}^{h-3}$ $h = 1,2,3$; $i = 1,2$; $\ell = 1,\dots,7$ [‡‡]
		$\overline{DM_{ELEC}}^{0} = .44$ (Line No. 95) are assumed
		$\overline{DM_{NONE}}^{0} = 1.44$ (Line No. 96)

‡‡For further information, see page 72.

Line No.	Change	Explanation
		The breakup for the initial conditions is done just like for the demand projection with the following exception: $\xi^{0}_{RCN} = .4/\overline{DM^{0}_{NONE}}$ is assumed in order to avoid that for h=1 the installed capacity is greater than the projected demand. Remark: This diligence in determining the initial conditions may not seem to be appropriate for the expected accuracy of the results. The reason for this complicated way of calculation is internal, namely it allows for a better comparison with the original Häfele-Manne model [3]. It is recommended to anybody who wants to change these initial conditions to determine these by hand. The contribution of a given set of initial conditions to the supply in the first three time periods may easily be calculated from formula (2.15).
107	*	BETA = β [|| ||]
109 - 116		Initializations
118 - 122		Setting of logical constants for electricity supply paths
124 - 127		Initializations
129 - 132	*	CUR(I,J1(L)) = $cur_{k(i,\ell)}$ (see page 81) [10^9\$/TW_{th-a}] (for definition of J1 see line 46)
134 - 140	s	NELE-supply paths (analogous to lines 45 & 46)
142 - 148	*	YN(I,L) = $\nu_{i(\ell)}/\nu_{PETG(\ell)}$ [§§]

[|| ||] For further information, see page 81.

[§§] For further information, see page 80.

Line No.	Change	Explanation
150 - 154		Setting of logical constants in connection with the bounds set.
156 - 163		Setting of logical constants in connection with the bounds set.
165-174		Initializations
176 - 188	*	$\left.\begin{array}{l} DF(I,J) = \delta F_{i,j} \\ AR(I,J) = \alpha R_{i,j} \\ AF(I,J) = \alpha F_{i,j} \\ DR(I,J) = \delta R_{i,j} \end{array}\right\\}$ $\begin{array}{l} i = 1 \triangleq NU;\ j = 1 \triangleq LWR \\ i = 2 \triangleq PU;\ j \triangleq 2 \triangleq FBR \\ i = 3 \triangleq U3;\ j = 3 \triangleq HTR \end{array}$ (See page 183 for definition and units)
190 - 203	*	$ETA(I) = ETB(I) = \eta_i$ [¶¶] ; with the exceptions: $ETA\ (5) = .4 = \eta_{HTR}$ $ETB\ (5) = .5 = \eta_{HTGR}$ $ETA(11) = \hat{\eta}_{ELHY}$ [÷÷]
205 - 208		Initializations
209 - 221	*	$BVAL(I,H,K) = UBDP_i^h$ in bounds set k $[TW_{th}/yr]$
223 - 230	s	$V(I,J)$ = coefficient of activity PC_i^h in equation CS_j^h (see pages 73 and 74)
232 - 261	*	$\left.\begin{array}{l} CAP(I,L) = cap_{k(i,\ell)}\ [10^9\$/TW_{th}] \\ CUR(I,L) = cur_{k(i,\ell)}\ [10^9\$/TW_{th-a}] \end{array}\right\\}$ (see page 81
263 - 268	s	$U(1,I,J)$ = coefficient of activity DP_{i+2}^h in constraint CS_{j+2}^h (see pages 179 and 180) $U(2,I,J)$ = coefficient of activity DP_{i+2}^h in constraint CS_{j+2}^{h+3} (see pages 179 and 180)

[¶¶] For further information, see page 23.

[÷÷] For further information, see page 181.

