EXCHANGE RATE DETERMINATION AND CURRENCY SUBSTITUTION: MICRO ANALYSIS AND MACRO IMPLICATIONS

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FOREWORD

This Collaborative Paper is one of a series embodying the outcome of a workshop and conference on Economic Structural Change: Analytical Issues, held at IIASA in July and August 1983. The conference and workshop formed part of the continuing IIASA program on Patterns of Economic Structural Change and Industrial Adjustment.

Structural change was interpreted very broadly: the topics covered included the nature and causes of changes in different sectors of the world economy, the relationship between international markets and national economies, and issues of organization and incentives in large economic systems.

There is a general consensus that important economic structural changes are occurring in the world economy. There are, however, several alternative approaches to measuring these changes, to modeling the process, and to devising appropriate responses in terms of policy measures and institutional redesign. Other interesting questions concern the role of the international economic system in transmitting such changes, and the merits of alternative modes of economic organization in responding to structural change. All of these issues were addressed by participants in the workshop and conference, and will be the focus of the continuation of the research program's work.

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1. Introduction

The purposes of this essay are threefold: first, to provide a microtheoretic framework that incorporates the transaction motive for holding money in a multicurrency world; second, to show how the recent "currency substitution theory" can be imbedded in the micro choice model we develop; third, to derive some comparative static results from a macro model that allows agents to hold different currencies.

Recently, Cuddington [1982] noted in a paper discussing the issue of currency substitution that we still lack an explicit microtheoretic framework clarifying the transaction roles of different currencies in a multicurrency world. Tobin [1982] also seems to call for an analysis of the service yields of different currencies. The next section discusses what properties a transaction technology describing the transaction roles of currencies should satisfy. Section 3.1 then formalizes the individual choice problem that determines the demand for the various currencies.

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The second part of section 3 describes how the currency substitution literature is related to our model. Several results on currency substitution are shown to be nested hypotheses of our general model.

A short-run general equilibrium model for two open economies is developed in the fourth section in order to derive some implications in the presence of currency substitution. The way in which expectations are introduced into the model can be desrcibed as a form of bounded rationality. The exchange rate in the model has the feature that it is an element of the price to holders of one currency of every asset and commodity denominated in the other currency.

In the last section we investigate the issue of currency substitution on a macro level. By comparing comparative static results for economies with and without currency substitution present, we are able to analyze the qualitative and quantitative aspects of currency substitution.

Mathematics is relegated to the appendices, together with a list of symbols.

2. The Transaction Technology

The purpose of this section is to discuss a transaction technology, which rationalizes agents' decisions to hold money as a way of economizing on the time spent in completing transactions. We start with an overview of the literature.

Patinkin's [1965, p.82] argument for including money in the utility function is the nonsynchronization between payments and receipts. This, however, implies that full monetization is implicitly assumed. Wallace

[1980, p.49] dismisses the approach because the inclusion of money in the utility function begs too many questions. For example: "What if there are several fiat moneys, those of different countries?" Wallace argues that some kind of friction has to be introduced to give fiat money value, and uses the intergenerational friction of the overlapping generations models. Another type of friction is the Clower [1967] constraint: "Only money buys goods." But as Hahn [1982, p.20] argues: "It assumes what should be explained."

Still another kind of friction stems from the transaction technology prevalent in the economy. Gale's [1982] basic assumption is that agents are not trust-worthy, and therefore quid pro quo characterizes each transaction. In such a sequence economy agents have to satisfy their budget constraints all the time, and this can be achieved more efficiently by using assets than by balancing with commodities only. The second assumption is that money has the least informational cost for enquiring into its future value, completing the argument for the positive value of money. The main argument in Alchian [1977] is that the costs of identifying qualities of a good determine wich good will be used as a money. Ignorance of qualities of goods will provoke efforts to reduce that ignorance in order to achieve more trade. In Alchian's words: "If some good were sufficiently and most cheaply identifiable so that everyone were like an expert in it, the cost of exchanging that good for any other good would be less than if a more costly to identify good were offered, and it will become a money." In Jones' article [1967] individuals try to minimize the time involved in transactions by minimizing the expected number of encounters to fulfill their trading plans. Jones gives a condition when indirect trade can effect the ultimate exchange in fewer expected encounters than can direct trade. The argument is then that indirect trade by using the most prevalent commodity saves transaction time in an exchange economy.

In our view these arguments can be reconciled as follows. The opportunity costs of enquiring into the trustworthiness of an agent and the future value of assets is the leisure time foregone. Similarily, one has to invest time in examining the qualities of a good. Lastly, Jones' argument is directly based on the time saved by using money. Hence, we take it that an essential feature of money is that it enables agents to reduce their time spent on completing transactions. For a clear account of this point of view, see Clower [1969].

If there is more than one currency we have to explain how agents choose their currency mix. We will develop a transaction technology that explains the demand for different currencies based on the time-saving value of money. We require the transaction technology to exhibit the following properties:

1. Foreign currencies are held domestically to facilitate international transactions. This line of thought has been developed by Swoboda [1968], based on the inventory argument for holding money.
2. Money is like a language. To see what we mean by this, we quote from Tobin [1980, p.86]: "Another time-honored observation of monetary economists is the analogy of money and language. Both are means of communication. The use of a particular language or a particular money by one individual increases its value to other actual or potential users. Increasing returns to scale, in this sense, limits the number of languages or moneys in a society..." If one currency is much more

commonly used than another currency at a certain date and place, then invoking Jones' and Tobin's arguments, the one currency will be the principal one trough which an agent can reduce his transaction-time costs. On the other hand, if another currency is the more prevalent then the roles of the currencies should be the reverse. In cases where both currencies are widely used their usefulness should be equal.

3. Differences in transaction technologies between countries can arise out of man-made constraints. For example, a government could require that taxes be paid in the national currency. This is then a kind of Clower constraint.
4. In priniciple, any currency should be able to fulfill a transactions role as well as any other currency. This has been argued for example by Hayek [1976] and is the subject of the recent currency substitution literature. Thus, the transaction technology should allow for changes in the habits discussed under property 2.
5. The resulting demand functions for money should exhibit the usual homogeneity properties.
6. We will require that holding more of a currency reduces the time involved in accomplishing transactions, but does so at a diminishing rate. Moreover, money is not essential in that the derived marginal utility from holding money does not go to infinity if no stock of money is held, i.e. if one engages in pure barter.

There are a few examples in the literature that use such a transaction technology if only one currency is present. See for example Arrow and Hahn [1971, chapter 14], McCallum [1982], and Greenwood

[1983]. A model of this type, in which more than one currency is held, is presented in the next section.

3. Micro Choices and Currency Substitution

3.1. The individual's choice problem

We start with a description of the economic setting in which the individual takes his decisions. The world consists of two countries, which produce two commodities, and each country's government supplies a national currency and a bond. Divide time up in discrete periods, which will be referred to as Hicksian weeks. Imagine that at the beginning of such a Hicksian week, prices are established in auctioneer markets and contracts for delivery are finalized. During the rest of the week, output is generated and deliveries are made. To settle transactions, agents will hold cash balances. We would like to give a summary account of the individual's economic behavior during the Hicksian week. To make such a description possible, we assume that an agent maximizes a utility function subject to his budget constraint. The individual's present choices are influenced by his expectations about the future state of the economy. In particular, expectations will influence the agent's demand for assets. Assume that the agent's capability of looking forward is limited to a finite number of weeks; for simplicity, we take the planning horizon to be one week. For the coming week the agent perceives different states of nature to be possible, and attaches some probability to the occurrence of each state.

A precise description of the individual's choice problem is as follows. The individual maximizes his intertemporally separable utility function subject to some restraints. All prices and income are known to the individual for the first period, but when planning for the second period these variables are still unknown. It is assumed, however, that the individual perceives a set of possible future states of the world, to the occurrence of which he attaches a subjective probability measure. The individual plans for the second period by maximizing his expected utility function. Arguments of the utility function are the consumption levels of the two commodities x and z, and the available leisure time. Assuming that the wage rate and labor hours are fixed, the individual divides his leftover time t between conducting transactions and leisure. Transactions are facilitated by the use of money and thus leisure time can be increased by holding more money. The transaction technology $s(.)$ models the exchange frictions. It indicates the time used in completing transactions as a function of the domestic currency m and the foreign currency l, as well as some variables representing payment habits h. A price index $n = n\,[p,q]$, satisfying the axioms discussed in Eichhorn [1978, p.153], such as homogeneity of degree one in p and q, is used to deflate money holdings. The budget constraints are denoted in the domestic currency, and e represents the exchange rate measured as the cost of the foreign currency. The agent's fixed wage income is denoted by y. Government taxes or subsidies to the individual are denoted by g. The individual can hold domestic or foreign bonds b and d with rates of return $(r-1)$ and $(i-1)$ respectively, so that, for example, one plus the domestic interest rate equals r. The individual's pure rate of time preference is indicated by the factor ρ, where $1 > \rho > 0$. Forward exchange purchases k can be made against the forward rate f. Possible capital market restrictions are modelled by limiting individual purchases of foreign bonds up to the amount $\bar{d}$. Situations in which forward markets are absent can be studied by setting k equal to zero. The states of the world are indicated by j, and each state has probability $\pi(j)$, $\Sigma_j \pi(j) = 1$. The time dimension of a variable is indicated between parentheses following the variable. The previous period is indicated by (-1), the present-period variables do not carry a time indicator, and the contingent future variables are indicated by (j).

Formally, the individual's choice problem may be expressed as:

(1) maximize:

$$V = U\left[x,z,t - s\left(\frac{m}{n},\frac{el}{n},h\right)\right] + \sum_{j=1}^{n} \pi(j)\, W\left[x(j),z(j),t - s\left(\frac{m(j)}{n(j)}, \frac{e(j)l(j)}{n(j)}, h\right)\right],$$

where

$W[.] = \rho U[.]$, with $1 > \rho > 0$, $n = n[p,q]$, and $s(.) < t$, $s'(.) < 0$,

subject to:

(2) $y + m(-1) + el(-1) + r(-1)b(-1) + ei(-1)d(-1) + ek(-1) -$
$f(-1)k(-1) - g - px - qz - m - el - b - ed \geq 0,$

and

$$(3) \quad y(j) + m + e(j)l + rb + e(j)id + e(j)k - fk - g(j) - p(j)x(j) - q(j)z(j) - m(j) - e(j)l(j) \geq 0, \text{ for all } j = 1,2,\ldots,n$$

and if relevant, the capital market constraint

$$(4) \quad \bar{d} - d \geq 0.$$

Somewhat similar choice problems have been discussed in the literature. The reader might consult, for example, the articles by Stockman [1978, 1980] or the paper by Stulz [1982]. In contrast with our setup, Stockman models the demand for money by way of Clower constraints. A major difference between Stulz' approach and ours is that Stulz assumes "... individuals produce consumption, using commodities, cash balances and labor as inputs." In this case a function of x, m and el would enter the utility function as the argument for the consumption of x. However, in this approach it is not clear how money is used for settling transactions in bonds.

The necessary first-order conditions for the optimization problem are relegated to Appendix A, but some are stated here because we make repeated use of them below (see Appendix A eqns. (A3) - (A8)):

$$(5) \quad U_{m/n} - \xi n + \Sigma\lambda(j)\pi(j)n \leq 0,$$

$$(6) \quad U_{el/n} - \xi n + \Sigma\lambda(j)\pi(j)\frac{e(j)}{e}n \leq 0,$$

$$(7) \quad - \xi + \Sigma\lambda(j)\pi(j)r \leq 0,$$

(8) $\quad - \xi e + \Sigma\lambda(j)\pi(j)e(j)i - \phi \leq 0,$

(9) $\quad \Sigma\lambda(j)\pi(j)e(j) - \Sigma\lambda(j)\pi(j)f \leq 0,$

(10) $\quad W_{x(j)} - \lambda(j)p(j) \leq 0,$ for all j,

where ξ, $\lambda(j)$ and ϕ are the Lagrange multipliers for the constraints (2), (3) and (4). In the following discussion we assume that the solution of the optimization problem is such that the demand for all goods is positive, unless explicitly stated to the contrary.

With unrestricted capital markets, the above conditions can be manipulated to arrive at the interest parity condition $f/e = r/i$. If the interest parity condition does not hold in the market, suppose for example that $f/e > r/i$, then no domestic bonds are demanded. The reason is, that the individual can make gains through riskless arbitrage. With free capital markets, a deviation from the interest parity will be exploited by individuals and can therefore be expected to evaporate rapidly. However, if individuals are restrained in their purchases of foreign bonds, $f/e > r/i$ can be compatible with the indivual's choice. With capital market restrictions present in the form of an individual quota on foreign bonds, we derive from the above conditions that $f/e = (1+\phi/\xi e)\ r/i$. The forward premium differs from the interest rate differential by a factor $\phi/\xi e$. The ratio ϕ/ξ gives the marginal change in present income due to a marginal change in the quota on foreign bonds.

