Working Paper

SIMPLIFICATION OF A COMPREHENSIVE HYDROLOGICAL MODEL FOR SCENARIO ANALYSIS

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WP-85-92

International Institute for Applied Systems Analysis
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PREFACE

Research by the *Regional Water Policies Project* of IIASA is focused on the design of decision support systems to assist in the analysis of rational water policies in regions with intense agriculture and with open-pit mining activities. One direction of this research is aimed at the elaboration of simplified models of interrelated groundwater processes, crop growth processes, basing on available comprehensive and other models.

One of the methods used by the project for this purpose is based on the development and application of the Interactive Modeling Support System for Model Simplification described recently by Y. Nakamori et al. in WP-85-77. This paper outlines a concrete application of this system in the context of the study for the Southern Peel region in the Netherlands

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SIMPLIFICATION OF A COMPREHENSIVE HYDROLOGICAL MODEL FOR SCENARIO ANALYSIS

P.E.V. van Walsum and Y. Nakamori

1. INTRODUCTION

Intense agricultural development in many regions of the world puts an increasing pressure on the environment both by consuming water resources and by discharging pollutants that are hazardous to the population and to natural ecosystems. Apart from being a resource that is vital for socio-economic development and for the evolution of natural ecosystems, a regional water system is a basic medium through which local human interventions penetrate to and are "felt" in other parts of a region. It is the latter aspect that lends regional water systems their complexity. This gives rise to a demand for the design of decision support systems that can help regional decision makers in formulating policies aimed at providing a satisfactory balance between the agricultural development on the one hand and the development of the environment on the other.

Using an example region in the Netherlands, a prototype of such a decision support system has been developed within the framework of IIASA's Regional Water Policies Project (RWP, in press). Methodologically the system is based on the use of a two-stage decomposition, with scenario analysis in the first stage and policy analysis in the second. The scenario analysis stage is directed towards generating scenarios of the potentially rational development of the regional system, as seen from the regional perspective.

A set of coupled "comprehensive" models that are state-of-the-art mathematical descriptions of relevant socio-economic and environmental processes is the best tool for evaluating scenarios in terms of regional objective function values (e.g. income from agriculture, nitrogen concentration of groundwater). However, due to their complexity and high computational demand, comprehensive models are not suitable for screening analyses using mathematical programming and interactive methods for multi-objective choice. For this reason it is necessary to develop reduced models of the same processes. The comprehensive and reduced models are then combined into a hierarchical system, with an integrated set of reduced models on the first level and coupled comprehensive ones on the second.

In the mentioned study the choice has fallen on the use of linear reduced models in order to take advantage of the fact that linear mathematical programming techniques are vastly better developed than nonlinear ones. For developing one such reduced model from an existing comprehensive model of a regional hydrologic system, use was made of the Interactive Modeling Support System (IMSS) that was developed by Nakamori et al.(1985). The model simplification procedure that was followed is the subject of this paper.

For regions hydrologically similar to the example region in the Netherlands, the Southern Peel Region, the comprehensive ("second level") model FEMSATP has been developed (Querner & Van Bakel, 1984). This model is based on a finite-element approximation of the partial differential equation describing the regional hydrologic system. Coupled to FEMSATP is the crop production model SIMCROP (Querner & Feddes, in press), which predicts the effects of solar radiation and the availability of moisture and nitrogen on the actual crop production.

After having given a short description of the example region in the Netherlands, with the emphasis on those aspects that are of relevance here, namely those pertaining to water quantity processes, we proceed by giving a brief outline of the models FEMSATP and SIMCROP and their application to the Southern Peel Region. Subsequently a specification is given of some of the characteristics that the reduced model should have - this specification follows from the intended way of implementing and using the reduced model. This specification is then followed by the description of the actual modeling exercise and the validation of the reduced model as a component of the scenario module.

2. THE SOUTHERN PEEL REGION

The Southern Peel is an undulating area of about 30.000 ha in the south of the Netherlands. The lie of the land varies in altitude between 17 and 35 m above sea level.

A large part of the area used to be covered by a layer of peat that grew as a consequence of extremely high groundwater levels. Most of the peat has been delved and used for heating. The remaining peat areas are now protected from exploitation, because of their value as recreation or nature areas. These nature areas can only keep their value if high enough groundwater levels are maintained.

Roughly half of the land is used as pasture for dairy cattle; the remaining area is used for growing a variety of crops, of which maize is the most important one, followed by sugar beets, potatoes and cereals. Farmers try to reduce moisture deficits by subirrigation and sprinkler irrigation. Subirrigation is the infiltration of water into the bottoms of ditches, thereby raising the groundwater level under the neighboring fields; this increases the availability of moisture for capillary rise to the rootzone. Sprinkling is a more direct way of supplying moisture to the soil. Water for sprinkling is pumped from the groundwater or taken from the surface water supply system. This pumping from groundwater affects agricultural production in other parts of the region and also the conditions in nature areas. In the Southern Peel the surface water supply system coincides with the drainage system. It consists of some larger canals and a network of ditches and brooks with a varying density (Figure 1).

