MODELING INTERACTING RESOURCE-ECONOMY SYSTEMS

Anatoli Propoi
Institute for Systems Studies, 29 Ryleyev Street, Moscow 119034, USSR

May 1984
CP-84-17

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

PREFACE

Many of today's most significant socioeconomic problems, such as slower economic growth, the decline of some established industries, and shifts in patterns of foreign trade, are international or transnational in nature. But these problems manifest themselves in a variety of ways; both the intensities and the perceptions of the problems differ from one country to another, so that intercountry comparative analyses of recent historical developments are necessary. Through these analyses we attempt to identify the underlying processes of economic structural change and formulate useful hypotheses concerning future developments. The understanding of these processes and future prospects provides the focus for IIASA's project on Comparative Analysis of Economic Structure and Growth.

Our research concentrates primarily on the empirical analysis of interregional and intertemporal economic structural change, on the sources of and constraints on economic growth, on problems of adaptation to sudden changes, and especially on problems arising from changing patterns of international trade, resource availability, and technology. The project relies on IIASA's accumulated expertise in related fields and, in particular, on the data bases and systems of models that have been developed in the recent past.

In this paper, Anatoli Propoi examines the interactions between economic systems and those sectors of the economy that produce resources, broadly defined. He presents a general scheme for modeling these interacting systems that can be used to analyze both the structural dynamics of the sector concerned and its interrelations with the overall development of the economy. The method links a process model of the sector of interest with other economic models within an optimization framework, although the optimization is an analytical means rather than an end in itself. The approach is flexible and has the advantage that the individual models can be built and used separately, whilst at the same time making the fullest possible use of information (particularly subsystem shadow prices) derived from individual runs.

Anatoli Smyshlyaev
Project Leader
Comparative Analysis of
Economic Structure and Growth

ACKNOWLEDGMENTS

The author wishes to thank Ingrid Teply-Baubinder and Julie Troutwine for their accurate and patient typing and Tim Devenport for his careful editorial assistance.

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INTRODUCTION

Among the most important problems of modern economic growth is that of structural change in resource production and consumption patterns. Here the term "resource" is used in a somewhat broader sense than usual, to imply not only energy or mineral resources, but skilled labor, food, water, fertilizers, etc., as well.

Two basic processes underly the problem. On the one hand, there has been some depletion of various kinds of resource. More precisely, economies can be said to be shifting from a situation of "unlimited" resources (where the problem of substitution may not need to be taken into account) to one of limited resources (where substitution assumes a crucial role). On the other hand, new, nonconventional technologies are being introduced into resource production and usage. Examples include solar and synfuel energy, biotechnology, robots and computer-based production, etc.

In short, we are now witnessing the transition from old or conventional structures of resource production and consumption (which we consider were "optimal" or "equilibrium" in the past) to new, nonconventional structures, which will be "optimal" in the future (Koopmans 1981).

In turn, the transition period can be analyzed on the basis of two main characteristics. First, there is the influence of many interdependent factors on the process. When we speak, for example, of the sufficiency of a certain kind of mineral resource in a region, it would be wrong to assess only the reserves of this resource: its availability and transportation costs, as well as ecological constraints, the possibility of substitution, and the world market situation must also be taken into consideration. Thus, a local problem formulated for a particular region and a specific kind of resource very quickly turns into a global problem. Second, we need to consider the dynamics of the process. The transition to the new structure of resource production and consumption requires research and development for the new technologies involved, the modification or replacement of capital stock, and retraining of the labor force. All of these stages need time, so we are clearly dealing with a long-term problem.

For these reasons conventional economic models are not appropriate for analyzing transition periods in an economy. For example, the production functions in econometric models are explicitly based on statistics of past trends and therefore cannot detect or project future structural changes.

This paper describes the approach and corresponding system of models that are being developed at the Institute for Systems Studies in Moscow to study these problems. The approach is oriented toward analysis of the transition period in a particular sector of the economy (which produces a certain kind of resource) of a region or country and its interrelations with the rest of the economy (which consumes the resource). We refer to the entire interacting structure as a "resource-economy system". Examples of such systems include energy supply-economy, mineral resources-economy, skilled labor supply-industrial production, water supply-agriculture, forage supply-livestock breeding, and so on (Carter *et al.* 1977; Propoi 1978, 1979; Kallio *et al.* 1980; Dantzig *et al.* 1981; Propoi and Zimin 1981; Csaki and Propoi 1982).

This paper considers a system of models of resource-economy interactions. First, we describe a general resource-supply model and then we discuss its linkage to a model of economic development. The linkage can be performed through a specially built, integrated resource-economy model or by iterative linking of the separate models. For the latter case two special "interface" models are often needed: the first model projects demand for the resource and the second model assesses the total costs of developing the resource-supply system.

It should be noted that such an approach has already been used in one way or another for a number of specific resource-development studies (primarily in the analysis of energy-economy interactions).[1] Nevertheless, much work remains to be done to unify this approach, develop the corresponding software, and make it available for routine applications.

1. A SYSTEM OF RESOURCE-ECONOMY MODELS

We shall single out the following stages of resource-economy modeling:

Resource-Supply Model. Let us consider the problem of the transition of a sector (resource subsystem) of a given economy to a new production structure; we may be dealing here with a regional or a national economy.