The forward rate f can be expressed as the sum of three terms. To do this, note that the expected future exchange rate is

$E[e(j)] = \Sigma\pi(j)e(j)$, and that by using (10) the forward rate is given by

$$f = \frac{\Sigma\pi(j)W_{x(j)}/p^*(j)}{\Sigma\pi(j)W_{x(j)}/p(j)},$$

where $p^*(j) = p(j)/e(j)$, i.e. the foreign price of $x(j)$. Hence

$$f = E[e(j)] + \left(\frac{\Sigma\pi(j)W_{x(j)}/p^*(j)}{\Sigma\pi(j)W_{x(j)}/p(j)} - \frac{\Sigma\pi(j)/p^*(j)}{\Sigma\pi(j)/p(j)}\right) + \left(\frac{\Sigma\pi(j)e(j)/p(j)}{\Sigma\pi(j)/p(j)}\right.$$

$$\left. - \Sigma\pi(j)e(j)\right)$$

$$= E[e(j)] + a + o.$$

The forward exchange rate equals the sum of the expected future exchange rate $E[e(j)]$ plus a risk premium a and a convexity term o.
The risk premium a stems from exchange rate uncertainty. Note that a is zero if $e(j) = e(1)$ for all j states, i.e. there is no exchange rate risk; but it is nonzero in general if $p(j) = p(1)$ for all j states, i.e. if there is no domestic commodity price risk. The convexity term o can be viewed as a risk premium for domestic commodity price uncertainty; it is zero if $p(j) = p(1)$ for all j states. Unlike the risk premium a, the convexity term o does not depend on people's attitudes toward risk. In case $p(j)$ has positive variance o is nonzero due to Jensen's inequality. The convexity term is also zero in the absence of exchange rate uncertainty.

The marginal rate of substitution between the foreign and the domestic currency can now be expressed in four different ways

$$(11) \qquad \frac{U_{el/n}}{U_{m/n}} = \frac{(i-1)}{(r-1)}\frac{f}{e},$$

$$(12) \qquad = \frac{(i-1)/i}{(r-1)/r},$$

$$(13) \qquad = \frac{(i-1)}{(r-1)}\frac{1}{e}(E[e(j)] + a + o),$$

$$(14) \qquad = \frac{(r-1) - (f/e-1)}{(r-1)}.$$

With unrestricted capital markets, the opportunity loss in allocating a unit of income to the holding of the domestic currency instead of bonds is the interest (r-1) foregone. The opportunity loss in allocating a unit of income to the holding of the foreign currency instead of foreign bonds is the foreign interest (i-1) foregone times the capital gain f/e induced by currency revaluations (see (11)). This latter capital gain is tied to the interest rate by the interest parity condition (12). The third interpretation (13) exhibits the individual's evaluation of the costs he associates with the elimination of exchange risk through forward sales. Still another interpretation is given by (14). The numerator in (14) most clearly exhibits the opportunity costs of holding the foreign currency. The opportunity loss equals the domestic rate of interest foregone minus the forward premium (or, plus the forward discount).

When the capital market restriction is binding, (11) becomes

$$\frac{U_{el/n}}{U_{m/n}} = \frac{(i-1)}{(r-1)}\frac{f}{e} - \frac{if - er}{(r-1)e}.$$

It was shown above that in the case of imperfect capital markets

$f/e > r/i$. Thus when trade in foreign assets is limited, the opportunity loss from holding the foreign currency relative to the domestic currency declines. This leads to a substitution of the foreign currency for the domestic currency. Stated differently, if one is unable to purchase and hold all the intended foreign interest bearing assets, one partially compensates for this restriction by holding more of the foreign currency, because in this way the capital gain due to currency revaluations can still be made. Note that (14) still holds, but, given the two interest rates, the forward premium $(f/e-1)$ has risen due to the quota on foreign bonds.

To arrive at our next result, recall the way in which cash balances enter the agent's utility function. Only the two commodities x and z and leisure time u are arguments of the utility function. But, part of the leisure time has to be used for completing transactions due to exchange frictions. The individual is therefore left with an amount of leisure time $u = t-s(.)$. Cash balances are arguments of $s(.)$ because they smooth the exchange process. The marginal utility of holding m can therefore be expressed as $U_{m/n} = - U_u s_{m/n}$. Hence, the marginal rate of substitution between the foreign and domestic currency in the presence of perfect capital markets (12) can be written simply as

$$(12') \quad \frac{s_{el/n}}{s_{m/n}} = \frac{(i-1)/i}{(r-1)/r}.$$

The only two choice variables that are arguments of the function $s(.)$ are the two currencies m and l. This leads to the following conclusion. The way in which the two currencies m and l are optimally combined in

<u>the presence of perfect capital markets does not depend on the individual's tastes, and particularly not on his attitudes towards risk.</u> The optimal combination of m and l is dictated by the transaction technology of the economy and the interest rates. For somewhat similar results, see Fama and Farber [1979] and Stulz [1982]. The reason we can write the marginal rate of substitution between m and l as in (12') is the fact that m and l are in a seperate branch of the utility tree. (See Strotz [1957] for a discussion of these concepts.) The reason the right-hand sides in (12) and (12') can be stated solely in terms of interest rates is that the bond markets are perfect. Were the bond markets and forward market absent or imperfect, the right-hand side in (12') would still depend on the present and future marginal utilities of income. This, in its turn, would imply that the optimal combination of the two currencies depends on the risk assessment of the individual.

In the case where no foreign bonds can be bought or held and forward purchases are not possible either, we can derive a specific condition for currency substitution to occur. Assume that the two currencies are perfectly substitutable for transaction purposes. In this case the function s(.) takes the specific form $s(.) = s(m/n+el/n, h)$, so that $s_{el/n} = s_{m/n}$. Divide (5) and (6), and find $e\Sigma\lambda(j)\pi(j) = \Sigma\lambda(j)\pi(j)e(j)$. Suppose that the sole cause of uncertainty is the exchange rate risk. In this case <u>the condition for diversification between the currencies at the point where no foreign currency is held initially, is $\Sigma\pi(j)e(j) > e$.</u> This is essentially Arrow's proposition [1974, p.100], that a risk averter always takes some part of a favourable bet.

3.2. A transaction technology example and relations with previous work.

The question we address here is whether the above micro model provides a unifying framework for the currency substitution literature. The articles by Girton and Roper [1981] and Miles [1978] are taken as examples, and we show that, if we choose the appropriate transaction technology, our model corresponds to those of Girton and Roper, and of Miles.

To specify our example transaction technology we define the following functions

(15) $s = s(c)$, with domain $c > 0$, range $t > s(c) > 0$, and derivatives $s'(c) < 0$ and $s''(c) > 0$,

(16) $$c = \left[\, g^m\left(\frac{m}{n}\right)^\tau + g^l\left(\frac{el}{n}\right)^\tau\right]^{1/\tau}, \quad \text{where } -\infty < \tau < 1,$$

(17) $$g^m = \left(\frac{M(-1)}{n}\right)^{\alpha_1}\left(\frac{M^*(-1)}{n}\right)^{\alpha_2}\left(\frac{eL(-1)}{n}\right)^{-\alpha_3}\left(\frac{eL^*(-1)}{n}\right)^{-\alpha_4}\left(N(-1)\right)^{\alpha_5}\left(N^*(-1)\right)^{-\alpha_6},$$

where all α's as well as $M(-1)$, $M^*(-1)$, $L(-1)$, $L^*(-1)$, $N(-1)$, and $N^*(-1)$, are positive, and

(18) $$g^l = \left(\frac{M(-1)}{n}\right)^{-\alpha_3}\left(\frac{M^*(-1)}{n}\right)^{-\alpha_4}\left(\frac{eL(-1)}{n}\right)^{\alpha_1}\left(\frac{eL^*(-1)}{n}\right)^{\alpha_2}\left(N(-1)\right)^{-\alpha_6}\left(N^*(-1)\right)^{\alpha_5}.$$

Substitution of all these functions into (15) gives the proposed transaction technology; and some further discussion of it is in order at this point.

Property 6 defined in the previous section is reflected by $s'(c) < 0$, $s''(c) > 0$, and $t > s(c) > 0$, i.e. s is a convex and bounded function on R^+. Because $t > s(c)$, the amount of leisure time u left, $u = t - s(c)$, is always positive. This implies that the derived marginal utility of money is always finite. The CES function in (16) is chosen because of the two particular articles we have chosen to discuss. Parameters τ, g^m, and g^l represent payment habits h. The parameter τ will be seen to define, in a certain sense, the elasticity of currency substitution σ. In the case of imperfect substitution, i.e. $-\infty < \tau < 1$, the ratio m/el will be relatively high if g^m/g^l is high. The parameters g^m and g^l try to capture Tobin's observation that the use of a particular currency by one individual increases its value to other actual or potential users; see property 2 defined above. This is done by means of the gravity equations (17) and (18). Let the total of domestically held stocks of currencies m and l in the previous period be denoted by M(-1) and L(-1), respectively, while abroad these stocks are indicated by M*(-1) and L*(-1). The transaction usefulness of the real stock of, say, currency m held by an individual, will be high if it represents a readily accepted means of payment; this in turn will presumably be so if the total real stock of currency m in the economy is high relative to the total real stock of l, and vice versa. Moreover, we have included the size of population in the two countries, N and N*, as factors that may possibly influence g^m and g^l. We assume that the agent has only past observations available on these aggregate variables, and therefore we lag them by one period. Above we assumed all M(-1), M*(-1), L(-1), and L*(-1) to be positive in the definition of g^m and g^l. If, for example, currency m is not held abroad, but l is, i.e. the asymmetric country assumption, then we define g^m and g^l as in (17) and (18) but omit the factors $(M^*(-1)/n)^{\alpha_2}$ and $(M^*(-1)/n)^{-\alpha_4}$. Inspecting equations (17) and (18), we see that the roles of the two currencies can be reversed symmetrically, so property 4 is satisfied. Moreover, payment patterns and thus g^m and g^l can change, because currency holdings change due to expectations as outlined in the micro optimization model. The homogeneity properties of this technology are easily established, i.e. property 5 is satisfied. The transaction technology defined by (15) - (18) allows for the usefulness of the foreign currency, and thus property 1 is satisfied.

The marginal rate of substitution between the foreign and domestic currency is, for this specific example,

$$(19) \qquad \frac{U_{el/n}}{U_{m/n}} = \frac{g^l}{g^m}\left(\frac{el}{m}\right)^{\tau-1} = \frac{(i-1)/i}{(r-1)/r}.$$

Define the elasticity of currency substitution, given perfect capital markets, as

$$(20) \qquad \sigma = \frac{d\ln(el/m)}{d\ln\frac{(i-1)/i}{(r-1)/r}} = \frac{1}{\tau - 1}.$$

This corroborates Miles' [1978] result. However, in our view, when measuring the opportunity costs of holding the foreign currency, account should also be taken of the forward premium. Therefore, the foreign interest rate (i-1) is multiplied by the factor f/e to measure capital gains through currency revaluations. Moreover, we have shown how this elasticity derives from a full-fledged theory of individual choice.

In the case where cash balances are measured as demand deposits on which interest is being paid, we have to measure the opportunity costs as follows. Let $(1-r^m)$ and $(1-r^l)$ be interest rates being paid on domestic and foreign demand deposits, respectively. Then the right-hand side of (11) becomes

$$(11') \quad \frac{U_{el/n}}{U_{m/n}} = \frac{(i-r^l)f/e}{(r-r^m)}.$$

Take logarithms of the above marginality condition (19) to obtain

$$(21) \quad \ln e = \ln\frac{m}{l} + \sigma[\ln(i-1) - \ln(r-1) + \ln\frac{r}{i}] + \sigma\ln\frac{g^m}{g^l}.$$

This expression corresponds closely to the basic equation (8) in Girton and Roper [1981]. We interprete $\ln(i-1) - \ln(r-1)$ as the nominal interest rate differential used by Girton and Roper. Secondly, $\ln(r/i)$ can be interpreted as an approximation for the anticipated rate of change in the exchange rate, because by the interest parity condition $r/i = f/e$. As eqn. (19) lends itself to aggregation if the ratio g^m/g^l is identical for all individuals, m and l can be interpreted as aggregate currency demands. One must be careful, however, not to deduce any causal macro relationship from eqn. (21). It only indicates how m and l should be optimally combined. In the aggregate both e, r and i are determined simultaneously. This will be shown in the next section. However, this fact was somewhat obscured in the presentation of Girton and Roper.

