RR-75-18

MAN-MACHINE PROCEDURE FOR MULTIOBJECTIVE CONTROL IN WATER RESOURCE SYSTEMS

Ilya V. Gouevsky

Ivan P. Popchev

June 1975

Man-Machine Procedure for Multiobjective Control in Water Resource Systems

Ilya V. Gouevsky[1] and Ivan P. Popchev[2]

Abstract

The formulation of a procedure for multiobjective control is presented. The procedure has been developed under the assumption that there are two subsets of objectives: the primary objective and the secondary objectives. The primary objective is used for defining a scalar-valued optimization problem. The secondary objectives are being improved by a DM through changing a set of parameters in the scalar-valued optimization problem. An illustrative example presented, includes the part of the Iscar River, Bulgaria. Results are very satisfactory when compared with the existing control policy.

1. Introduction

Water resource systems create special problems that make the application of classical decision making techniques quite difficult and, unless they are treated with considerable insight, two quite important characteristics of these systems still strongly influence their management.

The first characteristic creates the problem of control under uncertainty and risk in virtually all water resource decisions. Over the past thirty years, many papers have been devoted to investigating the stochastic nature of the processes in water control systems and many models have been suggested (e.g. [6,15,16,20,22,23]).

---

[1]International Institute for Applied Systems Analysis, Schloss Laxenburg, A-2361 Laxenburg, Austria.

[2]Institute of Engineering Cybernetics, Bulgarian Academy of Sciences, Sofia, Bulgaria.

The second characteristic creates the problem of control under multiobjectives. This problem has become quite interesting during the past ten years. The existence of a great number of noncommensurable objectives in water resource systems makes decision making more complex and requires applying the special techniques of the multiobjective optimization.

In multiobjective optimization, two basic approaches exist [13]. The first approach comes out of the assumption that one criterion dominates over the remaining criteria. Being primary, this criterion can be used in classical scalar optimization problem, where secondary objectives can be taken into account through constraints.

The second approach deals with criteria which could not be ordered (or at least to be divided into primary and secondary). The techniques used when this approach is being applied could be divided approximately into the following groups:

1) noninferior vector's technique [4,5,24,28], etc.;
2) ideal vector's technique [3,7,29], etc.;
3) the utility theory approaches [7,11,17], etc.;
4) the game theory approaches [1,14,19,27], etc.;
5) techniques using man-machine procedures [2,9,10,18, 25,26], etc.

For the time being, these five techniques have been insufficiently implemented in water control systems. There are several examples but they are still more methodologically oriented studies concerning the development of the water resources systems.

In this paper, an attempt is made to use an approach for multiobjective optimization when there is one primary criterion. In order to take into consideration the stochastic nature of the processes in an implicit way, and to reduce computational difficulties arising from treating the secondary criteria as constraints, a man-machine procedure is used which is described in detail below.

2. Basic Notations

Suppose, a decision maker (DM) in a water control system has to solve the following optimization problem:

$$\min_{x} \{\psi_1(x),\ \psi_2(x),\ldots,\psi_v(x)\} \tag{1}$$

subject to

$$G_j(x) \leq o \quad j = 1,\ldots,m \tag{2}$$

$$\psi_1(x) \quad \psi_i(x),\ i = 2,3,\ldots,v \tag{3}$$

where x is an n-dimensional vector of decision variables; $\psi_i(x)$, i = 1,2,...,v are v objective functions; and $G_j(x)$, j = 1,...,m are constraint functions.

The interrelation (3) denotes that the criterion $\psi_1(x)$ dominates over all the remaining criteria, but there is no order between the criteria $\psi_2(x),\ldots,\ \psi_v(x)$. The described optimization problem could be solved by the techniques already mentioned or by reducing it to the following scalar optimization problem:

$$\min_{x} \psi_1(x) \tag{4}$$

subject to

$$G_j(x) \leq o \qquad j = 1,\ldots,m \tag{5}$$

$$\psi_i(x) \leq \xi_i \qquad i = 2,3,\ldots,v \tag{6}$$

where ξ_i are the maximum admissible values of the criteria $\psi_i(x)$, $i = 2,\ldots,v$. The values of ξ_i should be determined by the DM on the basis of the DM's preferences about the satisfaction level of $\psi_i(x)$, past experience, etc.

In many cases, especially with high dimensional optimization problems and nonlinear functions $G_j(x)$ and $\psi_i(x)$, the DM would encounter a great number of computational difficulties. Furthermore, better decision making is needed to evaluate the sensitivity of the solution.

This means that the problem has to be solved many times, and if the criteria $\psi_i(x)$, $i = 2,v$ are far from their bounds ξ_i, and at the same time $\psi_1(x)$ has comparatively large value, a compromize between these values should be found.

So as to reduce the computational difficulties, a man-machine procedure is suggested in this paper. This procedure is based on the following general assumptions:

A) The DM could determine the values of ξ_i, $i = 2,3,\ldots,v$ in advance, or if values of ξ_i have been suggested to the DM, he can choose those which satisfy him.

B) The DM could state quantitative considerations which would improve the values of a given criterion $\psi_i(x)$, $i = \overline{1,v}$. This means that in the problem, there is a set of parameters $\{P_e\}$ which could be changed and the DM at least knows the direction the decision has moved when $\{P_e\}$ is changed.

C) The DM could better evaluate the decision in the functional space than in the decision space.

D) The DM has to have the possibility to find at least two different decisions obtained by different values of some of the parameters $\{P_e\}$ in order to make sure the decision is Pareto optimal.

3. Description of the Procedure EVAL

The decision made by following this procedure is called "rational decision." The main idea for finding it is based on the possibility for interactive dialogue between a DM and a computer.

The procedure consists of the following several steps.

1) At the first stage, optimization problem (7) and (8) is solved:

$$\min_x \quad \psi_1(x) \tag{7}$$

subject to

$$G_j(x) \leq 0 \quad j = 1,\ldots,m \tag{8}$$

(remember that $\psi_1(x)$ is the dominating criterion) as a result of this stage, the vector x^1 is obtained.

2) The value of all criteria $\psi_i(x)$, $i = 2,3,\ldots,v$ are computed using the vector x^1, obtained in step 1) above.

3) Choose the value of ξ_i, i = 1,v. If the DM has no idea about the magnitude of ξ_i he can continue the procedure assuming that $\xi_i = \psi_i(x^1)$.

4) The values $\psi_i(x^1)$ and ξ_i for i = 2,...,v are compared: If $\psi_i(x^1) \leq \xi_i$ for all i = 2,...,v the DM either can decide to accept it as a rational decision, or go to step 5) for improving it; if $\psi_i(x^1) > \xi_i$ for any i = 2,...,v go to step 5).

5) Analyzing the values of the function $\psi_i(x^1)$, i = 2,...,v the DM changes some of the parameters of the set $\{P_e\}$ in such a way that he believes the values could be improved.

6) The DM determines the maximum value of concession toward the value of the function $\psi_1(x)$. Let this value be denoted with ξ_1. Go to step 1).

This procedure will be implemented below for years control of a multipurpose reservoir.

4. Control of Multipurpose Reservoir

Let us consider a reservoir created for supplying water for industrial, municipal and irrigation needs, as well as for use in generating hydroelectric power and for providing facilities for fishing and recreation. The whole process involving control of a reservoir is considered in integer time s = 1,...,N. The structure of the reservoir for any stage (month) s is shown in Figure 1.

Control variables, in the investigated reservoir's node, are the amount of water y_k^s allocated to the k^{th} user at the s^{th} stage, $k = \overline{1,n+1}$; $s = \overline{1,N}$ and the amount of water in the reservoir at the s^{th} stage.

Upon the control variables, denoted by the vector

$x = \{y_k^s,\ z_s;\ s = \overline{1,N};\ k = \overline{1,n+1}\}$, the following constraints are imposed:

$$\sum_{k=1}^{n+1} y_k^s \leq R_s + \gamma_s z_{s-1} , \qquad s = \overline{1,N} \tag{9}$$

$$z_s = \min (M,\ R_s + \gamma_s z_{s-1} - \sum_{k=1}^{n+1} y_k^s) , \qquad s = \overline{1,N} \tag{10}$$

$$z_s \geq M_o \tag{11}$$

$$\nu_k^s \leq y_k^s \leq \mu_k^s , \qquad s = \overline{1,N};\ k = \overline{1,n+1} \tag{12}$$

$$y_{n+1}^s = \begin{cases} \mu_{n+1}^s , & \text{if } z_s \geq \mu_{n+1}^s \\ z_s , & \text{if } z_s < \mu_{n+1}^s \end{cases} , \ s = \overline{1,N} \tag{13}$$

where R_s is an input into the reservoir at the s^{th} month.

For determining the $R = \{R_s\}$ additional model, taking into account that the stochastic nature of the input is needed, γ_s is the evaporation coefficient at the s^{th} stage, $o < \gamma_s < 1$ (by γ_s other kinds of losses could also be taken into consideration),

- M is the capacity of the reservoir;
- M_o is the minimum admissible amount of water in the reservoir (under the value the released water could not be used for municipal supply);
- ν_k^s is the mandatory release for the k^{th} user at the s^{th} stage. This release allows for technological, social and other considerations;
- μ_k^s is the demand of the k^{th} user at the s^{th} stage;

μ^s_{n+1} is the demand for drinking water.

Constraint (9) means that the amount of released water at the s^{th} stage cannot be in excess of water available at this stage. Constraint (10) reflects the restriction capacity of the reservoir at every stage.

Constraints (9), (10), (11), (12) and (13) form the set G(x) and the set comprises all admissible solutions. The main problem of the DM is to choose subset $G^1(x)$ optimal in the sense of criteria $\psi_i(x)$, $i = 1,V$ described below and after that to try to reduce the subset $G^1(x)$ into a single decision x^*.

It is assumed that the DM can divide all of the criteria into two groups. The first group contains only the criterion $\psi_1(x)$, while the second group consists of six criteria $\psi_2(x),\ldots,\psi_7(x)$.

For convenience in comparing the results, all criteria are normalized in the interval (o,1). Because of this, the most desirable level of all criteria is 1.

