MULTILEVEL COMPUTER MODEL OF
WORLD DEVELOPMENT SYSTEM

Comments on the Implementation
of the Food Submodel and its
Driving Forces

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FOREWORD

This paper is intended to give some critical comments on the Food Submodel carried out under M. Mesarovic and E. Pestel ("Multilevel Computer Model of World Development Systems").

In the first part of this report the technical problems are dealt with, which were met during the implementation of this model, due to misprints, mistakes and shortcomings in the relevant publication [2].

In the second part, the extensive list of equations pertaining to the Food Submodel is reduced in an appropriate manner to gain some insight into the model behavior.
I.  PREFACE

This paper deals with the Food Submodel worked out in the framework of the "Multilevel Computer Model of World Development System" under the leadership of M. Mesarovic and E. Pestel.

In May 1974 the world model under consideration was presented at a seminar held by IIASA to scientists of various disciplines. In January 1975 the proceedings of this symposium were available (6 volumes; SP-74-1 to SP-74-6). In these volumes the various submodels pertaining to the world model and the underlying methodology are described in some detail. In accordance with IIASA's role as a switchboard to facilitate the international flow of scientific information, I have been concerned with implementing the various submodels at IIASA for initiating further discussion and investigation.

At the present state of the M.P. World Modelling Project, the Food Submodel seemed to be of special interest since it is the only module so far where a linkage between different submodels has been tried, i.e. it consists of a population model, an economic submodel, a land use part, a model for food production and distribution, and a pricing mechanism.

Although the verbal description in [1] and [2] seemed quite promising it turned out during the actual implementation that there were quite a lot of mistakes, shortcomings, and discrepancies present in the more mathematical part of [2].

In section II of this paper these deficiencies are pointed out and corrected in an appropriate manner. A "Revised List of Equations" is given that also eliminates those mistakes that have not been dealt with explicitly as they seemed to be merely due to misprints.

In Chapter III this revised list is reduced by eliminating some of the variables involved in the model and by leaving out any equation that does not contribute to the model dynamics.
Finally, the "Reduced List of Equations" resulting from III is used for some analysis of the Food Submodel in Chapter IV. The driving forces of the model are revealed and some comments on the actual model performance for the region of South East Asia are given.

II. TECHNICAL INADEQUACIES

This section mainly deals with shortcomings and instances that reveal a certain carelessness in the reports describing the M.P. Food Submodel. It is not intended to be a critical evaluation of this submodel but merely to point out the difficulties that had to be met during the implementation using the list of equations given in [2].

A. Population Submodel

In spite of the author's assertion in [2] (p. B557) that the Population Submodel used there is described in the respective report by Oehmen and Paul [4], one can observe only similarities between the model given in [4] and that actually used in [2]. The identification of the analogies is made even more difficult because of the partly different notation that is used in [4].

The first mistakes within the population sector appear in equations (1.3) and (1.4) on page B574. Isolating every particular age group AP(I), I=1,...,85, equation (1.3) does not account for the fact that the number of I - year-olds of year T + 1 derives from the number of people I - 1 years of age in year T that did not die during year T. In equation (1.4) the dead among the 85 - year-olds have not been considered.

The corrected equations are the following:

\[
\begin{align*}
AP(I) &= AP(I - 1) - DN(I - 1) , & I=1,\ldots,85 \\
AP(86) &= AP(86) - DN(86) + AP(85) - DN(85) .
\end{align*}
\]

Besides, it has been forgotten to update the array PPCSAV to account for the time lag between a decrease of protein and its impact on mortality.
In equation (4.7), $E(I)$ that is defined immediately before in (4.6) is to be used instead of $E$. Another detail that might be irritating is the definition given for PROPCN on page B576. There it says:

"PROPCN Multiplier denoting the amount by which age specific mortality is increased due to protein deficiencies"

which is not the case. The appropriate definition should be:

"PROPCN Daily per capita protein consumption of year $T - TIMLAG$"

Finally, in equation (5.3), the variable DEATHS is used without having been defined before. It can be inferred, however, that what is meant is:

$$DEATHS = \sum_{I=0}^{86} DN(I)$$

The population numbers by age-group of 1975 and the figures for age-specific fertility and mortality that are not reported have been calculated using the Population Model by Oehmen and Paul.

B. Economic Submodel

Considering the economic sector one may start with the same criticism as has been expressed before dealing with the population sector. There is a report given on "Specification of Structure for a Macro Economic World Model" [5] and a paper labelled "Computer Implementation of Micro Economic World Model" [6]. In [5] recommendations are given for disaggregating the Macro Economic Model into different economic sectors. In [6] a few details for building a Micro Economic Model are stated.
Turning to the two sector economic model used for food supply analysis, however, it seems to me that only the very basic ideas outlined in the reports mentioned above have been realized in the actual set of equations. To briefly mention some of the deficiencies that may be found at first sight: no labor market has been considered, no dynamic mechanisms have been built to cope with an insufficient food supply. In fact, the only tool to influence the economic system is to change IAKS, which has to be done exogenously, thus initiating additional investment to the agricultural sector.

But returning to the technical problems, one of the shortcomings that must be mentioned here is that some of the initial values and parameters necessary to run the model have been forgotten when it comes to giving figures (pp. B607, B608). The figures for the following items have been left out:

\[ UAFK, \text{SYSYNA}_1975, K1 \].

\[ UAFK \text{ and SYSYNA could easily be calculated from the computer results given on pp. B661 - B685, while K1 is a scenario variable and may be chosen in different ways.} \]

According to the definitions of QA and QNA as capital per output ratios, equations (1.2) and (1.5) do not hold but must be replaced by

\[ YAX = \frac{KA}{QA} \]

and

\[ YNA = \frac{KNA}{QNA} \]

The authors' carelessness also becomes evident through their inconsistent use of time subscripts. Such subscripts recur at various places throughout the report in spite of a footnote announcing that no such time subscripts will be given as all computations are annual.

Annoying as this may be even more serious deficiencies can be found. From the presentation on page B582, one erroneously gets the impression that the matrix AI used in
equation (2.1) and (2.2) and the inverse of the input/output matrix A are identical. The authors have failed to say explicitly that

$$ A^{-1} = (I - A)^{-1} $$

I denoting the 2 x 2 identity matrix.

Furthermore, equation (3.7) is used to compute investment in non-agricultural sector. However, since IA accounts only for a part (depending on Kl) of the investment shifted to the agricultural sector this equation does not fit.

The appropriate way to calculate INA would be

$$ INA = I - IA - (1 - Kl) \cdot IAS $$

By the way, the definition given for IAS on p. B584 should be the other way round, namely, IAS is the amount of investment shifted from non-agricultural sector to agricultural sector.

C. Land Use Submodel

The major mistakes in this section appear in equations (3.3) and (3.8). In (3.3), the annual land withdrawal is calculated to be

$$ TLAW = (BABIES - DN(0)) $$

which is obviously nonsense. But even the completed equation

$$ TLAW = (BABIES - DN(0)) \cdot TLWM \cdot TLWPCB $$

which the authors probably had in mind does not quite fit since the annual land withdrawal is not really a function of babies reaching an age of 6 months but rather a function of actual population growth which may be entirely different from the quantity used above. Nevertheless one should not pay too much attention to this equation since land withdrawal has practically no impact on the dynamics of the model.
In (3.8) the fraction of remaining cultivable land looks like the following:

\[ F_{CLR} = \frac{CL}{TLM \cdot (CLM/TLM) - CLW} \]

On page B619 in the section describing the parameters and functional relationships a different version is given:

\[ F_{CLR} = \frac{CLR}{CLM} \]

Both of them are wrong; a version that would make sense is:

\[ F_{CLR} = \frac{CLR}{CLM - CLW} \]

which I used in the actual implementation. Besides, FCLR is not the fraction of cultivated land remaining as it says on p. B588, but that of cultivable land remaining.

In addition to the above mentioned mistakes a mathematical formulation of CLWGR (cultivated grain land that is withdrawn), which is used in equation (1.1) of the land use submodel to compute the amount of available grain land, has been left out.

In our own implementation I used

\[ CLWGR = CLAW \cdot CLGR/CL \]

in accordance with the equation given for newly developed grain land, namely

\[ CLDGR = CLD \cdot CLGR/CL \]

In return for the missing equation concerning CLWGR, (4.2) defines a variable CLDNG (non-grain land developed) as
\[
\text{CLDNG} = \text{CLD} \cdot \frac{\text{CLNG}}{\text{CL}},
\]

although this variable does not occur in any other equation of the given set nor can it reasonably be used as an indicator. Moreover this equation is not only useless in this place but is also at variance with actual computations of the submodel. Since the amount of non-grain land is determined as a linear function of the cultivated grain land \( \text{CLGR} \):

\[
\text{CLNG} = \text{TA} + \text{TB} \cdot \text{CLGR},
\]

the amount of newly developed non-grain land that is implicitly used can accordingly be described as

\[
\text{CLDNG} = \text{TB} \cdot \text{CLDGR},
\]

which is different from (4.2).