Girton and Roper arrive at an equation like (21) by combining LM equations for the domestic and foreign currency. But the exchange rate e is not an argument of their LM-schedules, whereas in our theory it would be; and the exchange rate is merely determined by the two

LM schedules. This leads Girton and Roper to conclude that any exchange rate is an equilibrium rate when substitution is perfect, see Girton and Roper [1981, p.16]. Their argument is as follows: if $\sigma = -\infty$, then for e to be finite in (21), it is neccessary that $r = i$ and $g^l = g^m$. Assume this to be the case. Thus we get from (21) $\ln e = \ln(m/l)$. Because of perfect substitutability however, m and l are not determinate themselves, and therefore e can take on any value. Note now that we could never have derived eqn. (21) in the case where $\sigma = -\infty$, because the marginality condition (19) reduces in this case to $g^l/g^m = (1-1/i)/(1-1/r)$. Whether e is determinate or not has still to be settled for our model. One finds indeed that only the sum $m + el$ is determinate with perfect capital markets and perfect substitutability. However, <u>if capital markets are imperfect, because of, say, a quota on foreign bonds, then m and l will be determinate</u>. This can be inferred from the first-order conditions for the individual's optimization problem. Thus, even if m and l are perfectly substitutable from a transactions-facilitating point of view, their difference in risk properties renders them determinate in the absence of perfect capital markets. Girton and Roper allude to this possibility when they conjecture that "transactions costs," in the sense of conversion costs, would render m and l determinate. One could view the quota on foreign bonds as a way of modeling these transactions costs. We note that m and l in the case of perfect substitutability and imperfect capital markets still depend on e. We conjecture that e will be determinate in an equilibrium situation for the whole economy.

We are still puzzled by Girton and Roper's observation that the individual money demands are not defined in their model in the case of perfect substitutability. Inspecting their eqn. (3), it appears that the demand for real cash balances, M_1/P_1 in Girton and Roper's notation, are still determinate in the case of perfect substitutability. (Setting $r_1 = r_2$, we get: $M_1/P_1 = \Theta_1(\omega) \exp \alpha_1(r_1-r)$.) Presumably, in the case of imperfect substitutability real money demand is equated with real money supply to arrive at the LM equation (3) of Girton and Roper. This procedure would give us nominal balances M_1, from either knowing the domestic price level P_1 or the nominal money supply. We see no reason why this cannot also be done in the case of perfect substitutability. After all, if the currencies are perfect substitutes in all dimensions, then we do not expect demand theory to offer an explanation for the currency ratio M_1/M_2 ; but the exogenous supplies of both currencies would completely determine M_1/M_2 . It seems that indeterminacy of e is indeed possible, but has to be argued in a different way.

4. Macro Model

4.1. Introduction

This section sets out to develop a short-run macro model for an open economy with both commodity and asset markets present. In this way we try to combine the flow market model of exchange rate determination with the asset market approach. The individual whose choice problem was discussed in the previous section has limited foresight to the extent that he is able to plan one period ahead for each perceived contingency. In this section we assume that the individual expects, for each state of the world, those prices to prevail which would clear all markets. The approach taken here is a form of bounded rationality; see Tobin [1982]. Both, flexible and fixed exchange rate regimes are discussed. Some comparative statics results are derived in the next section.

4.2. Supply Side

We assume that both countries are completely specialized in production. The domestic country produces total output X^o and the foreign country produces Z^o. Suppose that a production function F, with the stock of capital K fixed, but with variable labour inputs N, is an adequate representation of the supply side for our short-run model. The functional form of F is of the neoclassical type, with implicit costs of adjustment of the labor force possibly added to it. With nominal wages y fixed, the macro short-run production function is conveniently summarized as

$$(22) \quad X^o = F(K, F_N^{-1}(y/p)) \\ = X^o(p), \text{ and } X_p^o > 0.$$

We assume for simplicity that the short-run profits $P^x = pX^o - Ny$ are retained by firms and used for investment purposes, but that these investments are not realized in the periods under consideration. Moreover, we assume that each industry uses the product of the other industry as in input for the investment process. This investment process transforms the input into physical capital, which is installed in a future period. We could have introduced a third sector producing physical capital explicitly. This, however, would not alter the analysis significantly but would certainly clutter up the calculations. For the same reason, we assume complete specialization.

To summarize, the budget constraint of the supply side reads

$$(23) \quad pX^o - Ny - P^x = 0.$$

4.3. Demand Side

Assume that all individuals have identical preferences as defined by the expected utility function in the previous section. When aggregating individual demand functions, we have to take account of the possibility that these demand functions may not be identical because of differences in individual wealth levels. The level of individual wealth is $m(-1) + el(-1) + r(-1)b(-1) + ei(-1)d(-1)$. Differences in wealth of individuals arise because the level of employment N is variable. However, it can be shown that while the magnitudes of the partial derivatives of the demand functions differ across individuals, the signs do not. It would only obscure the computation of the macro comparative statics and not affect the results qualitatively if we were to distinguish between different cohorts of agents. Therefore, we introduce a fictitious representative agent whose demand functions, when multiplied by the number employed N, give the aggregate demand functions. For the first period the aggregate household constraint is

$$(24)\quad Ny + N(-1)m(-1) + eN(-1)l(-1) + r(-1)N(-1)b(-1) + ei(-1)N(-1)d(-1) - Ng - pNx - qNz - Nm - eNl - vNb - ewNd = 0.$$

It will facilitate the discussion later on to introduce prices v and w for domestic and foreign bonds, respectively. These prices represent the premium or discount at which the bond is sold given a fixed interest rate. In the previous section, the premium or discount was captured by the yields r and i. Throughout this section and the following section it is assumed that no forward markets exist.

The next period's constraints are

$$(25)\quad N(j)y(j) + Nm + e(j)Nl + rNb + e(j)iNd - N(j)g(j) - p(j)N(j)x(j) - q(j)N(j)z(j) - N(j)m(j) - e(j)N(j)l(j) = 0,$$

for all j states.

4.4. Government

The government purchases domestic commodities G^x/p and foreign commodities G^z/q, which are supplied to the unemployed according to some rationing scheme. The government can finance this by collecting taxes Ng, by monetary financing ΔM^s, or by the issue of bonds vB^s.
Assume that these bonds are one-period bonds. Therefore, the government needs to pay the rate of interest plus the principal sum on its outstanding bonds: $r(-1)B^s(-1)$. The government's first-period budget constraint reads

$$(26)\quad Ng + \Delta M^s + vB^s - G^x - G^z - r(-1)[N(-1)b(-1) + N^*(-1)b^*(-1)] = 0.$$

The contingent second-period finnancing constraints are

$$(27)\quad N(j)g(j) + \Delta M^s(j) - G^x(j) - G^z(j) - r[Nb + N^*b^*] = 0,$$

for all j states. Note that no new bond issues are foreseen. This has to do with the limited planning horizon of the agents, as will be explained below.

4.5. Balance of Payments

We add up the above sectoral constraints to arrive at the excess supply functions that make up the balance of payments. The home country's first-period balance of payments is

$$\begin{aligned}(28)\quad & p\{X^o - Nx - \tfrac{1}{p}G^x\} + q\{-Nz - \tfrac{1}{q}G^z - \tfrac{1}{q}P^x\} + \{N(-1)m(-1) + \Delta M^s - Nm\} + \\ & e\{N(-1)l(-1) - Nl\} + v\{B^s - Nb\} + ew\{ - Nd\} + \\ & ei(-1)N(-1)d(-1) - r(-1)N^*(-1)b^*(-1) = 0.\end{aligned}$$

Along the same lines, the foreign balance of payments denoted in the domestic prices is written as

(29)

$$p\{- N^*x^* - \frac{e}{p}G^{x}* - \frac{e}{p}P^{z}\} + q\{Z^{o} - N^*z^* - \frac{e}{q}G^{z}*\} + \{N^*(-1)m^*(-1) - N^*m^*\}$$

$$+ e\{N^*(-1)l^*(-1) + \Delta L^{s} - N^*l^*\} + v\{ - N^*b^*\} + ew\{D^{s} - N^*d^*\}$$

$$r(-1)N^*(-1)b^*(-1) - ei(-1)N(-1)d(-1) = 0.$$

Because the contingent second-period balances are very similar to the ones presented above, they are not stated here.

4.6. World Budget Constraints

With flexible rates the first-period world budget constraint is found by adding the two countries' balances of payments:

$$(30) \quad \begin{aligned} &p\{X^{o} - Nx - \frac{1}{p}G^{x} - N^*x^* - \frac{e}{p}G^{x}* - \frac{e}{p}P^{z}\} + \\ &q\{ - Nz - \frac{1}{q}G^{z} - \frac{1}{q}P^{x} + Z^{o} - N^*z^* - \frac{e}{q}G^{z}*\} + \\ &\{N(-1)m(-1) + \Delta M^{s} - Nm + N^*(-1)m^*(-1) - N^*m^*\} + \\ &e\{N(-1)l(-1) - Nl + N^*(-1)l^*(-1) + \Delta L^{s} - N^*l^*\} + \\ &v\{B^{s} - Nb - N^*b^*\} + ew\{ - Nd + D^{s} - N^*d^*\} = 0. \end{aligned}$$

To keep the equations transparent, we introduce the following shorthand notation. Denote the domestic excess supply functions for x, z, m, l, b, and d, respectively, by $E^{s}X$, $E^{s}Z$, $E^{s}M$, $E^{s}L$, $E^{s}B$ and $E^{s}D$; a star again indicates the foreign variables. Using this notation, the world budget constraint (30) can be stated as

$$(31)\quad p(E^sX + E^sX^*) + q(E^sZ + E^sZ^*) + (E^sM + E^sM^*) + e(E^sL + E^sL^*) + v(E^sB + E^sB^*) + ew(E^sD + E^sD^*) = 0.$$

By a flexible rate regime we mean that no intervention takes place. Thus the amounts of each currency supplied are willingly held and the supply of each currency is fully exogenously determined. Note that, even with bond markets absent, the trade balance does not necessarily equal zero under a flexible exchange rate system. The reason is that both currencies can be freely traded by all agents.

In the case of fixed exchange rates we assume that an Exchange Stabilization Fund (ESF) intervenes to support the currencies; see for example Kemp [1962, p.317]. To see how the ESF works, suppose that it has bought a quantity of currency l with currency m. Then the ESF will be restocked by the domestic central bank, which prints currency m, through a swap of m for l against the fixed rate. The domestic central bank ends up with a decrease in its stock of m and an equivalent increase in its stock of foreign exchange. If, at a later stage, the ESF has to sell currency l in return for m, then it can also be restocked by the domestic central bank running down its stock of foreign exchange. With fixed rates, the supply of a currency does not necessarily equal the world private demand for that currency, but the activities of the ESF ensure that the world's private demand plus the fund's demand equal the supply; see Kemp [1962, p.318]. It follows that a decrease in one currency held in private hands because of intervention implies an increase in the other currency held in private hands by the same nominal amount. Denote by I the amount of currency m the ESF has to sell in return for currency l to stabilize the agreed rate. The first-period world budget constraint is then

$$(32)\quad p(E^sX + E^sX^*) + q(E^sZ + E^sZ^*) + (E^sM + E^sM^* + I) + (eE^sL + eE^sL^* - I) + v(E^sB + E^sB^*) + ew(E^sD + E^sD^*) = 0.$$

To save space, the second-period constraints are stated only in Appendix B.

4.7. Excess Supply Systems

Individual decisions in the first period depend on the agent's expectations with regard to the second period. We will assume that expectations are formed rationally in such a way that those prices are expected to prevail, in each state of the world, which would clear all markets. The full excess supply system describing the macro model therefore consists of the first-period excess supply functions and the contingent second-period excess supply functions.

In section 4.4 we noted that agents do not foresee new bond issues during the second period. The reason is that with full rationality and a limited planning horizon, agents do not plan to demand any bonds in the second period. Demand for bonds is only positive in the second period if we introduce a third period. If we drop the assumption of rationality for the third period, then positive planned bond demand for the second period can be introduced into the macro model. However, in this case all the demand functions depend upon the prices expected to prevail in the third period. To sum up, we could introduce positive bond demand by assuming that price expectations are formed rationally for the second period, and are determined in a fuzzy way for the third period. For the present discussion, however, we assume full rationality because we do not perceive any major gain from using the other approach.

In the first period there are six world markets for commodities x and z, currencies m and l, and bonds b and d. One market equilibrium condition can be eliminated from the excess supply system by invoking Walras' law. The perceived second-period markets are those of x(j), z(j), m(j), and l(j); again, one market equilibrium condition can be eliminated for each state of the world.

Under the fixed exchange rate regime the excess supply system that describes the world economy reads, for example,

$$
\begin{aligned}
&E^sX + E^sX^* &&= 0,\\
&E^sZ + E^sZ^* &&= 0,\\
&E^sM + E^sM^* + I &&= 0,\\
(33)\quad &E^sB + E^sB^* &&= 0,\\
&E^sD + E^sD^* &&= 0,\\
&E^sX(j) + E^sX^*(j) &&= 0, \text{ for all } j,\\
&E^sZ(j) + E^sZ^*(j) &&= 0, \text{ for all } j,\\
&E^sM(j) + E^sM^*(j) + I(j) &&= 0, \text{ for all } j.
\end{aligned}
$$

This is a system of 5 + 3n equations in the 5 + 3n endogenous variables p, q, I, v, w, p(j), q(j) and I(j), where n is the number of states of the world. Without international bond markets the excess supply functions E^sB^* and E^sD have to be omitted. Without any bond markets the fourth and fifth market equilibrium conditions are abandoned. The excess supply system under the flexible rates regime is given in Appendix B eqns. (B48). Appendix B also gives the above excess supply system (33) expressed in the original macro demand and supply functions; see eqns. (B4).

