MARS: DESCRIBING DYNAMICS OF MILITARY EXPENDITURES REDUCTION

V. Iakimets

September 1985
WP-85-61

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

FOREWORD

Even with globally adequate food availability, large numbers of people remain chronically undernourished today. Evaluation of alternative national and international policies that can help reduce rapidly hunger in the world has been a major theme of the FAP since its inception.

Though national redistributive policies may be essential to reduce hunger at a satisfactory rate, the resources available with the developing countries are limited. International capital transfers are thus needed.

Among the sources for such funds can be reduction in arms expenditure.

With the help of FAP's Basic Linked System (BLS) of national agricultural policy models we have explored consequences for economic development and reduction in hunger of mutual arms reduction and redistribution of parts of the resources thus saved.

In this paper, Vladimir Iakimets explores the dynamics of military expenditure reduction as a prelude to designing appropriate arms reduction scenarios.

Kirit S. Parikh
Program Leader
Food and Agriculture Program

ACKNOWLEDGMENTS

The fruitful discussions with Dr. J. P. Hrabovszky, and Prof. K.S. Parikh (IIASA), and Prof. F. Rabar (IIASA) who reviewed an earlier draft of this paper are greatly appreciated.

The author wishes to express his special thanks to Dr. I. Kouzminsky (VNIISI, Moscow) for his assistance and to Prof. O.A. Kossov (VNIISI, Moscow) for his support and discussions and to Cynthia Enzlberger who typed this paper.

ABSTRACT

In this paper four hypotheses on possible dynamics of reduction of military expenditures of a country are introduced and described. Illustrative examples of calculation of annually released resources for the BLS runs according to each hypothesis are given.

CONTENTS

1. Introduction

The main starting point for launching the Food and Agriculture Program at IIASA was in fact our desire to understand whether reasonable and speedy internationally and nationally acceptable paths exist to reduce the large number of (more than 400 million) hungry people in the world. The Basic Linked System (BLS) consisting of interacting national agricultural policy models with sectoral breakdown which was created at IIASA is a suitable tool for the study of world hunger problems from different points of view. One of the possible directions of such a study is to analyse the opportunities for the utilization of the huge reserves existing in individual countries, namely resources used for military purposes, for the improvement of food supplies. A set of scenarios which we call the Mutual Arms Reduction Scenarios (MARS) in the set of other scenarios for the FAP's hunger study was suggested for investigation. Objectives of the MARS, their importance, assumptions for their construction, problems to be solved as well as description of their structure were given in Iakimets (1985).

One of the problems to be solved for implementation of MARS with BLS is the problem of determining the annual amount of reduction of a country's military expenditure.

This paper contains the formalized statement of the problem, description and formalization of the hypotheses relating to the desired dynamics of reduction of military expenditure and illustrative examples for calculation of annually released resources.

2. The Problem Statement

Let us denote $E(t)$ as total military expenditures of a country in year t and μ_t as its reduction coefficient. The amount of the military expenditures reduction of a country in year t or the released fund created by this country, $F(t)$, is determined as

$$F(t) = \mu_t E(t). \quad (1)$$

Hence

$$E(t+1) = E(t) (1 - \mu_t). \quad (2)$$

In order to determine annual amount of reduction of military expenditures of a country F(t) and hence E (t+1), we need to know initial value E(o) and value of μ_t, for each t = 0,1,...T−1. It is naturally to assume that

$$\mu_t = \mu(\bar{\alpha}, t), \quad (3)$$

where $\bar{\alpha}$ is in general case vector of parameters $\bar{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_A)$ which helps us to make hypothesis on "behaviour" of a country with relation to its military expenditures reduction for the period T under consideration.

The problem is to identify $\bar{\alpha}$ and to find type of functions $\mu(\bar{\alpha}, t)$ which will reflect hypothetical kinds of this behaviour.

3. Description of the Hypotheses

To illustrate the possible hypotheses relating to the desired dynamics of reduction of military expenditures let us consider Figure 1.