For this, we normally start with the exogenously given demand for this resource (or for the final product) during the whole period considered. Note that the demand is usually given in generalized terms that somehow characterize the usefulness of the product (e.g., it might be units of electrical or nonelectrical energy, carbohydrates or protein), rather than in straightforward units of final product output.

Usually, there exists a number of alternative technologies that can, in principle, satisfy the given demand and there are initial production capacities associated with each of these technologies (for new technologies these are zero at the starting point of the analysis). Each alternative technology has its own advantages and drawbacks, concerning its input and output characteristics, impact on the environment, etc. Therefore, an optimal mix of the technologies

[1] See the bibliographic notes in Section 3.

phased over time must be found that satisfies the given constraints on demand and on the availability of external products and factors (which are needed for operating and developing the resource system) and minimizes some chosen criteria. One criterion frequently chosen for minimization is the total cost over the period considered; and it can be argued that this type of *optimization model* simulates a possible transition from the mix of technologies currently used (the initial state of the system) to a more progressive and, in a sense, optimal future mix of technologies.

These kind of models are also often called *process models* (Ayres 1978; Manne *et al.* 1979; Koopmans 1981) because they describe the process of transformation of a primary resource to a final product (or secondary resource). We use the terms resource-supply model or simply resource model to emphasize the role that the system modeled plays in the economy.

Formally speaking, resource-supply models are dynamic optimization models and in most practical cases they can be formulated in a dynamic linear programming framework (Propoi 1979, 1980). They are usually large-scale models[2] because many factors and constraints need to be taken into account. Another feature is that such models are typically formulated in real (quantity) terms because the associated price relations are only starting to be established during the transition period.

Note that there are two main groups of exogenous variables that influence the behavior of the model:

(i) demand for the output of the resource-supply system;

(ii) external factor requirements (labor, capital) that are needed for operating and developing the resource-supply system (see Figure 1).

For this reason, an isolated resource model is limited in its possibilities and linkage of the resource-supply model to a model of economic development is in order.

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[2]See the references cited in Section 3.

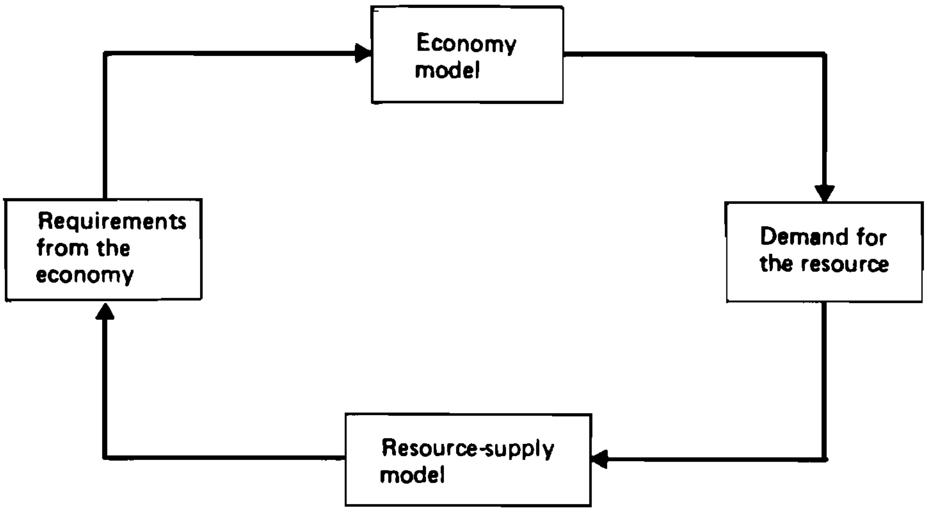

Figure 1. The general scheme of resource-economy models.

Economy Model. This represents the more inertial (or more conservative) part of the economy (in comparison with the resource subsystem). Hence this model can be described in a more conventional way. It can, for instance, be an input-output model or a macroeconometric model. Linking the resource model to an economy model in principle allows us to analyze the process of substitution, not only from the supply side but also from the demand side. For this reason it is expedient to formalize the economy model as a multisectoral optimization model. Therefore it becomes again a dynamic linear programming or, in other words, a dynamic optimization input-output model.

It is necessary, however, to underline the principal difference between the two models discussed above. The resource-supply model is a detailed optimization model in real terms: the use of value terms is inappropriate there because the production relations and consequently the values of the various production factors are only starting to be established in the transition period. In comparison with the resource model, the economy model may be more aggregated and may be written in value terms (see also the discussion of this issue in Ayres (1978)).

There are two alternative ways of linking the resource and economy models. First, we could build an integrated model of the resource-economy system; this might be a dynamic linear programming model with a detailed resource sector, such as the PILOT model of energy-economy interaction (Dantzig 1976) or a nonlinear optimization model with an aggregated macroeconomic model, like the ETA-MACRO model (Manne *et al.* 1979). The second approach is to link existing models of resource supply and economic development: one example is the IIASA system of energy models (Häfele *et al.* 1981).