5. Comparative Statics

The macro model is now complete and by total differentiation of the excess supply system we can derive some comparative statics results. In general the totally differentiated excess supply system looks like

(34) $A\, d\bar{p} = \bar{b}$,

where A is a square matrix with elements a_{ij} representing the partial derivatives of the excess supply functions with respect to the endogenous variables p_j; $\bar{p}$ is the vector with macro endogenous variables p_j as elements; and the vector $\bar{b}$ contains the partial derivatives of the excess supply functions with respect to the exogenous variables. To be able to tell in which direction an endogenous variable changes due to a change in an exogenous variable, one needs to know the signs of the determinants used when applying Cramer's rule. In the case where a matrix is totally stable it is a Hicksian matrix, see for example Quirk and Saposnik [1968, p.166], and the sign of its determinant can be determined. Sufficient conditions for a matrix to be totally stable are that is has a positive diagonal and is quasi-dominant-diagonal; see Quirk and Saposnik [1968, p.167]. Fortunately, we can establish that some of the matrices we need are totally stable under some additional assumptions. For example, the diagonal elements of the matrix of the differentiated excess supply system with fixed exchange rates and no bond markets are all positive, because of the negative own price effect on the demand side. Moreover, the quasi-dominant-diagonal property follows from the homogeneity properties of the demand functions. This result is elaborated in Appendix B, as well as the other results we need to sign the determinants.

Before we describe our results, we wish to mention two studies that have employed macro models similar to the one described above. Kemp's [1962] article is based upon an one-period, general equilibrium model with domestic money markets and internationally traded commodities. As such, the article is a formal precursor of what later was called the monetary approach to the balance of payments; see Frenkel and Johnson [1976]. Kemp analyzed the effects of a devaluation with this model. The model we employ here is an extension of Kemp's model in the following sense: employment effects are introduced, a public sector and its budget constraint are taken into account, we allow for the foreign currency to be held domestically, and we deal explicitly with expectations with regard to macro endogenous variables. Using this model we study the phenomenon of currency substitution.

A more recent article is that by Stockman [1980]; see also the discussion in section 3.1. Stockman's model is a multiperiod model that embodies uncertainty and expectations as in our model. Stockman is mainly concerned with the effects of a real shock. While reading Stockman's article we encountered some ambiguity in the way the comparative statics results are derived. Note that for derivation of comparative statics results, we differentiate the system of first-period and second-period contingent excess supply functions; see for example eqns. (33) and (34). The contingent second-period excess supply functions cannot be disregarded in the analysis because of the rational expectations assumption. Stockman, however, first reduces the full excess supply system to an excess supply system comprising only the first-period excess supply functions. This is done by relating the second-period macro endogenous variables to first-period variables. Then, the reduced excess supply system is differentiated totally, and comparative statics results are obtained. To obtain any definite results, additional assumptions have to be made, for example, normality of goods. However, from Stockman's presentation it is unclear whether these assumptions are also made with respect to the second-period variables. In particular, a partial derivative of a first-period macro variable with respect to one of the first-period endogenous variables, in the differentiated reduced excess supply system, contains indirectly the effects of changes in second-period endogenous variables. As shown in Appendix B, for example, in addition to the assumption of gross substitutability for goods of the same period, we also need the assumption of gross substitutability for goods across periods.

The purpose of this section is to analyze the issue of currency substitution on a macro level. We go some way toward answering the questions as to what are the qualitative and quantitative differences between macro models with and without the possibility of currency subsitution. These questions will be dealt with by comparing comparative statics results in both circumstances. In particular, we study the effects of monetary financed expenditure increases by the government under two exchange rate regimes.

5.1. Fixed Exchange Rates

We start with an analysis of the fixed rates system without bond markets, i.e. system (33) with the fourth and fifth equations omitted. Before we derive any specific result, we do some preliminary work. To derive the comparative statics result we use Cramer's rule. For any qualitative conclusions we need to know the signs of the two determinants. The determinant of matrix A in (34) is one of those we need to sign, and this will be discussed first.

Sufficient conditions for the negative of matrix A in (34) to be totally stable are:

(35) commodities of the same period and across periods are gross substitutes,

(36) all commodities are normal goods,

(37) commodity price induced employment and profit effects are small in the short run.

Conditions (35) and (36) are the common normality and gross substitutability assumptions employed to guarantee Hicksian stability. However, our short-run model allows for variations in the level of employment and profits due to fixed nominal wages and flexible commodity prices. Employment and profits are positively correlated with the commodity prices. Moreover, an increase in employment and profits increases the demand for commodities. Thus, there is a tendency for the partial market demand curves to become upward sloped. Condition (37) states that this tendency has to be small in a sense made precise in Appendix B, eqn. (B17). Conditions (35) - (37) imply that the determinant that appears in the denominator in application of Cramer's rule is positive. Next we turn to the determinant in the numerator.

We are specifically interested in the effect of a monetary financed subsidy to domestic residents upon the intervention of the ESF. First, we derive the effects under the usual assumption that no foreign currencies are held domestically. Second, we show what the implications are if we relax this assumption.

Given two other conditions, it turns out that the ESF is required to buy the home currency, supposing zero intervention initially, as a result of the monetary financed subsidy to domestic residents. The conditions are those stated above and in addition

(38) $$\frac{u-m}{m}\eta_u^m + \Sigma\eta_{uj}^m > \frac{m+g}{m}\phi_p^N,$$

and either

(39) all price elasticities with respect to money ε^m are positive and the income elasticities with respect to money η^m are positive,

or

(40) both income elasticities η^m_u and η^m_{uj} are positive and $\frac{u}{m} > \eta^m_u$.

In the above, income elasticities η^m_u are defined as $\eta^m_u = \frac{u}{m} m_u$, price elasticities like ε^m_p are defined as $\varepsilon^m_p = \frac{p}{m} m_p$ and the employment elasticity ϕ^N_p is given by $\phi^N_p = \frac{p}{N} N_p$. The set of conditions (38) and (39) is elaborated in Appendix B, see (B27) and (B28). Without employment effects, i.e. with $\phi^N_p = 0$, condition (38) just states that money is a normal good, which is already taken into account in (39). Hence, with short-run employment effects present, one reaches again the conventional conclusion that I declines if the government "rolls the presses" to finance its transfer increases, conditional upon the employment effect ϕ^N_p being small. Condition (39) can be replaced by the weaker condition (40). Condition (40) is weaker than (39) because it does not assume that money and all commodities are gross substitutes. By the homogeneity of degree one of m, see (B23), this is equivalent to allowing money to be a luxury good. In cases where the sum of the income elasticities $\eta^m_u + \Sigma\eta^m_{uj}$ exceeds one, condition (38) becomes more plausible. Empirically the estimates for the elasticity of the demand for money with respect to income tend to be larger than unity. (See Arrow [1974,p.103] and Intriligator [1978,p.309] for a discussion and an overview of studies that support this assertion.)

With foreign currency demand present in both countries, condition (38) has to be modified to

$$(41) \quad \frac{u-m}{u} \eta^m_u + \Sigma\eta^m_{uj} > \frac{m+g}{m} \phi^N_p + \frac{N^*m^*}{Nm} \{1-\eta^{m^*}_{u^*} - \Sigma\eta^{m^*}_{u^*j} + \phi^{N^*}_q\}.$$

The qualitative difference between (38) and (41) is the term

$(N^*m^*/Nm)\{1-\eta^{m^*}_{u^*} - \Sigma\eta^{m^*}_{u^*j} + \phi^{N^*}_{q}\}$. This term also indicates the possible effects of currency substitution. For an interpretation, it will pay us to disregard for a moment the employment effects, i.e. set $\phi^{N}_{p} = \phi^{N^*}_{q} = 0$. If money is a luxury good, i.e. we assume that (40) holds, then $(1 - \eta^{m^*}_{u^*} - \Sigma\eta^{m^*}_{u^*j})$ is negative, and the conclusion that intervention I on behalf of the home currency m becomes necessary is strengthened. However, if money is a gross substitute for all commodities, i.e. one relies on (39), then (41) is a stronger assumption than (38). In this case, if total foreign currency holdings abroad N*m* are large relative to the domestic holdings Nm, then the reverse of (41) might prevail instead. This does not necessarily imply that I increases, but neither can this possibility be ruled out.

Let us summarize our conclusions obtained thus far. Assume that all commodities are normal goods and are gross substitutes. Moreover, assume the price induced employment and profit effects upon demand to be small. Then, under the fixed parities regime without bond markets and if money is a luxury good, one finds that a monetary financed subsidy to domestic residents makes it necessary for the ESF to intervene on behalf of the home currency. This result can be interpreted as a version of Gresham's law, in the sense that the ESF receives the "officially overvalued" domestic currency in return for the "officially undervalued" foreign currency. The conclusion is strengthened, in the sense that one needs weaker conditions to obtain definite results, if one allows for currency substitution. However, if money is a necessary good, then currency substitution in principle introduces the possibility of a perverse reaction to the domestically pursued policy, i.e. one needs stronger conditions to rule this out.

With bond markets in the model we have to make some additional assumptions with regard to the bond price elasticities of the commodities and currencies, and the cross price elasticities of bonds to arrive at the same conclusion; see Appendix B.

5.2. Flexible Exchange Rates

The discussion of the flexible exchange rate case again centers around the question of what are the specific features of the phenomenon of currency substitution. The setting of the ensuing analysis is what we term the asymmetric country assumption. To anticipate possible misunderstandings, we caution the reader that our definition of asymmetry is different from the customary definition. By the asymmetric country assumption we will understand that no foreign exchange or foreign bonds are held domestically in private portfolios, but that both currencies and bonds are held abroad. The domestic country will, in this setting, also be referred to as the "large" country, the other country is the "small" country. As a motivation for studying this specific case, we quote the following passage from Frenkel and Johnson [1976,p.26]:

"Where the small-country assumption does become relevant is on the monetary side of the analysis; concretely, a large country - the United States, and to a lesser extent other international financial centres - may be able to operate its domestic policies on the assumption that its national money is internationally acceptable so that, say, an expansion of its domestic credit through a 'cheap money' policy will lead to an accumulation of its money in the hands of foreign holders - and eventually to world inflation - rather than to a loss of international reserves."

We will illustrate in what sense the phenomenon of currency substitution is important against this international background. As an example, we discuss the effect of a monetary financed government expenditure increase in x upon the exchange rate e. The comparative statics results are derived both for the case where the large country pursues this policy change and for the case where the small country implements the same change. We compare the effects upon e. It turns out that the difference is related to the issue of currency substitution.

We start the analysis with a simplified one-period model. Assume that foreigners do hold both currencies, but that domestic residents only hold the domestic currency, and that bond markets are absent. In the Appendix B we derive the following:

$$(42) \quad \frac{de}{d\Delta M^s} = \frac{\Delta q}{\Delta}(- N^*l^*_q - (l^*+g^*)N^*_q) - \frac{\Delta p}{\Delta}(- N^*l^*_p),$$

and

$$(43) \quad \frac{de}{d\Delta L^s} = - \frac{\Delta q}{\Delta}(- Nm_q - N^*m^*_q - m^*N^*_q) + \frac{\Delta p}{\Delta}(- Nm_p - (g+m)N_p - N^*m^*_p).$$

Equation (42) indicates how the exchange rate e changes due to a government expenditure increase upon x, which is financed by printing money ΔM^s. Similarly, (43) gives the effect on e when the foreign government pursues such a policy. The Δ's in the expressions denote the determinant in the denominator needed in the application of Cramer's rule. The determinants in the numerators of (42) and (43) are developed with respect to the one row where the two determinants differ; the Δ's with a subscript refer to the relevant cofacters.