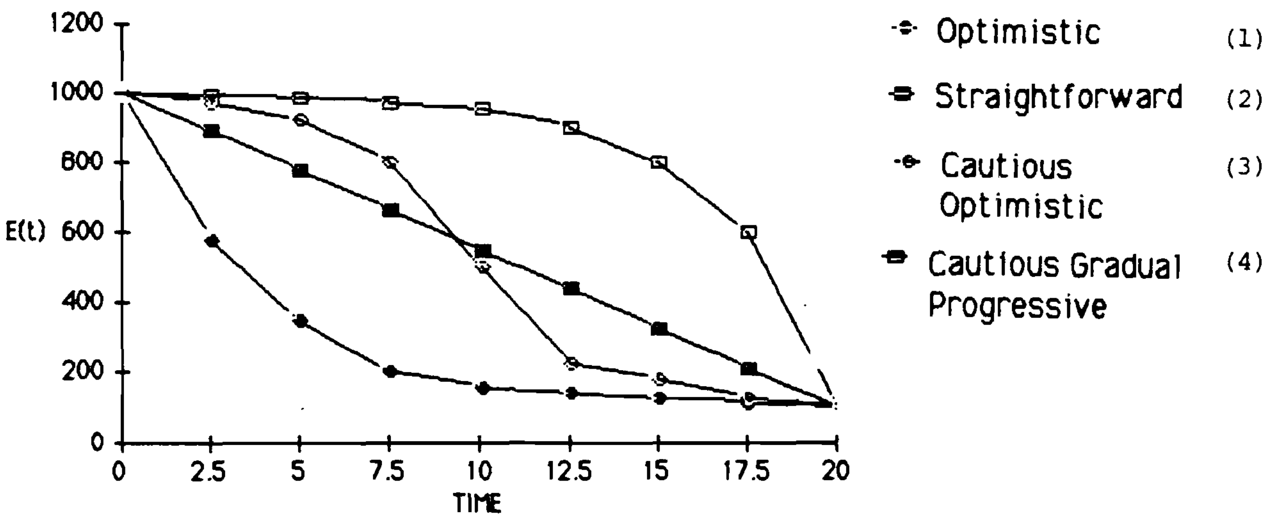

Figure 1. Four hypotheses of a country's behaviour

It is assumed that a country "wants" to have at the end of the period under consideration (T) the minimal admissible level of it's military expenditures E_T. Each of the four curves on Figure 1 explains one of the possible distinctive behaviours of a country with relation to reduction of its military expenditures.

Hypothesis 1 (optimistic)

This hypothesis is represented by curve 1. It can be described in the following way. A country has agreed to start with considerable initial reduction of its expenditures with subsequential deceleration of the amounts of reduction to the end of period [0, T]. This implies that this country can be sure of similar behaviour of other countries and of opportunities of rational utilization of resources released for internal purposes and for external aid.

Hypothesis 2 (constant)

This hypothesis is represented by the line 2. As in the previous case a country is absolutely sure of the same behaviour of other countries and rational utilization of resources released. However it "considers" that an initially fixed rate of reduction is maintained for the whole period under consideration.

Hypothesis 3 (cautious optimistic)

This hypothesis is represented by curve 3. At the beginning of period under consideration a country shows a cautious approach to reduction of its military expenditures both from the point of view of behaviour of other countries and opportunities for rational utilization of released resources, for example in developing countries because of their initial low absorptive capacity. However, after some time when the joint efforts of other countries' reduction of their military expenditures are realized, this country decides to reduce faster in the middle of the period considered with a subsequential decelerated reduction at the end.

Hypothesis 4 (cautious gradual progressive)

This hypothesis is represented by curve 4. It can be described as follows. A country has agreed to begin reduction of expenditures under consideration. However, it "prefers" to implement these with a gradually increasing rate of reduction.

This behaviour can be explained mainly by possible progressive growth of absorptive capacity of developing countries and in part by rates of structural changes in the economy both in developed and developing countries.

4. Formalized description of hypotheses

All of these hypotheses have very clear distinctive properties, however it is possible to describe these types of behaviour, for example, with the help of the following equations:

It is easy to see that (2) and (3) leads us to the differential equation:

$$\frac{1}{E_t}\ \frac{dE_t}{dt} = \mu\,(t,\bar{\alpha}). \tag{4}$$

Let us fix

$$E(o) = E_o, \tag{5}$$

$$E(T) = E_T, \tag{6}$$

and by definition*:

$$E_T < E_o \tag{7}$$

Solution of (4) if we take into account (5) and (6) is

$$E(t) = E_o \exp\left[-\int_o^t \mu(t,\bar{\alpha})\,dt\right] \tag{8}$$

* The behaviour with $E_T \geq E_o$ seems to be excluded from consideration. However, it is simple if necessary to make calculations for this case also.