The analytical expression of the criterion $\psi_1(x),\ldots,\psi_7(x)$ is as follows:

$$\psi_1(x) = 1 - \frac{\sum_{s=1}^{N} \sum_{k=1}^{n} f^s_k(y^s_k)}{\sum_{s=1}^{N} \sum_{k=1}^{n} f^s_k(\nu^s_k)} \tag{14}$$

where $f^s_k(y^s_k)$ denotes the relationship between the amount of water distributed to the k^{th} user at the s^{th} stage and the loss (in monetary units) obtained by the user. The loss defined as above is $f^s_k(\nu^s_k)$, when $y^s_k = \nu^s_k$. It is assumed in the paper that the functions $f_k^s(y_k^s)$, for all k and s, are convex and piece-wise nonlinear in the interval (ν_k^s, μ_k^s).

The second group of secondary objectives are more interesting for the DM when a single reservoir is operated and thus is described below.

<u>$\psi_2(x)$ - Users' Priority</u>

This criterion is based on the following assumptions. Let us denote by ξ_k the average degree of satisfying the demand $\{\mu_k^s,\ s = \overline{1,N}\}$ of the k^{th} user

$$\xi_k = \frac{\sum_{s=1}^{N} y_k^s}{\sum_{s=1}^{N} \mu_k^s} \quad , \qquad k = \overline{1,n+1} \quad , \tag{15}$$

and to each user an integer $C_k \varepsilon\ (\overline{1,n+1})$, is ascribed:

$$C_\ell = 1, \text{ if } \max_{k = \overline{1,n+1}} \xi_k = \xi_\ell$$

$$C_q = 2, \text{ if } \max_{\substack{k = \overline{1,n+1} \\ k \neq \ell}} \xi_k = \xi_q$$

$$C_p = n, \text{ if } \max\ (\xi_p, \xi_r) = \xi_p$$

$$C_r = n+1 \quad .$$

If the order of the users concerning the average degree of satisfying the demand, preferably for the DM, is $\overset{o}{C}_k$, $k = \overline{1,n+1}$, then the criterion $\psi_2(x)$ could be expressed in the form

$$\psi_2(x) = \psi(\overset{0}{C}_k, C_k) = 1 - \frac{\sum_{k=1}^{n+1} \left|\overset{0}{C}_k - C_k\right|}{1+2+\ldots+n+n+1} . \qquad (16)$$

$\psi_3(x)$ - Equability of Satisfying the Users' Demand

Because of a number of users' technological peculiarities, not only is the amount of water allocated to a given user of great importance, but also that this amount should be evenly distributed. Quantity evaluation of these requirements could be made by defining the function $\psi_3(x)$ in the following manner:

$$\psi_3(x) = 1 - \frac{\sum_{k=1}^{n+1} (\max_s g_k^s - \min_s g_k^s)}{\sum_{k=1}^{n+1} \max_s g_k^s} \qquad (17)$$

where $g_k^s = \frac{y_k^s}{\mu_k^s}$, $s = \overline{1,N}$.

$\psi_4(x)$ - Flexibility of the Decision

With this criterion, there is a possibility for reallocation (i.e. changing the solution of the problem (7) and (8) before making the final decision) in the amount of water among the users when some of the parameters $\{P_\ell\}$ are changed. This indicates that the DM does not have to restrain the set G(x) very much, i.e. to put a high boundary on ν_k^s and M_o (the higher ν_k^s and M_o the more restrained G(x)).

If q_k is the number of stages at which the amount of water y_k^s distributed to the k^{th} user is equal to the lower boundary ν_k^s, then $\psi_4(x)$ can be obtained by the expression:

$$\psi_4(x) = 1 - \frac{\sum_{k=1}^{n} q_k}{N.n} \quad . \tag{18}$$

ψ_5 - Stability of the Decision

It is assumed that for solving the problem (7) and (8), the mean value of the input $R = \{R_s\}$ obtained on the basis of historical data is used. By means of $\psi_5(x)$ the possibility for implementation of this decision is evaluated when vector R is a stochastic variable with a number of values $R^w = (R_1^w, \ldots, R_s^w, \ldots, R_N^w)$, $w = 1, \ldots, \theta$. θ may simply either denote the number of historical data available, or it can be determined using appropriate data processing.

To each vector R^w an admissible set $G^w(x)$ determined by (9), (10), (11), (12), and (13) corresponds. It is assumed that the decision x is admissible for the input R^w if $x \in G^w(x)$. If the variable h^w is introduced,

$$h^w = \begin{cases} 1 , & \text{if } x \in G^w(x) \\ 0 , & \text{if } x \notin G^w(x) \end{cases} ,$$

then the criterion $\psi_5(x)$ could be determined as follows:

$$\psi_5(x) = \frac{\sum_{w=1}^{\theta} h^w}{\theta} \quad . \tag{19}$$

$\psi_6(x)$ - End State of the Reservoir

In order to meet future demands it is of substantial interest to allow for the end reservoir's state. So to evaluate this criterion the following function is introduced:

$$\psi_6(x) = \frac{Z'_N + M_o}{M + M_o} , \tag{20}$$

where

$$Z'_N = \begin{cases} Z_N , & \text{if } Z_N \leq M \\ M , & \text{if } Z_N > M \end{cases} .$$

Z_N is the reservoir's state at the N^{th} stage.

$\psi_7(x)$ - Ecological Effect of the Decision

For a region where the reservoir is the main water source, the following factors influence basically the ecological equilibrium (from the point of view of water resources only):

a) reservoir level, or, which is the equivalent, the amount of water Z_s, $s = \overline{1,N}$.

b) the amount of waste water $y_k^s \cdot \theta_k^s$ discharged by the k^{th} user in the river below the reservoir at the s^{th} stage, and the level λ_k^s of its pollution ($k \in I_w$, where I_w is a set of indexes of users discharging waste water; θ_k^s is the coefficient between o and 1).

c) the irrigation regime of crops area--insufficiency of water could disturb the ecological equilibrium of the region. This regime depends on the value of g_k^s, a degree of satisfying users' demand for irrigation ($k \in I_{ir}$, where I_{ir} is a set of indexes of the crops).

Taking into account the above factors, the DM could estimate the ecological effect of any admissible decision x through the function $\psi_y(x)$ formulated, for instance, in the following manner:

$$\psi_7(x) = b_1 \frac{\sum_{s=1}^{N} \left| \overset{0}{Z}_s - Z_s \right|}{\sum_{s=1}^{N} \overset{0}{Z}_s} + b_2 \frac{\sum_{s=1}^{N} \sum_{k \in I_{ir}} y_k^s}{\sum_{s=1}^{N} \sum_{k \in I_{ir}} \overset{0}{y}{}_k^s} \tag{21}$$

$$+ b_3 \left(1 - \frac{\sum_{s=1}^{N} \sum_{k \in I_w} y_k^s \, \theta_k^s (1 - \lambda_k^s)}{\sum_{s=1}^{N} \sum_{k \in I_w} y_k^s} \right)$$

where

$\overset{0}{Z}_s$, $\overset{0}{X}{}_n^s$ and $\overset{0}{y}{}_n^s$ are respectively the necessary values, from the ecological equilibrium point of view, of Z_s, X_n^s, and y_k^s, $s = \overline{1,N}$, $k \in I_{ir}$;

λ_k^s is the level of pollution, $o \leq \lambda_k^s \leq 1$, determined following [8] (the most polluted water has a level $\lambda_k^s = o$);

b_1, b_2, b_3 are coefficients indicating the intensity of the above three terms on the ecological equilibrium; $\sum_{r=1}^{3} b_r = 1$; $b_r > 0$ for all r.

The stated seven criteria do not cover the great variety of possible criteria. If necessary, the DM could either complement this set of criteria or define another one, taking into account the specific conditions of the investigated system.

The optimization problem the DM encounters when he is seeking the best decision can be formalized as a multiobjective nonlinear problem, i.e.

$$\max_{x \in G(x)} \left\{\psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x), \psi_5(x), \psi_6(x), \psi_7(x)\right\} \quad (22)$$

where G(x) is determined by (9), (10), (11), (12) and (13).

Although it is possible to use conventional techniques for solving the problem mentioned in the beginning, the complexity of criteria, their substantial nonlinear character and the need of many runs of the problem so as to obtain a Pareto surface, make the usage of these techniques very difficult. For these reasons an attempt is made below to solve the problem (22) by the described man-machine procedure.

5. Illustrative Example

The described procedure for making the rational decision by interactive man-machine dialogue has been applied for control of a multipurpose reservoir for a time period of one year divided into twelve months. The amount of water released from the reservoir is distributed to the following users: industrial water supply (users No. 1 and No. 2); irrigation of different crops (user No. 3--wheat, No. 4--barley, No. 5--corn, No. 6--

corn for fodder, No. 7--vegetables, No. 8--lucerne, No. 9--meadows and pastures); an additional amount of water given for power generation (user No. 10) (except that one distributed to the users No. 1 - No. 9 and used already for power generation); and drinking water supply (user No. 11).

The data needed for carrying out the items 1 and 2 of the procedure EVAL are as follows:

1) input in the reservoir $R = \{R_s\}$, $s = \overline{1,12}$ (Table 1); the choice of the vector R has been done assuming the comparatively big shortage will take place during the optimization period;

2) vectors $R^w = \{R_s^w\}$, $s = \overline{1,12}$; $w = \overline{1,10}$, represent the historical data available for the input R of the reservoir (Table 1);

3) lower ν_k^s and upper μ_k^s bounds for all $s = \overline{1,12}$ and $k = \overline{1,10}$ (Table 3); evaporation and other losses $\gamma = \{\gamma_s\}$ are in the reservoir (Table 1);

4) the constants M, M_o, $\overset{0}{Z}_s$, λ_k, θ_k, b_1, b_2, b_3, $\overset{0}{c}_k$, $k = \overline{1,11}$ are shown in Table 2;

5) the loss functions $f_k^s(\bar{y}_k^s)$, all k and s are piecewise nonlinear and concave.

$$f_k^s(\bar{y}_k^s) = \begin{cases} p_k^s\, \bar{y}_k^s + a_k^s(\bar{y}_k^s)^2 + \delta_k^s\,, & \text{if } 0 \le \bar{y}_k^s \le r_k^s\,, \\ \beta_k^s\, \bar{y}_k^s + \alpha_k^s(\bar{y}_k^s)^2 + \xi_k^s\,, & \text{if } r_k^s \le \bar{y}_k^s \le \mu_{k}^s - \nu_k\,, \end{cases} \quad k = \overline{1,10},\ s = \overline{1,12}$$

where

$$\bar{y}_k^s = y_k^s - \nu_k^s \quad .$$

The coefficients of these functions are shown in Table 3.