We use a property of the utility function of the individual to rewrite (42). The property is that m* and l* are in a separate branch of the utility tree; see Strotz [1957]. This allows us to write, say,

$$(44)\qquad l^*_p/m^*_p = l^*_q/m^*_q = \Phi.$$

Hence, (42) can be expressed as

$$(45)\qquad \frac{de}{d\Delta M^s} = \Phi\{\frac{\Delta q}{\Delta}(-N^*m^*_q - \frac{1}{\Phi}(l^*+g^*)N^*_q) - \frac{\Delta p}{\Delta}(-\ N^*m^*_p)\}.$$

To give a heuristic explanation for (44), we consider the transaction technology example elaborated in section 3.2. From the first-order conditions (5) and (6) in a one-period model, we have

$$(46)\qquad \frac{U^*{}_{el^*/n}}{U^*{}_{m^*/n}} = \frac{\xi n}{\xi n}.$$

Combine this with the specific transaction technology of section 3.2 to obtain

$$\frac{g^{*l}}{g^{*m}}\left(\frac{el^*}{m^*}\right)^{\tau-1} = 1,$$

or equivalently

$$(47)\qquad m^* = e\left(g^{*m}/g^{*l}\right)^{\frac{1}{1-\tau}}l^*.$$

From (47), one easily establishes (44). Moreover, in this case

$$(48) \quad \frac{1}{\Phi} = e(g^{*m}/g^{*1})^{1/(1-\tau)}.$$

Combine (48) and (45), and substitute this into (43). This allows us to express (43) as a combination of (42) and some other variables:

$$(49) \quad \frac{de}{d\Delta L^s} = -\left(\frac{g^{*m}}{g^{*1}}\right)^{\frac{1}{1-\tau}} \frac{de}{\frac{1}{e}d\Delta M^s} - \frac{\Delta q}{\Delta}\left(-Nm_q + \frac{1^* + g^*}{\Phi}N_q^*\right) + \frac{\Delta p}{\Delta}\left(-Nm_p - (g+m)N_p\right).$$

If currency substitution does not prevail abroad either, i.e. foreigners do not hold currency m, then (49) would reduce to

$$(50) \quad \frac{de}{d\Delta L^s} = -\frac{\Delta q}{\Delta}\left(-Nm_q\right) + \frac{\Delta p}{\Delta}\left(-Nm_p - (g+m)N_p\right).$$

Compare (49) and (50) and note that the difference consists of two terms, $\left(\frac{g^{*m}}{g^{*1}}\right)^{1/1-\tau} \frac{de}{\frac{1}{e}d\Delta M^s}$ and $\frac{1^* + g^*}{\Phi} N_q^*$.

Abstract from the latter term by assuming that the employment effect is small. Suppose that e rises as a result of the domestic policy, i.e. $de/d\Delta M^s > 0$, and that e falls as a result of the foreign policy, i.e. $de/d\Delta L^s < 0$. This is what is commonly believed to happen as a result of such policies; see Mussa [1979]. Moreover, suppose that this would still result if currency substitution were absent abroad, i.e. $de/d\Delta L^s$ in (50) is negative too. Let payment habits be such that $g^{*m} \approx g^{*1}$, i.e. foreigners hold m and 1 in about equal amounts (measured in value); see (47).

Given this configuration, it follows that e would appreciate more as a result of the foreign policy than e would depreciate as a result of the domestic policy. The effect of currency substitution abroad is that the small country absorbs part of the money supply increase from the large country. Therefore, the "burden of adjustment" does not fall completely on the exchange rate. In contrast, if the foreign country pursues inflationary financing, the exchange rate has to do all the adjustment. This result can be generalized to our two-period model with bond markets; see Appendix B, eqn.(B55). One should realize that the above conclusion could still follow if currency substitution were completely absent. In this case one would have to compare (42) and (50). But, it follows immediately from (49) that the possibility of currency substitution in the small country certainly adds to the divergence.

It is tempting to draw the overall conclusion that currency substitution can be a matter of both degree and substance. The foregoing discussion shows that the comparative statics results are numerically evaluated differently with and without currency substitution. Therefore currency substitution is at least a matter of degree. In the previous subsection we showed that if money is a necessary good, then it is in principle possible that policy impacts have opposite signs in cases with and without currency substitution. The result of this subsection is that currency substitution causes the wedge between the large country's monetary policy impacts on the exchange rate and the small country's monetary policy impacts. Thus it seems that currency substitution can be a matter of substance too.

Appendix A

Domestic Residents Optimization Problem

Maximize:

$$V = U[x,z,t - s(\frac{m}{n},\frac{el}{n},h)] + \sum_{j=1}^{n} \pi(j)W[x(j),z(j),t - s(\frac{m(j)}{n(j)}, \frac{e(j)l(j)}{n(j)}, h)],$$

where

$W[.] = \rho U[.]$, with $1 > \rho > 0$, $n = n[p,q]$, and $s(.) < t$, $s'(.) < 0$,

subject to:

$$y + m(-1) + el(-1) + r(-1)b(-1) + ei(-1)d(-1) + ek(-1) -$$
$$f(-1)k(-1) - g - px - qz - m - el - b - ed \geq 0,$$

and

$$y(j) + m + e(j)l + rb + e(j)id + e(j)k - fk - g(j) - p(j)x(j)$$
$$- q(j)z(j) - m(j) - e(j)l(j) \geq 0, \quad \text{for all } j = 1,2,\ldots,n,$$

and if relevant, the capital market constraint

$$\bar{d} - d \geq 0.$$

Form the Langrangian function to solve this constrained optimization problem:

$$L = U[.] + \Sigma\pi(j)W[.] + \xi\{.\} + \Sigma\pi(j)\lambda(j)\{.\} + \phi\{\bar{d} - d\}.$$

Next we state the first-order conditions:

(A 1) $L_x \quad U_x - \xi p \leq 0,$

(A 2) $L_z \quad U_z - \xi q \leq 0,$

(A 3) $L_m \quad U_{m/n} - \xi n + \Sigma\lambda(j)\pi(j)n \leq 0,$

(A 4) $L_l \quad U_{el/n} - \xi n + \Sigma\lambda(j)\pi(j)\frac{e(j)}{e}n \leq 0,$

(A 5) $L_b \quad - \xi + \Sigma\lambda(j)\pi(j)r \leq 0,$

(A 6) $L_d \quad - \xi e + \Sigma\lambda(j)\pi(j)e(j)i - \phi \leq 0,$

(A 7) $L_k \quad \Sigma\lambda(j)\pi(j)e(j) - \Sigma\lambda(j)\pi(j)f \leq 0,$

(A 8) $L_{x(j)} \quad W_{x(j)} - \lambda(j)p(j) \leq 0, \ \forall\ j,$

(A 9) $L_{z(j)} \quad W_{z(j)} - \lambda(j)q(j) \leq 0, \ \forall\ j,$

(A10) $L_{m(j)} \quad W_{m(j)/n(j)} - \lambda(j)n(j) \leq 0, \ \forall\ j,$

(A11) $L_{l(j)} \quad W_{e(j)l(j)/n(j)} - \lambda(j)n(j) \leq 0, \ \forall\ j,$

(A12) L_{ξ} $y + m(-1) + el(-1) + r(-1)b(-1) + ei(-1)d(-1) + ek(-1)$

$- f(-1)k(-1) - g - px - qz - m - el - b - ed \geq 0,$

(A13) $L_{\lambda(j)}$ $y(j) + m + e(j)l + rb + e(j)id + e(j)k - fk - g(j)$

$- p(j)x(j) - q(j)z(j) - m(j) - e(j)l(j) \geq 0, \forall j,$

(A13) L_{ϕ} $\bar{d} - d \geq 0,$

(A15) $\xi\{.\} + \Sigma\pi(j)\lambda(j)\{.\} + \phi\{\bar{d} - d\} = 0,$

(A16) $\xi \geq 0, \quad \lambda(j) \geq 0 \ \forall j, \quad \phi \geq 0.$

We assume that the different branches of the utility function are strictly concave functions; this implies, together with assumption that the constraint qualification condition is met, that the above problem has a solution.

Appendix B

Comparative Statics

The purpose of this appendix is to derive some of the comparative statics results in detail. We start by recapitulating some of the budget constraints. With fixed parities, the money supplies of both currencies are endogenous because of intervention I by the Exchange Stabilization Fund (ESF). The world budget constraint reads in this case:

$$\text{(B1)}\quad p(E^sX + E^sX^*) + q(E^sZ + E^sZ^*) + (E^sM + E^sM^* + I) + (eE^sL + eE^sL^* - I) + (E^sB + E^sB^*) + e(E^sD + E^sD^*) = 0.$$

The contingent second-period world budget constraints are, under the fixed rate regime:

$$\text{(B2)}\quad p(j)(E^sX(j) + E^sX^*(j)) + q(j)(E^sZ(j) + E^sZ^*(j)) + (E^sM(j) + E^sM^*(j) + I(j)) + (e(j)E^sL(j) + e(j)E^sL^*(j) - I(j)) = 0,$$

for all j states.

Using Walras' law, the following market equilibrium conditions describe the world economy under the fixed exchange rate regime:

$$
\begin{aligned}
&E^sX + E^sX^* &&= 0,\\
&E^sZ + E^sZ^* &&= 0,\\
&E^sM + E^sM^* + I &&= 0,\\
&E^sB + E^sB^* &&= 0,\\
\text{(B3)}\quad &E^sD + E^sD^* &&= 0,\\
&E^sX(j) + E^sX^*(j) &&= 0, \quad \text{for all } j,\\
&E^sZ(j) + E^sZ^*(j) &&= 0, \quad \text{for all } j,\\
&E^sM(j) + E^sM^*(j) + I(j) &&= 0, \quad \text{for all } j.
\end{aligned}
$$

This is a system of 5 + 3n equations in the 5 + 3n endogenous variables $p,q,I,v,w,p(j),q(j)$ and $I(j)$, and n is the number of states of the world. Without international bond markets, the excess supply functions E^sB^* and E^sD have to be omitted. Without any bond markets, the fourth and fifth market equilibrium conditions are abandoned.

To obtain comparative statics results we need to differentiate totally the excess supply system. To permit differentiation of , e.g. (B3), we express (B3) in the original macro supply and demand functions

(B4)

$$
\begin{aligned}
&X^o - Nx - \frac{1}{p}G^x - N^*x^* - \frac{e}{p}G^{x^*} - \frac{e}{p}P^z &&= 0,\\
&- Nz - \frac{1}{q}G^z - \frac{1}{q}P^x + Z^o - N^*z^* - \frac{e}{q}G^{z^*} &&= 0,\\
&N(-1)m(-1) + \Delta M^s - Nm + N^*(-1)m^*(-1) - N^*m^* + I &&= 0,\\
&B^s - Nb - N^*b^* &&= 0,\\
&-Nd + D^s - N^*d^* &&= 0,\\
&X^o(j) - N(j)x(j) - \frac{1}{p(j)}G^x(j) - N^*(j)x^*(j) - \frac{e(j)}{p(j)}G^{x^*}(j) - \frac{e(j)}{p(j)}P^z(j) &&= 0, \ \forall\, j,\\
&- N(j)z(j) - \frac{1}{q(j)}G^z(j) - \frac{1}{q(j)}P^x(j) + Z^o(j) - N^*(j)z^*(j) - \frac{e(j)}{q(j)}G^{z^*}(j) &&= 0, \ \forall\, j,\\
&Nm + \Delta M^s(j) - N(j)m(j) + N^*m^* - N^*(j)m^*(j) + I(j) &&= 0, \ \forall\, j,
\end{aligned}
$$

Upon differentiaton of the excess supply system (B4) one obtains

$$\text{(B5)}\quad A\, d\bar{p} = \bar{b},$$

where A is the (5+3n) x (5+3n) matrix with elements a_{ij} representing the partial derivatives of the excess supply functions with respect to the endogenous variables $\bar{p}_j$; $\bar{p}$ is the vector with macro endogenous variables p_j as elements; and the vector $\bar{b}$ contains the partial derivatives of the excess supply functions with respect to the exogenous variables. We give here some of the elements of A for the reader's convenience. The first row elements of A are

$$
\text{(B6)}\quad
\begin{aligned}
a_{11} &= X_p^o - N_p x - Nx_p + G^x/p^2 - N^* x_p^* + eG^{x*}/p^2 + eP^z/p^2,\\
a_{12} &= - Nx_q - N_q^* x^* - N^* x_q^* - eP_q^z/p,\\
a_{13} &= 0,\\
a_{14} &= - Nx_v - N^* x_v^*,\\
a_{15} &= - Nx_w - N^* x_w^*,\\
a_{16}(j) &= - Nx_{pj} - N^* x^*{}_{pj}, \ \forall\ j,\\
a_{17}(j) &= - Nx_{qj} - N^* x^*{}_{qj}, \ \forall\ j,\\
a_{18}(j) &= 0, \ \forall\ j.
\end{aligned}
$$

Subscripts indicate with respect to which variable the derivative has been taken. The second row is very similar to the first and is not stated here; the third row reads

$$
\text{(B7)}\quad
\begin{aligned}
a_{31} &= - N_p m - N m_p - N^* m^*{}_p, \\
a_{32} &= - N m_q - N^*{}_q m^* - N^* m^*{}_q, \\
a_{33} &= 1, \\
a_{34} &= - N m_v - N^* m^*{}_v, \\
a_{35} &= - N m_w - N^* m^*{}_w, \\
a_{36}(j) &= - N m_{pj} - N^* m^*{}_{pj}, \ \forall\ j, \\
a_{37}(j) &= - N m_{qj} - N^* m^*{}_{qj}, \ \forall\ j, \\
a_{38}(j) &= 0, \ \forall\ j,
\end{aligned}
$$

and the fourth row is

$$
\text{(B8)}\quad
\begin{aligned}
a_{41} &= - N_p b - N b_p - N^* b^*{}_p, \\
a_{42} &= - N b_q - N^*{}_q b^* - N^* b^*{}_q, \\
a_{43} &= 0, \\
a_{44} &= - N b_v - N^* b^*{}_v, \\
a_{45} &= - N b_w - N^* b^*{}_w, \\
a_{46}(j) &= - N b_{pj} - N^* b^*{}_{pj}, \ \forall\ j, \\
a_{47}(j) &= - N b_{qj} - N^* b^*{}_{qj}, \ \forall\ j, \\
a_{48}(j) &= 0, \ \forall\ j.
\end{aligned}
$$

As the other rows are similar to those stated above, they are left to the reader.