Let us assume that $\overline{\alpha}$ is not a vector, but some parameter: $\overline{\alpha} := \alpha$.

If we select the determined type of function $\mu(\alpha, t)$, then using (5), (6), (7) and (8) it is possible to calculate the value of alpha. It enables us to immediately calculate of time series E (t) for the corresponding hypothesis.

4.1. Optimistic Hypothesis

The family of functions $\mu^{(1)}(t, \alpha)$ to describe the hypothesis 1 can be written with the help of

$$\mu^{(1)}(t, \alpha) = \mu_o (1 + t \cdot \alpha),$$

and

$$E_t^{(1)} = E_o \exp\left[-\mu_o \cdot t \left(1 + \frac{\alpha t}{2}\right)\right], \tag{9}$$

where μ_o and α have the determined "physical" sense. Namely μ_o is the initial value of reduction coefficient (for t = o) and α is parameter which determines the rate of decreasing $\mu^1(t, \alpha)$ in time for the whole period under consideration. Let us consider for certainty that T = 20.

For various values of α we have

for $\alpha = 0$

$$\mu^{(1)}(t, \alpha) = \mu_o \tag{10}$$

for $\alpha < 0$

$$\mu^{(1)}(t, \alpha) = \mu_o (1 - t \cdot \alpha_1) \tag{11}$$

for $\alpha > 0$

$$\mu^{(1)}(t, \alpha) = \mu_o (1 - t \cdot \overline{\alpha}_1) \tag{12}$$

The behaviour of $\mu^{(1)}(t, \alpha)$ for (10), (11) and (12) is shown in Figure 2.

After substitution of one of (10), (11), (12) in (8), and taking into account (6), we can solve (8) in relation to α. So we have:

for (10)

$$E_T = E_o \exp(-\mu_o \cdot T) \ , \tag{13}$$

for (11)

$$E_T = E_o \exp\left[-\mu_o \cdot T(1 - \alpha_1 \frac{T}{2})\right] \ , \tag{14}$$

for (12)

$$E_T = E_o \exp\left[-\mu_o \cdot T(1 - \overline{\alpha}_1 \frac{T}{2})\right] \ , \tag{15}$$

In other words for fixed values of T, E_o, E_T and μ_o, $\alpha_1 = \overline{\alpha}_1$ and we have for these two cases the same curve on Figure 3.

When we need to have in $t = T$, $E_t = E_T =$ constant, then the value of μ_o from (13) is calculated as follows:

$$\mu_o = -\frac{1}{T} \ln \frac{E_T}{E_o} \tag{16}$$

The value of α for cases (14) and (15) are calculated as follows (let us assume that $\mu_o =$ constant):

for (14)

$$\alpha_1 = \frac{2}{T}\left[\frac{1}{\mu_o \cdot T} \ln \frac{E_T}{E_o} + 1\right] \ , \tag{17}$$

and for (15)

$$\overline{\alpha}_1 = -\frac{2}{T}\left[\frac{1}{\mu_o \cdot T} \ln \frac{E_T}{E_o} + 1\right] \ . \tag{18}$$

The qualitative behaviour of E_t for all the abovementioned cases is shown on Figure 3.