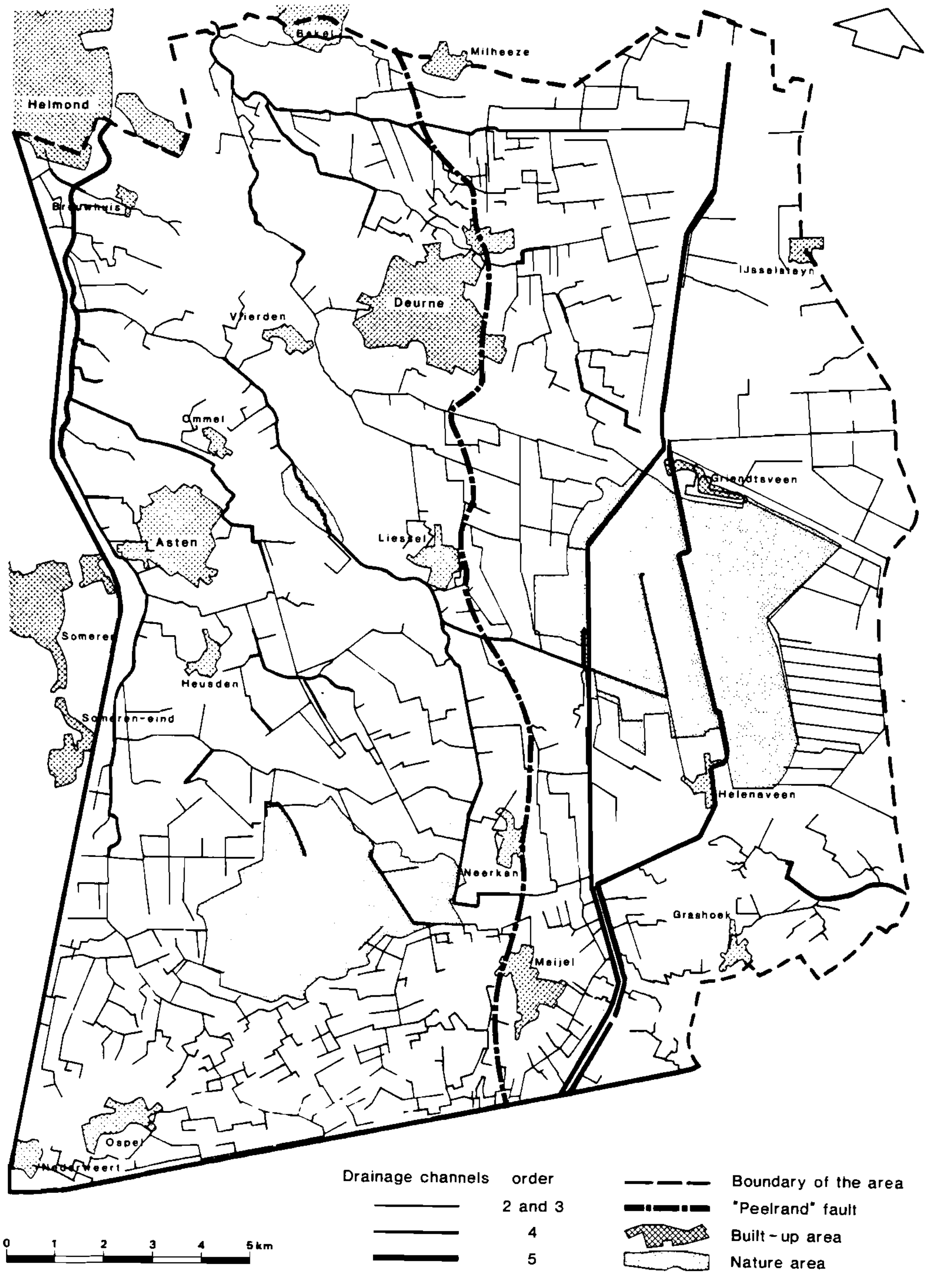

Figure 1: Surface water system of the Southern Peel

3. OUTLINE OF FEMSATP-SIMCROP

3.1. FEMSATP

FEMSATP is a finite-element model that is quasi three-dimensional (i.e it uses a schematization into purely vertical flows and purely horizontal flows). For advancing through time it uses a Crank-Nicholson implicit calculation scheme, meaning that the flows are calculated using the average of the hydraulic heads"* at the beginning and at the end of a time-step (Querner & Van Bakel, 1984). Using the recommended time-step of one week, FEMSATP requires for a one-year run about 20 min of CPU time on a VAX 11/780 under Unix.

In FEMSATP the saturated groundwater flow is schematized into purely vertical flow in flow-resisting layers (aquitards) and purely horizontal flow in permeable layers (aquifers). The phreatic layer in the Southern Peel is modeled as an aquitard (Figure 2).

The first aquifer is present in both the Eastern and Western part of the region, but differs in thickness. In the Eastern part this aquifer lies on the hydrological basis that serves as the lower boundary of the groundwater flow system. This lower boundary is present at a much shallower depth in the Eastern part than in the Western part due to a geological fault that runs through the middle of the region. In the Western part a second aquitard is

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*For the convenience of the hydrologically non-informed reader, a glossary of terms is provided:

aquifer - a geological layer with a relatively high permeability, thus with a low resistance to the flow of groundwater through the pores between the subsoil particles.

aquitard - a geological layer with a relatively low permeability, thus with a relatively high resistance to groundwater flow.

evapotranspiration - the combined evaporation from the soil surface and from the surfaces of crop leaves; by *potential* evapotranspiration is meant the evapotranspiration that would take place under optimal conditions of moisture supply to the soil; by *actual* evapotranspiration is meant the amount that occurs under the actual moisture supply conditions - the actual value is lower than the potential one.

hydraulic head - the potential energy of water; water flows in the direction of the steepest (downward) gradient of the hydraulic head.

infrastructure - the combined outlay of canals, hydraulic structures etc.

phreatic layer - the geological layer in which the groundwater table is.

solar radiation - the amount of energy in sun rays.

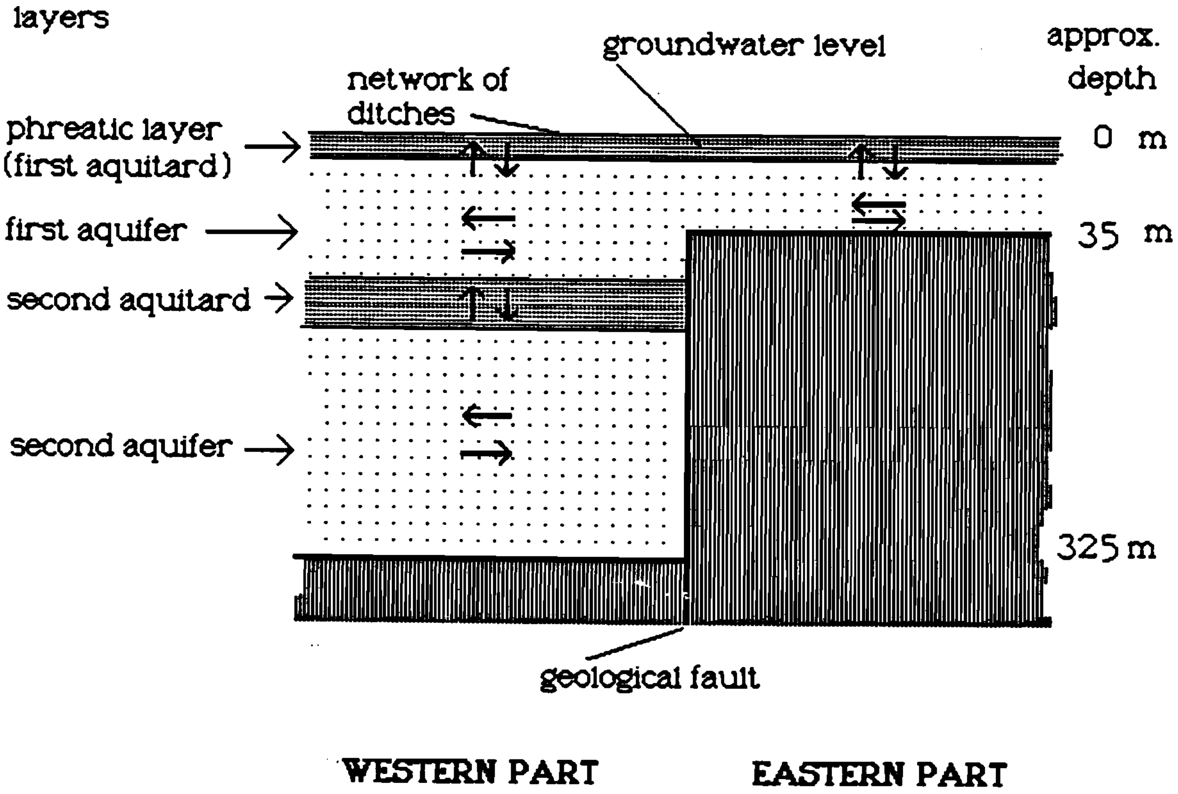

Figure 2: Hydrogeological schematization of the Southern Peel

followed by a second aquifer that reaches to an average depth of about 325 *m* below ground level.

For different aspects of a regional hydrologic system, FEMSATP uses different aggregation levels. A region is divided into subregions, each with relatively homogeneous soil properties and hydrogeological schematization. The description of the water movements in a second-level model requires an accurate representation of the geohydrological situation. Therefore the subregions are subdivided into triangular finite-elements. The Southern Peel has been divided into 31 subregions and into 748 finite-elements.

A subregion is also subdivided into areas characterized by different types of land use. These types of land use are here termed "technologies". Apart from agricultural technologies, the model allows for the specification of built-up areas, nature reserves and forests. The typification of an agricultural technology includes among other things whether it involves sprinkling or not. Of each technology the area has only to be known as a percentage of the subregion, either from collected data about the current state or from a target scenario that is generated by a "scenario module". The model abstracts, however, from the geometrical position(s) of a technology within a subregion: the total area of a technology may in reality be present as numerous portions of land scattered over a subregion.