In the main text we are concerned with the effects of a monetary financed subsidy to the domestic residents. For simplicity we first consider the situation when no bond markets exist. Equation (B5) reads in this case

$$(B9)\quad \begin{pmatrix} a_{11} & a_{12} & 0 & a_{16}(j) & a_{17}(j) & 0 \\ a_{21} & a_{22} & 0 & a_{26}(j) & a_{27}(j) & 0 \\ a_{31} & a_{32} & 1 & a_{36}(j) & a_{37}(j) & 0 \\ a_{61} & a_{62} & 0 & A_{66}(j) & A_{67}(j) & 0 \\ a_{71} & a_{72} & 0 & A_{76}(j) & A_{77}(j) & 0 \\ a_{81} & a_{82} & 0 & A_{86}(j) & A_{87}(j) & I^n \end{pmatrix} \begin{pmatrix} dp \\ dq \\ dI \\ dp(j) \\ dq(j) \\ dI(j) \end{pmatrix} = \begin{pmatrix} Nx_g dg \\ Nz_g dg \\ Nm_g dg - d(\Delta M^s) \\ N(j)x(j)_g dg \\ N(j)z(j)_g dg \\ N(j)m(j)_g dg \end{pmatrix},$$

where underlines denote a row vector, overbars indicate column vectors, and the capital letters in the matrix are matrices of the order n x n. On the right-hand side appear the changes in the exogenous variables considered in the main text. By applying Cramer's rule we want to establish the direction of change in the endogenous variables caused by changes in the exogenous variables, and hence we need to determine the signs of several determinants. To obtain those signs we use the following theorem, see Quirk and Saposnik [1968, p.167].

Theorem. If a real nxn matrix A has a negative diagonal and is quasi-dominant-diagonal, then A is totally stable. Moreover, if A is totally stable it is a Hicksian matrix, and the sign of its determinant can be found, see Quirk and Saposnik [1968,p.166].

Under one minor additional assumption, it follows that the diagonal elements of the two determinants we need are all positive. Consider for example the determinant in the denominator, i.e. the determinant of the matrix which appears in (B9). Recall a_{11} from (B6). Ruling out Giffen goods, it follows that the only negative factor in a_{11} is $-N_p x$. We will assume that this employment effect upon demand is small compared to the output effect and the other demand effects. Similar arguments can be given to sign a_{22} and the diagonal elements of $A_{66}(j)$ and $A_{77}(j)$.

To give sufficient conditions for the matrices to be quasi-dominant-diagonal requires a little more effort. We start with the determinant in the denominator.

From the first-order conditions of the micro optimization problem it follows that the demand functions x, z, x(j), z(j) are homogeneous of degree zero in the variables u, p, q, u(j), p(j), q(j),

$$\text{where } u = y + m(-1) + el(-1) - g, \text{ and } u(j) = y(j) - g(j).$$

Moreover, the demand functions m, l, m(j), l(j) are homogenous of degree one in these variables. Hence, by Euler's law,

$$\text{(B10)} \qquad ux_u + px_p + qx_q + \Sigma u(j)x_{uj} + \Sigma p(j)x_{pj} + \Sigma q(j)x_{qj} = 0.$$

We make a series of additional assumptions:

(B11) all goods are normal goods,

(B12) goods of the same period or state of the world are gross substitutes,

(B13) at least one good of one period or state of the world is a gross substitute with a good of another period or state of the world.

Assumptions (B12) and (B13) imply that all cross period goods are gross substitutes. The reason is that goods of different periods are in different branches of the utility tree; see Strotz [1957]. The above three assumptions, together with (B10), enable us to write

$$\text{(B14)}\qquad p\left|x_p\right| > m\left|x_g\right| + q\left|x_q\right| + \Sigma u(j)x_{uj} + \Sigma p(j)\left|x_{pj}\right| + \Sigma q(j)\left|x_{qj}\right| .$$

Note that in any nontrivial case $u > m > 0$, and that $x_u = x_y = -x_g$. Similarly, abroad we have

$$\text{(B15)}\qquad p\left|x^*_p\right| = u^*x^*_{u*} + q\left|x^*_q\right| + \Sigma u^*(j)x^*_{uj} + \Sigma p(j)\left|x^*_{pj}\right| + \Sigma q(j)\left|x^*_{qj}\right| ,$$

where by definition $u^* = ey^* + m^*(-1) + el^*(-1) - eg^*$, and $u^*(j) = e(j)y^*(j) - e(j)g^*(j)$. Premultiply (B14) and (B15), respectively, by N and N*, and add them up to obtain:

$$\text{(B16)}\qquad p\left|Nx_p + N^*x^*_p\right| > m\left|Nx_g\right| + u^*N^*x^*_{u*} + q\left|Nx_q + N^*x^*_q\right| + N\Sigma u(j)x_{uj} + N^*\ \Sigma u^*(j)x^*_{uj} + \Sigma p(j)\left|a_{16}(j)\right| + \Sigma q(j)\left|a_{17}(j)\right| ,$$

where we have used the notation of (B6) for the last two terms.

Suppose that the following inequality holds

$$\text{(B17)}\qquad pX^o_p + G^x/p + eG^{x*}/p + eP^z/p + u^*N^*x^*_{u*} + N\Sigma u(j)x_{uj} + N^*\Sigma u^*(j)x^*_{uj} > pN_px + qN^*_qx^* + eq\ P^z_q/p.$$

Sufficient conditions to conclude that

$$\text{(B18)}\qquad p\left|a_{11}\right| > q\left|a_{12}\right| + m\left|Nx_g\right| + \Sigma p(j)\left|a_{16}(j)\right| + \Sigma q(j)\left|a_{17}(j)\right|$$

are (B11), (B12), (B13), and (B17). Conditions (B11), (B12), and (B13) are the common normality and gross substitutability assumptions employed to guarantee Hicksian stability. However, our short-run model also allows for variations in the level of employment and profits due to commodity price changes. The effects thereof upon the demand for x are captured in the terms on the right-hand side of inequality (B17). Condition (B17) then states that these effects are, in some sense, small.

Under similar sets of conditions the following inequalities can be obtained:

(B19) $$q\left| a_{22}\right| > Nm\left| z_g\right| + p\left| a_{21}\right| + \Sigma p(j)\left| a_{26}(j)\right| + \Sigma q(j)\left| a_{27}(j)\right| ,$$

(B20) $$p(j)\left| a_{66}(j)\right| > p\left| a_{61}\right| + q\left| a_{62}\right| + \Sigma N(j)m\left| x(j)_g\right| + \sum_{i\neq j} p(i) \left| a_{66}(i)\right| + \Sigma q(j)\left| a_{67}(j)\right| ,$$

(B21) $$q(j)\left| a_{77}(j)\right| > p\left| a_{71}\right| + q\left| a_{72}\right| + \Sigma N(j)m\left| z(j)_g\right| + \Sigma p(j) \left| a_{76}(j)\right| + \sum_{i\neq j} q(i)\left| a_{77}(i)\right| .$$

These inequalities, i.e. (B18) - (B21), effectively imply that the negative of the matrix appearing in (B9) is totally stable. Therefore, the determinant of the matrix in (B9) is positive.

Next, we turn to the determinant in the numerator. We are interested in the effect of a monetary financed subsidy to domestic residents upon the intervention activities of the ESF. To establish this, one needs to substitute the righthand-side vector in (B9) into the third column of the matrix in (B9) and compute the sign of its determinant. Inspecting this determinant, we can again employ inequalities (B18) - (B21), but we now need one for the third row too before we can establish the quasi-dominant-diagonal property.

Suppose that short term government deficits or surpluses, that arise due to changes in the endogenous variables, are covered by changes in the money supply to keep the budget balanced. Hence, $Nm_g - d\Delta M^s$ can be expressed differently as $N(1+m_g)dg + N_p g dp$. This follows directly from differentiation of the government budget constraint (26), given that G^x and G^z are kept constant. The term $N_p g dp$ has to be moved to the left-hand side in (B9) before applying Cramer's rule. The third row of the determinant in the numerator therefore reads

$$\text{(B22)} \qquad (a_{31} - N_p g,\ a_{32},\ N(1+m_g),\ a_{36}(j),\ a_{37}(j),\ 0).$$

Above we noticed that m is homogeneous of degree one in u, p, q, u(j), p(j) and q(j). Hence, by Euler's law:

$$\text{(B23)} \qquad m = um_n + pm_p + qm_q + \Sigma u(j)m_{uj} + \Sigma p(j)m_{pj} + \Sigma q(j)m_{qj}$$

Recall that $m_u = -m_g$. We rewrite (B23) into elasticity form:

$$\text{(B24)} \qquad (1+m_g) - \varepsilon_p^m - \varepsilon_q^m - \Sigma\varepsilon_{pj}^m - \Sigma\varepsilon_{qj}^m = \frac{u-m}{u}\,\eta_n^m + \Sigma\eta_{uj}^m,$$

where ε denotes a price elasticity, such as $\varepsilon_p^m = \frac{p}{m}m_p$; and η denotes an income elasticity, such as $\eta_u^m = \frac{u}{m}m_u$. For simplicity, we first study the case when foreign demand for currency m is absent. In this situation we would like to investigate under what conditions the following inequality holds:

$$\text{(B25)} \quad m\left| N(1+m_g)\right| > p\left| -Nm_p - N_p(m+g)\right| + q\left| -Nm_q\right| + \Sigma p(j)\left| -Nm_{pj}\right| + \Sigma q(j)\left| -Nm_{qj}\right| .$$

Rewrite (B25) into elasticity form:

$$\text{(B26)} \quad \left| 1+m_g \right| > \left| -\varepsilon_p^m - \left(\frac{m+g}{m}\right)\phi_p^N \right| + \left| \varepsilon_q^m \right| + \Sigma \left| \varepsilon_{pj}^m \right| + \Sigma \left| \varepsilon_{qj}^m \right| ,$$

where ϕ_p^N denotes the labor demand elasticity $(p/N)N_p$. Using (B24), (B26) will certainly hold if

$$\text{(B27)} \quad \frac{u-m}{u}\eta_u^m + \Sigma\eta_{uj}^m > \frac{m+g}{m}\phi_p^N$$

and

(B28) all price elasticities ε^m are positive and the income elasticities η^m are positive.

Thus, it turns out that sufficient conditions for (B26) to hold are that money is a gross substitute for all other commodities, it is normal with respect to income, and that the employment effects ϕ_p^N are small. With foreign demand for currency m present, it is straightforward to show that condition (B27) has to be amended, such that the following holds:

$$\text{(B29)} \quad \frac{u-m}{u}\eta_u^m + \Sigma\eta_{uj}^m > \frac{m+g}{m}\phi_p^N + \frac{N^*m^*}{Nm}\{1 - \eta_{u^*}^{m^*} - \Sigma\eta_{u^*j}^{m^*} + \phi_q^{N^*}\}.$$

If in addition to (B11), (B12), (B13) and (B17), assumptions (B27) and (B28) or (B29) and (B28) are made, then we can conclude that the determinant in the numerator is positive. Hence, I declines as a result of the monetary financed subsidy to domestic residents. To recapitulate, sufficient conditions for this result are basically that all goods are gross substitutes and that short-run employment effects are small.

Our next task is to establish under which conditions we can arrive at the above conclusion if bonds markets do exist. First we investigate the quasi-dominant-diagonal property of the matrix appearing in (B5). From the micro part it follows that the demand functions x, z, x(j), z(j), b and d are homogeneous of degree zero in the variables u, p, q, v, w, u(j), p(j), q(j), r and i, where u = y + m(-1) + el(-1) + r(-1)b(-1) + ei(-1)d(-1) - g and u(j) = y(j) -g(j). The demand functions m, l, m(j), l(j) are homogeneous of degree one in the same variables. By Euler's law,

$$(B30) \qquad ux_u + px_p + qx_q + vx_v + wx_w + \Sigma u(j)x_{uj} + \Sigma p(j)x_{pj} + \Sigma q(j)x_{qj} + rx_r + ix_i = 0.$$

From the micro part one can obtain that

$$(B31) \qquad \begin{aligned} &\text{sign } x_v = - \text{ sign } x_r, \\ &\text{sign } x_w = - \text{ sign } x_i. \end{aligned}$$

Moreover, one can show that

(B32) $x_r < 0$, and $x_i < 0$.