**D. Food Production Submodel**

Again mistakes of varying severity appear within this section. A mistake that I would classify to be a minor one in the sense that it is quite easy to be detected appears in equation (1.5) defining \( \text{ZPHG} \) (per hectare use of fertilizers and related productive factors), where a factor 1000 that has to appear due to the dimensions of the variables used has been omitted. So, (1.5) should be:

\[
\text{ZPHG} = 1000 \cdot \text{TPF} \cdot \frac{\text{GZPHK}}{\text{CLGR}}.
\]

Subsequently \( \text{TPF} \) (total use of fertilizer and related productive factors) is given in (1.6) as

\[
\text{TPF} = \frac{\text{UAF}}{\text{PXPF}}.
\]

Although this relationship makes sense it does not quite
match the authors' ideas outlined in the verbal description of the model and the flow charts shown there. Since UAF as it is calculated in (2.6) of the economic model does not account for additional investment shifted to the agricultural sector for increased use of fertilizer and related productive factors the adjusted value SUAF (equation (3.6) of the economic model) has to be used instead of UAF.

The most crucial point with regard to the implementation of the model in this section, however, is the definition of FA (saturation level for grain production) in (1.4)

\[
FA = PMCI + PTFC,
\]

which is not at all clear since only incomplete and inexact verbal descriptions of PMCI (productivity coefficient from infrastructure) and PTFC (productivity coefficient from capital investment) are given on pp. B613 - B615. Apart from this improper mathematical treatment there is a kind of absurdity involved in this equation. If one carefully checks the output of the standard computer run for the region of South East Asia given at the end of [3] (pp B661 - B685) one will recognize that the reported values of PMCI and PTFC do not sum up to the values given for FA, but that obviously the following equation has been used

\[
FA = PMCI + PTFC + 1.5.
\]

In order to circumvent the above mentioned lack of information concerning the evaluation of FA I used a log-linear approach as proposed in the report and performed a least squares fitting of the values reported for PMCI and PTFC on page B680. The results I obtained are the following:
A minor mistake I found is the wrong use of subscripts in equations (1.2.1) and (2.2.2). From the verbal descriptions given for \( SLV(J), J = 1, \ldots, 9 \), \( SLVMK(J), J = 1, \ldots, 12 \), and \( SLVP(J), J = 1, \ldots, 12 \), the right subscripts can be deduced as

\[
SLVP(J) = SLV(J) \cdot \frac{SLVMK(J)}{1000}, \quad J = 1, \ldots, 8,
\]
\[
SLVP(J) = SLV(9) \cdot \frac{SLVMK(J)}{1000}, \quad J = 9, \ldots, 12.
\]

Concerning the remaining three sections of the food production submodel the following remarks seem to be appropriate.

In the "Fish Production" sector figures are given neither for \( FWMM \) (maximum possible catch of marine fish) nor for \( UFWPM \) (maximum area of land under pond fish culture).

In the section "Distribution of Food to Alternative Uses," an array \( FGP(J), J = 1, \ldots, 26 \) defined as "Gross Regional Food Production by Category," is used without being assigned values anywhere in the set of equations. Given the verbal definition of \( FGP \) one can infer the following specification to be the most likely one:

\[
FGP(J) = SLVP(J), \quad J = 1, \ldots, 12
\]
\[
FGP(13) = FWT, \quad J = 13
\]
\[
FGP(J) = PLGP(J), \quad J = 14, \ldots, 26.
\]

Calculating "Gross Regional Food Supply, by Category" \( FTS(J), J = 1, \ldots, 26 \), withdrawal for seed has to be accounted for and thus (4.4) should be

\[
FTS(J) = FGP(J) \cdot (1.0 - SPFTK(J)), \quad J = 1, \ldots, 26.
\]
Last but not least there is a mistake in the section "Regional Food Supply Compared with Population Needs" that is worth mentioning. In equation (1.6) regional animal protein per capita (PTAPCR) is defined using calories (VCLPCR) instead of protein (VPTPCR). The corrected version of (1.6) is

$$PTAPCR = \sum_{I=1}^{13} VPTPCR$$

E. Pricing Mechanism

Although there are only a few equations pertaining to the pricing mechanism some inadequacies can still be found. First of all there is a price coefficient PXK. Since there is no equation given with PXK on its left side one assumes PXK to remain constant during the model computations. Checking the computer results of pp. B661 - B685 it turns out, however, that PXK increases with an annual growth rate of 2.5%. To serve this purpose the following equation has to be added

$$PXK = PXK \cdot (1.0 + RPXK)$$

where RPXK is the annual growth rate of PXK and may be treated as a scenario variable. In our computations it has been assigned a value of 0.025 in accordance with the observations mentioned above.

Furthermore, it is not clear why equation (2.2) has been reported within this section since an identical equation is given in the food production submodel (equation (1.6)).

The dubious character of the definition of FCLR has already been dealt with. An additional problem arises from equation (2.4). In (2.4) per hectare capital cost for land development is defined as a function of FCLR. On pages B621 and B622 the corresponding curves for South Asia and North America are
graphed. While the function chosen for South East Asia seems quite reasonable the one for North America appears to make no sense at first sight. However, the plot given might quite easily be explained by the fact that the scale actually used on the abscissa is for $1 - FCLR$ and not for $FCLR$.

F. Revised List of Equations

Besides the mistakes I have pointed out so far there is quite an extensive list of errors that I have not dealt with simply because these mistakes seemed to be due to misprints in the report. On the following pages a "Revised List of Equations" is given. In this list all mistakes that appeared to me have been eliminated. Time subscripts are omitted in accordance with a footnote given in [2] on page B573. To facilitate a further citing of the equations a different and unique numbering has been chosen to avoid equations having the same number.
Revised List of Equations

A. Population Submodel

(1) Total Population

\[
\text{POP} = \sum_{I=0}^{86} \text{AP}(I) \quad (1.1)
\]

\[
\text{AP}(0) = \text{BABIES} - \text{DN}(0) \quad (1.2)
\]

\[
\text{AP}(I) = \text{AP}(I - 1) - \text{DN}(I - 1) \quad I = 1, \ldots, 85 \quad (1.3)
\]

\[
\text{AP}(86) = \text{AP}(85) - \text{DN}(85) + \text{AP}(86) - \text{DN}(86) \quad (1.4)
\]

(2) Births

\[
\text{BABIES} = \text{FERT} \cdot \sum_{I=1}^{86} (\text{AP}(I) \cdot \text{AF}(I)) \quad (1.5)
\]

(3) Deaths

\[
\text{DN}(0) = \text{BABIES} \cdot \text{AMPF}(0) \cdot \text{AM}(0) \cdot 0.5 \cdot \text{MORT} \quad (1.6)
\]

\[
\text{DN}(I) = \text{AP}(I) \cdot \text{AMPF}(I) \cdot \text{AM}(I) \cdot \text{MORT} \quad I = 1, \ldots, 85 \quad (1.7)
\]

\[
\text{DN}(86) = \text{AP}(86) \cdot \text{AMPF}(86) \cdot \text{AM}(86) \cdot \text{MORT} \quad (1.8)
\]

(4) Effects of Protein Starvation on Mortality

\[
\text{PROPCI} = \text{PTPCR} \cdot \text{PRODST} \cdot 1000/365 \quad (1.9)
\]

\[
\text{I} = \text{TIMLAG} + 0.5 \quad (1.0)
\]

\[
\text{PPCSAV}(J) = \text{PPCSAV}(J + 1) \quad J = 1, \ldots, I-1 \quad (1.11.a)
\]

\[
\text{PPCSAV}(I) = \text{PROPCI} \quad (1.11.b)
\]

\[
\text{PROPCN} = \text{PPCSAV}(1) \quad (1.12)
\]

\[
\text{PROFAC} = (\text{PRONOR} - \text{PROO})/(\text{PROPCN} - \text{PROO}) - 1.0 \quad (1.13)
\]
\[ E(I) = (E_O - E_U) \cdot \exp(-I/\tau) + E_U \quad I = 1,\ldots,85 \quad (1.14) \]

\[ AMPF(I) = PROFAC \cdot E(I) + 1 \quad I = 1,\ldots,85 \quad (1.15) \]

\[ AMPF(0) = PROFAC \cdot E_O + 1 \quad (1.16) \]

\[ AMPF(86) = PROFAC \cdot E_U + 1 \quad (1.17) \]

(5) Indicators

\[ DCHLD = \sum_{I=0}^{15} D_N(I) \quad (1.18) \]

\[ DEATHS = \sum_{I=0}^{86} D_N(I) \quad (1.19) \]

\[ CBR = \frac{Babies}{Pop} \quad (1.20) \]

\[ CDR = \frac{Deaths}{Pop} \quad (1.21) \]

\[ POPGR = \frac{(Babies - Deaths)}{Pop} \quad (1.22) \]