Line No.	Change	Explanation
269 - 477		Writing of input matrix in MPS-format
479 - 511		Control program for APEX

3.2.2. How to Read the Output

Two kinds of information are necessary to understand the output of the APEX optimization routine. The first one is general information which is specific to APEX and could be obtained from a manual, and the second kind concerns the names of the constraints and variables used in the program, namely how they correspond to the names used in Section 2.2. This correspondence is described as follows:

I. Constraint Names

Row No. in APEX Output	Row Name in Program		Name used in Section 2.2
1	COST		objective function
2 - 43	CS j, h	j = COAL,	CS_j^h
		PETG,	
	h = 10(for j =	NULC,	
	COAL, PETG)	NATU,	
	h = 1,...,10	PLUT,	
	otherwise	U233	
44 - 113	DMND ℓ h	ℓ = 1,...,7	$DMND_\ell^h$
		h = 1,...,10	
114 - 123	DIBRGN h	h = 1,...,10	$DIBRGN^h$
124 - 403	UT i ℓ h	i = 1,...,11	$UT_{k(i,\ell)}^h$
		ℓ = 1,...,7	
		h = 1,...,10	
	only for those pairs (i,ℓ) which represent possible paths: source kind i → demand sector ℓ		
404 = 483	AGPC i h	i = 1,...,11	$AGPC_i^h$
		(excl. 7,8,9)	
		h = 1, 10	

Row No. in APEX Output	Row Name in Program		Name used in Section 2.2
484 - 563	AGDP i h	i = 1,...11 (excl. 7,8,9) h = 1,...,10	$AGCP_i^h$

II. Column Names

Column No. in APEX Output	Column Name in Program		Name used in Section 2.2	
1 - 280	DP i ℓ h	i = 1,...,11 ℓ = 1,...,7 h = 1,...,10	$DP^h_{k(i,\ell)}$	
	only for those pairs (i,ℓ) which represent possible paths: source kind i → demand sector ℓ			
281 - 360	DP i h	h = 1,...,10 i = COAL, PETG, LWRX, FBRX, HTGR, STEC, SHYD, ELHY	DP^i_h	i = 1,...,11 (excl. 7,8,9) h = 1,...,10
361 - 381	DP i ℓ h	i = CL, PG ℓ = 1,...,7 h = -2,-1,0	$DP^h_{k(i,\ell)}$	h = -2,-1,0 i = 1,2 ℓ = 1,7
	(of the specification of the initial conditions on page 187)			
382 - 661	PC i ℓ h	i = 1,...,11 ℓ = 1,...,7 h = 1,...,10	$PC^h_{k(i,\ell)}$	
	only for those pairs (i,ℓ) which represent possible paths: source kind i → demand sector ℓ			
662 - 741	PC i h	h = 1,...,10 i = COAL, PETG, LWRX, FBRX, HTGR, STEC, SHYD, ELHY	PC^h_i	i = 1,...,11 (excl. 7,8,9) h = 1,...,10
742 - 751	ANNUHC h	h = 1,...,10	$ANNUHC^h$	
752 - 761	PCFBPL h	h = 1,...,10	$PCFBPL^h$	