The set of parameters $\{P_\ell\}$ used by the DM in the procedure EVAL are: lower ν_k^s and upper μ_k^s boundaries and sometimes the coefficients of the loss functions $f_k^s(\bar{y}_k^s)$ and the input R. Also all of the rest variables and coefficients can be included in $\{P_\ell\}$ if they are considered as inaccurately defined.

Using the method described in [18], the following results have been obtained:

a) the optimum amount of water, according to the first item of EVAL, which should be distributed among the users during the optimization period. In Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 the graphical results are shown (using right hatch). The analytical solution is shown in Table 4.

b) the state of the reservoir during the optimization period (Figure 13). The state of the reservoir when the vector R^w, $w = \overline{1,10}$ is changed is also shown in this figure. This state is computed under the assumptions that the allocation which have been already accepted could not be changed, i.e. the question is answered, what will happen if R^w changes but vector x, and respectively its components y_k^s, does not.

c) the values of the criteria $\psi_1(x^1),\ldots,\psi_7(x^1)$ (the bigger value of $\psi_i(x^1)$ the more acceptable vector x^1) (Table 6).

Having all this information, as well as some of the intermediate computational results, the DM goes to item three of the procedure EVAL, namely evaluation of the results which have already been obtained.

What do the results obtaining at the first stage of the procedure EVAL for this particular system show?

1) Users N1 and N2 (industrial water supply) obtain in the months I, II, III and X the whole demanded water. During the remaining months the maximum demand deviation is .98%.

2) User N11 (drinking water supply) is fully satisfied all the time.

3) User N3 gets 18.86% as its demand occurs only in October. User N9 does not get water at all. There are at least two reasons for this: a) these crops are not important for the system considered; b) if a) is not true, then the objective functions of these crops were not determined precisely.

4) Some of the users, i.e. N4 and N5, do not have a uniform satisfaction of their demands.

5) The end state of the reservoir equals the minimum admissible amount of water in it.

6) Figure 13 indicates the influence of the stochastic input represented by the vector R^w, $w = \overline{1,10}$. If w = 3,4,6,7,8 and 9, then after August the reservoir will be empty. If

$w = 1,2,5$ and 10, then there will be enough water even for complete satisfaction of the users' demands in the reservoir.

7) It can be seen that there is a difference between a predetermined order of the users and this one already obtained concerning the users Nos. 3, 4, 5, 6, 7 and 9. Such a difference may be caused by the following:

a) the predetermined order reflecting the subjunctive opinion of the DM is wrong;

b) the objective function of the users are not determined exactly.

8) User No. 10 (hydro power generation) obtains $20.5475 \cdot 10^6\ m^3$ additional water. This additional amount of water cannot be used by the other users during the "nonirrigated" months and in the condition of shortage could be "a peculiar luxury" to produce energy by water.

Taking into consideration the results obtained at the first stage of procedure EVAL, the DM has decided first of all to reduce the upper bound μ_{10}^{s} of user No. 10. The new solution is shown in Table 5 and in Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 14 using left hatch. At that, the second solution is better with respect to the criterion $\psi_3(x)$, i.e. equability in satisfying users' demands has been improved.

Using other parameters of the set $\{P_\ell\}$, the DM could improve the solution of the problem. This process could go on until the DM finds the satisfying compromise among the values of the criteria $\psi_1(x),\ldots,\psi_7(x)$.

6. Computational Time

For carrying out the procedure EVAL a computer program was developed. The program consists of three subroutines: preparative subroutine, subroutine for optimization and subroutine for introducing the functions $f_k^s(y_k^s)$. The latter permits the introduction of piece-wise, concave, nonlinear functions of any type.

A single reservoir's optimization model comprising 120 variables was executed in about 0.25 minutes on CDC 6600. This included all computations needed for carrying out the first and the second items in the procedure EVAL.

7. Conclusions

The procedure EVAL presented here can be used when multiobjectives are divided into two subsets: a primary objective and the secondary objectives. In many practical applications in water resource systems, economic efficiency usually can be asserted as being primary and the secondary objectives imply additional goals have to be achieved.

Using this procedure, instead of conventional techniques for multiobjective programming, a closer contact between a system investigator (SI) and the DM can be established. Furthermore, such an interactive approach enables both SI and DM to define parameters more precisely in the optimization model. Another advantage of this approach is that the complexity of an optimization problem does not depend on the mathematical structure of the secondary objectives at all.

The procedure can also be implemented in a multireservoir water resource system and in more general gaming situations between different DM's who agree that one of the criteria is primary. After changing parameters of the optimization model in a direction they think is appropriate and after evaluation of results, a compromise between them could be achieved.

Table 1.

	1	2	3	4	5	6	7	8	9	10	11	12
R_s	15.00	19.00	20.00	44.00	40.00	25.00	5.00	2.00	2.00	2.00	5.00	11.00
R^1	19.53	17.67	32.86	44.10	87.63	75.60	26.97	21.70	7.80	10.23	17.70	18.29
R^2	16.00	20.00	32.00	43.00	75.00	52.00	21.00	6.00	4.00	6.00	12.00	13.00
R^3	5.86	7.59	8.96	16.10	32.06	29.71	17.22	9.10	5.13	6.76	8.00	4.78
R^4	8.04	5.74	5.47	8.90	37.51	32.42	19.02	10.93	3.96	3.94	16.67	8.21
R^5	4.65	8.71	32.07	59.07	67.78	24.00	19.79	7.29	4.03	22.14	5.37	4.10
R^6	4.35	12.62	16.01	18.19	28.29	22.71	8.17	7.83	2.16	4.60	4.14	3.99
R^7	15.54	16.04	6.70	12.33	19.85	40.46	15.98	7.57	3.40	4.13	7.87	2.76
R^8	12.13	6.58	20.69	31.87	26.45	14.50	4.53	3.13	2.87	2.19	6.29	13.35
R^9	3.48	3.07	7.89	5.53	17.69	31.86	19.63	6.38	7.53	5.84	2.17	2.36
R^{10}	12.24	10.50	16.40	21.57	57.50	34.85	9.92	4.70	3.98	4.88	14.75	12.15
γ_s	0.9995	0.9990	0.9985	0.9980	0.9970	0.9970	0.9890	0.9800	0.9890	0.9900	0.9990	0.9995
$\overset{0}{Z}_s \times 10^6$	200	200	220	250	250	250	250	220	220	200	200	200

R_s - input in the reservoir

$R^1,\ldots,R^{10}$ - input in the reservoir (available historical data)

γ_s - evaporation coefficient

$\overset{0}{Z}_s$ - desirable state of the reservoir

Table 2.

k	1	2	3	4	5	6	7	8	9	10
λ_k^s	0.30	0.20	0.50	0.60	0.50	0.60	0.60	0.60	0.60	1.0
θ_k^s	0.95	0.80	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.0
$\overset{0}{C}_k$	1	2	4	5	3	7	6	9	10	8

$\lambda_k^1=\ldots=\lambda_k^s=\ldots=\lambda_k^{12}$ for all k

$\theta_k^1=\ldots=\theta_k^s=\ldots=\theta_k^{12}$ for all k

$b_1=0.22;\ b_2=0.31;\ b_3=0.47$

$M = 473.10^6;\ M_o = 200.10^6;\ Z_o = 92.10^6$

λ_k^s - level of pollution

θ_k^s - ratio between supplied water and discharged waste water

$\overset{0}{C}_k$ - rank of the users

Table 3.

(Loss of the k^{th} user, $k = \overline{1,10}$ during the optimization period)

	s	1	2	3	4	5	6	7	8	9	10	11	12
	P_1^s	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92	-3.92
	$a_1^s x 10^{-6}$	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714	0.8714
	$\delta_1^s x 10^6$	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
	β_1^s	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00
$f_1^s(y_1^s)$	$\alpha_1^s x 10^{-6}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	$\xi_1^s x 10^6$	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	$\nu_1^s x 10^6$	4.6	4.0	4.6	4.5	4.6	4.6	4.8	4.8	4.6	4.8	4.6	4.8
	$r_1^s x 10^6$	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	$\mu_1^s x 10^6$	6.1	5.5	6.1	6.0	6.1	6.1	6.3	6.3	6.1	6.3	6.1	6.3
	P_2^s	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875
	$a_2^s x 10^{-6}$	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833	2.5833
	$\delta_2^s x 10^6$	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03
	β_2^s	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5
$f_2^s(y_2^s)$	$\alpha_2^s x 10^{-6}$	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	$\xi_2^s x 10^6$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	$\nu_2^s x 10^6$	1.4	1.2	1.4	1.4	1.5	1.4	1.6	1.6	1.6	1.6	1.6	1.6
	$r_2^s x 10^6$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	$\mu_2^s x 10^6$	1.9	1.7	1.9	1.9	2.0	1.9	2.1	2.1	2.1	2.1	2.1	2.1

Table 3 (cont.)

$f_3^s(y_3^s)$	P_3^s	-	-	-	-0.0615	-0.0655	-	-	-	-0.0615	-0.0655	-	-
	$a_3^s \times 10^{-6}$	-	-	-	0.0040	0.0053	-	-	-	0.0040	0.0053	-	-
	$\delta_3^s \times 10^6$	-	-	-	0.1792	0.1408	-	-	-	0.1792	0.1408	-	-
	β_3^s	-	-	-	-0.0425	-0.0311	-	-	-	-0.0425	-0.0311	-	-
	$\alpha_3^s \times 10^{-6}$	-	-	-	0.0062	0.0655	-	-	-	0.0062	0.0655	-	-
	$\xi_3^s \times 10^6$	-	-	-	0.01	0.01	-	-	-	0.01	0.01	-	-
	$\nu_3^s \times 10^6$	-	-	-	0	0	-	-	-	0	0	-	-
	$r_3^s \times 10^6$	-	-	-	3.6	2.5	-	-	-	3.6	2.5	-	-
	$\mu_3^s \times 10^6$	-	-	-	5.1	4.128	-	-	-	5.1	4.128	-	-
$f_4^s(y_4^s)$	P_4^s	-	-	-	-0.1043	-0.1045	-	-	-	-0.1043	-0.1157	-	-
	$a_4^s \times 10^{-6}$	-	-	-	0.0112	0.0124	-	-	-	0.0112	0.0214	-	-
	$\delta_4^s \times 10^6$	-	-	-	0.1823	0.1693	-	-	-	0.1823	0.1172	-	-
	β_4^s	-	-	-	-0.0930	-0.0874	-	-	-	-0.0930	-0.1005	-	-
	$\alpha_4^s \times 10^{-6}$	-	-	-	0.0150	0.0156	-	-	-	0.0150	0.0279	-	-
	$\xi_4^s \times 10^6$	-	-	-	0.0150	0.0100	-	-	-	0.0150	0.0100	-	-
	$\nu_4^s \times 10^6$	-	-	-	0	0	-	-	-	0	0	-	-
	$r_4^s \times 10^6$	-	-	-	2.100	2.000	-	-	-	2.100	1.200	-	-
	$\mu_4^s \times 10^6$	-	-	-	3.100	2.800	-	-	-	3.100	1.798	-	-

Table 3 (cont.)