B. Economic Submodel

(1) Production

\[ K_A = K_A \cdot (1 - D_A) + I_A \quad (2.1) \]

\[ YAX = \frac{K_A}{Q_A} \quad (2.2) \]

\[ KDA = K_A \cdot D_A \quad (2.3) \]

\[ KNA = KNA \cdot (1 - D_NA) + I_NA \quad (2.4) \]

\[ YNA = \frac{KNA}{QNA} \quad (2.5) \]

\[ KDNA = KNA \cdot D_NA \quad (2.6) \]

\[ Y = YNA + YA \quad (2.7) \]

(2) Gross Output and Intermediate Demand

\[ Z(1) = AI(1,1) \cdot Y_A + AI(2,1) \cdot YNA \quad (2.8) \]

\[ Z(2) = AI(1,2) \cdot Y_A + AI(2,2) \cdot YNA \quad (2.9) \]
\begin{align*}
U(1) &= A(1,1) \cdot Z(1) + A(2,1) \cdot Z(2) \\
U(2) &= A(1,2) \cdot Z(1) + A(2,2) \cdot Z(2) \\
UA &= A(2,1) \cdot Z(2) \\
UAF &= UA \cdot UAFK
\end{align*}

(3) Investment
\begin{align*}
I &= GI \cdot YNA \cdot SYSYNA \\
SYSYNA &= Y/YNA \\
IAS &= (IAKS - IAK) \cdot I \\
IAS &= \text{MAX}(IAS, 0.0) \\
IA &= IAK \cdot I + K1 \cdot IAS \\
SUAF &= UAF + (1 - K1) \cdot IAS \\
INA &= I - IA - (1 - K1) \cdot IAS \\
IR &= I \cdot IRK \\
IMN &= I - IR \\
IAP &= IAPK \cdot IA \\
IALV &= IALVK \cdot IA \\
IALD &= IA - IAP - IALV
\end{align*}

(4) Consumption, Governmental Expenditure, Imports
\begin{align*}
C &= GC \cdot Y \\
G &= GG \cdot Y \\
M &= GM \cdot Y \\
MA &= MAK \cdot M
\end{align*}
IF(IMN - M + MA) 10,10,11

11 MI = M - MA

GO TO 12

10 MI = IMN

12 CONTINUE

MC = M - MA - MI

MC = MAX(MC, 0.0)

C. Land Use Submodel

(1) Cultivated Land

CLGR = CLGR + CLDGR - CLWGR (3.2)

CLNG = TA + TB • CLGR (3.3)

CL = CLGR + CLNG (3.4)

(2) Livestock Land

GL = GLM - GLW (3.5)

TLLS = GL + CLR (3.6)

CLR = TLM • (CLM/TLM) - CL - CLW (3.7)

(3) Withdrawal of Land for Urban and Economic Development

TLWR = TLW/TLM (3.8)

TLWM = TLWMF(TLWR) (3.9)

* TLA W = (BABIES - DN(0)) • TLWM • TLWPCB (3.10)

CLAW = TLA W • (CLM/TLM) (3.11)

CLW = TLW • (CLM/TLM) (3.12)

GLW = TLW - CLW (3.13)

*Labels 10 and 11 had to be interchanged in the original list of equations in order to make sense.
TLW = TLW + TLAW  \hspace{1cm} (3.14)  
FCLR = CLR / (TLM \cdot (CLM/TLM) - CLW)  \hspace{1cm} (3.15)

(4) Development of Cultivated Land

CLD = IALD / KCLDH  \hspace{1cm} (3.16)  
CLDNG = CLD \cdot CLNG/CL  \hspace{1cm} (3.17)  
CLDGR = CLD \cdot CLGR/CL  \hspace{1cm} (3.18)  
CLWGR = CLAW \cdot CLGR/CL  \hspace{1cm} (3.19)

D. Food Production Submodel

(1) Crop Production

PMCI = PMCIF(YNAPC)  \hspace{1cm} (4.1)  
KAPH = KA / CL  \hspace{1cm} (4.2)  
PTFC = PTFCF(KAPH)  \hspace{1cm} (4.3)  
FA = PMCI + PTFC + 1.5  \hspace{1cm} (4.4)  
ZPHG = TPF \cdot GZPHK \cdot 1000 / CLGR  \hspace{1cm} (4.5)  
TPF = SUAF / PXPF  \hspace{1cm} (4.6)  
TEMP = FA - FB  \hspace{1cm} (4.7)  
GRPH = FA - TEMP \cdot \exp\left(-FC / TEMP \cdot ZPHG\right)  \hspace{1cm} (4.8)  
GRGP = CLGR \cdot GRPH  \hspace{1cm} (4.9)  
NGGP = FD + FE \cdot GRGP  \hspace{1cm} (4.10)

(2) Livestock Production

(2.1) Livestock "On the Hoof"

SLVMA = RLLVS \cdot TLLS  \hspace{1cm} (4.11)  
SLVA = SLV(2) \cdot SLVK(2) + SLV(4) + SLV(5) + SLV(9)  \hspace{1cm} (4.12)
$$\text{SLVAR} = \frac{\text{SLVA}}{\text{SLVMA}} \quad (4.13)$$

$$\text{LVPLM} = \text{XLVPLMF} (\text{SLVAR}) \quad (4.14)$$

$$\text{UALV}(J) = \text{IALV} \cdot \frac{\text{SLV}(J) \cdot \text{SLVK}(J)}{\text{SLVA}}$$
$$J = 1, \ldots, 9 \quad (4.15)$$

$$\text{ALVI}(J) = \frac{\text{UALV}(J)}{\text{LVPL}(J)}$$
$$J = 1, \ldots, 9 \quad (4.16)$$

$$\text{SLV}(J) = \text{SLV}(J) + \text{ALVI}(J)$$
$$J = 1, \ldots, 9 \quad (4.17)$$

(2.2) Production of Meat and Livestock Products

$$\text{SLVP}(J) = \frac{\text{SLV}(J) \cdot \text{SLVMK}(J)}{1000}$$
$$J = 1, \ldots, 8 \quad (4.18)$$

$$\text{SLVP}(J) = \frac{\text{SLV}(9) \cdot \text{SLVMK}(J)}{1000}$$
$$J = 9, \ldots, 12 \quad (4.19)$$

(3) Fish Production

$$\text{AWFM} = \text{FWCM} \cdot \text{AWFMK} \quad (4.20)$$

$$\text{AUFWP} = \text{UFWP} \cdot \text{WB} \quad (4.21)$$

$$\text{FWCP} = \text{UFWP} \cdot \text{UFWPK} \quad (4.22)$$

$$\text{FWCT} = \text{FWCM} + \text{FWCP} \quad (4.23)$$

$$\text{FWT} = \text{FWCT} \cdot \text{FWCNTK} \quad (4.24)$$

$$\text{FWCM} = \text{MIN} (\text{FWCM} + \text{AWFM}, \text{FWMM}) \quad (4.25)$$

$$\text{UFWP} = \text{MIN} (\text{UFWP} + \text{AUFWP}, \text{UFWPM}) \quad (4.26)$$

(4) Production of Crop Food Types and Distribution of Food to Alternative Uses

$$\text{PLGP}(J) = \text{NGGP} \cdot \text{NGGPK}(J)$$
$$J = 14, \ldots, 21 \quad (4.27)$$

$$\text{PLGP}(J) = \text{GRGP} \cdot \text{GRGPK}(J)$$
$$J = 22, \ldots, 26 \quad (4.28)$$
E. Regional Food Supply Compared with Population Needs

(1) Supply

FSRPC(J) = FTN(J)/POP · 1000
\[ J = 1, \ldots, 26 \]  
(5.1)

VCLPCR(J) = FSRPC(J) · CLK(J)
\[ J = 1, \ldots, 26 \]  
(5.2)

VPTPCR(J) = FSRPC(J) · PTK(J)
\[ J = 1, \ldots, 26 \]  
(5.3)

CLPCR = \[ \sum_{J=1}^{26} VCLPCR(J) \]  
(5.4)

PTPCR = \[ \sum_{J=1}^{26} VPTPCR(J) \]  
(5.5)

PTAPCR = \[ \sum_{J=1}^{13} VPTPCR(J) \]  
(5.6)
PTAR = PTAPCR \cdot \frac{POP}{1000} \quad (5.7)

(2) Comparison with Needs

PTNM = PTKF(YPC) \quad (5.8)
PTPCN = PTPCB \cdot PTNM \quad (5.9)
SPTPC = \text{MAX}(PTPCR - PTPCN, 0.0) \quad (5.10)
DPTPC = \text{MAX}(PTPCN - PTPCR, 0.0) \quad (5.11)
PTPCSN = \frac{PTPCR}{PTPCN} \quad (5.12)
PTN = PTPCN \cdot \frac{POP}{1000} \quad (5.13)
PTR = PTPCR \cdot \frac{POP}{1000} \quad (5.14)
DPT = DPTPC \cdot \frac{POP}{1000} \quad (5.15)
PTPCDR = \frac{PTPCR}{365.0} \quad (5.16)