APPENDIX F

Computer Program

```
$SEQUENCE,W39.
$CHARGE,GA700  -E03.
JOB,CM50000,CL100000,P0,T50,IO500.
REDUCE.
FTN.
MAP(OFF)
LGO.
REWIND(TAPE6)
COPY(TAPE6,TAPE1)
REWIND(TAPE1)
RFL,21000.
APPLIC,APEX.
RFL,100000.
APEX.
END OF RECORD
      PROGRAM SOL(INPUT,TAPE5=INPUT,TAPE6)
      REAL YN(11,7),BVAL(11,10,5),DM(7,10),HP(2,5),FC(4)
      REAL CUR(11,7),CAP(11,7),RHO(7)
      REAL V(5,6),BK(11),RS(6),RTP(6) ,
     &ETA(11),ETB(11),FOSS(7),DEP(7,3)
      REAL DF(3,3),AF(3,3),AR(3,3),DR(3,3)
      INTEGER I1(6),J1(4),H,HI,HX,H1
      INTEGER INI(7)
      REAL NUHC
      REAL U(2,3,4)
      REAL ZE(2),WR(2),FO(7)
      LOGICAL LC(11,7),LUB(11),T,F
C
      DATA (FO(L),L=1,7)/2HCL,2HCL,2HPG,2HCL,2HCL,2HPG,2HPG/
      DATA CP,CS,DP,UB,RI,AN,DI/2HCP,2HCS,2HDP,2HUB,2HRI,2HAN,2HDI/
      DATA DQ,UU,PC,UT,DU/2HDQ,2HUU,2HPC,2HUT,2HDU/
      DATA PNAME,DMND,NUHC,FBPL/4HSONU,4HDMND,4HNUHC,4HFBPL/
      DATA G,BRGN/1HG,4HBRGN/
      DATA AGDP,AGPC/4HAGDP,4HAGPC/
      DATA FX,UP/2HFX,2HUP/
      DATA  (FOSS(L),L=1,7)/2*(4HCOAL),4HPETG,2*(4HCOAL),2*(4HPETG)/
      DATA (BK(I), I=1,11)/4HCOAL,4HPETG,4HLWRX,4HFBRX,4HHTGR,4HSTEC
     X4HOBSE,4HPVSE, 4HSBIO,4HSHYD,4HELHY/
      DATA (RTP(J), J=1,6)/3*(1HL),1HN,2*(1HL)/
      DATA (RS(J), J=1,6)/4HCOAL,4HPETG,4HNULC,4HNATU,4HPLUT,4HU233/
C
      DATA HX,IX,JX,LX,JD,ITAU/10,11,6,7,4,2/
      DATA WA,AS,AQ,AW,AA/-10.,1.,-1.,-.5,.3/
C
      DATA (I1(I),I=1,6)/1,3,4,6,7,8/
      DATA (J1(L),L=1,4)/1,2,4,5/
      DATA (INI(I),I=1,7)/1,1,2,1,1,2,2/
C
      DATA DM(1,1),DM(1,2),DM(1,3),DM(1,4)/.93,1.73,2.62,3.33/
      DATA DM(1,5),DM(1,6),DM(1,7),DM(1,8)/3.62,3.62,3.62,3.62/
      DATA DM(1,9),DM(1,10),DM(3,9),DM(3,10)/3.62,3.62,3.62,3.62/
      DATA DM(3,1),DM(3,2),DM(3,3),DM(3,4)/2.27,2.89,3.31,3.55/
      DATA DM(3,5),DM(3,6),DM(3,7),DM(3,8)/3.62,3.62,3.62,3.62/
      DATA (FC(I),I=1,4)/5.,10.,10.,5./
C
      NRHS=4
      NBND=1
C
      DO 10 K=1,NRHS
   10 HP(1,K)=50.
```

```
      HP(2,1)=2.25
      HP(2,2)=3.375
      HP(2,3)=4.5
      HP(2,4)=5.625
C
      ZE(1)=.3
      ZE(2)=.1
      GR(1)=1.08
      GR(2)=1.04
C
      DO 124 L=1,LX
  124 RHO(L)=1.
      RHO(2)=.5
      RHO(5)=.5
C
      DO 176 H=1,HX
      DEM=DM(1,H)*.33
      DM(1,H)=DEM*.6*(1.-ZE(1))
      DM(2,H)=DEM*.6*ZE(1)
      DM(4,H)=DEM*.4*(1.-ZE(2))
      DM(5,H)=DEM*.4*ZE(2)
      DM(6,H)=(DM(3,H)-.5)*6./11.
      DM(7,H)=(DM(3,H)-.5)*5./11.
  176 DM(3,H)=.5
      DO 177 L=1,LX
      DO 177 H=1,HX
  177 DM(L,H)=DM(L,H)*RHO(L)
