

GLOBAL MODELS AND GLOBAL MECHANISMS II:  
THE GENERAL PRODUCTION FUNCTION

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The General Production Function<sup>(1)</sup>

H. Millendorfer

1. Setting of the Investigation

"In some cases old terms must be given new meanings or be defined or combined in new ways. In other cases more explicit steps are needed to develop new terms...we must... embed some of our presently sophisticated language (particularly that of economics and econometrics) in a broader language of inter- or supra-disciplinary scientific discourse."<sup>(2)</sup>

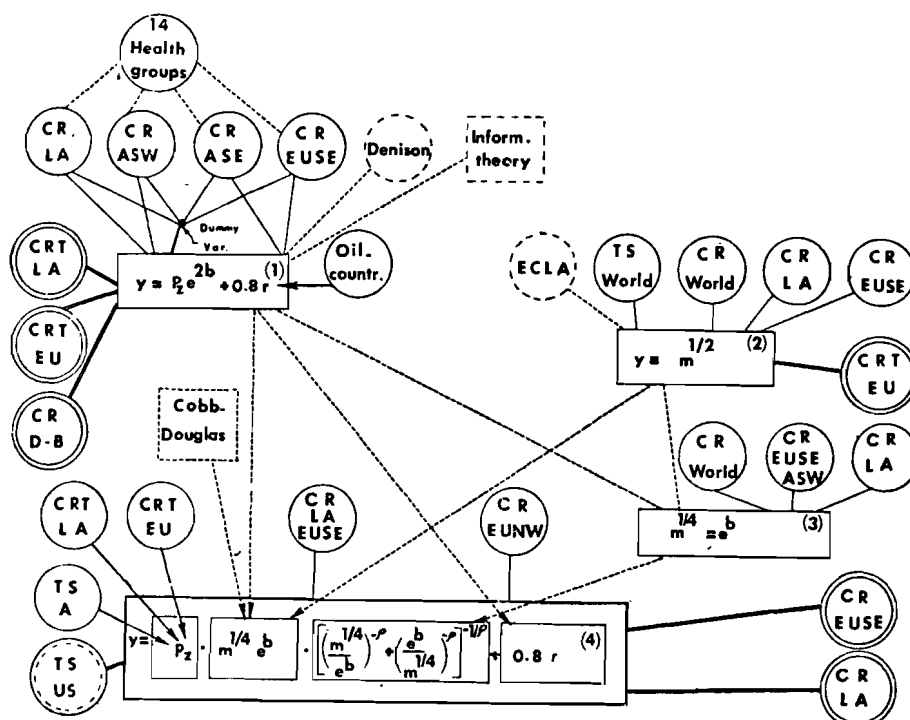
This assertion by B. Gross is significant in an investigation of the process of development, for there exist numerous studies on questions of societal development which ought to be collated to form a comprehensive picture, using a more general framework of conceptual relationships.

The investigations leading to the general production function were based on results and reflection arising from economics, educational economics and a system theoretical understanding of the society as an energy and information processing cybernetic system with a certain goal setting motivation structure and social form.

The methodology of the investigations is based on an iterative process of interactions between empirical observations

and theoretical considerations. In this interaction theoretical hypotheses have two functions: prior to the empirical observation a heuristic function and afterwards an interpretative function. Both follow in alternative turns in a process which starting with first heuristic preliminary hypotheses lead to empirical observations and in their preliminary interpretation to improved hypotheses. These again initiate new empirical observations and the interpretation of which corrects and improves the original hypotheses and so on. This process<sup>(3)</sup> leads to a step by step approximation of model and reality and is continued until the theoretical interpretation of empirical observation is no longer changed by further empirical observation. The role of existing theories is in the first steps that of heuristic hypotheses. Afterwards existing theories become sometime parts of the new theory for a special case. (For example the Cobb-Douglas-Production function became a special case of the general production function for constant education and structure).

Fig. 1 shows the research strategy used which is an application of the principle described.



Every circle is one investigation (CR, cross section analysis; TR, investigation of time series; ERT, subsequent cross section analyses for different years; circles with two solid lines, subsequent tests; solid line, interdependence of results; interrupted line, formating and relating hypotheses). The number of data used for one investigation, i.e. corresponding to one circle ranges from several hundred (i.e. economic and educational data) to several thousand (i.e. health data).

## 2. First Preliminary Hypotheses and Observations

### 2.1 Basic Concept

The development of the General Production Function is based on a certain understanding of the functioning of societal systems in general. Society can be thought of as a complex, multihierarchical system. This system is characterized by the interaction of a great number of subsystems, and it is this interaction which enables the overall system to function. Economy is one of these subsystems. Therefore, if we want to understand how this subsystem works, we must go beyond the economic sphere proper and consider also the interrelations with other subsystems.

Society has another characteristic feature in common with other systems, namely its ability of processing energy and information. The economic output of a social system is

therefore governed by its capacity to cope with these two fundamental processes. Superimposed on an energy processing system, in which the material processes of the production take place, there is a system of information receptors, information channels, information storing units and logical units. This network receives information on the actual situation, on environmental changes and on the internal structure of the system. It then processes this information using additional information stored in the past. This process gives rise to new information which regulates the measures to be taken, that is to say the commands given to the control mechanism. (4)

The energy processing capacity of a system as well as its capability of mastering information are not sufficient to determine a system's economic output. A third component is needed to describe adequately the complex processes just mentioned. On the one hand, this third factor reflects the objectives toward which the efforts of this system are directed and which is determined by the motivation structure. On the other hand, however, the way the system is organized, i.e. the regulation and coordination of the energy and, in particular, the information processing operations, also ought to play a certain role. These two components are the chief determinants of the third factor, i.e. structure. This factor thus describes the objectives of the system and the organisation of its energy and data processing subsystems, that corresponds to these goals.

This model, of which only a short outline has been given here, has served as a basis for the investigation of the societal subsystem economy. The chapters which follow will show how a country's economic performance can be described as a function of its energy and data processing capacity and of its structure.

Applying the concept described above to the study of economic development we are led to formulate the following hypothesis:

$$y = F(m, b, p, r)$$

per capita income (y) = economic output

energy consumption (m)  
per capita (capital) = energy processing capacity

education (b) = data processing capacity

structure (p) = values, type of organisation,  
types of behavior

and mineral resources (r)

## 2.2 Empirical Observation on Economic Performance , Capital and Structure

The factor capital - which, as can be shown, shows a close correlation with energy consumption - constitutes a common component of almost any economic production function. The relationship between capital input and economic performance has been dealt with in a great number of previous studies, and we find also attempts to measure this relation on the basis of international comparisons. A study by Galenson

and Pyatt (5) is such an attempt to test the assumptions underlying the theory of growth by means of an international comparison. Thereby it has become apparent that *too simple assumptions are not suited to explain the international differences in economic growth.*

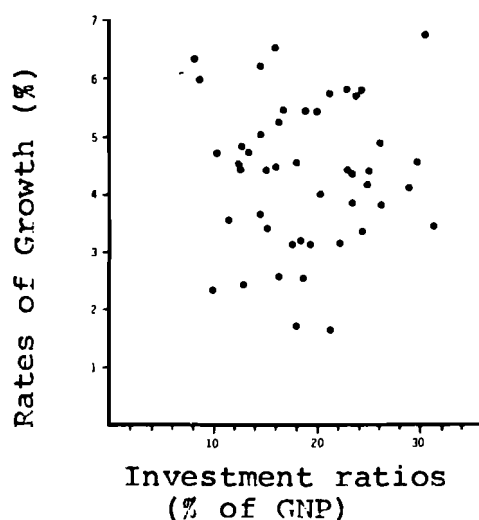


Fig. 2: *Investments and growth rates in the countries of the world.*

Obviously this study has run into difficulties. The authors are confronted with problems which are very similar to those encountered in many other studies, relying on international comparison as a means of testing their hypotheses. For the most part these studies have not succeeded in taking account of the great number of geographical, cultural, climatic and political differences among the individual countries of their sample. In other words, the observed differences in income (dependent variable) result from heterogeneous influences which cannot be adequately explained by differences in capital inputs. They must be explicitly controlled and

entered separately into the study. Not taking care of these influences corresponds to a non-fulfilment of the *ceteris-paribus* (the rest remains equal) - condition.

This gives rise to the demand for an efficient methodology which takes account of these various influences. One such method would undoubtedly consist of forming groups of countries within which the combined effect of these non-observed but relevant influences on each country is nearly constant.

An important clue for the formation of such groups of countries could be provided by observing the health development in the different countries of the world since the beginning of this century. The time series from all countries of the world for which figures are available have shown that the development of health is fairly uniform within groups of countries, these groups being at the same time sensible geographical aggregates. The differences between the groups are big enough to decide to which group a specific country actually belongs.

Without going into details of the problems encountered here, we may interpret this observation as follows: Health statistics provide one very fundamental indicator of the development of a society. Here we are measuring the "pulsation" rate of the societal system as it were. It is governed by many factors which cannot adequately be measured directly. We may consider the adoption of medical innovations as one part



of innovations in general and which we call "learning process". Thus, the regions differ as to their readiness and capability of adopting innovations. This general statement is based on the observations made in one subsystem of the society, namely the health system. For it is the greater or smaller readiness to adopt medical innovations that is responsible for a more rapid or slower improvement of health conditions.

We can assume that the "structure", i.e. the value system, the patterns of behaviour and the types of organisations they constitute is an important factor responsible for the differences observed with regard to the development of health.

Using this methodology the European countries can be classified into three, respectively two, zones according to their different health development.