$f_5^s(y_5^s)$	p_5^s	-	-	-	-	-	-0.1579	-0.1179	-0.1473	-	-	-	-
	$a_5^s \times 10^{-6}$	-	-	-	-	-	0.0261	0.0091	0.0087	-	-	-	-
	$\delta_5^s \times 10^6$	-	-	-	-	-	0.1834	0.3884	0.5071	-	-	-	-
	β_5^s	-	-	-	-	-	-0.1598	-0.1593	-0.1680	-	-	-	-
	$\alpha_5^s \times 10^{-6}$	-	-	-	-	-	0.0397	0.0185	0.0150	-	-	-	-
	$\xi_5^s \times 10^6$	-	-	-	-	-	0.0150	0.0150	0.0150	-	-	-	-
	$\nu_5^s \times 10^6$	-	-	-	-	-	0	0	0	-	-	-	-
	$r_5^s \times 10^6$	-	-	-	-	-	1.400	3.400	4.600	-	-	-	-
	$\mu_5^s \times 10^6$	-	-	-	-	-	2.015	4.300	5.600	-	-	-	-
$f_6^s(y_6^s)$	p_6^s	-	-	-	-	-	-	-0.1948	-0.2612	-0.2629	-	-	-
	$a_6^s \times 10^{-6}$	-	-	-	-	-	-	0.0129	0.0106	0.0149	-	-	-
	$\delta_6^s \times 10^6$	-	-	-	-	-	-	0.7966	1.2869	0.9805	-	-	-
	β_6^s	-	-	-	-	-	-	-0.2222	-0.2531	-0.2917	-	-	-
	$\alpha_6^s \times 10^{-6}$	-	-	-	-	-	-	0.0222	0.0154	0.0235	-	-	-
	$\xi_6^s \times 10^6$	-	-	-	-	-	-	0.05	0.05	0.05	-	-	-
	$\nu_6^s \times 10^6$	-	-	-	-	-	-	0	0	0	-	-	-
	$r_6^s \times 10^6$	-	-	-	-	-	-	3.500	6.400	4.750	-	-	-
	$\mu_6^s \times 10^6$	-	-	-	-	-	-	5.000	8.200	6.209	-	-	-

Table 3 (cont.)

	p_7^s	-	-	-	-	-0.6304	-0.6855	-0.6440	-0.6440	-0.7004	-	-	-
	$a_7^s x 10^{-6}$	-	-	-	-	0.0964	0.1632	0.1431	0.1431	0.2335	-	-	-
	$\delta_7^s x 10^6$	-	-	-	-	0.8280	0.5323	0.5914	0.5914	0.4140	-	-	-
	β_7^s	-	-	-	-	-0.4716	-0.1750	-0.4800	-0.4800	-0.6400	-	-	-
$f_7^s(y_7^s)$	$\alpha_7^s x 10^{-6}$	-	-	-	-	0.1082	0.0625	0.1600	0.1600	0.3200	-	-	-
	ξ_7^s x 10^6	-	-	-	-	0.02	0.01	0.01	0.01	0.02	-	-	-
	$\nu_7^s x 10^6$	-	-	-	-	0	0	0	0	0	-	-	-
	$r_7^s x 10^6$	-	-	-	-	1.7500	1.0000	1.2500	1.2500	0.7500	-	-	-
	$\mu_7^s x 10^6$	-	-	-	-	2.1800	1.4000	1.5000	1.5000	1.0000	-	-	-
	p_8^s	-	-	-	-	-0.1747	-0.1747	-0.1878	-0.1747	-0.1878	-	-	-
	$a_8^s x 10^{-6}$	-	-	-	-	0.0144	0.0144	0.0123	0.0144	0.0123	-	-	-
	$\delta_8^s x 10^6$	-	-	-	-	0.4359	0.4259	0.5571	0.4359	0.5571	-	-	-
	β_8^s	-	-	-	-	-0.1678	-0.1678	-0.1992	-0.1678	-0.1992	-	-	-
$f_8^s(y_8^s)$	$\alpha_8^s x 10^{-6}$	-	-	-	-	0.0207	0.0207	0.0195	0.0207	0.0195	-	-	-
	$\xi_8^s x 10^6$	-	-	-	-	0.0500	0.0500	0.0500	0.0500	0.0500	-	-	-
	$\nu_8^s x 10^6$	-	-	-	-	0	0	0	0	0	-	-	-
	$r_8^s x 10^6$	-	-	-	-	2.900	2.900	3.500	2.900	3.500	-	-	-
	$\mu_8^s x 10^6$	-	-	-	-	4.054	4.100	5.100	4.100	5.100	-	-	-

Table 3 (cont.)

$f_9^s(y_9^s)$	P_9^s	-	-	-	-	-	-0.0228	-0.0443	-0.0468	-	-	-	-
	$a_9^s x 10^{-6}$	-	-	-	-	-	0.0011	0.0049	0.0039	-	-	-	-
	$\delta_9^s x 10^6$	-	-	-	-	-	0.1460	0.0656	0.0864	-	-	-	-
	β_9^s	-	-	-	-	-	-0.0414	-0.0256	-0.0320	-	-	-	-
	$\alpha_9^s x 10^{-6}$	-	-	-	-	-	0.0029	0.0042	0.0400	-	-	-	-
	$\xi_9^s x 10^6$	-	-	-	-	-	0.0200	0.0100	0.0250	-	-	-	-
	$\nu_9^s x 10^6$	-	-	-	-	-	0	0	0	-	-	-	-
	$r_9^s x 10^6$	-	-	-	-	-	4.400	1.500	1.500	-	-	-	-
	$\mu_9^s x 10^6$	-	-	-	-	-	7.000	3.040	4.000	-	-	-	-
$f_{10}^s(y_{10}^s)$	P_{10}^s	0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840	-0.1840
	$a_{10}^s x 10^{-6}$	0.0097	0.0114	0.0209	0.0204	0.0151	0.0110	0.0107	0.0097	0.0105	0.0103	0.0105	0.0088
	$\delta_{10}^s x 10^6$	0.8740	0.7452	0.4048	0.4140	0.5612	0.7728	0.7912	0.8740	0.8096	0.8188	0.8096	0.9660
	β_{10}^s	-	-	-	-	-	-	-	-	-	-	-	-
	α_{10}^s	-	-	-	-	-	-	-	-	-	-	-	-
	ξ_{10}^s	-	-	-	-	-	-	-	-	-	-	-	-
	ν_{10}^s	0	0	0	0	0	0	0	0	0	0	0	0
	r_{10}^s	-	-	-	-	-	-	-	-	-	-	-	-
	μ_{10}^s	9.500	8.100	4.400	4.500	6.100	8.400	8.600	9.500	8.800	8.900	8.800	10.50
Demand of the 11^{th} user (drinking water supply)	$\mu_{11}^s x 10^6$	11.00	9.90	11.00	10.60	11.00	10.60	11.50	11.50	11.20	11.50	11.20	11.50

Table 4.

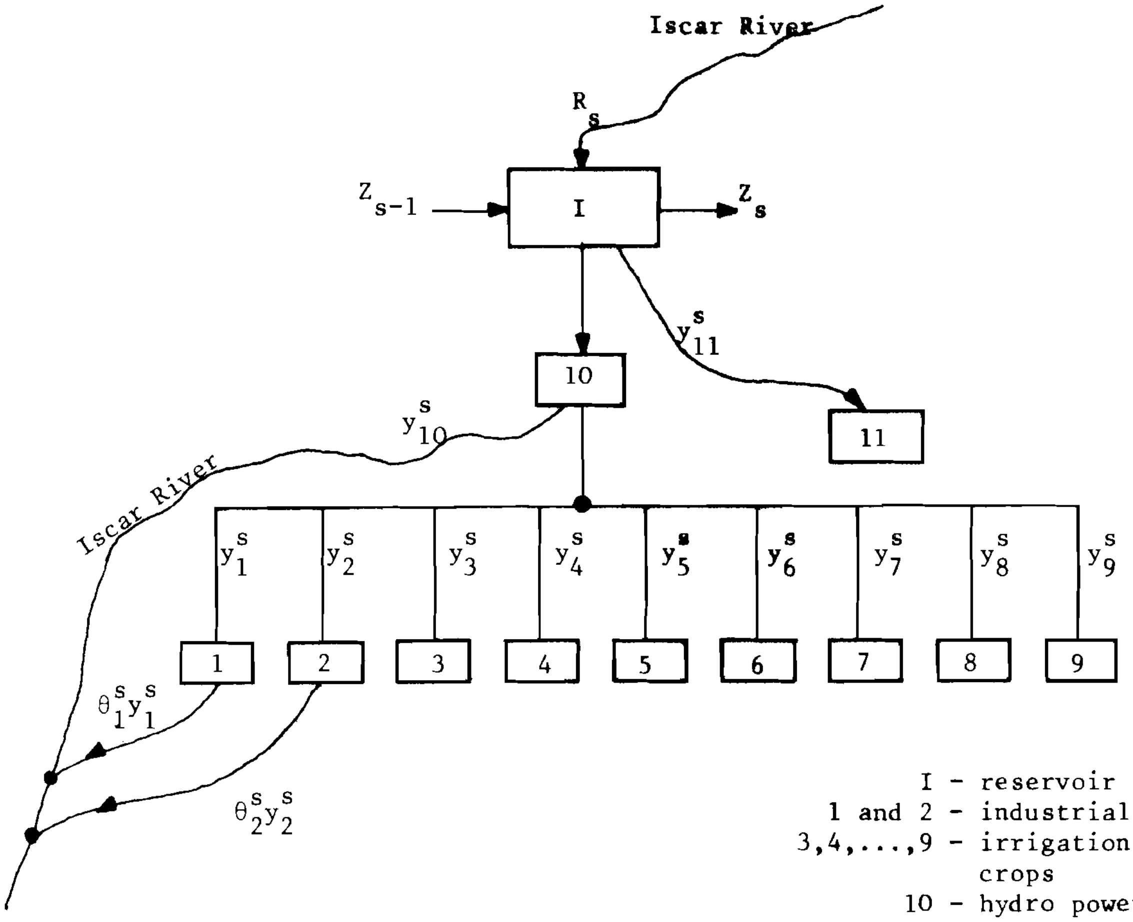
months / users	1	2	3	4	5	6	7	8	9	10	11	12
1	6.1	5.5	6.1	5.9606	6.0499	6.0499	6.2427	6.2386	6.0484	6.2713	6.1	6.2325
2	1.9	1.7	1.9	1.8842	1.9799	1.8799	2.0771	2.0754	2.0754	2.0885	2.1	2.073
3	-	-	-	0	0	-	-	-	0	0.7779	-	-
4	-	-	-	1.1366	0.17009	-	-	-	0.0516	1.2000	-	-
5	-	-	-	-	-	1.0992	1.7844	1.3958	-	-	-	-
6	-	-	-	-	-	-	3.0856	6.4000	4.7500	-	-	-
7	-	-	-	-	1.7500	1.000	1.2500	1.2500	0.8388	-	-	-
8	-	-	-	-	2.5921	2.5921	2.9812	1.8067	3.4494	-	-	-
9	-	-	-	-	-	0	0	0	-	-	-	-
10	2.3732	1.9918	1.0848	1.1092	1.4940	2.0447	2.0 9	2.2474	2.0533	2.0722	2.0307	0
11	11.00	9.90	11.00	10.60	11.00	10.60	11.50	11.50	11.20	11.50	11.20	11.50

