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THE SYSTEM OF DEMAND EQUATIONS IN A STATE OF DISEQUILIBRIUM

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FOREWORD

Many of today's most significant socioeconomic problems, such as slower economic growth, the decline of some established industries, and shifts in patterns of foreign trade, are inter- or transnational in nature so that the intercountry comparative analyses of recent historical developments are necessary. The understanding of these processes and future prospects provides the focus for IIASA's project on Comparative Analysis of Economic Structure and Growth.

Our research concentrates primarily on the empirical analysis of interregional and intertemporal economic structural change, on the sources of and constraints on economic growth, on problems of adaptation to sudden changes, and especially on problems arising from changing patterns of international trade, resource availability, and technology. This paper deals with the problem of econometric modeling of personal consumption expenditures under the conditions of disequilibria.

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1. PREFACE

Along with the increase in incomes and the decrease in supply the economic crisis in Poland of 1980-82 has been accompanied by a series of breakdowns in the markets of consumer goods and by the appearance of unsatisfied demand. As a result a total imbalance was created, manifested by the rise of an inflationary gap, as well as by the symptoms of market panic, lack of confidence in money and in savings and loan institutions, and by a growth of the black market and illegal or semi-legal trade on a scale unheard-of before. The emergence of these processes had very serious implications on the consumers' behavior. They had to adjust to their notional demand. Firstly, they were anticipating the expected price increases and a further deterioration of the markets, and bought those commodities, which were available at the moment, instead of those, which were in short supply (forced substitution). Secondly, they were trying to spend not only their current incomes but also parts of their savings, especially those which they had involuntarily accumulated in the past (forced savings). Thus an econometric analysis of consumer demand should take into account the features mentioned above. Otherwise objections could be raised with regard to the completeness and to the correctness of the conclusions drawn on the basis of such an analysis. These considerations will affect the estimation of demand function parameters in such cases where the fact is ignored that in some periods the actual quantity of consumption is the supply and not the demand. The aim of the present paper is to construct a system of demand functions that is not subject to these difficulties.

Our analysis deals with a short-term and a medium-term time period, and we assume that the shortage of supply does not cause any permanent distortions of the consumers' preferences. This means that after the equilibrium state is reestablished, the consumers would buy the same quantities of goods as before the market breakdown, and we assume that incomes and relative prices are unchanged.

A brief introduction to the problems of disequilibrium model building constitutes the first part of the paper. The second part is devoted to the analysis of monetary incomes flowing to the consumer goods market; here we shall consider both voluntary and forced savings. In the third part we specify a demand function that takes into account factors associated with disequilibrium. In the fourth part we present a proposal of building disequilibrium indicators which permits us to avoid the problems of unobservable variables. In the fifth part we specify and analyze a system of demand functions; the results of its estimation are shown in the last part.[1)]

2. THE DISEQUILIBRIUM MODEL. INTRODUCTION

An econometric disequilibrium model is most frequently described by means of the following equations (see [1], p. 156 ff.):

$$D_t = f_D(x_{D_t}, \alpha_D) + \xi_{D_t} \quad , \tag{1}$$

$$S_t = f_S(x_{S_t}, \alpha_S) + \xi_{S_t} \quad , \tag{2}$$

$$Q_t = \min\{D_t, S_t\} \tag{3}$$

where

D_t, S_t, Q_t - represent demand, supply, and transacted quantity,

x_{D_t}, x_{S_t} - vectors of explanatory variables of the demand and supply equations, respectively,

α_D, α_S - vectors of the parameters,

1) The estimation was performed on the basis of annual data for the period 1961-1982, published in the Yearbooks of the Central Statistical Office (GUS), and the Yearbooks of National Income published by GUS. The documentation of the source materials is contained in [13].

ξ_{D_t}, ξ_{S_t} - additionally introduced disturbance terms.

$t = 1,\ldots T.$

The total consumer demand D_t is understood as the quantity of goods the consumers want to buy in a unit period of time, given their disposable income and within a given price system. The demand is realized in whole or in part--in case of deficient supply--in the form of household expenditures. The supply of consumer goods is identical with the market offer by the socialized and private sectors of the economy.

The basic problem we have to face in estimating of model (1)-(3) consists in the fact that the use of ordinary least squares for the estimation of its parameters leads to biased and inconsistent estimators of these parameters (see [9], pp.164-174, and [2] p. 66 ff.), since the example contains subperiods of supply but not of demand realization. Three solutions are commonly used to avoid such consequences. The first one is to exclude from the sample periods in which supply restrictions took place, e.g. to exclude the years of supply crisis during war times.[2] The second solution uses estimation procedures based on the maximum likelihood method (see [8], pp. 164-174, and [2], pp. 66-70). The third one--the one we have accepted--extends the model (1)-(3) by the definitional and stochastic equation of excess demand:

$$DE_t = D_t - S_t \quad , \qquad (4)$$

$$DE_t = f_E(x_{E_t}, \alpha_E) + \xi_{E_t} \quad , \qquad (5)$$

where

x_{E_t} - vector of explanatory variables,

α_E - vector of parameters,

ξE_t - disturbance term.

If (1) and (2) hold, then DE_t (4) is by definition a function of the factors determining both demand and supply. On the other hand, DE_t can be determined indirectly, by means of inverting functions, where excess demand is an explanatory variable. A classical example of this approach is to define

[2] Such an operation was performed by the authors of one of the first models of the United States economy (see [5]).

price changes as a function of excess demand and then to invert the function, solving for excess demand.