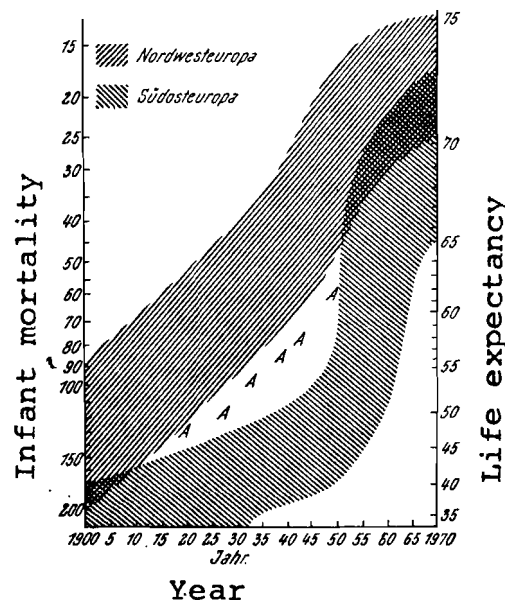


Fig. 3: Health development in Europe

There are three zones, Northwestern Europe, Southern Europe and Eastern Europe. Southern Europe and Eastern Europe show much the same development in the past but differ now more and more.

Applying the classification in groups of countries as presented in Fig. 3 to the data of Fig. 2 in order to separate the two groups of European countries we arrive at a meaningful relationship:

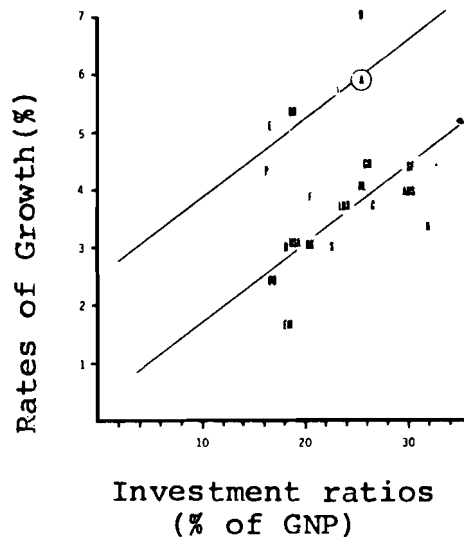


Fig. 4: Investments and growth rates in two groups of European countries.

The observations in this diagram are data taken from Fig. 2. They show that meaningful relationships exist within groups of similar countries. Krelle<sup>(4)</sup> found empirically a relationship between investment and growth, similar to the Lower Regression Line. He obtained it by the unconscious use of a group of countries with the same structure (NW Europe and N. America) and interpreted it - obviously without knowing the ILO observation of Fig. 2 - as a support for the theory of growth.

The regression lines showing economic growth per unit of investment lie on different levels in countries with different political and cultural systems. This observation suggests, that the differences of the political and cultural structure affect the effectiveness with which capital is used, thus the energy processing capacity, and that these differences are reflected also in the health development and therefore can be

identified by the use of health indicators. In addition to this factual finding, the evidence presented gives rise to the methodological remark, that reliable results and meaningful interpretations in international comparisons can only be obtained if due account is taken of the ceteris-paribus-condition. If this is carefully done, a lot of contradictions of results could be avoided.

A striking example is the contradiction between Fig. 1 and Fig. 2 of the IIASA working paper, Millendorfer, "Global Mechanisms and Global Models I: Methodological Considerations", which can be explained now after the fulfillment of the ceteris-paribus-condition.

### 2.3 Empirical Observations on Economic Performance, Education and Structure

The majority of the studies made so far have considered man as an element of an undifferentiated factor "Labor" and have used just the number of people in the working force as one of the input factors. Various abilities, differences in the level of education (which correspond to differences in information processing capacity) have so far been neglected, but if we understand the information processing capacity of society as a determining factor of the performance of the system we should expect a close correlation between a country's level of education and its economic performance.

Closer examination of the relationship between education and income in a cross-section of countries reveals that the aggregation of countries according to the criterion of similar health development is also a good criterion for this purpose. Under this *ceteris-paribus*-condition we find empirically the expected relationship.

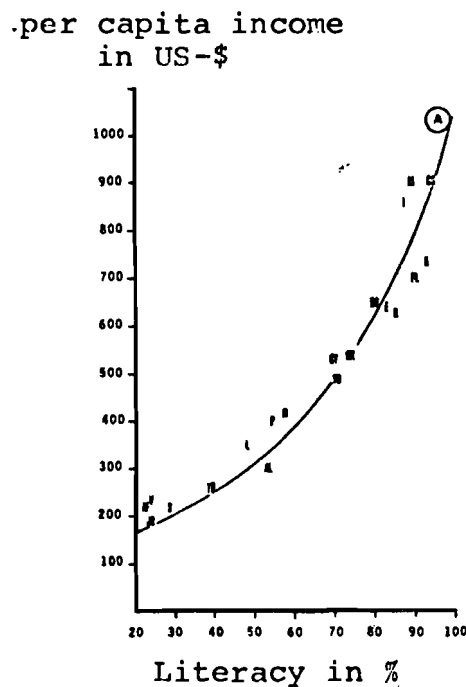


Fig. 5: Per capita income and literacy rates in groups of countries with similar structure.

*The diagram shows the close fit to the exponential relation between the economic output and level of education.*

The regression coefficient of the education is practically the same in all the regions, defined by the criterion of equal health development. The intercepts of the regression lines, however, differ from one zone to the other. Significant

differences, however, do occur only between five large zones of the world, as will be shown below.

The evidence presented thus far can be summarized as follows: There is empirical support for the hypothesis that a country's *economic performance is mainly determined by three factors, namely capital (energy), education (information), and structure.*

#### 2.4 A Modification of the First Preliminary Hypothesis: Natural Resources as an Additional Factor

Examination of the output differences among the countries of the world reveals that, apart from the factors discussed so far, the occurrence of *mineral resources is an additional relevant factor.*

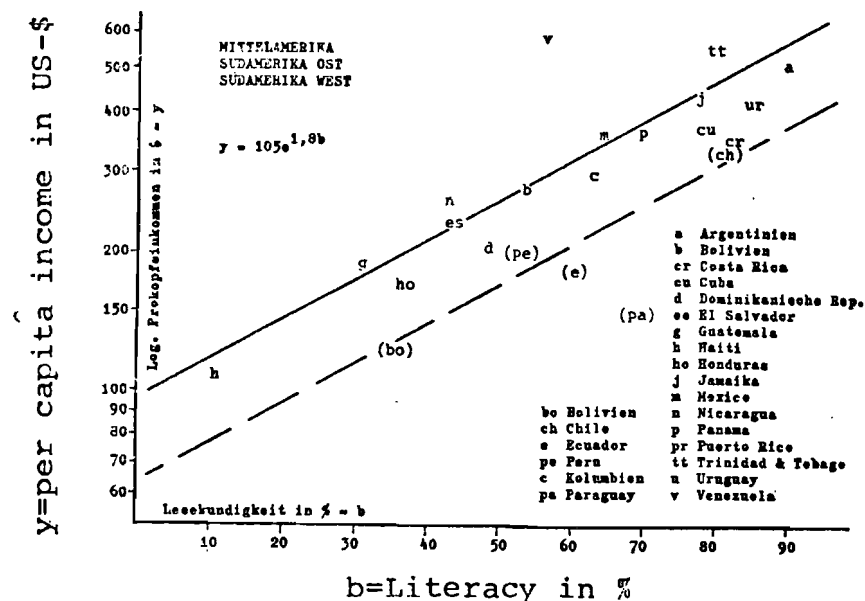


Fig. 6: The special position of Venezuela, Trinidad and Tobago in the comparison of per capita income and literacy in South America.

Venezuela's large deviation from the regression line is very striking and can be explained, even quantitatively, by this country's important oil production.

The important influence of mineral resources on the level of economic output is supported by observations in other regions, in particular in the countries of the Near East and in Africa.

### 3. Extending Hypotheses and Observations

The concept of society as an information and energy processing system is a heuristic basic hypothesis from which different individual hypotheses of heuristic nature were derived. Some examples of deriving such a hypothesis for interpretation of observed relationships or for stimulation of new empirical observations is outlined briefly.

#### 3.1 Additional Hypotheses and Observations on Structure

The information processing section of the societal system exerts in addition to a learning function also a self-organizing function. According to this concept, development is a self-organizing of a learning system. If the system is closely connected with another system through the respective communication channels, mutual adaptation occurs in this self-organizing process; structures develop in a similar way. For actual society structures and mean values, modes of behaviour and organisational patterns, the hypothesis that the similarity of the health development of countries within one group reflects similar structures is supported by different empirical observations.

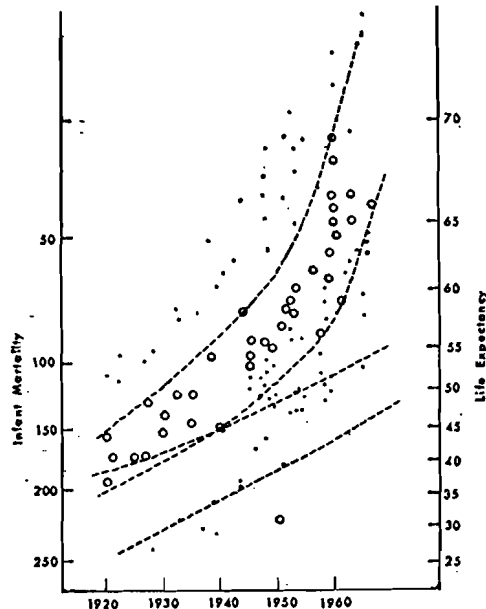


Fig. 7 Health Development in the Carribean

The interrupted lines signify the borders between two areas, within which the health data analogous to Fig. 3 of two Latin American groups of countries are located. The slowly rising zone applies to the West Coast states. The fast rising zone applies to the Central American states. Within the last group we find as a sub-group the Carribean Islands, the larger ones with a population of over one million, being represented by circles and the smaller islands represented as dots. The dots are much more far-spread than would be explained by the smaller number of random samples taken on the small islands. If we assume that the larger islands have more communication with each other than the small ones, this could be the underlying reason for their more similar development which is reflected in the fewer variations of health data.