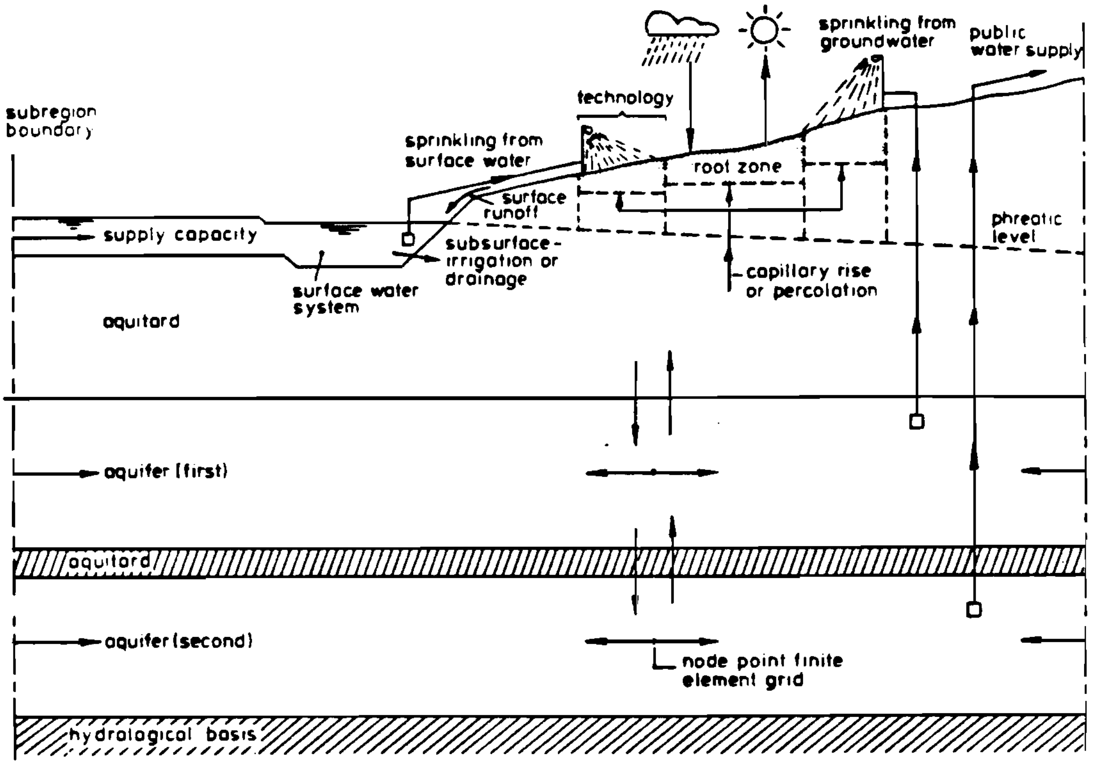

Figure 3: Schematization of flows in a subregion

The various water transport and storage processes are simulated by three different submodels. They represent the saturated zone, the unsaturated zone, and the surface water system. The various water movements allowed for within the schematization of a subregion and between the three submodels are shown in Figure 3. In this figure the summer situation is shown, with subirrigation and a supply of water towards the subregion.

3.2. SIMCROP

SIMCROP is a crop growth model that requires as input data the actual evapotranspiration data from FEMSATP and the nitrogen application values from the nitrogen submodel of the scenario module (that is not described here). Data of solar radiation are also needed. Output of the model is in terms of dry matter production. If certain economic data are also provided (yield per kg dry matter, and fixed cost per unit of area), then the model also supplies the monetary yields of the crops and the totals of income for the subregions and for the whole region.

4. SCENARIO ANALYSIS USING FEMSATP-SIMCROP

4.1. Scenario analysis procedure

The scenario analysis procedure that has been described in RWP(in press) is schematically depicted in Figure 5. The "scenario requirements" that the "user" has to specify pertain to the requirements on multi-objectives for the target scenario of regional development. The used procedure for multi-objective choice consists simply of asking the user to specify bounds on $N-1$ of the N objectives. An integrated set of (linear) models coupled to the linear programming system GEMINI-MINOS (Lebedev,1984) then optimizes the $N-th$ objective, provided that the $N-1$ requirements are feasible. The $N-th$ objective has been taken as the sum of the investments that would be required to instantaneously transit from the current state to the target scenario. These investments are minimized, because the less the required investments, the higher the probability that the scenario is reachable through taking policy measures.

After obtaining an "optimized" scenario a run is made with the second-level models, in order to obtain a more accurate estimate of the scenario obtained at the first level. Of special interest to the user are of course the objective function values obtained at the second level.

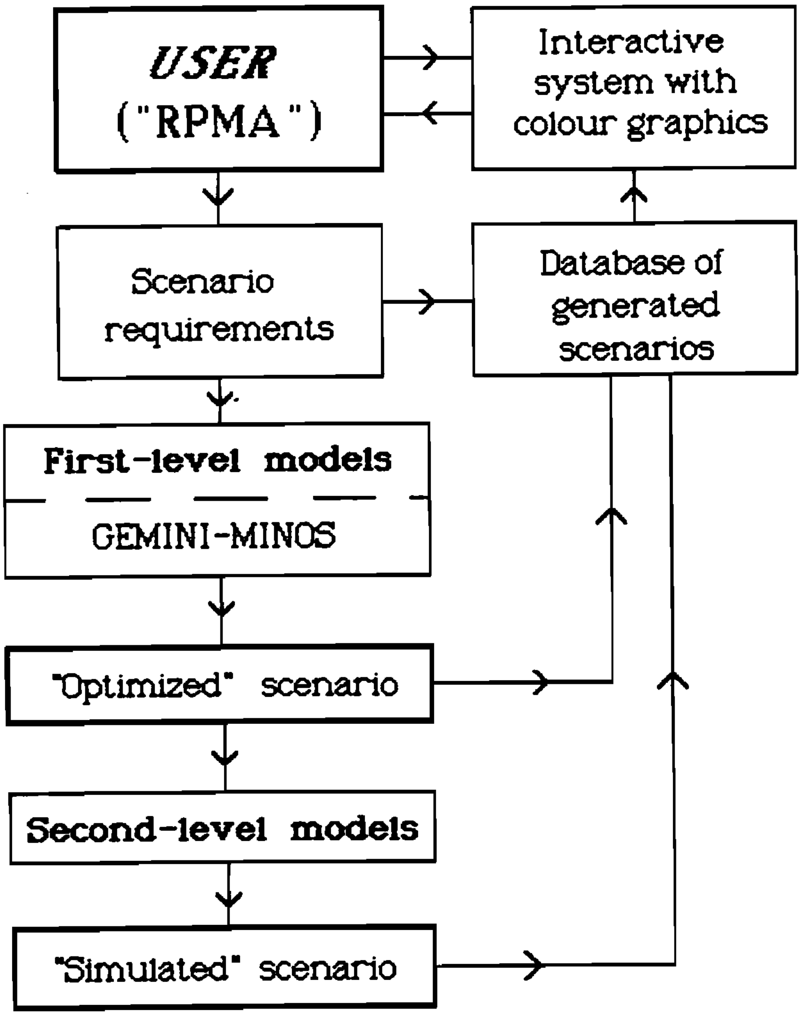

Figure 4: Scenario analysis proc

In the subsequent sections, descriptions are given of various types of variables that play a role in using FEMSATP-SIMCROP for scenario analysis and that also are of relevance for the reduced model.

4.2. Fixed parameters and control variables

Of the regional characteristics some are rigidly fixed, and are not modifiable by a regional authority (i.e. our "User"). Fixed parameters are for instance the aquifer permeabilities; also the infrastructure for surface water supply to the subregions is considered to be non-modifiable. Conditions that can be modified, here denoted by "control variables", are for instance the surface water supply to the region as a whole and the allocation to the different subregions (which is, however, subject to the constraint imposed by the surface water supply infrastructure). The following control variables are relevant here:

- area percentages of technologies, $x(j,r,k)$;

- capacities of sprinkling from surface water and groundwater, $s_c(r)$ and $g_c(r)$;

- allocation of surface water supply to a subregion, $S_c(r)$;

The index k indicates whether a technology involves sprinkling ($k=1$) or not ($k=0$). By capacities of sprinkling are meant the available flow-rate capacities of sprinkler-cannons and accompanying pumps. An increase of these capacities in comparison to the capacities in the current state, requires of course a certain amount of investments.