Both approaches have their own advantages and disadvantages, which will be discussed later. We will now turn to a more formal description of the models.

2. THE RESOURCE-SUPPLY MODEL

The verbal description of the resource-supply model given in the preceding section implies in a quite straightforward way its formal structure.

Let us consider the production of a resource (a final product) by K^R alternative technologies. The state of the system in each period t is described by the values of production capacities at the beginning of the period for each of the k technologies.

The state equations define the change of structure of the production capacities in relation to investments, which are considered as the control elements in the system. These equations are of the form

$$y_k^R(t+1) = (1 - \mu_k^R(t))y_k^R(t) + v_k^R(t) \tag{1}$$

$$(k = 1,2,\ldots,K^R\ ;\quad t = 0,1,\ldots,T-1)$$

where

$y_k^R(t)$	is the production capacity of the k-th technology in period t;
$v_k^R(t)$	is the increase in production capacity of the k-th technology over period t;
$\mu_k^R(t)$	is the depreciation factor of the k-th technology during period t;
K^R	is the total number of alternative technologies for resource production to be considered in the model;
T	is the time horizon; and the superscript
R	refers to the resource system.

An alternative description of the dynamics of production capacities involves the assumption that the capacities are constant during their lifetime τ_k, and that thereafter they are zero. In this case

$$y_k^R(t+1) = y_k^R(t) + v_k^R(t) - v_k^R(t-\tau_k) \tag{1a}$$

The initial state of the production capacities is given by

$$y_k^R(0) = y_k^{oR} \quad (k = 1,2,\ldots,K^R) \tag{2}$$

In the case of (1a) it is also assumed that the increases in capacities during the period preceding the time horizon ($t < 0$) are given as well

$$\begin{aligned} v_k^R(-\tau_k) &= v_k^{oR}(-\tau_k) \; ; \\ &\qquad\qquad (2a) \\ v_k^R(-1) &= v_k^{oR}(-1) \end{aligned}$$

By choosing different controls $\{v_k^R(t)\}$ in (1) or (1a) one can calculate the corresponding trajectories $\{y_k^R(t)\}$ of the production capacities of the resource system. However, not all of these trajectories will be admissible because of the constraints on the process. We can identify a number of groups of constraints. First, we consider constraints on output.

Constraints on the Utilization of the Production Capacities. Let $x_k^R(t)$ be the output of the production capacity for the k-th technology at time t. Evidently, for each t

$$x_k^R(t) \leq y_k^R(t) \quad (k = 1,2,\ldots,K^R) \tag{3}$$

Demand Constraints. We consider J^R components by which the output of the resource system is measured (for example, these might be units of electrical or nonelectrical energy; calories or units of protein or amino acids, etc.). It is assumed that the demand for the final product $d_j^R(t)$ $(j = 1,2,\ldots,J^R)$ is given in these terms for the whole period $t = 0,1,\ldots,T-1$. Total production should satisfy the demand

$$\sum_{k=1}^{K^R} d_{jk}^R(t) x_k^R(t) \geq d_j^R(t) \quad (j = 1,2,\ldots,J^R) \tag{4}$$

where $d_{jk}^R(t)$ is the output of final product j per unit intensity of technology k.

Requirements Constraints. For operating and developing the resource-supply system, external factors (capital, labor, raw materials) are required. Let $f_s^R(t)$ be the available amount of factor s, $s = 1,2,\ldots,S^R$, in period t. Then the constraints can be written as follows

$$\sum_{k=1}^{K^R} f_{sk}^{1R}(t) x_k^R(t) + \sum_{k=1}^{K^R} f_{sk}^{2R}(t) v_k^R(t) \leq f_s^R(t) \quad (s = 1,2,\ldots,S^R) \tag{5}$$

Here the coefficients $f_{sk}^{1R}(t)$ and $f_{sk}^{2K}(t)$ denote, respectively, the amounts of external factors required for the operation and construction of one unit of production capacity of technology k.

Constraints on primary raw materials (for example oil, coal, gas, uranium, in energy systems) are frequently singled out from (5):

$$\sum_{k=1}^{K} r_{ik}(t) x_k^R(t) \leq r_i(t) \quad (i = 1,2,\ldots,I^R) \tag{6}$$

where $r_i(t)$ is the available amount of the i-th category of the raw material (primary resource) at time t.

Usually, cumulative consumption is bounded by available reserves. So, instead of (6), we write

$$\bar{r}_i(t+1) = \bar{r}_i(t) - \sum_{k=1}^{K^R} r_{ik}(t) x_k^R(t) \tag{7}$$

$$\bar{r}_i(0) = \bar{r}_i^o \text{ (the given initial reserves of primary resource } i) \tag{8}$$

$$\bar{r}_i(t) \geq 0$$

or

$$\sum_{\tau=0}^{t} \sum_{k=1}^{K^R} r_{ik}(\tau) x_k^R(\tau) \leq \bar{r}_i^o \tag{7a}$$

for all t.

Ecological Constraints. Let $p_q^o(t)$ be the admissible level of the q-th pollutant and $p_{qk}(t)$ the amount of this pollutant emitted per unit intensity of the k-th technology. Then

$$\sum_{k=1}^{K^R} p_{qk}(t) x_k^R(t) \leq p_q^o(t) \quad (q = 1,2,\ldots,Q^R) \tag{9}$$

for each $t = 0,1,\ldots,T-1$.