These observations and hypotheses lead to the interpretation that the criterion of equal health development indicates something like closer connections between the countries of a group according to this criteria. As this closer communication leads to a mutual adaptation of the structure, the countries in a group developed similar structures, i.e. values, modes of behaviour and organisational patterns.

The equality of groups of countries aggregated according to the criterion of health development or the economic efficiency of education supports also the hypothesis that the health criterion comprises a much larger area. It is likely that this area is related to the ability of incorporating innovations into the social framework and the structural preconditions related to it. We applied the criteria of equal health development to build meaningful aggregations of countries in the world and found using this methodology fourteen groups of countries with different "structure".

### 3.2 Additional hypotheses and observations on capital

The first industrial revolution was characterized not so much by the rising amount of capital in monetary terms - capital accumulations occurred also long before in the form of palaces, castles, bridges, etc. - but more by the rising use of artificial energy in the production process, i.e., the tremendous extension of the energy processing capacity of the society; modern capital became identical with the application of artificial energy. So it is not surprising that time series of indices for capital stocks of Belgium, Denmark, the Netherlands, France, Germany, Norway, the United Kingdom, and the United States, published by Denison<sup>(6)</sup> can be fairly well approximated by a combination of energy indicators like total energy consumption and electricity consumption. In this context, we can understand energy as indicator for capital and observations on energy as information on capital. As an example, the following observation may be interesting.



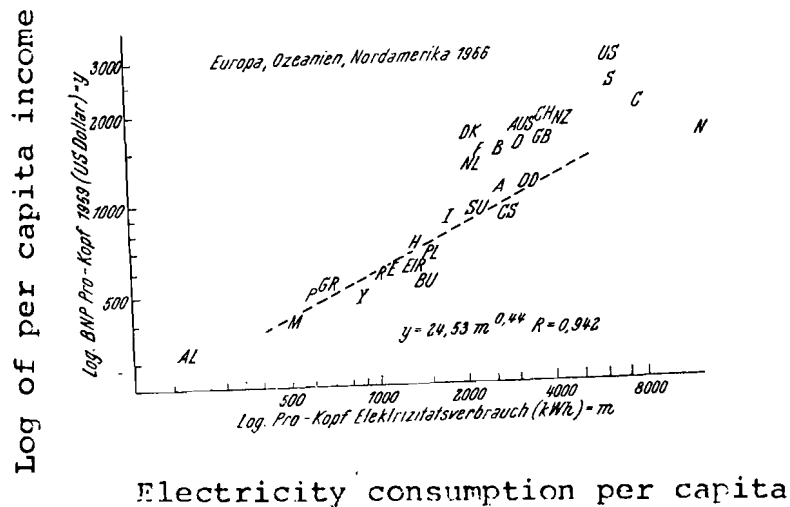


Fig. 8 Electricity Consumption and Income Per Capita in Europe.

Two zones are clearly evident, Northwestern Europe and Southern and Eastern Europe. Unlike Fig. 6, the zone of Northwestern Europe is at a higher level.

The different efficiency of how energy respectively capital is employed is another criterion for aggregation, that leads, as Fig. 8 shows, to the same grouping as the criterion of similar health development. This and the significant differences in the investment-growth ratio (Fig. 4) the different economic efficiency of education (as shown in the next paragraph) or other observations like the different share of agriculture or industry in the total economy confirm the aggregation achieved by the first criterion.

Using this aggregation, we can calculate the regression coefficient for every zone respectively for the corresponding

clusters in the diagrams.

Combining all zones by the use of dummy variables, we obtain in different cross-section analyses for different years very stable relationships between the variables with high correlation-coefficients. The relation between per capita electricity consumption (m) and per capita income (y) is then

$$y = m^{\frac{1}{2}} \quad (1)$$

(While the exponent is not exactly  $\frac{1}{2}$ , it does not depart statistically from this value used for the sake of simplicity).

### 3.3 Additional Observations and Hypotheses on Education.

The level of education was computed as stock of educated people in the primary, secondary and tertiary level by adding the figures of school enrolment of these categories of education over a longer period of time. Partly the various existing studies on literacy were collected and from the time series of literacy of various countries, data for a given period in time were determined.

Literacy was used initially as a measure of primary education. It turns out that relationships can only be found with sufficient accuracy if the data quoted for the different years is related to exactly the same year for all countries. As an indicator for economic potential we have used the real per capita income of the individual countries, which has in turn been related to a unitary point in time. The two variables

for the separate groups of countries mentioned above were put in relationship. The result was encouraging: within each group, a surprisingly significant connection was shown between literacy and per capita income following at a time-lag.

The following picture emerged: if the per capita income is measured on a logarithmic scale an approximately uniform gradient of the regression line can be established for each group of countries; the level of the regression lines, however, varies from area to area. The most consistent results can be obtained for a time-lag of thirteen years. The relationships illustrated in Fig.9-12 are based on income values for 1966 and literacy for 1953.

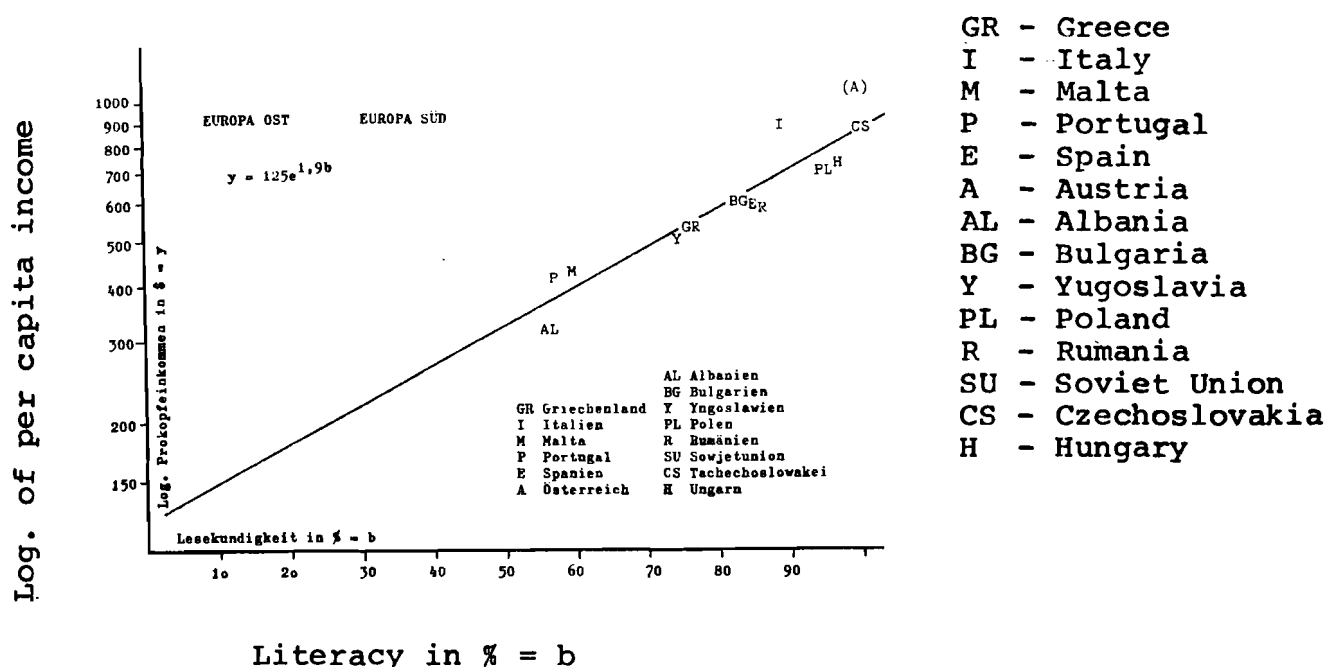


Fig. 9

Figs. 9 to 12. Literacy and per capita income in the various groups of countries

y = per capita income (Log)