F. The Pricing Mechanism

(1) Prices

\text{PXLVP} = \text{PXLV} \cdot \text{PXLVK} \cdot \text{PXK} \quad (6.1)
\text{GRV} = \text{GRGP} \cdot \text{PXGR} \cdot \text{PXK} \quad (6.2)
\text{NGV} = \text{NGGP} \cdot \text{PXNG} \cdot \text{PXK} \quad (6.3)
\text{SLVV} = \frac{5}{\sum_{J=1}^{5} \text{SLVP}(J)} \quad (6.4)
\text{LVV} = \text{SLVV} \cdot \text{PXLVP} \quad (6.5)
\text{FSV} = \text{FWT} \cdot \text{PXFS} \cdot \text{PXK} \quad (6.6)
\text{YA} = \text{GRV} + \text{NGV} + \text{LVV} + \text{FSV} \quad (6.7)
\text{YAPC} = \frac{\text{YA}}{\text{POP}} \quad (6.8)
\text{PXPTM} = \text{PXPTM} \cdot (1.0 + \text{RPXPTM}) \quad (6.9)
\text{PXK} = \text{PXK} \cdot (1.0 + \text{RPXK}) \quad (6.10)

(2) Cases

\text{PXPF} = \text{PXPF} \cdot (1.0 + \text{RPXPF}) \quad (6.11)
TPF = SUAF/PXPF \hspace{1cm} (6.12)

LVPL(J) = PXLVB(J) \cdot LVLM \hspace{1cm} (6.13)

KCLDH = XKLDF(FCLR) \hspace{1cm} (6.14)

**G. Energy Consumption**

ENZ = TPF \cdot ENZPLK \hspace{1cm} (7.1)

ENZPR = CLPCR \cdot POP/ENZ/1000.0 \hspace{1cm} (7.2)

**H. Import Sector**

FDMV = DPT \cdot PXPTM \hspace{1cm} (8.1)

FDMAR = FDMV/YA \hspace{1cm} (8.2)

FDMYR = FDMV/Y \hspace{1cm} (8.3)

FDMMR = FDMV/M \hspace{1cm} (8.4)

**I. Export Sector**

FDXV = YA \cdot (SPT/PTR) \hspace{1cm} (9.1)

FDXAR = FDXV/YA \hspace{1cm} (9.2)

FDXYR = FDXV/Y \hspace{1cm} (9.3)

FDXXR = FDXV/X \hspace{1cm} (9.4)

FDX9YR = FDXV9/Y \hspace{1cm} (9.5)

FDX9AR = FDXV9/YA \hspace{1cm} (9.6)

PTX9RR = PTX9/PTR \hspace{1cm} (9.7)

PTX9SR = PTX9/SPT \hspace{1cm} (9.8)

The functions TLWMF, PMCIF, PTFCF, XLVPLMF, PTKF, and XKLDF that are used throughout the List of Equations are described in [2] pp. B609 - B622.
J. Conclusions

Considering the fact that this paper has so far been a mere enumeration of various kinds of shortcomings (omissions, imprecise definitions, discrepancies between verbal and mathematical presentation, etc.), it will hardly be surprising that a critical judgement of the report on the Food Submodel—at least as far as its technical side is concerned—cannot do without attributes such as "highly defective" and "carelessly done." It is of course impossible to assess to what extent the authors themselves are responsible for all the deficiencies shown but even assuming that the original was less defective than the version eventually published one will have to remark that they did not bestow enough care upon the publication of their report.

Furthermore one cannot help regretting that a research project which would seem to be important enough on account of its general relevance and immediate interest is made to appear valueless through deficient and incompetent presentation.

III. REDUCTION OF THE MODEL EQUATIONS

This section relates to the "Revised List of Equations" given just before within this paper, since I felt that there was no sense in analysing the obviously wrong set of equations given in [2] on pp. B573 - B604.

The reduction of the equations is intended to provide a basis for further discussion of the food model thus revealing the driving forces and assumptions underlying the mathematical formulation of the model and to find discrepancies between these and the verbal description given by the authors.

To meet the above mentioned intention, equations and relationships that do not contribute to the dynamics of the model have been omitted. Equations are reduced to a level which satisfies two criteria:

1. The driving forces of the model should be revealed;
2. The reduced equations should still be easy to survey.

The reduced equations will then be used in the following chapter (IV), when it comes to analysing the model in greater detail. In order to achieve more precision time-subscripts are given from now on.

A. Population Submodel

In the population sector substituting some of the variables and reducing the equations appears to be quite simple. Incorporating (1.5) - (1.8) into (1.2) - (1.4) respectively you get

\[
\begin{align*}
\text{POP}_t &= \sum_{I=0}^{86} \text{AP}_t(I) \\
\text{AP}_{t+1}(0) &= [1.0 - \text{AMPF}_t(0) \cdot \text{AM}(0) \cdot 0.5 \cdot \text{MORT}] \\
&\quad \cdot \text{FERT} \cdot \sum_{I=1}^{86} \text{AP}_t(I) \cdot \text{AF}(I) \\
\text{AP}_{t+1}(I) &= \text{AP}_t(I-1) \cdot [1.0 - \text{AMPF}_t(I-1) \\
&\quad \cdot \text{AM}(I-1) \cdot \text{MORT}] , \quad I = 1, \ldots, 85 \\
\text{AP}_{t+1}(86) &= \text{AP}_t(86) \cdot [1.0 - \text{AMPF}_t(86) \cdot \text{AM}(86) \\
&\quad \cdot \text{MORT}] + \text{AP}_t(85) \cdot [1.0 - \text{AMPF}_t(85) \\
&\quad \cdot \text{AM}(85) \cdot \text{MORT}] .
\end{align*}
\]

From (1.9) - (1.14) the mortality multipliers AMPF_t(I) can be calculated as
AMPF_{t+1}(I) = \left[ \frac{\text{PRONOR} - \text{PROO}}{\text{PTPCR}_{t-TIMLAG} \cdot \text{PRODST} \cdot \left(\frac{1000}{365} - \text{PROO}\right) - 1.0} \right] \cdot \left[ (\text{EO} - \text{EU}) \cdot \exp\left(-\frac{I}{\text{EA}}\right) + \text{EU} \right] + 1.0 \quad I = 0, \ldots, 85

AMPF_{t+1}(86) = \left[ \frac{\text{PRONOR} - \text{PROO}}{\text{PRPCR}_{t-TIMLAG} \cdot \text{PRODST} \cdot \left(\frac{1000}{365} - \text{PROO}\right) - 1.0} \right] \cdot \text{EU} + 1.0

The remaining variables, although of interest when used as indicators, have no influence on the model and thus can be omitted.

B. Economic Submodel

Using equations (2.7), (2.14), and (2.15) one gets the following expression for annual investment:

\[ I_{t+1} = \text{GI} \cdot \text{YNA}_t \cdot \text{SYSYN}_t = \text{GI} \cdot \text{YNA}_t \]

\[ = \text{GI} \cdot \left( \frac{\text{YNA}_{t-1} + \text{YA}_{t-1}}{\text{YNA}_{t-1}} \right) \cdot \text{YNA}_t \]

Substituting KNA (from (2.5)) in (2.4) gives:

\[ \text{YNA}_t \cdot \text{QNA} = \text{YNA}_{t-1} \cdot \text{QNA} \cdot (1 - \text{DNA}) + \text{INA}_t \]

Provided IAKS - IAK \geq 0 holds one can deduce from equations (3.3) - (3.7) the following identity for INA_t:

\[ \text{INA}_t = (1 - \text{IAKS}) \cdot I_t \]

Inserting (1) and (3) into (2) yields:

\[ \text{YNA}_t \cdot \text{QNA} = \text{YNA}_{t-1} \cdot \text{QNA} \cdot (1 - \text{DNA}) + (1 - \text{IAKS}) \cdot \text{GI} \]

\[ \cdot \left(1 + \frac{\text{YA}_{t-2}/\text{YNA}_{t-2}}{\text{YNA}_{t-1}}\right) \cdot \text{YNA}_{t-1} \]
Dividing both sides of the equation by QNA gives:

\[ Y_{NA_t} = \left[ (1 - DNA) + (1 - IAKS) \cdot GI/QNA \right] \cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \cdot Y_{NA_{t-1}}. \quad \ldots (4) \]

From (1) easily derives

\[ IA_t = [IAK + K1 \cdot (IAKS - IAK)] \cdot GI \cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \cdot Y_{NA_{t-1}}. \quad \ldots (5) \]

and

\[ SUAF_{t+1} = UAPK \cdot A(2,1) \cdot [AI(1,2) \cdot YA_t + AI(2,2) \cdot YNA_t] + (1 - KL) \cdot (IAKS - IAK) \cdot GI \cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \cdot Y_{NA_{t-1}}. \quad \ldots (6) \]

\[ KA_t = (1 - DA) \cdot KA_{t-1} + IA_t. \quad \ldots (7) \]

It turns out that only equations (4) - (7) are necessary for the calculations of the model.