For some types of pollutant it is necessary to limit cumulative pollution. In this case

$$\bar{p}_q(t+1) = (1 - \vartheta_q)\bar{p}_q(t) + \sum_{k=1}^{K} p_{qk}(t) x_k^R(t) \tag{9a}$$

$$\bar{p}_q(t+1) \leq \bar{p}_q^o(t) \,. \tag{9b}$$

where $\bar{p}_q(0)$ is given.

Objective Function. The constraints (1)-(9) determine the set of feasible strategies for the development of the resource system. This set may still be rather broad and in order to formulate a strategy it is necessary to specify an objective function for the system. The most frequently used objective function for these models involves minimization of the total cost discounted over time

$$J = \sum_{t=0}^{T-1} \sum_{k=1}^{K^R} \beta(t)[c_k^{1R}(t) x_k^R(t) + c_k^{2R}(t) v_k^R(t)] \tag{10}$$

where $c_k^{1R}(t)$ represents operating and maintenance costs for technology k at time t, $c_k^{2R}(t)$ is the investment required for constructing one unit of production capacity of the k-th technology, and $\beta(t)$ is the discount rate.

An alternative objective might be maximization of the system output

$$J' = \sum_{t=0}^{T-1} \sum_{j=1}^{J^R} \sum_{k=1}^{K^R} \alpha_j(t) d_{jk}^R(t) x_k^R(t) \tag{10a}$$

where the $\alpha_j(t)$ are weight coefficients.

It must be stressed that the optimization procedure should not be viewed as the final stage of the modeling activity (yielding a unique, "optimal" solution) but rather as a tool for analyzing the interdependence of different policy alternatives and system performance.

Thus the problem is to find controls $\{v_k^R(t)\}$ (the strategies for resource system development) and corresponding state trajectories $\{y_k^R(t)\}$ (the dynamics of production capacities) that satisfy all the constraints on the system (for example, (1)-(9)) and that, for the given initial state $\{y_k^{oR}(t)\}$ (the initial structure of production capacities), optimize the chosen objective function ((10) or (10a)).

The model is particularly oriented toward analyzing structural changes in production capacities for a given resource.

Viewed in another way, the model describes the process of transformation of a primary resource into a final product. For this reason it is often convenient to present this process as a material flow with several stages of transformation.

Let N be the total number of stages of the process and let $x_{kij}^{(n)}(t)$ be the volume of flow at time t of product $i \in J^n$, which is used at stage $n \in N$ for the production of an (intermediate) product $j \in J^{(n)}$ using technology $k \in K^{(n)}$. And let $x_i^{(n)}(t)$ be the volume of intermediate product i input to stage n, with $x_j^{(n+1)}(t)$ as the volume of output of the same stage.

Then the following balance equations hold:

$$\sum_{k,i} \alpha_{kij}^{(n)} x_{kij}^{(n)}(t) = x_j^{n+1}(t) \tag{11}$$

$$\sum_{k,j} \beta_{kij}^{(n)} x_{kij}^{(n)}(t) = x_i^{(n)}(t) \tag{12}$$

$$\sum_{i,j} \gamma_{kij}^{(n)} x_{kij}^{(n)}(t) = u_k^{(n)}(t) \tag{13}$$

Here $u_k^{(n)}(t)$ is the intensity of the k-th technology in stage n at time t; the summations in (11)-(13) are taken over all possible flows in the system; and the technical coefficients α, β, and γ show the efficiency of the transformation process.

For the initial stage

$$\sum_{k,j} \beta_{kij}^{(0)} x_{kij}^{(0)}(t) \leq r_i(t) \tag{14}$$

where the $r_i(t)$ have the same meanings as in (7).

For the final stage

$$\sum_{kij} \alpha_{kij}^{(N)} x_{kij}^{(N)}(t) \geq d_j(t) \tag{15}$$

where the $d_i(t)$ were introduced in (4).

For the intensities we have

$$\sum_{i,j} \gamma_{kij}^{(n)} x_{kij}^{(n)}(t) \leq y_K^{(n)}(t) \tag{16}$$

where $y_k^{(n)}(t)$ is the production capacity k of stage n at time t.

This constraint is similar to (3) above.

The flow representation is very convenient because it clearly shows all the interconnections in the system for a given t (say, for the years 1980, 1990, 2000). This type of representation has been used in energy models (the so-called Reference Energy System (Markuse 1976)), and other examples can be found in Ayres (1978).

3. BIBLIOGRAPHIC NOTES

This section does not pretend to give a full or comprehensive list of references on resource-supply models; rather, it attempts to provide a few examples of real-life models that can be formulated within the framework outlined above. The most representative type are probably the energy-supply models, some of which are reviewed in Manne *et al.* (1979) and Propoi and Zimin (1981). Mineral resource models are described in Manne *et al.* (1979), Hibbard *et al.* (1979), and Golobin *et al.* (1979). Manpower and educational models have been studied fairly extensively (Grihold and Marshall 1977; Propoi 1978), and there is a good deal of literature devoted to models of agriculture and food supply (Carter *et al.* 1977; Il'ushnok 1980; Csaki and Propoi 1982) and the forest sector (Kallio *et al.* 1980). Even health-care systems (Propoi 1977) may be thought of as resource-supply systems producing a special type of "resource"--the health of the population.