C
      DEM=.44*.204
      DEP(1,3)=DEM*.33*.6*(1.-ZE(1))
      DEP(2,3)=DEM*.33*ZE(1)*.6*RHO(2)
      DEP(4,3)=DEM*.33*.4*(1.-ZE(2))
      DEP(5,3)=DEM*.33*.4*ZE(2)*RHO(5)
C
      DEP(3,3)=.4*.056
      DEM=(1.44-.4)*.056
      DEP(6,3)=6./11.*DEM
      DEP(7,3)=5./11.*DEM
C
      DO 6 I=1,2
      DO 6 L=1,LX
 6    DEP(L,I)=DEP(L,3)*GR(INI(L))**(10*I-30)
C
      T=.TRUE.
      F=.FALSE.
C
      BETA=1./1.1
C
      DO 500 I=1,2
      DO 500 J=1,3
      DO 500 K=1,4
  500 U(I,J,K)=0.
C
      DO 60 I=1,IX
      DO 60 L=1,LX
   60 LC(I,L)=T
C
      DO 62 K=1,4
      I=I1(K)
      DO 62 J=1,4
```

```
      L=J1(J)
   62 LC(I,L)=F
C
      DO 142 I=1,IX
      DO 142 L=1,LX
      CUR(I,L)=0.
  142 CAP(I,L)=0.
      DO 144 I=1,4
      CUR(1,J1(I))=30.
      CUR(3,J1(I))=5.8
      CUR(4,J1(I))=3.5
  144 CUR(6,J1(I))=2.2
C
      I1(1)=2
      I1(2)=5
      I1(3)=10
      I1(4)=11
      J1(1)=3
      J1(2)=6
      J1(3)=7
C
      DO 170 L=1,LX
      DO 170 I=1,IX
  170 YN(I,L)=1.
      DO 172 I=2,4
      YN(I1(I),3)=1.2
      YN(I1(I),6)=1.5
  172 YN(I1(I),7)=2.
C
      DO 64 K=1,4
      I=I1(K)
      DO 64 J=1,3
      L=J1(J)
   64 LC(I,L)=F
C
      DO 65 I=1,IX
   65 LUB(I)=F
      LUB(1)=T
      LUB(2)=T
      LUB(7)=T
      LUB(8)=T
      LUB(9)=T
      LUB(11)=T
C
      DO 66 I=1,5
      DO 66 J=1,JX
   66 V(I,J)=0.
C
      DO 126 I=1,3
      DO 126 K=1,3
      AR(I,K)=0.
      AF(I,K)=0.
      DR(I,K)=0.
  126 DF(I,K)=0.
C
      DF(1,1)=.5
      DF(2,2)=2.
      DF(1,3)=.54
      AR(1,1)=.03
      AR(2,1)=.17
```

```
      AR(2,2)=.16
      AR(3,2)=.29
      AR(3,3)=.19
      AF(1,1)=.21
      AF(3,3)=.48
      DR(1,1)=.1
      DR(2,2)=2.
      DR(3,3)=1.35
C
      ETA(1)=.4
      ETA(2)=1.
      ETA(3)=.33
      ETA(4)=.4
      ETA(5)=.4
      ETA(6)=.19
      ETA(7)=0.
      ETA(8)=0.
      ETA(9)=0.
      ETA(10)=.35
      ETA(11)=.26
      DO 156 I=1,IX
  156 ETB(I)=ETA(I)
      ETB(5)=.5
C
      DO 514 K=1,NBND
      DO 514 I=1,IX
      DO 514 H=1,HX
  514 BVAL(I,H,K)=0.
      DO 182 K=1,NBND
      DO 182 H=1,HX
      BVAL(3,H,K)=.04+.06*FLOAT(H-1)
      BVAL(4,H,K)=.04+.06*FLOAT(H-3)
      BVAL(5,H,K)=.04+.06*FLOAT(H-3)
      BVAL(6,H,K)=ETB(4)/ETB(6)*(.04+.06*FLOAT(H-3))
  182 BVAL(10,H,K)=ETB(5)/ETB(10)*(.04+.06*FLOAT(H-3))
      DO 184 K=1,NBND
      DO 184 H=1,HX
      DO 184 I=1,IX
      IF (BVAL(I,H,K).GE.0.) GOTO 184
      BVAL(I,H,K)=0.
  184 CONTINUE
C
      V(1,1)=.3
      V(2,2)=.3