We could now proceed using (4) and (5) to estimate from observable realizations either the demand function (1) or the supply function (2). The stochastic nature of equation (5) does not allow for an explicit re-specification of the model (1)-(3) (see [2], pp. 64-65). Replacing it by a deterministic relation leads to the following, directly estimable, functions of realizations

from the demand side:

$$Q_t = f_D(x_{D_t},\alpha_D) - f_E(x_{E_t},\alpha_E)U_{D_t} + \xi_{D_t} \quad , \tag{6}$$

from the supply side:

$$Q_t = f_S(x_{S_t},\alpha_S) + f_E(x_{E_t},\alpha_E)U_{S_t} + \xi_{S_t} \quad , \tag{7}$$

where

$$U_{D_t} = \begin{cases} 1 \text{ when } D_t > S_t \\ 0 \text{ when } D_t \leq S_t \end{cases} \quad , \quad U_{S_t} = \begin{cases} 1 \text{ when } S_t > D_t \\ 0 \text{ when } S_t \leq D_t \end{cases} \quad .$$

The relations, expressed by the above formulae, are of a non-linear character. However, it seems that linear approximations might be acceptable in view of the limited range of changes of incomes, prices, and supply. Thus, if we assume that the functions f_D and f_E are linear, model (6), which allows for direct estimation of parameters of the demand function, can be rewritten as

$$Q_t = x_{D_t}\alpha_D - x_{E_t}\alpha_E\, U_{D_t} + \xi_{D_t} \quad , \tag{8}$$

and this formula will be analyzed further below.

3. MONEY FLOWS IN A DISEQUILIBRIUM STATE

Let us consider at the beginning the situation of deficient supply in relation to the manifested consumer demand, i.e. $D_t > S_t$. As a consequence, a positive excess demand (DE_t) appears at the end of a time period:

$$DE_t = D_t - S_t \quad , \quad DE_t > 0 \iff D_t > S_t \quad . \tag{9}$$

Forced savings are its financial equivalent. The latter can be treated as finances accumulated in the savings and banking institutions as well as in cash, which would--under an unchanged price system--be spent by households if any scarce goods appeared on the market. It is of particular importance to stress that forced savings express the degree of imbalance after all possible demand transfers between the particular groups of goods have taken place.

The transacted quantity (expenditures) can also be written as:

$$Q_t = D_t - DE_t \tag{10}$$

During the next time period the households make further decisions concerning the use of the forced savings: part of them may be "neutralized", i.e. transformed into voluntary savings, thus increasing their total amount, and the rest will add to the funds allocated to the purchase of goods.

Normally voluntary savings remain untouched. These are the past voluntary savings accumulated as old-age insurance, or to be allocated to purchase (mainly of durable consumer goods) in the future, and as a necessary financial reserve. However, in some particularly unfavorable cases of a loss of confidence in banking institutions, they may also be treated by households as part of spendable funds. Thus it has to be considered that the total disposable money funds are composed not only of current personal income, but also of a certain portion of previously accumulated voluntary and forced savings.[3)]

$$YX_t = Y_t + \xi NS_{t-1} + \kappa FS_{t-1} \quad , \tag{11}$$

where

YX - total disposable monetary funds,

Y - personal monetary income,

NS - voluntary savings at the end of a period,

[3)] All values are treated as if they were expressed in constant prices, although the calculation concerning savings is usually presented in current prices.

FS - forced savings at the end of a period,

ξ - coefficient of utilization of the voluntary savings funds for the purchase of goods = coefficient of dissavings, $\xi \in <0,1>$,

η - coefficient of neutralization of forced savings, $\eta \in <0,1>$.

$\kappa = 1 - \eta$.

The above coefficients are unknown and must be estimated from the available data. It seems realistic to assume that the processes of neutralization of forced savings and dissavings exclude each other. Then the above formula can be rewritten as

$$\eta \neq 0 \Rightarrow \xi = 0 \quad , \qquad YX_t = Y_t + \kappa FS_{t-1} \tag{12a}$$

or

$$\xi \neq 0 \Rightarrow \eta = 0 \quad , \qquad YX_t = Y_t + \xi NS_{t-1} + FS_{t-1} \tag{12b}$$

To simplify the notation, we shall use (11), keeping (12a) and (12b) in mind.

The amount of total savings (GS) has been divided above into two parts: voluntary savings funds (NS), and forced savings funds (FS):

$$GS_t = NS_t + FS_t \quad . \tag{13}$$

Obviously in a state of a global equilibrium the second term of the above forumula is equal to zero.

On the other hand, total savings at the end of period t are equal to the savings from the end of the previous period (GS_{t-1}), decreased by a part of voluntary savings (ξNS_{t-1}) and forced savings (κFS_{t-1}) allocated to the purchase of goods, and increased by the difference between the purchasing fund YX_t and the transacted expenditure volume (Q_t).

$$GS_t = GS_{t-1} - \xi NS_{t-1} - \kappa FS_{t-1} + YX_t - Q_t \quad . \tag{14}$$

Using equation (11) we can easily chek that the well known identity holds:

$$\Delta GS_t = Y_t - Q_t \quad , \tag{15}$$

where Δ denotes the first difference.