Reasoning along the same lines as we did to obtain (B16), we arrive at

(B33) $p\left| Nx_p + N^*x^*_p\right| + r\left| Nx_r + N^*x^*_r\right| + i\left| Nx_i + N^*x^*_i\right| > m\left| Nx_g\right| + u^*N^*x^*_{u^*} + q\left| Nx_q + N^*x^*_q\right| + v\left| a_{14}\right| + w\left| a_{15}\right| + \Sigma p(j) \left|a_{16}(j)\right| + \Sigma q(j)\left| a_{17}(j)\right| + N\ \Sigma u(j)x_{uj} + N^*\ \Sigma u^*(j)x^*_{uj}$.

A sufficient condition for

(B34) $p\left| a_{11}\right| > q\left| a_{12}\right| + m\left| Nx_g\right| + v\left| a_{14}\right| + w\left| a_{15}\right| + \Sigma p(j) \left|a_{16}(j)\right| + \Sigma q(j)\left| a_{17}(j)\right|$

to hold, is the following:

(B35) $pX^o_p + G^x/p + eG^{x^*}/p + eP^z/p + u^*N^*x^*_{u^*} + N\ \Sigma u(j)x_{uj} + N^*\ \Sigma u^*(j)x^*_{uj} > pN_px + qN^*_qx^* + eq\ P^z_q/p + r\left| Nx_r + N^*x^*_r\right| + i \left|Nx_i + N^*x^*_i\right|$.

Condition (B35) is very similar to (B17), except that two terms with interest effects upon the demand for x had to be added. Similar conditions can be derived for the second, sixth, and seventh rows.

Next, we inspect the fourth row; see (B8). A sufficient condition for

(B36) $v\left| a_{44}\right| > p\left| a_{41}\right| + q\left| a_{42}\right| + m\left| Nb_g\right| + w\left| a_{45}\right| + \Sigma p(j)\left| a_{46}(j)\right| + \Sigma q(j)\left| a_{47}(j)\right|$

to hold, is that per country the following inequality is satisfied:

$$\text{(B37)}\quad v\left|b_v\right| > p\left|\frac{1}{N}N_p b + b_p\right| + q\left|b_q\right| + m\left|b_g\right| + w\left|b_w\right| + \Sigma p(j)\left|b_{pj}\right| + \Sigma q(j)\left|b_{qj}\right| .$$

From the homogeneity of degree zero mentioned earlier we have

$$\text{(B38)}\quad 0 = ub_u + pb_p + qb_q + vb_v + wb_w + \Sigma u(j)b_{uj} + \Sigma p(j)b_{pj} + \Sigma q(j)b_{qj} + rb_r + ib_i.$$

Moreover, b is homogeneous of degree one in u, p, q, u(j), p(j) and q(j); i.e. we hold v, w, r and i constant. Thus,

$$\text{(B39)}\quad b = ub_u + pb_p + qb_q + \Sigma u(j)b_{uj} + \Sigma p(j)b_{pj} + \Sigma q(j)b_{qj}.$$

Hence, substracting (B39) from (B38)

$$\text{(B40)}\quad -b = vb_v + wb_w + rb_r + ib_i$$

Combine (B40) and (B37) into

$$\left|b + wb_w + rb_r + ib_i\right| - \left|wb_w\right| > p\left|\frac{1}{N}N_p b + b_p\right| + q\left|b_q\right| + m\left|b_g\right| + \Sigma p(j)\left|b_{pj}\right| + \Sigma q(j)\left|b_{qj}\right| ,$$

or in elasticity form

$$\left|1 + \varepsilon_w^b + \varepsilon_r^b + \varepsilon_i^b\right| - \left|\varepsilon_w^b\right| > \left|\phi_p^N + \varepsilon_p^b\right| + \left|\varepsilon_q^b\right| + \frac{m}{g}\left|\eta_g^b\right| + \Sigma\left|\varepsilon_{pj}^b\right| + \Sigma\left|\varepsilon_{qj}^b\right|.$$

It seems plausible to assume that

(B41) $\quad \varepsilon_w^b > 0,\ \varepsilon_r^b > 0,\ \varepsilon_v^b < 0,\ \varepsilon_i^b < 0.$

Moreover, we will assume

(B42) $\quad \left|\varepsilon_v^b\right| > \varepsilon_w^b,$

i.e. the absolute value of the price elasticity with respect to the own bondprice exceeds the cross price elasticity with respect to the other bondprice. Inequality (B42) implies $\varepsilon_r^b + 1 > \varepsilon_i^b$; hence we can write the above as

(B43) $\quad 1 + \varepsilon_r^b + \varepsilon_i^b > \left|\phi_p^N + \varepsilon_p^b\right| + \left|\varepsilon_q^b\right| + \frac{m}{g}\left|\eta_g^b\right| + \Sigma\left|\varepsilon_{pj}^b\right| + \Sigma\left|\varepsilon_{qj}^b\right|.$

If (B43) is satisfied in both countries, then (B36) holds. Because it is likely that for example sign $\varepsilon_p^b \neq$ sign ε_{pj}^b, condition (B43) does not follow directly from the homogeneity property as expressed in (B38). This makes (B43) a stronger assumption than the other conditions we gave above. In cases where the price elasticities are small compared with the one on the left-hand side, then (B43) might be acceptable. At any rate, (B43) was the best we could find.

Finally, we turn to the third row. From the homogeneity properties we have

$$m = um_u + pm_p + qm_q + vm_v + wm_w + \Sigma u(j)m_{uj} + \Sigma p(j)m_{pj}$$
$$+ \Sigma q(j)m_{qj} + rm_r + im_i,$$

or, in elasticity form,

$$(1+m_g) - \varepsilon^m_p - \varepsilon^m_q - \Sigma\varepsilon^m_{pj} - \Sigma\varepsilon^m_{qj} - \varepsilon^m_v - \varepsilon^m_w = \frac{u-m}{u}\eta^m_u + \Sigma\eta^m_{uj} + \varepsilon^m_r + \varepsilon^m_i.$$

The question is, under what conditions does the following inequality hold:

$$m\left| N(1+m_g)\right| > p\left| -Nm_p - N_p(m+g)\right| + q\left| Nm_q\right| + v\left| -Nm_v - B^s\right| + w$$
$$\left|Nm_w\right| + \Sigma p(j)\left| Nm_{pj}\right| + \Sigma q(j)\left| Nm_{qj}\right| ,$$

or, in elasticity form,

(B44)
$$1+m_g > \left| -\varepsilon^m_p - \phi^N_p(m+g)/m\right| + \left|\varepsilon^m_q\right| + \left|-\varepsilon^m_v - \frac{vB^s}{Nm}\right| + \left|\varepsilon^m_w\right| + \Sigma\left|\varepsilon^m_{pj}\right| + \Sigma\left|\varepsilon^m_{qj}\right| .$$

For simplicity, we discuss here the case when foreigners do not demand the domestic currency. Now, (B44) will certainly hold if

$$\text{(B45)} \qquad \frac{u-m}{u}\eta^m_u + \Sigma\eta^m_{uj} > \frac{m+g}{m}\phi^N_p + \frac{vB^s}{Nm} - \varepsilon^m_r - \varepsilon^m_i.$$

Compare (B45) with (B27).

The last part of this appendix deals with the flexible rate system. We are specifically interested in the asymmetric country assumption: by this we mean that no foreign exchange or bonds are held domestically, but both currencies and bonds are held abroad. Given these circumstances, the first-period world budget constraint reads, using the convenient shorthand notation employed earlier,

$$\text{(B46)}\qquad p(E^sX + E^sX^*) + q(E^sZ + E^sZ^*) + (E^sM + E^sM^*) + e(E^sL^*) + v(E^sB + E^sB^*) + ew(E^sD^*) = 0.$$

The second-period world budget constraints read

$$\text{(B47)}\qquad p(j)(E^sX(j) + E^sX^*(j)) + q(j)(E^sZ(j) + E^sZ^*(j)) + (E^sM(j) + E^sM^*(j)) + e(j)(E^sL^*(j)) = 0,$$

for all j states. Now we invoke Walras' law to eliminate one market per period. The following market equilibrium conditions describe a world trade equilibrium under the flexible rate regime:

$$\text{(B48)}\qquad \begin{aligned} E^sZ + E^sZ^* &= 0, \\ E^sM + E^sM^* &= 0, \\ E^sL^* &= 0, \\ E^sB + E^sB^* &= 0, \\ E^sD^* &= 0, \\ E^sX(j) + E^sX^*(j) &= 0, \quad \text{for all } j, \\ E^sZ(j) + E^sZ^*(j) &= 0, \quad \text{for all } j, \\ E^sL^*(j) &= 0, \quad \text{for all } j. \end{aligned}$$

The above system contains 5 + 3n equations in the 5 + 3n variables p, q, e, v, w, p(j), q(j) and e(j).

We introduce the following shorthand vector notation $(p,q,v,p(j),q(j)) = \alpha$, $(e,w,e(j)) = \varepsilon$, and let β denote the vector of all exogenous variables. Note that the domestic excess supply functions depend only upon α and β, but the foreign excess supply functions depend on α, β^* and ε as well. The reason is our asymmetric country assumption. Differentiate the above system totally:

(B49)

$$\begin{pmatrix} E^sZ_\alpha + E^sZ^*_\alpha & E^sZ^*_\varepsilon \\ E^sM_\alpha + E^sM^*_\alpha & E^sM^*_\varepsilon \\ E^sL^*_\alpha & E^sL^*_\varepsilon \\ E^sB_\alpha + E^sB^*_\alpha & E^sB^*_\varepsilon \\ E^sD^*_\alpha & E^sD^*_\varepsilon \\ E^sX(j)_\alpha + E^s\overset{*}{X}(j)_\alpha & E^s\overset{*}{X}(j)_\varepsilon \\ E^sZ(j)_\alpha + E^s\overset{*}{Z}(j)_\alpha & E^s\overset{*}{Z}(j)_\varepsilon \\ E^s\overset{*}{L}(j)_\alpha & E^s\overset{*}{L}(j)_\varepsilon \end{pmatrix} \begin{pmatrix} d\alpha \\ \\ d\varepsilon \end{pmatrix} = - \begin{pmatrix} E^sZ_\beta \\ E^sM_\beta \\ 0 \\ E^sB_\beta \\ 0 \\ E^sX(j)_\beta \\ E^sZ(j)_\beta \\ 0 \end{pmatrix} d\beta - \begin{pmatrix} E^sZ^*_{\beta*} \\ E^sM^*_{\beta*} \\ E^sL^*_{\beta*} \\ E^sB^*{}_{\beta*} \\ E^sD^*_{\beta*} \\ E^sX^*(j)\beta* \\ E^s\overset{*}{Z}(j)_{\beta*} \\ E^s\overset{*}{L}(j)_{\beta*} \end{pmatrix} d\beta^* .$$

Suppose that short-term government budget deficits or surpluses that arise due to changes in the endogenous variables are covered by changes in the bond supply. Domestically this means that the government sets $vdB^s = -gN_pdp - B^sdv$; see eqn. (26). The terms $-gN_p$ and $-B^s$ have to be included in E^sB_α in (B49). Similarly, abroad the government sets $ewdD^s = -eg^*N^*_ede - eg^*N^*_qdq - eD^sdw$ and the right-hand side terms are included in $E^sD^*_\alpha$ and $E^sD^*_\varepsilon$ in (B49).

We wish to compare the policy effects of domestic or foreign monetary financed government expenditure increases in x upon the exchange rate e. Below we compute these effects using Cramer's rule:

(B50)

$$\frac{de}{d\Delta M^s} = -\frac{\Delta_p}{\Delta}N^*l^*_p + \frac{\Delta_q}{\Delta}(N^*l^*_q + l^*N^*_q) - \frac{\Delta_v}{\Delta}N^*l^*_v + \Sigma\frac{\Delta_{pj}}{\Delta}N^*l^*_{pj} - \Sigma\frac{\Delta_{qj}}{\Delta}N^*l^*_{qj}$$

$$+ \frac{\Delta_w}{\Delta}N^*l^*_w - \Sigma\frac{\Delta_{ej}}{\Delta}N^*l^*_{ej},$$

(B51)

$$\frac{de}{d\Delta L^s} = \frac{\Delta_p}{\Delta}(Nm_p + mN_p + N^*m^*_p) - \frac{\Delta_q}{\Delta}(Nm_q + N^*m^*_q + m^*N^*_q) + \frac{\Delta_v}{\Delta}(Nm_v + N^*m^*_v) -$$

$$\Sigma\frac{\Delta_{pj}}{\Delta}(Nm_{pj} + N^*m^*_{pj}) + \Sigma\frac{\Delta_{qj}}{\Delta}(Nm_{qj} + N^*m^*_{qj}) - \frac{\Delta_w}{\Delta}N^*m^*_w + \Sigma\frac{\Delta_{ej}}{\Delta}N^*m^*_{ej},$$

where Δ represents the determinant of the matrix in (B49), and the Δ's with subscripts are the relevant cofactors. The two expressions (B50) and (B51) only differ from each other by the second row in the determinants in the numerator. The determinants in the numerator have been developed with respect to these second rows; therefore the cofactors in the two expressions are identical.