The listed control variables are subject to constraints that derive from the fixed parameters. These constraints are described in Kettun et al. (in press).

4.3. State variables

The pair of models FEMSATP-SIMCROP compute a whole host of state variables for each time-step; for the Southern Peel a time-step of one week has been used. Various operating rules are included in the model, like:

- a soil moisture threshold for applying sprinkling;

- a water-level threshold for supplying surface water to a subregion.

(When the soil moisture depletes to below the threshold value, sprinkling is applied; when the water level drops below the threshold value surface water supply is activated.) Since the results obtained from FEMSATP were found to be not very sensitive to the criteria used in the operating rules, the optimization of these rules is not considered here.

Only a limited number of state variables are of direct interest in the described scenario analysis procedure. These are:

- crop evapotranspirations and crop productions;
- volumes of sprinkling water extracted from surface water and groundwater;
- volumes of sprinkling water applied to arable land and grassland;
- amount of subirrigation by infiltration of surface water in the ditches;
- groundwater levels in nature areas at the end of summer.

The evapotranspirations are needed for the interpretation of the results; the crop productions are also needed for this purpose, but the main reason for needing them is of course for computing the income from agriculture. The volumes of sprinkling water and subirrigation water are of interest for interpretation and also for other aspects of the set of models describing the whole regional system. The groundwater levels in nature areas at the end of summer are required as objective functions in the model: a procedure has been developed for interpreting these levels in terms of their effect on natural ecosystems of the type present in the Southern Peel.

4.4. Uncontrollable variables: method of dealing with uncertainty

Lastly, there are the "uncontrollable" variables, namely the meteorological conditions. Owing to these uncontrollable variables, in the (intermediate) formulation of the mathematical problem that is to be solved in looking for a scenario, there are chance constraints for the agricultural income and the groundwater levels in the nature areas. These are dealt with by means of the so-called deterministic equivalent approach to chance constraints containing stochastic variables. This implies that when we use the reduced models for scenario-analysis, the values of *uncertain* parameters are fixed prior to actual running of the mathematical programming algorithm (in this case the Simplex algorithm for linear programming). So the simplified models only have to be linear in *control* variables; the coefficients of these variables may however be functions of the uncontrollable variables, because prior to a run with the scenario module the values of these uncontrollable variables are fixed, thus making the model linear after all.

5. MODEL SIMPLIFICATION

5.1. Introduction

For the derivation of a reduced model from the comprehensive model FEMSATP-SIMCROP, we make use of a computer-assisted modeling procedure called the Interactive Modeling Support System (IMSS) that was developed by Nakamori et al (1985). IMSS is implemented on a micro-computer; the present version consists of 50 subprograms and requires for storage more than 600 KB computer memory. It combines algebraic and graph-theoretic approaches to extract a trade-off between human mental models and regression-type models based on the use of numerical data. The modeling process of IMSS consists of three separate stages of dialogues. The first stage is for preparation of the modeling, including input of measurement data and the initial version of the cause-effect relation on the set of variables, transformation of variables, data screening, and refinement of the cause-effect relation. The second stage is devoted to finding a trade-off between the measurement data and the modeler's knowledge about dependencies between the variables. The third stage dialogue is related to simplification or elaboration of the model obtained at the second stage.

Prior the actual use of IMSS, a decision had to be made with respect to the way of dealing with the uncontrollable variables, i.e. the meteorological conditions, and based on this decision a series of simulation experiments were performed with the comprehensive model. These experiments could not be performed on the micro-computer, and had therefore to be done on the mini-computer that the model is now resident in (VAX 11/780 of IIASA). And before the simulation data could be transferred from the mini- to the micro-computer, a decision had to made with respect to the time-step to be used in the reduced model, which determined the temporal aggregation that is applied to the results of the simulation before they got transferred to the micro-computer.

Lastly, since we were dealing with a large-scale hydrological system, it was necessary to decompose in the system into smaller components; otherwise the modeling system IMSS could not "digest" the masses of data that even remain after temporal aggregation; such a decomposition also has advantages with respect to the interpretability of the results that are produced when the developed model gets used. In effect, this decomposition is a structuring of the reduced model. This is in line with the emphasis that the system IMSS places on structural considerations.

5.2. Preparation of data for IMSS

5.2.1. Treatment of uncontrollable variables

Since the values of uncontrollable variables are fixed before making a run with the scenario module, it would be possible to use a different reduced model for each possible combination of uncontrollable variables. The method would, however, require the construction of a large number of such models, which is time-consuming and not very practical. Also, such a series can not provide answers to "questions" with respect to meteorological conditions that occurred after the construction of models was completed. So here the choice was made to construct a single model.

5.2.2. Design of simulation experiments with FEMSATP-SIMCROP

For designing a series of simulation experiments with FEMSATP-SIMCROP, we used the following procedures.

Because the series of available data were judged to be too short for the purpose of deriving a reduced model that is valid over a wide range of conditions, the available "real" data for 12 years were expanded to a series for 33 years. This was done by perturbing the "real data" by adding random variables with normal distributions; if an extremely unrealistic value happened to be obtained, it was discarded.

For the controllable variables, pseudo-random numbers were generated within the constraints that derive from the fixed parameters. The complete description of the algorithm used for generating these numbers is given in a separate publication (Kettun et al., in press).

Although FEMSATP-SIMCROP distinguishes a number of different arable land use technologies, only one was used for the derivation of the reduced model, namely potatoes. The justification for this is that the arable land technologies differ mainly in the length of the growing season. For computing crop productions in SIMCROP, the ratio between actual and potential evapotranspiration is the main determining factor; this ratio is, however, not very sensitive to the length of the growing season, because both the actual and potential evapotranspiration increase if the length of the season is increased. So in our experiments with FEMSATP-SIMCROP, we used only the following four control variables (per subregion) for areas of technologies:

- $x(r,1,0)$: area of arable land, non-sprinkled;
- $x(r,1,1)$: area of arable land, sprinkled;
- $x(r,2,0)$: area of grassland, non-sprinkled;
- $x(r,2,1)$: area of grassland, sprinkled;

The notations for the remaining control variables are

- $s_c(r)$ and $g_c(r)$: capacities of sprinkling from surface water and groundwater;
- $S_c(r)$: allocation of surface water to a subregion .

The notations used for the *state* variables mentioned in Section 4. are ($j=1$ for arable land, $j=2$ for grassland, $k=0$ for non-sprinkled, $k=1$ for sprinkled):

- $e_a(r,j,k)$ and $c_p(r,j,k)$: crop evapotranspirations and productions:
- $i_s(r)$: volume of sprinkling from surface water;
- $i_g(r)$: volume of sprinkling from groundwater;
- $i_{ar}(r)$: volume of sprinkling on arable land;
- $i_{gr}(r)$: volume of sprinkling on grassland;
- $s_i(r)$: amount of subirrigation;
- $h_w(r)$: groundwater levels in nature areas at the end of summer (r=10,16,27).

5.2.3. Choice of time step for uncontrollable variables

Though the problem of model simplification can be viewed as a process in the course of which the most appropriate time-step for the reduced model is chosen and then iteratively adjusted till a point has been reached where the simplification by increasing the time-step (cf. FEMSATP's seven days) is in "balance" with simplification through other means, we here have chosen the time-step prior to other steps of model simplification. We simply split the year into two halves: the winter half preceding a growing season, taken from 1st October till 1st April, and the growing season itself. For the "uncontrollable" variables this then gives:

- precipitation during winter, p_1, and during summer, p_2 ;
- potential evapotranspiration during winter, $e_{p,1}$, and during summer, $e_{p,2}$.