      DO 548 I=3,5,2
      DO 550 J=4,6
  550 V(I,J)=(AF(J-3,I-2)-AR(J-3,I-2))*10.*ETA(I)
  548 V(I,3)=V(I,4)
      V(4,5)=ETA(4)*AF(2,2)*10.
      V(4,6)=-ETA(4)*AR(3,2)*10.
C
      DO 146 L=1,4,3
      CAP(1,L)=192.
      CAP(3,L)=200.
      CAP(4,L)=264.
  146 CAP(6,L)=245.
      DO 148 L=2,5,3
      CAP(1,L)=384.
      CAP(3,L)=400.
      CAP(4,L)=528.
```

```
  148 CAP(6,L)=354.
      CUR(2,3)=74.4
      CUR(5,3)=34.3
      CUR(10,3)=23.1
      CUR(11,3)=14.4
      CUR (2,6)=40.3
      CUR(5,6)=15.4
      CUR(10,6)=9.9
      CUR(11,6)=4.4
      CUR(2,7)=91.9
      CUR(5,7)=40.3
      CUR(10,7)=27.3
      CUR(11,7)=17.6
C
      DO 150 L=3,6,3
      CAP(5,L)=220.
      CAP(10,L)=270.
  150 CAP(11,L)=20.
      CAP(5,7)=376.
      CAP(10,7)=379.
      CAP(11,7)=102.
C
      DO 544 I=1,3
      DO 546 J=2,4
      U(1,I,J)=10.*ETA(I+2)*DF(J-1,I)
  546 U(2,I,J)=-10.*ETA(I+2)*DR(J-1,I)
      U(1,I,1)=U(1,I,2)
 544  U(2,I,1)=U(2,I,2)
C
C                         PROBLEM NAME
C
      WRITE (6,200) PNAME
C
C                         ROW - IDENTIFICATION
C
      WRITE (6,202)
      WRITE (6,212)
      DO 300 J=1,JX
      DO 300 H=1,HX
      IF (J.LE.2.AND.H.NE.HX) GOTO 300
      WRITE (6,214) RTP(J),CS,RS(J),H
  300 CONTINUE
      DO 302 L=1,LX
      DO 302 H=1,HX
      WRITE (6,216) DMND,L,H
  302 CONTINUE
      DO 306 H=1,HX
  306 WRITE (6,214) G,DI,BRGN,H
C
      DO 312 I=1,IX
      DO 312 L=1,LX
      DO 312 H=1,HX
      IF (LC(I,L)) GOTO 312
      WRITE (6,244) UT,I,L,H
  312 CONTINUE
      DO 304 I=1,IX
      DO 304 H=1,HX
      DO 560 L=1,LX
      IF (.NOT.LC(I,L)) GOTO 562
  560 CONTINUE
```

```
      GOTO 304
  562 WRITE (6,260) AGPC,I,H
  304 CONTINUE
      DO 310 I=1,IX
      DO 310 H=1,HX
      DO 564 L=1,LX
      IF (.NOT.LC(I,L)) GOTO 566
  564 CONTINUE
      GOTO 310
  566 WRITE (6,260) AGDP,I,H
  310 CONTINUE
C
C                     DP - COLUMNS DISAGGREGATED
C
      WRITE (6,204)
C
      DO 530 I=1,IX
      DO 530 L=1,LX
      DO 530 H=1,HX
      IF (LC(I,L)) GOTO 530
      WRITE (6,220) DP,I,L,H,AGDP,I,H,AS
      QW=BETA**(10*H-ITAU-5)*CAP(I,L)*10.
      IF (H.GT.HX-3) QW=QW*(1.-BETA**(10*(HX-H+1)))
      WRITE (6,218) DP,I,L,H,QW
      HI=H+3
      IF (H+3.GT.HX) HI=HX
      DO 532 H1=H,HI
  532 WRITE (6,242) DP,I,L,H,UT,I,L,H1,FC(H1-H+1)
  530 CONTINUE
C
C                     DP - COLUMNS AGGREGATED
C
      DO 534 I=1,IX
      DO 534 H=1,HX
      DO 556 L=1,LX
      IF (.NOT.LC(I,L)) GOTO 558