The change of the total savings is decomposed into the change of voluntary and foced savings

$$\Delta GS_t = \Delta NS_t + \Delta FS_t \quad , \tag{16}$$

where $\Delta NS_t > 0$, as even in a state of disequlibirium a portion of the current incomes will be saved (e.g., special saving arrangements motivating the accumulation of financial resources for future purchases of cars, apartments, etc.).

Let us assume that there exists a long-term propensity to save. Then obviously the "notional" increase of voluntary savings will depend on the current income

$$\Delta NS_t^* = h(Y_t) \quad , \tag{17}$$

where

ΔNS^* - increase of voluntary savings deposits resulting from the inclination to save.

To arrive at the total level of the voluntary savings we will also have to include the neutralized forced savings and deduct dissavings analogously to (14):

$$NS_t = NS_{t-1} - \xi NS_{t-1} + \eta FS_{t-1} + \Delta NS_t^* \quad . \tag{18}$$

The total level of the forced savings is given by the identity:

$$FS_t = FS_{t-1} - \eta FS_{t-1} - \kappa FS_{t-1} + DE_t \quad . \tag{19}$$

Then the assumption follows that the households allocate their previously accumulated forced savings FS_{t-1} completely--either to purchase goods (expressed by the term κFS_{t-1}), or they neutralize them (expressed by the term ηFS_{t-1}):

$$FS_{t-1} = \eta FS_{t-1} + \kappa FS_{t-1} \quad . \tag{20}$$

From this assumption and from (19) we get

$$FS_t = DE_t \quad . \tag{21}$$

Forumla (21) is of great practical significance as the total excess demand was often identified with the increase of forced savings. The formula clearly indicates that the value of the excess demand, which appeared in a given period, is equal to the total level of forced savings at the end of that period. Let us stress once again that this is a result of the assumption that the consumers invest the whole amount of their non-neutralized forced savings into possible purchases.

Identity (11), using (10) and (15), implies also that

$$YX_t = D_t - DE_t + \Delta GS_t + \xi NS_{t-1} + \kappa FS_{t-1} \quad . \tag{22}$$

Subsequently, using (18), (19), and (16), the above formula may be rewritten in a well known form, i.e.:

$$YX_t = D_t + \Delta NS_t^* \quad , \tag{23}$$

which expresses the allocation of the purchasing fund among potential consumer expenditures and the increase of the voluntary savings. In case of deficient supply ($D_t > S_t$, formula (9)), the budget constraint stands as follows:

$$YX_t = Q_t + \Delta NS_t^* + FS_t \quad . \tag{24}$$

In a case contrary to the one discussed above (described by formula (9)), i.e. if the supply is sufficient and satisfies the consumer demand ($D_t \leq S_t$), we can accept the value of the subtraction result as being negative:

$$DE_t = D_t - S_t \quad , \qquad DE_t \leq 0 \quad . \tag{25}$$

This is due to the assumption that the excess supply will increase inventories. Even if such a hypothesis is not true for a short period of time, it does not have any implications with respect to the accumulation and allocation of the financial resources the households have at their disposal. The purchasing funds, using (11), then equal

$$YX_t = Y_t + \xi NS_{t-1} \quad . \tag{26}$$

In this context it should be expected that $\xi \cong 0$, provided that we exclude two possible but unlikely situations. The first of them occurs when the interest rate on bank deposits is significantly lower than the inflation rate. The consumers may then protect their savings against depreciation by investing them in goods. The second one is connected with a significant slowdown of the growth rate, or with a decrease in real incomes (Y). The households' policy of protecting the so far achieved consumption level and living standards can be manifested by a process of dissaving.

4. BUILDING OF DEMAND FUNCTIONS

The demand, which we shall call notional demand (DN^j), is the main component of the effective demand in a market of good j (D^j). In a state of market equilibrium and with given real personal incomes and prices, the notional demand is a consequence of a system of consumer preferences, and thus corresponds to the conventional understanding of demand (see [10], pp. 215-229]. The aggregation of demand micro-functions, taking into account the reduction of variables, describing modifications of the demographic and social structure of the population, leads to the following macro-function[4]:

$$DN_t^j = \beta_0^j + \beta_1^j Y_t - \beta_2^j PC_t^j + \beta_3^j t + \xi_{Nt}^j \quad , \tag{27}$$

where

- Y - real personal monetary income,
- PC^j - ratio of the price index of the examined articles to the prices of other articles (the relative price index),
- t - trend, denoting the modifications of the demographic-social structure,
- ξ_N^j - disturbance term.

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[4] See [3], pp. 16-18. For a better clarification of the notation we use the convention that parameters marked with Greek letters have positive values, and the direction of relation is indicated by a mark in front of each parameter.