5.2.4. Decomposition of the regional system

For the purpose of decomposing the regional system into a set of subsystems that are connected to each other through the aquifer system in the subsoil, we defined the following *intermediate* state variable:

$$g_k(r) = i_g(r) - l_k(r)$$

where $g_k(r)$ - intermediate variable

$i_g(r)$ - volume of sprinkling from groundwater during one summer

$l_k(r)$ - volume of "leakage" from the phreatic layer to the first aquifer.

The defined intermediate variable $g_k(r)$ can be seen as the "impact" that a subregion has on the regional system: $i_g(r)$ is an extraction from the first aquifer, and $l_k(r)$ is a flow to that aquifer. So $[i_g(r)-l_k(r)]$ is the combined (negative) effect of $i_g(r)$ and $l_k(r)$ on the (summer) water balance of the part of the first aquifer that is directly beneath a subregion. The leakage $l_k(r)$, however, is not only influenced by the activities in a subregion itself, but also by the activities in the surrounding subregions: an increase of the values of $g_k(r)$ in the surrounding subregions will "induce" also a larger value of the leakage due to the "sucking away" of water caused by the increased (negative) impacts on the water balances. A larger value of the leakage then means a lower $g_k(r)$ in the case that it is positive, or a more negative one in the case that it is negative (a change of sign is of course also possible). So in the reduced model, the relationships describing the $g_k(r)$'s should have the form:

$$g_k(r) = f\{ v_{r,1}, \cdots, v_{r,n}, g_k(1), \cdots, g_k(r-1), g_k(r+1), \cdots, g_k(n_s)\}$$

where $v_{r,i}$, $i=1,n$ are the variables (of all types) describing the subregional system; n_s is the number of subregions - in the Southern Peel the number is 31. The set of n_s equations of this type together provide the "linking" of the subregional systems.

5.2.5. Linearity requirement for reduced model

Since the scenario analysis procedure requires the agricultural income to be a *linear* function of the control variables, the crop productions and evapotranspirations must be in volumes and not in volumes per unit area: In the latter case the values would have to be multiplied by the respective areas in the objective function, which would lead to a quadratic form. So, in order to avoid this, the evapotranspirations and the crop productions obtained from FEMSATP-SIMCROP are first multiplied by the respective areas, and only then presented to the modeling system IMSS as state variables for which a reduced model has to be derived. Since the state variables are in volumes, the values of uncontrollable variables should not only be in volumes per unit area, but also in volumes as possible explanatory variables for the system IMSS to use. Each of the four uncontrollable variables thus gets expanded to 5 values (per subregion):

- the value per unit area (which is the same for all subregions);

- the value per unit area, times the area $x(r,1,0)$, the value times $x(r,1,1)$,

- the value times $x(r,2,0)$ and the value times $x(r,2,1)$.

5.3. Application of IMSS

5.3.1. Introduction

The system IMSS includes the submodules shown in Figure 5. These modules are implemented in an integrated manner on a microcomputer with a color graphical display.

The system includes facilities for

- data transformation;

- structural analysis;

- linear modeling;

- model verification and validation.

The modeling process of using IMSS consists of three different but interdependent stages of dialogues as shown in Figure 6. Of the facilities mentioned above, structural analysis is used in all three stages, and is the most emphasized feature of the system.

The *first stage dialogue* is required for preparation of the modeling, including input of measurement data and the initial version of the cause-effect relation on the set of variables, transformation of variables, data screening, and refinement of the cause-effect relation.

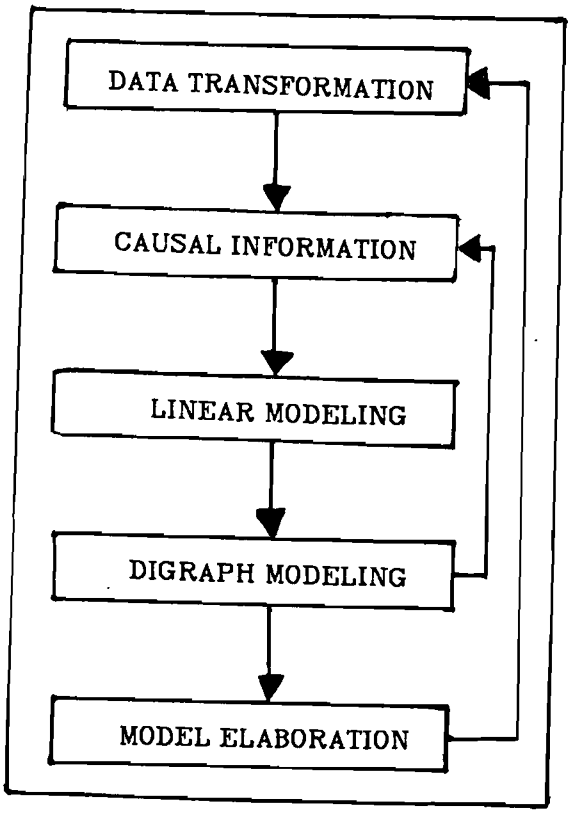

Figure 5: Submodules of IMSS

The ***second stage dialogue*** is devoted to finding a trade-off between the measurement data and the modeler's knowledge about dependencies between variables. Based on the measurement data and the initial version of the cause-effect relation, using an option of the regression method, the computer finds a model that is linear in estimated coefficients. The model is, however, not necessarily linear in the variables themselves: they could have been transformed in the first stage. Then the corresponding digraph models are drawn in order to facilitate the understanding and elaboration of the obtained model. If the structure of the model is modified, the affected parts of the model are again tested by means of regression methods. A series of reciprocal considerations and calculations by the analyst and the computer are repeated until the structure of the model becomes satisfactory in the eyes of the analyst.

The *third stage dialogue* is related to model simplification and elaboration. Model simplification is based on the use of the equivalence relation, and model elaboration is an application of regression analysis including the hypothesis testing on estimated coefficients, and examinations of the explanatory and predictive powers of the model.

5.3.2. Structural considerations

One of the main advantages of using the system IMSS is the facility for the structuring of reduced models; this corresponds to "causal information" and "digraph modeling" modules in Figures 5 and 6. For our purpose of finding a simple linear model, the structuring of the system is mathematically redundant. This is because the statistical closeness between the comprehensive and reduced models is the dominant requirement on the solution to our problem. But in systems analysis, mathematical redundancy is certainly not synonymous to uselessness : One of the great benefits of structural consideration is that it provides a learning exercise about the underlying system (which is here equated with the available comprehensive model of it). The complexity and ambiguity of a system is in the eye of the beholder. Put differently, the complexity and ambiguity that is perceived depends on the quality of the mental model that the perceiver uses for understanding a system, which in this case is a comprehensive model. Digraph modeling can provide a visualization that assists the construction of such a mental model. So the tracing of causation with the aid of a digraph model is a great help for understanding a comprehensive model and thus also for obtaining a simple model that is suitable for implementation within the framework of a scenario analysis procedure. As a byproduct it can even sometimes help to refine the original comprehensive model itself.