4. THE COST-ASSESSMENT MODEL

Suppose we have performed a run of the resource-supply model, and hence have arrived at a strategy for the development of the resource system. (At first, this is usually done without the direct requirements constraints (5).) In order to assess this strategy it is necessary to know the direct and indirect requirements from the economy as a whole that are needed for the implementation of the strategy.

The direct requirements $Z_s^R(t)$ can be calculated using the left-hand side of inequality (5), that is

$$Z_s^R(t) = \sum_{k=1}^{K} [f_{sk}^{1R}(t) x_k^R(t) + f_{sk}^{2R}(t) v_k^R(t)] \tag{17}$$

where $\{x_k^R(t)\}$ and $\{v_k^R(t)\}$ are obtained from the resource model run.

However, in many cases it is very important to know the indirect requirements from other sectors of the economy that support the resource-supply system: we shall refer to these formally as "supporting" sectors. Let $s = 1,2,\ldots,S^o$ be the number of supporting sectors and let $y_s^o(t)$ be the capital stock or production capacity of the j-th sector at time t. Then

$$y_s^o(t+1) = (1-\mu_s^o(t))y_s^o(t) + v_s^o(t) \tag{18}$$

where $v_s^0(t)$ is the increase in capital stock j during period t.

The bill-of-goods equation for the supporting sectors can be written in the form

$$x_s^o(t) - \sum_j a_{sj}^o(t)x_j^o(t) = \sum_j b_{sj}^o(t)v_j^o(t) + Z_s^R(t) + C_s^o(t) + e_s^o(t) \tag{19}$$

where $Z_s^R(t)$ is calculated from (17), $C_s^o(t)$ is the remainder of the final demand, and e_s^o is the net export.

Evidently

$$x_s^o(t) \le y_s^o(t) \tag{20}$$

Indirect requirements are defined by the outputs $x_s^o(t)$ of the corresponding supporting sectors. In order to calculate the values of $x_s^o(t)$ one can procede in two ways.

First, it is possible to define the increase $v_s^o(t)$ in production capacities for the supporting industries using the following nonlinear equation

$$v_s^o(t) = \max\{\min_{\tau \le t}[x_s^o(t) - x_s^o(\tau)]\,;0\} \tag{21}$$

which simply means that investments are made only if a consequent increase in output is expected. This approach and a corresponding model (the IMPACT model) were developed by Kononov and Por (1979) to assess the direct and indirect requirements of the energy-supply system. To run the model it is necessary to solve a nonlinear system of equations at each iteration.

The second approach has been developed at the Institute for Systems Studies. In this case a linear programming (LP) problem is solved at each iteration. That is, instead of (21) we introduce an objective function

$$J = \sum_{t=0}^{T-1} \sum_{s=1}^{S^o} c_s^o(t)v_s^o(t) \to \min \tag{22}$$

subject to constraints (18)-(20). The solution of this LP problem means that only the minimum amount of new production capacity needs to be constructed in order to meet the required demands $Z_s^R(t)$ of the resource-supply system.

Clearly, the set of feasible strategies $\{v_s^o(t)\}$ for the supporting sectors produced by optimization model (18)-(20), (22), also includes the strategy that is obtained from (21). However, the optimization version of the model seems to be more flexible. In particular, if we have some upper constraints on the increase of the capacities

$$v_s^o(t) \leq \bar{v}_s^o(t) \tag{23}$$

then the optimization model may still have a solution (which means that the required increase can be achieved for several periods, none of which violate (23)), but eqn. (21) may give values of $v_s^o(t)$ that are inconsistent with (23).

Note that to ensure that investments are not made earlier than necessary, it should be assumed that

$$c_s(t+1) < c_s(t)$$

Note also that the real-life model differs from the above description in certain details; in particular, it can include delays in construction, assessment of environmental constraints, etc. (Kononov and Por 1979).

5. THE DEMAND MODEL

The other link between the resource-supply system and the economy is the demand for the resource from the economy. There are two basic approaches for long-term resource demand evaluation.

The first approach uses direct calculation of the demand for the resource from each sector of the economy. Formally, this approach is based on the relation

$$d_j^R(t) = \sum_i a_{ji}(t)x_i(t) + \sum_i b_{ij}(t)v_i(t) + C_j^R(t) + e_j^R(t) \tag{24}$$

where the $a_{ji}(t)$ are the intermediate requirements for the j-th component of the resource per unit intensity of the i-th sector of the economy, the $b_{ji}(t)$ are the corresponding requirements for construction in the i-th sector, $C_j^k(t)$ is the final demand, and $e_j^k(t)$ is the net export.