  556 CONTINUE
      GOTO 534
  558 WRITE (6,256) DP,BK(I),H,AGDP,I,H,AQ
      IF (I.LT.3.OR.I.GT.5) GOTO  534
      H1=H+2
      IF (H+2.GT.HX) H1=HX
      DO 536 HI=H,H1
      DO 536 J=3,JX
      IF (U(1,I-2,J-2)) 538,536,538
  538 WRITE (6,226) DP,BK(I),H,CS,RS(J),HI,U(1,I-2,J-2)
  536 CONTINUE
      HI =H+3
      IF (HI.GT.HX) GOTO 534
      DO 540 H1=HI,HX
      DO 540 J=3,JX
      HT=U(1,I-2,J-2)+U(2,I-2,J-2)
      IF (HT) 542,540,542
  542 WRITE  (6,226) DP,BK(I),H,CS,RS(J),H1,HT
  540 CONTINUE
  534 CONTINUE
C
C                     INITIAL CONDITION COLUMNS
C
      DO 2 L=1,LX
```

```
      DO 2 H=1,3
      H1=H-3
      DO 2 HI=1,H
    2 WRITE (6,102) DP,FO(L),  L,H1,UT,INI(L),L,HI,FC(HI-H1+1)
C
C                        PC - COLUMNS DISAGGREGATED
C
      DO 520 I=1,IX
      DO 520 L=1,LX
      DO 520 H=1,HX
      IF (LC(I,L)) GOTO 520
      WRITE (6,242) PC,I,L,H,UT,I,L,H,AQ
      WRITE (6,220) PC,I,L,H,AGPC,I,H,AS
      HT=ETB(I)*YN(I,L)
      WRITE (6,220) PC,I,L,H,DMND,L,H,HT
      QQ=BETA**(10*H-5)*CUR(I,L)*10.
      WRITE (6,218) PC,I,L,H,QG
      IF (I.NE.11) GOTO 520
      IF (L.NE.3.AND.L.NE.6.AND.L.NE.7) GOTO 520
      HT=-ETA(3)
      WRITE (6,220) PC,I,L,H,DMND,JD,H,HT
  520 CONTINUE
C
C                        PC - COLUMNS AGGREGATED
C
      DO 524 I=1,IX
      DO 524 H=1,HX
      DO 552 L=1,LX
      IF (.NOT.LC(I,L)) GOTO 554
  552 CONTINUE
      GOTO 524
  554 IF (I.EQ.4) WRITE (6,226) PC,BK(I),H,DI,BRGN,H,AS
      WRITE  (6,256) PC,BK(I),H,AGPC,I,H,AQ
      IF (I.GT.5) GOTO 524
      DO 526 H1=H,HX
      DO 526 J=1,JX
      IF (J.LE.2.AND.H1.NE.HX) GOTO 526
      IF (V(I,J)) 528,526,528
  528 WRITE (6,226) PC,BK(I),H,CS,RS(J),H1,V(I,J)
  526 CONTINUE
  524 CONTINUE
C
C                          ANNUHC - COLUMNS
      DO 112 H=1,HX
      HT=770.*BETA**(10*H-5)
      WRITE (6,234) H,HT
      DO 112 H1=H,HX
  112 WRITE (6,226) AN,NUHC,H,CS,RS(3),H1,WA
C
C                          PCFBPL - COLUMNS
C
      DO 116 H=1,HX
      WRITE (6,226) PC,FBPL,H,DI,BRGN,H,AQ
      DO 116 H1=H,HX
      HT=-10.*ETA(4)*AR(2,2)
      WRITE (6,226) PC,FBPL,H,CS,RS(5),H1,HT
      HT=10.*ETA(4)*AR(3,2)
      WRITE (6,226) PC,FBPL,H,CS,RS(6),H1,HT
  116 CONTINUE
C
```

```
C                         RIGHT HAND SIDE
C
      WRITE (6,206)
      DO 192 K=1,NRHS
      DO 186 L=1,LX
      DO 186 H=1,HX
  186 WRITE (6,238) K,DMND,L,H,DM(L,H)
      DO 190 I=1,2
  190 WRITE (6,236) K,CS,BK(I),HX,HP(I,K)
      DO 192 H=1,HX