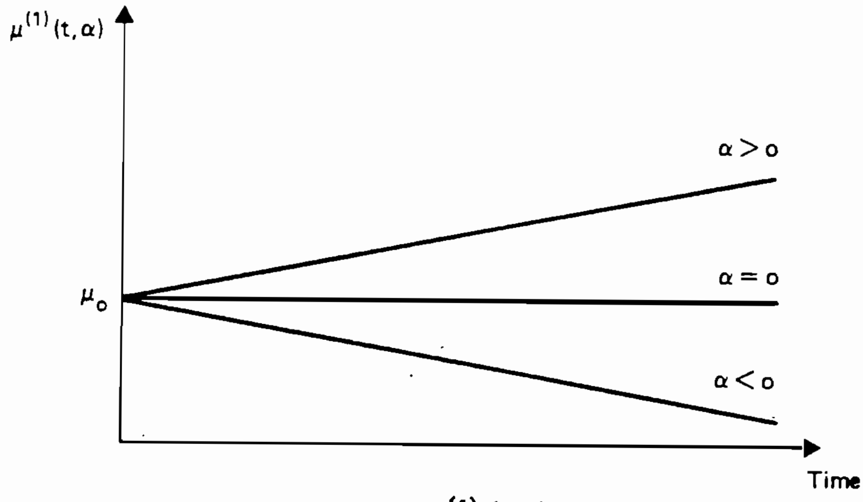

Figure 2. Behaviour of $\mu^{(1)}(t,\alpha)$

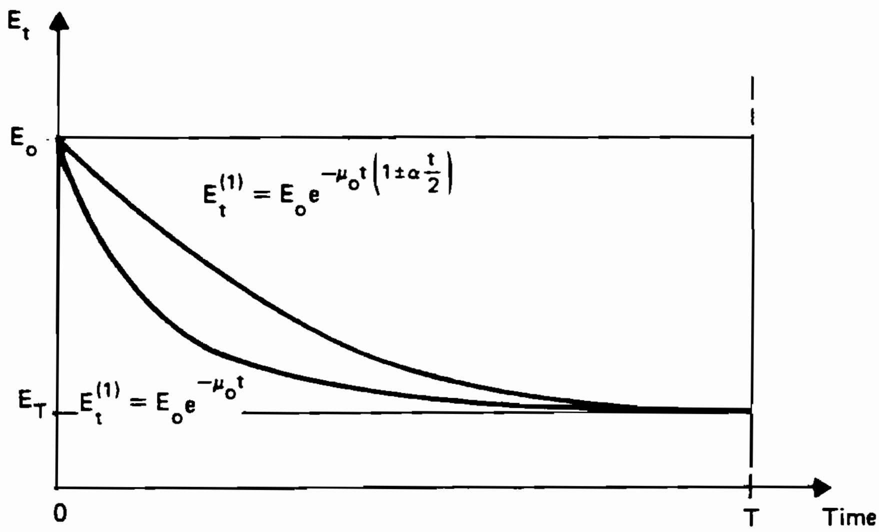

Figure 3. Qualitative behaviour of E_t for optimistic hypothesis.

4.2. The example of calculation of $E_t^{(1)}$ for hypothesis 1.

Let us assume that T = 20, E_o = 1000, E_T = 100. Results of calculation of E_T are given in Table 1 and shown in Figure 4.

Table 1. Example of calculation $E_t^{(1)}$ for hypothesis 1.

t	$E_t^{(1)}$ (13) $\mu_o = 0.115$, $\alpha_1 = 0$	$E_t^{(1)}$ (14), $\alpha_1 = -0.0151$ $E_t^{(1)}$ (15), $\overline{\alpha}_1 = 0.0151$ $\mu_o = 0.1$
0	1000	1000
1	891	904.1
2	794	816.2
3	707.9	735.8
4	630.9	662.2
5	562.3	595.2
6	501.2	548.8
7	446.7	477.7
8	398.1	449.3
9	354.8	406.6
10	316.2	341.1
11	281.2	303.7
12	251.2	270.1
13	223.8	239.8
14	199.5	212.6
15	177.8	188.2
16	158.5	166.3
17	141.2	146.8
18	125.8	129.4
19	112.2	113.8
20	100	100

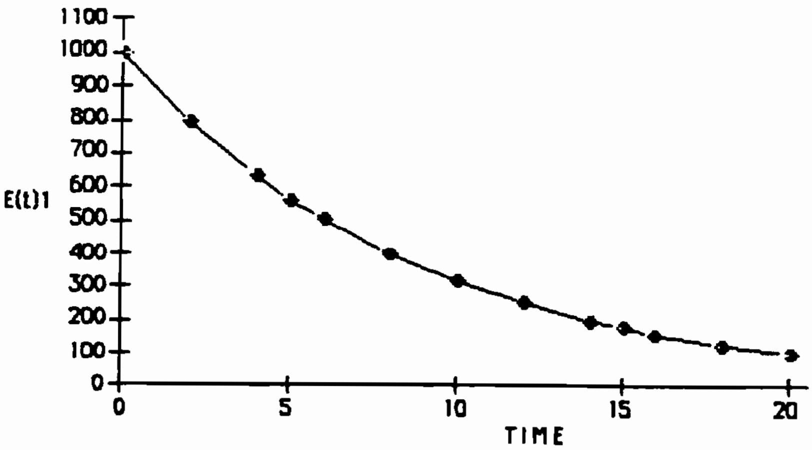

Figure 4. Calculated curve for hypothesis 1

4.3. Constant Hypothesis

It is easy to see from general considerations that the behaviour of E_t in this case is described as follows