C. Land Use Submodel

Starting from equation (3.1) one can proceed as follows:

\[ CLGR_t = CLGR_{t-1} + CLDGR_t - CLWGR_t \]
\[ = CLGR_{t-1} + (CLD - CLAW) \cdot CLGR_{t-1}/CL_{t-1} \]
\[ = CLGR_{t-1} + [(1 - IAPK - IALVK) \cdot \frac{IA_t}{KCLDH_t}] \]
In this expression $\text{TLM}_{t-1}$ denotes a function $\text{TLM}(\text{CLM}/\text{CLW}_t)$ that is graphed in [2] on page B611. $\text{KCLDH}_t$ is a function $\text{KCLDH}(\text{FCLR}_{t-1})$ shown on pp B620 - B622 in [2]. $\text{FCLR}_t$ in turn can be calculated from (3.15) in the following manner:

$$\text{FCLR}_t = \frac{\text{CLR}_t}{(\text{TLM} \cdot (\text{CLM}/\text{TLM}) - \text{CLW}_t)}$$

$$= \frac{(\text{TLM} \cdot (\text{CLM}/\text{TLM}) - \text{CL}_t - \text{CLW}_t)/(\text{CLM} - \text{CLW}_t)}{\text{CLM} - \text{CLW}_t}$$

$$= 1 - \frac{\text{CL}_t}{\text{CLM} - \text{CLW}_t} = 1 - \frac{\text{CLGR}_t \cdot (1 + \text{TB}) + \text{TA}}{\text{CLM} - \text{CLW}_t}$$

To work out $\text{CLW}_t$ the following procedure can be used:

$$\text{CLW}_t = \text{TLW}_t \cdot (\text{CLM}/\text{TLM})$$

$$= (\text{CLM}/\text{TLM}) \cdot [\text{TLW}_{t-1} + \text{TLWM}_{t-1} \cdot \text{TLWPCB} \cdot \text{FERT} \cdot (1 - \text{AMPF}_{t-1}(0) \cdot \text{AM}(0) \cdot 0.5 \cdot \text{MORT})$$

$$\sum_{I=1}^{86} \text{AP}_{t-1}(I) \cdot \text{AF}(I)] \} .$$
Finally we can calculate TLLS\_t to be:

\[
TLLS_t = GL_{t-1} + CLR_{t-1} = GLM + CLM - [CLGR_{t-1} \\
\cdot (1 + TB) + TA] - CLW_{t-1} .
\]

D. Food Submodel

Although the Food Submodel seems to be a complex one at first sight, there are quite a lot of equations that can be reduced.

Using equations (4.4) - (4.7) the following result for grain production per hectare is obtained:

\[
GRPH_t = FA_t - (FA_t - FB) \\
\cdot \exp \left(-\frac{(FC \cdot 1000 \cdot SUAF_t)}{(FA_t - FB)}\right) \\
\cdot (CLGR_t \cdot (1 + TB) + TA) \\
\cdot PXPF \cdot (1 + RPXF)\_t^{1-ISTAT}
\]

where

\[FA_t = FA \cdot \frac{YNAt/POP_t}{KA_t/\text{CLGR}_t \cdot (1 + TB) + TA} ,\]

is the grain production saturation level.

Since the amounts of grain crops and non-grain crops depend on GRPH and CLGR in a simple way both GRGP and NGGP need not be evaluated explicitly. As for the fish production sector only a single equation turns out to be necessary, namely
Concerning livestock production we have to focus on equations (4.11) - (4.19) which leave us with:

\[ \text{SLV}_t(J) = \text{SLV}_{t-1}(J) + \sum_{L=2,4,5,9} \frac{\text{SLV}_{t-1}(L) \cdot \text{SLV}(L)}{\text{LVPLM}_t} \cdot \text{LVPLM}(J) \]

where \( \text{LVPLM}_t \) denotes a function of the ratio of maximum possible livestock to be carried to actual livestock, thus:

\[ \text{LVPLM}_t = \text{LVPLM} (RLLVS \cdot TLLS_t / \left( \sum_{L=2,4,5,9} \text{SLV}_{t-1}(L) \cdot \text{SLV}(L) \right) ) \]

Gross food production by category then appears as:

\[
\begin{align*}
\text{SLV}_t(J) \cdot \text{SLVMK}(J)/1000, & \quad J = 1, \ldots, 12 \\
\text{FWT}_t, & \quad J = 13 \\
(FD + FE \cdot \text{CLGR}_t \cdot \text{GRPH}_t) \cdot \text{NGGPK}(J), & \quad J = 14, \ldots, 21 \\
\text{CLGR}_t \cdot \text{GRPH}_t \cdot \text{GRGPK}(J), & \quad J = 22, \ldots, 26
\end{align*}
\]
E. Regional Food Supply Compared with Population Needs

Although almost any variable calculated within this part of equations provides valuable information for examining the food supply situation of the region under consideration, it turns out, however, that only regional protein per capita has an influence on further computations. Therefore, everything else has been omitted. The equation of relevance aggregating (4.29) - (5.5) is found to be:

$$PTPCR_t = \frac{1}{POP_t} \cdot \sum_{J=1}^{26} (1 - HMLF(J)) \cdot FFTK(J)$$
$$\cdot (1 - SPFTK(J)) \cdot PTK(J) \cdot 1000 \cdot FGP_t(J).$$

F. Price Sector

The main purpose of the pricing mechanism is to compute the gross regional agricultural product. From equations (6.1) - (6.8) the subsequent relationship can be derived:

$$YA_t = PXK_t \cdot \{CLGR_t \cdot GRPH_t \cdot [PXGR + FE \cdot PXNG]$$
$$+ FD \cdot PXNG + PXLVK \cdot PXLV \cdot \sum_{J=1}^{5} FGP(J)$$
$$+ FWT_t \cdot PXFS\} ,$$

$$PXK_t = PXK_{t-1} \cdot (1 + RPXK).$$

The equations given in sections 7, 8 and 9 are only used as indicators and therefore have been left out. Thus the Reduced List of Equations is the following:

IV. REDUCED LIST OF EQUATIONS

A. Population Submodel

$$POP_t = \sum_{I=0}^{86} AP_t(I) ,$$
\[ AP_{t+1}(0) = [1 - AMPF_t(0) \cdot AM(0) \cdot 0.5 \cdot MORT] \]

\[
\sum_{I=1}^{86} FERT \cdot AP_t(I) \cdot AP(I).
\]

\[ AP_{t+1}(I) = AP_t(I - 1) \cdot [1 - AMPF_t(I - 1) \cdot AM(I - 1) \cdot MORT], \quad I = 1, \ldots, 85 \]

\[ AP_{t+1}(86) = AP_t(85) \cdot [1 - AMPF_t(85) \cdot AM(85) \cdot MORT] \]

\[ + AP_t(86) \cdot [1 - AMPF_t(86) \cdot AM(86) \cdot MORT]. \]

\[ AMPF_{t+1}(I) = [(PRONOR - PROO)/(PTPCR_t \cdot TIMLAG \cdot PRODST \cdot \frac{1000}{365} - PROO) - 1.0] \]

\[
\cdot [(EO - EU) \cdot \exp(-I/EA) + EU] + 1.0, \quad I = 0, \ldots, 85
\]

\[ AMPF_{t+1}(86) = [(PRONOR - PROO)/(PTPCR_t \cdot TIMLAG \cdot PRODST \cdot \frac{1000}{365} - PROO) - 1.0] \]

\[
\cdot EU + 1.0.
\]

B. Economic Submodel

\[ YNA_t = [(1 - DNA) + (1 - IAKS) \cdot GI/QNA \]

\[ \cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}})] \cdot YNA_{t-1}. \]

\[ IA_t = [IAK + K1 \cdot (IAKS - IAK)] \cdot GI \cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \]

\[ \cdot YNA_{t-1}. \]
SUAF_{t+1} = UAFK \cdot A(2,1) \cdot [AI(1,2) \cdot YA_t
+ AI(2,2) \cdot YNA_t] + (1 - K1)
\cdot (IAKS - IAK) \cdot GI \cdot (1 + \frac{YA_t - 1}{YNA_t}) \cdot YNA_t

\quad KA_t = (1 - DA) \cdot KA_{t-1} + IA_t

C. Land Use Model

CLGR_t = CLGR_{t-1} \cdot \{1 + \frac{1}{CLGR_{t-1} \cdot (1 + TB) + TA}
\cdot [(1 - IAPK - IALVK) \cdot \frac{IA_t}{KCLDH_{t}} - CLM
\quad TLM
\quad TLWPCB \cdot TLW_{t-1} \cdot FERT \cdot (1 - AMPF_{t-1}(0))
\quad AM(0) \cdot 0.5 \cdot MORT \cdot \sum_{I=1}^{86} AP_{t-1}(I) \cdot AF(I)\}