However, implementation of the model based on direct calculation runs into several difficulties. First, the level of detail of the sectoral outputs on the right-hand side of (24) needs to be rather high in order to take into account all the resource users. Second, there might be alternative ways of using the resource. Therefore, expert selection or some other form of optimization of these alternatives is also necessary. Third, in addition to the model structure, other factors influencing demand exist, which in most cases are difficult to formalize but which are still too important to neglect.

The second approach evaluates the behavior of resource consumers and is based on the econometric technique. However, because econometric methods rely upon historical statistics and thus on past trends and phenomena, it is difficult to use this approach straightforwardly to detect structural changes in the future demand pattern.

An example of a model for the long-term evaluation of the demand for energy is MEDEE (Lapillone 1978), which was used in the Energy Systems Program at the International Institute for Applied Systems Analysis (IIASA) (Häfele *et al.* 1981).

6. THE ECONOMY MODEL

For the reasons discussed above, it is most appropriate to link the resource model with a dynamic input-output model of economic development. We describe here very briefly an optimization version of such a model.

The state equations of the model are

$$y(t-1) = (I - \mu(t))y(t) + v(t)\,; \quad y(0) = y^0 \tag{25}$$

where $y(t)$ is the vector of capital stock or production capacities at time t, $v(t)$ is the increase in these capacities between times t and $t+1$, $\mu(t)$ is the depreciation diagonal matrix, and I is the identity matrix.

The bill-of-goods equations are of the form

$$(I - A(t))x(t) = B(t)v(t) + C(t) + e(t) \tag{26}$$

where $C(t)$ is the vector of final demand and $e(t)$ is the vector of net export.

The basic constraints on the variables are

$$x(t) \le y(t) \tag{27}$$

and

$$L(t)x(t) \le l(t) \tag{28}$$

where matrix $L(t)$ shows the requirement for a certain type of labor per unit activity in each sector of the economy and $l(t)$ is the vector of the available labor force.

In order to specify the solutions of the system (25)-(28) it is necessary either to set an investment function or to introduce an objective function, which may take a number of different forms. For example, it may be maximization of a function of final consumption

$$J = \sum_{t=0}^{T-1} \varphi(t, C(t))$$

or minimization of the difference between the given behavior of certain variables, for example macrovariables (scenario assumptions), and their model values.

7. LINKING THE MODELS

Four basic models have been considered above. Each of these models can be used individually; but carrying out separate runs of each model has, however, only limited usefulness because many important features of the system as a whole are neglected by such an analysis. Therefore we need to build a connected system of models in order to analyze interactions between the resource and economic systems. This can be done either iteratively or by building a new, integrated model.

Consider the first approach. The iterative linking of the resource and economy models might be done as follows.

At the beginning we define some initial estimate of the resource demand $d^k(t)$ for a given time horizon $t = 0,1,...,T-1$ and for given scenario assumptions on the development of the economy. By running the resource-supply model (Section 3) we can find the optimal mix of resource-supply technologies to meet this demand. Then the cost-assessment model (Section 4) gives the requirements $Z^R(t)$ that the resource system places on the rest of the economy. Now we can run the economy model (Section 5) with fixed requirements $(Z^R(t))$ for the resource system. If this run of the economy model is satisfactory (and there may be many different criteria for such an evaluation), but in particular if the model gives a demand for the resource that is consistent with the initial estimates, then the linkage procedure is terminated. If not, the demand value should be updated and the iterations repeated.

The advantage of the iterative approach is the possibility of experts constructively "interfering" at different stages of the linking process. At the same time it may still be unclear whether the solution obtained is optimal from the point of view of criteria by which the performance of the system is evaluated. Or, in other words, have all the possibilities of interactions between the resource and economic systems been used?

8. THE INTEGRATED RESOURCE-ECONOMY MODEL

The type of optimum or equilibrium described above can also be identified by using an integrated resource-economy model. Once again, it is however advisable to build the model in such a way that experts can play some role in the linking process (Kallio *et al.* 1979).

Let us begin by partitioning the economy model (Section 6) into four parts (see Figure 2):

- -- the resource-supply model (denoted by superscript *RR*);
- -- the economy model, which represents the rest of the economy (superscript *EE*);
- -- the demand model, showing the demand for resource R by the economy E (superscript *RE*);
- -- the cost-assessment model, evaluating the requirements of the resource-supply system R from the economy E (superscript *ER*).

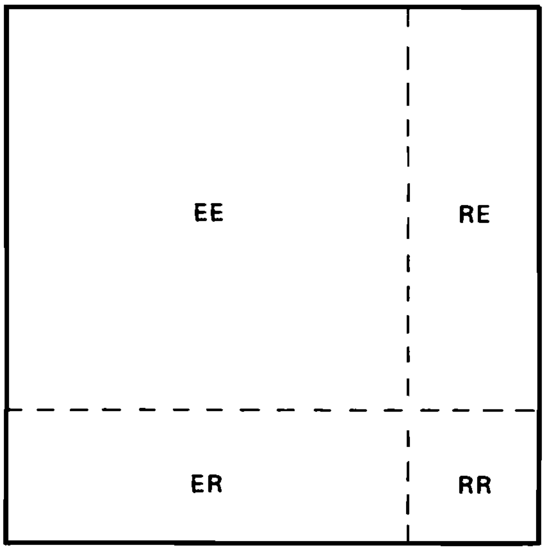

Figure 2. Decomposition of the economy model.