b = measure of education, measured here in percentage of literacy

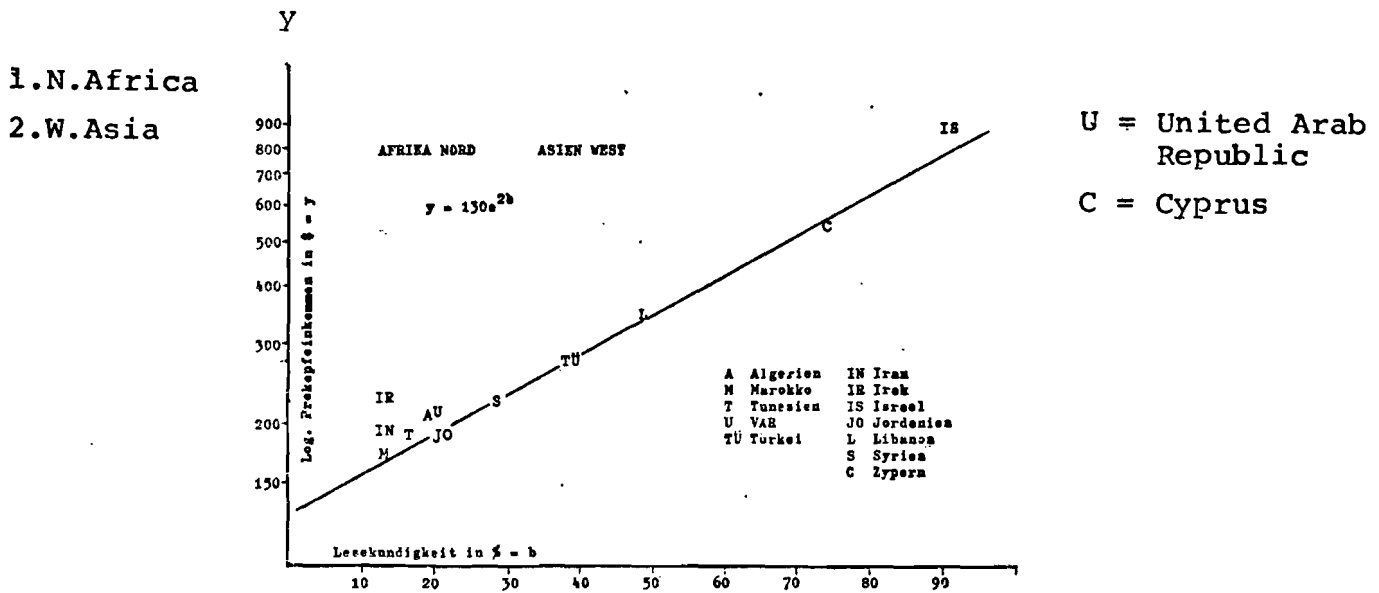


Fig. 10

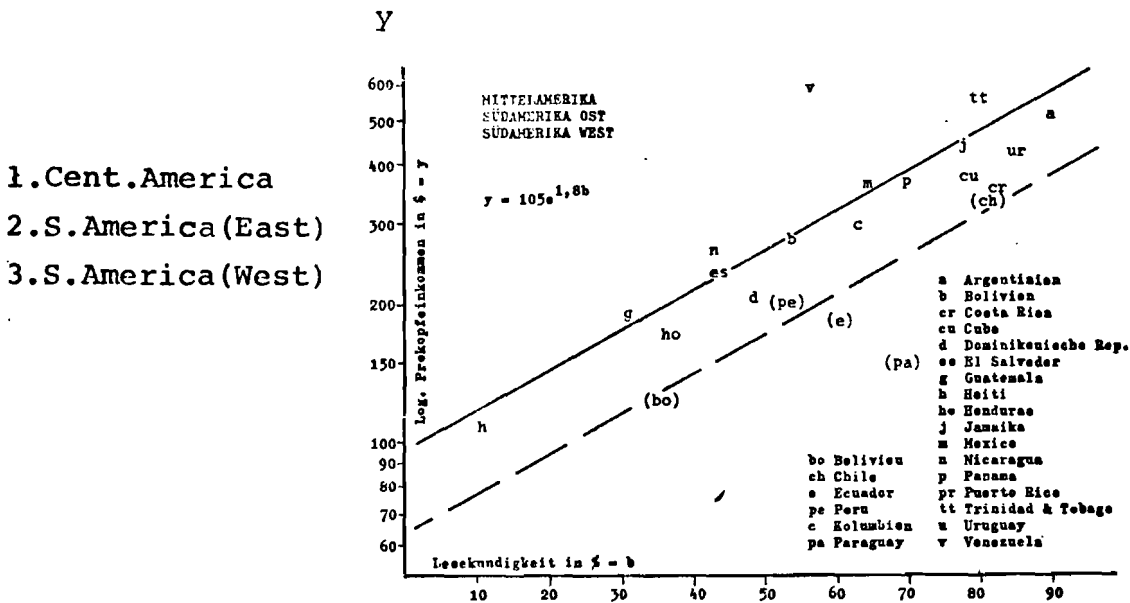


Fig. 11

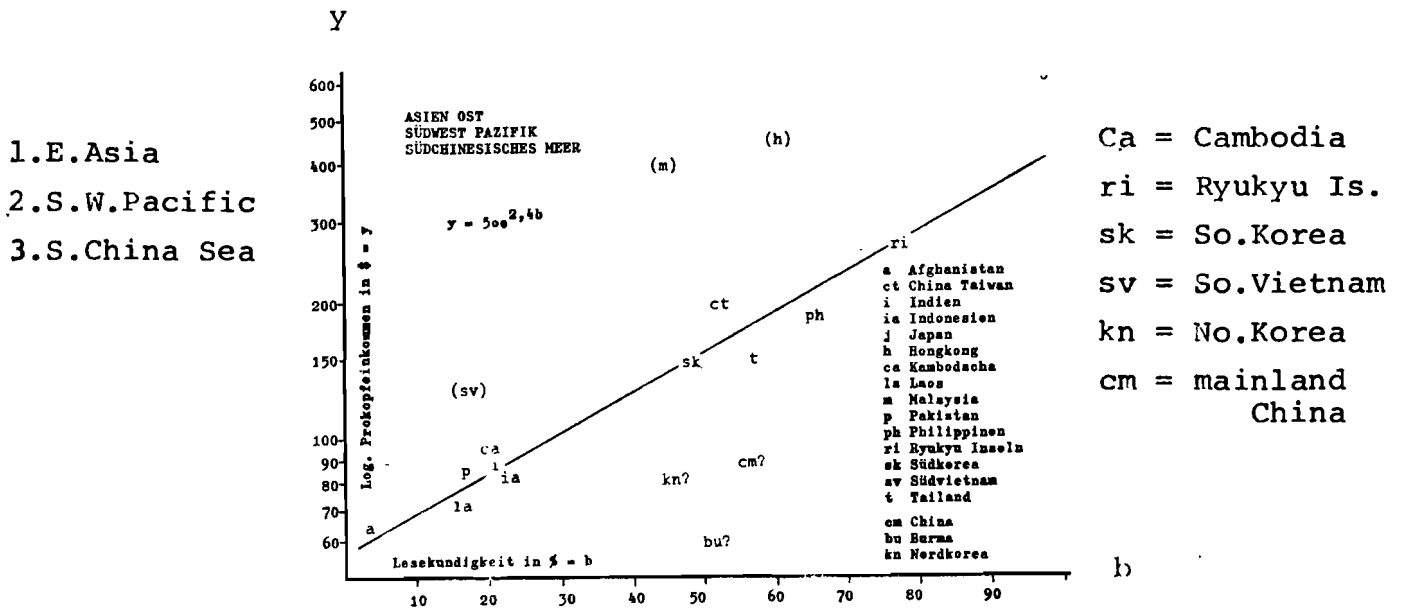


Fig. 12

In the equations recorded in Figs. 9 to 12, each of the groups of countries comprised in the separate diagrams has been assessed separately, except for South America (West) and East Asia, which have been combined. Hence, a somewhat different  $k$  emerges for each group. In equation (2), on the other hand, all the groups of countries were assessed simultaneously and thus only one  $K$  was established for the whole world, the various intercepts being determined with the help of dummy variables; the distribution of groups remained the same as before, but West Asia and North Africa were combined with Southeast Europe.

Countries in round brackets were disregarded in the assessment because they stand out as truly exceptional cases; Hong Kong consists almost entirely of an urban region and has a

special position commercially, Malaysia possesses rubber as a natural resource which is not calculable in the mining index, and the special situation of Vietnam is well known. Paraguay is excluded owing to an extremely low value for the material factor described below. The same in part applies to minor exceptions like Albania, Costa Rica and the Philippines, which are included in the assessment.

Regression analyses were carried out for sixty countries with a literacy rate of less than 95%. The African states south of the Sahara and Cambodia were not included in the calculations, because data for these countries does not appear to be reliable enough. It transpired that the average exponent for all groups of countries is close to 2 and that in no single group of countries does the exponent vary significantly from this value.

$$y = c_z e^{2b} \quad (2)$$

$y$  = per capita income

$c_z$  = constant of the zone, varying from one group of countries to another

The level of the regression lines does not have a different value for each of the fourteen groups of countries; on the contrary, one arrives at the unexpected result that the point of intersection of the separate regression lines for the whole world, determined with the aid of dummy variables, assumes only three significantly different values. A fourth regression line

may lie below the three that have been measured, and include the area of China which could not, however, be determined for lack of precise data. A fifth area lies above the three regression lines and includes Northwest Europe, the USA and Australasia. (Here no meaningful measurements for the present day can be made with literacy; the application of a different education indicator will be described below.)

For the period when 100% literacy in Europe was still far short of being achieved and when literacy, therefore, which can now be used as a measure of education for developing countries only, was a meaningful measure of education for Europe too, the same relationship applies as has been found in a comparison of countries for the present day. The only difference is that before the First World War, Europe cannot be classified into differing areas (Fig.7). Bearing in mind the quality of the material data, this comparatively good relationship is all the more surprising. The validity of the connection between a society's education and its economic potential over such a long period suggests that it may be a question of a fundamental relationship.

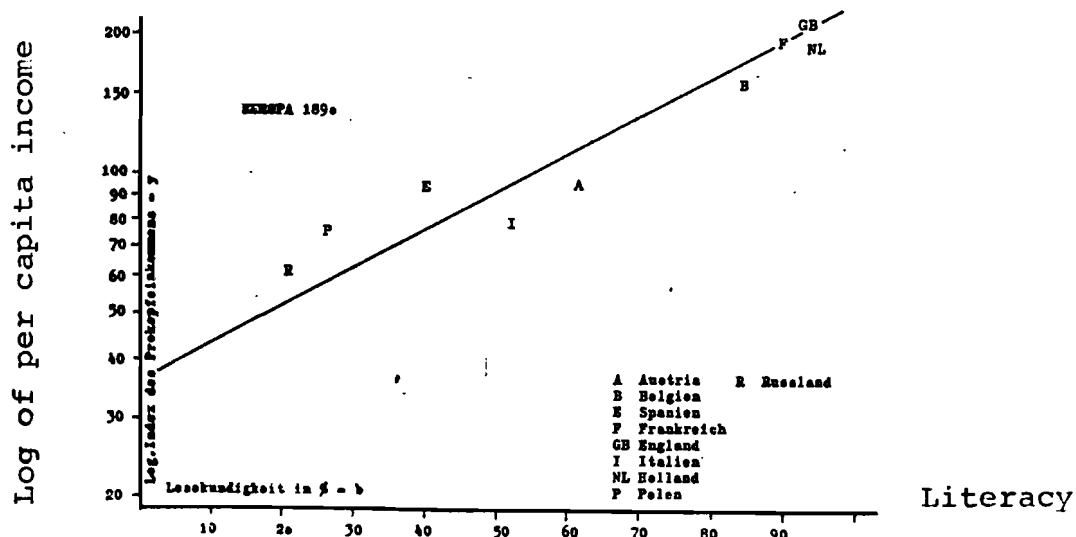


Fig. 13 Europe 1890: literacy - per capita income.