$$E_t^{(2)} = E_o \exp\left[\ln(1 - \mu_o^{(2)} \cdot t)\right], \tag{19}$$

where

$$\mu_o^{(2)} = \frac{E^o - E_T}{E_o \cdot T}, \tag{20}$$

because

$$E_t^{(2)} = E_o - \frac{E^o - E_T}{T} \cdot t\ , \tag{21}$$

when E_o and E_T are known, and besides in (8)

$$\mu_t^{(2)} = \frac{\mu_o^{(2)}}{1 - \mu_o^{(2)} \cdot t} \tag{22}$$

Here $\mu_o^{(2)}$ has sense as relative coefficient of military expenditures reduction.

Behaviour of $\mu_t^{(2)}$ and $E_t^{(2)}$ is shown in Figures 5 and 6 correspondingly.

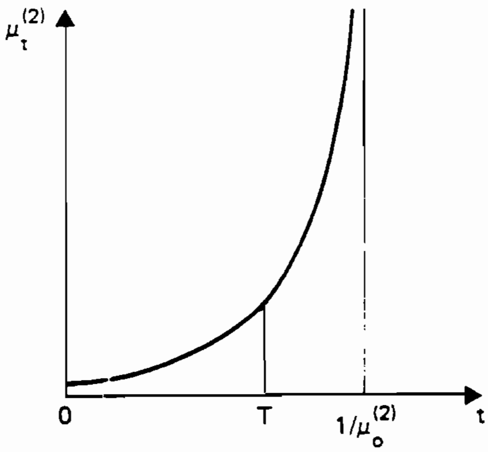

Figure 5

Qualitative behaviour of $\mu_t^{(2)}$

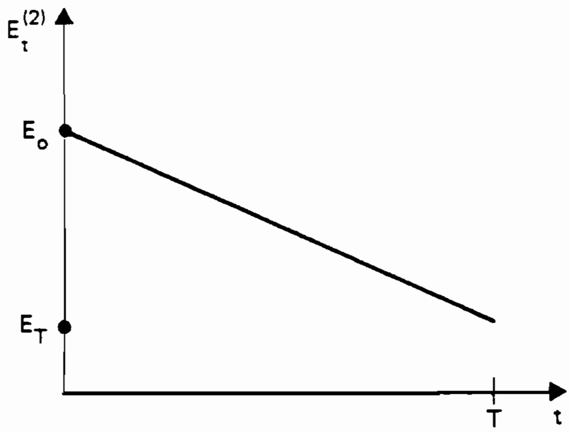

Figure 6

Qualitative behaviour of $E_t^{(2)}$

4.4. Example of calculation of $E_t^{(2)}$

Let us assume that $E_o = 1000$, $E_T = 100$, $T = 20$. Then $\mu_o^{(2)}$ according to (20) is equalled to 0.045. Values of $E_t^{(2)}$ for this case are given in Table 2 and shown in Figure 7.

Table 2. Values of $E_t^{(2)}$

t	0	1	2	3	4	5	6	7	8	9
$E_t^{(2)}$	1000	955	910	865	820	775	730	685	640	595

t	10	11	12	13	14	15	16	17	18	19	20
$E_t^{(2)}$	550	505	460	415	370	325	280	235	190	145	100