A comparison between $de/d\Delta M^s$ and $de/d\Delta L^s$ is facilitated by the following. In section 3.1 we observed that the way in which the two currencies m and l are optimally combined is independent of the individual's tastes and his attitudes towards risk. The reasons are that m and l are in a separate branch of the utility tree, together with the fact that the bond markets are perfect. Therefore, we can write

(B52) $$\frac{U_{el/n}}{U_{m/n}} = \frac{s_{el/n}}{s_{m/n}} = \frac{1 - w/i}{1 - v/r}.$$

This implies a string of equalities,

$$\text{(B53)} \qquad l^*_p/m^*_p = l^*_q/m^*_q = l^*_{pj}/m^*_{pj} = l^*_{qj}/m^*_{qj} = l^*_{ej}/m^*_{ej} = \Phi,$$

say. We can now express (B50) as

(B54)

$$\frac{de}{d\Delta M^s} = \Phi\Big\{-\frac{\Delta_p}{\Delta}N^*m^*_p + \frac{\Delta_q}{\Delta}N^*m^*_q + \frac{1}{\Phi}\frac{\Delta_q}{\Delta}l^*N^*_q - \frac{1}{\Phi}\frac{\Delta_v}{\Delta}N^*l^*_v + \Sigma\frac{\Delta_{pj}}{\Delta}N^*m^*_{pj} - \Sigma\frac{\Delta_{qj}}{\Delta}N^*m^*_{qj}$$

$$+ \frac{1}{\Phi}\frac{\Delta_w}{\Delta}N^*l^*_w - \Sigma\frac{\Delta_{ej}}{\Delta}N^*m^*_{ej}\Big\}.$$

Lastly, we write (B51) as a combination of (B50) and some other terms

(B55)

$$\frac{de}{d\Delta L^s} = -\frac{1}{\Phi}\frac{de}{d\Delta M^s} + \frac{\Delta_p}{\Delta}(Nm_p + mN_p) - \frac{\Delta_q}{\Delta}(Nm_q + m^*N^*_q - \frac{1}{\Phi}l^*N^*_q)$$

$$+ \frac{\Delta_v}{\Delta}(Nm_v + N^*m^*_v - \frac{1}{\Phi}N^*l^*_v) - \Sigma\frac{\Delta_{pj}}{\Delta}Nm_{pj} + \Sigma\frac{\Delta_{qj}}{\Delta}Nm_{qj} - \frac{\Delta_w}{\Delta}(N^*m^*_w - \frac{1}{\Phi}N^*l^*_w).$$

For an interpretation of (B55), it pays to consider a simplified one-period model. Suppose that the choice problem of foreign agents can be formulates as

(B56)

$$\text{maximize:} \quad U^* = U^*[x^*, z^*, t-s^* \{\frac{m^*}{n}, \frac{el^*}{n}, h^*\}],$$

$$\text{subject to:} \quad ey^* + el^*(-1) + m^*(-1) - eg^* - px^* - qz^* - el^* - m^* = 0.$$

The choice problem of domestic agents reads

(B57)

$$\text{maximize:} \quad U = [x, z, t-s \{\frac{m}{n}, h\}],$$

$$\text{subject to:} \quad y + m(-1) - g - px - qz - m = 0.$$

The asymmetric country assumption is reflected by the fact that only foreign agents hold both currencies. Note that the demand for both currencies abroad is determinate, as long as m* and l* are not perfect substitutes in the sense discussed above. The world budget constraint is found to be

$$\begin{aligned} & p\{X^o - Nx - \frac{1}{p}G^x - N^*x^* - \frac{e}{p}G^{x^*} - \frac{e}{p}P^z\} + \\ \text{(B58)} \quad & q\{-Nz - \frac{1}{q}G^z - \frac{1}{q}P^x + Z^o - N^*z^* - \frac{e}{q}G^{z^*}\} + \\ & \{N(-1)m(-1) + \Delta M^s - Nm + N^*(-1)m^*(-1) - N^*m^*\} + \\ & e\{N^*(-1)l^*(-1) + \Delta L^s - N^*l^*\} = 0, \end{aligned}$$

or, in terms of the notation employed before,

$$\text{(B59)} \quad p\{E^sX + E^sX^*\} + q\{E^sZ + E^sZ^*\} + \{E^sM + E^sM^*\} + e\{E^sL^*\} = 0.$$

By Walras's law, the following market equilibrium conditions completely characterize a world trade equilibrium:

$$
\text{(B60)}\qquad
\begin{aligned}
E^sZ + E^sZ^* &= 0,\\
E^sM + E^sM^* &= 0,\\
E^sL^* &= 0.
\end{aligned}
$$

There are three equations in (B60) in the three varaibles p,q, and e. Suppose that short term government surpluses or deficits due to changes in the endogenous variables are covered by changes in the money supply. Thus, the government sets $d(\Delta M^s) = -\, gN_p dp$. Bearing this in mind, differentiate (B60) totally

(B61)

$$
\begin{pmatrix}
E^sZ_p + E^sZ^*_p & E^sZ_q + E^sZ^*_q & E^sZ^*_e \\
-Nm_p - (g+m)N_p - N^*m^*_p & -Nm_q - N^*m^*_q - m^*N^*_q & -N^*m^*_e - m^*N^*_e \\
-N^*l^*_p & -N^*l^*_q - (l^*+g^*)N^*_q & -N^*l^*_e - (l^*+g^*)N^*_e
\end{pmatrix}
\begin{pmatrix} dp \\ dq \\ de \end{pmatrix}
= -\begin{pmatrix} E^sZ_\beta \\ E^sM_\beta \\ 0 \end{pmatrix} d\beta
- \begin{pmatrix} E^sZ^*_{\beta^*} \\ E^sM^*_{\beta^*} \\ E^sL^*_{\beta^*} \end{pmatrix} d\beta^*.
$$

We wish to consider, as before, the effects of monetary financed increases in government expenditures in x upon e. More precisely, consider $dG^x = d\Delta M^s + gN_p dp$ and $dG^{x^*} = d\Delta L^s + g^*N^*_q dq + g^*N^*_e de$. Computation by

Cramer's rule gives

(B62) $$\frac{de}{d\Delta M^s} = \frac{\Delta_q}{\Delta}(-N^* l^*_q - (l^*+g^*)N^*_q) - \frac{\Delta_p}{\Delta}(-N^* l^*_p),$$

and

(B63) $$\frac{de}{d\Delta L^s} = -\frac{\Delta_q}{\Delta}(-Nm_q - N^* m^*_q - m^* N^*_q) + \frac{\Delta_p}{\Delta}(-Nm_p - (g+m)N_p - N^* m^*_p).$$

From the first-order conditions for the optimization problem (B56) it is straightforward to show that Strotz' proposition holds, i.e. we have

(B64) $$l^*_p/m^*_p = l^*_q/m^*_q = \Phi,$$

say. Hence, (B62) can be written as

(B65) $$\frac{de}{d\Delta M^s} = \Phi\{\frac{\Delta_q}{\Delta}(-N^* m^*_q - \frac{1}{\Phi}(l^*+g^*)N^*_q) - \frac{\Delta_p}{\Delta}(-Nm^*_p)\}.$$

Combine (B65) and (B63) into

(B66) $$\frac{de}{d\Delta L^s} = -\frac{1}{\Phi}\frac{de}{d\Delta M^s} - \frac{\Delta_q}{\Delta}(-Nm_q - N^*_q[m^* - \frac{1}{\Phi}(l^*+g^*)])$$
$$+ \frac{\Delta_p}{\Delta}(-Nm_p - (g+m)N_p).$$

To be even more specific, consider the transaction technology example elaborated in section 3.2. From the first-order conditions for (B56) one finds

$$\text{(B67)} \qquad \frac{U^*_{el^*/n}}{U^*_{m^*/n}} = \frac{g^{*l}}{g^{*m}}\left(\frac{el^*}{m^*}\right)^{\tau-1} = 1,$$

or equivalently

$$\text{(B68)} \qquad m^* = \left(g^{*m}/g^{*l}\right)^{\frac{1}{1-\tau}} el^*.$$

In this case, Φ defined above in (B64) is given by

$$1/\Phi = e\left(g^{*m}/g^{*l}\right)^{1/1-\tau}$$

Hence, $de/d\Delta L^s$ in (B66) can be written as

$$\text{(B69)} \; \frac{de}{d\Delta L^s} = -e\left(\frac{g^{*m}}{g^{*l}}\right)^{\frac{1}{1-\tau}} \frac{de}{d\Delta M^s} - \frac{\Delta_q}{\Delta}\left(-Nm_q + \frac{l^* + g^*}{\Phi} N^*_q\right) + \frac{\Delta_p}{\Delta}\left(-Nm_p - (g+m)N_p\right).$$

Glossary of Mathematical Symbols

In general capital letters refer to macro variables and lower case letters refer to micro variables. First the Latin letters are given, followed by the Greek letters. Some other symbols are given at the end.

A, A_{ij}	=	matrix and submatrix of the totally differentiated excess supply system
a_{ij}	=	matrix elements
$\bar{a}_{ij}$	=	matrix column
$\underline{a}_{ij}$	=	matrix row
a	=	exchange rate risk premium
$\bar{b}$	=	vector of the totally differentiated excess supply system
$b(-1)$, b	=	domestic demand for domestic treasury notes at times t-1 and t
B^s	=	total supply of domestic treasury notes at time t
c	=	argument of the transaction technology
D^s	=	total supply of foreign treasury notes at time t
$d(-1)$, d	=	domestic demand for foreign treasury notes at times t-1 and t
$\bar{d}$	=	quota on domestic holdings of foreign treasury notes

e, $e(j)$ = the exchange rate at times t and $t+1$

E^s = excess supply function; for example, E^sX indicates the domestic excess supply of x

f = forward rate at time t for $t+1$

$F(.)$ = macro production function of x

g, $g(j)$ = government taxes if positive (or subsidies when negative) at times t and $t+1$

G^x, G^z = governement expenditures on x and z

g^m, g^l = gravity variables

h = habits of invoicing and paying

$i(-1)-1$, $i-1$ = foreign interest rate at times $t-1$ and t

I = intervention by the Exchange Stabilization Fund (ESF); when negative it indicates that the ESF buys currency m

I^n = identity matrix of order nxn

j = state of the world at time $t+1$, $j=1,2,...,n$

k = amount of forward purchases of the foreign currency contracted at time t for time $t+1$

K = amount of fixed captial

$l(-1)$, l, $l(j)$ = domestic demand for the foreign currency at times $t-1$, t, and $t+1$

L	=	the Langrangian in the micro part
L	=	total domestic demand for l in the macro part
ΔL^s	=	change in the foreign money supply
M	=	total domestic demand for m
m(-1), m, m(j)	=	domestic demand for the domestic currency at times t-1, t and t+1
ΔM^s	=	change in the domestic currency supply
n	=	price index, which is a function of prices p and q
N	=	the number of employed
o	=	convexity term
p, p(j)	=	domestic price of x at times t and t+1
$\bar{p}$	=	vector of macro endogenous variables
P^x, P^z	=	profits, where the superscript refers to the relevant industry
q, q(j)	=	domestic price of z at times t and t+1
r(-1)-1, r-1	=	domestic interest rate at times t-1 and t
s	=	time involved in completing transactions
t	=	time constraint to the individual after deleting the amount of hours worked

U	=	first-period branch of the utility function
u	=	leisure time in micro part
u	=	wealth in macro part
v	=	domestic bond premium
V	=	utility function, which is additively separable over time
W	=	second-period branch of the utility function
w	=	foreign bond premium
x, x(j)	=	domestic demand for commodity x at times t and t+1
X^o	=	domestic total output of x
y	=	fixed nominal wage rate
z, z(j)	=	domestic demand for commodity z at times t and t+1
Z^o	=	foreign total output of z
α	=	coefficient in gravity equation in the micro part
α	=	price vector in macro part
β	=	vector of all exogenous variables
Δ	=	determinant; and when Δ carries a subscript it denotes a cofactor

ε	=	price vector
ε	=	with subscripts and superscripts it denotes a price elasticity
η	=	income elasticity
$\lambda(j)$	=	Lagrangian multiplier
ξ	=	Lagrangian multiplier
$\pi(j)$	=	subjective probability measure indicating the chance that state j will occur
ρ	=	pure rate of time preference of the individual
σ	=	elasticity of currency substitution
τ	=	coefficient of transaction technology
ϕ	=	Lagrangian multiplier in micro part
ϕ	=	employment elasticity in macropart
Φ	=	ratio of partial derivatives
$-$	=	overbars denote column vectors, underlines indicate row vectors in the macro part
$\forall$	=	for all
$\lvert\cdot\rvert$	=	absolute value sign
*	=	superscript indicating foreign variables

(-1)	=	time indicator for the previous period
(j)	=	time indicator for the coming period

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