According to this decomposition, the state equation (25) may be rewritten as

$$y^R(t+1) = (1 - \mu^R(t))y^R(t) + v^R(t)\ ; \quad y^R(0) = y^{oR} \tag{29}$$

and

$$y^E(t+1) = (1-\mu^E(t))y^E(t) + v^E(t)\,; \quad y^E(0) = y^{oE} \tag{30}$$

The bill-of-goods equations now become:

$$(I - A^{RR}(t))x^R(t) = A^{RE}(t)x^E(t) + B^{RE}v^E(t) + B^{RR}v^R(t) + \dot{C}^R(t) + e^R(t) \tag{31}$$

for the resource outputs, and

$$(I - A^{EE}(t))x^E(t) = B^{EE}(t)v^E(t) + A^{ER}(t)x^R(t) + B^{ER}(t)v^R(t) + \dot{C}^E(t) + e^E(t) \tag{32}$$

for the outputs of the economy. The meaning of eqns. (31) and (32) should be quite evident and needs no further explanation.

Let us denote by $d^R(t)$ the economy's demand for resource R. From (32) we obtain (cf. eqn. (24))

$$d^R(t) = A^{RE}(t)x^E(t) + B^{RE}(t)v^E(t) + C^R(t) + e^R(t) + B^{RR}(t)v^R(t) \tag{33}$$

Note that the last term on the right-hand side of eqn. (33) is usually small and can be neglected. In this case, eqns. (24) and (33) become identical.

Also, let us denote by $Z^{ER}(t)$ the requirements of the resource-supply system for products of the economy. From (32)

$$Z^{ER}(t) = A^{ER}(t)x^R(t) + B^{ER}(t)v^R(t) \tag{34}$$

Again, apart from differences of notation, eqn. (34) coincides with eqn. (17).

Using (33) and (34), the balance equations (31) and (32) can be rewritten as

$$D^R(t)x^R(t) = d^R(t) \tag{35}$$

and

$$(I - A^{EE}(t))x^E(t) = B^{EE}(t)v^E(t) + Z^{ER}(t) + C^E(t) + e^E(t) \tag{36}$$

Equations (35) and (36) are analogues to (4) and (19).

Partitioning constraints (27) and (28) gives

$$x^R(t) \le y^R(t) \tag{37}$$

$$x^E(t) \le y^E(t) \tag{38}$$

$$L^E(t)x^E(t) \le l^E(t) \tag{39}$$

$$L^R(t)x^R(t) \le l^R(t) \tag{40}$$

$$l^E(t) + l^R(t) \le l(t) \tag{41}$$

Thus we have obtained equations that represent explicitly, and to a certain degree independently, four submodels (see Figure 3):

-- the resource model RR - eqns. (29), (35), (37), (39);

-- the economy model EE - eqns. (30), (36), (38), (40);

-- the demand model RE - eqn. (33);

-- the direct requirements model ER - eqn. (34).

Now we will make some concluding remarks. The decomposition of the economic system described above is only the first stage in building the resource-economy model. To analyze structural changes in the resource system, the corresponding blocks of the economic system should be disaggregated (see Figure 3). Moreover, to make it possible to investigate the process of substitution between different technologies, the corresponding parts of the matrices $A(t)$ and $B(t)$ should be made rectangular (see Figure 3) and the variables related to the resource parts of the model measured in real terms. (This is not apparent in the matrix notations (29)-(41).)

This decomposition, together with the disaggregation, also makes it possible to introduce into each submodel nonlinearities and scenario assumptions (as in the demand model RE of Section 4) or to take into consideration the indirect requirements of the resource subsystem in an explicit form (as in the model in Section 5).

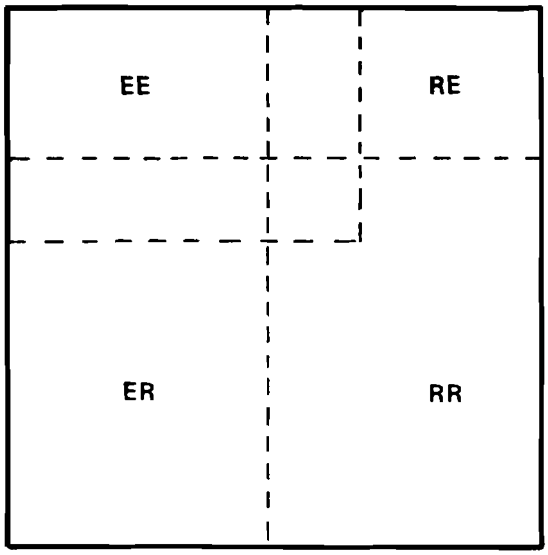

Figure 3. The resource-economy model with a detailed resource submodel.

From a formal standpoint the model described above constitutes a linear or nonlinear programming problem of block structure. The problem has coupling variables $d^R(t)$ and $Z^{ER}(t)$ and coupling constraints (41) (Figure 4). A special algorithm has been developed that is suitable for this particular model structure (Kallio *et al.* 1979).