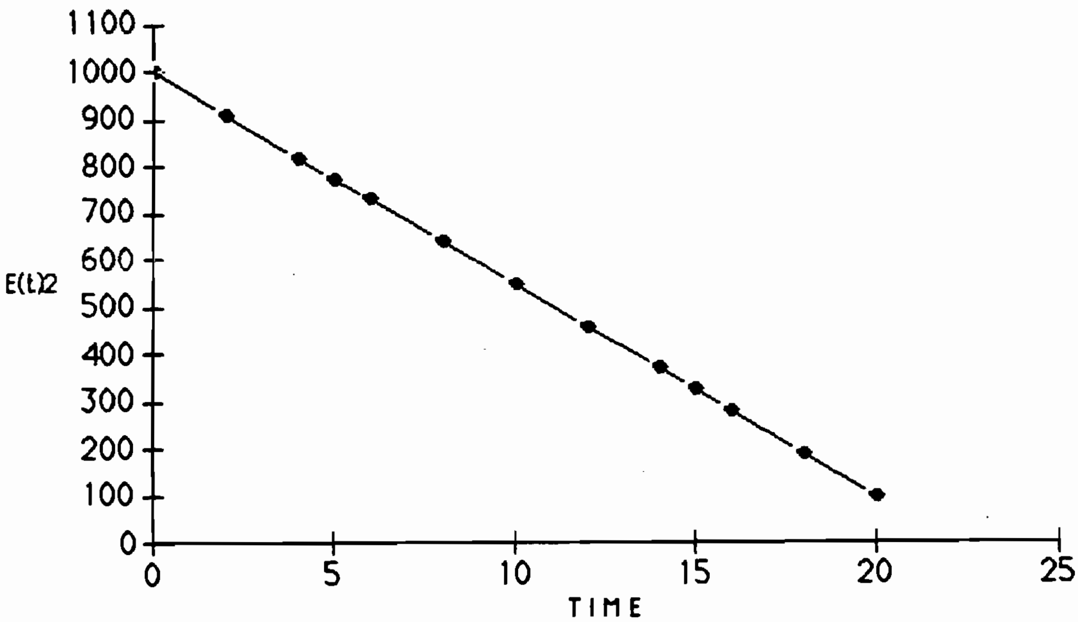

Figure 7. Calculated curve for hypothesis 2

4.5. Cautious optimistic and cautious gradual progressive hypotheses.

The family of $\mu_t^{(3)} = \mu_t^{(3)}(\alpha, t)$ with help of which we can describe hypotheses 3 and 4 is determined as follows:

$$\mu_t^{(3)} = \alpha_3 t^a, \qquad a>0 \tag{23}$$

Hence according to (8):

$$E_t^{(3)} = E_o \exp\left[-\frac{\alpha_3}{a} t^{a+1}\right] \tag{24}$$

Behaviour of coefficient μ_t^3 and $E_t^{(3)}$ is shown in Figures 8 and 9 correspondingly (respectively)

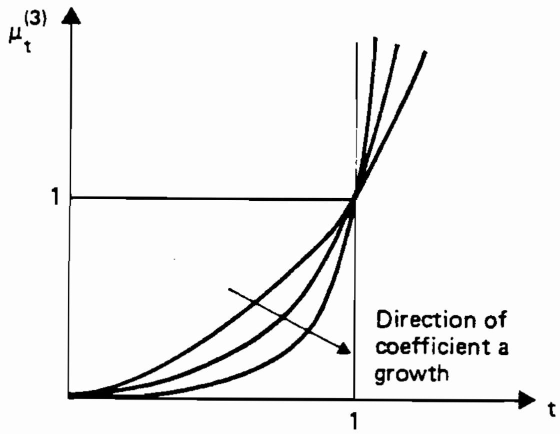

Figure 8

Qualitative behaviour of $\mu_t^{(3)}$

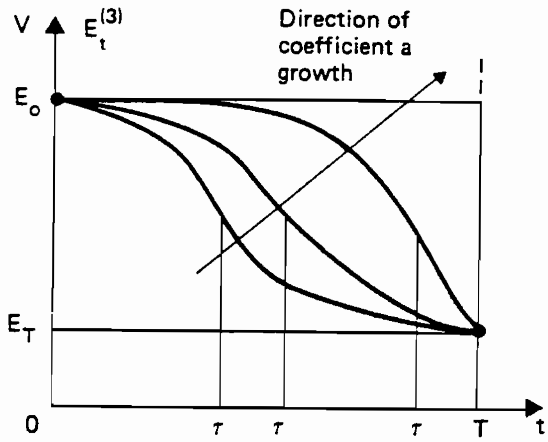

Figure 9

Qualitative behaviour of $E_t^{(3)}$

Values of α_3 for different values of coefficient a are determined from (24)