\quad FCLR_t = 1 - \frac{CLR_t \cdot (1 + TB) + TA}{CLM - CLW_t}

CLW_t = CLW_{t-1} + \frac{CLM}{TLM} \cdot [TLW_{t-1} \cdot TLWPCB \cdot FERT
\quad (1 - AMPF_{t-1}(0) \cdot AM(0) \cdot 0.5 \cdot MORT)
\quad \sum_{I=1}^{86} AP_{t-1}(I) \cdot AF(I)]

TLWM_t = TLWM(CLW_{t-1}/CLM)

KCLDH_t = KCLDH(FCLR_{t-1})

TLLS_t = GLM + CLM - [CLGR_{t-1} \cdot (1 + TB) + TA] - CLW_{t-1}
D. Food Submodel

\[ \text{FA}_t = \text{FA}(\text{YNA}_t/\text{POP}_t, \text{KA}_t/[\text{CLGR}_t \cdot (1 + \text{TB}) + \text{TA}]) , \]

\[ \text{GRPH}_t = \text{FA}_t - (\text{FA}_t - \text{FB}) \]
\[ \cdot \exp \left( - \left( \text{FC} \cdot 1000 \cdot \text{SUAF}_t \right)/\left( (\text{FA}_t - \text{FB}) \cdot (\text{CLGR}_t \cdot (1 + \text{TB}) + \text{TA}) \cdot \text{PXPF} \cdot (1 + \text{RPXPF})^{t-\text{ISTAT}} \right) \right) , \]

\[ \text{FWT}_t = [\text{FWCM} \cdot (1 + \text{AWFK})^{t-\text{ISTAT}} + \text{UFWP} \]
\[ \cdot (1 + \text{WB})^{t-\text{ISTAT}} \cdot \text{UFWPK}] \cdot \text{FWCNTK} , \]

subject to

\[ \text{FWCM} \cdot (1 + \text{AWFMK})^{t-\text{ISTAT}} \leq \text{FWMM} , \]

\[ \text{UFWP} \cdot (1 + \text{WB})^{t-\text{ISTAT}} \leq \text{UFWM} . \]

\[ \text{SLV}_t(J) = \text{SLV}_{t-1}(J) \]
\[ = \text{SLV}_{t-1}(J) \cdot \text{SKV}(J) \cdot \text{IALVK} \cdot \text{IA}_t \]
\[ + \sum_{L=2,4,5,9} \frac{\text{SLV}_{t-1}(L) \cdot \text{SLV}(L) \cdot \text{LVPLM}_t \cdot \text{PXLVB}(J)}{\text{LVPLM}_t = \text{LVPLM}(\text{RLLV} \cdot \text{TLLS}_t/ \sum_{L=2,4,5,9} \text{SLV}_{t-1}(L) \cdot \text{SLV}(L))} , \]

\[ J = 1, \ldots , 9 , \]

\[ \text{FGP}_t(J) = \begin{cases} \text{SLV}_t(J) \cdot \text{SLVMK}(J)/1000 & J = 1, \ldots , 21 \\ \text{FWT}_t & J = 13 \\ (\text{FD} + \text{FE} \cdot \text{CLGR}_t \cdot \text{GRPH}_t) \cdot \text{NGGPK}(J) & J = 14, \ldots , 21 \\ \text{CLGR}_t \cdot \text{GRPH}_t \cdot \text{GRPH}_t \cdot \text{GRGPK}(J) & J = 22, \ldots , 26 \end{cases} \]
E. Regional Food Supply Compared with Population Needs

\[ \text{PTPCR}_t = \frac{1}{\text{POP}_t} \cdot \frac{26}{\sum_{J=1}^{26} (1 - \text{HMLF}(J)) \cdot \text{PTK}(J)} \cdot \left(1 - \text{SPFTK}(J)\right) \cdot \text{PTK}(J) \cdot 1000 \cdot \text{FGP}_t(J). \]

F. Pricing Mechanism

\[ \text{YA}_t = \text{PXK}_t \cdot \left\{\text{CLGR}_t \cdot \text{GRPH}_t \cdot \left[\text{PXGR} + \text{FE} + \text{PXNG}\right]\right. \]
\[ + \text{FD} \cdot \text{PXNG} + \text{PXLK} \cdot \text{PXLV} \cdot \sum_{J=1}^{5} \text{FGP}_t(J) \]
\[ + \text{FWT}_t \cdot \text{PXFS}\right\}, \]

\[ \text{PXK}_t = \text{PXK}_{t-1} \cdot (1 + \text{RPXK}). \]

IV. MODEL BEHAVIOUR

The actual performance of the model has been studied in some detail for the region of South East Asia. The reasons for choosing this region were two-fold. First of all, South East Asia faces a very critical food supply situation, and secondly, a more technical reason, computer results for South East Asia are given in [3] on pp. B661 - B685 so that initial values and parameter estimates which have been forgotten in the model description could be taken from this output. Nevertheless, some of the results do not only relate to South East Asia but are valid for any region.

1. Population Sector

Considering the equations of the population sector there is especially one calculation which needs further discussion. Besides FERT which may be treated as a scenario variable
MORTALITY MULTIPLIER

FOR FRESH = 25 GR PROTEIN / DAY

AND FRESH = 35 / 45 / 55 GR PROTEIN / DAY
(prescribed by a time-series or by other means) population development is mainly influenced by the mortality multiplier AMPF(I), I = 0, ..., 86 provided the region under consideration has to face protein deficiencies.

The corresponding equation is:

$$\text{AMPF}_t = \frac{(\text{PRONOR} - \text{PROO})}{(\text{PTPCR}_t \cdot \text{TIMLAG} \cdot \text{PRODST}} \cdot \frac{1000}{365} - \text{PROO}) - 1.0] \cdot E + 1.0,$$

where E accounts for the individual age specific sensitivity to protein deficiency which actually varies between EU and EO which in turn are specified as scenario variables. For our discussion we may assume E to be constant. The same assumption is valid for PRODST since PRODST is used to adjust the value of PTPCR rather than as a variable in the above equation. As for South East Asia PRODST is fixed at a level of 0.7. Thus there are two parameters left, namely PRONOR (level of daily protein per capita consumption below which starvation occurs) and PROO (minimum level of per capita protein consumption below which there is now survival). The equation defining AMPF is a hyperbola with a vertical asymptote at PROO. On figures 1 and 2 plots of AMPF are given for some choices of PROO and PRONOR. From these plots it is quite obvious that in order to get reasonable results a careful choice of PROO has to be made since PROO may become a powerful constraint for population development in the model. The impact of PRONOR is by far not as critical as that of PROO provided PRONOR is chosen within reasonable limits.

2. Economic Sector

As may be seen from the "Reduced List of Equations" there are only a few equations left in this section. Among these the equations for YNA and IA are of special interest. For the non-agricultural regional product we have
\[ Y_{NA_t} = [(1 - DNA) + (1 - IAKS) \cdot GI/QNA \cdot (1 + \frac{YA_{t-2}}{Y_{NA_{t-2}}})] \cdot Y_{NA_{t-1}}, \]

and for the agricultural investment

\[ I_{A_t} = [IAK + KL \cdot (IAKS - IAK)] \cdot GI \cdot (1 + \frac{YA_{t-2}}{Y_{NA_{t-2}}}) \cdot Y_{NA_{t-1}} \]

\[ = [IAK + KL \cdot (IAKS - IAK)] \cdot GI \cdot \frac{Y_{NA_{t-1}}}{Y_{NA_{t-2}}} \cdot Y_{t-2}. \]

Since DNA, GI, and QNA are fixed for each region there are only three quantities to be examined, namely IAKS, the ratio \( \frac{YA_t}{Y_{NA_t}} \) and the scenario variable KL which is used to distribute the additional investment (which is due to IAKS) among the agricultural capital stock and the expenditures on fertilizer and related productive factors.

Incidentally, it should be noticed that the term

\[ (1 + \frac{YA_t}{Y_{NA_t}}) \]

equals the ratio \( \frac{Y_t}{Y_{NA_t}} \),

\( Y_t \) denoting the gross regional product of year \( t \).

\[ Y_t = Y_{NA_t} + YA_t. \]

Due to the non-linearities involved in the evaluation of \( YA \) one cannot easily make quantitative statements about the ratio \( YA/Y_{NA} \), but obviously this ratio depends very much on the choice of IAKS. An increase of IAKS leads to a considerable increase of the agricultural capital stock (depending on KL) and to an increased use of fertilizer both of which are inputs.
for the crop production function and thus lead to an increase of \( YA \). On the other hand, the annual growth rate of YNA depends essentially on IAKS.