Let us denote the set of variables by

$$S = \{ x_1, x_2, \ldots\ldots, x_m \}.$$

The structural consideration of the reduced model is important for verifying whether the model behaves grossly in the fashion we intend it to. By the structure of the model is meant the cause-effect relation between variables. To introduce the cause-effect relation, the adjacency matrix

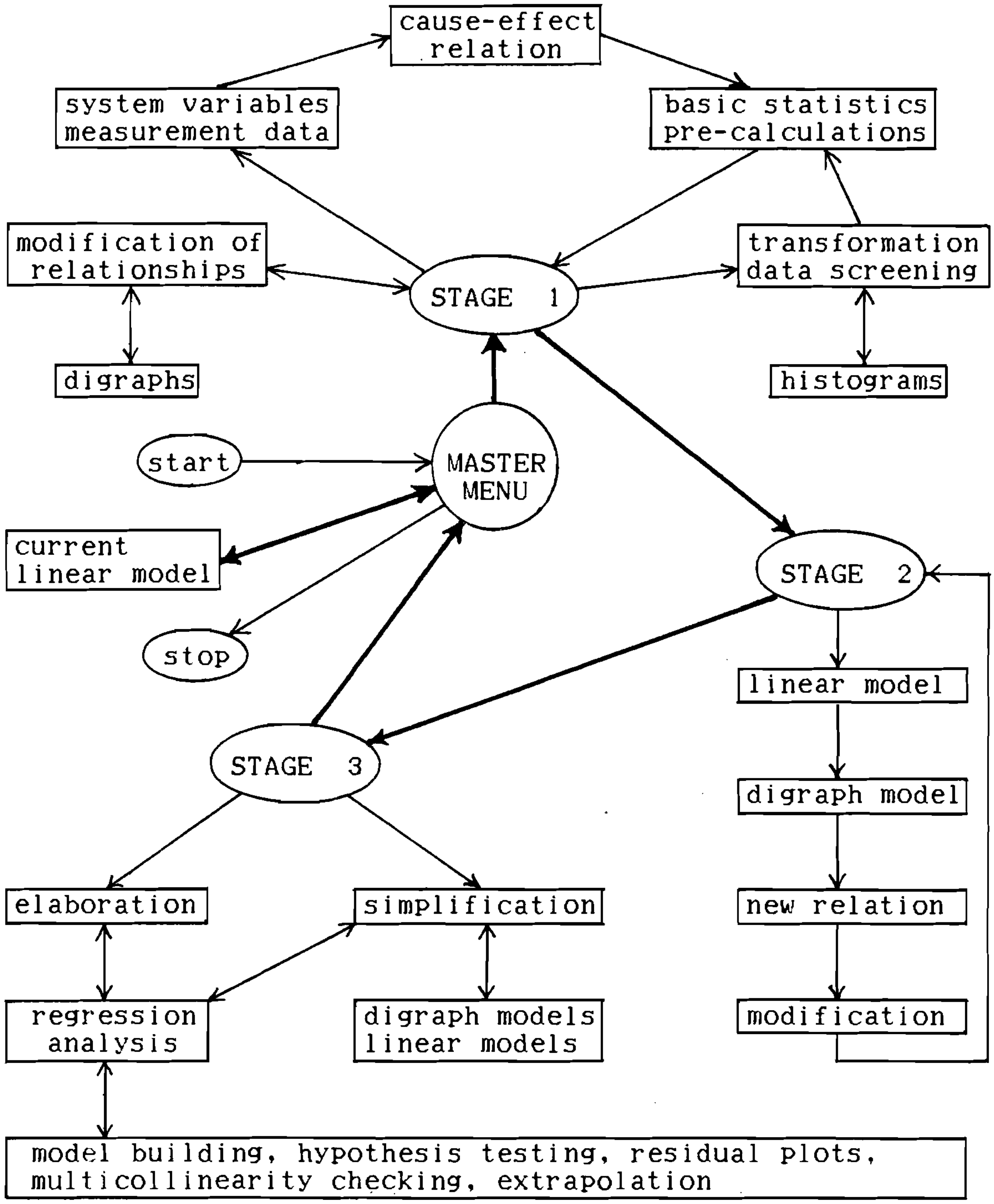

Figure 6: The interactive modeling support system

$A=(a_{ij})$, $i,j=1,2,\ldots,m$. is prepared; the entries are defined by

$$a_{ij} = \begin{cases} 2 \text{ if } x_i \text{ certainly affects } x_j \\ 1 \text{ if } x_i \text{ possibly affects } x_j \\ 0 \text{ if } x_i \text{ never affects } x_j \end{cases} .$$

To fill in entries of this matrix is sometimes quite difficult because the state variables (including the intermediate ones) often influence each other in such a manner that it is difficult to separate causes and effects. So the work requires a deep insight into the comprehensive model and the real world under study. The burden of entering the adjacency matrix is reduced, however, by initially assuming the validity of transitive inference. It is then possible to subsequently check the resulting adjacency matrix by drawing a digraph corresponding to it and modifying it if necessary. Mathematically the process of deriving a digraph is as follows.

Let B be the binary relation on $S \times S$ defined by

$$(x_i , x_j) \in B \text{ if and } \textit{only} \text{ if } a_{ij} \neq 0.$$

We introduce a digraph $D=(S,B)$ where the elements of S are identified as vertices and those of B as arcs. The vertices are represented by points and there is an arc heading from x_i to x_j if and only if (x_i,x_j) is in B. If there is a path from x_i to x_j, we say x_j is reachable from x_i . Apparently the digraph D is transitive, i.e., if x_j is reachable from x_i and x_k is reachable from x_j , then x_k is reachable from x_i. Therefore, we can reduce D to the condensation digraph D_C by grouping mutually reachable variables and selecting a so-called ***proxy*** variable in each group. Such a variable "represents" itself and the other variables belonging to the group.

Finally, we obtain a skeleton digraph D_S by removing arcs as long as the reachability present in D_C is not destroyed. If this digraph D_S is still complicated, format amendments can be carried out to facilitate interpretation. Those amendments include replacement of vertices, pooling of vertices of the same level and contraction of vertices between adjacent levels. This digraph D_S is usually highly aggregated and less informative, but visualizes the system structure in a clear manner. However, because the cause-effect relation is not necessarily transitive, we often have to modify the digraph D_S and the corresponding entries in the adjacency matrix A. It should be noted that if the digraph D_S is highly condensed, we should look at the original digraph D so that the modification of A can be done as we intend.

After several iterations, the structure of a subregional model was drawn as shown in Figure 7, where the full lines indicate the unconditional influences in the direction of the arrowheads and the dotted lines indicate the conditional influences. The digraph indicates for instance that the amount of infiltration of surface water s_u depends on the "pool" of meteorological variables and on $[S_c - s_c]$ - being the surface water supply capacity minus the capacity required for supporting the sprinkling from surface water that can take place. By "conditional influence" is meant that for instance the size of area $x(r,1,0)$ affects only $e_a(r,1,0)$, etc. (The evapotranspirations are taken in *volumes* as explained earlier.)

The crop productions and groundwater levels in nature areas are not shown in the diagram in order to avoid it being cluttered. Crop productions are assumed to depend on both the actual and potential evapotranspirations; groundwater levels in nature areas are assumed to depend on the values of the intermediate variables $g_k(r)$ in subregions surrounding a nature area, and also on the meteorological variables.

The use of intermediate variables in the structure serves not only the purpose of making full use of the structural modeling features of IMSS, but also to provide what one could call stepping-stones for the regression modeling that takes place in the second and third stage dialogues. Such stepping-stones help deal with non-linearities in the comprehensive model: It is easier to derive linear equations for two slightly non-linear relationships than for a very non-linear relationship that is the composite of the two slightly non-linear ones.

In the digraph of Figure 7 one would perhaps expect the total irrigation capacity $[s_c + g_c]$ to appear. However, this is not necessary because the information with respect to the size of this total capacity is already contained by the sum of the sprinkled areas $[x(r,1,1)+x(r,2,1)]$: the sum of this area multiplied by the sprinkling capacity per unit area yields the total capacity. Since this information is already contained in the mentioned sum, the method of regression modeling "finds this out" when a search is made for explanatory variables.