Table 5.

months / users	1	2	3	4	5	6	7	8	9	10	11	12
1	6.1	5.5	6.1	5.9560	6.0536	6.0508	6.2499	6.2493	6.0545	6.2606	6.1	6.3
2	1.9	1.7	1.9	1.8824	1.9815	1.8803	2.0799	2.0797	2.0818	2.0842	2.1	2.1
3	-	-	-	0	0	-	-	-	0	0	-	-
4	-	-	-	0.7222	0.4725	-	-	-	0.5941	0.8603	-	-
5	-	-	-	-	-	1.1333	0.9644	2.6203	-	-	-	-
6	-	-	-	-	-	-	3.5000	6.4000	4.7500	-	-	-
7	-	-	-	-	1.7513	1.0000	1.2500	1.2500	0.8578	-	-	-
8	-	-	-	-	2.8539	2.6539	3.5000	2.5539	3.5000	-	-	-
9	-	-	-	-	-	0	0	0	-	-	-	-
10	0.9	0.8	0.7	0.6	0.5	0.001	0.001	0.001	0.5	0.6	0.7	0.8
11	11.00	9.90	11.00	10.60	11.00	10.60	11.50	11.50	11.20	11.50	11.20	11.50

Table 6.

	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$	$\psi_4(x)$	$\psi_5(x)$	$\psi_6(x)$	$\psi_7(x)$
1st run	0.85725	0.63636	0.90476	0.29718	0.20559	0.4000	0.48969
2nd run	0.80771	0.45455	0.88889	0.29721	0.29257	0.40000	0.47803



I - reservoir
1 and 2 - industrial water supply
3,4,...,9 - irrigation of different crops
10 - hydro power station
11 - drinking water supply

Figure 1.