From above we have

\[
\frac{YNA_t}{YNA_{t-1}} = 1 + (1 - IAKS) \cdot GI/QNA
\]

\[\cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) - DNA.
\]

To investigate the effect of an investment shift to the agricultural sector one has to concentrate on the product

\[(1 - IAKS) \cdot (1 + R)
\]

\( R \) denoting the ratio of agricultural to non-agricultural regional product. An increase \( \Delta IAKS \) of IAKS causes an increase \( \Delta R \) of \( R \) which has a positive effect on the growth of YNA only if

\[(1 - (IAKS + \Delta IAKS)) \cdot (1 + (R + \Delta R)) > (1 - IAKS) (1 + R)
\]

which implies

\[
\frac{\Delta R}{\Delta IAKS} > \frac{1 + R}{1 - IAKS}
\]

Computations have shown that this ratio which is an approximation for the partial derivative \( \frac{\partial R}{\partial IAKS} \) is always less than 1 at any time during the model computations for any level of \( IAKS \geq IAK \). Therefore any investment shift to the agricultural sector must injure the non-agricultural sector (at least in the model).
There is zero growth or even decrease of YNA in year $t$ if

$$\text{DNA} \geq (1 - \text{IAKS}) \cdot \text{GI/QNA} \cdot (1 + \frac{\text{YA}_{t-2}}{\text{YNA}_{t-2}}),$$

i.e. if

$$1 > \frac{\text{DNA}}{\text{GI/QNA}} \cdot \frac{\text{YNA}_{t-2}}{\text{Y}_{t-2}}.$$

To give an example: for South East Asia the ratio $\text{YNA}/\text{Y}$ is about 0.53 for 1975 and therefore we have

$$\text{IAKS} \geq 1 - \frac{0.0285702}{0.1285934 \cdot 1/2.5} \cdot 0.53 \approx 0.706.$$

Considering investment as a whole (without that share of the additional investment to agriculture which goes to expenditures on fertilizers etc.) we have

$$\text{IA}_t + \text{INA}_t = [\text{IAK} + \text{K1} \cdot (\text{IAKS} - \text{IAK}) + (1 - \text{IAKS})]$$

$$\cdot \text{GI} \cdot \frac{\text{YNA}_{t-1}}{\text{YNA}_{t-2}} \cdot \frac{\text{Y}_{t-2}}{\text{Y}_{t-2}} = [1 - (\text{IAKS} - \text{IAK}) \cdot (1 - \text{K1})] \cdot \text{GI} \cdot \frac{\text{YNA}_{t-1}}{\text{YNA}_{t-2}} \cdot \frac{\text{Y}_{t-2}}{\text{Y}_{t-2}}.$$

The above expression makes clear that the actual amount of investment depends not only on the level of the gross regional product but also on the growth of YNA. This feature of the investment function together with the properties of the agricultural production function dealt with subsequently explains the results that were obtained in [3], namely, that
any investment shift to the agricultural sector leads to a worse performance of the model, at least in the long run. Besides, it does not seem reasonable to treat quantities like the investment coefficient GI and the capital per output ratio QNA constant over a period of 50 years which is at variance with historical data.

3. Agricultural Production

According to the Reduced List of Equations the gross regional product is given by

\[ Y_A(t) = PX_K(t) \cdot \left\{ CLGR_t \cdot GRPH_t \cdot [PXGR + FE \cdot PXNG] ight\} + FD \cdot PXNG + PXLVK \cdot PXLV \cdot \sum_{j=1}^{5} FGP_t(j) + FWT_t \cdot PXFS \} . \]

As may be seen from the "Revised List of Equations" this production function covers three sectors: the crop production, the livestock sector, and the fish production.

a) Crop Production

Dealing with South East Asia the crop production is by far the most important contribution to the agricultural production. This must be taken into consideration when speaking about the model behaviour. In accordance with the above equation for \( Y_A \) the dollar value of the crop production (grain crops and non-grain crops) is

\[ GRV_t + NGV_t = PX_K_t \cdot \left\{ CLGR_t \cdot GRPH_t \cdot [PXGR + FE \cdot PXNG] + FD \cdot PXNG \} . \]

For \( PX_K \) we have

\[ PX_K_t = PX_K_{t-1} \cdot (1 + RPXK) . \]
i.e. the price coefficient $PX_K$ has a steady-state growth at the specified rate $RP_K$ and is not linked to any other sector of the model. As for the model output shown in [3] $RP_K$ has been fixed to be 0.025.

So, irrespective of any economic considerations $YA$ grows annually by 2.5% in addition to the growth of the actual agricultural production. The authors of [2] do not seriously attempt to explain this peculiar aspect in their model. The only reason they give in [2] is that the price coefficient $PX_K$ has been introduced in addition to the coefficients $PX_{GR}$, $PX_{NG}$, $PX_{LV}$ and $PX_{FS}$ "to allow experimentation with the model."

Turning to the other variables of the above production function the following remarks seem to be appropriate for South East Asia:

$CL_{GR_t}$ may be treated as constant in spite of the rather extensive land use model. From 1980 onward $CL_{GR}$ is predicted to remain constant at a level of 178.398 million ha. So no increase of the crop production is possible by means of land development. This behavior is due to the underlying (low) estimates of maximum cultivable land. Therefore, any attempt to increase the crop production has to focus on $GRPH$ which is defined as

$$
GRPH_t = FA_t - (FA_t - FB) 
\cdot \exp\left(- (FC \cdot 1000 \cdot SUAF_t)/(FA_t \cdot FB)\right) 
\cdot (CL_{GR_t} \cdot (1 + TB) + TA) 
\cdot PXPF \cdot (1 + RPXF) t-ISTAT 
$$

where the grain production saturation level is

$$
FA_t = 0.15280 \cdot \ln(YNA_t/POP_t) 
+ 0.21714 \cdot \ln(KA_t/[CL_{GR_t} \cdot (1 + TB) + TA]) 
+ 3.861
$$
and the expenditures on fertilizer and related productive factors \( SUAF_t \):

\[
SUAF_t = UAFK \cdot A(2,1) \cdot [A11(1,2) \cdot YA_{t-1} + A11(2,2) \\
\cdot YNA_{t-1}] + (1 - K1) \cdot (IAKS - IAK) \cdot GI \\
\cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \cdot YNA_{t-1}.
\]

To facilitate further investigation we assume \( \text{RPXPF} = 0 \) and \( CLGR_t = 178.398 \), inserting of which yields

\[
GRPH_t = FA_t - (FA_t - 0.919983) \cdot \exp \left( - \frac{0.58604 \cdot SUAF_t}{FA_t - 0.919983} \right),
\]

and the partial derivatives

\[
\frac{\partial GRPH}{\partial FA} = 1 - \left( 1 + (FA - 0.919983) \cdot \left( \frac{0.58604 \cdot SUAF}{(FA - 0.919983)^2} \right) \right) \\
\cdot \exp \left( - \frac{0.58604 \cdot SUAF}{FA - 0.919983} \right) \\
= 1 - \left( 1 + \frac{0.58604 \cdot SUAF}{FA - 0.919983} \right) \\
\cdot \exp \left( - \frac{0.58604 \cdot SUAF}{FA - 0.919983} \right),
\]

\[
\frac{\partial GRPH}{\partial SUAF} = 0.58604 \cdot \exp \left( - \frac{0.58604 \cdot SUAF}{FA - 0.919983} \right).
\]

The above derivatives show that the sensitivity of \( GRPH \) to changes in \( SUAF \) decreases very rapidly when \( SUAF \) increases. Once a certain level of \( SUAF \) (depending on \( FA \)) is reached a significant increase of \( GRPH \) can be achieved only if \( FA \) goes up too. This must be taken into consideration when choosing \( K1 \). The
Figure 3. Surface of the crop production function
(GRPH varies between 1 and 4.5)
possibilities of increasing FA, however, are limited. Since the non-agricultural production per capita is not very likely to increase dramatically—at least not if an investment shift to the agricultural sector is performed—the only way to increase FA is by means of increasing KA. From the function used to define FA it is clear that only an exponential growth of KA can lead to a constant increase of FA. Assuming that the ratio YNA/POP increases at an annual rate R1 and KA at an annual rate R2 then $FA_t$ is accordingly

$$FA_t = FA_{t_0} + t \cdot \ln [(1 + R1) \cdot (1 + R2)] .$$

Due to

$$KA_t = KA_{t-1} \cdot (1 - DA) + IA_t ,$$

an exponential growth of KA can only be achieved through a steady-state growth of IA, which has already been dealt with.