$$\alpha_3 = -\frac{a}{T^{a+1}} \ln \frac{E_T}{E_o} \tag{25}$$

Let τ be the value of point of inflection where the derivative of $E_t^{(3)}$ changes its sign. τ is determined as follows:

$$\tau = \left(\frac{a}{\alpha_3}\right)^{\frac{1}{a+1}} \tag{26}$$

or

$$\tau = \frac{T}{\left(\ln \frac{E_o}{E_T}\right)^{\frac{1}{a+1}}} \qquad (27)$$

So, for $T = 20$, $E_o = 1000$, $E_T = 100$, we have:

for	$a = 0.5$	$\alpha_3 \approx$	0.012	$\tau \approx$	11.5
for	$a = 1$	$\alpha_3 \approx$	0.0057	$\tau \approx$	13.2
for	$a = 2$	$\alpha_3 \approx$	0.00057	$\tau \approx$	15.15
for	$a = 3$	$\alpha_3 \approx$	0.000043	$\tau \approx$	16.24
for	$a = 5$	$\alpha_3 \approx$	2×10^{-8}	$\tau \approx$	17.4

So by choosing the value of coefficient a and by calculating the value of α_3 we can determine curves corresponding to hypothesis 3 or 4. It is clear, however, that these curves will be non-symmetrical ones and values of a more than 2 gives us curves for 4th hypothesis description. If values of a are less than 2 ($a > 0$) we will construct curves for third hypothesis.

4.6. Examples of calculation $E_t^{(3)}$ and $E_t^{(4)}$

Let us fix again $T = 20$, $E_o = 1000$, $E_T = 100$ and calculate $E_t^{(3)}$ and $E_t^{(4)}$ being chosen $a = 1$ for third hypothesis and $a = 3$ and 5 for fourth one. Results of calculation are given in Table 3.

Table 3. Values of $E_t^{(3)}$ and $E_t^{(4)}$

t	$E_t^{(3)}$, $\tau = 13.2$ $a = 1$, $\alpha \approx 0.0057$	$E_t^{(4)}$, $\tau = 16.24$ $a = 3$, $\alpha = 0.000043$	$E_t^{(4)}$, $\tau = 17.4$ $a = 5$, $\alpha = 2 \times 10^{-8}$
0	1000	1000	1000
1	994	999.9	1000
2	977	999.7	999.9
3	949.5	998.8	999.9
4	912.0	996.3	999.8
5	865.9	991.0	999.4
6	812.8	981.5	998.3
7	754.2	966.0	995.8
8	691.8	942.7	990.6
9	627.3	909.9	981.1
10	562.3	865.9	964.7
11	498.3	810.0	938.3
12	436.5	742.0	898.1
13	378.0	663.0	840.6
14	323.6*	575.3	762.7
15	273.8	482.6	663.8
16	229.1	389.4	546.8
17	189.4	300.6*	419.6*
18	154.9	220.7	294.1
19	125.2	153.3	184.0
20	100	100	100

The asterisks on this table correspond to points of inflection. Corresponding curves $E_t^{(3)}$ and $E_t^{(4)}$ for these cases are shown in Figure 10.