Anticipated demand (DA^j) is the second component of the effective demand. It appears to reflect the consequences of the consumers' anticipations; they are manifested by the growth of purchases above or below a standard level. On the one hand the expectations of households pertain to price changes, on the other they pertain to the appearance of a disequilibrium. In the first case, the increase in purchases and stocking of goods are intended as a means of protecting themselves against a depreciation of savings; in the second case, against a drop in consumption due to the expected shortages in supplies, i.e. against the growth in the supply deficiency. A mixed situation can often take place, when an expected rise in prices is not sufficient for market balancing. Then the households may expect either a growing or a delcining supply deficiency. Thus, both sources of the anticipated demand have to be taken into account. Consequently, it can be assumed that the anticipated demand is a function of the following three variables: the observable rate of price changes, which reflects the simple hypothesis that some of the consumers expect a similar fall or rise in prices in the future (see [7], pp. 130-132, and [4]); the expected rate of price changes related to the past behavior or prices (which can be defined in terms of rational expectations); and the rate of disequilibrium changes in a given market.

$$DA_t^j = \rho_0^j + \rho_1^j/1-\lambda/TP_t^j + \rho_2^j\lambda/TP_t^j/R_{t+1} + $$

$$+ \rho_3^j\, TDE_t^j + \xi_{A_t}^j \qquad (28)$$

where

$\lambda \in < 0,1 >$.

$/TP_t^j/R_{t+1}$ - is a variable of the rational expectation type,

TP^j - the rate of price changes,

TDE^j - the rate of disequilibrium changes, to be defined below,

ξ_A^j - disturbance term.

Two subsequent components of the total effective demand have been distinguished in connection with the appearance of deficient supply. Let us assume that the supply of a good j is not sufficient to satisfy both the normal and the anticipated demand. As a consequence, an unsatisfied demand appears. We shall call it the initial excess demand (DEi^j).

Should the consumers consider the disequilibrium in this market as a temporary state, the financial means allocated to the purchase of good j will be saved in the form of forced savings (FS^j), and in the next period they will appear with a total demand increased by a further component, namely the postponed demand (DP^j):

$$DP_t^j = \gamma^j \kappa FS_{t-1}^j \quad , \tag{29}$$

where $\gamma^j \in <0,1>$, an unknown coefficient to be estimated.

However, should consumers conclude that the disequilibrium state has a permanent character, they will transfer (partially or totally) the demand from the market of good j into another market. In an extreme case, such a process may bring about a situation in which the excess demand (DE^j), described by formula (4), will be equal to zero, even if the initial excess demand (DEi^j) has been positive. The direction of these demand transfers is determined by the character of the disequilibrium: positive transfers appear in markets characterized by a sufficient supply, or in such markets, which the consumers expect to regain the equilibrium state fairly soon; negative transfers, causing an "outflow" of demand, are expected to appear in such markets, in which the initial unsatisfied demand has occurred. In the system as a whole, i.e. in the total consumer goods market, the sum of the positive and negative transfers is equal to zero.

Summing up, we have distinguished the following components of total demand: notional demand (DN^j), anticipated demand (DA^j), positive transfer demand (DTp^j), and negative transfer demand (DTn^j), which exclude each other, as well as postponed demand (DP^j), which occurs only in periods following those of short supply. Thus, the total effective demand can be written as follows:

$$D_t^j = DN_t^j + DA_t^j + DTp_t^j/1-U_t^j/ + DTn_t^j \cdot U_t^j +$$
$$+ DP_t^j \cdot U_{D_{t-1}}^j + \xi_{D_t}^j \quad , \tag{30}$$

where

$$U_t^j = \begin{cases} 1 \text{ when } DEi^j > 0 \\ 0 \text{ when } DEi^j \leq 0 \end{cases} , \qquad U_{D_t}^j = \begin{cases} 1 \text{ when } DE_t^j > 0 \\ 0 \text{ when } DE_t^j \leq 0 \end{cases} ,$$

and the equation of the transacted quantity (see [14]) can be written as:

$$Q_t^j = DN_t^j + DA_t^j + DTp_t^j/1-U_t^j/ + DTn_t^j \cdot U_t^j +$$

$$+ DP_t^j \cdot U_{D,t-1}^j + DE_t^j \cdot U_{D_t}^j + \xi_{D_t}^j \quad . \qquad (31)$$

5. INDICATORS OF DISEQUILIBRIUM

Formulae (30)-(31) contain unobservable variables, some of which are related to the excess demand (which is also unobservable). As mentioned in Section 2 (compare formula (5)), the latter can be regarded either as a function of variables, changes of which are consequences of disequilibrium intensity (e.g. price changes), or as a function of variables that are the basic determinants of demand and supply. We use the second approach in our study.

By the notion of a disequilibrium indicator we understand a variable, or a collection of variables, by means of which both the sign and the value of an excess demand can be determined (compare formula (5)).[5] To find an operation disequilibrium indicator, identity (4) can be transformed as follows:

$$\frac{DE_t}{S_t} = \frac{D_t}{S_t} - 1 \quad ,$$

$$DE_t = S_t \cdot \Delta(\frac{D_t}{S_t}) + S_t \cdot \frac{DE_{t-1}}{S_{t-1}}$$

$$\cdots$$

$$DE_t = S_t \sum_{u=0}^{n} \Delta(\frac{D_{t-u}}{S_{t-u}}) + S_t \frac{DE_{t-n-1}}{S_{t-n-1}} \qquad (32)$$

$$u = 0,1,\ldots,n \quad .$$

[5] For further information on disequilibrium indicators, see [2], pp. 39-41.

The above decomposition means that the excess demand in period t can be presented as a sum of two components: the sum of values of the first differences of the demand/supply ratios in the successive time periods, and a relative value of the excess demand of the period t-n-1; the two components are subsequently multiplied by a common factor--the value of supply in period t. For the purposes of estimation of (8), the only interesting case occurs when $DE_t > 0$ and, consequently, $D_t/S_t > 1$. Then the sum in equation (32) will be specified for each subsequence of the variable sequence D_t/S_t, whose terms satisfy the relation $D_{t-u}/S_{t-u} > 1$.