### 3.4 Additional Observation: Influence of Minerals on Income; Improvement of Formula (2).

If one observes the way in which the observation points are spread around the regression lines, one is struck directly by the extremely strong positive deviations of the oil producing countries such as Iraq, Iran, Libya, Kuwait, Venezuela, Trinidad and Tobago (Fig.6). These observations show clearly that possession of oil reserves makes it possible for a country to have an income higher than would correspond to its level of education. This fact, coupled with observations in some African countries which are particularly rich in gold, copper and other minerals, suggested the conclusion that in general the quantitative recording of mineral production and its inclusion in the regression function would give a sharp boost to the declared value. For this purpose an index of minerals produced was calculated (see Appendix) and included in the reckonings. It turns out that the correlation is thereby improved and that the per capita value of minerals produced, with a fixed percentage, is supplementary to the income derived from education.

$$y = c_z e^{1.99 b} + 0.8q \quad R = 0.957 \quad (3)$$

$y$  = per capita income

$b$  = measure of education, here measured by literacy

$q$  = index of minerals produced

$c_z$  = zonal constant ("efficiency parameter")

(2)  $c_z = 122.1$  for these groups of countries: S.Europe, E.Europe, W.Asia, N.Africa, S.Africa <sup>(7)</sup> and Gold Coast <sup>(7)</sup>

(3)  $c_z = 91.8$  for these groups of countries: Cent. America, S.America (East)

(4)  $c_z = 63.7$  for these groups of countries: S.America (West), E.Asia, S.Asia and Cent.Asia, S.China Sea, S.E.Africa <sup>(7)</sup> and Cent. N.Africa <sup>(7)</sup>.



# Developing Countries 1966

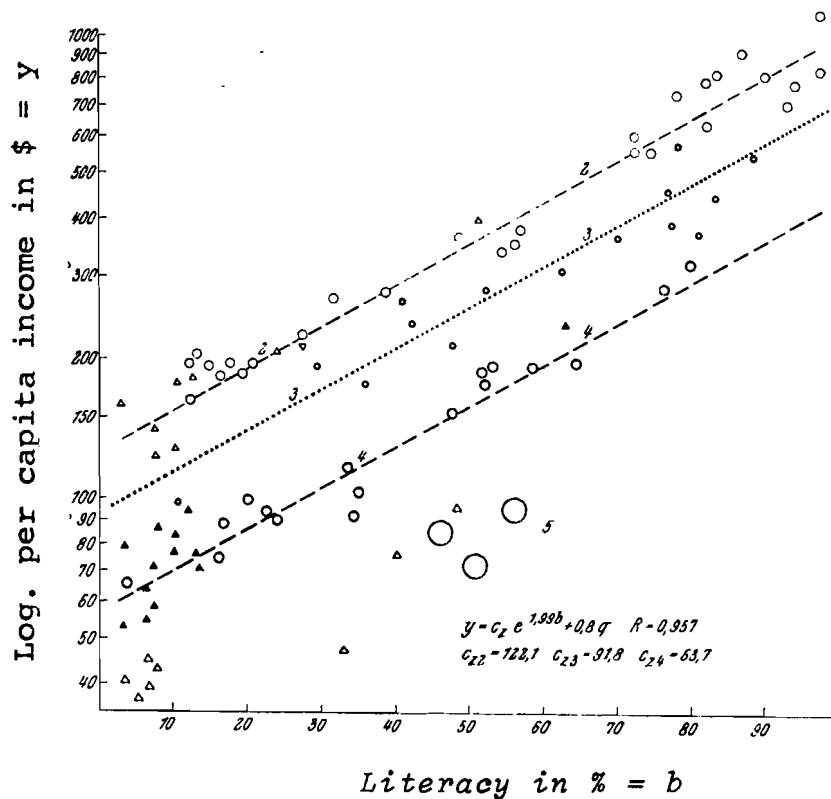


Fig. 14 Literacy 1953 and per capita income 1966 developing countries.

This diagram represents a condensation of Figs.6-12. The various heights of the regression lines correspond to the different values of the constant  $c_z$ , which can be interpreted as efficiency parameters. <sup>z</sup> The main areas corresponding to these different values of  $c_z$  are illustrated in Fig. 15. (The triangles represent African countries situated south of the Sahara, which have not been included in the calculations because for the majority of these countries the statistical data do not appear to be sufficiently reliable).

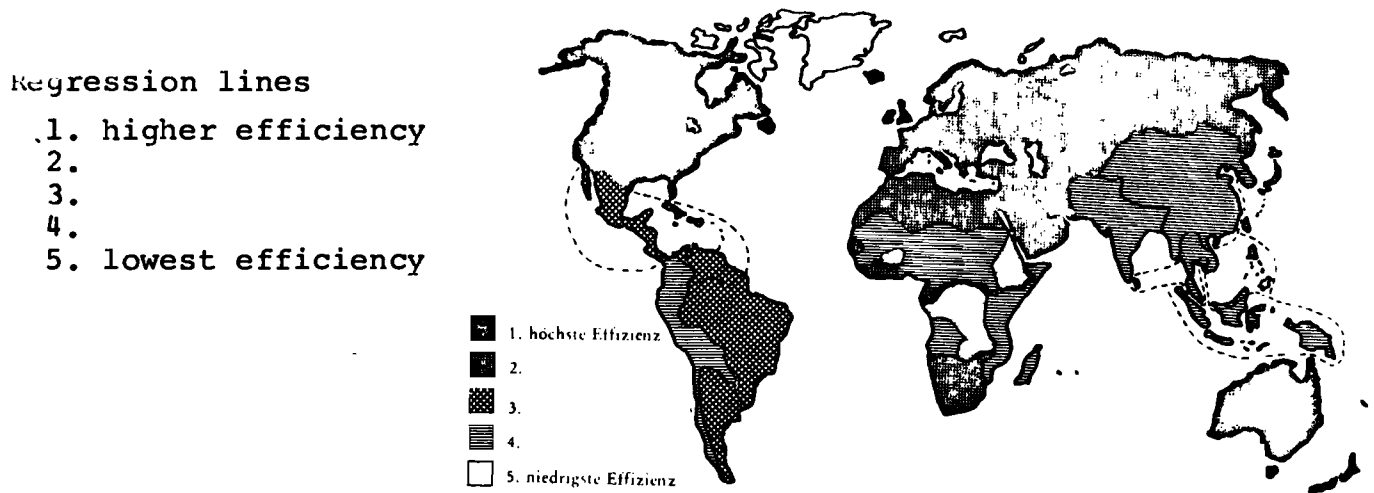


Fig. 15

Fig. 15 Main areas with different economic efficiency of education.

The main areas correspond to certain values of the constant  $c$  in equation (3), or to different regression lines in Fig. 14. The demarcation in Africa south of the Sahara has been undertaken corresponding to the position of these countries with relation to the regression lines which were calculated without them.

Equation (3) is illustrated in Fig. 14; the main areas belonging to it can be traced from Fig. 15.

### 3.5 System Theoretical Interpretation of Formula (2) resp. (3).

The ability to read and write is of fundamental importance for social relations <sup>(8)</sup>. It enables everyone who is master of this skill to get in contact with anyone else. Whoever can read and write is no longer dependent

on personal contacts for receiving information; he can rather disseminate, with or without personal contact, knowledge coming from outside; consequently he is in a certain sense an ancillary channel of information. What is more, he has the ability to store information, or to turn stored information to account if need be. This function is particularly important where technical or other instructions have to be carried out repeatedly at intervals. Literacy can consequently be construed as a fundamentally important measure for social transmission and storing of information.

If one interprets the literacy indicator as such a measure of information, the exponential relationship with economic potential then appears in a new light. For this, a few considerations are necessary which proceed from the basic concept mentioned above, that society is a learning system.

By the use of terms from information theory and systematology, let us outline a theoretical derivation of the relationship between the potential of a system and the information that causes it, which may be able to explain our empirical observations.

(a) We interpret the potential of a system as ability, in the sense of mastery of successful strategies. The potential is set in proportion to the number of successful strategies available.

(b) The information whether an available strategy is successful can be gained from the system itself or transferred from another system; we are dealing with the latter case.

(c) The number of strategies from which a successful strategy is chosen depends exponentially upon the information transferred from the teaching system to the learning system. This corresponds with a proposition of information theory which states that the information necessary in order to select an element from a certain quantity is proportional to the (dual) logarithm of the number of elements.

(d) From (a) to (c) it follows that the logarithm of the potential of a system is proportional to the information which is necessary in order to produce this potential. To put it another way, the potential of a system rises exponentially with the number of informations about successful strategies.

According to what was said above concerning literacy, the number of informations transferred can be fixed as a proportion of literacy. Hence it follows, having regard to (d), that the potential of a society is exponentially dependent upon the literacy.