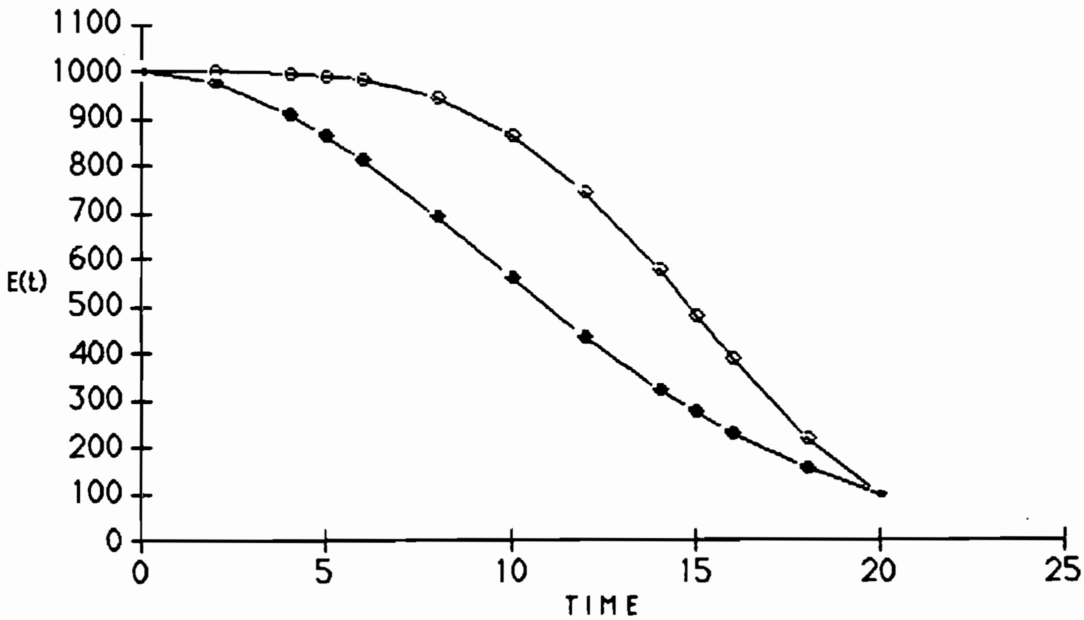

Figure 10. Calculated curves for hypotheses 3 and 4

4.7. Summary

The above given description of ways for exogenous calculation of the annual amount of reduction of a country's military expenditures in relation to the BLS was done for the case when E_T is fixed. Figure 11 contains calculated curves for all hypothesis.

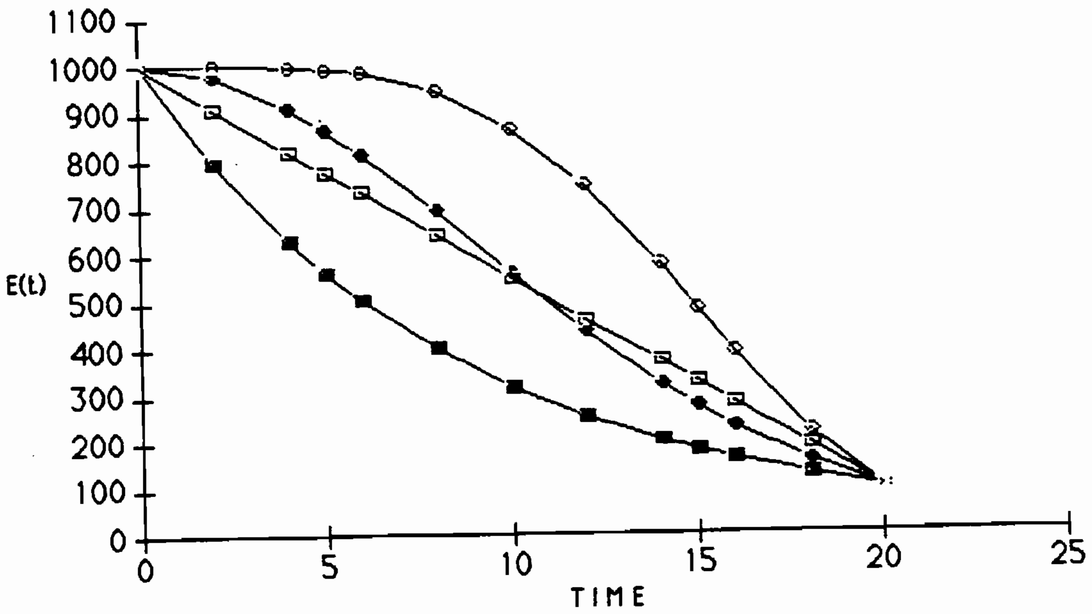

Figure 11. Calculated curves for all hypotheses.

Formulas (9, (19), (24) for calculation of $E_t^{(i)}$ for i-th hypothesis can be used also for calculation when E_T is not fixed. However, in such cases we need to be careful when values of corresponding parameters are determined.

REFERENCES

Iakimets V. (1985) Mutual Arms Reduction Scenarios (MARS) for the FAP's study "Hunger, Growth and Equity", IIASA, Laxenburg (forthcoming).