9. DISCUSSION: METHODOLOGICAL ASPECTS

This paper has shown how to link a process model of a certain sector of the economy (the resource model) with other models in order to analyze the structural dynamics of the sector and its interrelations with the development of the economy as a whole. The approach is flexible and makes it possible to investigate many different aspects of these interactions.

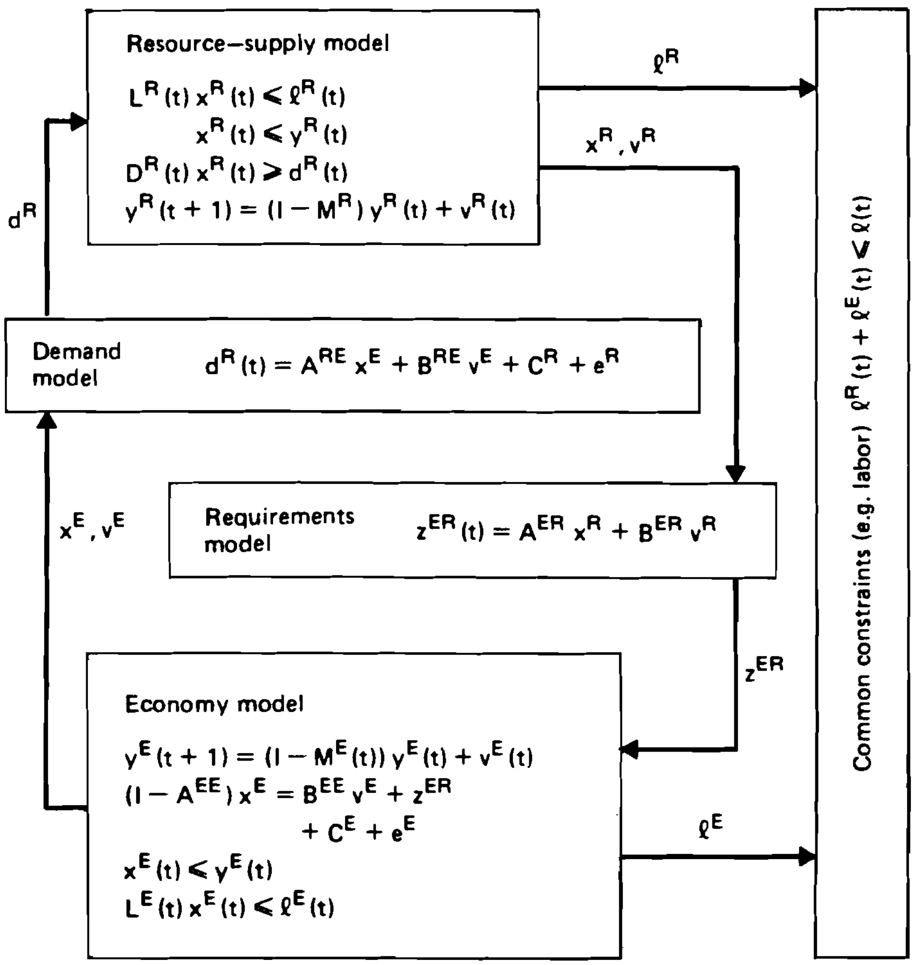

Figure 4. Interrelations between the resource-supply and economy submodels.

It can be seen that three of the four models are written in a unified optimization framework. (Even the demand model can be reduced to an optimization problem if substitution of resource uses is allowed.) However, the optimization procedure serves here not to obtain a single, "optimal" solution but rather for distinct modeling purposes, namely exposing the most important interrelations and connections between separate factors and blocks of the system. Therefore a flexible optimization modeling system is required. Generally, such a system should comprise subsystems for data management, model generation, optimization, and postmodeling analysis. The modeling system can also include subsystems for aggregation and parameter identification; it also makes it possible to extend the research beyond running a single model so that different problem-oriented models can be built, based on the same common data base (Propoi *et al.* 1982).

A major advantage of the approach described is that individual models can be separately built and used (even on different computers), whilst at the same time utilizing the information obtained from these separate runs to the fullest extent. In particular, it allows very effective use of the shadow prices obtained from each subsystem. For example, if the marginal or "local shadow price" of a factor (say, labor) has been obtained from running the resource model then this value can be taken into account when running the economy model, and vice versa.

Simultaneous runs of several large-scale models can be rather troublesome and expensive in computer time. Therefore, in some cases it is more expedient to link the resource model with a macroeconometric model, as was done in the ETA-MACRO model for energy-economy interactions (Manne *et al.* 1979). On the other hand, when running a "conventional" input-output model of the whole economy, structural changes in the different sectors can be taken into account from preceding runs of the corresponding sectoral or resource models. The data obtained from such runs can be used for evaluating the future dynamics of the technical coefficients of the input-output model or the parameters of the production function.

10. CONCLUSIONS

A general scheme for modeling resource-economy interactions has been presented and discussed. Modeling such complex, large-scale systems requires thorough research into many methodological questions, some of which have been raised here (see also Thrall *et al.* 1983). The work certainly appears to be worthwhile, because incorporating resource process models into the framework of economic modeling will improve the flexibility and predictive power of the modeling effort.

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