      HT=2.
  192 WRITE (6,236) K,CS,RS(3),H,HT
C
C                         BOUNDS SECTION
C
      WRITE (6,208)
      DO 8 K=1,NBND
      DO 174 I=1,IX
      DO 174 H=1,HX
      IF (LUB(I)) GOTO 174
      HT=FX
      IF (BVAL(I,H,K).GT.0.) HT=UP
      WRITE (6,258) HT,K,DP,BK(I),H,BVAL(I,H,K)
  174 CONTINUE
C
      DO 4 L=1,LX
      DO 4 H=1,3
      H1=H-3
    4 WRITE (6,104) FX,K,DP,FO(L),L,H1,DEP(L,H)
    8 CONTINUE
C
      WRITE (6,210)
C
      STOP
  102 FORMAT (T5,A2,A2,I1,I2,T15,A2,3I2,T25,F12.5)
  104 FORMAT (1X,A2,4H BND,I1,6X,A2,A2,I1,I2,3X,F12.5)
  200 FORMAT (4HNAME,10X,A10)
  202 FORMAT (4HROWS)
  204 FORMAT (7HCOLUMNS)
  206 FORMAT (3HRHS)
  208 FORMAT (6HBOUNDS)
  210 FORMAT (6HENDATA)
  212 FORMAT (2H N,2X,4HCOST)
  214 FORMAT (1X,A1,2X,A2,A4,I2)
  216 FORMAT (2H G,2X,A4,2I2)
  218 FORMAT (4X,A2,3I2,2X,4HCOST,6X,F12.5)
  220 FORMAT (4X,A2,3I2,2X,A4,2I2,2X,F12.5)
  226 FORMAT (4X,A2,A4,I2,2X,A2,A4,I2,2X,F12.5)
  234 FORMAT (4X,6HANNUHC,I2,2X,4HCOST,6X,F12.5)
  236 FORMAT (4X,3HRHS,I1,6X,A2,A4,I2,2X,F12.5)
  238 FORMAT (4X,3HRHS,I1,6X,A4,2I2,2X,F12.5)
  242 FORMAT (4X,A2,3I2,2X,A2,3I2,2X,F12.5)
  244 FORMAT (2H G,2X,A2,3I2)
  256 FORMAT (4X,A2,A4,I2,2X,A4,2I2,2X,F12.5)
  258 FORMAT (1X,A2,4H BND,I1,6X,A2,A4,I2,2X,F12.5)
  260 FORMAT (4H E   ,A4,2I2)
      END
END OF RECORD
            INPUT      $
            SET        LCUSER1    1
```

```
DOLP        SET         LBRANCH     LCUSER1
            PERFORM     SET11       SET12       SET13       SET14
            SELECT
            SET         LBRANCH     LCUSER1
            PERFORM     BASIS       BS          BS          BS
            PRIMAL
            BASISOUT    $
            OUTPUT      FULL
            STEP        LCUSER1     1
            TEST        LCUSER1     5
            BRANCH      DOLP        *
            EXIT
SET11       SET         KNDIR       MIN
            SET         KNOBJ       COST
            SET         KNBND       BND1
            SET         KNRHS       RHS1
            TITLE       RS 8.1.40 BASE CASE
            NEXT
SET12       SET         KNRHS       RHS2
            TITLE       RS 8.1.60 BASE CASE
            NEXT
SET13       SET         KNRHS       RHS3
            TITLE       RS 8.1.80 BASE CASE
            NEXT
SET14       SET         KNRHS       RHS4
            TITLE       RS 8.1.100 BASE CASE
            NEXT
BASIS       CRASH
            NEXT
BS          BASISIN     $           TAPE4
            NEXT
END OF FILE
```