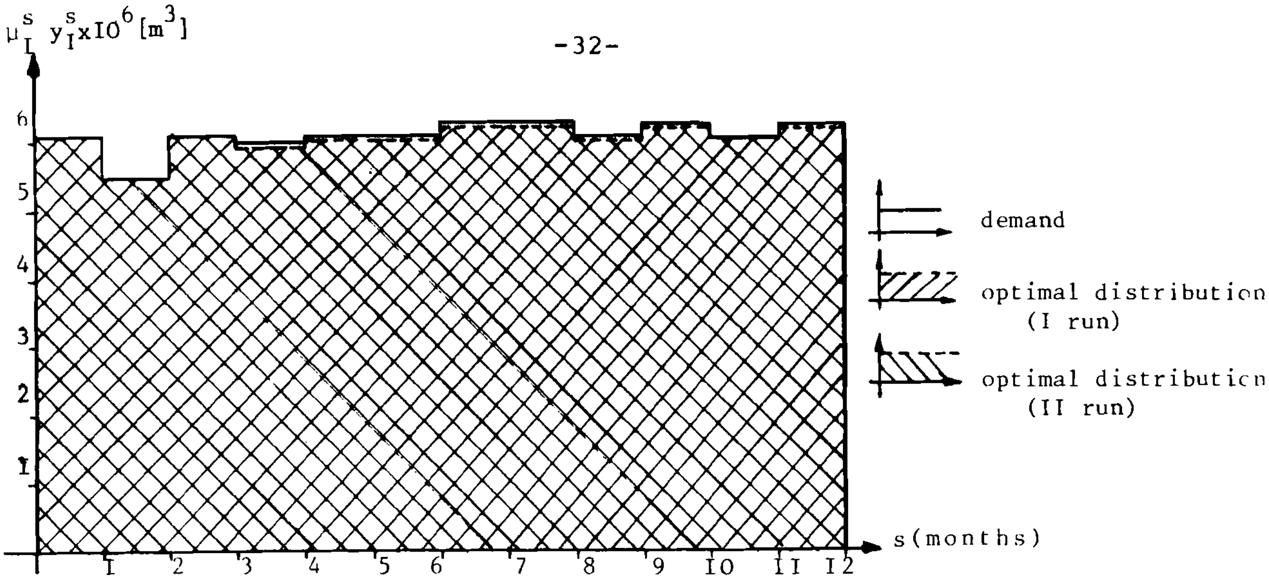


Figure 2. Demand and distributed amount of water to the 1st user

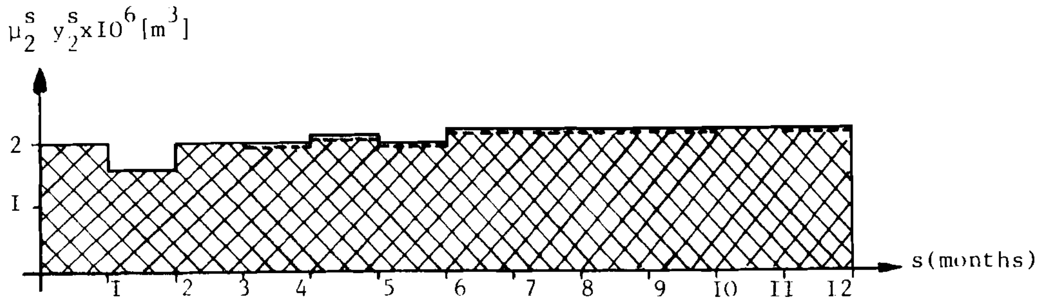


Figure 3. Demand and distrubuted amount of water to the 2nd user

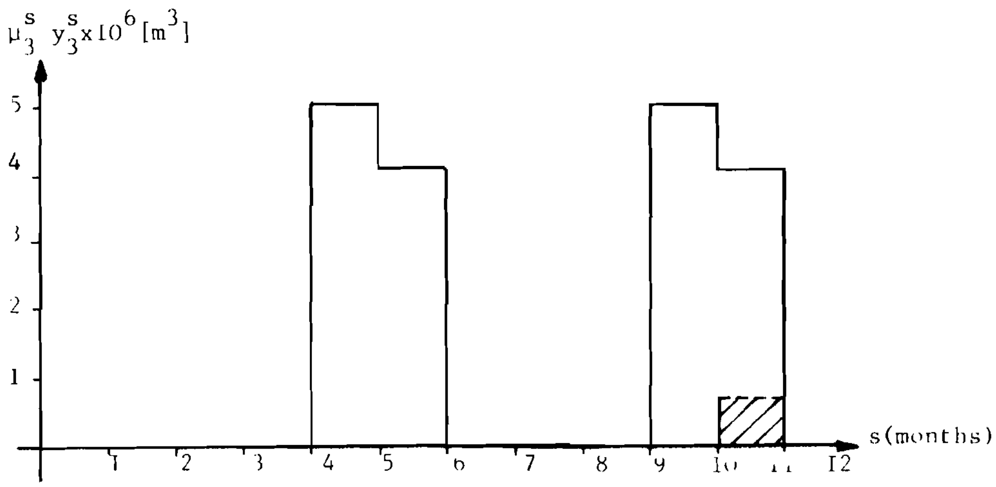


Figure 4. Demand and distrubuted amount of water to the 3rd user

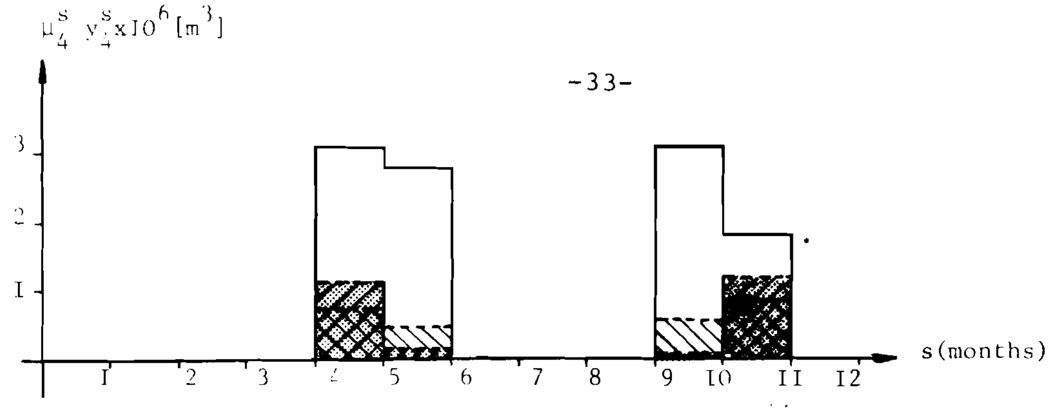


Figure 5. Demand and distributed amount of water to the 4th user

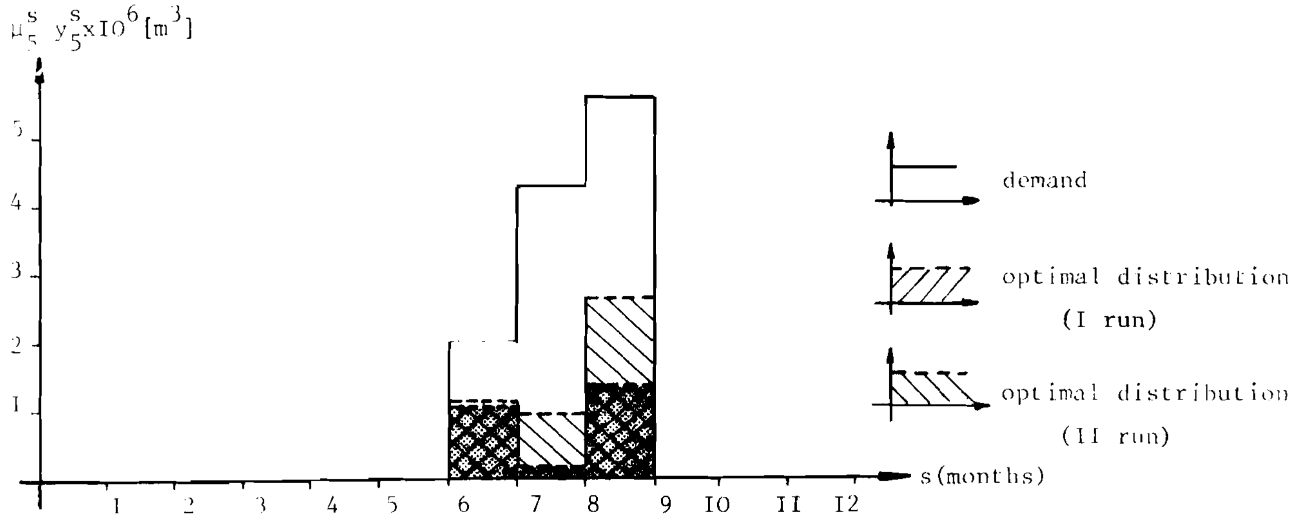


Figure 6. Demand and distributed amount of water to the 5th user

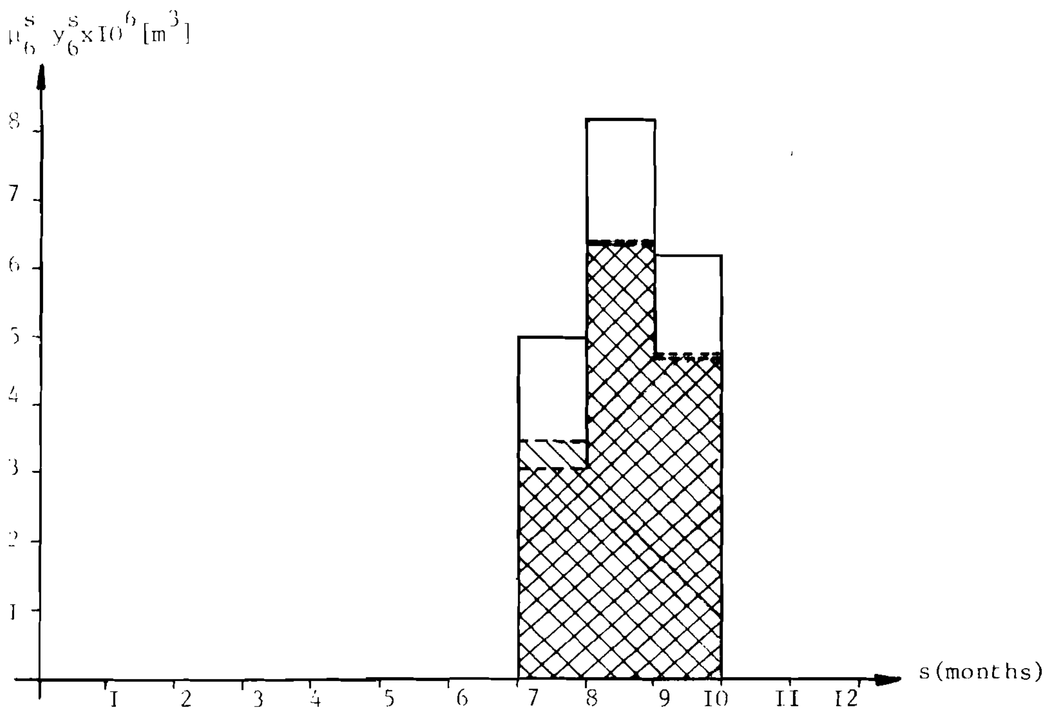


Figure 7. Demand and distributed amount of water to the 6th user

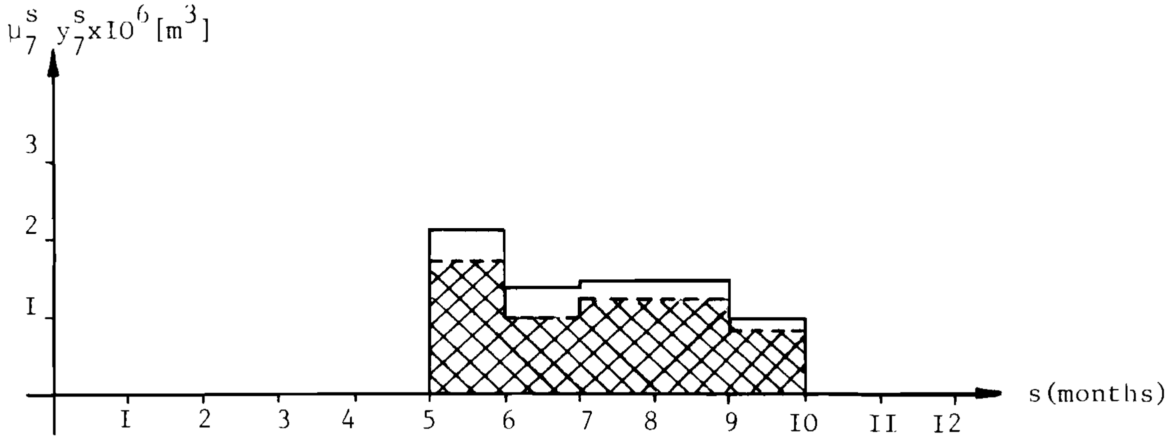


Figure 8. Demand and distributed amount of water to the 7[th] user

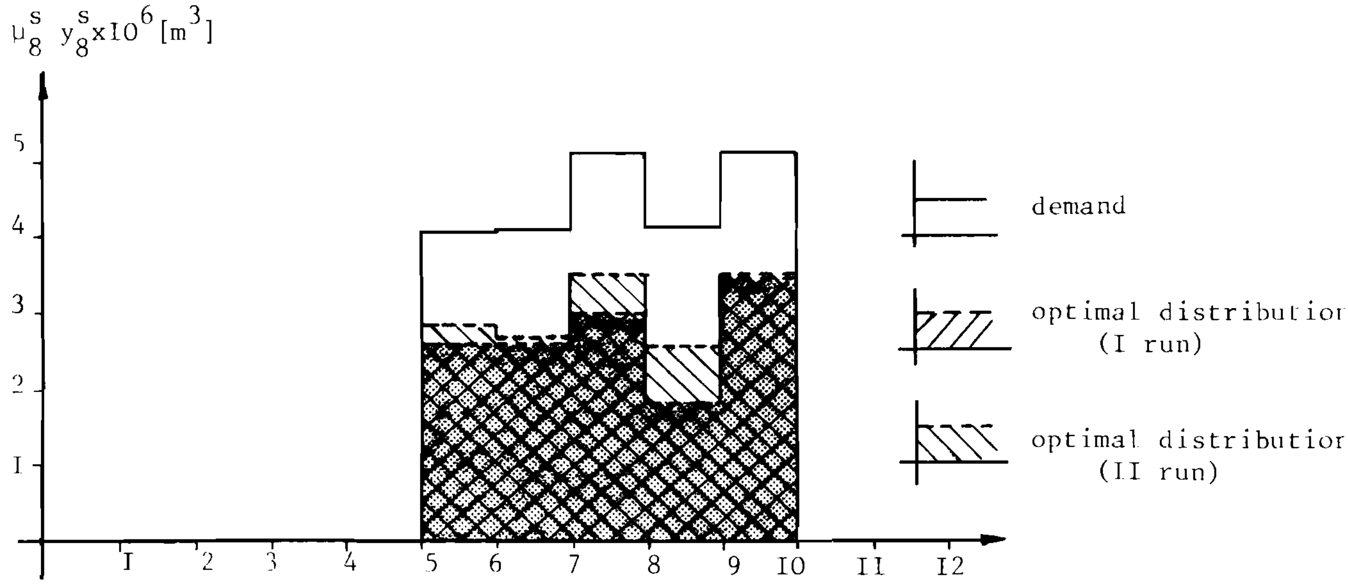


Figure 9. Demand and distributed amount of water to the 8[th] user

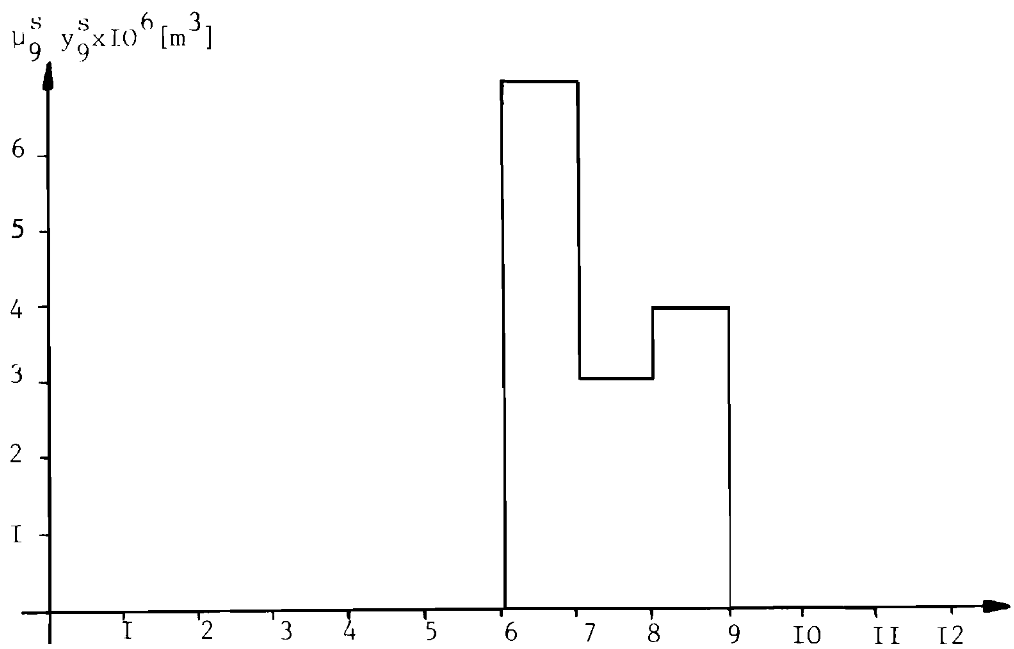


Figure IO. Demand and distributed amount of water to the 9[th] user

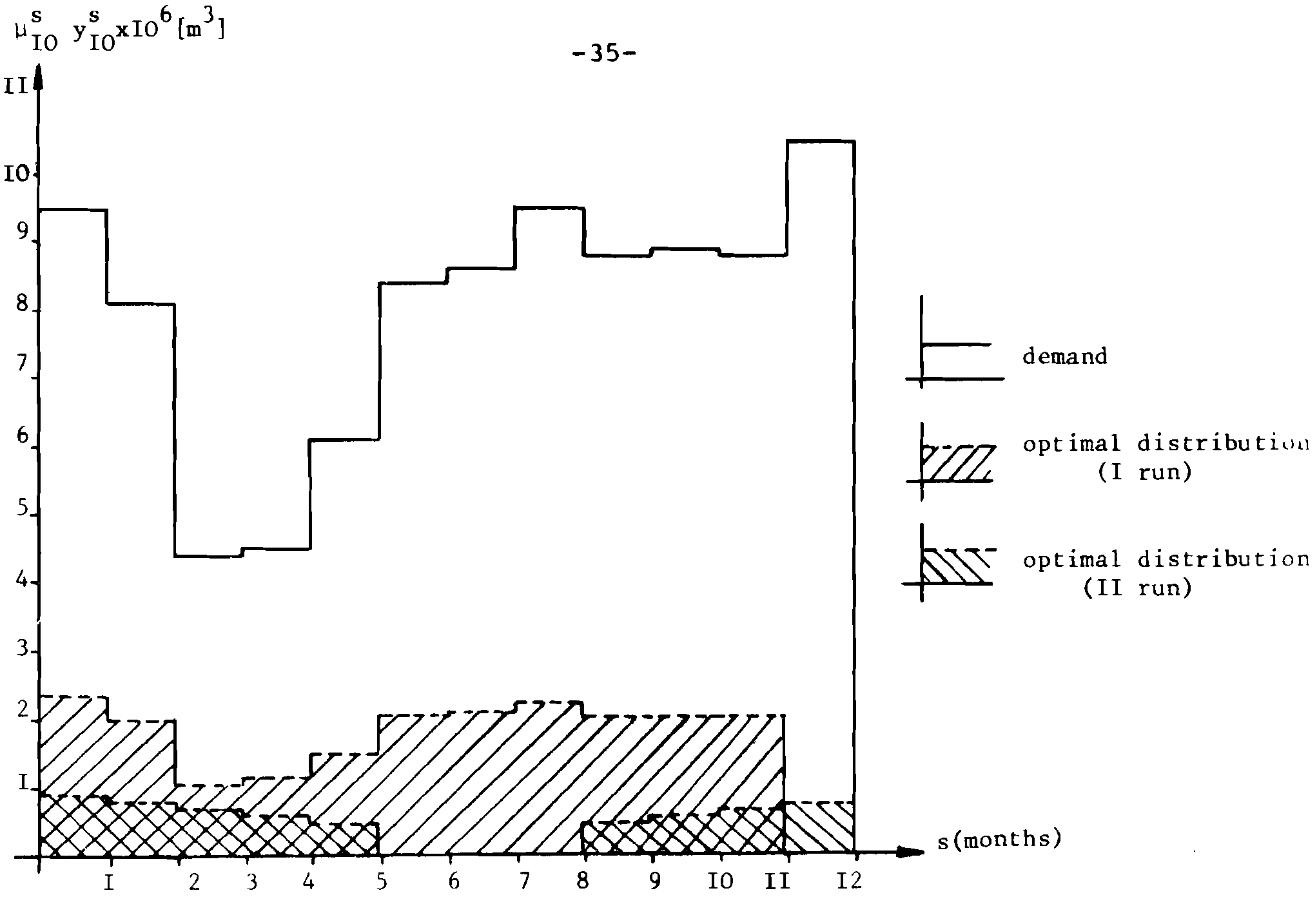


Figure II. Demand and distributed amount of water to the IOth user

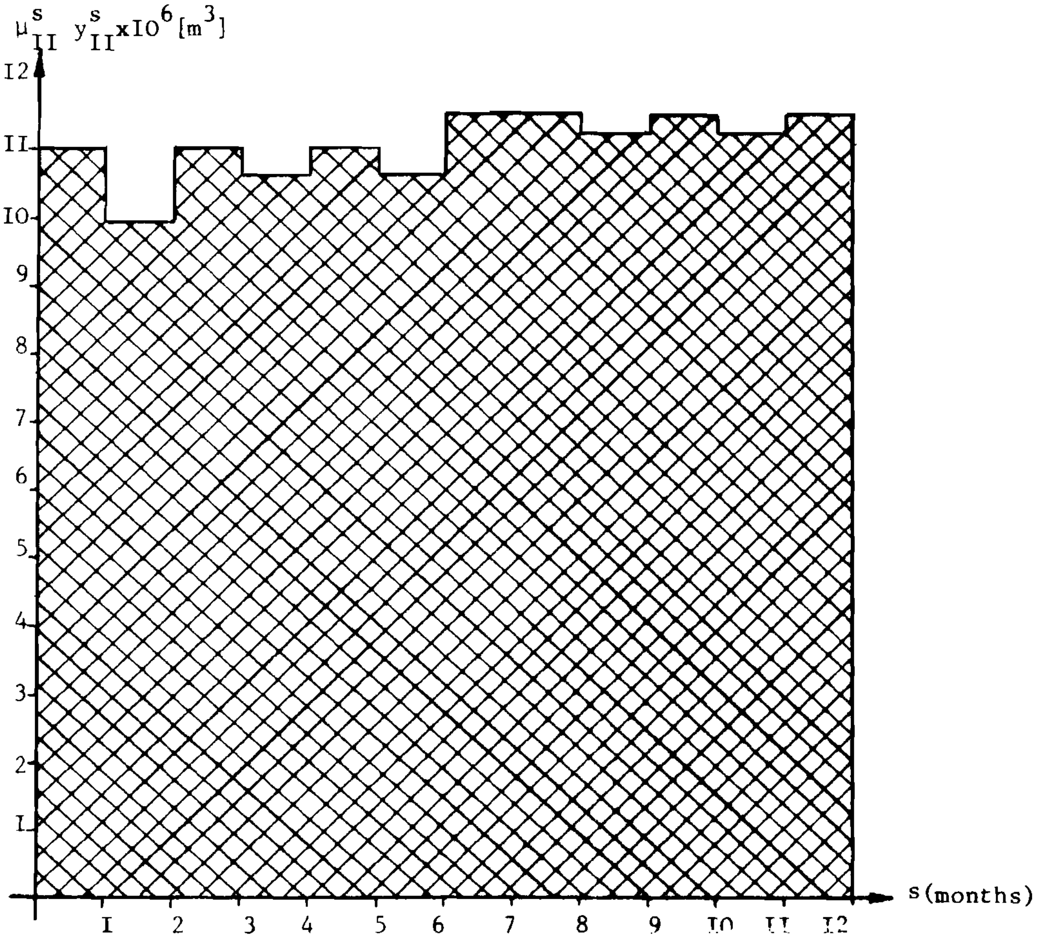


Figure I2. Demand and distributed amount of water to the IIth user

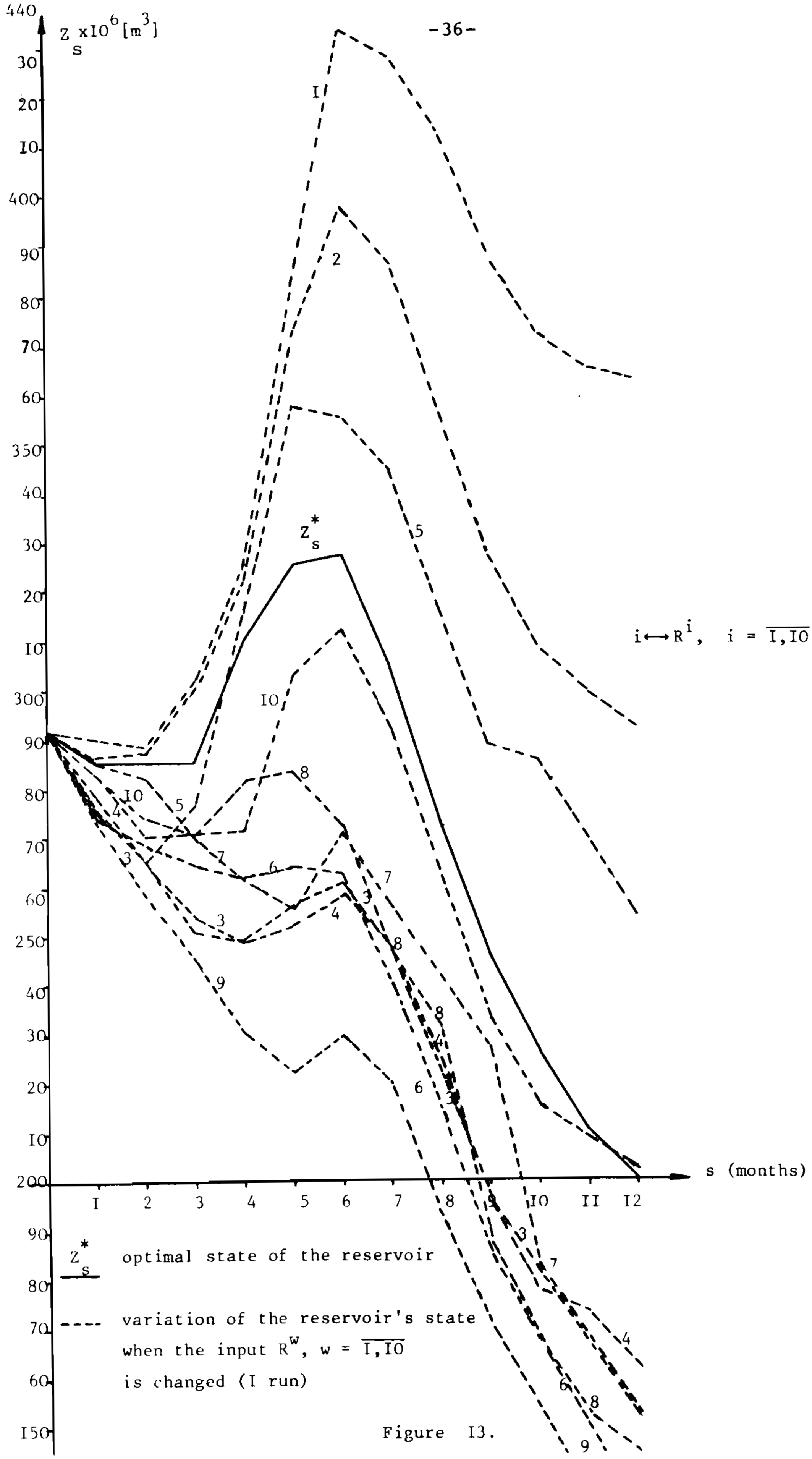
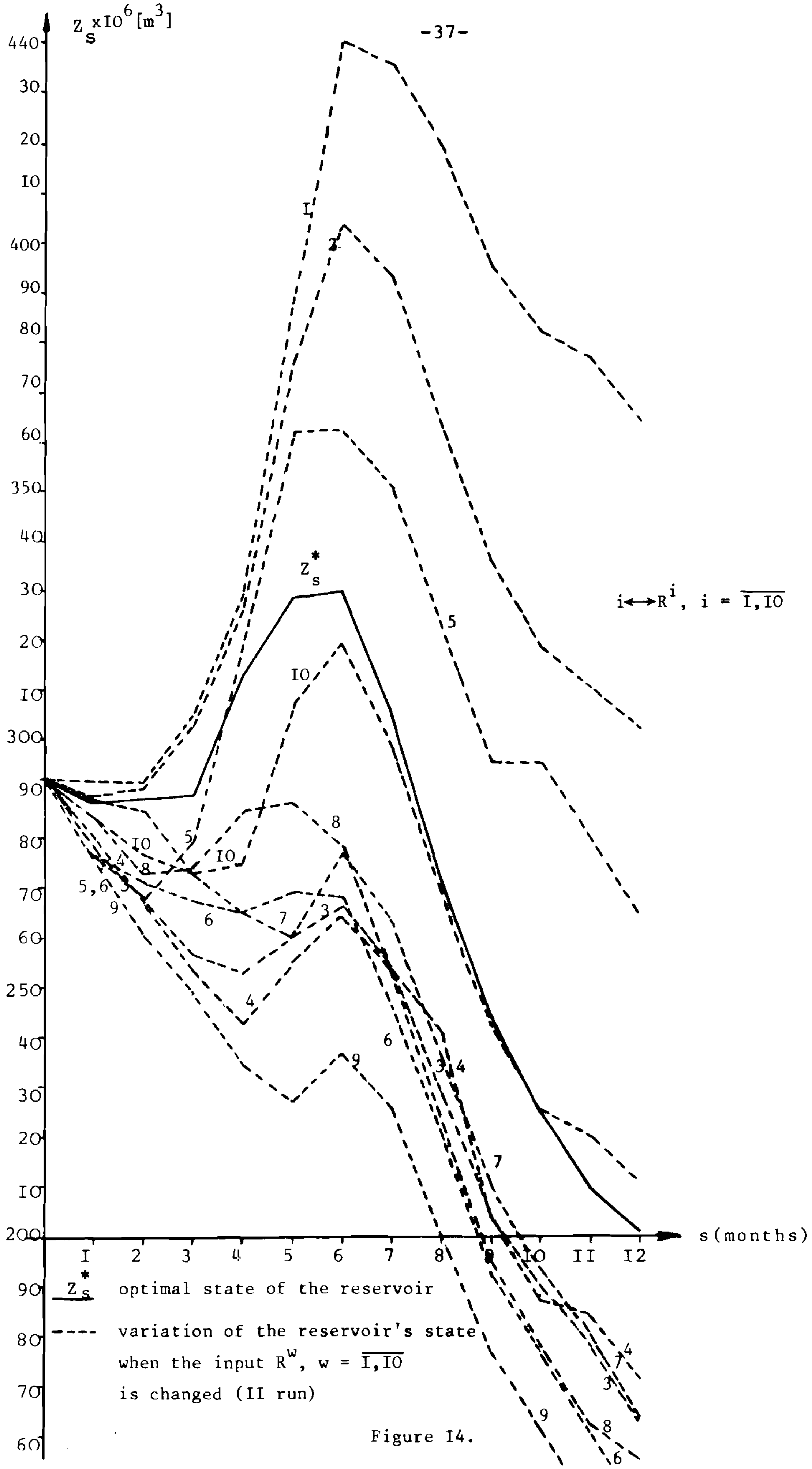

Z_s xI0^6[m^3]
440
30
20
I0
400
90
80
70
60
350
40
30
20
I0
300
90
80
70
60
250
40
30
20
I0
200
90
80
70
60
150
s (months)
I
2
3
4
5
6
7
8
9
I0
II
I2
Z_s^*
i ⟷ R^i, i = $\overline{I,I0}$
Z_s^* optimal state of the reservoir
variation of the reservoir's state when the input R^w, w = $\overline{I,I0}$ is changed (I run)

Figure I3.



$Z_s \times 10^6 [m^3]$
440
30
20
10
400
90
80
70
60
350
40
30
20
10
300
90
80
70
60
250
40
30
20
10
200
90
80
70
60
s(months)
I 2 3 4 5 6 7 8 9 10 II 12
Z_s^*
$i \leftrightarrow R^i$, $i = \overline{I,IO}$
Z_s^* optimal state of the reservoir
variation of the reservoir's state when the input R^w, $w = \overline{I,IO}$ is changed (II run)

Figure I4.

References