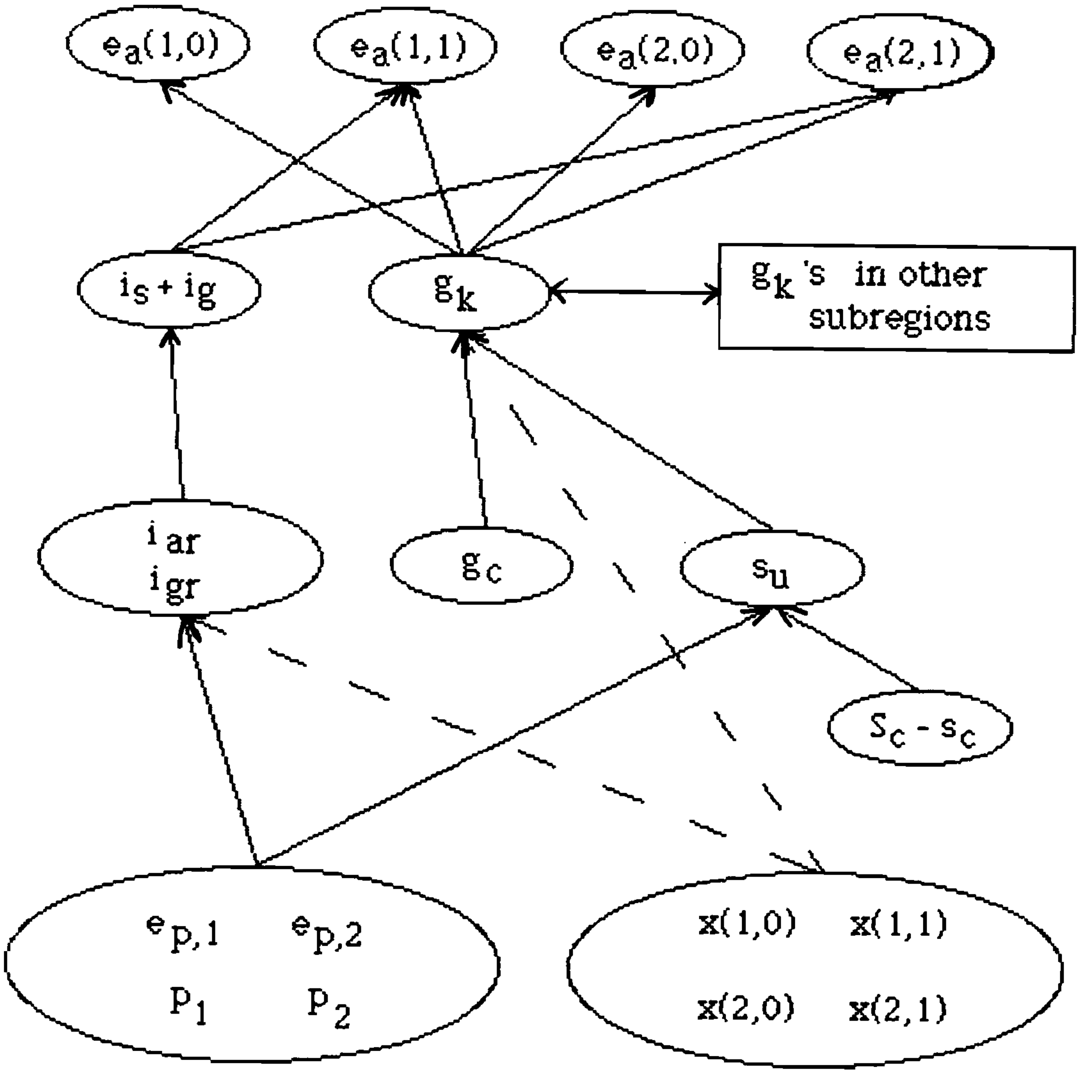

Figure 7: Structure of reduced model. The notation used is the same as introduced in Section 5.2.2.1 5.2.2.

5.3.3. Finding trade-off structures

The purpose of this step, which is the main objective of the "second stage dialogue", is to find a trade-off structure between the computer model and the mental model. First a reduced subregional model is obtained by the methods of stepwise or all-subset regression. (See for instance Mosteller & Tukey, 1977). Then the corresponding digraphs are drawn to facilitate the understanding and elaboration of the obtained model. If the structure of the model is modified, the affected parts of the model are again tested by the regression methods. A series of reciprocal considerations and calculations by the analysts and the computer are repeated until the structure of the model becomes satisfactory with respect to the current problem. This process is summarized in the following.

Let us define two subsets S_i^c and S_i^o of S for each x_i :

$$S_i^c = \{ x_j \; ; \; a_{ji} = 2 \},$$

$$S_i^o = \{ x_j \; ; \; a_{ji} = 1 \}.$$

Following the terminologies in statistics, we call S_i^c the *core* variable set and S_i^o the *optional* variable set for x_i. The elements of S_i^c are always chosen as the explanatory variables for x_i and those of S_i^o are *candidates*.

For each x_i , if $S_i^c \cup S_i^o \neq \phi$, then the coefficients of the equation:

$$x_i = c_{i0} + \sum_{x_j \in S_i^c \cup S_i^o} c_{ij} \, x_j$$

are identified using the simulation data and a regression method. The criterion of goodness of fit used here is the *controlled determination coefficient*, i.e., the square of the modified coefficient of multiple correlation:

$$R^2 = 1 - \frac{\sum_k (x_{ik} - \hat{x}_{ik})^2 / (n - p - 1)}{\sum_k (x_{ik} - \bar{x}_i)^2 / (n - 1)},$$

where $\hat{x}_{ik}$ is estimates of the kth data x_{ik} of the variable x_i, $\bar{x}_i$ the sample mean of x_i, n the number of data points and p the number of selected explanatory variables (x_j's). The set of selected variables (which in any case includes all the *core* variables) includes the combination of *candidate* variables that yields the value of $\bar{R}^2$ nearest to unity.

Table 1. An example of a subregional model (for r=24). The notation is the same as introduced in Section 5.2.2.

$e_a(1,0)$	=	9.7660D+02	+ 2.8546D+01*$(S_c - s_c)$	+ 7.6173D−01*g_k	5
			+ 3.5709D−01*$e_{p,2}$*x(1,0)	+ 5.0973D−01*p_2*x (1,0)	
$e_a(1,1)$	=	3.6005D+03	− 5.2861D−01*$(i_s + i_g)$	+ 1.5349D+00*g_k	6
			+ 2.8349D−01*$e_{p,2}$*x(1,1)	+ 7.6437D−02*p_1*a(1,1)	
			+ 3.7506D−01*p_2*x(1,1)	+ 7.7206D−01*	
$e_a(2,0)$	=	4.1776D+03	− 1.7472D+00*s_u	+ 1.0155D+00*(g_k)	7
			+ 7.2597D−01*$e_{p,2}$*x(2,0)	− 2.1579D−01*p_1*x(2,0)	
			+ 3.5611D−01*p_2*x(2,0)		
$e_a(2,1)$	=	1.77366D+02	+ 1.6014D+01*g_c	+ 5.3155D−01*s_u	8
			− 3.0734D−01*$(i_s + i_g)$	+ 4.0412D−01*g_k	
			+ 9.9655D−01*$e_{p,2}$*x(2,1)	− 5.5351D−02*p_1*x(2,1)	
g_k	=	−1.1705D+03	+ 4.8338D+00*$(S_c - s_c)$	+ 4.8205D+00*g_c	−
			− 1.8653D+00*p_2	+ 3.2445D−01*g_k(25)	
			− 4.4502D−01*g_k(28)		
$i_s + i_g$	=	−7.2322D−01	− 1.1371D−04*p_1x(1,0)	+ 1.6037D−04*p_2*x(1,1)	4
			+ 5.2728D−05*p_2*x(2,1)	+ 2.1755D−01*i_{ar}	
			+ 2.1730D−01*i_{gr}		
S_u	=	1.2503D+03	+ 1.2916D+01*$(S_c - s_c)$	+ 1.7803D+00*e sub p.2	1
			− 3.0936D+00*p sub 2		
i_{ar}	=	1.2761D+01	+ 1.0924D+00*s_u	+ 4.5345D−01*g_k	3
			+ 6.9894D−01*$e_{p,2}$*x(1,0)	− 3.6521D−02*p_1*x(2,1)	
			− 5.8669D−01*p_2*x(1,0)		
i_{gr}	=	4.4441D+03	+ 2.4207D+00*$(S_c - s_c)$	− 8.1116D−01*g_k	2
			+ 6.9407D−01*p_2*x(2,1)	− 5.1673D−02*p_1*x(2,2)	
			− 4.7922D−01*p_2*x(2,1)		

An example of subregional model is shown in Table 1. The individual interpretation of the coefficients is quite difficult or impossible. Therefore, at this step, we should check by the digraph whether the structure of the model is suited for the purpose of scenario analysis. The presented result shown in Table 1 is actually the one that is obtained after several repetitions of this step and intensive discussions to modify the model structure using the digraphs. In a subsequent section more will be said about the nature of the relationships given in Table 1.