If in an initial period the demand was satisfied, i.e., $DE_{t-n-1} \leq 0$, or if the excess demand had the "normal" value according to J. Kornai's concept of normal state of disequilibrium (see [6], p. 21 ff.), then the second term in (32) can be dropped. Formula (32) can be rewritten as:

$$DE_t = S_t \sum_{u=0}^{n} \Delta \frac{D_{t-u}}{S_{t-u}} \quad , \tag{33}$$

if either

$$\frac{D_{t-u}}{S_{t-u}} > 1 + \frac{DE_{t-n-1}}{S_{t-n-1}} \quad ,$$

or, in the second case,

$$\frac{D_{t-n}}{S_{t-n}} > \frac{D_{t-n-1}}{S_{t-n-1}} = 1 + \varepsilon \quad ,$$

where ε is the "normal" value of the relative excess demand.

Thus, the above formula expresses the changes of an excess demand, measured against a state of disequilibrium, which is considered as normal. However, the exact values of demand and supply in the above formula are unknown. To estimate an excess demand they can be approximated by the respective proxy variables $\tilde{D}$ and $\tilde{S}$. We are first of all concerned with the variables which are the main factors affecting demand (incomes) and supply (deliveries), as their changes must be closely correlated with the changes of demand and supply. The value of ε can be accepted at a minimum historically observed level of the ratio D_t/S_t, which is, in a sense, a reversal of the so-called peak method. Thus formula (33) can be approximated by

$$DE_t = \alpha\tilde{S}_t\left[\frac{\tilde{D}_t}{\tilde{S}_t} - \min\left\{\frac{\tilde{D}_u}{\tilde{S}_u}\right\}\right] \tag{34}$$

where

$$\tilde{\varepsilon} = \min \frac{\tilde{D}_u}{\tilde{S}_u} - 1 \quad ;$$

and

$$\sum_{u=0}^{n} \Delta(\tilde{D}_{t-u}/\tilde{S}_{t-u}) = \tilde{D}_t/\tilde{S}_t - \tilde{D}_{t-n-1}/\tilde{S}_{t-n-1} \quad ,$$

$$\tilde{D}_{t-n-1}/\tilde{S}_{t-n-1} = \min\{\tilde{D}_u/\tilde{S}_u\} \quad .$$

The variable $\tilde{S}$ can either be taken as the supply directed to the socialized trade, i.e. market supplies, or as the sales in socialized trade, if we only assume that outside the socialized trade prices play an equilibrating role and the unsatisfied demand does not appear there. The purchasing fund described by formula (11)[6] can be treated as a determinant of the consumer demand ($\tilde{D}$).

6. A SYSTEM OF DEMAND FUNCTIONS

In this section we shall discuss the conditions that must be met so that the demand functions for the commodity groups defined above satisfy the adding-up criterion.

Assume that the consumers will make a partial neuralization of forced savings, and that, for example, this part of the savings will equal 6% of the FS value ($\eta = 0.06$), which would be an approximate reflection of their marginal propensity to save. The remaining part can be used to finance the postponed demand. However, it may occur that even if some needs

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[6] Paper [14] presents a discussion on the use of socialized trade stocks as a disequilibrium indicator, together with an analytical introduction of the appropriate relations.

have not been satisfied in the previous period, they will not cause an increase in the consumer demand in the current period. This refers, for example, to food, services, and many other, non-durable goods. Formally, this means that the coefficient γ^j in formula (29) is different from 1, which causes

$$\kappa FS_{t-1} - \kappa \sum_{l} \gamma^l FS^l_{t-1} > 0 \quad , \tag{35}$$

where

$$FS_{t-1} = \sum_{l} FS^l_{t-1}$$

and summing takes place in all l markets.

Let us first assume that the value obtained as a result of the subtraction of postponed demand from total adjusted forced savings as described by (35) has been considered by the consumers as an extraneous income. Then we have

$$DN^j_t = \beta^j_0 + \beta^j_1 Y_t + \beta^j_4 (FS_{t-1} - \kappa \sum_{l} \gamma^l FS^l_{t-1}) V_t - \\ - \beta^j_2 PC^j_t + \beta^j_3 + \xi^j_{Nt} \quad , \tag{36}$$

where

$$V_t = \begin{cases} 1 \text{ if } \kappa FS_{t-1} - \kappa \sum_{l} \gamma^l FS^l_{t-1} > 0 \\ 0 \text{ in all other cases} \end{cases}$$

Using formulae (29), (30), and (36), we can write for the total effective demand

$$D_t^j = \beta_0^j + \beta_1^j Y_t + \beta_4^j \kappa FS_{t-1} \cdot V_t - \beta_2^j PC_t^j + \beta_3^j t +$$

$$+ DA_t^j + DTp_t^j /1 - U_t^j/ + DTn_t^j \cdot U_t^j -$$

$$- \beta_4^j \kappa \sum_{1 \neq j} \gamma^1 FS_{t-1}^1 \cdot V_t + /1 - \beta_4^j \cdot V_t/$$

$$\gamma^j \kappa FS_{t-1}^j + \xi_{Dt}^j \quad . \qquad (37)$$

The first two components show the dependence on income (current and from forced savings), whereas the last two terms indicate, respectively, the postponed demand for other commodities (1 ≠ j), and for the commodity j in question, which must be subtracted from foreced savings.