[1] Belenson, S.M. and Kapur, C.K. "An Algorithm for Solving Multicriterion Programming Problem with Examples." Operational Research Quarterly, 24, 1 (1973).

[2] Benayoun, R. and Terguy, J. " Criteres Multiples et Programation Mathematique: une Solution dans le cas lineaire." Rev. Franc. Inform. et Rech. Oper., 3, V-2 (1969).

[3] Borisov, V.I. "On Vector-Valued Optimization Problems." (In Russian). Proceedings of the 1st Operation Research Conference. Minsk, 1972.

[4] Boychuck, L.M. and Ovchinnikov, V.O. "Basic Approaches for Solving Multiobjective Optimization Problems (Survey)." (In Russian). Automatica, 1 (1973).

[5] Ching, C.K. "On the Noninferior Set of the Systems with Vector-Valued Objective Function." IEEE Transactions on Automatic Control, 15, 5 (1970).

[6] Clarke, R.T. "Mathematical Models in Hydrology, Food and Agriculture Organization of the United Nations." Rome, 1973.

[7] Dinkelbach, W. "Über einen Lösungsansatz zum Vector-maximumproblem." Lect. Notes Oper. Res. and Math. Syst., 50 (1971).

[8] Dinus, S.H. "Social Accounting System for Evaluating Water Resources." Water Resources Research, 8, 5 (1972).

[9] Duckstein, L., Monarchi, D., and Kisiel, C.C. "Interactive Multi-Objective Decision Making Under Uncertainty." NATO Conference on "The Role and Effectiveness of Decision Theories in Practice." Luxembourg, August 1973.

[10] English, R., Kisiel, C.C., and Duckstein, L. "An Interactive Algorithm for the Multipurpose Control of a Natural Lake." Symposium on the Hydrology of Lakes. Helsinki, 1973.

[11] Fischburn, P.C. Utility Theory for Decision Making. New York, Academic Press, 1970.

[12] Gouevsky, I. "Decomposition Algorithm for a Class of Nonlinear Programming Problems." Proceedings of the Institute of Engineering Cybernetics, Bulgarian Academy of Sciences, Vol. 18, Sofia, 1974.

[13] Haimes, Y.Y. and Hall, W.A. "Multiobjectives in Water Resource Systems Analysis: The Surragate Worth Trade Off Method." Water Resources Research, 10, 4 (August 1974).

[14] Haurie, A. "On Pareto Optimal Decisions for a Coalition of a Subset of Players." IEEE Transactions on Automatic Control, AC-18, 2 (1973).