If this interpretation is taken as a basis, this empirically found relationship could be of fundamental importance. It provides a possible description of the relationship between the capacity of a system and its information potential or data-processing potential, on principle - irrespective of the type of system. Certainly this relationship in the case of social systems is empirically verified for the majority of developing countries in the mid-20th century and appears also to be valid for the Europe of around 1890. (See Fig. 13).

#### 4. The Final Result: Combining the Factors to the General Production Function

According to the first hypothesis, the understanding of the society as an energy and information processing system, the empirically observed relationships between economic performance and capital with respect to energy on the one hand and education on the other hand should be combined in an appropriate way. An empirical test of this theoretical consideration shows that *the combination of capital, education, structure and natural resources has a much higher explanatory power than the simple regression on a single factor*. This is demonstrated graphically in Fig. 16.

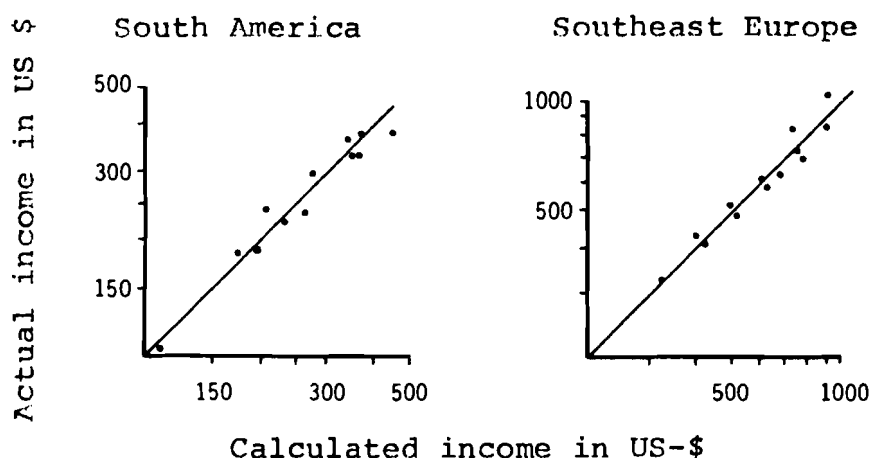


Fig. 16: Comparison of observed per capita income and its estimated as calculated value when account is being taken of all the factors

All factors combined yield a much higher explanatory power. Discrepancies between observed and calculated values are insignificant.

The individual factors are combined by means of the so-called General Production Function, which has the following mathematical form:

$$y = P_{t,z} m^{\frac{1}{4}} e^b \left[ \frac{1}{2} \left( \frac{m^{0,25}}{e^b} \right)^{-\rho} + \frac{1}{2} \left( \frac{e^b}{m^{0,25}} \right)^{-\rho} \right]^{-\frac{1}{\rho}} + 0,8 r \quad (4)$$

y ... per capita income, measured in US-dollars  
per inhabitant (annual)

$P_{t,z}$  ... efficiency parameter

m ... per capita capital input, measured by means  
of energy indicators

- b ... qualification of labor, measured by using educational indicators
- r ... natural resources index, measured by the value produced by mining operations, etc.

Expression in parenthesis: limitationality parameter. This describes the diminishing efficiency of additional input units of capital or educational effort when departing from the optimal relationship (see below).

The exponent of  $m$  and the coefficient of  $b$  have been estimated in a cross-section of countries by econometric means. (While the parameters are not exactly  $1/4$  or  $1$ , respectively, they do not depart statistically significantly from these values used for the sake of simplicity).

A reason why equation (4) is called General Production Function is because of an interesting relation between this equation and the well known Cobb-Douglas Function which is of the type most frequently used in computing and estimating macro-economic production functions. It has been estimated for the first time for the United States and describes the relationship between both capital and labor inputs and income. *The Cobb-Douglas Function can be regarded as a special case of the General Production Function: if the two factors education and structure remain constant (social technology, organisation and management, values system, etc. remaining static). In this static case the General Production Function becomes formally identical with the Cobb-Douglas function.*

If we convert the per capita relationships of equation (4) to total values, while assuming no major departure from the optimal relationship (leaving out the expression for complementarity) and disregarding natural resources, we get

$$y = \frac{Y}{V} = P_{tz} \left( \frac{M}{V} \right)^{\frac{1}{4}} \left( \frac{A}{V} \right)^{\frac{3}{4}} e^b ; Y = \underbrace{P_{tz} e^b}_{\text{constant}} \cdot M^{\frac{1}{4}} A^{\frac{3}{4}} \quad (4a)$$

capital letter ..... total value

V ..... population

Thus the time-series of American production from 1899 to 1922 explained by Cobb and Douglas could equally well be explained by the General Production Function, assuming education and structure to have remained more or less constant over this period. As a matter of fact (as can be seen in Fig.17) the education explosion in the USA started after 1922. Similar observations might be true about the structure which possibly began to crumble and become dynamic only with the onset of the Depression.

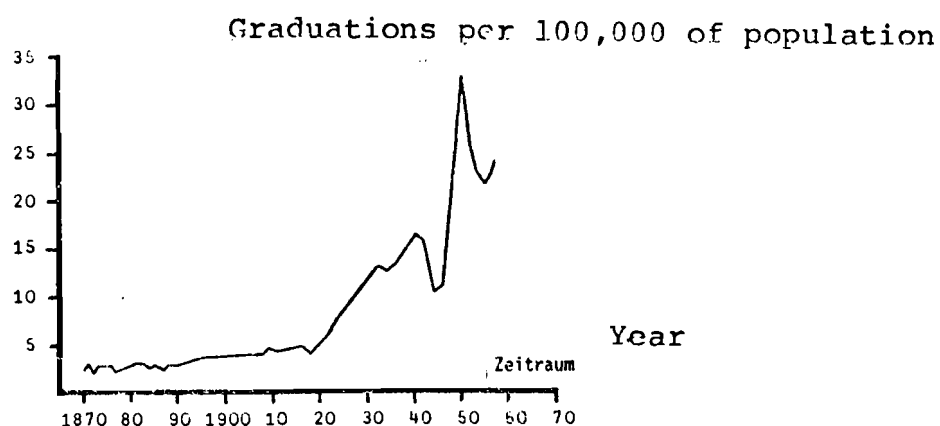


Fig. 17: Number of college graduates in the USA for the years 1870 - 1970

*It is clearly evident from this diagram that the education explosion in the USA began only after 1920.*



## 5. Essential Characteristic Traits of the General Production Function

### 5.1 Capital as a factor of production

In order to estimate the General Production Function the energy processing capacity is measured on the basis of data for the total energy consumption of a country as well as by using the figures for electric power consumption. In using these measures it became apparent that for countries for which capital stock time series exist, these time series have a remarkably high degree of correlation with an indicator reflecting the changes in total energy consumption and in the use of electric power. This can be expressed in the following relationship:

$$k = m_1^{\frac{1}{2}} \cdot m_2^{\frac{1}{4}} \quad (5)$$

$k$  = capital per capita

$m_1$  = energy consumption per capita

$m_2$  = consumption of electricity per capita

(while the parameters are not exactly  $\frac{1}{2}$  or  $\frac{1}{4}$  respectively they do not depart statistically significantly from these values used for the sake of simplicity).

The fact that energy consumption turns out to be such a useful capital indicator has its deep-lying causes. Introduction of "artificial" energy into the production process was the characteristic feature of the First Industrial Revolution.

If one wanted to put a point on it, one might even go as far as saying that capital stock were an indicator for the utilization of "artificial" energy in the production process. In any case, however, "artificial" energy turned out to be the prerequisite of mechanisation. This - at least in part - explains the close relationship between energy utilization and capital.

The contribution capital makes to the economic output remains more or less the same, regardless of whether we use the General Production Function or the Cobb-Douglas Function in its computation. Furthermore, in the General Production it corresponds at least approximately to the contribution of capital to output arrived at by using the Marginal Productivity Theory and assuming perfect competition, when viewing capital's empirically observed returns.

Another such observation relates to technological development in the production process. Looking at the world-wide development over time, the relation of electric power used to total energy consumption is changing, since the use of the former increases by a square ratio to the increase in total power needs. This relation of electric to total energy consumption is frequently used as a measure for the technological level of the stock of capital(machinery, etc.). Thus it is related to capital-induced technological progress. One study by P.B. Du Boff,<sup>(9)</sup> among others, seems to support this interpretation. In dealing with the development of the

United States in the years from 1879 to 1962, Du Boff states: "... technological innovation frequently is difficult to represent quantitatively. Output per unit input does rise for non-technological reasons ... but there must be important cases where the reverse is true. One such case seems to be electrification of manufacturing industries. Here the rate of technical change can be reasonably well measured in terms of horse power capacity of power equipment and consumption of power. Furthermore, this 'revolution' in the application of power can be viewed against the background of clear, known changes in manufacturing productivity ...". And, after dealing with the various functions and uses of electric power in plant modernization, he states: "Electric power affected the whole scope of these processing services, by revolutionizing the application of energy to materials."

Summarizing we can say that *energy indicators can be used for measuring capital stock*. Doing this, we take, moreover, into account actual utilization of capital, which corresponds to what Solow calls "Employed Capital". It encompasses only that part of capital defined as "active" capital stock (viz. plant and machinery excluding buildings).

## 5.2 Education as a factor of production

When it came to the task of measuring the information processing capacity of the system - which is closely tied to the qualification of labor - figures pertaining to the educational structure were used. These figures were derived from

enrolment data in three different levels. They represent the amount of primary, secondary and higher education in the respective countries.

Education appears to be a more reliable measure of qualification of labor than, say, length of employment - which could be taken as an indicator for learning-by-doing. This is so because the latter is not simply time-dependent, but is highly modified by the native intelligence, educational level, and willingness to learn of the person subjected to this process. If one, on the other hand, uses educational statistics as indicators, one arrives at a measure probably containing all the components just mentioned. Such educational data not only state mere length of training, but also - as a result of drop-outs and retardation - the components "adaptability" and "willingness to learn".