To determine under what conditions the budget constraint will be satisfied, we shall aggregate the above function with respect ot particular markets. Then we have

$$\sum_1 D_t^1 = \sum_1 \beta_0^1 + Y_t \sum_1 \beta_1^1 + \kappa FS_{t-1} \sum_1 \beta_4^1 \cdot V_t -$$

$$- \sum_1 \beta_2^1 PC_t^1 + t \sum_1 \beta_3^1 + \sum_1 DA_t^1 + \sum_1 DTp_t^1 /1 - U_t^1/ +$$

$$+ \sum_1 DTn_t^1 \cdot U_t^1 - \kappa \sum_1 \beta_4^1 \sum_1 \gamma^1 FS_{t-1}^1 \cdot V_t +$$

$$+ \kappa \sum_1 \gamma^1 FS_{t-1}^1 + \sum_1 \xi_{Dt}^1 \quad . \qquad (38)$$

Under the assumptions accepted so far, and taking into account the definitions of the demand transfers, we have

$$\sum_1 DTp_t^1 /1 - U_t^1/ + \sum_1 DTn_t^1 \cdot U_t^1 = 0 \quad . \qquad (39)$$

From the arguments presented above (compare equations (23) and (26)) it can be observed that the following adding-up conditions should be satisfied with regard to the marginal propensities to consume:

$$\sum_{l} \beta_1^l = 1 - \Delta NS_t^*/Y_t \quad , \tag{40}$$

and

$$\sum_{l} \beta_4^l = 1 \quad . \tag{41}$$

The assumption of the symmetry of price effects leads to the conclusion that

$$\sum_{l} \beta_2^l \, PC_t^l = 0 \quad . \tag{42}$$

If we accept an additional assumption that

$$\sum_{l} \beta_0^l + t \sum_{l} \beta_3^l = 0 \quad , \tag{43}$$

then we can rewrite formula (38) as follows:

$$D_t = Y_t - \Delta NS_t^* + \kappa FS_{t-1} + DA_t \quad , \tag{44}$$

where

$$D_t = \sum_{l} D_t^l \quad ,$$

$$DA_t = \sum_{l} DA_t^l \quad .$$

On the other hand, formulae (11) and (23) give us

$$D_t = Y_t - \Delta NS_t^* + \kappa FS_{t-1} + \xi NS_{t-1} \quad , \tag{45}$$

which finally results in the conclusion that the budget constraint will be satisfied if

$$DA_t = \xi NS_{t-1} \quad . \tag{46}$$

Let us now suppose that the remaining part of forced savings, i.e. the difference defined by formula (35), first of all covers the anticipated demand, and only the part that remains afterwards, is treated by consumers as an extra income; then we have

$$D_t^j = \beta_0^j + \beta_1^j Y_t + \beta_4^j \kappa FS_{t-1} \cdot Z_t - \beta_2^j PC_t^j + \beta_3^j t +$$

$$+ /1 - \beta_4^j \cdot Z_t/ DA_t^j + \beta_4^j \sum_{l \neq j} DA_t^l Z_t +$$

$$+ DTp_t^j /1 - U_t^j/ + DTn_t^j U_t^j + \beta_4^j \kappa \sum_{l \neq j} \gamma^l FS_{t-1}^l \cdot$$

$$\cdot Z_t + /1 - \beta_4^j \cdot Z_t/ \gamma^j \kappa FS_{t-1}^j + \xi_{Dt}^i \quad , \qquad (47)$$

where

$$Z_t = \begin{cases} 1 \text{ if } \kappa FS_{t-1} - \kappa \sum_l \gamma^l FS_t^l - \sum_l DA_t^l > 0 \\ 0 \text{ in all other cases} \end{cases}$$

Having aggregated the demand function according to the above formulation and accepting the assumptions described by formulae (39)-(43), we have

$$D_t = Y_t - \Delta NS_t^* + \kappa FS_{t-1} \cdot Z_t + /1 - Z_t/ DA_t +$$

$$+ /1-Z_t/ \kappa \sum_l \gamma^l FS_{t-1}^l \quad . \qquad (48)$$

Thus, if $Z_t = 1$, then we conclude on the basis of formulae (45) and (48) that $\xi = 0$; if $Z_t = 0$, then

$$DA_t = \kappa FS_{t-1} - \kappa \sum_l \gamma^l FS_{t-1} + \xi NS_{t-1} \quad , \qquad (49)$$

which simply means that the anticipated demand is financed from both forced and voluntary savings after allowing for postponed demand.

7. AN ATTEMPT OF EMPIRICAL VERIFICATION

The system of demand equations described by formula (37) has been estimated. However, some further simplifications need to be included.

The first of them consists in the elimination of demand transfers between the groups of commodities--we have made the hypothesis that they occur inside the analyzed aggregates. The second simplification is based on taking no account of the anticipated demand and on the assumption that γ^1 are equal to zero. As a result, the functions of (real) expenditures were estimated in the following form (compare formulae (6), (34), and (37)):

$$Q_t^j = \beta_0^j + \beta_1^j Y_t + \beta_4^j \kappa FS_{t-1} - \beta_2^j PC_t^j + \beta_3^j t - \\ - \alpha^j I_t^j + \xi_{Dt}^j \quad , \qquad (50)$$

where

I^j - is the applied disequilibrium indicator (see forumula (34)).