Expenditures on fertilizers, SUAF, which is an essential input to the crop production function is defined as:

$$SUAF_t = UAFK \cdot A(2,1) \cdot [AI(1,2) \cdot YA_{t-1} + AI(2,2) \cdot YNA_{t-1}] + (1 - K1) \cdot (IAKS - IAK) \cdot GI$$

$$\cdot (1 + \frac{YA_{t-2}}{YNA_{t-2}}) \cdot YNA_{t-1} ,$$

the second term of which is due to the investment shift and thus depends on IAKS and K1. Assuming that there is no investment shift the above equation may be written as

$$SUAF_t = UAFK \cdot A(2,1) \cdot AI(1,2)$$

$$\cdot [YA_{t-1} + \frac{AI(2,2)}{AI(1,2)} \cdot YNA_{t-1}] .$$
Speaking about South East Asia the ratio $\frac{AI(2,2)}{AI(1,2)}$ is

$$\frac{AI(2,2)}{AI(1,2)} = \frac{1.4176}{0.0984764} \approx 14.40 .$$

This means that the growth of SUAF is primarily related to the growth of YNA.

b) Livestock Production

From III.D one gets the dollar value of livestock production

$$LVV_t = PXLVK \cdot PXLV \cdot \sum_{j=1}^{5} FGP_t(J) \cdot PXK_t ,$$

and

$$FGP_t(J) = SLV_t(J) \cdot SLVMK(J)/1000 \quad J = 1, \ldots, 5 ,$$

$$SLV_t(J) = SLV_{t-1}(J)$$

$$+ \left( \sum_{L=2,4,5,9} \frac{SLV_{t-1}(J) \cdot SLV_K(J) \cdot IA_t}{SLV_{t-1}(L) \cdot SLV_K(L) \cdot LVPLM_t \cdot PXLVB(J)} \right) \quad J = 1, \ldots, 5 .$$

In this expression $LVPLM_t$ accounts for the carrying capacity of the available land and thus is a function of the ratio of actual livestock to maximum possible livestock which in turn is a function of TLLS (total land for livestock support). Concerning South East Asia $LVPLM$ remains almost constant at a level of 0.90 and thus may be omitted from our considerations. So, the only input variable remaining is $IA_t$ which has been treated before.
The formulation of livestock development assumes implicitly that livestock sectors cannot develop independently since money going to the livestock sector is split among the various livestock categories proportional to the adjusted level of each category (i.e. \( SLV(J) \cdot SLVK(J) \); here "adjusted" means that the number of individuals in each category is corrected for purposes of aggregating to a single base unit) divided by the corresponding livestock price \( PXLVB(J) \), \( J = 1, \ldots, 9 \). For South East Asia the ratios \( SLVK(J)/PXLVB(J) \), \( J = 1, \ldots, 9 \) are the following:

<table>
<thead>
<tr>
<th>Livestock Category</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cattle</td>
<td>0.649979/0.124996 = 5.200</td>
</tr>
<tr>
<td>Pigs</td>
<td>0.199997/0.0429983 = 4.651</td>
</tr>
<tr>
<td>Sheep and Goats</td>
<td>0.099999/0.0149996 = 6.666</td>
</tr>
<tr>
<td>Horses</td>
<td>1.000000/0.099999 = 10.000</td>
</tr>
<tr>
<td>Mules, Asses, Buffaloes, and Camels</td>
<td>1.000000/0.080998 = 12.346</td>
</tr>
<tr>
<td>Chickens</td>
<td>0.009999/0.000500 = 20.000</td>
</tr>
<tr>
<td>All Poultry</td>
<td>0.009999/0.000500 = 20.000</td>
</tr>
<tr>
<td>Dairy Animals</td>
<td>1.000000/0.080998 = 12.346</td>
</tr>
</tbody>
</table>

From the above ratios one can conclude that in the opinion of the model writers money is used more efficiently for breeding poultry and dairy animals than for breeding pigs, sheep and cattle.

Taking the time-series for IA shown on page B678 in [3] the development of livestock categories from 1975 - 2025 is the following:
<table>
<thead>
<tr>
<th>Category</th>
<th>Livestock Numbers in Millions</th>
<th>Percent Increase</th>
<th>Percentage of Corresponding Annual Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1975</td>
<td>2025</td>
<td></td>
</tr>
<tr>
<td>Cattle</td>
<td>266.26</td>
<td>344.35</td>
<td>29.3</td>
</tr>
<tr>
<td>Pigs</td>
<td>33.11</td>
<td>41.68</td>
<td>25.9</td>
</tr>
<tr>
<td>Sheeps &amp; Goats</td>
<td>200.24</td>
<td>278.42</td>
<td>39.0</td>
</tr>
<tr>
<td>Horses</td>
<td>3.13</td>
<td>5.13</td>
<td>63.8</td>
</tr>
<tr>
<td>Mules, Asses, Buffaloes and Camels</td>
<td>100.65</td>
<td>184.90</td>
<td>83.7</td>
</tr>
<tr>
<td>Chickens</td>
<td>612.86</td>
<td>1634.30</td>
<td>166.7</td>
</tr>
<tr>
<td>Miscellaneous Poultry</td>
<td>537.0</td>
<td>1432.01</td>
<td>166.7</td>
</tr>
<tr>
<td>Dairy Animals</td>
<td>327.71</td>
<td>601.98</td>
<td>83.7</td>
</tr>
</tbody>
</table>

Resuming the table given above one may conclude two things:

- The overall development of the livestock sector depends on the growth of IA;
- The development of each livestock category is determined by the ratio \( \frac{SLVK(J)}{PXLVB(J)} \) which may be interpreted as a reciprocal adjusted livestock price.

c) Fish Production

The fish production sector of the M.P. Food Model is fully deterministic in the sense that once a scenario has been fixed no interactions occur and the fish production is computed in the following way

\[
F_{WT_t} = [FWCM \cdot (1 + AWFMK)^{t-ISTAT} + UFWP] \cdot (1 + WB)^{t-ISTAT} \cdot UFWPK] \cdot FWCNTK ,
\]
subject to

\[ \text{FWCM} \cdot (1 + \text{AWFMK})^{t-\text{STAT}} \leq \text{FWMM} , \]

\[ \text{UFWP} \cdot (1 + \text{WB})^{t-\text{STAT}} \leq \text{UFWPM} . \]

Accordingly the dollar value of fish production is

\[ FSV_t = PXK_t \cdot FWT_t \cdot PXFS . \]

As for South East Asia the fish production remains constant from about 1980 on since FWMM acts as a constraint for marine fish catch.

4. Conclusions

Although one might get the impression from [2] that the M.P. Food Model is a fairly complex and disaggregated one the "Reduced List of Equations" shows that this is not at all true. Focusing on South East Asia one may state the following:

Population development depends very much on the chosen scenario. Among the scenario variables of the population sector FERT and PROO are of special interest. If a lack of protein occurs population growth is limited by the growth of food production. The population sector in turn has practically no impact on the model dynamics.

The economic model is dominated by the non-agricultural sector, the development of which most of all depends on the choice of IAKS. If an investment shift is performed the computations are also rather sensitive to the choice of KL.

The food sector is governed by the crop production which by and by stagnates due to the lack of cultivable land to be
developed, which in turn results from the underlying estimates for maximum cultivable land. Livestock development depends essentially on IA (livestock categories cannot develop independently), whereas the fish production is fully deterministic.

The steady-state growth of the price coefficient PXK which is not sufficiently explained by the authors of [2] makes up for the stagnation of the crop production so that most of the growth of YA is due to PXK. Again referring to the model output listed on pages B661 - B685 there is an increase of YA from 80.559 billion US dollars up to 469.836 billion US dollars which means an increase of 483% which corresponds to an annual growth of 3.59%. In spite of this considerable growth the actual increase of agricultural production from 1975 - 2025 is 70% (only!) which is equivalent to an annual growth of 1.067%. In any case, this is considerably less than the growth of YNA (about 4.6% per year). The increases of the agricultural and the non-agricultural capital stock correspond to an annual growth of 3.85% and 4.6% respectively. This indicates that the available capital is used less effectively in the agricultural sector, which might be another explanation of the results obtained by the authors in [3] where they comment on an investment shift to the agricultural sector (p. B639).

Roughly speaking the mechanisms involved appear to be the following:

Depending on the level of IA, the growth of YNA is more or less truncated. By means of the additional money that is going to the agricultural sector YA can be increased considerably, but after a short period of rapid growing the agricultural production gradually stagnates due to the saturating crop production function. The capital use in the agricultural sector becomes more and more inefficient. In the long run this leads to a worse performance of the model than without an investment shift.
In addition, the rapid increase of YA due to the investment shift leads to an improved diet and therefore no starvation occurs which in turn makes the population grow at a high rate. The stagnation of the agricultural production then causes an even worse food supply situation.

After all the question remains whether the output produced by the M.P. Food Model can be taken seriously. As I have tried to point out, there are some underlying assumptions that are both too simplifying and unreasonable. This reproach applies especially to the economic part and the food production submodel (constant capital per output ratio and investment coefficient; unreasonable livestock development and fish production). On the other hand, the model pretends a level of complexity and disaggregation that actually does not exist and moreover would not make sense compared to the simple mechanisms that in fact drive the model. In my opinion there is no sense in having 13 different crop categories which are all depending on one crop production function in a trivial linear relationship.

Thus as a resume of this paper one might say that the M.P. Food Model can be looked upon as a first attempt towards modelling the relations between population growth, agricultural production and economic development. One must stress, however, that the model as it is published in [2] leaves quite a lot of problems that seem to be approached in an inappropriate and insufficient way.
References