5.3.4. Model validation

In this step the explanatory and predictive powers of the subregional model are examined by the following statistics:

- standard errors of estimated coefficients,
- t-ratios of estimated coefficients,
- standard deviation of residuals,
- F-ratio against a null hypothesis,
- controlled deterministic coefficient,
- correlation coefficients, and
- residuals and predictions.

Although the simulation experiment with FEMSATP-SIMCROP was designed carefully so that the values of of control variables had a low correlation with each other, it is possible that some intermediate variables are highly correlated with each other because of the properties of the comprehensive model. Also, it is possible that the correlation could have been introduced by the transformation of the control variables; e.g. multiplication of all $x(r,j,k)$ with the summer precipitation p_2 introduces correlation between the 4 newly created explanatory variables. If the presence of correlation means that in the relationships for which certain variables are the explanatory variables we can eliminate some of them as long as the reduction does not destroy the cause-effect relation structure necessary for the intended use of the model. This means that relations that in the eyes of the analyst "must" be there but that are not strongly supported by the "statistics", nevertheless get retained in the model.

Table 2. Example of a linear relationship and some statistics

Subregion 24	Regressand ==> e_a(1,0)			Equation No. 1
variable	coefficient	standard error	t-ratio	correlation
$S_c - s_c$	0.2855D+02	0.2675D+02	0.1067D+01	-.0461
g_k	0.7617D+00	0.6178D+00	0.1233D+01	-.3140
$e_{p,2}$*x(1,0)	0.3571D+00	0.4305D-01	0.8295D+01	0.9827
p_2*x(1,0)	0.5097D+00	0.5241D-01	0.9726D+01	0.9848
constant	0.9766D+03			

Degrees of Freedom = 23 Adjusted R-Square=0.9932
S.D. of Residual = 0.6465D+03 F-Ratio = 0.9810D+03
T(23,0.05) = 2.0687 F(4,23,0.05)=2.7955

Table 3. Illustration of the predictive powers of relationship given in Table 2.

Subregion 24	Regressand ==> e_a(1,0)		Equation No. 1
Case Number	Measurement	Prediction	Standard Error
No. 29	0.1566D+05	0.1533D+05	0.6784D+03
No. 30	0.3930D+04	0.4975D+04	0.7164D+03
No. 31	0.9383D+03	0.5715D+03	0.7572D+03
No. 32	0.1914D+04	0.1704D+04	0.6988D+03
No. 33	0.1436D+05	0.1328D+05	0.6683D+03

The Number of Cases = 5 Correlation (meas,pre) =0.9821
Mean Square Error = 0.5090D+06 Mean Absolute Error =0.1725D+00

An example of a linear relationship and some relevant statistics are given in Table 2. For a complete description of the meaning of the statistical indicators, the reader is referred to Nakamori et al. (1985). Table 3. gives an example of the predictive powers of the derived relationship. The wide range of values of the evapotranspiration is due to the fact that the values in *mm* have been multiplied by [*% of agricultural area*]; the area of subregion 24 is 2175. ha. The actual values of explanatory variables are, however, not those obtained through using other derived relationships that together comprise the whole model, but values taken from the *data*. This leads to a too favorable impression of the predictive powers, because there is of course a certain cumulation of errors upwards through the model hierarchy.

In the derived model a large amount of correlation is present due to the fact that the model structure was more based on "human expertise" than on "statistical evidence". This was done in order to obtain a model that could be a prototype for other regions and not just for the considered one : it turned out that for the considered region the evapotranspiration of non-sprinkled land could very well be explained just by the potential evapotranspiration and precipitation - this can be suspected when one sees how high the respective correlation coefficients are as given in Table 2. (both correlation coefficients are higher than 0.98)

5.3.5. Implementation of reduced model in the scenario module

Before implementing the subregional models, the equations given in Table 1 were ordered in such a manner that they form a lower triangular matrix - the numbers in the column O indicate the order. These equations can then be solved by means of forward substitution, which can very easily be done using the "matrix generator generator" system GEMINI, that was developed by Lebedev (1984). The equations connecting the subregional models, the equations for $g_k(r)$, can not be ordered in such a way, however. So in the LP constraint-matrix these equations were implemented as equality constraints containing in total 31 unknown $g_k(r)$'s. Since, however, this set of equations was derived for certain ranges of $g_k(r)$-values, these variables are not left completely free in the model. Instead, lower and and upper bounds are introduced that are derived from respectively the minimum and maximum values (per subregion) present in the data set that was supplied to IMSS. In order to leave the model some freedom to "extrapolate" the derived relationships beyond the ranges for which they were derived, the ranges of the $g_k(r)$'s were extended by 20% on both the lower and upper end; so in total the range was broadened by 40%.

The equations giving the actual evapotranspirations (which are in *volumes*) also include intercepts. These intercepts imply that even if the area of a technology is zero, there is still some evapotranspiration. This paradox is explained by the fact that we are here dealing with a *statistical* model, that has "maximum validity" for the average value of the area of a technology for which the evapotranspiration equation was derived. Since the simulation experiments with FEMSATP-SIMCRIP were done using pseudo-random numbers using four technologies (arable land non-irrigated, arable land irrigated, grassland non-irrigated, and grassland irrigated) these average values are roughly 25% of the agricultural area. The question arose what to do with the intercepts when one has 9 agricultural technologies (each with a non-irrigated subtechnology and an irrigated one) instead of 1. The decision was made to divide the intercepts by 9, because the average values obtained by scenario analysis can be expected to also be proportionally less. A similar procedure was applied for the 3 grassland technologies. It should, however, be noted that this procedure was only applied to the explicit intercepts given in Table 1, and not to the implicit ones that are obtained through the forward substitution.

A comparative sample of results obtained with the comprehensive model and with the reduced model are given in Table 4.

5.3.6. Conclusion

Though the advantages of using IMSS for model simplfication only become fully apparent after having actually used this *interactive* system personally, it is hoped that this paper will have given the reader a complete enough description of its use in order to appreciate the following main advantages :

- the data-screening features provide a powerful tool for debugging the data-set;

Table 4. Validation of the reduced model through comparison with comprehensive model

Evapotranspiration and Crop Production (subregion 24)

* e_a = Evapotranspiration * q_a = Crop Production
* R = Reduced Model * C = Comprehensive Model

		Eact (mm)		Qact (mm)	
		R	C	R	C
potatoes	(non-irrigated)	336			
	(irrigated)	399	406	48.7	48.0
grass land	(non-irrigated)	344	381	62.3	69.3
	(irrigated)	453	455	81.4	80.4

Water Quantity Subregional Variabls (mm) (subregion 24)

		R	C
sprinkling	from groundwater	102	91
	from surface water	7	6
subirrigation		61	41

- the structural modeling features are helpful for organizing one's thinking with

respect to the reduced model and also to the comprehensive itself.

- it enables rapid access to the set of relationships comprise a reduced model;

- it enables rapid validation of the reduced model using input data that were not

used for the modeling itself;

- it makes possible the easy refinement of the reduced model.

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