The estimates were performed for seven groups of goods: food (CBZ), alcoholic beverages (CA), tobacco and cigarettes (CW), textiles, clothing, and shoes (CT), durable goods (CR), other, perishable goods (CV), and services (COO). The sample covered the period from 1963 to 1982, and all variables were expressed in constant prices of the year 1982[7]. The estimation was made by the least squares method (OLS)[8].

The preliminary results have already pointed to the necessity of eliminating the time variable, because of the collinearity with income (Y_t).

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7) The choice of the sample period, the type of marking, and the disaggregation of goods are connected with the fact that the equation system has been included in the macro-model of the national economy of Poland, W5, which is being built at the Institute of Econometrics and Statistics of Lodz University, under the guidance of Prof. Wladyslaw Welfe; this has given rise to some additional requirements and limitations we have had to accept.

8) The modifications of OLS and TSLS methods, and the results of the Monte-Carlo experiments are discussed in [2]. They will be applied in our next study.

In addition, an analysis of the rate of real income changes (Y) demonstrated the fast acceleration in the years 1972-1975 (see [12]). For that reason we have introduced an appropriate interaction variable ($Y_t \cdot U7275$) to reduce the bias in the estimator of the marginal propensity to save. The introduced changes modify function (50) in the following way:

$$Q_t^j = \beta_0^j + \beta_1^j Y_t - \beta^{*j}_1 Y_t \cdot U7275 + \beta_4^j \kappa FS_{t-1} - - \beta_2^j PC_t^j - \alpha^j I_t^j + \xi_{Dt}^j \quad , \qquad (51)$$

where

U7275 - is a dummy variable which, in 1972-1975, had the value = 1, otherwise 0.

Disequilibrium indicators (I^j) have been built according to formula (34). As the supply determinant ($\tilde{S}^j$) we have accepted the quantity of goods belonging to a given group, purchased from real personal income, assuming that the excess supply is set aside in stock, and that in the sample period only supply is realized. As the demand determinant we have used the sum of real, personal monetary income and the real value of the inflationary gap represented by forced savings at the end of the previous time period. However, because of differences in the marginal propensities to consume among particular groups of goods, and their non-stability during the sample period, the variable determined in this way has been modified by raising it to the power equal to the income elasticity of demand for each of the examined groups of goods. The elasticities have been derived from earlier studies (see [11]; they are presented in Table 1.

The value of the inflationary gap has been obtained by estimation of the following function of total savings:

$$GS_t = \Psi_0 + \Psi_1 \overline{Y}_t + \Psi_1^* \overline{Y}_t \cdot U7275 + \Psi_2 \Delta\overline{Y}_t + + \Psi_3 \Delta\overline{I}_t + \xi_t \quad , \qquad (52)$$

where

all values are expressed in current prices, marked by a dash above the symbols of the variables,

Table 1. Income elasticities of demand

Variable	Food CBZ	Alcoholic beverages CA	Tobacco, cigarettes CW	Textiles, clothing, & shoes CT	Durable goods CR	Other non-durable goods CW	Services COO
Income elasticity	0.779	1.252	0.483	0.775	1.510	1.285	1.010

I - is an indicator built according to formula (34); here we have accepted $\overline{Y}_t$ for $\overline{D}_t$; $\tilde{S}_t$ is the consumption of goods and services financed from personal monetary income,

ξ - disturbance term.

In addition we have assumed with regard to function (51) that $\kappa \cong 1$, and $\beta_4^j = \beta_1^j$, which permits us to rewrite the function in the form:

$$Q_t^j = \beta_0^j + \beta_1^j \,/Y_t + FS_{t-1}/ - \beta_1^{*j}\, Y_t \cdot U7275 -$$

$$- \beta_2^j\, PC_t - \alpha^j\, I_t^j + \xi_{DT}^j \quad . \tag{53}$$

Table 2 contains the results of estimation of the above function.

It is necessary to stress the significance of all explanatory variables (except for the price variable in the equation for durable goods). An additional variable has been introduced in this function, namely the amount of voluntary savings which are partially allocated by the consumers to the purchase of durable goods. In two cases (equations CT and COO), the interaction variable proved to be insignificant, which led to its omission. The disequilibrium indicator we have used has not shown any signs of excess demand in the tobacco market. However, it proved to be necessary to distinguish--by means of dummy variables--the effects of anticipated purchases in the year of rationing of tobacco and cigarettes.

Figure 1 shows the dynamics of effective demand and the purchased quantities for particular goods.

The empirical results obtained should, although they seem satisfactory, be treated as the first approximation of the proposed functions of demand (formulae (37) and (47). Further studies follow the direction of quantification of the remaining components of demand; their purpose is to build a complete model of demand.