Summarizing the empirical evidence concerning the relation between education and the other factors of the General Production Function we may state: first we find an *exponential relationship between the qualification of labor (i.e. education) and economic performance*; then we observe that there exists a certain amount of *complementarity between qualification and capital*; and finally we see that a similar relationship holds for the subfactors of education, i.e. *primary, secondary and higher education are substitutable only in a very narrow range*.

As opposed to these concepts which understand the contribution of education to income as the rates of return to human capital (i.e. capital accumulated in the process of education), the empirical evidence presented supports very strongly the existence of an exponential relationship between the two variables. This is to say, that *a linear increase in education produces an exponential increase in the economic output*. If we interpret, moreover, education as an indicator for the information processing capacity of the system, we see that this exponential relationship can be deduced from the basic concepts of information theory.

Another important aspect to be dealt with when scrutinizing the relationship between education and economic output is the fact that *a time lag of ten to fifteen years exists* between an "injection" in one of the categories of education and its eventual effect on the economy. It is this time lag which makes a trial-and-error control of the educational system via the labor market so extremely difficult and which makes this type of empirical market feed-back so prone to oscillations.

### 5.3 Complementarity of factors

The factors education and capital do not occur in random combination, but we observe only a limited degree of substitutability between these factors. This implies that *the information processing capacity is tightly connected to the energy processing capacity*. An expansion in one sector only produces a deviation from the optimum relationship. Such a

deviation is characterized by a suboptimal use of the factor which is in surplus.

Empirically, we have arrived at the following optimal relationship:

$$m^{1/4} = e^b \quad (6)$$

A departure of either education or capital from this optimal relationship results in the relatively smaller of the two inputs becoming a bottleneck for the economic performance of the system. In such a case, increasing the proportionately larger input will not yield a significant increase in production.

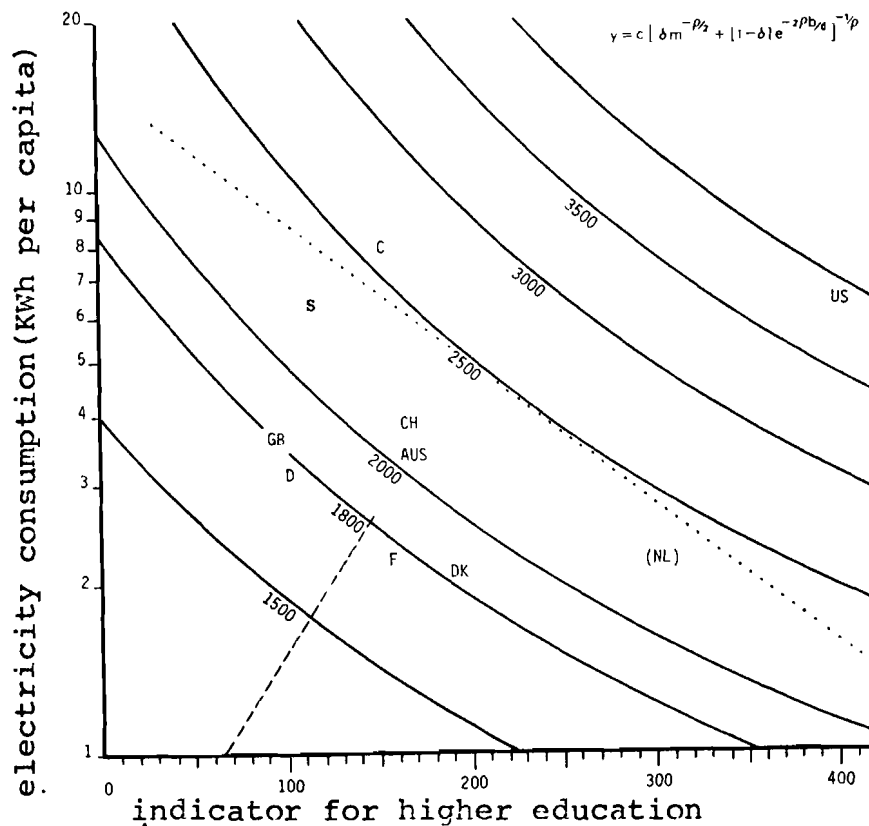


Fig. 18: Limited substitutability of education and capital. The curvature of isoquant describes the limitation of substitutability of the two factors. Unlimited substitutability would correspond approximately to the dotted tangent of curve 2500. The interrupted line represents optimal relation.

The concept of limited complementarity can be meaningfully applied also to the various categories of education. It turned out that the numerical relation between the various educational categories differed from one group of countries to the other. Furthermore, *there exists an optimal relationship, in the case of which the economic effect of education (as a whole) is maximized.* If one of the educational categories is less developed than this optimal relationship demands, that level becomes a bottleneck of education. Such a bottleneck exists, e.g. in the Arabic countries in the field of primary education, while in the Latin-American nations the shortage is in the secondary education. The latter applies to the countries of Southern and Eastern Europe, too. Investing in that category of education which represents a bottleneck yields the highest returns. An identical investment in a "surplus category" produces much smaller gains.

#### 5.4 Structure as a factor influencing production

##### 5.4.1 Static features

Measured in cross-section the efficiency parameter  $P_{z,t}$  of the General Production Function is the same for all countries belonging to one group at a given time. As previously stated, analyzing efforts in the field of health makes a division of countries in the fourteen distinct groups possible. Some of these groups have very similar efficiency parameters and thus can be aggregated into Greater Zones. The characteristic trait

common to all component states of each Greater Zone is a constant relationship between capital and education effort on one and economic performance on the other hand. *Countries within each zone are using capital and education with equal efficiency.*

Using these criteria for the mid-sixties, the world could be divided into five Greater Zones with differing efficiency parameters. This Greater Zone is identical with the main areas with different economic efficiency of education as shown in Fig. 10. It is interesting to note that Greater Zones with differing efficiency parameters - which were calculated using only the data for per capita income, education, capital and natural resources - strongly resemble the various groups of countries and areas of a map showing dominant religions. The zone of maximum efficiency consists almost exclusively of Protestant nations, the one ranking just below it of Catholic countries - including Greek Orthodox and formerly Christian countries of the Mediterranean region. This would seem to indicate that Max Weber's theories could be to some extent empirically verified. The results of McClelland's research can be interpreted in a similar way.

The delineation of these Greater Zones of varying efficiency leads one to suspect that religion,



and especially the system of values, is of decisive importance when it comes to economic development, i.e. that marked differences in efficiency are determined by cultural behavior patterns and their appropriate structure of social organisation.<sup>(10)</sup>

#### 5.4.2 Dynamic features

Gross section analyses have been extended for different years in the past since 1900 and in some cases before 1900 (see Fig. 13). In this chronological analysis of world-wide development, or of the development of a specific group of countries, changes of the efficiency parameter over time become apparent. This - compared to the old residual factor of the Cobb-Douglas Function - *new residual factor of the General Production Function reflects world-wide increases in efficiency*. These increases result on one hand from the development of new technologies and their adaption to the production process and on the other from improvements in the organisation structure. According to the usual terminology we call this residual "technological progress". But this technological progress is different in every Greater Zone.

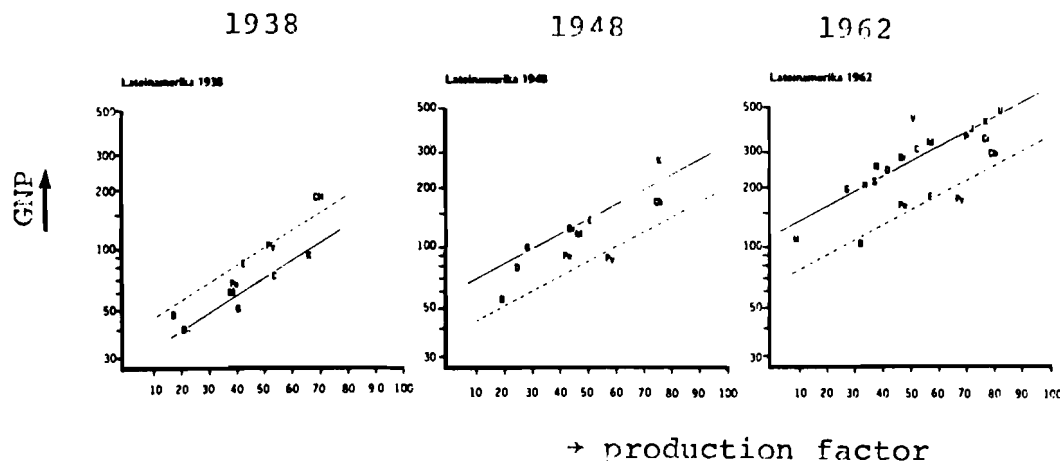
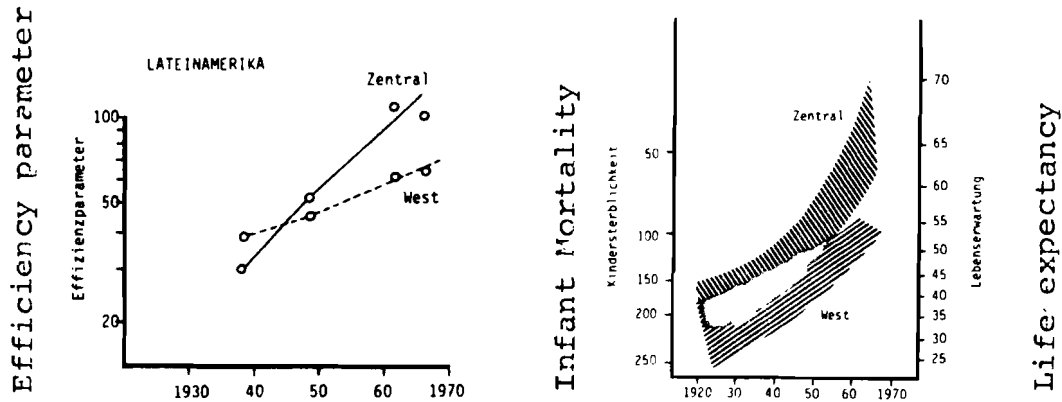


Fig. 19: Different speed of growth of the efficiency parameter of the General Production Function in the two Latin American zones.