[15] Hufschmidt, M.M. and Fiering, M.B. Simulation Techniques for Design of Water Resource Systems. Cambridge, Mass., Harvard University Press, 1966.

[16] Kaczmarek, Z. "Some Problems of Stochastic Storage with Correlated Inflow." Proceedings of the International Symposium on Mathematical Models in Hydrology. 26-31 July 1971. Vol. 1, Part 1. Warsaw.

[17] Kapur, K.C. "Mathematical Methods of Optimization for Multiobjective Transportation Systems." Socio. Econ. Plan. Sci., 4, 4 (1970).

[18] Larichev, O.I. "Man-Machine Procedure for Decision Making (Survey)." (In Russian). Automation and Telemechanics, 12 (1971).

[19] Luce, R.D. and Raiffa, H. Games and Decisions. New York, John Wiley & Sons, 1958.

[20] Moran, P.A.P. The Theory of Storage. London, Methuen, 1959.

[21] Polyak, R.A. "An Approach to Multiobjective Programming Problems." (In Russian). Proceedings of the 4th Winter School on Mathematical Programming. Vol. 2, 1971.

[22] Prabhu, N.U. "Time-Dependent Results in Storage Theory." Methuen's Review Series in Applied Probability, Vol. 1, 1964.

[23] Proceedings of the International Symposium on Uncertainties in Hydrology and Water Resource Systems. December 11-14, 1972. Vol. 3, Tucson, Arizona, University of Arizona.

[24] Reid, R.W. and Vemuri, V. "On the Noninferior Index Approach to Large-Scale Multi-Criteria Systems." J. Franklin Inst., 291, 2 (1971).

[25] Roy, B. "Classement et choix en presence de points de vue multiples (la methode ELECTRE)." Rev. Franc. Inform. et Rech. Operat., 2 (1968).

[26] Savir, D. "Multiobjective Linear Programming." Operations Research Center, University of California, November 1966.

[27] Schmitendorf, W.E. "Co-operative Games and Vector-Valued Criteria Problems." IEEE Transactions on Automatic Control, AC-18, 2 (1973).

[28] Zadeh, L.A. "Optimality and Non-Scalar-Valued Performance Criteria." IEEE Transactions on Automatic Control, AC-8, 1 (1963).

[29] Zak, F.A. "Models and Methods for Finding Noninferior Solutions in Vector-Valued Optimization Problems." (In Russian). Cybernetics, 4 (1972).