Table 2. Estimation of demand functions

Explanatory variable	Con-stant	$Y_t + FS_{t-1}$	$Y_t \cdot U7275$	PC_t^j	I^j	Others			R^2	D - W
						U80	U81	NS_{t-1}		
Food (CBZ)	424.23 (44.05)	0.290 (71.23)	-0.007 (2.43)	-250.0*	-1,750.0 (9.35)	-	-	-	0.998	1.718
Alcoholic beverages (CA)	44.88 (1.17)	0.170 (51.01)	-0,012 (6.11)	-162.830 (3.02)	-0.016 (12.72)	-	-	-	0,997	1.235
Tobacco, cigarettes (CW)	36.00 (7.14)	0.015 (17.94)	-0.002 (3.87)	-13.420 (2.64)	-	5.211 (1.82)	12.44 (4.26)	-	0.983	1.004
Testiles, clothing & shoes (CT)	74.740 (6.85)	0.085 (76.89)	-	-7.652 (1.32)	-0.695 (34.10)	- -	- -	-	0.999	1.522
Durable goods (CR)	-8.800 (0.0)	0.091 (3.35)	-0.010 (5.47)	-40.33 (0.81)	-0.002 (5.19)	-	-	0.045 (1.12)	0.996	2.304
Other, non-durable goods (CV)	68.940 (2.06)	0.195 (124.86)	-0.020 (16.176)	-26.76 (1.01)	-0.018 (12.12)	-	-	-	0.999	1.340
Services (COO)	-0.734 (1.81)	0.037 (170.21)	-	-0.214 (1.00)	-0.034 (51.90)	-	-	-	1.000	2.353

Notes: *Parameter value ascribed a priori.
Under the parameter estimates there are, in parentheses, the values of t-Student's statistics.
Symbols U80, U81 denote dummy variables whose value is, in the distinguished years, equal to 1, otherwise it is 0.
NS denotes the amount of voluntary savings.

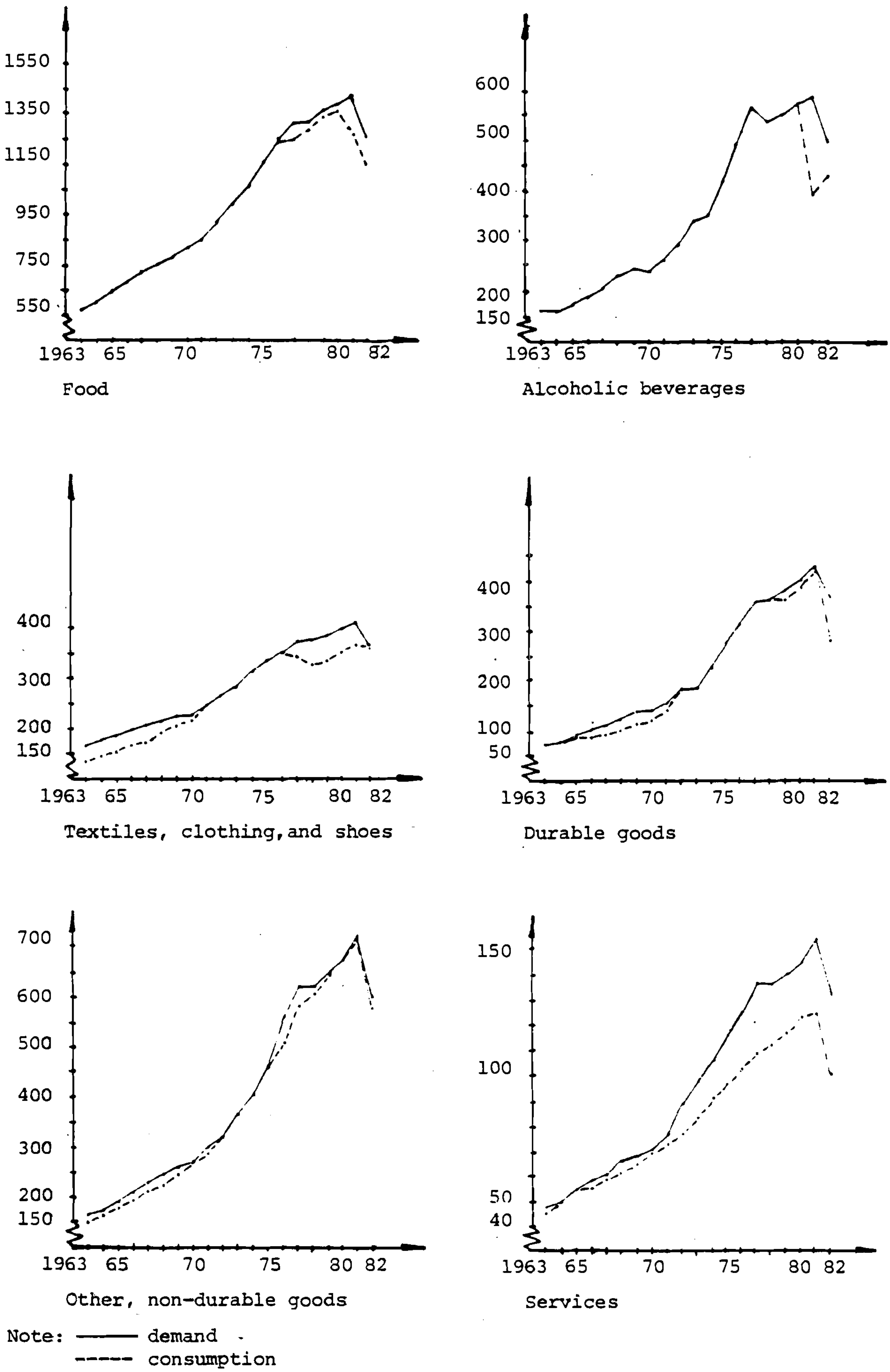

Figure 1. Demand and consumption curves.

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