The efficiency parameter is represented by the level (intercept) of the regression lines (solid line: Latin America Central and East; dotted line: Latin America West)

This could be explained by the following hypothesis:

The differences in the "technological progress" - i.e. the differences in the development of the efficiency parameters over time respectively the differences in the residuals of the General Production Function applied to time-series - that exist between the separate group of countries result from varying changes in the structure, i.e. the culturally determined behavior patterns and corresponding forms of social organisation. Differences in the "technological progress" are connected with differences in the structure. In this context is an interesting observation, that in a Greater Zone the "technological progress" measured by the growth of the efficiency parameter of the General Production Function is similar to the "medical progress" measured by time series of health indicators



Technological progress  
(efficiency parameter =  
level resp. intercept  
of the regression lines  
of Fig. 19).

Medical progress

Fig. 20: Similarity of technological and medical progress  
in the two Latin American zones

This observation and the interpretation that behind both kinds of progress lies the culturally influenced ability of the Great Zones to accept the world wide advances of knowledge supports the hypothesis that the speed of the technological (and also medical) progress depends on culturally determined behavior patterns, systems of values and organisational structures. The technological progress has two components: the world wide - say exponentially-growing advances of knowledge, and the culturally determined ability of the different regions to accept and transform these advances for regional concepts of concrete development. It is highly likely that the complementarity mentioned in 5.3 also exists between the

production factors capital and education on one side and the structure of the system on the other. This means that *a quantitative increase in the factors capital and education must be matched by corresponding progressive changes in the structure of organisation* in order to enable the system to process and utilize effectively ever increasing quantities of data and energy. Changes in values and attitudes on one hand make changes in the structure of organisation possible, on the other hand they themselves actively contribute to such changes. When structures have become rigid, and values are formally maintained, they soon become either hollowed out or changed from inside.

#### 5.4.2 Natural resources as an input factor

Natural resources are taken into account according to an index based on the total value of mining and crude oil production. Using regression calculation, one finds that *approximately 80% of the value of natural resources are added to the income determined by the other factors.*

### SUMMARY OF THE RESULTS HITHERTO OBTAINED

The results thus far obtained by our method can be summarized as follows:

The effectiveness of the social sub-system economy can be described by the General Production Function, which shows certain characteristic traits. It describes economic output in terms of both material and non-material inputs, as well as of systemic structure. The input factors are related to each other according to limited substitutability (partial complementarity).

The concept of defining society as an energy and information processing system with varying structures and of applying an iterative empirical theoretical methodology of investigations, has turned out to be extremely useful in international comparisons on country level, aimed at describing and explaining their varying economic performance. It is only natural that one should want to broaden this successful concept by adapting it to investigations of other mechanisms of societal development. Encouraged by different institutions like UNIDO, IAEA, Chamber of Commerce, etc., this concept is recently applied to investigations of various economic sectors, of the structure of education in European countries and its impact on economic performance, of the economic impact of sociopsychological conditions, of the main factors of health development, etc.

FOOTNOTES

- 1) Background publications - See list on page 47.
- 2) Gross, B., "The Coming General Systems Models of Social Systems" in Human Relations, p.20, (1967).
- 3) This process is described in IIASA working paper, H. Millendorfer, "Global Models and Global Mechanisms I: Methodological Considerations".  
  
See also Klaus, G., Wörterbuch der Kybernetik, Berlin 1968.
  - Stachowiak, H., Allgemeine Modelltheorie Wien, New York, 1973.
- 4) Argumenting in such a way we are applying a concept which clearly shows the influence of K.W. Deutsch who understands society as a self-organizing system. Deutsch, K.W., The Nerves of Government, London 1963.
- 5) Galenson, W., and Pyatt, G., The Quality of Labour and Economic Development in Certain Countries, Geneva 1964
- 6) Denison, E.F., The Sources of Economic Growth in the United States and the Alternative Before Us, New York, 1962  
  
Denison, E.F., Why Growth Rates Differ, Washington, D.C., 1967
- 7) Group of countries not taken into account in the regression analysis owing to unreliability of the data; subsequent coordination to the calculated regression lines.
- 8) "Literacy has a key role in the development process. It provides both general and specific benefits, it broadens the range of contact and expands the range of stimuli to which our individual is exposed." Brown, L.R., Increasing World Food Output, Washington, D.C. 1965
- 9) Du Boff, R.B. "Electrification and Capital Productivity: A Suggested Approach." The Review of Economics and Statistics, Vol. XLVIII (1966), pp. 426-431.
- 10) Various sociologists came to similar results, namely that system of values, culturally determined attitudes and behavior patterns play a decisive role in the process of development:

- Weber, M. "Soziologie-Weltgeschichtliche Analyse-Politik", Stuttgart, 1956.
- Behrendt, R. "Soziale Stragegien für Entwicklungs Ländes", Frankfurt, 1965.
- McClelland, D. The Achieving Society, Princeton, 1961.

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- a) Attinger, E.O. and Millendorfer, H., "Performance Control of Biological and Social Systems." Persp.Biol.Med. 12, 103-123, 1968
- b) Millendorfer, H., "Entstehung und Übernahme von Fortschritt als wesentliche Wirtschaftsfaktoren", Wirtschaftspol. Blätter, Nr. 5-6, 1967
- c) Millendorfer, H. and Attinger, E.O., "Global Systems Dynamics", Medical Care 6, 467-489, 1968
- d) Millendorfer, H., "Input-Output Relations of Societal Systems" in Global Systems Dynamics, Attinger, E.O., Ed., S. Karger, A.G., Baslo-New York, 1970
- e) Millendorfer, H., "Kommunikation" in "Gesellschaft und Politik", Schriftenreihe Inst. Socialpolitik und Social-reform 5, 15-27, 1969
- f) Millendorfer, H., "Von der Kunst der Prognose zum kybernetischen System", Quartalhefte Girozentrale, 1, 1969
- g) Millendorfer, H. und Gaspari, C., Entwicklung als gesellschaftlicher Lernprozess, Int.Rep.Studiengruppe für Internat.Analysen, Feb. 1970
- h) Millendorfer, H., "Systemtheoretische Aspekte der mittelfristigen Prognose", Zeitschr.f.Nat.Ökonomie 29, 371-384, 1969
- i) Millendorfer, H. und Gaspari, C., "Immaterielle und materielle Faktoren der Entwicklung. Ansätze zu einer Allgemeinen Produktionsfunktion." Zeitschr.f.Nat.Ökonomie 31, 81-120, 1971 (Translated by the British Government: "Immaterial and material functions of development; Formulation of a general production function".)
- j) Millendorfer, H., "Die Frage nach der Zukunft, Wort und Wahrheit", Zeitschrift für Religion und Kultur, 52-64, Nr.1, Thomas-Morus-Presse T.Verlag Herder, Wien, 1972
- k) Millendorfer, H., "Die Rolle der Bildung im Entwicklungsprozess", Internationale Entwicklung, III, 1972
- l) Millendorfer, H. und Gaspari, C., "Kapital, Bildung, Struktur in der längerfristigen Entwicklung, Österreich 1918-1980", Forschungsbericht d.Studiengruppe F.Internat.Analysen, Veröffentlicht unter dem Titel "Prognosen für Österreich", Verlag f.Geschichte und Politik, Wien 1972, 2nd Ed., 1973



- m) Millendorfer, H., "Innere und Äussere Grenzen des Wachstums", Wort und Wahrheit, Nr. 5, Wien 1973
- n) Millendorfer, H., "Differenzierte Entwicklungshilfe", in Report, Zeitschrift für Jugendarbeit und Jugendforschung in Österreich, 15, 1973
- o) Millendorfer, H., "Methodische und inhaltliche Aspekte des Berichtes des Club of Rome", Wirtschaftspol.Blätter 9-10, 1973
- p) Millendorfer, H., "Österreich 1980-Methoden und Ergebnisse längerfristiger Prognosen", Österr. Ingenieurzeitschrift, 1974 (in conjunction with Part 12)
- q) Millendorfer, H. und Gaspari, C., "Consideration on Sectoral Growth in the Manufacturing Industry", UNIDO Working Paper ID/WG.160/10 for the Expert Group Meeting on Projection of Industrial Development, Vienna 27-31 August, 1973
- r) Millendorfer, H., "Religion from the Point of View of the General Production Function", Working Paper for the Rome Special World Conference on Futures Research 1973 (Continuation of Part 17)
- s) Millendorfer, H., "Metamorphose des Fortschritts-ein Modell für Österreich", Von der Quantität zur Qualität, Europa-Verlag, Wien 1974
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- w) Millendorfer, H., Gaspari, C., "Globale Mechanismen und Weltmodelle", Österr. Zeitschrift f. Politikwissenschaft, 2, Wien, 1975

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\* Contribution of the methodology for future planning given by Prof. Bruckmann.

\*\* Contribution given by Prof. Zsifkovits.

- x) Millendorfer, H., Gaspari, C., "Non-economic and Economic Factors in Societal Development", The General Production Function paper presented at the Second IIASA Conference on Global Modelling